# Debt as Safe Asset: Mining the Bubble

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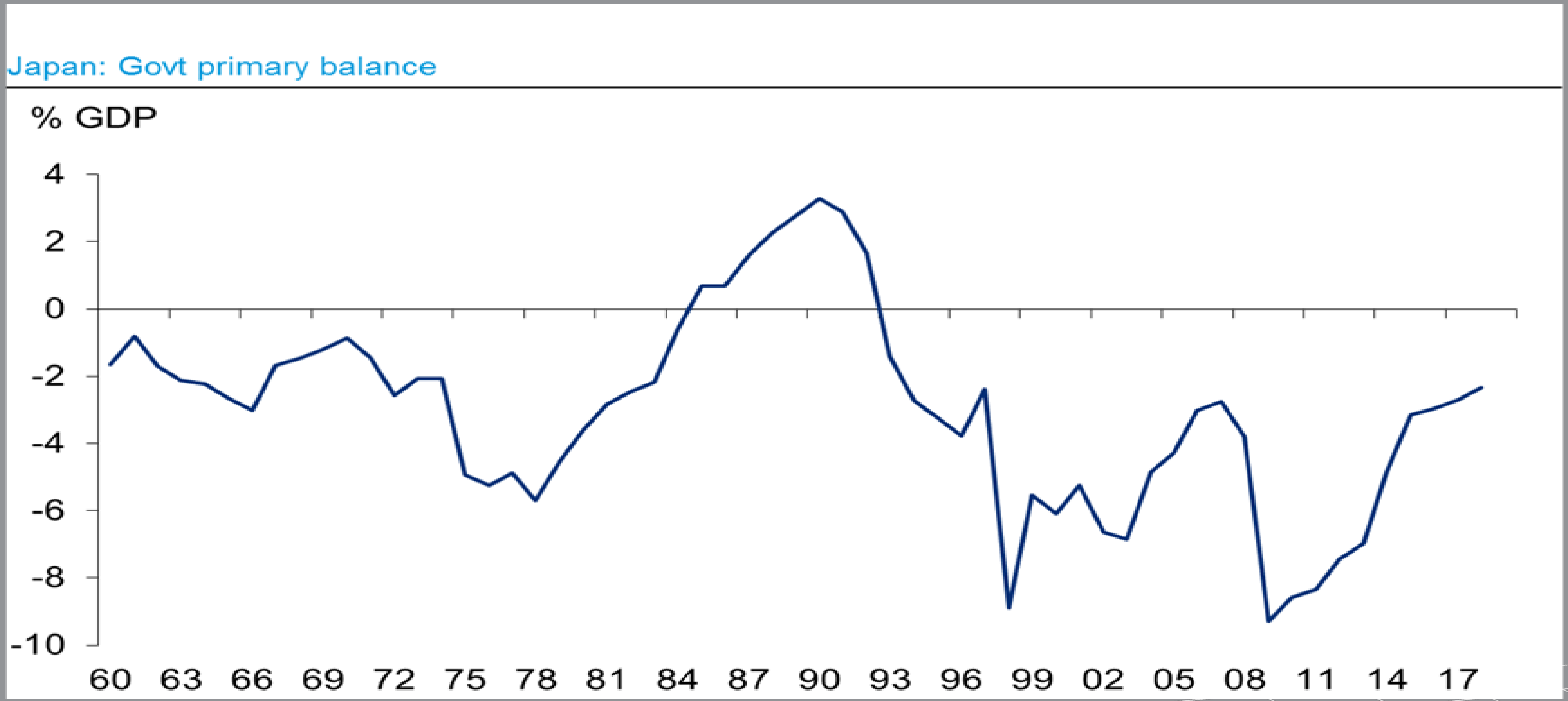
# Questions of our times

- How much government debt can the market absorb?
- At what interest rate?
- Is there a limit, a “Debt Laffer Curve”?
- What is the impact on inflation?
- When can governments run a deficit without ever pa-ying back its debt, like a Ponzi scheme, and nevertheless individual citizens’ transversality conditions hold?
- What is a safe asset? What are its features? Retrading?
- Why is government debt a safe asset?
- When do you lose safe asset status?
- Why is there debt valuation puzzle for US, Japanese?
- How do we have to modify representative agent asset pricing and the FTPL?

# Valuating Government Debt

- Think of a representative agent holding all gov. debt
  - His cash flow is primary surplus
  - $\frac{\mathcal{B}_t}{\wp_t} = E_t \int_t^\infty \frac{\xi_s}{\xi_t} (T_s - G_s) ds$ ($\xi_t$= SDF)
  - … but Japan primary surplus was negative for 50 out of 60 years
  - Can surpluses be negative forever? Yes, if gov. debt is safe asset



# What's a Safe Asset?

- Asset Price = E[PV(cash flows)] + E[PV(service flows)]

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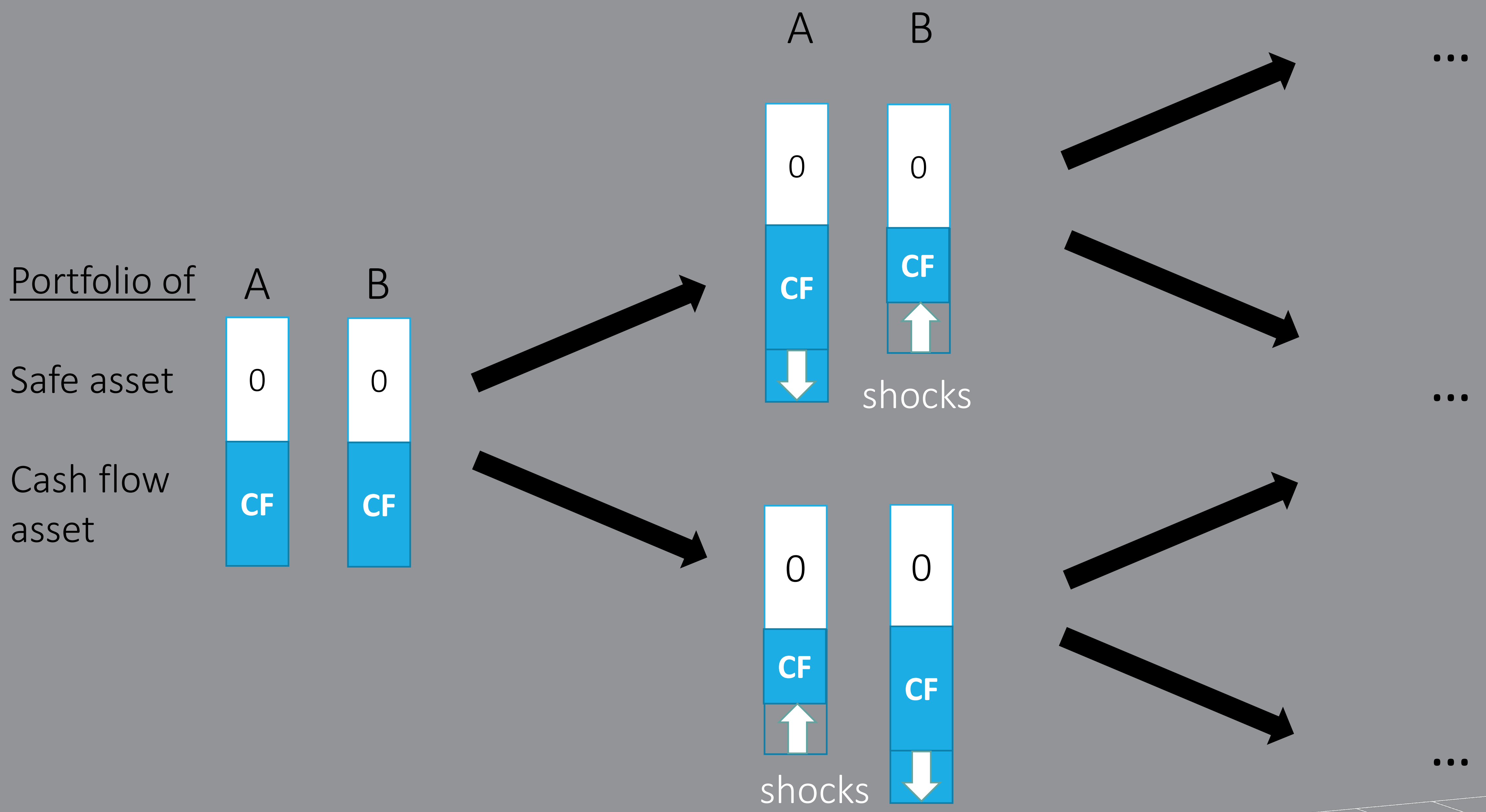

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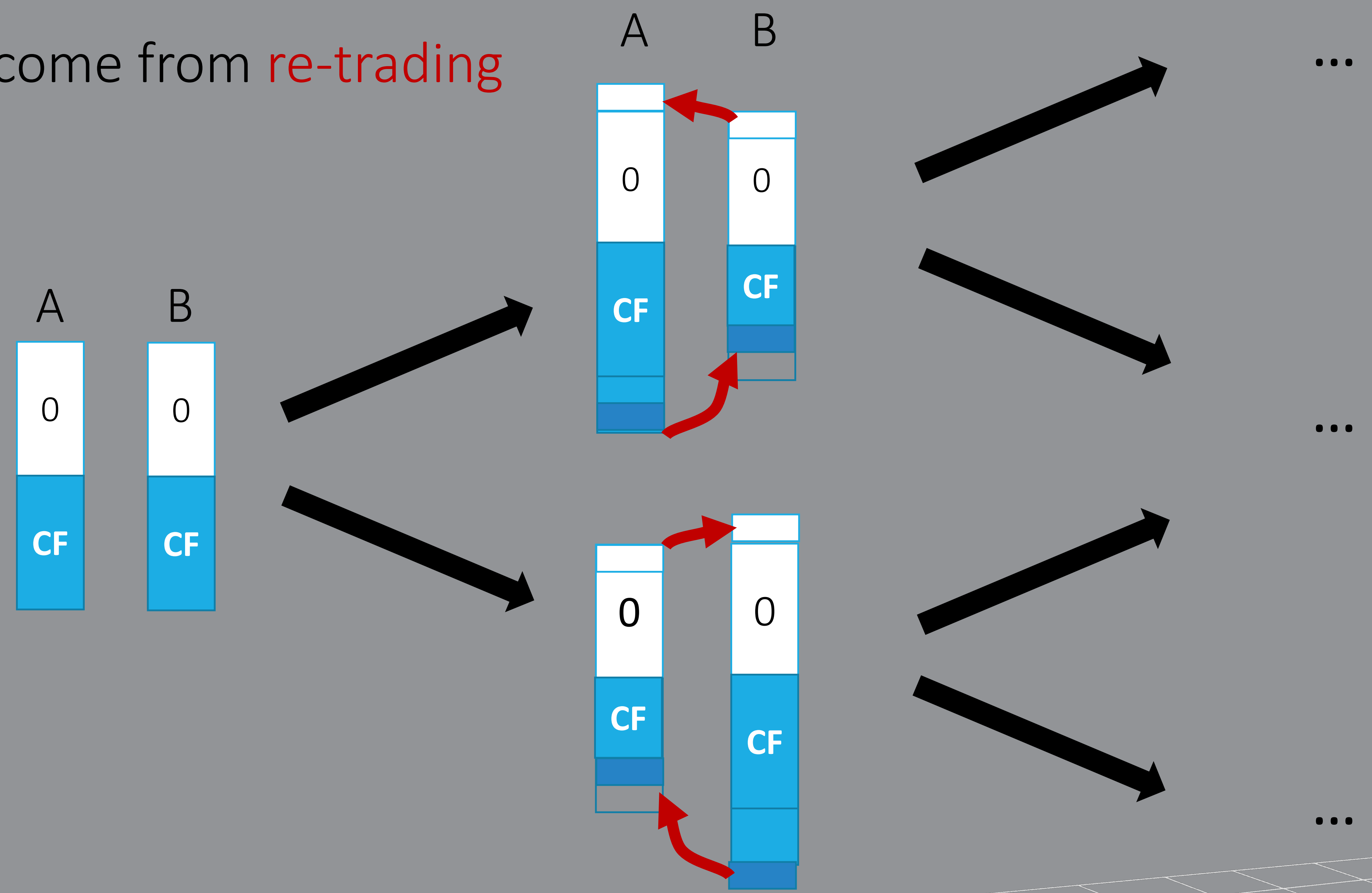

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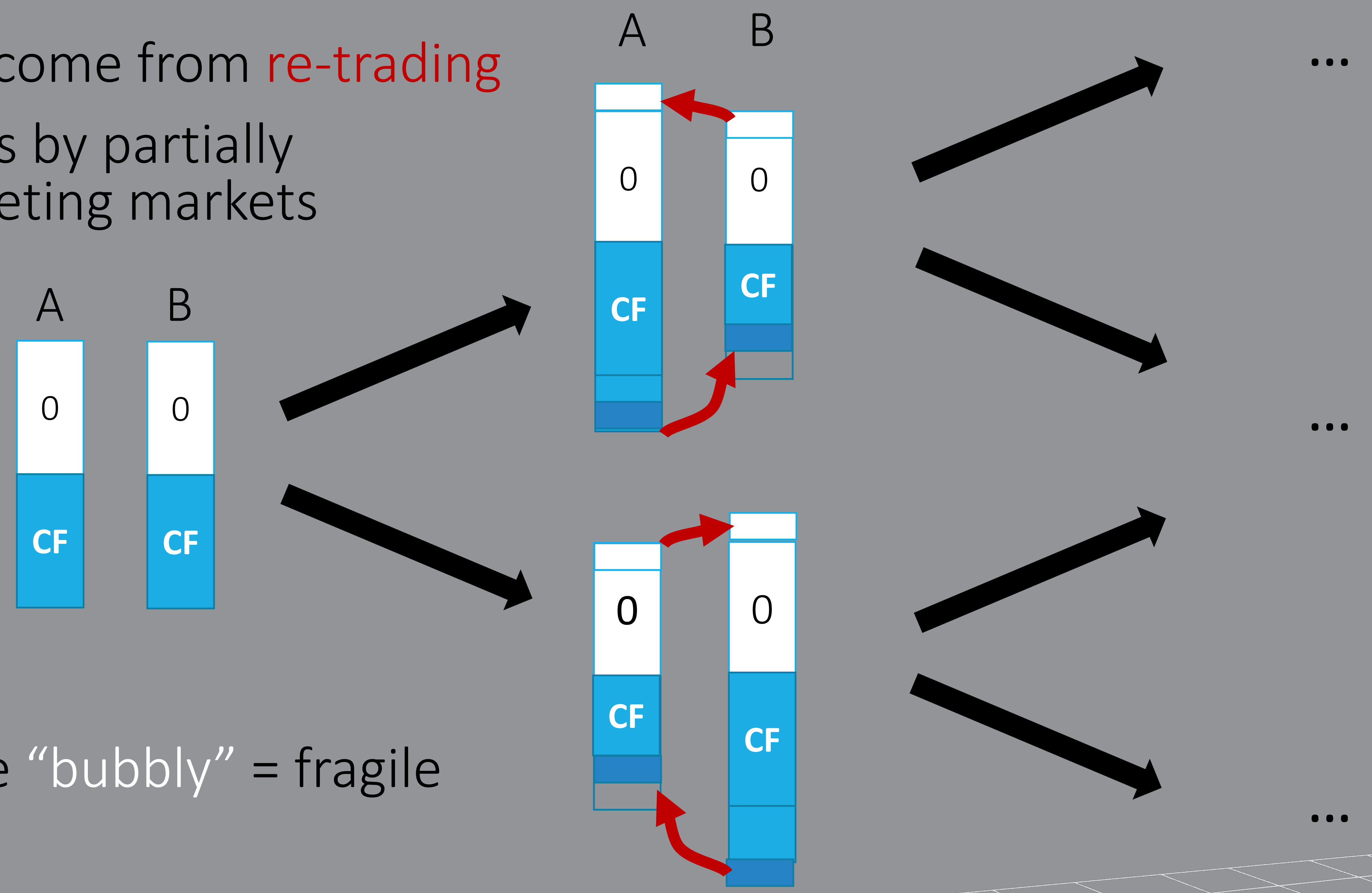
- Value come from re-trading
- Insures by partially completing markets




- Can be “bubbly” = fragile

# Safe Asset Pricing Equation, 2 $\beta s$, Fragility

- Asset Price = E[PV(cash flows)] + E[PV(service flows)]

  dividends/interest — convenience yield

  - 2 $\beta$s — $\beta^{cf} > 0$ — $\beta^{sf} < 0$

1. Good friend analogy (Brunnermeier Haddad, 2012)
   - When one needs funds, one can sell at stable price… since others buy
     - **Idiosyncratic shock:**
       - Partial insurance through retrading - low bid-ask spread
     - **Aggregate (volatility) shock:**
       - Appreciate in value – negative $\beta = \omega\beta^{cf} + (1-\omega)\beta^{sf} < 0$
2. Safe Asset Tautology
   - Safe asset is a bubble from aggregate perspective - fragility

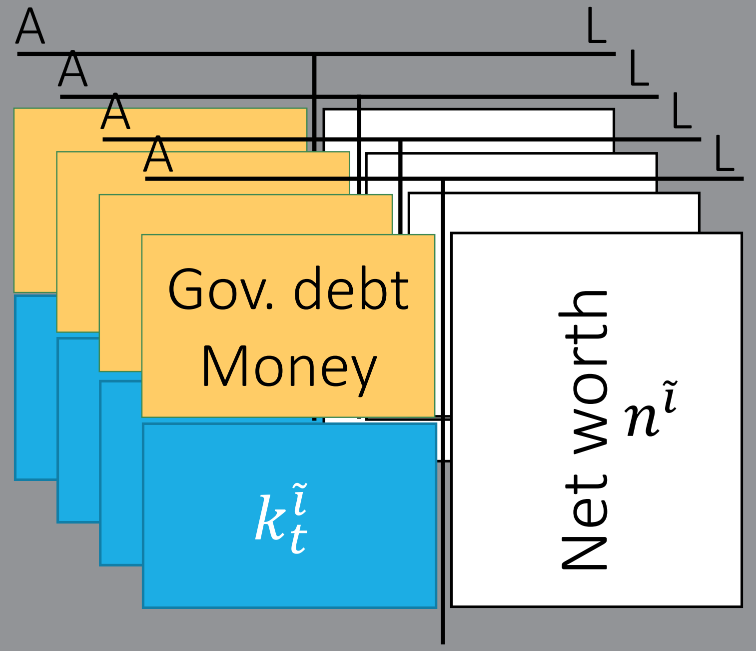
- Other service flows: collateral constraint, double-coincidence of wants

# Model with Capital + Safe Asset

- Each heterogenous citizen $\tilde{\imath} \in [0,1]$

$$E\left[\int_0^\infty e^{-\rho t} \log c_t^{\tilde{\imath}}\, dt\right] \text{ s.t.} \frac{dn_t^{\tilde{\imath}}}{n_t^{\tilde{\imath}}} = -\frac{c_t^{\tilde{\imath}}}{n_t^{\tilde{\imath}}} dt + dr_t^{\mathcal{B}} + (1-\theta_t^{\tilde{\imath}})\left(dr_t^{K,\tilde{\imath}}(\iota_t^{\tilde{\imath}}) - dr_t^{\mathcal{B}}\right)$$

- Each citizen operates one firm
  - Output $y_t^{\tilde{\imath}} = a_t k_t^{\tilde{\imath}}$
  - Physical capital $k_t^{\tilde{\imath}}$
  - $\frac{dk_t^{\tilde{\imath}}}{k_t^{\tilde{\imath}}} = (\Phi(\iota_t^{\tilde{\imath}}) - \delta)dt + \tilde{\sigma}_t d\tilde{Z}_t^{\tilde{\imath}}$



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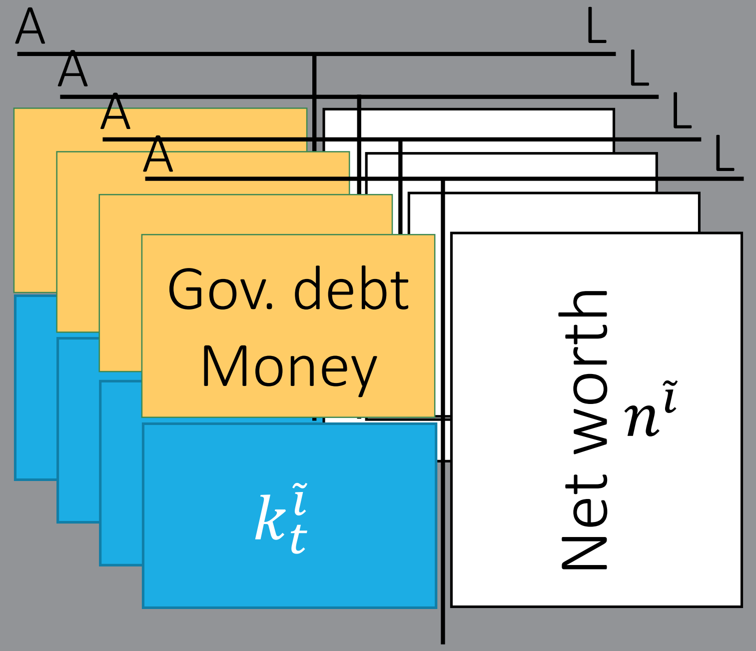


- Aggregate risk:
  $\tilde{\sigma}_t$ , $a_t$, $g_t$ exogenous process with aggregate shock $dZ_t$

- Financial Friction: Incomplete markets:
  citizens cannot trade claims on $d\tilde{Z}_t^{\tilde{\imath}}$

# Taxes, Bond/Money Supply, Gov. Budget

- Government policy Instruments
  - Government spending $\mathcal{g}_t K_t$
  - Proportional tax $\tau_t k_t$ on capital
  - Nominal government debt supply

$$\frac{d\mathcal{B}_t}{\mathcal{B}_t} = \mu_t^{\mathcal{B}} dt$$

  - Nominal interest rate $i_t$

- Government budget constraint (BC)

$$\underbrace{\left(\mu_t^{\mathcal{B}} - i_t\right)}_{\breve{\mu}_t^{B} :=} \mathcal{B}_t + \wp_t K_t \underbrace{(\tau_t - \mathcal{g}_t)}_{\substack{s_t := \\ \text{Primary surplus (per } K_t\text{)}}} = 0$$

- Assume here:
  - Gov. chooses $\mu^{\mathcal{B}}$, $i$; while $\tau_t$ adjusts to satisfy (BC)
- Goods market clearing:

$$C_t + \mathcal{g}_t K_t = (a_t - \iota_t) K_t$$

Let $\breve{a}_t := a_t - \mathcal{g}_t$

# Real prices and returns

- $q_t^K K_t$ value of physical capital
  - Return $dr_t^{K,\tilde{i}} = \Big(\underbrace{\frac{a(1-\tau)-\iota_t^{\tilde{i}}}{q_t^K}}_{\text{Dividend Yield}} + \underbrace{\Phi(\iota_t^{\tilde{i}}) - \delta + \mu_t^{q^K}}_{\text{Capital gains}}\Big) dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^{\tilde{i}}$
- $q_t^B K_t$ real value of gov. debt
  - $\mathcal{B}_t / \wp_t = q_t^B K_t$
  - Return $dr_t^B = (\overbrace{\underbrace{i - \mu_t^{\mathcal{B}}}_{-\breve{\mu}_t^{\mathcal{B}}} + \underbrace{\Phi(\iota_t) - \delta}_{g=} - \mu_t^{q^B}}^{-\text{ inflation}})dt + \sigma_t^{q^B} dZ_t$
- $\tilde{\iota}$'s dynamic trading strategy of gov. bond
  - Inflow (outflow) from selling (buying) bond
  - Reduces (increases) future payoffs

# Optimality and market clearings

- Optimal real investment rate $\iota_t$: (Tobin's q)

- Optimal consumption: $c_t = \rho n_t$

- Optimal portfolio choice: $1 - \theta_t = \frac{(a_t - \iota_t)/q_t^K + \breve{\mu}^B}{\gamma \tilde{\sigma}_t{}^2} = 1 - \vartheta_t$

# Two Stationary Equilibria (for $K_0 = 1$)

| Gordon-Growth Formula | Closed Form Solution |
|---|---|
| $q^K = \frac{(1-\tau)\breve{a}-\iota}{E[dr^K]/dt-g}$ | $q^K = \frac{\sqrt{\rho+\breve{\mu}^B}\ (1+\phi\breve{a})}{\sqrt{\rho+\breve{\mu}^B}+\phi\tilde{\sigma}\rho}$ |
| $\frac{\mathcal{B}}{\wp} = \frac{s}{E[dr^n]/dt-g} + \frac{(1-\vartheta)^2\tilde{\sigma}^2\frac{\mathcal{B}}{\wp}}{E[dr^n]/dt-g}$ | $q^B K_t = \frac{\left(\tilde{\sigma}-\sqrt{\rho+\breve{\mu}^B}\right)(1+\phi\breve{a})}{\sqrt{\rho+\breve{\mu}^B}+\phi\tilde{\sigma}\rho}K_t$ |
| | $\iota = \frac{a\sqrt{\rho+\breve{\mu}^B}-\tilde{\sigma}\rho}{\sqrt{\rho+\breve{\mu}^B}+\phi\tilde{\sigma}\rho}$ |

$$dr^n = \theta dr^B + (1-\theta)dr^K$$

$\rho$ time preference rate
$\phi$ adjustment cost for investment rate
$\breve{\mu}_t^B = \mu_t^B - i$ bond issuance rate beyond interest rate
$\breve{a} = a - \mathfrak{g}$ part of TFP not spend on gov.)

# **Safe Asset Valuation Equation:** 2 Perspectives

- Individual Perspective $d\xi_t^{\tilde{\imath}}/\xi_t^{\tilde{\imath}} \quad = -r_t^f dt - \varsigma_t dZ_t - \tilde{\varsigma}_t^{\tilde{\imath}} d\tilde{Z}_t^{\tilde{\imath}}$
  - Bond as part of a dynamic trading strategy
    - Cash flow from selling (buying) after negative (positive) idiosyncratic shock

- Aggregate Perspective $d\bar{\xi}_t/\bar{\xi}_t = -r_t^f dt - \varsigma_t dZ_t$

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    - Price "bond-part" of portfolio
    - Integrate over citizens weighted by net worth share $\eta_t^i$
      - $\xi^i$ and $\eta^i$ are negatively correlated ⇒ depresses weighted SDF (higher discount rate) $E[dr^n]/dt = r^f + \varsigma\sigma + \tilde{\varsigma}\tilde{\sigma}$

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E}\left[\int_0^\infty \left(\int \xi_t^i \eta_t^i di\right) s_t K_t dt\right] + \mathbb{E}\left[\int_0^\infty \left(\int \xi_t^i \eta_t^i di\right) \underbrace{(1-\vartheta_t)^2 \tilde{\sigma}_t^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt}_{\text{"Partial insurance Service"}}\right].$$

$$\frac{\mathcal{B}}{\wp} = \frac{s}{E[dr^n]/dt - g} + \frac{(1-\vartheta)^2 \tilde{\sigma}^2 \frac{\mathcal{B}}{\wp}}{E[dr^n]/dt - g}$$

$\rho + g$ = discount rate

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"Partial insurance Service"

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$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E}\left[\int_0^T \bar{\xi}_t s_t K_t dt\right] + \mathbb{E}\left[\bar{\xi} \frac{\mathcal{B}_T}{\mathcal{P}_T}\right].$$

$r^f + \varsigma\sigma$

Only for $s>0$

$$\frac{\mathcal{B}}{\wp} = \frac{s}{r^f - g}$$

$g - \check{\mu}^B$ = discount rate

  - Without aggregate risk $\bar{\xi}_t = e^{-r^f t}$
  - Lower social discount rate + Bubble term

# Bubble/Ponzi Scheme and **Transversality**

- Gov. Debt is a Ponzi scheme/bubble (in aggregate perspective)
  - Service flow – partial insurance to overcome market incompleteness

- Why does transversality condition not rule out the bubble?
  - Individual Perspective
    High individual discount rate (low SDF) since net worth

$$\lim_{T\to\infty} E\left[\xi_T n_T^{\tilde{i}}\right] = 0$$

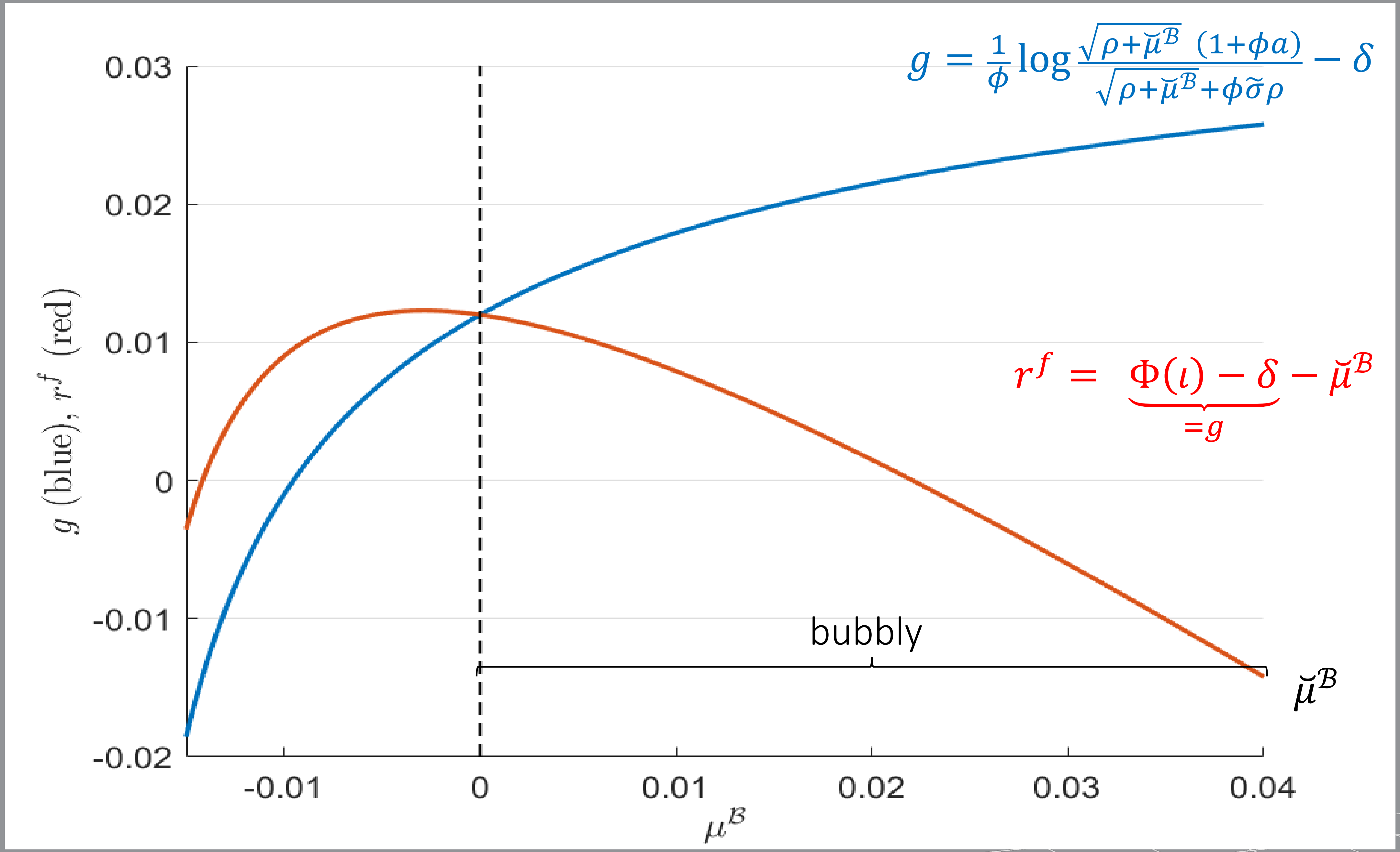
  - Aggregate perspective
    Low "social" discount rate (high SDF)

$$\lim_{T\to\infty} E\left[\bar{\xi}_T n_T^{\tilde{i}}\right] > 0$$

# $r^f$ versus $g$ for different $\breve{\mu}^{\mathcal{B}}$

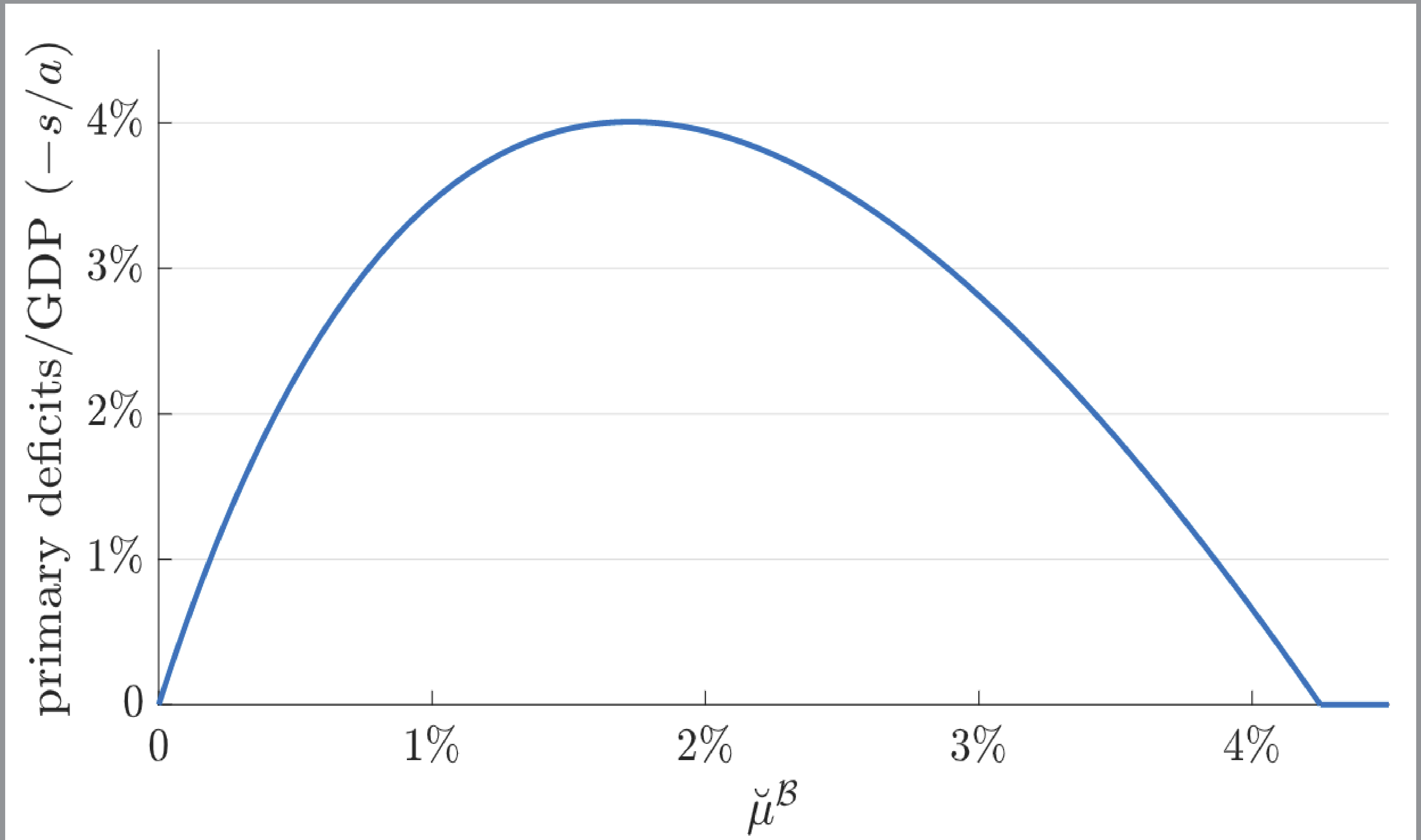
$a = .27, g = \frac{a}{3}, \delta = .1,$
$\rho = .02, \tilde{\sigma} = .25, \phi = 3$ ,

- When primary deficit forever $s < 0\ \forall t \Leftrightarrow \breve{\mu}^B > 0$? Japan?
  - Higher issuance rate ⇒ higher inflation tax ⇒ lower real return ⇒ $r^f < g$



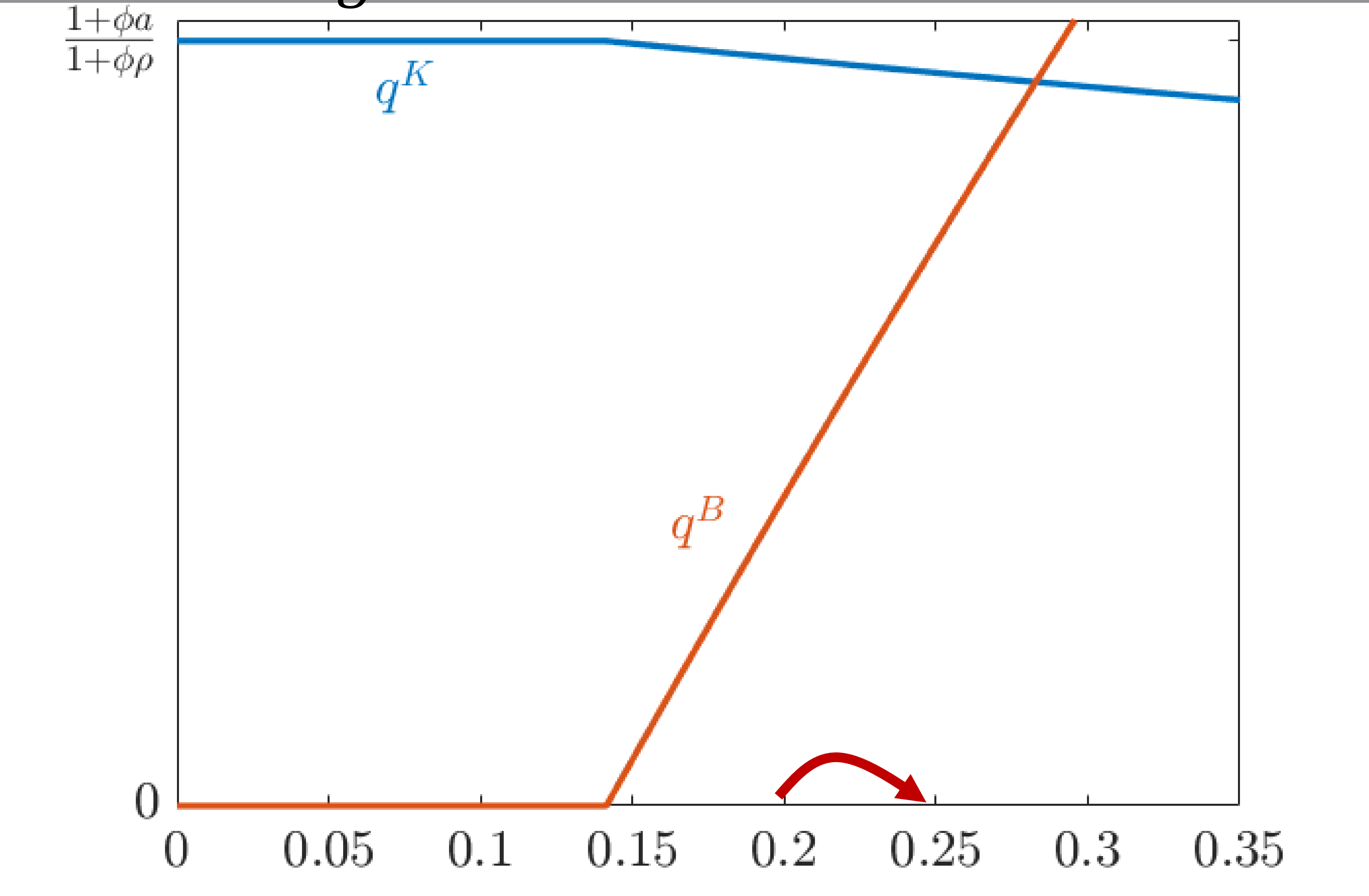
# Debt Laffer Curve

- Higher issuance rate, $\breve{\mu}^B$ ⇒ higher inflation tax
- But real value of bonds, $\frac{\mathcal{B}}{\wp}$, declines ⇒ lower "tax base"



# **Flight to Safety:** Comparative static w.r.t. $\tilde{\sigma}$

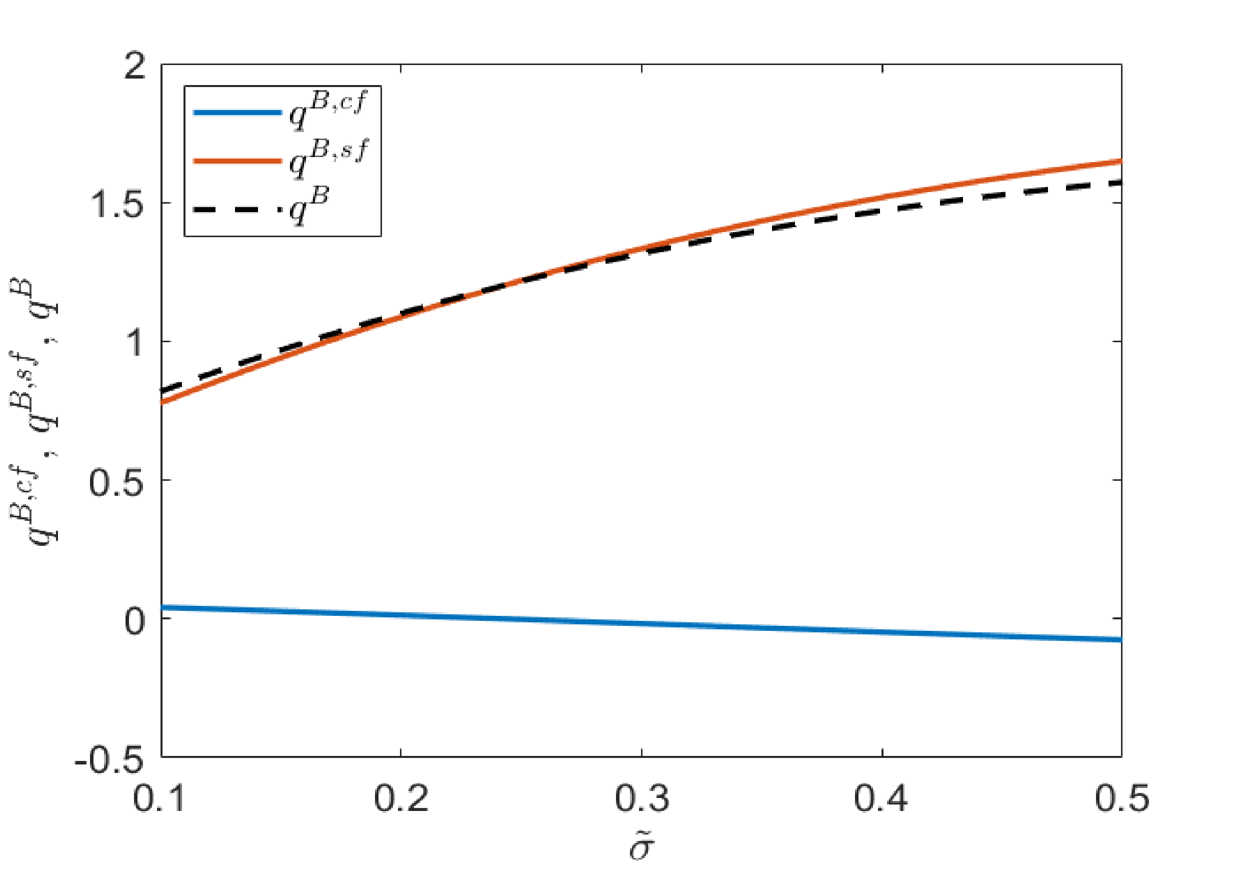
- Flight to safety into bubbly gov. debt
  - $q^B$ rises (disinflation)
  - $q^K$ falls and so does $\iota$ and $g$




- Similar with stochastic idiosyncratic volatility

# Countercyclical Safe Asset

- Aggregate risk state variable:
  - Stochastic idiosyncratic volatility: $d \log \tilde{\sigma}_t = -\psi \log \frac{\tilde{\sigma}_t}{\tilde{\sigma}^0} dt + \sigma^x dZ_t$
  - Stochastic TFP: $a_t = a(\tilde{\sigma}_t)$ s.t. $\frac{C}{K}(\tilde{\sigma}_t) = \alpha^0 - \alpha^1 \tilde{\sigma}_t$ linear
- Policy (surpluses decrease in $\tilde{\sigma}_t$): $\check{\mu}_t^{\mathcal{B}} = -\nu_0 + \nu_1 \tilde{\sigma}_t$
- Individual perspective:



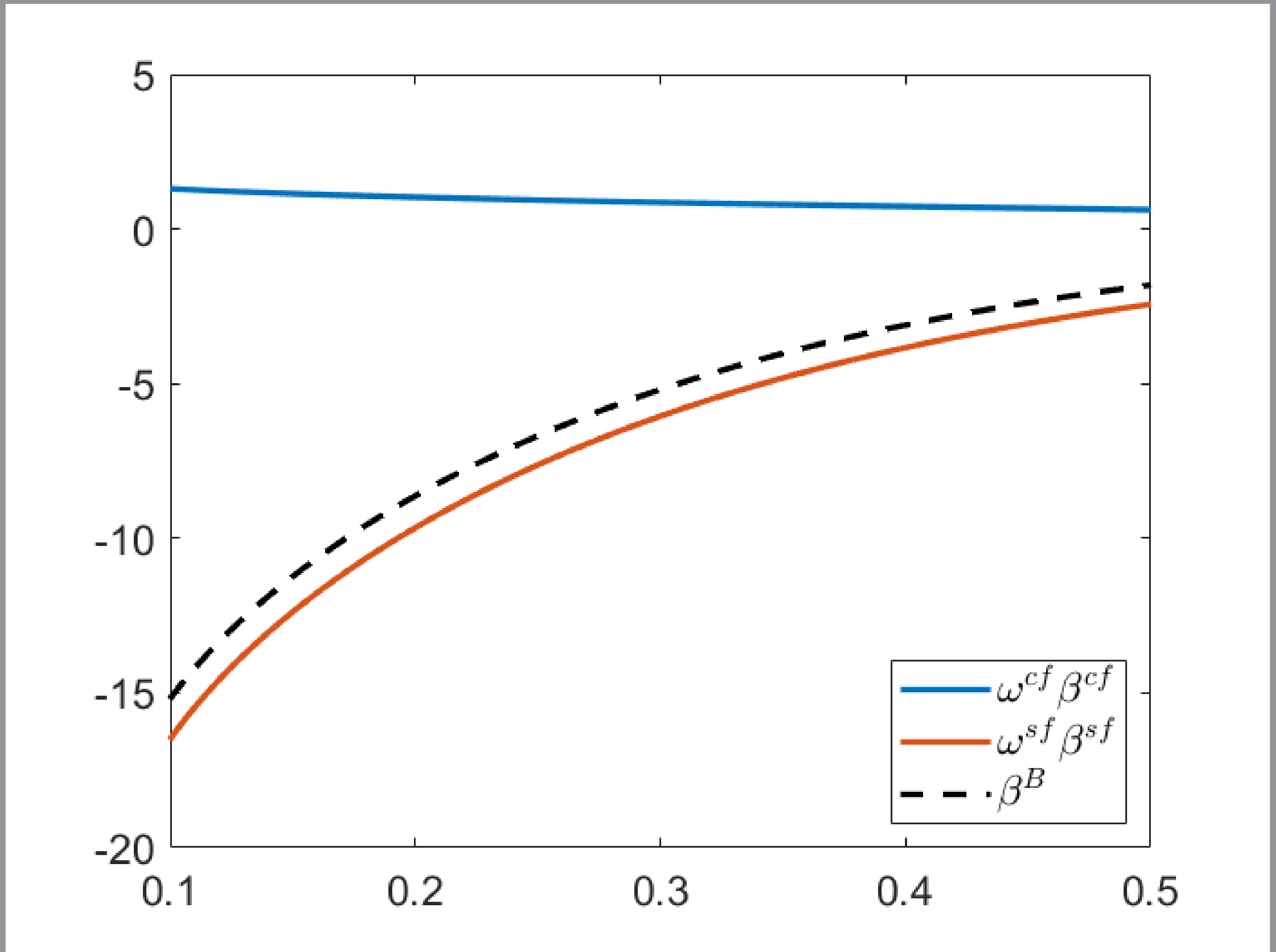

2 terms of valuation equation

- cash flow term around 0

- safe asset service flow term dominates

# Countercyclical Safe Asset – 2 Betas

- $\beta^{B,cf} > 0$ for cash flow term (primary surplus term)
- $\beta^{B,sf} < 0$ for service flow term (due to risk sharing)



# Loss of Safe Asset Status

- Bubbles can pop
- Able to prop up the bubble/safe-asset status by (off-equilibrium) hiking taxes (fiscal space)
- Market maker of last resort to secure low bid-ask spread
  - 10 year US Treasury in March 2020
- Competing safe asset
  - Interest rate policy of competing central banks
  - “least ugly horse”

# Conclusion

- Asset Pricing
  - Safe asset is different – provides service flow
    - Risk sharing via precautionary saving and constant retrading
  - 2 terms: cash flow + service flow
    - Split depends on perspective (individual vs. aggregate)
      - different discount rates
    - 2 $\beta s$
- Flight to safety creates countercyclical Safe Asset Valuations
  - negative $\beta$

- Bubble mining for government
  - Negative primary surpluses for decades (like in Japan)
  - But has its limits (unlike MMT)

- Bubbles can pop: Loss of flight to safe asset status
    - Fiscal capacity to fend off + Market maker of last resort