Asset Pricing under Asymmetric Information
Modeling Information & Solution Concepts

Markus K. Brunnermeier

Princeton University

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References

**Books:**

**Articles:**
many others - see syllabus

Some parts of these slides rely on Princeton lecture notes by Nöldeke (1993)
Two Interpretations of Asymmetric Information

- different information
- different interpretation of the same information (different background information)
Modeling information I

- **State space** $\Omega$
  - state $\omega \in \Omega =$ full description of reality
    - fundamentals
    - signals
  - state space is common knowledge and fully agreed among agents
Modeling information II

• Partition
  • \((\omega_1, \omega_2, \omega_3), (\omega_4, \omega_5), (\omega_6, \omega_7, \omega_8)\)
  • \(P_i^1, P_i^2, P_i^3\) (partition cells)
  • later more about ‘knowledge operators’ etc.

• Field (Sigma-Algebra) \(\mathcal{F}_i\)

• Probability measure/distribution \(P\)
Modeling information III

- Prior distribution
  - Common prior assumption (CPA) (Harsanyi doctrine)
    - any difference in beliefs is due to differences in info
    - has strong implications
  - Rational Expectations
    - prior_i = objective distribution \( \forall i \)
    - implies CPA
  - Non-common priors
    - Problem: almost everything goes
    - Way out: Optimal Expectations
      (structure model of endogenous priors)
- Updating/Signal Extraction
Modeling information III

- Updating (general)
  - Bayes’ Rule

\[
P^i (E_n|D) = \frac{P^i (D|E_n) P^i (E_n)}{P^i (D)},
\]

- if events \(E_1, E_2, \ldots, E_N\) are a partition

\[
P^i (E_n|D) = \frac{P^i (D|E_n) P^i (E_n)}{\sum_{n=1}^{N} P^i (D|E_n) P^i (E_n)},
\]
Updating - Signal Extraction - general case

- Updating - Signal Extraction
  - $\omega = \{v, S\}$
  - desired property: signal realization $S^H$ is always more favorable than $S^L$
  - formally: $G(v|S^H) \text{ FOSD } G(v|S^L)$
  - Milgrom (1981) shows that this is equivalent to $f_S(S|v)$ satisfies monotone likelihood ratio property (MLRP)
  - $f_S(S|v)/f_S(S|\tilde{v})$ is increasing (decreasing) in $S$ if $v > (<)\tilde{v}$
    \[
    \frac{f_S(S|v)}{f_S(S|v')} > \frac{f_S(S'|v)}{f_S(S'|v')} \forall v' > v \text{ and } S' > S. 
    \]
  - another property: hazard rate $\frac{f_S(S|v)}{1-F(S|v)}$ is declining in $v$
Updating - Signal Extraction - Normal distributions

• updating normal variable $X$ after receiving signal $S = s$

$$E[X|S = s] = E[X] + \frac{\text{Cov}[X,S]}{\text{Var}[S]} (s - E[S])$$

$$\text{Var}[X|S = s] = \text{Var}[X] - \frac{\text{Cov}[X,S]^2}{\text{Var}[S]}$$

• $n$ multidimensional random variable $\left(\vec{X}, \vec{S}\right) \sim \mathcal{N}(\mu, \Sigma)$

$$\mu = \begin{bmatrix} \mu_X \\ \mu_S \end{bmatrix}_{n \times 1}; \Sigma = \begin{bmatrix} \Sigma_{X,X} & \Sigma_{X,S} \\ \Sigma_{S,X} & \Sigma_{S,S} \end{bmatrix}_{n \times n}$$

• Projection Theorem $(X|S = s)$

$$\sim \mathcal{N} \left( \mu_X + \Sigma_{X,S} \Sigma_{S,S}^{-1} (s - \mu_S), \Sigma_{X,X} - \Sigma_{X,S} \Sigma_{S,S}^{-1} \Sigma_{S,X} \right)$$
Special Signal Structures

- $\mathcal{N}$-Signals of form: $S_n = X + \varepsilon_n$
  
  (Let $X$ be a scalar and $\tau_y = \frac{1}{\text{Var}[y]}$),

\[
E[X|s_1, \ldots, s_N] = \mu_X + \frac{1}{\tau_X + \sum_{n=1}^{N} \tau_{\varepsilon_n}} \sum_{n=1}^{N} \tau_{\varepsilon_n} (s_n - \mu_X)
\]

\[
\text{Var}[X|s_1, \ldots, s_N] = \frac{1}{\tau_X + \sum_{n=1}^{N} \tau_{\varepsilon_n}} = \frac{1}{\tau_X|s_1, \ldots, s_N}
\]

- If, in addition, all $\varepsilon_n$ i.i.d. then

\[
E[X|s_1, \ldots, s_N] = \mu_X + \frac{1}{\tau_X + N\tau_{\varepsilon_n}} N\tau_{\varepsilon_n} \left( \sum_{n=1}^{N} \frac{1}{N} s_n - \mu_X \right),
\]

where $\bar{s} := \sum_{n=1}^{N} \left( \frac{1}{N} \right) s_n$ is a sufficient statistic
Special Signal Structures

- **N-Signals of form:** \( X = S + \varepsilon \)
  
  \[
  E [X|S = s] = s \\
  Var [X|S = s] = Var[\varepsilon]
  \]

- **Binary Signal:** Updating with binary state space/signal
  - \( q = \) precision = \( \text{prob}(X = H|S = S^H) \)

- **“Truncating signals”:** \( v \in [\bar{S}, S] \)
  - \( v \) is Laplace (double exponentially) distributed or uniform
  - posterior is a truncated exponential or uniform

(see e.g. Abreu & Brunnermeier 2002, 2003)
Solution/Equilibrium Concepts

- **Rational Expectations Equilibrium**
  - Competitive environment
  - agents take prices as given (price takers)
  - Rational Expectations (RE) $\Rightarrow$ CPA
  - *General Equilibrium Theory*

- **Bayesian Nash Equilibrium**
  - Strategic environment
  - agents take strategies of others as given
  - CPA (RE) is typically assumed
  - *Game Theory*
  - distinction between normal and extensive form games
    simultaneous move versus sequential move
## The 5 Step Approach

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<thead>
<tr>
<th>Step</th>
<th>REE</th>
<th>BNE (sim. moves)</th>
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| **Step 1** | Specify joint priors  
Conjecture price mappings  
\( P : \{S^1, \ldots, S^I, u\} \rightarrow \mathbb{R}_+^J \) | Specify joint priors  
Conjecture strategy profiles |
| **Step 2** | Derive posteriors | Derive posteriors |
| **Step 3** | Derive individual demand | Derive best response |
| **Step 4** | Impose market clearing | Impose Rationality |
| **Step 5** | Impose Rationality  
Equate undet. coeff. | No-one deviates |
A little more abstract

- **REE**
  Fixed Point of Mapping: $M_P(P(\cdot)) \mapsto P(\cdot)$

- **BNE** (simultaneous moves)
  Fixed Point of Mapping:
  strategy profiles $\mapsto$ strategy profiles

- What’s different for sequential move games?
  - late movers react to deviation
  - equilibrium might rely on ‘strange’ out of equilibrium moves
  - refinement: subgame perfection

- Extensive form move games with asymmetric information
  - Sequential equilibrium (agents act sequentially rational)
  - Perfect BNE (for certain games)
    - nature makes a move in the beginning (chooses type)
    - action of agents are observable
A Classification of Market Microstructure Models

- simultaneous submission of demand schedules
  - competitive rational expectation models
  - strategic share auctions
- sequential move models
  - screening models: (uninformed) market maker submits a supply schedule first
    - static
      - uniform price setting
      - limit order book analysis
  - dynamic sequential trade models with multiple trading rounds
- signalling models: informed traders move first, market maker second