Problem 1

[Based on 4.3 of Vives(2008)] Consider a market with a single risky asset, with random fundamental value \( \theta \sim N(\bar{\theta}, \sigma_\theta^2) \), and a riskless asset (with unitary return). There are 3 types of traders: informed traders indexed in the interval \([0, 1]\), noise traders, and risk-neutral market makers. Informed traders have CARA utility function with risk aversion coefficient \( \rho \). Each informed traders \( i \) receives a private signal \( s_i = \theta + \varepsilon_i \) about \( \theta \), where \( \theta \) and \( \varepsilon_i \) are uncorrelated, errors are uncorrelated across agents and normally distributed with zero mean and variance \( \sigma^2_\varepsilon \). \( u \sim N(0, \sigma^2_u) \) is noisy traders’ total demand for the risk asset.

Informed traders and noisy traders move first. A proportion \( \nu \) of informed traders submit demand schedules \( X(s_i, p) = a(s_i - \bar{\theta}) + \zeta(p) \) and the rest of informed traders place market orders \( Y(s_i) = c(s_i - \bar{\theta}) \), where \( a, c, \) and \( \zeta(p) \) are determined endogenously. Their orders are accumulated in a limit-order book \( L(\cdot) \). Based on this limit-order book, competitive risk-neutral market makers set price informational efficiently:

\[
p = E[\theta | L(\cdot)]
\]

(a) Derive \( L(\cdot) \) and argue that \( p = E[\theta | p] \);

(b) Derive \( \text{var}[p] \) and show \( \text{var}[p - \bar{\theta}] + \text{var}[\theta - p] = \text{var}[\theta] \). Provide some comment.

(c) Express \( a \) explicitly and derive \( c \) as a root of a cubic equation.

(d) Set \( \nu = 1 \), derive the expected volume traded by informed agents,

\[
E \left[ \int_0^1 X(s_i, p) \, di \right]
\]

(e) (optional) set \( \nu = 0 \), perform a comparative statics analysis of the market parameters \( \rho, \sigma^2_\varepsilon, \sigma^2_\theta, \) and \( \sigma^2_u \).
Problem 2

[Based on 7.1 of Veldkamp(2011)] There is a continuum of ex ante identical traders, indexed by $i$, with CARA utility function and risk averse coefficient $\rho$. There are two assets. One offers a riskless return $r$. The other pays a risky amount $f = \theta + \varepsilon$, where $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and $\theta$ and $\varepsilon$ are uncorrelated. Traders can observe $\theta$ at a cost of $c$. The supply of the risky asset is $x \sim N(\bar{x}, \sigma_x^2)$. Solve the equilibrium asset price $p$ and the proportion of traders who the acquire information about $f$. 