Debt as Safe Asset: Mining the Bubble

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Questions of our times

- How much government debt can the market absorb?
- At what interest rate?
- Is there a limit, a “Debt Laffer Curve”?
- What is the impact on inflation?
- When can governments run a deficit without ever paying back its debt, like a Ponzi scheme, and nevertheless individual citizens’ transversality conditions hold?
- What is a safe asset? What are its features? Retrading?
- Why is government debt a safe asset?
- When do you lose safe asset status?
- Why is there debt valuation puzzle for US, Japanese?
- How do we have to modify representative agent asset pricing and the FTPL?
Valuating Government Debt

- Think of a representative agent holding all gov. debt
  - His cash flow is primary surplus
  \[ B_t = \frac{\mathbb{E}_t}{\gamma_t} \int_t^\infty \frac{\xi_s}{\xi_t} (T_s - G_s) ds \quad (\xi_t = \text{SDF}) \]
  - ... but Japan primary surplus was negative for 50 out of 60 years
  - Can surpluses be negative forever? Yes, if gov. debt is safe asset

Japan: Govt primary balance
What’s a Safe Asset?

- Asset Price = E[PV(cash flows)] + E[PV(service flows)]
  
  dividends/interest
  convenience yield
What’s a Safe Asset?

- Asset Price = \( E[\text{PV(cash flows)}] + E[\text{PV(service flows)}] \)
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  - convenience yield

Portfolio of Safe asset

Cash flow asset

<table>
<thead>
<tr>
<th>Portfolio of</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cash flow asset</td>
<td>CF</td>
<td>CF</td>
</tr>
</tbody>
</table>

shocks

0

CF

0

CF

0

CF

0

CF

0

CF

0

CF

0

CF

0

CF

...
What’s a Safe Asset?

- Asset Price = \( E[\text{PV(cash flows)}] + E[\text{PV(service flows)}] \)
  
  dividends/interest  convenience yield

- Value come from **re-trading**

A

0

CF

B

0

CF

...
What’s a Safe Asset?

- **Asset Price** = \( E[PV(cash \ flows)] + E[PV(service \ flows)] \)
  
  dividends/interest  
  convenience yield

- Value come from **re-trading**
- Insures by partially completing markets

- Can be "bubbly" = fragile

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Diagram:

- A
  - 0
  - CF
- B
  - 0
  - CF

---

...
Safe Asset Pricing Equation, 2 $\beta$s, Fragility

- Asset Price = $E[\text{PV(cash flows)}] + E[\text{PV(service flows)}]$
  
  - dividends/interest $\beta^c = 0$
  - convenience yield $\beta^s < 0$

1. Good friend analogy (Brunnermeier Haddad, 2012)
   - When one needs funds, one can sell at stable price... since others buy
     - Idiosyncratic shock:
       - Partial insurance through retrading - low bid-ask spread
     - Aggregate (volatility) shock:
       - Appreciate in value – negative $\beta = \omega\beta^c + (1 - \omega)\beta^s < 0$

2. Safe Asset Tautology
   - Safe asset is a bubble from aggregate perspective - fragility
     - Other service flows: collateral constraint, double-coincidence of wants
Model with Capital + Safe Asset

- Each heterogenous citizen $\tilde{i} \in [0,1]$
  
  \[
  E \left[ \int_0^\infty e^{-\rho t} \log c_t^{\tilde{i}} \, dt \right] \text{ s.t. } \frac{dn_t^{\tilde{i}}}{n_t^{\tilde{i}}} = -\frac{c_t^{\tilde{i}}}{n_t^{\tilde{i}}} \, dt + dr_t^B + \left(1 - \theta_t^{\tilde{i}}\right) \left(dr_t^{K,\tilde{i}}(i_t^{\tilde{i}}) - dr_t^B\right)
  \]

- Each citizen operates one firm
  - Output
    \[ y_t^{\tilde{i}} = a_t k_t^{\tilde{i}} \]
  - Physical capital
    \[ k_t^{\tilde{i}} \]
  - \[ \frac{dk_t^{\tilde{i}}}{k_t^{\tilde{i}}} = (\Phi(i_t^{\tilde{i}}) - \delta)dt + \bar{\sigma}_t d\tilde{Z}_t^{\tilde{i}} \]
Model with Capital + Safe Asset

- Each heterogenous citizen $\tilde{i} \in [0,1]$
  
  \[ E \left[ \int_0^\infty e^{-\rho t} \log c_t^\tilde{i} \, dt \right] \text{ s.t. } \frac{dn_t^\tilde{i}}{n_t^\tilde{i}} = -\frac{c_t^\tilde{i}}{n_t^\tilde{i}} \, dt + dr_t^B + (1 - \theta_t^\tilde{i}) \left( dr_t^{K,i}(i_t^\tilde{i}) - dr_t^B \right) \]

- Each citizen operates one firm
  - Output
    \[ y_t^\tilde{i} = a_t k_t^\tilde{i} \]
  - Physical capital
    \[ k_t^\tilde{i} \]
  - Change in physical capital
    \[ \frac{dk_t^\tilde{i}}{k_t^\tilde{i}} = (\Phi(i_t^\tilde{i}) - \delta) \, dt + \tilde{\sigma}_t \, d\tilde{Z}_t \]

- Aggregate risk:
  - $\tilde{\sigma}_t, a_t, g_t$ exogenous process with aggregate shock $dZ_t$

- Financial Friction: Incomplete markets:
  citizens cannot trade claims on $d\tilde{Z}_t$
Taxes, Bond/Money Supply, Gov. Budget

- **Government policy Instruments**
  - Government spending $g_t K_t$
  - Proportional tax $\tau_t k_t$ on capital
  - Nominal government debt supply
    \[
    \frac{d B_t}{B_t} = \mu_t^B dt
    \]
- Nominal interest rate $i_t$
- **Government budget constraint (BC)**
  \[
  (\mu_t^B - i_t) B_t + g_t K_t (\tau_t - g_t) = 0
  \]
- **Assume here:**
  - Gov. chooses $\mu^B, i$; while $\tau_t$ adjusts to satisfy (BC)
- **Goods market clearing:**
  \[
  C_t + g_t K_t = (a_t - i_t) K_t
  \]
  Let $\bar{a}_t := a_t - g_t$
Real prices and returns

- \( q_t^K K_t \) value of physical capital
  
  \[
  \text{Return } dr_t^{K,i} = \left( \frac{a(1-\tau) - \frac{1}{q_t^K} i_t}{K_t} + \Phi(i_t) - \delta + \mu_t q^K \right) dt + \sigma_t q^K dZ_t + \tilde{\sigma}_t d\tilde{Z}_t
  \]

- Dividend Yield
- Capital gains

- \( q_t^B K_t \) real value of gov. debt
  
  \[
  \frac{B_t}{\phi_t} = q_t^B K_t
  \]

- Inflow (outflow) from selling (buying) bond
- Reduces (increases) future payoffs

- \( \tilde{\iota}'s \) dynamic trading strategy of gov. bond
  
  \[
  \text{Return } dr_t^B = \left( i - \mu_t^B + \Phi(i_t) - \delta - \mu_t q^B \right) dt + \sigma_t q^B dZ_t
  \]

- Dividend Yield
- Capital gains

- Inflow (outflow) from selling (buying) bond
- Reduces (increases) future payoffs
Optimality and market clearings

- Optimal real investment rate $\iota_t$: (Tobin’s q)

- Optimal consumption: $c_t = \rho n_t$

- Optimal portfolio choice: $1 - \theta_t = \frac{(a_t - \iota_t)/q_t^K + \bar{\mu}^B}{\gamma \sigma_t^2} = 1 - \vartheta_t$
### Two Stationary Equilibria (for $K_0 = 1$)

<table>
<thead>
<tr>
<th>Gordon-Growth Formula</th>
<th>Closed Form Solution</th>
</tr>
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<tbody>
<tr>
<td>$q^K = \frac{(1-\tau)\bar{a} - \lambda}{E[dr^K]/dt - g}$</td>
<td>$q^K = \frac{\sqrt{\rho + \bar{\mu}^B} (1 + \phi \bar{a})}{\sqrt{\rho + \bar{\mu}^B + \phi \bar{\sigma} \rho}}$</td>
</tr>
<tr>
<td>$\frac{B}{\phi} = \frac{s}{E[dr^m]/dt - g} + \frac{(1 - \theta)^2 \bar{\sigma}^2 B}{\bar{\phi}}$</td>
<td>$q^K B K_t = \frac{\left( \bar{\sigma} - \sqrt{\rho + \bar{\mu}^B} \right) (1 + \phi \bar{a})}{\sqrt{\rho + \bar{\mu}^B + \phi \bar{\sigma} \rho}} K_t$</td>
</tr>
<tr>
<td>$\frac{\tilde{\sigma}}{\rho} = \frac{a \sqrt{\rho + \bar{\mu}^B - \bar{\sigma} \rho}}{\sqrt{\rho + \bar{\mu}^B + \phi \bar{\sigma} \rho}}$</td>
<td>$t = \frac{a \sqrt{\rho + \bar{\mu}^B - \bar{\sigma} \rho}}{\sqrt{\rho + \bar{\mu}^B + \phi \bar{\sigma} \rho}}$</td>
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$$dr^m = \theta dr^B + (1 - \theta) dr^K$$

- $\rho$ time preference rate
- $\phi$ adjustment cost for investment rate
- $\tilde{\mu}_t^B = \mu_t^B - i$ bond issuance rate beyond interest rate
- $\bar{a} = a - g$ part of TFP not spend on gov.)
Safe Asset Valuation Equation: 2 Perspectives

- **Individual Perspective**
  \[ \frac{d\xi_t}{\xi_t} = -r_t^f dt - \varsigma_t dZ_t - \varsigma_t \tilde{d}Z_t \]
  - Bond as part of a dynamic trading strategy
    - Cash flow from selling (buying) after negative (positive) idiosyncratic shock

- **Aggregate Perspective**
  \[ d\bar{\xi}_t/\bar{\xi}_t = -r_t^f dt - \varsigma_t dZ_t \]

- \( \eta_t i \) and \( \xi i \) are negatively correlated \( \Rightarrow \) depresses weighted SDF (higher discount rate)

- Without aggregate risk 
  \( \bar{\xi}_t = e^{-r_f t} \)

- Lower social discount rate + Bubble term

Safe Asset Valuation Equation: 2 Perspectives
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- **Individual Perspective**
  \[ \frac{d \xi^i_t}{\xi^i_t} = -r_t^f dt - \zeta_t dZ_t - \tilde{\zeta}_t^i d\tilde{Z}_t^i \]
  - Bond as part of a dynamic trading strategy
    - Cash flow from selling (buying) after negative (positive) idiosyncratic shock
    - Price “bond-part” of portfolio
    - Integrate over citizens weighted by net worth share \( \eta^i_t \)
      - \( \xi^i \) and \( \eta^i \) are negatively correlated \( \Rightarrow \) depresses weighted SDF
        - (higher discount rate) \( E[dr^n]/dt = r_f^f + \zeta \sigma + \tilde{\zeta} \sigma \)

- **Aggregate Perspective**
  \[ \frac{d \bar{\xi}_t}{\bar{\xi}_t} = -r_t dt - \bar{\zeta}_t d\bar{Z}_t \]
  - Without aggregate risk
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Safe Asset Valuation Equation:

\[
\frac{B_0}{P_0} = \mathbb{E} \left[ \int_0^\infty \left( \int \tilde{\xi}^i \eta^i dt \right) s_t K_t dt \right] + \mathbb{E} \left[ \int_0^\infty \left( \int \tilde{\xi}^i \eta^i dt \right) (1 - \tilde{\eta}_t) \tilde{\sigma}^2 \frac{B_t}{P_t} dt \right].
\]

\[
\frac{s}{\rho + g} = \frac{s}{E[dr^n]/dt - g} + \frac{(1 - \tilde{\eta})^2 \tilde{\sigma}^2 B_t}{E[dr^n]/dt - g}
\]
Safe Asset Valuation Equation: 2 Perspectives

- **Individual Perspective**
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    - \( \xi^i \) and \( \eta^i \) are negatively correlated \( \Rightarrow \) depresses weighted SDF
    - (higher discount rate) \( E[d r^n]/dt = r^f + \varsigma \sigma + \dot{\varsigma} \)

\[ \frac{B_0}{P_0} = \mathbb{E} \left[ \int_0^\infty \left( \int \xi_t^i \eta_t^i di \right) s_t K_t dt \right] + \mathbb{E} \left[ \int_0^\infty \left( \int \xi_t^i \eta_t^i di \right) (1 - \dot{\theta}_t)^2 \dot{\sigma}_t^2 B_t/\tilde{P}_t dt \right] \]

- **Aggregate Perspective**
  \[ \frac{d\bar{\xi}_t}{\bar{\xi}_t} = -r^f_t dt - \zeta_t dZ_t \]
  - Without aggregate risk \( \bar{\xi}_t = e^{-r^f t} \)
  - Lower social discount rate + Bubble term

\[ \frac{B_0}{P_0} = \mathbb{E} \left[ \int_0^T \bar{\xi}_t s_t K_t dt \right] + \mathbb{E} \left[ \frac{\bar{\xi}_T}{\bar{P}_T} \right] \]

\[ \frac{\bar{B}}{\bar{P}} = \frac{s}{E[dr^n]/dt - g} + \frac{(1 - \theta)^2 \dot{\sigma}^2 \bar{B}}{E[dr^n]/dt - g} \]

\[ \rho + g = \text{discount rate} \]

\[ \frac{\bar{B}}{\bar{P}} = \frac{s}{r^f - g} \]

\[ g - \mu_B = \text{discount rate} \]
Bubble/Ponzi Scheme and **Transversality**

- **Gov. Debt is a Ponzi scheme/bubble (in aggregate perspective)**
  - Service flow – partial insurance to overcome market incompleteness

- Why does transversality condition not rule out the bubble?
  - **Individual Perspective**
    - High individual discount rate (low SDF) since net worth
    
    \[
    \lim_{T \to \infty} E[\xi_T n_T^i] = 0
    \]

  - **Aggregate perspective**
    - Low “social” discount rate (high SDF)
    
    \[
    \lim_{T \to \infty} E[\bar{\xi}_T n_T^i] > 0
    \]
\( r^f \) versus \( g \) for different \( \tilde{\mu}^B \)

- When primary deficit forever \( s < 0 \ \forall t \iff \tilde{\mu}^B > 0? \) Japan?
  - Higher issuance rate \( \Rightarrow \) higher inflation tax \( \Rightarrow \) lower real return \( \Rightarrow r^f < g \)

\[
g = \frac{1}{\phi} \log \left( \frac{\sqrt{\rho + \tilde{\mu}^B} (1 + \phi a)}{\sqrt{\rho + \tilde{\mu}^B} + \phi \tilde{\sigma} \rho} \right) - \delta
\]

\[
r^f = \frac{\Phi(i) - \delta - \tilde{\mu}^B}{g}
\]

\( a = .27, g = \frac{a}{3}, \delta = .1, \rho = .02, \tilde{\sigma} = .25, \phi = 3 \)
- Higher issuance rate, $\ddot{\mu}^B \Rightarrow$ higher inflation tax
- But real value of bonds, $\frac{B}{\sigma'}$, declines $\Rightarrow$ lower “tax base”
**Flight to Safety**: Comparative static w.r.t. $\tilde{\sigma}$

- Flight to safety into bubbly gov. debt
  - $q^B$ rises (disinflation)
  - $q^K$ falls and so does $\iota$ and $g$

- Similar with stochastic idiosyncratic volatility
**Countercyclical Safe Asset**

- **Aggregate risk state variable:**
  - Stochastic idiosyncratic volatility: 
    \[ d \log \tilde{\sigma}_t = -\psi \log \frac{\tilde{\sigma}_t}{\bar{\sigma}_0} dt + \sigma^x dZ_t \]
  - Stochastic TFP:
    \[ a_t = a(\tilde{\sigma}_t) \text{ s.t. } \frac{C}{K}(\tilde{\sigma}_t) = \alpha^0 - \alpha^1 \tilde{\sigma}_t \text{ linear} \]

- **Policy (surpluses decrease in }\tilde{\sigma}_t):**
  \[ \ddot{\mu}_t^B = -\nu_0 + \nu_1 \tilde{\sigma}_t \]

- **Individual perspective:**
  - 2 terms of valuation equation
    - cash flow term around 0
    - safe asset service flow term dominates
Countercyclical Safe Asset – 2 Betas

- $\beta^{B,cf} > 0$ for cash flow term (primary surplus term)
- $\beta^{B,sf} < 0$ for service flow term (due to risk sharing)
Loss of Safe Asset Status

- Bubbles can pop

- Able to prop up the bubble/safe-asset status by (off-equilibrium) hiking taxes (fiscal space)

- Market maker of last resort to secure low bid-ask spread
  - 10 year US Treasury in March 2020

- Competing safe asset
  - Interest rate policy of competing central banks
  - “least ugly horse”
Conclusion

- **Asset Pricing**
  - Safe asset is different – provides service flow
    - Risk sharing via precautionary saving and constant retrading
  - 2 terms: cash flow + service flow
    - Split depends on perspective (individual vs. aggregate)
      - different discount rates
      - $2 \beta$s
  - Flight to safety creates countercyclical Safe Asset Valuations
    - negative $\beta$

- **Bubble mining for government**
  - Negative primary surpluses for decades (like in Japan)
  - But has its limits (unlike MMT)

- **Bubbles can pop: Loss of flight to safe asset status**
  - Fiscal capacity to fend off + Market maker of last resort