Why the Active Management Industry Grew: Learning about Heterogeneity in Skills

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Abstract

We argue that active management’s growth is not puzzling despite the industry’s poor track record. Our explanation features learning about heterogeneity in skills and decreasing returns to scale at both the fund level and the aggregate level. Investors are uncertain about parameters governing fund returns, and they learn about them from realized returns. After observing a fund’s negative performance, investors infer that the fund manager’s skill is lower than expected rather than that the aggregate-level decreasing returns to scale is higher than expected. Optimism about the industry as a whole comes at the expense of disappointment about existing individual funds. But this disappointment is significantly muted away by the sustained entry of new funds. If this force is strong enough, investors increase their allocation to active management. On the other hand, fund-level decreasing returns to scale will imply that the average unit cost associated with investing in active management is lower as the number of funds increases and, ceteris paribus, make the industry grow even bigger. Quantitatively, our story can keep the whole fund industry growing even if its performance is unimpressive and can reproduce salient features of the time series of industry size. It can also rationalize the empirical fact that this industry growth coincides with net fund entry over time.

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1 Introduction

One of the central facts facing financial economics is that during the last 30 years, the financial services sector has grown enormously. Research into the growth of the financial sector found that a large part of this growth came from the dramatic increase in the value of financial assets under professional management, with the total fees charged to manage these assets growing at approximately the same pace (see Greenwood and Scharfstein 2013; Malkiel 2013). One social benefit of active management is more accurate (“efficient”) securities prices, which enable firms to raise new capital at prices that better reflect their fundamental value. On the other hand, numerous studies have shown that these funds have provided investors with average returns significantly below those on passive benchmarks.\(^1\) Although it may be socially beneficial for active managers to acquire information, it is puzzling that they are able to attract funds despite their underperformance. Moreover, what is less clear is whether we need nearly as much active money management as exists.

Why has the active management industry grown? By definition, industry size fluctuates over time as a result of changes in the number of active mutual funds and changes in the average fund size. As illustrated in Figure 1, we observe that the active management industry has grown steadily over time. This industry growth coincides with sustained entry of skilled competitors. The top panel of Fig. 1 shows the number of funds over time. The number of funds increases from 104 in 1980 to 1,279 in 2014. The bottom panel of Fig. 1 plots the industry’s size as a fraction of total stock market capitalization over time. It starts at 1.7% in January 1980, peaks at 15.4% in July 2008, and finishes at 13.2% in December 2014. Indeed, the time-series correlation between industry size and the number of active mutual funds is 0.98, whereas the time-series correlation between industry size and the average fund size, however, is −0.78. Therefore, the increase in the measure of industry size was itself driven essentially by an increase in the number of active mutual funds.

These facts raise interesting new questions. First, what is the mechanism behind the almost perfect positive correlation between industry size and the number of active mutual funds? Second, what is the mechanism behind the strongly negative correlation between industry size and the average fund size?

In this paper, we develop a model of active management based on learning, which can quantitatively capture the historical fluctuations in the industry size. Interestingly, the model can rationalize not only the almost perfect positive correlation between industry size and the number of active mutual funds but also the strongly negative correlation between industry size and the average fund size.

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\(^1\) See Jensen (1968), Malkiel (1995), Gruber (1996), Fama and French (2010), and others.
Recently, Pástor and Stambaugh (2012) show that the presence of industry-level decreasing returns to scale presents investors with an inference problem that can involve slow learning about the degree of decreasing returns and thus slow convergence to the correct allocation. Pástor and Stambaugh analyze this learning problem with a constant true underlying relation between the size of the active management industry and active management’s alpha. They argue that the popularity of active management is not puzzling despite its poor track record if investors must arrive at the appropriate investment level by inferring empirically the effect of scale on performance. In that case, adjustment to the correct level of investment can be slow.

We argue that the simple model of Pástor and Stambaugh (2012) abstracts from various important considerations in explaining the observed steady growth in industry size. To highlight these issues, we simulate historical paths of industry size from our estimated version of the Pástor and Stambaugh model. We find that their simple model cannot qualitatively (or quantitatively) capture the historical fluctuations in the industry size.\(^2\)

One reason is difference in how we measure the industry’s size. They measure the active management industry’s size as a fraction of the total amount managed actively and passively, whereas we measure it as a fraction of total stock market capitalization that active mutual funds own at that time. Their measure of industry size trends downward for the full sample period, and this trend is monotonic. Empirically, we find that the active management industry has grown over time, which is the stylized fact we focus on. This industry growth coincides with an even stronger growth of indexing, which explains why the time trend in industry size depends on how we measure it. While active management’s negative track record relative to passive benchmarks and investors’ growing awareness of indexing render the reduction of actively managed funds relative to index funds not quite puzzling, we consider its growth relative to the stock market the real "active-management puzzle".

On the other hand, there also are economically significant reasons behind these shortcomings. Key to overcoming these shortcomings is to realize that the active management industry has faced a sustained entry of fresh competitors, and the implications of this fact on the growth of active management industry depends critically on not only the nature of

\(^2\)Pástor and Stambaugh (2012) do not hope that their simple model can quantitatively capture the historical fluctuations in the industry size and write that "our model delivers a year-by-year dependence between [the active return] \(r_A\) and [the equilibrium active allocation] \((S/W)\), generally implying that an unexpectedly high \(r_A\) in a given year causes a higher \((S/W)\) going into the next year. In principle, one could also look for this dependence in the year-by-year historical data in table 1, but we do not believe that such an exercise would be very informative. Indexing was novel when it emerged on the investment landscape during the 1970s. Understanding subsequent year-by-year fluctuations in its share relative to active management must surely have much to do with the dissemination and adoption of financial innovation, which we cannot hope to capture in our simple model" (p. 771), so the fact that their model deliver a rather poor fit of the data is understandable.
learning problem that investors face, but also the nature of decreasing returns to scale.

Incorporating heterogeneity in skills and learning about it help us understand the growing popularity of active management. Investors are uncertain about parameters governing active funds’ alphas, and they learn about them from realized returns. After observing an active fund’s negative performance, investors infer that either the idiosyncratic factor of the fund’s raw skill is lower than expected or the industry’s common factors are not as optimistic as expected. If investors’ prior beliefs about the idiosyncratic factor of the fund’s raw skill is relatively more optimistic and their prior uncertainty relatively larger, they will interpret the fund’s underperformance as evidence that this fund manager lacks skill rather than that the industry as a whole lacks skill. If this novel aspect of learning about active management is significant enough, it will cause investors’ posterior beliefs about the industry’s common factors to be more optimistic at the expense of investors growing cynical of the idiosyncratic factors of the incumbent funds’ raw skill. At the same time, the sustained entry of new funds effectively mutes away the negative expectations about incumbent funds’ idiosyncratic parameters. Take together, sustained entry of new funds, in the presence of investors learning about fund heterogeneity, will lead the industry to grow bigger.

Decreasing returns to scale at the fund level also help us understand the growing popularity of active management. In the presence of fund-level decreasing returns to scale, the equilibrium industry size increases as the number of funds increases when investors’ beliefs about the parameters governing expected gross returns at an aggregated level are held constant. In the limit with infinite number of managers, the industry’s fully competitive equilibrium size is determined by the posterior means of the average of the fund fixed effects (net of proportional costs and managerial compensation) across all funds operating in that period in each sector and the posterior means of sector-level decreasing returns to scale for each sector. If investors’ prior beliefs about fund fixed effects were more optimistic or they believed a priori in more weakly decreasing returns to scale, the industry’s fully competitive equilibrium size would be too big, so that the entry of new funds over time induces a steady growth in industry size.

To explore the quantitative implications of the above mechanisms, we develop a model of active management featuring investors who competitively supply funds to managers based on their perceptions of skill and decreasing returns to scale. We model skill and decreasing returns to scale, with investors learning about unknown parameters both at an aggregated level and at the fund level. We derive the model’s implications for the equilibrium sizes of each segment of the active management industry, measured in relative terms. Quantitatively, our simulation exercise shows that the model succeeds in rationalizing the salient trends in the growth of active management industry in absolute terms.
We also find that the implications of net fund entry on the industry’s equilibrium size depends critically on the subjective learning model of active returns that investors use to make their investment decisions. To clearly delineate this point, we will specify an exogenous entry probability and an exit process with the process parameter as functions of the lagged number of funds and time, to mimic the entry and exit observed in the data. While time proxies for the trend in fixed cost, while stock market capitalization proxies for the opportunities generated by growing financial markets. Together, they predict the profitability of entering this industry to compete for investors quite well. Taking as given the entry and exit processes, we compare multiple specifications of investors’ learning problem to infer the key mechanisms for the observed relationship between intensity of competition and sizes in the active management industry.

Interestingly, in the absence of learning and if investors knew the true values of the parameters governing active returns, the equilibrium industry size would have been significantly smaller than the size in reality. In particular, our counterfactual exercise shows that the industry size would have shrunk rather than grow. We interpret this result as a telling evidence that the increased complexity of investors’ inference problem is the key reason why the industry has historically grown and as a suggestive evidence that this industry is too big. This, in turn, has clear policy implications.

Both our model and that of Pástor and Stambaugh (2012) build on the influential work of Berk and Green (2004). While the model of Pástor and Stambaugh assumes homogeneity across funds and in turn, is necessarily inconsistent or indeterminate with respect to cross-sectional facts on mutual funds, our model allows for heterogeneity in skill, similar to Berk and Green, so that it is at least consistent with cross-sectional facts which Berk and Green reproduce. For example, fund flows respond to past performance in my model, but not in the Pástor and Stambaugh model. In addition, as mentioned already, the increase in the number of funds has been an important driver of growth in this industry. In the Pástor and Stambaugh model, this extensive margin of growth is completely muted in driving the aggregate dynamics.

Quintessentially, our contributions are the following. First, we propose a new learning mechanism that help us understand the historic growth of active management industry relative to the stock market. Second, we show the importance of not simply industry-level decreasing returns to scale, but fund-level decreasing returns to scale in understanding the fluctuations in the active management industry size.

Taken together, our results are consistent with the following narrative. When a fund’s performance turns out disappointing, it may reflect that the particular manager is incapable, or it may reflect upon the value of active management in general. Investors blame the
past poor performance on the existing fund managers rather than on the notion of active management, and this, combined with the entry of new funds, end up investors to have an even more positive assessment of the industry size. Additionally, fund-level decreasing returns to scale allows the observed industry size to this optimistic competitive limit size to make the growth more dramatic. These effects would not be there, were it not for investor learning.

We are not alone in trying to explain the puzzling growth of active management in spite of its poor track record. In our story, similar to those in Berk and Green (2004) or Pástor and Stambaugh (2012), investors do not expect negative past performance to persist, but in other explanations they do. Gruber (1996) suggests that some "disadvantaged" investors are influenced by advertising and brokers, institutional restrictions, or tax considerations. Glode (2011) investors expect negative future performance as a fair trade-off for active management policies that insure investors against bad states of the economy. We do not imply that such alternative explanations play no role in explaining the growth. We simply suggest that investor learning in the active management industry is a critical element and such learning models featuring decreasing returns to scale in recent literature are missing two interesting aspects of learning and returns to scale, which are our contribution.

A number of studies address learning about managerial skill, but none of them analyze the size of the active management industry. Baks, Metrick, and Wachter (2001) analyze mutual-fund performance from an investor's perspective and find that even extremely skeptical prior beliefs about skill would lead to economically significant allocations to active managers. Other studies that model learning about managerial skill with a focus different from ours include Lynch and Musto (2003), Berk and Green (2004), Huang, Wei, and Yan (2012), and Brown and Wu (forthcoming).

There are also a few studies using mutual fund flows to infer investor preferences, similar to how we use them to infer the parameters governing investors' learning problem. Berk and van Binsbergen (forthcoming) and Barber, Huang, and Odean (2015) use fund flows to infer investor risk preferences and find that investors use the CAPM. While my paper and these are about investors' subjective model, there is a large literature that estimates the objective regression models for mutual fund returns and the true distribution of managerial skills. A recent example is Pástor, Stambaugh, and Taylor (2015), which empirically analyzes the nature of returns to scale in active mutual fund management. Other examples include Chen et al. (2004), Fama and French (2010), and Ferreira et al. (2015).

Our study relates to a number of other directions in recent research. More broadly, the study adds to a growing literature addressing contentious issues related to the size of the financial industry (e.g., Philippon 2015; Bolton, Santos, and Scheinkman forthcoming).
Our approach is partial equilibrium, similar to that in Berk and Green (2004), Pástor and Stambaugh (2012), and He and Xiong (2013), in the sense that asset prices are not determined endogenously in the model. On the other hand, Gârleanu and Pedersen (2015) introduce asset managers into the Grossman-Stiglitz model, so that the efficiency of asset prices is linked to the efficiency of the asset management market.\(^3\) Finally, Khorana, Servaes, and Tufano (2005) empirically analyze the determinants of the size of the mutual fund industry across countries.

The paper is organized as follows. Section 2 presents our model. After describing the general model, we show how the models of Pástor and Stambaugh (2012) are obtained as special cases of our model. Section 3 describes our mutual fund dataset and sketches the estimation technique, based on Bayesian Markov chain Monte Carlo (MCMC) method. Sections 4 discuss the estimation results and provides interpretations of our results. Section 5 conducts a number of robustness checks and compares the fit of our baseline model relative to alternative specifications, including the models of Pástor and Stambaugh. Section 6 presents conclusions.

### 2 The Model

If decreasing returns to scale are driven by competition, a fund’s performance should be more closely related to the sizes and performance of funds in the same sector, which presumably follow similar investment strategies, than to the size and performance of a typical fund. To model this idea, we use the nine sectors corresponding to Morningstar’s 3 × 3 stylebox (small growth, mid-cap value, etc.) to label funds’ investment strategies. We assume that, for any given sector \( j \in N \) and period \( t \), \( M_{j,t} \) active mutual fund managers construct (presumably similar) investment portfolios from the primitive assets in the economy. Investors reward managers by paying a given time-varying percent of assets under management every period, where \( f_{j,t} \) is the \( M_{j,t} \times 1 \) vector of rates at which managers in sector \( j \in N \) at time \( t \) charge proportional fees.

All participants in the model are symmetrically informed. Funds differ in their managers’ ability to generate expected returns in excess of those provided by a passive benchmark—an alternative investment opportunity available to all investors. The model is partial equilibrium. Managers’ actions do not affect the benchmark returns, and we do not model other investors at the expense of whose decisions the managers’ potential outperformance comes.

\(^3\)Other recent examples of studies that analyze the implications of delegated portfolio management on equilibrium asset prices include García and Vanden (2009), Cuoco and Kaniel (2011), and Guerrieri and Kondor (2012).
A manager’s ability to beat this benchmark is unknown to both the manager and investors, who learn about this ability by observing the history of returns of the fund and other funds in the same sector. Let

\[ R_{i,j,t+1} = \alpha_{i,j,t} + u_{i,j,t+1} \]

(1)
denote the return, in excess of the passive benchmark, on actively managed fund \( i \) in sector \( j \), without costs and fees. This is not the return actually earned and paid out by the fund, which is net of costs and fees (see below). The fund alpha \( \alpha_{i,j,t} \) is the source of differential performance across funds at time \( t \). The error terms, \( u_{i,j,t+1} \), have the following factor structure:

\[ u_{i,j,t+1} = x_{j,t+1} + \epsilon_{i,j,t+1} \]

(2)

for \( i \in M_{j,t}, j \in J \), where all \( \epsilon_{i,j,t+1} \)'s have a mean of zero, a variance of \( \sigma_{j,e}^2 \), and zero correlation with each other. The common factor \( x_{j,t+1} \) for sector \( j \) has mean zero and variance \( \sigma_{j,x}^2 \). The values of \( \sigma_{j,x} \) and \( \sigma_{j,e} \) are constants known to both investors and managers.

The factor structure in equation (2) means that the benchmark-adjusted returns of skilled managers in the same sector are correlated as long as \( \sigma_{j,x} > 0 \). Multiple skilled managers who follow similar investment strategies are likely to identify the same opportunities, so multiple managers in the same sector are likely to hold some of the same positions, resulting in correlated benchmark-adjusted returns among funds belonging to the same sector. As a result, investors’ posteriors on the abilities of active managers in the same sector will exhibit more and more correlation over time.

Participants learn about the parameters governing equilibrium alphas by observing the realized excess returns the managers produce. This learning is the source of the relationship between performance and the flow of funds as in several recent models of learning about managerial skill.

Assume that fund manager \( i \) in sector \( j \) is simply paid a fixed management fee, \( f_{i,j,t+1} \), expressed as a fraction of the assets under management, \( s_{i,j,t} \). (We shall discuss further this assumption shortly.) Managers accordingly accept and invest all the funds investors are willing to allocate to them. The amount investors will invest in the fund depends on their subjective assessment of the managers’ ability and on the costs they perceive managers face in expanding the fund’s scale.

The excess total payout to investors over what would be earned on the passive benchmark is

\[ TP_{i,j,t+1} = q_{i,j,t} (R_{i,j,t+1} - f_{i,j,t+1}) . \]

Let \( r_{i,j,t} \) denote the excess return over the benchmark that investors in fund \( i \) in sector \( j \)
receive in period \( t \). Then
\[
  r_{i,j,t+1} = \frac{TP_{i,j,t+1}}{q_t} = R_{i,j,t+1} - f_{i,j,t+1}. \tag{3}
\]

The return \( r_{i,j,t} \) corresponds to the return empirically observed.

We assume that investors supply capital with infinite elasticity to funds that have positive excess expected returns from their subjective perspective. This can be justified as long as investors are all risk neutral. Our assumption of risk neutrality is conservative in that, given active management’s popularity despite its historical underperformance, allowing for risk aversion and estimating the risk-aversion parameter would presumably lead to risk neutrality anyhow, if not risk seeking. Similarly, they remove all funds from any fund that has a negative excess expected return from their subjective perspective. At each point in time, then, funds flow to and from each fund so that the expected excess return to investing in any surviving fund is zero with respect to investors’ subjective probability distribution of next period’s returns:
\[
  \tilde{E}_t(r_{i,j,t+1}) = 0. \tag{4}
\]

We can complete the description of a learning model of active management by specifying the objective distributions of next period’s returns and fees, and specifying parameters in it, which are unknown to investors and subject to learning.

### 2.1 The Benchmark Model: Pástor and Stambaugh (2012)

We begin by describing the model of Pástor and Stambaugh (2012) that we take as the benchmark model against which to evaluate our general model. We shall then combine the model with two new elements, which are necessary to capture the stylized trends in industry-size time series and to reproduce the time-series correlation between the number of funds and industry size.

Pástor and Stambaugh model decreasing returns to scale as follows:
\[
  \alpha_{i,j,t} = a_j - b_j \left( \frac{S}{W} \right)_{j,t}, \tag{5}
\]

where \( R_{i,j,t} \) is the expected return gross of fees and costs at time \( t \) in excess of passive benchmarks generated by fund \( i \) in sector \( j \) and \( (S/W)_{j,t} \) is the sector size as a fraction of total stock market capitalization. Sector-level decreasing returns to scale are captured by \( b_j > 0 \). The parameters \( a_j \) and \( b_j \) in equation (5) are unknown. We denote their first and
second conditional moments by

\[
\begin{align*}
E \left( \begin{bmatrix} a_j \\ b_j \end{bmatrix} \mid D_{t-1} \right) &= \begin{bmatrix} \tilde{a}_{j,t-1} \\ \tilde{b}_{j,t-1} \end{bmatrix}, \\
\text{Var} \left( \begin{bmatrix} a_j \\ b_j \end{bmatrix} \mid D_{t-1} \right) &= \begin{bmatrix} \sigma_{a,j,t-1}^2 & \sigma_{ab,j,t-1} \\ \sigma_{ab,j,t-1} & \sigma_{b,j,t-1}^2 \end{bmatrix},
\end{align*}
\]

(6)

where \( D_{t-1} \) denotes the set of information available to investors at time \( t - 1 \).

The parameter \( a_j \) represents the expected return on the initial small fraction of wealth invested in sector \( j \) of the active management industry, without proportional costs and managerial compensation. It seems likely that \( a_j > 0 \), although we do not preclude \( a_j < 0 \). If no money were invested in sector \( j \), some opportunities to outperform the passive benchmarks by following a typical investment strategy in sector \( j \) would likely be present, so it is likely to have a positive expected benchmark-adjusted return.

The parameter \( b_j \) determines the degree to which the expected benchmark-adjusted return for any fund declines as the relative size of the fund’s sector increases. We allow \( b_j > 0 \) to capture decreasing returns to scale at the sector level. As more money chases opportunities to outperform, prices move (unless markets are perfectly liquid), making such opportunities more elusive. If decreasing returns to scale at an aggregated level are driven by competition with other funds, a fund’s performance should be related to the size of the fund’s sector rather than the size of the entire industry.

Their primary focus is on the fully competitive case (\( M_{j,t} \to \infty \)). They compare the model-implied equilibrium size of the active management industry with the actual size.

With competing managers, the equilibrium fee is \( f_{i,j,t} = 0 \). If the fee were instead equal to some positive value, any fund manager setting an infinitesimally lower fee would attract all investment from other funds to that lower-fee fund.

Let \( r_{j,t} \) denote the benchmark-adjusted net return on the aggregate portfolio of all funds:

\[
r_{j,t} = a_j - b_j \left( \frac{S}{W} \right)_{j,t} + x_{j,t} + \frac{1}{M_{j,t}} \sum \epsilon_{i,j,t},
\]

(8)

using (1), (2), (3), and the result that \( f_{i,j,t} = 0 \) in equilibrium. Thus, as \( M_{j,t} \to \infty \),

\[
r_{j,t} = a_j - b_j \left( \frac{S}{W} \right)_{j,t} + x_{j,t}
\]

(9)
since the variance of the last term in (8) goes to zero. It follows from (9) that

\[ \text{E} (r_{j,t}|D_t) = \tilde{a}_{j,t} - \tilde{b}_{j,t} \left( \frac{S}{W} \right)_{j,t} \]  

(10)

Equation (4) can then be rewritten as

\[ \left( \frac{S}{W} \right)_{j,t} = \frac{\tilde{a}_{j,t}}{\tilde{b}_{j,t}} \]  

(11)

We specify a bivariate normal joint prior distribution for \( a_j \) and \( b_j \):

\[ \begin{bmatrix} a_j \\ b_j \end{bmatrix} \sim N (E_{j,0}, V_{j,0}) \]  

(12)

where \( N (E_{j,0}, V_{j,0}) \) denotes a bivariate normal distribution with mean \( E_{j,0} \) and covariance matrix \( V_{j,0} \). Denote

\[ E_{j,0} = \begin{bmatrix} \tilde{a}_{j,0} \\ \tilde{b}_{j,0} \end{bmatrix}, \quad V_{j,0} = \begin{bmatrix} \sigma_{a,j,0}^2 & \sigma_{ab,j,0} \\ \sigma_{ab,j,0} & \sigma_{b,j,0}^2 \end{bmatrix}. \]  

(13)

To capture beliefs that the industry faces decreasing returns to scale, we specify \( \tilde{b}_{j,0} > 0 \). We also specify \( \sigma_{ab,j,0} = 0 \) for simplicity.

These moments are updated by using standard results for the conditional distributions of a multivariate normal

\[ V_{j,t} = V_{j,t-1} - V_{j,t-1}Z_{j,t-1}' (Z_{j,t-1}V_{j,t-1}^{-1}Z_{j,t-1}')^{-1} Z_{j,t-1}V_{j,t-1}, \]  

(14)

\[ E_{j,t} = E_{j,t-1} + V_{j,t-1}Z_{j,t-1}'r_{j,t} + z_{j,t}, \]  

(15)

where \( Z_{j,t-1} = \begin{bmatrix} 1 & - (S/W)_{j,t-1} \end{bmatrix} \). %Here, \( z_{j,t} \) is a belief shock. Given the complexity of this industry, it is most probable that there are other factors influencing investors’ perception of the parameters governing this industry beyond the simple learning mechanism described above. To capture this and to proceed with likelihood estimation of the parameters, we encapsulate these other variations in \( z_{j,t} \). Technically, it serves to avoid stochastic singularity, much like measurement error or shocks do in macroeconomic DSGE models. % The posterior distribution of \( a_j \) and \( b_j \) is bivariate normal as in equation (12), except that \( E_{j,0} \) and \( V_{j,0} \) are replaced by \( E_{j,t} \) and \( V_{j,t} \) from equations (14) and (15).

In the model of Pástor and Stambaugh (2012), the industry dynamics is fully described by equations (9), (11), and (13)-(15). Note that the number of funds do not enter in any
of these equations. Hence it follows straightforwardly that there is no relationship between the growth in the number of competitors and the growth of the industry size, which is inconsistent with the empirical fact documented in the introduction. We will now embed this simple model into our model, which can not only generate industry size growth, but also relate this growth to the growth in the number of funds.

2.2 Heterogeneity in Skill and Fund-Level Returns to Scale

Our model combines the quintessential spirit of the Pástor and Stambaugh model and two other elements. There is heterogeneity in skill across funds, which investors learn about. Finally, there is decreasing returns to scale at the fund level.

Our key assumption is that $\alpha_{i,j,t}$ is decreasing in $(S/W)_{j,t}$ and $(s/W)_{i,j,t}$, where $S_j$ is the sum of fund sizes across all funds within a given sector and $s_{i,j}$ is the size of fund $i$ in sector $j$. Here $W$ is equal to the total stock market capitalization in the same period. Dividing $S_j$ and $s_{i,j}$ by $W$ reflects the notion that the relative (rather than absolute) sizes are relevant for capturing decreasing returns to scale in active management. In order to obtain closed-form equilibrium results, we assume the functional relation

$$
\alpha_{i,j,t} = a_j - b_j \left( \frac{S}{W} \right)_{j,t} + a_{i,j} - c_j \left( \frac{s}{W} \right)_{i,j,t},
$$

(16)

with $S_{j,t} = \sum_{i \in M_{j,t}} s_{i,j,t}$. Analogously, $S$ is the aggregate size of the active management industry, with $S_t = \sum_{j \in J} S_{j,t}$. The parameters $a_j, b_j$ and $a_{i,j}$ in equation (16) are unknown.

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4 An earlier version of the model included decreasing returns at the industry level. Empirically, we do not find evidence of decreasing returns at the industry level. The estimation results indicate that industry-level decreasing returns to scale play an insignificant role in determining the industry’s size. All things considered, the addition of decreasing returns to scale at the industry level do not affect the conclusions discussed in this paper.

Both sector-level and industry-level decreasing returns to scale are plausible hypotheses of how a fund’s performance could depend on other funds, due to competition among active funds. Moreover, these alternative hypotheses are not mutually exclusive. These results suggest essentially that our proxy for sector size accurately measures the size of a fund’s competition.

5 We specify the relation (??) exogenously, but decreasing returns to aggregate scale can also arise endogenously in a richer model. In canonical rational-expectations equilibrium models of financial markets (see, e.g., Grossman and Stiglitz 1980; Garleanu and Pedersen 2015), agents choose whether to pay the cost of becoming informed, and the benefit from informed trading is decreasing in the proportion of informed traders, just like equation (??).
We denote their first and second conditional moments by

$$E \left( \begin{bmatrix} a_j \\ b_j \\ \{a_{i,j}\}_{i \in M_{j,t-1}} \end{bmatrix} | D_{t-1} \right) = \begin{bmatrix} \tilde{a}_{j,t-1} \\ \tilde{b}_{j,t-1} \\ \{\tilde{a}_{i,j,t-1}\}_{i \in M_{j,t-1}} \end{bmatrix},$$

$$\text{Var} \left( \begin{bmatrix} a_j \\ b_j \\ \{a_{i,j}\}_{i \in M_{j,t-1}} \end{bmatrix} | D_{t-1} \right) = \begin{bmatrix} \sigma^2_{a,j,t-1} & \sigma_{ab,j,t-1} & \{\sigma_{ai,j,t-1}\}_{i \in M_{j,t-1}} \\ \sigma_{ab,j,t-1} & \sigma^2_{b,j,t-1} & \{\sigma_{bi,j,t-1}\}_{i \in M_{j,t-1}} \\ \{\sigma_{ai,j,t-1}\}_{i \in M_{j,t-1}} & \{\sigma_{bi,j,t-1}\}_{i \in M_{j,t-1}} & \{\sigma_{i_1i_2,j,t-1}\}_{i_1,i_2 \in M_{j,t-1}} \end{bmatrix},$$

where $D_{t-1}$ denotes the set of information available to investors at time $t - 1$.

The parameter $a_j$ is the source of differential effectiveness across sectors, and the parameter $a_{i,j}$ is the source of differential ability across managers within a given sector. The sum of these two parameters, $a_j + a_{i,j}$, represents, thus, the expected return on the initial small fraction of wealth invested in actively managed fund $i$ in sector $j$, without proportional costs and managerial compensation. Again, it seems likely that $a_j + a_{i,j} > 0$, although we do not preclude $a_j + a_{i,j} < 0$.\(^6\)

The parameter $c_j$ determines the degree to which the expected benchmark-adjusted return for any fund in sector $j$ declines as the relative size of the fund increases. We allow $c_j > 0$ to capture decreasing returns to scale at the fund level. Like industry-level decreasing returns to scale, fund-level decreasing returns to scale has been motivated by liquidity constraints. At the fund level, a larger fund’s trades have a larger impact on asset prices, eroding the fund’s performance. Unlike the parameter $b_j$ governing sector-level returns to scale, we assume that the costs associated with fund-level decreasing returns to scale faced by the funds are common knowledge. We allow learning about $b_j$ so as to embed the Pastor and Stambaugh model. Our novel mechanism is how learning about fund-specific factors interact with the sector-level parameters and our arguments related to fund-level decreasing returns to scale do not require uncertainty about it. So according to Occam’s razor, we choose the simplest model which accommodates the Pastor and Stambaugh model as a special case.

Assume that the fund managers do not optimally choose the fee they charge at each point in time and, instead, are simply paid an exogenous time-varying management fee, $f_{i,j,t}$, expressed as a fraction of the assets under management, $q_{i,j,t}$. Each element of $f_{j,t}, j \in N$

\(^6\)If no money were invested in a skilled active fund, some opportunities (to outperform the passive benchmarks) would likely be present for that manager whoever he is (given that he essentially is an informed trader), so it is likely to have a positive expected benchmark-adjusted return.
evolves (independently) according to the following stochastic processes:

\[
\log f_{i,j,t} = \log f_{i,j,t-1} + \nu_{i,j,t}, \quad i \in M_{j,t}, j \in N,
\]

where \( \nu_{i,j,t} \sim N(0, \sigma^2_f) \). As discussed in Berk and Green (2004), the fixed fee contract comes closer to the institutional setting for retail mutual funds, which is the source of data for our empirical analysis. Empirical evidence suggests that mutual funds show relatively little variation in fees, through time and across funds.\(^7\) On the basis of monthly data for the January 1993–December 2012 period, the natural logarithm of fund expense ratio in CRSP has a beta of 0.99 with respect to its one-month-lag variables. Our assumption of the exogenous time-varying expense ratio throughout the sample is clearly appropriate in that the \( R \)-squared \((R^2)\) for the regression is 0.99.

Then

\[
r_{i,j,t+1} = a_j + a_{i,j} - f_{i,j,t+1} - k_j \left( \left( \frac{S}{W} \right)_{j,t} \cdot \left( \frac{S}{W} \right)_{i,j,t} \right) + u_{i,j,t+1},
\]

where

\[
k_j \left( \left( \frac{S}{W} \right)_{j,t} \cdot \left( \frac{S}{W} \right)_{i,j,t} \right) = b_j \left( \frac{S}{W} \right)_{j,t} + c_j \left( \frac{S}{W} \right)_{i,j,t}
\]

denotes the extent to which the fund’s gross alpha is eroded by decreasing returns to scale. For any given sector \( j \), investors learn about \( a_j, b_j \) and \( \{a_{i,j}\}_{i \in M_{j,t}} \) by observing realized returns of and active allocations to all funds belonging to that sector.

We do not impose the assumption that investors have rational expectations. At the birth of a fund in sector \( j \), the participants’ prior about the fund-specific factor of the fund skill is that \( a_{i,j} \) is normally distributed with mean \( \phi_{j,0} \) and variance \( \eta^2_{j,0} \). Since investors are not restricted to have rational expectations, this is not necessarily the distribution of fund-specific skills across new funds. Investors (and the managers) update their posteriors on the basis of the history of observed returns as Bayesians. Let the posterior mean of management ability at time \( t \) be denoted

\[
\Phi_{i,j,t} \equiv E(a_j + a_{i,j}|D_t).
\]

The timing convention is as follows.

1. Each incumbent fund \( i \in M_{j,t-1} \) in sector \( j \in N \) enters period \( t \) with \( s_{i,j,t-1} \) funds under management and subjective estimates of parameters governing fund returns, \( \{\tilde{a}_{j,t-1}, \tilde{b}_{j,t-1}, \{\tilde{a}_{i,j,t-1}\}_{i \in M_{j,t-1}}\}_{j \in N} \).

\(^7\) Christoffersen (2001) describes both historical practice and regulatory constraints which limit the ability of retail mutual funds to use performance-based fees.
2. Managers and investors observe \( \{r_{i,j,t}, f_{i,j,t}\}_{i \in M_{j,t-1}, j \in N} \) (from which they can infer \( \{R_{i,j,t}\}_{i \in M_{j,t-1}, j \in N} \)) and update their estimates of the parameters governing fund returns by calculating \( \{\tilde{a}_{j,t}, \tilde{b}_{j,t}, \{\tilde{a}_{i,j,t}\}_{i \in M_{j,t-1}}\}_{j \in N} \).

3. Each incumbent fund \( i \in M_{j,t-1} \) in sector \( j \in N \) makes exit decision.

4. The number of entering funds in each sector \( j \in N \) is determined, and the set of time-\( t \) incumbents, \( M_{j,t} \), is determined.

5. Belief shocks \( \{z_{i,j,t}\}_{i \in M_{j,t}, j \in N} \) are drawn for each incumbent funds at time \( t \).

6. Then capital flows into or out of exiting funds to determine \( \{s_{i,j,t}\}_{i \in M_{j,t-1}, j \in N} \).

Next, we shall calculate this flow of funds explicitly, and derive from the model’s equilibrium relationships a likelihood function which the estimation is based on.

Recall that there is competitive provision of capital by investors to mutual funds, imposing the participation condition, (4).

Taking expectations of both sides of (20), and requiring expected excess returns of zero as in (4), gives

\[
\Phi_{i,j,t} - E_t[f_{i,j,t+1}] = E_t \left[ k_j \left( \left( \frac{S}{W} \right)_{j,t}, \left( \frac{s}{W} \right)_{i,j,t} \right) \right] = \tilde{b}_{j,t} \left( \frac{S}{W} \right)_{j,t} + c_j \left( \frac{s}{W} \right)_{i,j,t},
\]

As \( \{\Phi_{i,j,t}\}_{i \in M_{j,t}}, \tilde{b}_{j,t}, \) change, \( \{(s/W)_{i,j,t}\}_{i \in M_{j,t}} \) change to ensure that this equation is satisfied for all funds at all points in time.

In equilibrium, then \( (S/W)_{j,t} \) is given by the (unique) real positive solution to the equation

\[
\frac{1}{M_{j,t}} \sum_{i \in M_{j,t}} (\Phi_{i,j,t} - E_t[f_{i,j,t+1}]) = \left( \tilde{b}_{j,t} + c_j/M_{j,t} \right) \left( \frac{S}{W} \right)_{j,t}
\]

Equation (23) can then be rewritten as

\[
\left( \frac{S}{W} \right)_{j,t} = \frac{\frac{1}{M_{j,t}} \sum_{i \in M_{j,t}} (\Phi_{i,j,t} - E_t[f_{i,j,t+1}])}{\tilde{b}_{j,t} + c_j/M_{j,t}} = \frac{\Phi_{j,t} - E_t[f_{j,t+1}]}{K_{j,t}},
\]
where
\[
\Phi_{j,t} = \frac{1}{M_{j,t}} \sum_{i \in M_{j,t}} \Phi_{i,j,t},
\]
\[
f_{j,t+1} = \frac{1}{M_{j,t}} \sum_{i \in M_{j,t}} f_{i,j,t+1},
\]
\[
K_{j,t} = \tilde{b}_{j,t} + c_j / M_{j,t}.
\]

Similarly, we can compute individual fund sizes by rewriting (22),
\[
\left( \frac{s}{W} \right)_{i,j,t} = \frac{1}{c_j} \left( \Phi_{i,j,t} - E_t [f_{i,j,t+1}] - \tilde{b}_{j,t} \frac{\Phi_{j,t} - E_t [f_{j,t+1}]}{K_{j,t}} \right). \tag{24}
\]

Recall that the priors for the sector-level parameters, \(a, b\), are given by (12), i.e., a bivariate normal distribution. Recall, further, that investors’ priors (before drawing belief shock) for the fund-specific skill, \(a_{i,j}\), is that \(a_{i,j} \sim N(\phi_{i,0}, \eta_i^2)\). These prior specifications, together with the fact that each active return is a linear transformation of the parameter vector, imply that investors’ posteriors at each point of time is also a multivariate normal distribution.

The moments for \(a, b, \{a_{i,j}\}_{i \in M_{j,t-1}}\) (i.e., fund-specific skills for funds that operated at time \(t - 1\)) are updated using standard results for the conditional distributions of a multivariate normal
\[
\text{Var} \left( \begin{bmatrix} a_{j,t} \\ b_{j,t} \\ \{a_{i,j}\}_{i \in M_{j,t-1}} \end{bmatrix} \right) | D_t = V_{j,t-1} - V_{j,t-1} Z_{j,t-1} V_{R,t-1}^{-1} Z_{j,t-1} V_{j,t-1} \tag{25}
\]
\[
\text{E} \left( \begin{bmatrix} a_{j,t} \\ b_{j,t} \\ \{a_{i,j}\}_{i \in M_{j,t} \cap M_{j,t-1}} \\ \{a_{i,j}\}_{i \in M_{j,t} \setminus M_{j,t-1}} \end{bmatrix} \right) | D_t = E_{j,t-1} + V_{j,t-1} Z_{j,t-1} V_{R,t-1}^{-1} (R_{j,t} - E_{t-1}(f_{j,t})) + \begin{bmatrix} z_{j,t} \\ 0 \\ \{z_{i,j,t} - z_{j,t}\}_{i \in M_{j,t} \cap M_{j,t-1}} \\ 0 \end{bmatrix} \tag{26}
\]
where $Z_{j,t-1} = \left[ I_{M_j,t-1} \left[ 1 - \frac{(Q_j/W)_{t-1}}{I_{M_j,t-1}} \right] \right]$, $z_{j,t} = \frac{1}{K_{M_j,t}} \sum_{i \in M_j,t} z_i, j, t$ and

$$V_{R,t-1} = Z_{j,t-1}V_{j,t-1}Z_{j,t-1}' + \sigma^2_{Z_{j,t-1}} + \sigma^2_{t M_j,t-1}.$$  (27)

2.3 Entry and Exit of Funds

Our primary focus is on the nature of investors’ inference problem in this industry and on the nature of decreasing returns to scale. Indeed, depending on the particular specification of these two elements, implications of fund entry and exit on the equilibrium industry size dynamics changes substantially. In order to focus on the roles of these two factors independent of the entry and exit dynamics, we will keep the entry and exit processes simple and exogenous.

Our focus is on analyzing the nature of investors’ subjective learning model that decomposes the growth of the industry size into fluctuations in its two components, the number of funds and the average fund size. To do this, we will assume that entry and exit are exogenous stochastic processes and determine the industry and average fund sizes in equilibrium.

At the beginning of time $t$, each incumbent fund $i \in M_{j,t-1}$, which was in operation at time $t-1$, will make an exit with probability $p_j$. At the same time, a cohort of $N_{j,t}$ new funds enter the industry. For simplicity, we will assume that $N_{j,t}$ is a poisson random variable with intensity

$$\log (\lambda_{j,t}) = \beta_{j,0} + \beta_{j,1}t + \beta_{j,2}t^2 + \beta_{j,M} \log (M_{j,t-1}).$$

We will estimate the coefficients $\beta_{j,0}, \beta_{j,1}, \beta_{j,2}, \beta_{j,M}$ by running poisson regressions in our data.

This specification for entry and exit assumes that entry and exit processes are not affected by fund returns or fund sizes. In particular, it simplifies our problem substantially, for it allows us to separately estimate the entry-exit processes and the investors’ learning problem. This allows us to study investors’ learning problem in isolation, and the specification of entry and exit becomes important only when we do counterfactual simulations. There, we will see that this simple specification captures reasonably the entry and exit time series observed in the data.
3 Inference

3.1 The Data


We now define the variables used in our analysis. Summary statistics are in Table 2.

Our measure of fund performance is $NetR$, the fund’s monthly benchmark-adjusted net return, which corresponds to $r_{i,j,t}$ in Section 2. $NetR$ equals the fund’s gross return minus its monthly expense ratio minus the return on the benchmark index portfolio for actively managed US equity mutual funds. We take expense ratios from CRSP because Morningstar is ambiguous about their timing. The average of $NetR$ is $-9$ bps per month, and the average benchmark-adjusted gross return is $+1$ bps per month.

The benchmark against which we judge a fund’s performance is the CRSP value-weighted market portfolio. For the market portfolio to represent a fair benchmark for active funds, we need to take into account the small but nontrivial cost of holding the market. We do so by subtracting 15 basis points from each annual market return.\(^8\) We follow Pástor and Stambaugh (2012) in choosing our benchmark.

We construct $NetR$ by subtracting the index benchmark return from the fund’s gross return, effectively assuming that the fund’s benchmark beta is equal to one.\(^9\)

$FundSize$ corresponds to $(s/W)_{i,j,t-1}$ in Section 2. $FundSize$ equals the fund’s AUM at the end of the previous month, divided by the total market value of all stocks at the end of the previous month. We fill in missing values of $FundSize$ by taking the fund’s most recent reported size and updating it by using interim realized total fund returns. There is considerable dispersion in $FundSize$: The coefficient of variation is 344%.

We measure $SectorSize$ by adding up fund sizes across all funds within a given sector, divided by the total market value of all stocks (i.e., the sum of $FundSize$ across all sample funds within a given sector). We use the nine sectors corresponding to Morningstar’s 3 × 3 stylebox (small growth, mid-cap value, etc.). The number of funds in these sectors ranges

\(^8\) asdf

\(^9\) This assumption is consistent with empirical evidence for active equity mutual funds. On the basis of monthly data for the January 1980–December 2014 period, US actively managed mutual funds have an average beta of 1.00 with respect to the value-weighted market index.
from 88 in small value to 479 in large growth.

*IndustrySize* is the sum of AUM across all funds in our sample, divided by the total market value of all stocks (i.e., the sum of *FundSize* across all sample funds). It is the fraction of total stock market capitalization that the sample’s mutual funds own at that time.

The variables defined above—*NetR*, *ExpenseRatio*, *FundSize*, *SectorSize*, and *IndustrySize*—are the main variables used in our maximum likelihood estimation of the model presented in Section 2.

The average pairwise correlation in GrossR between funds belonging to the same Morningstar Category is 0.17. To account for these cross-sectional correlations in our subsequent regressions, we cluster standard errors by Morningstar Category month. The average correlation between funds from different categories is only 0.02. Therefore, we do not cluster by month to avoid adding noise to standard errors.

### 3.2 Bayesian Inference

No serious quantitative model of the size of the active management industry, similar to dynamic equilibrium models in macroeconomics, has, as far as I know, been developed or examined. Therefore, following a growing literature in macroeconomics, we adopt a Bayesian approach to inference, integrating the sample information with weakly informative priors, which summarize additional information about the parameters (see, for instance, Levin et al. 2005, Del Negro et al. 2007, or Justiniano and Primiceri 2008). One advantage of this approach is that it ameliorates common numerical problems related to both the flatness of the likelihood function in some regions of the parameter space and the existence of multiple local maxima. On the other hand, it also allows us to constrain some parameter values to ensure that the prior beliefs of investors are reasonable.

The estimation algorithm is a random walk Metropolis MCMC procedure based on An and Schorfheide (2007):

1. Use a maximization algorithm (specifically, a simulated annealing algorithm) to maximize \( \ln L(\theta|Y) + \ln p(\theta) \). This is done for multiple initial values drawn at random from our prior to ensure convergence of this initial search to a unique mode. Denote the posterior mode by \( \hat{\theta} \).

2. Obtain an inverse Hessian at the posterior mode \( \hat{\theta} \), which then becomes the dispersion measure for our proposal distribution. Denote the inverse Hessian by \( \hat{\Sigma} \).

3. Draw \( \theta^{(0)} \) from \( N(\hat{\theta}, c_0^2\hat{\Sigma}) \).
4. For $s = 1, \ldots, n_{\text{sim}}$, draw from the proposal distribution $\mathcal{N}\left(\theta^{(s-1)}, c^2\tilde{\Sigma}\right)$, where we scale $\tilde{\Sigma}$ to attain an acceptance rate close to 0.25, as it is usually suggested. The jump from $\theta^{(s-1)}$ is accepted ($\theta^{(s)} = \vartheta$) with probability $\min\left\{1, r\left(\theta^{(s-1)}, \vartheta|Y\right)\right\}$ and rejected ($\theta^{(s)} = \theta^{(s-1)}$) otherwise. Here

$$r\left(\theta^{(s-1)}, \vartheta|Y\right) = \frac{\mathcal{L}\left(\vartheta|Y\right) p\left(\vartheta\right)}{\mathcal{L}\left(\theta^{(s-1)}|Y\right) p\left(\theta^{(s-1)}\right)}.$$ 

As it is standard in the literature, for the computation of the marginal likelihood of these models, we use the modified harmonic mean method of Gelfand and Dey (1994) and Geweke (1999). Appendix B discusses checks for the convergence of the algorithm.

### 3.3 Priors

Priors for the nondiversifiable and idiosyncratic risks of the managed portfolios’ returns ($\sigma_{j,x}$ and $\sigma_{j,e}$), the parameters that govern the subjective distribution of skill level ($\tilde{a}_{j,0}, \sigma_{a,j,0}$ and $\phi_{j,0}, \eta_{j}^2$) perceived by investors, as well as investors’ perception of the parameter governing returns to scale ($\tilde{b}_{j,0}, \sigma_{b,j,0}$) and the fund-level decreasing returns to scale ($c_{j}$), are quantified based on panel regressions of funds’ benchmark-adjusted returns on lagged fund size, sector size, and industry size. Further details of the regressions are relegated to Appendix A.

We begin by tying down the model parameters that can be inferred directly from the data. In particular, we set the sector-level return volatility ($\sigma_{j,x}$) to an average of 0.016, or 1.6 percent per month, which is approximately equal to the average of monthly cluster-level standard deviation component from our performance regressions in Appendix A, where we cluster standard errors by Morningstar Category $\times$ month. Moreover, we set the fund-specific return volatility ($\sigma_{e}$) to an average of 0.019, or 1.9 percent per month, which is approximately equal to the average of monthly residual standard deviation from the same panel regressions in Appendix A. We will also set the fund-level decreasing returns to scale parameter ($c_{j}$) to the slope coefficient on $FundSize$. Our key mechanism is not affected and we prefer not to consider biased perception of decreasing returns at this point.

Table 3 report our priors for the remaining parameters of the model. First, priors for the parameters that govern investors’ initial prior over managerial ability ($\phi_{0}$ and $\eta^2$) are then centered at the average and variance of the estimated fund fixed effects across all funds operating in our sample, respectively. While the prior for $\sigma_{j,0}^2, \eta_{j}^2$ is relatively disperse, we use the standard error of the mean fixed effect for the standard deviation of the prior for $\tilde{a}_{j,0}\phi_{j,0}$. These priors reflect our view that the regression analysis are informative about
investors’ subjective prior estimates (which we impose ought to be reasonable), but that
they substantially understate investors’ subjective prior uncertainty about each fund’s skill
\(a_i\) (or at least produce a rough proxy for rational prior uncertainty over managerial ability).
Second, prior for the parameters that govern investors’ subjective assessment of the nature
of returns to scale \((\tilde{b}_j, \sigma_{b,j,0})\) are centered at the estimated slopes on \(SectorSize\). For all
those scale parameters \(\tilde{b}_j\) we use the standard errors of regression slope estimates on the size
variables, which correspond to \((S/W)_{j,t}\), and \((s/W)_{i,j,t}\) in Section 2, respectively.
Finally, following Del Negro et al. (2007), the priors for the standard deviations of the
belief shocks are fairly disperse and chosen in order to generate realistic volatilities for the
endogenous variables.

4 Estimation Results

4.1 Parameter Estimates

Table 4 summarizes the posterior distribution of the model coefficients, reporting posterior
medians, standard deviations, and fifth and ninety-fifth percentiles computed with the draws
of our posterior simulator. All coefficient estimates are fairly tight and seem sensible.

We note three observations with respect to the parameter estimates. First, there is
substantial heterogeneity in not only objective skills, but also investors’ prior beliefs. Hence
it is necessary to take into account for such heterogeneity by modeling common factor in fixed
effects and aggregated-level decreasing returns at the sector level rather than at the industry
level. Second, investors perceive substantial prior uncertainty about the fund-specific factor
in the skill. Finally, the standard deviation for the belief shocks are not large. This implies
that our specification of investors’ learning problem (over fund fixed effects and rates of
decreasing returns to scale) well approximates the actual investors’ inference problem.

For comparison, Table ?? shows the coefficient estimates of the Pástor-Stambaugh model.
Notice that most of the coefficient estimates are similar to our general model.

4.2 Model Fit

This section has two objectives. First, we evaluate the fit of our model relative to the Pástor-
Stambaugh model specification. From a Bayesian perspective, the marginal likelihood is the
most comprehensive and accurate measure of fit, as it can be used to construct posterior
odds on competing models. The first row corresponding to each model specification in Table
XXXXXX reports the log-marginal data density for our model with fund-level heterogeneity
in skill and decreasing returns to scale. As evident, the value of the log-marginal likelihood
is conclusively in favor of our model that allows for investors’ learning about heterogeneity in skills and the presence of fund-level decreasing returns to scale.

Beyond the likelihood ratio comparison, we will simulate the model economy. For a given model specification, we will use our data from January 1980 to December 1992 to initialize the economy, and then simulate 3,000 samples of data for the periods January 1993-December 2014.

We will set the model parameters to the median parameter estimates. We will then update investors’ prior beliefs at the beginning of January 1980 to initialize investors’ posterior beliefs at the end of December 1992 for the sector-level parameters and fund-specific parameters for incumbent funds. We also draw fund fixed effects for incumbent funds in December 1992.

Then in each period during January 1993-December 2014, the following events happen sequentially:

1. Each incumbent fund $i \in M_{j,t-1}$ in sector $j \in N$ enters period $t$ with $s_{i,j,t-1}$ funds under management and subjective estimates of parameters governing fund returns, $\left\{\tilde{a}_{j,t-1}, \tilde{b}_{j,t-1}, \{\tilde{a}_{i,j,t-1}\}_{i \in M_{j,t-1}}\right\}_{j \in N}$.

2. Draw returns according to
   \[ R_{i,j,t} = a_{i,j} - b_{j}\left(\frac{S}{W}\right)_{j,t} - c_{j}\left(\frac{s}{W}\right)_{j,t} + x_{j,t} + \epsilon_{i,j,t} \]
   where $a_{i,j}$ is a randomly drawn fund-fixed effect, $b_{j}, c_{j}$ are estimated from the fixed effect regression, and the standard deviations for $x_{j,t}, \epsilon_{i,j,t}$ are set from the standard errors of the same regression. Draw, in addition, fees according to
   \[ \log(f_{i,j,t}) = \log(f_{i,j,t-1}) + \nu_{i,j,t} \]
   where the standard deviation of $\nu_{i,j,t}$ is set from running this regression in the data.

3. Managers and investors observe $\{R_{i,j,t}, f_{i,j,t}\}_{i \in M_{j,t-1}, j \in N}$ and update their estimates of the parameters governing fund returns by calculating $\left\{\tilde{a}_{j,t}, \tilde{b}_{j,t}, \{\tilde{a}_{i,j,t}\}_{i \in M_{j,t-1}}\right\}_{j \in N}$ (using formulas (25)-(27)).

4. Each incumbent fund $i \in M_{j,t-1}$ in sector $j \in N$ makes an exit with probability $p_{j}$. We set $p_{j}$ to match the average life length of a fund in sector $j$.

5. Draw the number of entering funds in each sector $j \in N$ according to the Poisson
distribution with intensity

$$\log(\lambda_{j,t}) = \beta_{j,0} + \beta_{j,1} t + \beta_{j,2} t^2 + \beta_{j,M} M_{j,t-1},$$

where we estimate the parameters $\beta_{j,0}, \beta_{j,1}, \beta_{j,2}, \beta_{j,M}$ from a poisson regression of this specification.

6. Then capital flows into or out of exiting funds to determine $\{s_{i,j,t}\}_{i \in M_{j,t-1}, j \in N}$.

7. The number of entering funds in each sector $j \in N$ is determined using equation (24).

As already discussed, an important stylized fact on the growth of active management industry is that the growth coincided with a steady entry of new competitors. Hence, beyond the likelihood ratio comparison, whether the model can generate this positive correlation will provide another dimension for whether the model performs well. The rest of the Table XXXX reports the result. We see that our model captures the joint occurrence of industry growth and the increase in the number of funds.

## 5 Conclusion

The size of active management industry is a big part of recent literature on the "size of finance". In this paper, we developed a model of active management, emphasizing the complexity of investors’ inference problem and the nature of decreasing returns to scale in this industry, so as to understand the sources of this industry’s historical growth fluctuations. We started from the observation that this dramatic growth coincides with steady entry of new funds, and showed that the relationship between the two is highly sensitive to the specification of investors’ learning problem and to the nature of decreasing returns to scale. In particular, our Bayesian estimation shows that learning about heterogeneity in managerial skills and the presence of decreasing returns to scale at the fund level play an important role in reproducing the observed relationship. Furthermore, the fact that the industry size would have significantly smaller without learning (according to our counterfactual experiment) shows that investors’ learning about the correct allocation to active management is complex and slow, due to the proliferation of new funds. This has clear policy implications that there should be active efforts to increase investors’ awareness of the parameters governing active fund returns, which would help the industry size converge to the correct level of allocation much faster.
6 Appendix

6.1 Data-Cleaning Steps

We require that funds appear in both CRSP and Morningstar, which allows us to check data accuracy by comparing the two databases, as detailed below. Morningstar assigns each fund a category (e.g., large growth, mid-cap blend, small value), which we use to categorize funds. We start the sample in 1980, the first decade after Vanguard introduced its S&P 500 index fund (i.e., May 1975). We merge CRSP and Morningstar using funds’ tickers, CUSIPs, and names. We check the accuracy of each match by comparing assets and returns across the two databases.

We use keywords in the Primary Prospectus Benchmark variable to exclude bond funds, money market funds, international funds, funds of funds, industry funds, real estate funds, target retirement funds, and other non-equity funds. We also exclude funds identified by CRSP or Morningstar as index funds, as well as funds whose name contains the word “index”. We exclude fund-month observations with expense ratios below 0.1% per year, since it is extremely unlikely that any actively managed funds would charge such low fees. Finally, we exclude fund-month observations with lagged fund size below $15 million in 2011 dollars.

First, we follow Berk and van Binsbergen in reconciling return data between CRSP and Morningstar. Returns differ across the two databases by at least 10 bps per month in 1.6% of observations. By applying Berk and van Binsbergen’s algorithm we reduce the discrepancy rate to 0.5%. We set the remaining return discrepancies to missing. Similarly, total assets under management (AUM) differ between CRSP and Morningstar in 7.0% of observations. The average of these discrepancies is $31.3 million. AUM differs by at least $100,000 and 5% across databases in 1.1% of observations. We set these AUM values to missing; otherwise we use CRSP’s value.

We use FundID in Morningstar to aggregate share classes. 10

Finally, we exclude the returns of funds younger than three years to address the incubation bias 11 documented by Evans (2010). Evans (2010) reports that "removing the first three years of return data for all funds eliminates the bias" (p. 1584).

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10 Many mutual funds offer multiple share classes, which represent claims on the same underlying assets but have different fee structures. Different share classes of the same fund have the same Morningstar FundID. We aggregate all share classes of the same fund. Specifically, we compute a fund’s AUM by summing AUM across the fund’s share classes, and we compute the fund’s returns and expense ratios by asset-weighting across share classes.

11 Incubation is a strategy for initiating new funds, where multiple funds are started privately, and, at the end of an evaluation period, some are opened to the public. Evans finds that the incubated funds outperform the non-incubated funds during the incubation period.
6.2 Regression Analysis for Prior

We will describe the regressions we run to formulate priors for our Bayesian MCMC. To be updated

6.3 Convergence Diagnostics

We assess the convergence of our posterior simulators using a battery of diagnostics. To be updated

References


Table 1
Summary Statistics

This table shows summary statistics for our sample of active equity mutual funds from 1980–2014. The unit of observation is the fund/month. All returns and expense ratios are in units of fraction per month. \( NetR \) is the return received by investors. Benchmark-adjusted net return equals net return minus the return on benchmark portfolio. Gross return is the benchmark-adjusted net return plus \( \frac{1}{12} \)th of the annual expense ratio. \( FundSize \) is the fund’s total AUM aggregated across share classes, divided by the total stock market capitalization in the same month, imputing \( FundSize \) when missing. \( IndustrySize \) is the sum of all funds’ AUM divided by the total stock market capitalization in the same month. \( FundAge \) is the number of years since the fund’s first offer date. \( SectorSize \) is the sum of AUM (in nominal dollars) across all funds within a given sector, divided by the total stock market capitalization in the same month. We use the nine sectors in Morningstar’s 3 × 3 StyleBox.

XXX%To be updated%XXX
The dependent variable in each regression model is $NetR$, the fund’s benchmark-adjusted net return. $FundSize$ is the fund’s total AUM at the end of the previous month, divided by the total market cap of stocks in CRSP. $IndustrySize$ is the total AUM of all active equity mutual funds divided by the total market cap of all stocks in CRSP. $SectorSize$ is the total AUM of all funds within a given sector, divided by the total market cap of all CRSP stocks. The OLS FE estimator includes fund fixed effects. The RD estimator recursively forward-demeans all variables and instruments for forward-demeaned $FundSize$ using backward-demeaned $FundSize$. Heteroskedasticity-robust $t$-statistics clustered by sector × month are in parentheses.

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Table 4
Posterior Estimates for the Model with Fund-Level Heterogeneity in Skill and Decreasing Returns to Scale

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### Posterior Estimates for the Model with Fund-Level Heterogeneity in Skill and Decreasing Returns to Scale (continued)

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<td>.7411</td>
<td>.000664</td>
<td>.4776</td>
<td>.0038</td>
<td>.0026</td>
<td>.000028</td>
</tr>
</tbody>
</table>
Table 6
Likelihood Ratio Comparison between Our Baseline Model and the Pastor-Stambaugh Model

The first column reports the log-likelihood of each model at the median parameter estimates. The rest of the columns report the time series correlation between the industry size and the number of funds. The actual value of correlation in the data is 0.980. The two tests show that our model uncontroversially performs better on the two metrics.

<table>
<thead>
<tr>
<th></th>
<th>(log) Likelihood at median</th>
<th>Median</th>
<th>Std</th>
<th>( \rho ((S/W)_t, M_t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pastor and Stambaugh (2012)</td>
<td>17353</td>
<td>.114</td>
<td>.596</td>
<td>[.820, .915]</td>
</tr>
<tr>
<td>Our Baseline Model</td>
<td>18460</td>
<td>.925</td>
<td>.064</td>
<td>[.800, .983]</td>
</tr>
</tbody>
</table>
Figure 1.

The first figure plots the number of funds over time in our sample for the period 1980-2014. The second figure plots the average expense ratio, which has hardly moved. The third figure plots the industry size (measured relative to the stock market capitalization), which has dramatically increased over time.
Figure 2.

This figure plots the actual size and the simulated industry size from the Pastor-Stambaugh model. The key take-away is that the Pastor-Stambaugh model generates no significant trend in industry size.
This figure plots the actual size and the simulated industry size from our model featuring learning about heterogeneity in fund skills and fund-level decreasing returns to scale. As it is, our model cannot generate exact quantitative trend. However, the average correlation between the actual and the simulated industry size is about 0.91. In this sense, the model captures the salient features of historical fluctuations in the industry size.