

Symbolic Victory
Signaling Strength through Battlefield Choice
(Paper in Progress)

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Abstract

Strong states can fight riskier battles in order to signal their strength and conclude wars. I open with the broad question, why do some battles end wars, while others do not? I argue that Clausewitz' conception of decisive victory does not describe the majority of war-deciding engagements. The larger literature on bargaining and war proves equally unhelpful: while it predicts war termination should correlate with its duration (since fighting causes beliefs to converge), empirical tests have failed to corroborate this hypothesis. Focusing on the game theoretic literature, I suggest this failing results from the assumption that generals cannot choose their battlefields. I argue that where and how a general fights conveys information: stronger states can (and will) fight on more difficult ground and for more difficult objectives than weak states. This decision conveys information to an enemy; this additional information increases our ability to predict war termination. In short, strategic risk is a costly signal in war. After developing this logic informally, I develop it further in a formal, game theoretic model of symbolic victory. Finally, I briefly illustrate the model with cases from the Second World War and the Second Punic War.

In 1800 Daniel Steibelt, a pianist of some renown, challenged the young Ludwig van Beethoven to an improvisational duel. Confident in his abilities, Steibelt played a quintet of his own composition and then—in an unmistakable insult—improvised on one of Beethoven’s own themes. Furious, Beethoven strode to the piano, on his way snatching the cello part of Steibelt’s quintet. Setting it upside down upon the rack, the great master proceeded first to pick out the theme, then embellish it, outdoing Steibelt’s own tempestuous tremolos and reducing to parody the heights of his rival’s skill.

Humiliated, Steibelt fled Vienna, never to return.

After this resounding victory Beethoven never again had to prove himself in the salons of Vienna. He had established his reputation beyond dispute, and he would never again be called to defend it. Henceforth he was Vienna’s preeminent virtuoso.

Of course, Beethoven did not need to improvise on an unfamiliar (and inferior) piece of music, much less in the style of his enemy: he was, after all, the more talented musician, and likely the Viennese aristocrats would correctly have identified him as the victor. Yet he deliberately chose the riskier option. Why? The answer to this question discovers powerful truths, not only about amusing musical contests, but about the mortal struggles of nations and the course and onset of their wars.

Theories of War, Bargaining, and Battlefield Victory

Why are some battles significant and others not? That is, why do some victories end wars while others simply continue hostilities? The question has puzzled military theorists for millenia. Not all important battles are decisive; neither are they especially violent, lopsided, or surprising. Indeed, many of the most celebrated victories do not change the course of a conflict, while others which lack even a semblance of artistry prove indispensable. From ancient authors to modern game theorists none explain why some victories matter. This paper offers a partial answer: by fighting for unnecessarily difficult objectives, a state can signal its strength to terminate a war more quickly. In the remainder of this section, I discuss previous bargaining theories of war from Clausewitz to game theory, highlighting the insufficiency of each to account for the varying importance of battlefield victories. The second

section develops the paper's thesis and formalizes the argument, and the third discusses the empirical implications of the model. The fourth and fifth sections illustrate the strategic logic with cases from the Second World War and the Second Punic War. A final section concludes.

How do victories gain significance? Intuitively, we recognize that some battles matter far more than others, but we struggle to move beyond this intuition. Does significance depend on scale or counterfactuals; does it follow when battles turn the tide of war, or when one side inflicts far more losses than it receives; are important victories always decisive, and what does that even mean? Scholars have traditionally used some (or all) of these as metrics, but, I argue, none of them satisfy. Contemporary political science is equally unhelpful; indeed, formal modelers have ignored the question entirely—a disregard which jeopardizes the empirical utility of their constructions. But before engaging game theory, I turn first to sources of a somewhat more reverend wisdom.

The great prophet of decisive victory is Clausewitz. His monumental *On War* opens: the aim of war “is to *throw* [an] opponent in order to make him incapable of further resistance. *War is thus an act of force to compel our enemy to do our will.*” (Clausewitz, 1976, p.75) To thus incapacitate an enemy, a general should seek his ‘center of gravity,’ “the hub of all power and movement, on which everything depends. That is the point against which all our energies should be directed.” (Clausewitz, 1976, p.595-6) While this center varies from enemy to enemy, typically it resides in his capital, his army, or his people. Before one falls the war's outcome remains uncertain; only victory over this crucial center will decide the war. Because war knows no logical limit, it will end only with the total defeat of an enemy. That is, war will end only in decisive victory.

But Clausewitz should never be read simply, for his work is dialectic: he explores war *in extremum* only to contrast it with war in practice. While war in its ideal form knows no limit, its practical form is the instrument of the state, and the goals of the state will constrain the strategy of war. To some extent even decisive victory belongs more to the absolute than the actual, for Clausewitz recognizes that “an analysis of probabilities may lead to peace itself. Not every war need be fought until one side collapses.” (Clausewitz, 1976, p.91)

Unfortunately, Clausewitz died before finishing *On War*, leaving no developed theory connecting political ends and non-decisive victory. Clausewitz correctly argues that war can end before its decision, but he neither explains how war reaches an early termination nor advises states how to fight a war to achieve such an end. Even non-decisive battles can matter—but how?

An immediate answer might look to stunning victories, ones where victors obliterate their foes in lopsided routs. If one side loses twice, thrice, or tenfold the men as another, surely such a victory will change the war. But this intuition is wrong. Consider the American Civil War: few battles can rival the tactical brilliance of Chancellorsville, a stunning victory in which a force half its enemy's size inflicted a third again the losses it received, yet Chancellorsville changed nothing, while Atlanta, a meat-grinding exercise with neither a surprising nor an imbalanced outcome, helped determine the war.

Another answer might turn to scale. More important battles are more massive: the clash of a hundred thousand surely outranks the encounter of a few brigades.¹ But this will not do, either. Often armies of the most colossal proportions engage to no purpose, while smaller ones clash to decide the fate of centuries. Still another might suggest that important battles dramatically shift one side's odds of victory. Yet if the outcomes of battle are predictable—at least in the sense that states know how strong versus weak opponents would fare—then such a battle should not alter the course of a war, since it has not revealed any new information; that is, it has not resolved the underlying cause of conflict. Finally, some battles seem counterfactually important—a different outcome could have changed all of history.² Yet on further inspection this criterion fails, at least regarding the war's course, for it leaves unanswered the most important question—how do battlefield outcomes gain

¹Even Clausewitz cannot resist the allure of scale: “the scale of a victory does not increase simply at a rate commensurate with the increase in size of the defeated armies, but progressively.” Or again: “the decision that is brought about by the battle partly depends on the battle itself—its scale, and the size of the forces involved—and partly on the magnitude of the success” (Clausewitz, 1976, p.253,261).

²This criterion is probably the most common. It leads Fuller to rank the Battle of the Metaurus as the most important of the Second Punic War, and in general it nominally underpins various compilations of important battles. For instance, it is the second criterion Davis uses in identifying what he calls decisive battles (the first is the vague ‘political importance,’ the third the first use of revolutionary technology), and it is the sole criterion of Mitchell and Creasy. (Davis, 1999), (Fuller, 1954), (Creasy and Mitchell, 1964)

political importance, i.e. how do they cause war to end?

These military accounts do not satisfy. Not all important victories are decisive victories, at least in the sense Clausewitz employs the term: more victories are significant than simply those fought for an enemy's center of gravity, as Clausewitz himself recognized. But historians and military theorists have not filled the gap, for they turn to indicators like scale or skew that do not in fact speak to the issue. The reason is simple: these measures look only to battlefield numbers and ignore the context of conflict—that is, they ignore the political.

War is a political act, not mere violence, an instrument not of brute force but of coercion. Brute force breaks an opponent, but coercion convinces him to yield. And where brute force achieves decisive victory, coercion can culminate in something else—a symbolic victory. The former leaves an opponent incapable of further resistance; the latter leaves him unwilling to further resist. To work coercion requires a credible threat and a credible assurance; the former a battle can supply, proving a state's strength beyond question. If war begins because an enemy does not appreciate a general's strength, then a credible demonstration of strength can end it.

This theme runs throughout the formal literature on war. While once game theorists talked of conflict as a one-shot lottery, now the literature extenuates war into a costly bargaining process, seeking to understand when and why wars terminate.³ A rich literature exists on signaling and crisis bargaining, yet this literature focuses almost exclusively on the signals which domestic constraints or institutions allow a leader to convey;⁴ rarely does it examine how actual fighting signals strength. Even when scholars examine battles as signals they depict their informational content as having only two sources: the information revealed from the fact that a state won (or lost) a battle, and the information revealed from its willingness to fight the battle in the first place. The actual battlefields and the character of the engagement are taken as given: states choose whether to fight a battle, but not which battles to fight. This

³Fearon famously shows that rational war has two primary sources—problems of information (alongside incentives to misrepresent) and problems of commitment. (He largely dismisses problems of issue indivisibility, whose theoretical purchase in any case (Powell, 2006) exploded.) After his article the literature splits into two tracks, developing models of each in isolation from the other. I here focus exclusively on informational accounts of war, since this literature is most relevant to my argument. (Fearon, 1995)

⁴Fearon's 1994 piece is probably the most famous in this vein. (Fearon, 1994)

simplifying assumption leads theorists to neglect the most interesting source of information during war.

Most formal treatments of information and war assume two actors with common priors and private information about their relative strength; generally, this private information is asymmetric, with one actor fully informed about the state of the world, the other to some degree ignorant. These models commonly share two features: first, they all exhibit a screening mechanism whereby weak states accept bargains early in war and strong states hold out for better settlements; second, they all convey information primarily through bargaining, i.e. the offers states are willing to make and accept, not through battles themselves.⁵ Two conclusions follow: first, because stronger opponents wait longer to settle, wars between more mismatched enemies should last longer; second, because information primarily flows from states' willingness to continue fighting, war termination should correlate with its length. Neither prediction describes reality.

The first prediction, while *prima facie* implausible, is difficult to disprove. The problem arises from the temptation of *post hoc* rationalizations: since the concept of state strength lacks a clear definition, evaluating it *ex ante* is difficult. Indeed, studies of asymmetric warfare find that since WWII states traditionally identified as 'weak' win more often, highlighting the term's lack of clarity (Arregun-Toft, 2001, p.97). Moreover, intuitively most would suspect that unbalanced conflicts end faster, since, while the power disparity might not have been apparent at the beginning, the gap in strength quickly becomes obvious—as in the First Gulf War.

The second prediction has received more conclusive disproof. Because these models primarily confine themselves to theory, offering only minimal empirical work, in an important statistical study Ramsay (2008) attempts to discover whether these games help explain the world. He tests their claims in the form of two hypotheses: first, that as battlefield informa-

⁵The most forceful exponent of this proposition is (Slantchev, 2003); see also (Filson and Werner, 2002; Powell, 2004) or (Admati and Perry, 1987). It should be noted that within this literature there exists an important dissent: Langlois and Langlois challenge the basic claim of all these models. They assume mutual uncertainty, but unlike all other models they allow their actors to keep silent. This, combined with assigning a continuum of types, time, and the possibility of stalemate, show that a unique equilibrium exists in which players wait until the last moment to come to terms, a result in sharp contrast to earlier work but, perhaps, more in line with actual state experience. (Langlois and Langlois, 2012)

tion (measured in battle days) increases, so does the probability of war termination; second, that more shocking victories (measured by the difference of expected and actual losses) increase the probability of war termination. Ramsay finds no support for either: shocking victories have some effect, but the variable's magnitude remains statistically insignificant, while battlefield information, though significant, in fact decreases the probability war ends. That is, wars are not more likely to end as they continue, the chief finding of the formal literature on bargaining during war. In light of these discouraging findings, Ramsay suggests that explanations for war which rely on information may be unfruitful.⁶

The judgment is, perhaps, premature. Nevertheless, Ramsay's findings carry a clear charge, and the weight rests squarely upon those who would pursue this approach. Further theorizing requires a clarification of informative versus uninformative battle. That is, because war termination does not simply correlate with its duration, a revised model must show in theory how some kinds of battles can convey more information than others, and it must show in practice how such battles tend to end wars. This paper attempts to offer such a revision.

The two errors I highlight do not inhere in the formalization of war. Rather, they result from an assumption common to almost all game theoretic treatments of conflict. Whether models of private or complete information, most models treat war as a line, with the tide of war drifting back and forth but never side to side: countries' campaigns run on rails from which they cannot deviate, and generals have no choice of approach or attack. In effect, these models eliminate military strategy. When once we reintroduce strategy to theory, these errors disappear.

This point deserves elaboration. All bargaining models of war assume a line of N forts (or objectives); if ever a player captures all the forts, he wins, seizing the pie in full. (In one-shot models of war, $N = 1$.) In each round, players bargain and, if that fails, they fight for the next fort (or, in the most sophisticated models, they do nothing). And now the problem becomes clear. While at first glance that bargaining conveys more information than battlefield victory surprises us, in fact it should not: since all types of player must attack the

⁶(Ramsay, 2008). (Reiter, 2003, p.32) echoes the concern, citing the previous work of (Bennett and Stam III, 1996) and (Goemans, 2000)

same target, information only updates through victory, which is ever uncertain; strategies can only separate during the bargaining phase. Battles cannot signal because there is no choice: because both weak and strong states must attack the same targets, in effect these models' assumptions impose pooling strategies on states. If we allow for choice, permitting states to attack one target and not another—if we reintroduce military strategy—then battlefield victories regain their importance.

This reintroduction is long overdue. To this end, the following sections attempt to discriminate significant from insignificant victories, arguing that the difference can result when a strong state deliberately pursues a riskier strategy in order to signal its strength; that is, a strong state chooses riskier targets than a weak type. Thus, symbolic victories are battles which only a strong state would fight—and win.

Argument and Model

I argue a strong state will voluntarily diminish its strategic odds of victory in order to signal its strength. This signal creates the opportunity for a symbolic battle, which, unlike ordinary battles, terminates the war. In essence, the argument builds upon three uncontroversial premises: first, that generals choose their battles; second, that generals do not know others' strength; third, that war is costly.

The fields of war are not foreordained: a general must choose his battles, deciding where, when, and how he will fight. A fixed line of forts does not tie one capital to a rival; rather, an expanse of forts lies before the strategist, and he must cut his own path. If war were cheap, he would take the path of least resistance; but war is costly. A general must therefore balance the cost of his campaign against the odds of its eventual victory, for the more quickly the parties can conclude a war, the better off are both. Because neither can trust the other, such a peaceful solution often eludes commanders: when words are cheap only deeds can tell. Fortunately, battle supplies its own language. A general cannot credibly talk of his strength, but he can credibly demonstrate it. If he takes the field in a place or with a force distinctly to his disadvantage, he can signal to an opponent his power; that is, he can take a risk only a strong state can afford.

This unintuitive claim builds from an obvious intuition. Suppose that a man wishes to prove his skill at sharpshooting; suppose he also wants to save bullets, since they cost money, and that his friends have set up targets for him at increasing distances. Of course, the man will first shoot the farthest target he can hit, since this success will prove he can hit the closer targets without wasting ammunition. Thus, when shows of force have costs an actor will at the start undertake the most difficult he reasonably can. The same principle applies to nations: when Richelieu wished to quell the French Protestants he besieged La Rochelle, their strongest city, knowing that if it fell others would capitulate. The action was not without its risk—if he failed Richelieu might well have found his head a few steps from his body—but the significance of victory, if achieved, would be well worth the cost.

I extend this simple intuition to a much wider application. A strong state, I suggest, will undertake risk even unnecessarily in order to demonstrate its strength. In effect, strong states will sometimes seek out danger in order to prove their mettle.⁷

An Intuition with Numbers

Before turning to the actual model, consider a stylized situation where an aggressor may invade a state either through a plain or over a mountain. After a successful invasion, an aggressor proposes a binding ultimatum division of his victim's treasure, and his victim may accept or reject this ultimatum; if his victim rejects, the aggressor then besieges the capital, destroying $\frac{1}{4}$ of the total treasure; the winner of the siege keeps the treasure in full, less the portion destroyed.

Now, suppose that an invader is either strong or weak. A strong invader always wins on the plain, successfully besieges the city with probability $\frac{7}{8}$, but he only wins $\frac{7}{8}$ of the time in the mountains; a weak invader wins on the plain half the time and successfully besieges the capital $\frac{1}{4}$ of the time, but he never wins in the mountains. Finally, suppose that the defender believes his opponent is strong with prior probability $\frac{1}{4}$.

⁷Throughout this paper the term 'risk' is always relative: it describes not the riskiness of a strategy but its riskiness compared to an alternative. Many generals undertake risk because they have no other choice: Hannibal crosses the Alps, an exceptionally dangerous stratagem, not to signal his strength but because Rome controls the sea. His strategy is therefore not 'risky' in the sense in which I use the term, since crossing the Alps, however absolutely dangerous, was the relatively safest route.

If both strong and weak players always invade through the plain, then after a loss there the defender believes he faces a strong opponent with probability $\frac{2}{5}$. The minimum ultimatum which would satisfy both strong and weak aggressors is $\frac{21}{32}$ of the total treasure, which leaves the defender with $\frac{11}{32}$, but the defender's utility from fighting is $\frac{57}{160}$, and so he would reject such an offer. Instead, when both invade over the plain, weak types will propose an ultimatum $\frac{7}{16}$, which the victim will accept, while strong types will unseriously propose some ultimatum in excess of $\frac{3}{32}$ (for instance, they will demand the whole treasure).

Now consider what happens if a strong player deviates to invade over the mountain: he decreases his odds of reaching the capital, but if he does, he can impose an ultimatum of $\frac{29}{32}$ (since the defender knows he must be facing a strong opponent), giving him an expected utility of $\frac{203}{256}$, which exceeds his utility from invading the plain ($\frac{21}{32}$) by $\frac{35}{256}$. In other words, a strong player can increase his expected utility by decreasing his ultimate odds of victory, for such a decrease can signal to an opponent his strength and thus avoid the costs of war.

The Formal Model

I present a simple game which follows the standard bargaining model of war with asymmetric information over capabilities, except that a state chooses whether to fight more or less difficult battles. Though Fearon identifies three rationalist causes of war, I abstract from problems of commitment and indivisibility in order to examine information in isolation. Finally, though the model deals with uncertainty over capabilities it should apply equally well to uncertainty over resolve.

Suppose an aggressive state, A, desires to seize from some other state B a prize with value standardized to 1. State A is either weak or strong, $\theta \in \{W, S\}$, with a prior probability of strength $\Pr(\theta = S) = p \in (0, 1)$. (Note that weakness contains no information about A's strength *vis-a-vis* state B: state A could be weak and yet still be more likely to win a battle than B.) State A knows whether he is weak or strong, but B does not. States win their easiest battles with probability ϕ_θ , where $\phi_S > \phi_W$.

In addition, suppose it takes only two battles to win a war: first, the aggressor A must successfully invade B; second, he must capture B's capital. If A fails in either battle he loses

the war, gaining nothing, but if he wins both he gains the entire prize. Between battles A presents an ultimatum which B may accept or reject.

The game comprises four stages. First, state A chooses where to invade B. He may invade at any location a within the interval $[0, 1]$, where his possible invasion points are arranged from easiest to most difficult; both states observe a . The probability a player wins at location a is given by a continuous, weakly decreasing function α which maps a onto $[0, 1]$; for tractability, let $\alpha(a) = \{\phi_\theta - a \ \forall a \leq \phi_\theta; 0 \text{ else}\}$. This function is known to all states. For convenience, define $s \equiv \alpha(0)$ when $\theta = S$ and $w \equiv \alpha(0)$ when $\theta = W$.

Second, the players fight the chosen battle; if A loses, the game terminates and B keeps the prize in full, payoffs $(0,1)$, but if A wins, the game continues.

Third, state A proposes some division of the prize $(x, 1 - x)$, $x \in [0, 1]$ which B may accept or reject. If he accepts, the game terminates with the proposed payoffs; if he rejects, the game continues.

Fourth, the players fight for B's capital, with cost $c \in (0, 1)$. The winner receives the entire prize. Thus the payoffs if A wins are $(1 - c, -c)$; if B wins, $(-c, 1 - c)$. To simplify, assume that the probability A wins this battle is ϕ_θ , though the results will hold for many values.

The model involves a minimum of moving parts. Its key elements are the variable strength of the aggressor, the nonzero cost of war, the ignorance of the defender, and the aggressor's ability to choose more or less dangerous battlefields.

In order to demonstrate the intuition that risk can convey an army's strength I have assigned to such risk only the most dire consequences: in the model, each decrease in the probability of a battlefield victory equally risks the loss of the entire war. Moreover, none can mistake the decision to fight a riskier battle as an attempt to conserve resources or incur fewer casualties, since none of these variables enter the equation. There exists only one tradeoff: a strong state must weigh the decreased probability of ultimate victory against the increased returns from a risky show of strength. I argue that above a quite minimal cost-of-war threshold an equilibrium exists in which a strong state always stands to gain by undertaking unnecessary risks.

Obviously, if $a \geq w$ and A successfully invades then B must update his beliefs to $Pr(\theta = S) = 1$, since a weak aggressor could not have won. But a separating equilibrium does not require such an extreme measure. Rather, a strong aggressor has only to choose a risky enough battle a which a weak aggressor would be unwilling to imitate. This paper's central claim is that, as long as war is sufficiently costly, a strong aggressor will seek to demonstrate his strength through an early, risky battle in order to more quickly conclude a war.

Consider the case where a strong state wins with probability $s = .875$, a weak state wins with probability $w = .5$, and the cost of the final battle is $c = .125$. A strong state invades at a location $a = \frac{3}{16}$, giving him a probability of victory in the initial invasion equal to $\frac{11}{16}$, while the weak state invades at $a = 0$. If he wins, a strong aggressor then proposes $x = s + c = 1$, and a weak aggressor proposes $x = w + c = .625$. For his part, the defender believes that only a strong state would take such an unnecessary risk and accepts the offer; given that the strong state always takes this risk, he also rejects any offer greater than .625 if A attacks at $a < \frac{3}{16}$. If a weak state tries to pretend to be strong (that is, he chooses $a = \frac{3}{16}$), his expected payoff becomes $(\frac{1}{2} - \frac{3}{16}) * 1 = \frac{5}{16}$, which is equal to his current payoff of $\frac{1}{2} * \frac{5}{8} = \frac{5}{16}$. (In other words, a strong A takes just enough risk to make a weak player indifferent.) Similarly, if a strong state decides not to take the unnecessary risk he would receive an expected payoff of $.875 * (.875 - .125) = \frac{21}{32}$, which is less than his current payoff of $\frac{11}{16}$. The situation is thus a separating perfect Bayesian Nash equilibrium. This result is not unusual, and in fact generalizes to all cases where the cost of war is sufficiently high:

Proposition 1. $\forall c, s, w$ s.t. $c \geq w(s - w)/(2s)$ and $1 \geq s + c$,

\exists a separating eqb in which A of type $\theta = S$ invades at $a > 0$
and B accepts x with probability 1. (For proof see appendix.)

A few points are worth noting immediately. First, ordinarily a separating equilibrium would not exist for a game with fewer bargaining rounds than player types; the possibility of choosing an $a > 0$ creates the potential for such an outcome. Second, the existence of a risky equilibrium does not depend upon p , the prior probability that a state is strong. Third, a state's willingness to undertake a riskier battle, not its battlefield success, signals a state's strength. Fourth, war always terminates before a final confrontation, which in general

cannot occur in the standard model. Fifth, the threshold requirement for c relative to s and w is quite lax, never exceeding $\frac{1}{8}$, and usually approaching 0; in other words, most actual circumstances will have a 'risky victory' equilibrium. Finally, and perhaps most importantly, the equilibrium is never pareto inferior to an equilibrium where all players choose $a = 0$, and, in fact, is pareto superior to the typical equilibrium where $a = 0 \forall \theta$ (see Appendix).

This equilibrium is immune from the most common critiques of bargaining models of war. For instance, the limitation in many bargaining models of θ to a finite number of types helps create the screening property criticized above; as Langlois and Langlois demonstrate, with a few plausible assumptions this property will not generalize to a continuum of types. Fortunately, this criticism does not apply to the model I present: in equilibrium, the separation derives from information conveyed through risky behavior, not from a state's willingness to continue fighting; in fact, because the number of bargaining periods is less than the number of types, states cannot make fully informative offers during the bargaining phase.

Caveats

Still, a few caveats are in order—after all, generals rarely leave their tanks at home. The model succeeds because both players observe a accurately and because a has no effects other than reducing A's probability of victory. These simplifications do not always hold in reality. Often, conserving resources in one battle means a general has more in another; reducing the odds of victory in one place increases the odds in another. This tradeoff can obscure the signal a strong state intends to send, since his opponent might mistake unnecessary risk for calculated caution. Furthermore, players cannot always observe a : battlefields are noisome, and the fog of war obscures much. If a rival cannot recognize risky behavior, such behavior loses its symbolic utility.

Fortunately, rather than undermine the model these limitations in fact illuminate the real world: while risky behavior might not make sense on the battlefield, it will in the campaign; symbolic victory is a tool, not of tactics, but of strategy. A general signals his strength, not in how he fights a particular battle, but in which battles he fights. In his choice of cities to besiege and strongholds to capture a commander can deliberately and obviously choose

a riskier approach, one his opponent cannot mistake. While the model will not accurately predict tactical encounters, it should speak to the shape of the broader campaign. How a general fights his battles may signal little, but how he chooses them will signal much.

A few final caveats. First, more difficult battles will often be the most valuable, confusing the signal a strong state can send: as with a Hail Mary, great risk often entails great reward. Second, the equilibrium depends upon actors' beliefs: theoretically, states could live in a world where an enemy regards such risky action, not as a costly signal, but as simple stupidity. Third, in some sense battlefields are occasionally foreordained—there may exist only one feasible route for an army to take. Fourth, the model cannot apply to absolute wars: if states expect to fight even until unconditional surrender (as modern democracies increasingly incline to do), then a general can have no reason to pursue symbolic victory, for he aims at a decision; in these circumstances, strategists will always pursue the low-hanging fruit. Finally, the choice of battlefields is not infinite. While an army is large enough to effectively approximate a continuum, the small number of cities restricts a commander's options; as a result, there may no longer exist an a such that a strong state could profitably signal its strength (this is akin to arguments about indivisibility and the onset of war). But again, far from undermining the model, these caveats in fact simply show why wars do not terminate after the first encounter—the opportunity for symbolic victory has not yet arisen.

Empirical Implications

I couch the model in the language of battle, but its applications are far wider. Most obviously, it applies to morale: if the hopes of soldiers and citizens depend in part on their expectations of victory, bolstering their expectations can strengthen morale. An ordinary victory does not much change these expectations, but a risky victory, a victory against the odds, can change them dramatically. As well, though the model I present deals only with two states, its dynamics strengthen exponentially with more actors. The desire to establish a reputation, to persuade a reluctant ally, or to deter a restive enemy all increase the incentives to credibly demonstrate strength through a symbolic victory. For instance, during the American Revolution, French strategy required a demonstration of rebel strength

before committing its support. More spectacularly, during the Gallic wars Caesar crosses into Germany for no other purpose than to prove that Roman might could reach any tribe, anywhere; he bridges the Rhine—a feat not replicated for a millenium—not to seize territory, not to achieve a military objective, but for the purely strategic purpose of demonstrating to a sceptical barbarian population the matchless reach of Roman arms (Caesar, 1951, IV.16).

Besides these general implications, the model supports at least two related hypotheses:

Hypothesis 1. *Unnecessarily risky victories reduce the length of wars.*

Hypothesis 2. *Unnecessarily risky victories convey more information than other victories.*

The first hypothesis claims that wars are more likely to end after a risky victory than before. To some extent wars continue because states disagree on their relative strength; since risky victories convey information about this strength, they should increase the probability that a war terminates.

The second hypothesis claims that during wars a risky victory will determine a war's outcome more than other victories, regardless of scale or duration. In essence, a battle can derive additional significance from its superfluous risk. Corroborating evidence would a) identify the strategies of the opposing states, i.e. how they intended to win the war; b) identify victories that would signal the strength (weakness) of such strategies, i.e. victories only a strong (weak) state could afford to win (lose); c) show that a strong state sought such victories; and d) demonstrate that the consequences of such victory reached farther than those of less risky battles with similar tactical results.

With these implications and hypotheses in mind, I briefly turn to two cases to demonstrate the argument at work. The first case, the Doolittle Raid, shows that the logic I outline actually passes through the minds of military strategists; I canvass this case only briefly to illuminate the argument. To examine the second hypothesis, this paper considers the Second Punic War. In particular, it contrasts the effects of two battles, one the most shocking victory of the war, the other an unremarkable but risky siege, showing that the latter, not the former, signaled the actors' relative strengths. I draw three conclusions: first, that symbolic victory figures in the strategies of states; second, that the logic of symbolic victory extends beyond

the chief competitors to their allies; and third, that the importance of victory depends far more on its symbolism than its figures.

The Doolittle Raid

After the bombing of Pearl Harbor came a series of Allied setbacks: the loss of Hong Kong, Singapore, Manila, Guam, and the Dutch East Indies, to name a few. These setbacks only entrenched the intention of President Roosevelt, who had determined in the weeks after December 7 to strike the Japanese mainland. Although such a strike, if it was even possible, could cause only minimal damage, Roosevelt recognized that it could have an indispensable psychological impact on the American and Japanese psyche. And so on April 18, 1942, America bombed Tokyo.

The Doolittle Raid is well-known, enjoying numerous treatments in popular histories and on the silver screen (all, naturally, of varying accuracy). Unsurprisingly: its planners intended the raid to thrill the American heart and chill the Japanese; its purpose was not so much to inflict damage as to demonstrate capability. It did. I include it here not to offer an original take on its events but rather because it epitomizes the argument I present. Moreover, as its details are fairly well-established, I hope that a survey of the salient facts will demonstrate my argument without arousing controversy. Most agree that the highest levels of command, including FDR himself, called for the raid in order to boost American morale and worry the Japanese; similarly, they all agree that it achieved both ends. That is, the American Air Force undertook a great risk for comparatively little reward and for almost no other purpose than to signal its awesome strength. As this story accords precisely with the argument of this paper, I briefly relate it here.

The raid was the deliberate policy of Washington. No mere tactical decision, the assault had its genesis and purpose in the highest strategy of the war. By late December Roosevelt had directed his commanders to plan an attack on the Japanese mainland. The President first approached his Russian allies, whom he hoped would bomb Tokyo for him or, when that tack was rebuffed, at least permit him the use of Vladivostok. The Russians declined, but Roosevelt remained undeterred. In January, he once again pressured his chiefs to devise a way

to attack the Japanese mainland, unaware that such plans had progressed steadily since his initial decision. Again and again, Roosevelt emphasized the psychological importance of such a victory (Schultz, 1988, p.47), both for Americans distressed by a string of early losses and for Japanese who had not yet tasted serious defeat.⁸ A similar logic echoes in the minds of the raid's architects, Duncan and Low, who both agreed that American soldiers and citizens needed a dramatic victory to improve their morale.(Schultz, 1988, p.14) America needed to demonstrate its strength, both to its allies and its enemies, and no better way existed than a risky, flagrant attack on a homeland thought untouchable.⁹

It fulfilled these intentions. Domestically, the attack lifted ebbing American spirits and captured the popular imagination (hence the many movie adaptations); internationally, it shattered the confidence of the Japanese people.¹⁰ It had two primary effects on Japanese strategy. First, it led them to withhold for the defense of their islands planes needed elsewhere, chiefly in the Solomon Islands (Merrill, 1964, p.180). Second, it convinced Japanese leaders of the need to extend their defense. As many have observed, before the raid Japan focused its efforts to the south, despite the arguments of Yamamoto; after, they turned east, determined to deny the United States the wherewithal to ever again attack Japan. The result was Midway—the turning point of the Pacific War.¹¹

The Doolittle Raid dramatically altered the war—and yet, it inflicted almost no damage. Indeed, upon landing Doolittle himself thought that the attack had failed.¹² The tactical futility of the raid has perhaps no better witness than the absence of any intention to repeat

⁸While some early histories could not ascertain whether Roosevelt originated the policy (Craven and Cate, 1948, p.438), with all the evidence gathered and at hand modern texts unanimously agree on his personal involvement. The most often-cited quotation is from Hap Arnold's notes, "find ways and means of carrying home to Japan proper, in the form of a bombing raid, the real meaning of war." (Nelson, 2002, qtd. on p.106)

⁹As an aside, I would note that the raid itself was partially the result of Henry ("Hap") Arnold's determination to prove the air force's worth (Nelson, 2002, p.107)—and what better way to prove strength, he thought, than by a particularly daring and *risky* mission?

¹⁰"That raid by Doolittle was one of the greatest psychological tricks ever used....The results of the Doolittle Raid are still evident in Japan. They are stamped into the daily living habits of the Japanese people...[who] search the skies each morning and each night." (Lawson, 1943, p.210)

¹¹"During their discussions, the Doolittle raid of 18 April 1942 struck Tokyo. Although the damage caused was inconsequential, the reach of the attack supported a growing feeling that the Japanese perimeter would gain in strength if it had greater defense in depth." (USSBS, 1946, p.3); similarly, "The bombs caused relatively little physical destruction...but the psychological damage was severe and long-lasting to Yamamoto and other military leaders." (Schultz, 1988, p.3)

¹²Upon landing, Doolittle groaned, "It's been a complete failure" (Schultz, 1988, p.3)

it: if its planners had thought it a viable way to inflict material damage, they would have continued bombing; instead, though overjoyed at its results, no one, including Roosevelt, shows any inclination to replicate the feat (Glines, 1988). Once the raid demonstrated American strength, repeating such a show would serve no purpose. The victory was symbolic, not tactical, and its symbolism could not be denied.

Leaders think in terms of symbols and signals. They seek to demonstrate strength, and they will undertake even great risk for little material reward if it will signify their power. In light of this illustration, I now turn to the Second Punic War, which demonstrates how such symbolic victories can outweigh even the most spectacular of military triumphs.

The Second Punic War

In 216 BC Hannibal Barca achieved the greatest tactical encirclement in the history of warfare. His genius annihilated a Roman force almost twice the size of his own, a defeat that would haunt the Roman imagination for generations. To overstate Hannibal's tactical achievement would be almost impossible: facing over eighty thousand Roman soldiers with perhaps forty thousand of his own, Hannibal set a trap which would kill over forty-eight thousand of his enemy, more than fought in his entire army.¹³ In early August on the banks of the Aufidus Carthage inflicted the rising Republic's bloodiest and most grievous loss, with tens of thousands dead and entire legions obliterated in a single battle—Cannae.

The third century before Christ witnessed the clash of two superpowers for dominance of the Mediterranean world. In three Punic Wars Carthage, an ancient and rich city in modern-day Tunisia, fought with Rome, then a rising power who dominated a league of Italian states. Though lasting two decades and at the time the bloodiest war in antiquity, like the First World War the First Punic War would prove but a prelude to its sequel's grim enormity. The Second Punic War convulsed the Mediterranean world from the shores of Greece to the suns of Gibraltar; its determining influence on Western history cannot be

¹³According to (Polybius, 2010, III.105,113), Rome fielded eight legions, each expanded beyond the norm to 5000 men apiece, with an approximately equal number of allied troops and an additional force of 6000 cavalry. Some scholars favor Livy's figures because they are smaller, but most agree with (Lazenby, 1978, p.75-6) that Polybius' totals should be accepted. Finally, it is worth noting that even of the 30,000 not killed, only 15,000 escaped.

understated.

In this brief treatment, I contrast two battles from the war, the battle of Cannae and the siege of Capua. I argue that, although the former excels the latter in every measure of tactical military success, the latter, not the former, changes the course of the war, because Capua, not Cannae, was a symbolic victory. To advance this argument, I first identify the antagonists' strategies; second, I highlight the striking failure of Cannae to alter the war; third, I discuss the decision to besiege Capua; and finally, I contrast the strategic significance of the fall Capua with the strategic unimportance of Cannae.

The strategies of the two states are well documented. Rome derived her strength, not from wealth, but from her ability to raise almost limitless armies; so long as she could field men throughout the peninsula, she could maintain her position. She might lose temporarily in one place, but she would retain control everywhere else. Roman strategy passed through three main phases in the Italian theater: after initial defeats in the north, culminating in Hannibal's victory at Lake Trasimene, Rome, having appointed Fabius dictator, followed his eponymous scorched-earth strategy, seeking to deny Hannibal the resources to sustain his army in a foreign land while avoiding open battle. Then, embittered by the cruel necessities and apparent cowardice of this approach, Rome decided to engage the Carthaginians directly, leading to her stunning defeat at Cannae. In the wake of this debacle Rome returned to a Fabian approach, eschewing open confrontation while attempting to confine the damage Hannibal could inflict. Rome recognized she did not need to beat Hannibal to win: so long as she could maintain control over the larger peninsula, Hannibal could win every battle and yet lose the war.

Carthaginian strategy was even simpler. Rome won the First Punic War through her almost inexhaustible reserves of manpower, reserves which none of her enemies could match.¹⁴ So long as the city of Rome itself survived, and so long as Rome held her Italian alliances and possessions, Rome would eventually win.¹⁵ Rome's defenses ensured she would fall only

¹⁴ "The lack of citizen manpower ensured that Carthagian armies were recruited from foreign soldiers...[by contrast,] As Rome expanded so too did her citizen population which, combined with her allies, gave Rome vast resources of military manpower, far greater than those of Carthage" (Goldsworthy, 2000, p.32,39)

¹⁵ "To Hannibal it was obvious that so long as Rome possessed the huge manpower reserves of Italy, there was no question but that Rome would win in the end." (Mackay, 2004, p.67)

to a protracted siege, for which Hannibal was utterly unprepared; more importantly, were Hannibal to invest Rome, he would lose the key to his tactical victories, his freedom of maneuver, without which he could not triumph over the larger armies Rome could raise. Thus, Hannibal's only hope for victory lay in turning Rome's allies to his cause. He therefore traversed the countryside, appearing like lightning and wreaking havoc on the peninsula, hoping to bring Rome to its knees and to seduce her allies. Indeed, in the early years of the war, Hannibal consistently released prisoners of war who had fought as Rome's allies, hoping to divide Rome from her friends.¹⁶ His entire strategy depended on their widespread defection. His strategy failed. Rome maintained her alliances, and with it she retained her mastery of Italy, regardless the tactical victories Hannibal achieved.

The Second Punic War began in 218 with Hannibal's fabled crossing of the Alps. As he made his way southward, Hannibal soundly defeated every force he encountered. Determined to eliminate this threat before he could wreak more serious damage, Rome raised an immense army of almost ninety thousand, dispatching it to meet and crush the Carthaginian general. Hannibal met the Roman force on a field near the town of Cannae. Placing his weaker troops in the center of his line, as the armies clashed he lured the stronger Roman force into driving too far forward; swinging his line into a vice, he encircled the Roman legions on three sides, his cavalry sweeping around to close the rear. Thus surrounded, the Roman army was annihilated. The Carthaginian victory was total. Few escaped the massacre, and by nightfall almost fifty thousand lay dead. Of those left alive, Hannibal released without ransom many of the Italians, hoping to attract Rome's allies to his cause, largely without success.

Following his victory, Hannibal sent an emissary to Rome, expecting his spectacular victory would force the Romans to sue for peace.¹⁷ Rome refused even to meet him. Instead, Rome began to prepare for a long war, resolved to defeat Hannibal despite her staggering defeat. Hannibal hoped that Rome's allies would flock to him, but most did not: Rome loses the Samnites and a few Apulians, but little else. The Gauls had rebelled as soon as

¹⁶For instance, after Trebia, Trasimene and, of course, Cannae, Hannibal released his non-Roman captives in an explicit show of goodwill. (Livy, 2006, XXII.13,58)

¹⁷While commentators to some extent differ on Hannibal's expectations following Cannae, it does not seem farfetched to conclude that the "addition of Carthalo to the delegation of prisoners suggests that Hannibal had expected to begin peace negotiations with the Roman Senate" (Goldsworthy, 2000, p.217)

Hannibal entered Italy, Syracuse would not desert until after Hiero's death in 215, and the Greek towns defect only when Tarentum falls in 212, not after Cannae. With one exception, no major city defects after Cannae, and many cities, most prominently Naples, summarily rebuff Hannibal's overtures and choose to weather his assaults rather than betray Rome.¹⁸

Thus, unsurprisingly Cannae did not change the war, because it did not change perception of Roman power. How could it? Roman strength lay, not in her ability to win field battles, but her ability to maintain widespread control. Hannibal had proven he could beat Rome in open battle; he had not proven he could defeat her in war. After Cannae, Rome lost several allies, mostly in southern Italy, yet not so many as the defeat's magnitude would at first glance suggest. In fact, most astounding in the aftermath of Cannae was Roman and allied resolve. Hannibal had expected Rome to quickly sue for peace; he had not expected her to escalate the war. Hanno, Hannibal's domestic rival, put it best:

'But just what is this jubilation all about anyway...since the battle at Cannae has meant the annihilation of Roman power, and since it is an established fact that all Italy is up in arms, has any people from the Latin league gone over to us? And, secondly, has any individual from the thirty-five Roman tribes deserted to Hannibal?' When Mago [Hannibal's ally in Carthage] answered 'no' to both questions, Hanno continued...'So we have a war in which we are no further forward than the day Hannibal crossed into Italy.' (Livy, 2006, XXIII.12-13)

Still, Rome did lose Capua. Among all Italian cities, Capua ranked second only to Rome.¹⁹ Capua had long yearned to supplant Rome in Italy, and when Hannibal offered her the prize

¹⁸The resilience of the Roman alliance is well established. The scholars who contend otherwise generally cite declining allied contributions to the war effort, but fail to note that such declines reflect more the strains of war than a collapsing alliance. For instance, Eckstein suggests that after Cannae "numerous polities in southern Italy defected," citing Lazenby to corroborate "the large scale of defections" (? , p175); yet Lazenby derives his figures from Livy's list at 22.61, which catalogues all Italian defections over the next five years (Livy 22.61f), *not* those which result from the defeat at Cannae; moreover, when Lazenby maintains that "by 212 over 40% of the allies were no longer available to Rome" (Lazenby, 1996, p.44), he includes the sheer physical impossibility of most cities to replace the loss of over 100,000 young men; in other words, the diminished number of soldiers Rome could field reflects as much the exhaustion of the Italian population as allied disaffection.

¹⁹Polybius notes the wealth of the entire region, and even comments that Capua was once the most prosperous city in the world (Polybius, 2010, III.91). This wealth engendered a certain degeneracy (Livy, 2006, XXIII.4), to the point where Livy even says (with great exaggeration) that Capua was Hannibal's Cannae.

she seized the opportunity. She forsook Rome, a decision which would prove her destruction.

Cannae changed Roman strategy, forcing it to abjure aggressive confrontation with Hannibal, but it did not change Roman odds of victory. Afterward Rome knew it could not best Hannibal in the field, for if it could not win with even doubly superior numbers it likely could not win at all. But Rome also knew that Hannibal could not be everywhere at once. Therefore Rome adopted a strategy for the long-term, harrying Hannibal but not offering battle, using her superior numbers to retake any cities the moment Hannibal departed. Rome would prove that Hannibal could not succeed: he could not defend his gains, and Rome would retain her mastery of the peninsula.

In this spirit Rome turned to Capua. The strongest of the war's defectors, among the Italian cities Capua stood second only to Rome. Her defection despite ties of law and blood galled the Republic, and Rome would make of her treachery an example to all Italy. Rome did not need to take Capua: other cities were more crucial to Hannibal, and these could be had for less effort. After all, Rome had already attempted once (unsuccessfully) to besiege the city, and a second attempt would thus seem all the less promising; more importantly, Tarentum, which had just recently fallen to Hannibal, was of far greater strategic importance, since it allowed Hannibal access to the sea and a degree of maneuver in Sicilian waters.²⁰ But Rome besieged Capua. Indeed, this needlessness, far from vain excess, in fact underlined the siege's symbolism: Capua, as the second city in Italy and Hannibal's greatest prize, would prove beyond doubt that Rome was mightier than Hannibal. If Hannibal could not protect Capua, he could not protect anyone; above all Capua would symbolize indefatigable Roman might.

The siege was unremarkable. Rome invested Capua as it would most any other city, weakening its defenses through starvation and the gradual collapse of morale. By the close of 212 almost half of Rome's Italian legions lay encamped about the city, surrounding it with a double line of siege works and their own defensive perimeters.²¹ These last effectively

²⁰See (Goldsworthy, 2000). In addition, the occasion for the first siege of Capua had been Hannibal's absence to take Tarentum, a prize he thought of greater worth. Moreover, the city was at least as open as Capua to attack, since Hannibal had left both in hopes of capturing Brundisium, and Roman garrison maintained control of the citadel, to boot.

²¹See (Lazenby, 1978, p.121). Even in the third century BC, Roman engineering was such that these works were almost impregnable

immunized the Roman position from Hannibal, and his sallies all came to naught.

Sensing the mortal blow Capua's loss would deliver him, Hannibal played every card in his hand, increasing the risk of the siege and marching to the very walls of Rome. Wild rumors flew, claiming that Rome had lost the siege, that all was lost, and citizens panicked in the streets at Hannibal's terrible approach. Yet Rome persevered. The Senate persisted in the siege, risking even Rome itself, and despite Capuan strength and the peril to her capital Rome triumphed. She took the city, driving the aristocrats to suicide and selling the population into slavery. Rome had chosen to take the riskiest, most difficult city, and with Rome itself in the balance she won, proving to all that she was still the master of Italy.²²

The fall of Capua effectively ended Hannibal's hopes in Italy.²³ To win, Carthage had to convince Roman allies to defect, but the fall of Capua symbolized to all what would happen to those that did. Though he remained undefeated in open battle, no Italian city would now join his cause, and he found himself confined to the narrowest tip of the peninsula. Hannibal had lost the Italian campaign, and soon he would lose the war.

In retrospect, Cannae, a battle whose consummate artistry military officers still study today, determined the outcome of the war far less than Capua, a straightforward siege of far less dramatic resolution. The Roman war effort—indeed, the entire Roman international system—depended on its ability to maintain its alliance and raise armies from the Italian populace. Cannae did not significantly alter the perception of Rome's ability to do either, while Capua confirmed beyond all doubt Roman strength. Cannae was a great, a monumental victory, one which survives even now in the military imagination. Yet it was not symbolic. By contrast, Capua was a lesson to all Italian states: Hannibal can win on the battlefield, but he cannot protect his gains; defect and you will be destroyed. Until the siege Roman

²²In his *Art of War* Machiavelli cannot praise this Roman steadfastness enough: "In the war against Hannibal, nothing did the Romans so much honor as their constancy. For in any of the most hostile and adverse fortune, they never asked for peace and never made any sign of fear. Indeed, when Hannibal was near Rome...they did not want to leave off their attacks [on Capua] so as to defend Rome" (Machiavelli, 2003, VII.153). The importance of this risk cannot be overstated; it proved to all that Rome was strong and utterly beyond Hannibal's reach.

²³O'Connell summarizes the situation well: "Rome persevered and would persist in replacing armies lost...Symbolically and actually, all of this was epitomized by the wretched fate of Capua...after Capua's fall it became clear to all that Hannibal could not watch over widespread allies...the war was far from over, but its outcome in Italy was all but decided" (O'Connell, 2010, 195-6)

mettle remained untested against a powerful Italian city; in choosing the crown jewel of Hannibal's Italian possessions, one he would defend with every ounce of his resources, Rome undertook the riskiest move of her Italian campaign, and her victory signaled to all her unrivaled strength.

Conclusion

What separates symbolic from ordinary victory? Why do some victories, even seemingly minor ones, lead wars to end, while other victories, even colossal ones, do not? Answers which rely upon concepts of decisive victory or military importance do not satisfy, in large part because these answers ignore the political aspect of war. Unfortunately, current political science fails even to ask the question. Yet clearly some battles matter. Neither numbers of deaths, lengths of time, nor lopsided victories predict war termination. But where these fail, reckless danger may succeed. A victory's significance can lie, not in the numbers or proportions dead, but in its risk. Even small victories, if unnecessarily risky, can signal strength, while even massive victories might not. Capua, for all its banality, settled the war in Italy, while Cannae, for all its brilliance, signified—nothing.

In war, the primary source of information is the campaign, but our informational theories of war omit precisely this source. Different types of states will pursue different paths to victory, and these separate paths will signal their strength. In particular, a state can increase its risk of losing a conflict to communicate its strength, enabling an early settlement and avoiding the costs of prolonged hostilities. If a strong state takes an action which a weak state dare not, it signals its capability indisputably. Of course, in real war, the signals and risk a state can take are rarely clear. But, while the signals will be mixed, they will exist. Stronger states will more often go for more difficult, more valuable objectives, and this tendency will drive at least some separation in the equilibria. I abstract from these complications to capture the essence of the signal, but I would note that in the real world far more signals are being sent than my stylized model would suggest. And our current bargaining models of war cannot begin to describe these signals, because we do not allow for strategy.

Thus far the literature on bargaining during war misses a crucial signal—the war itself. Whether states fight and whether they win are not the only sources of information battles convey: where and how states fight figure even more. A battlefield victory can end a war, not because it destroys a general's enemies, but because it convinces his enemies that he is strong. Such a symbolic victory crowns the strategy of a strong state and concludes a conflict. The greatest minds and might have fought battles they need not, and these battles have ended wars. This phenomenon was previously inexplicable. I hope to have provided at least the beginnings of an explanation.

Appendix

This appendix proves the proposition and contrasts the separating equilibrium with a typical (non-risky victory) equilibrium.

Model Recap

1. Player A is type $\theta \in \{W, S\}$. Player A wins his easiest battle with probability $\phi_\theta \in (0, 1)$, with $\phi_S > \phi_W$.
2. Player A invades at $a \in [0, 1]$, and his probability of victory in this first battle is given by α , where $\alpha(a) = \max(\phi_\theta - a, 0)$.
3. Player A proposes a division of the pie $(x, 1 - x)$, with $x \in [0, 1]$. B accepts or rejects. If he accepts, game terminates with payoffs $(x, 1 - x)$.
4. If B rejects, player A attacks the capital and wins with probability ϕ_θ .
5. The winner receives the full prize (value 1), but both players pay a cost c .

For simplicity, let $s \equiv \phi_S$ and $w \equiv \phi_W$; as well, let $\bar{x} = s + c$ and $\underline{x} = w + c$.

Also, note the utility of an aggressor of type θ for invading at a and offering x is given by

$$u_\theta(a, x) = (s - a)[\Pr(\text{B accepts } x)x + \Pr(\text{B rejects } x)(\phi_\theta - c)]$$

Finally, denote the strategy of player i with type θ as σ_i^θ .

Proof of Proposition

Recall the **Proposition**: $\forall c, s, w$ s.t. $c \geq w(s - w)/(2s)$ and $1 \geq s + c$, \exists a separating eqb in which A of type $\theta = S$ invades at $a > 0$ and B accepts x with probability 1

The Proof proceeds in three sections. First, I define the strategies; second, I establish that the strategies are optimal; third, I establish that the beliefs are consistent.

1) The equilibrium strategies are given by:

- σ_A^S : invade at $a = 0$ and offer \underline{x} .

- σ_A^W : invade at $a = a_S$ and offer \bar{x} ,

where a_S satisfies the inequality $\frac{s(s-w)}{s+c} \geq a_S \geq \frac{w(s-w)}{s+c}$ and $\frac{2sc}{s+c} \geq a_S$.

Such an a_S exists because $1 > s > w > 0$ and $\frac{2sc}{s+c} \geq \frac{w(s-w)}{s+c}$ since $c \geq \frac{w(s-w)}{2s}$.

- σ_B : if $a \neq a_S$, accept all $x \leq \underline{x}$ and reject all $x > \underline{x}$ with belief $\Pr(\theta = W) = 1$; if $a = a_S$, accept all $x \leq \bar{x}$ and reject all $x > \bar{x}$ with belief $\Pr(\theta = S) = 1$.

2) The strategies are optimal:

Note that $u_S(a_S, \bar{x}) = (s - a)\bar{x}$.

$u_S(a_S, \bar{x}) \geq u_S(a_S, x < \bar{x})$, obviously.

$u_S(a_S, \bar{x}) \geq u_S(a > a_S, \bar{x})$, obviously.

$u_S(0, x > \underline{x}) \geq u_S(a \neq a_S \text{ and } a \neq 0, x > \underline{x})$, obviously.

$u_S(0, \underline{x}) \geq u_S(a, x < \underline{x})$, obviously.

$u_W(0, \underline{x}) \geq u_W(a > 0 \text{ and } a \neq a_S, \underline{x})$, obviously.

$u_W(a = a_S, x = \bar{x}) \geq u_W(a = a_S, x \neq \bar{x})$, obviously.

Finally, given the assumptions of the model, the following utility comparisons also hold

$$s(s - w)/(s + c) \geq a_S$$

$$s(1 - (w + c)/(s + c)) \geq a_S$$

$$(s - a_S)(s + c) \geq s(w + c)$$

$$(s - a_S)\bar{x} \geq s\underline{x}$$

$$u_S(a_S, \bar{x}) \geq u_S(0, x = \underline{x})$$

$$s(2c)/(s + c) \geq a_S$$

$$(s - a_S)(s + c) \geq s(s - c)$$

$$u_S(a_S, \bar{x}) \geq u_S(0, x > \underline{x})$$

$$a_S \geq w(s - w)/(s + c)$$

$$w(w + c) \geq (w - a_S)(s + c)$$

$$u_W(0, \underline{x}) \geq u_W(a = a_S, \bar{x})$$

Thus, the strong type has no incentive to deviate and either pretend to be weak or fight for the prize, and the weak type has no incentive to pretend to be strong.

3) The beliefs are consistent:

if $a = a_S$ then A must be type $\theta = S$.

if $a = 0$ then A must be type $\theta = W$.

and the off-path beliefs (if $a \neq a_S$ then $\theta = W$, if $a = a_S$ then $\theta = S$)

are feasible. \square

Characterization of Equilibria where $a = 0$

Separating Equilibrium when all invade at $a = 0$

There is only one separating equilibrium where all types invade at $a = 0$.

In the typical bargaining story, a weak player bargains his way out early while a strong player makes a nonserious offer and continues fighting. In the model I present, such an equilibrium exists so long as $\underline{x} \leq s - c$. The equilibrium strategies are thus

- σ_A^S : invade at $a=0$, propose some $x > \bar{x}$.
- σ_A^W : invade at $a=0$, propose $x = \underline{x}$.
- σ_B : $\forall a, x \leq \bar{x}$ accept any $x \leq \underline{x}$ and reject any $x > \bar{x}$ with belief $\Pr(\theta = W) = 1$; $\forall a, x > \bar{x}$, reject x with belief $\Pr(\theta = S) = 1$.

Note that the expected utility to a strong type in this equilibrium is $EU_S = s(s - c)$, while his expected utility in the proposition's equilibrium is $EU_S = (s - a_S)(s + c)$, which, assuming a_S is no larger than necessary, equals $s(s + c) - w(s - w)$. Thus, as long as $2sc > w(s - w)$ (which is guaranteed by the assumptions of the proposition), a strong type could do better in the separating than the typical equilibrium. As the weak player would do just as well, and the defender would also do better (since $p(s - a_S)(1 - s - c) + (1 - p)w(w + c) > ps(1 - s - c) + (1 - p)w(w + c)$), *the typical equilibrium is NOT renegotiation proof*. Thus, in some sense we should expect the symbolic victory equilibrium over the typical one.

Semi-Separating Equilibria

Claim: if S does not mix, then W can mix over at most two proposals. Proof: suppose W proposes $x_1 < \dots < x_n$, $n \geq 3$, all with $\Pr > 0$, while S only proposes some \hat{x} . Note that $\hat{x} \geq x_n$. Then $u_B(\text{accept } x_1) > u_B(\text{accept } x_{n-1})$.

Semi-Separating Equilibria when $a = 0$, W mixes with $\Pr > 0$, and S does not mix

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