Macro-literature on Frictions

1. Net worth effects:
   a. Persistence: Carlstrom & Fuerst
   b. Amplification: Bernanke, Gertler & Gilchrist
   c. Instability: Brunnermeier & Sannikov

2. Volatility effects: impact credit quantity constraints
   a. Margin spirals: Brunnermeier & Pederson
   b. Endogenous constraints: Geanakoplos

3. Demand for liquid assets & Bubbles – “self insurance”
   a. OLG, Aiyagari, Bewley, Krusell-Smith, Holmstrom-Tirole, ...

4. Financial intermediaries & Theory of Money
Demand for Liquid Assets, Bubbles, ...

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Demand for Liquid Assets

- So far: Technological & market illiquidity → amplification
  - Liquidity spirals:
    - Depressed price, $q_t$, tightens debt constraint, which in turn ...
    - Higher volatility of $q_t$, tightens debt constraint, which in turn ...
- Now: “safety cushion” + self-insurance
- Focus on demand for liquid instruments
  - No amplification effects:
    - perfect techn. liquidity due to reversibility of investment
      - constant price of capital $q$
      - Borrowing constraint = collateral constraint
  - Steps: Introduce (i) idiosyncratic risk, (ii) aggregate risk, (iii) amplification (revisited)
Outline – Demand for Liquid Assets

- Deterministic Fluctuations
  - Overlapping generations
  - Completing markets with liquid asset

- Idiosyncratic Risk
  - Precautionary savings
  - Constrained efficiency

- Aggregate Risk
  - Bounded rationality

- Amplification Revisited
Samuelson (1958) considers an infinite-horizon economy with two-period lived overlapping agents
- Population growth rate $n$

Preferences given by $u(c^t_t, c^t_{t+1})$
- Pareto optimal allocation satisfies $\frac{u_1}{u_2} = 1 + n$

OLG economies have multiple equilibria that can be Pareto ranked
Assume $u(c_t^t, c_{t+1}^t) = \log c_t^t + \beta \log c_{t+1}^t$

- Endowment $y_t^t = e, y_{t+1}^t = 1 - e$

Assume complete markets and interest rate $r$

Agent’s FOC implies that $\frac{c_{t+1}^t}{\beta c_t^t} = 1 + r$

- For $r = n$, this corresponds to the Pareto solution
- For $r = \frac{1-e}{\beta e} - 1$, agents will consume their endowment

Autarky solution is clearly Pareto inferior
Autarky solution is the unique equilibrium implemented in a sequential exchange economy:
- Young agents cannot transfer wealth to next period.

A durable asset provides a store of value:
- Effective store of value reflects market liquidity.
- Pareto solution can be attained as a competitive equilibrium in which the price level grows at the same rate as the population, i.e. \( b_{t+1} = (1 + n)b_t \).
- Old agents trade durable asset for young agents’ consumption goods.
OLG: Production

- Diamond (1965) introduces a capital good and production
  - Constant-returns-to-scale production $Y_t = F(K_t, L_t)$
- Optimal level of capital is given by the golden rule, i.e. $f'(k^*) = n$
  - Here, lowercase letters signify per capita values
- Individual (and firm) optimization implies that
  - $\frac{u_1}{u_2} = 1 + r = 1 + f'(k)$
  - It is possible that $r < n \Rightarrow k > k^* \Rightarrow$ Pareto inefficient
Diamond recommends issuing government debt at interest rate $r$.

Tirole (1985) introduces a rational bubble asset trading at price $b_t$:

$$b_{t+1} = \frac{1+r_{t+1}}{1+n} b_t$$

Both solutions crowd out investment and increase $r$.

If baseline economy is inefficient, then an appropriately chosen debt issuance or bubble size can achieve Pareto optimum with $r = n$. 

OLG: Production & Efficiency
OLG: Crowding Out vs. Crowding In

- Depending on the framework, government debt and presence of bubbles can have two opposite effects
  - **Crowding out** refers to the decreased real investment
  - **Crowding in** refers to increased investment due to improved risk transfer
- Woodford (1990) explores both of these effects
Consider a model with two types of agents
- Per capita production $f(k)$
- Alternating endowments $\bar{e} > e > 0$
- No borrowing

Stationary solution
- High endowment agents are *unconstrained*, consuming $\bar{c}$ and saving part of endowment
- Low endowment agents are *constrained*, consuming $c \leq \bar{c}$ and depleting savings
**OLG: Crowding Out**

- **Euler equations**
  - Unconstrained: \( u'(\bar{c}) = \beta (1 + r)u'(\bar{c}) \)
  - Constrained: \( u'(\underline{c}) \geq \beta (1 + r)u'({\bar{c}}) \)

- **Interest rate is lower than discount rate**
  - \( f'(k) - 1 = r \leq \beta^{-1} - 1 \equiv \rho \Rightarrow \text{Pareto inefficient} \)

- **Increasing debt provides market liquidity**
  - This increases interest rate and reduces capital stock
  - With \( r = \rho \Rightarrow \underline{c} = \bar{c} \) (full insurance)
Assume agents now have alternating *opportunities* (instead of endowments)

- Unproductive agents can only hold government debt
- Productive agents can hold debt *and* capital

**Stationary solution**

- Unproductive agents are *unconstrained*, consuming \( \bar{c} \) and saving part of endowment (as debt)
- Productive agents are *constrained*, consuming \( c \leq \bar{c} \) and investing savings and part of endowment in capital
**OLG: Crowding In**

- **Euler equations**
  - Unconstrained: \( u'(\bar{c}) = \beta (1 + r)u'(c) \)
  - Constrained: \( u'(\bar{c}) = \beta f'(k)u'(\bar{c}) \)
  - Interest rate satisfies \( 1 + r \leq f'(k) \)

- **Increasing debt provides market liquidity**
  - This increases \( r \) and \( k \) since \( \beta (1 + r) = \frac{1}{\beta f'(k)} \)
  - Transfer from unproductive periods to productive periods
  - Increase debt until both agents are unconstrained
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- Amplification Revisited
Precautionary Savings

- Consumption smoothing implies that agents will save in high income states and borrow in low income states
  - If markets are incomplete, agents may not be able to efficiently transfer consumption between these outcomes
- Additional precautionary savings motive arises when agents cannot insure against uncertainty
  - Shape of utility function $u'''$
  - Borrowing constraint $a_t \geq -b$
Utility maximization $E_0[\sum_{t=0}^{\infty} \beta^t u(c_t)]$

- Budget constraint: $c_t + a_{t+1} = e_t + (1 + r)a_t$
- Standard Euler equation: $u'(c_t) = \beta (1 + r)E_t[u'(c_{t+1})]$

If $u''' > 0$, then Jensen’s inequality implies:

- $\frac{1}{\beta(1+r)} = \frac{E_t[u'(c_{t+1})]}{u'(c_t)} > \frac{u'(E_t[c_{t+1}])}{u'(c_t)}$
- Marginal value is greater due to uncertainty in $c_{t+1}$
- Difference is attributed to precautionary savings

**Prudence** refers to curvature of $u'$, i.e. $P = -\frac{u'''}{u''}$
With *incomplete markets* and *borrowing constraints*, agents engage in precautionary savings in the presence of *idiosyncratic income shocks*

Following Bewley (1977), mean asset holdings $E[a]$ result from individual optimization.
In an exchange economy, aggregate supply of assets must be zero
- Huggett (1993)

Equilibrium interest rate always satisfies $r < \rho$
Aiyagari (1994) combines the previous setup with standard production function $F(K, L)$

- Constant aggregate labor $L$

Demand for capital is given by $f'(k) - \delta = r$

- Efficient level of capital $f'(k^*) - \delta = \rho \Rightarrow k^* < k$
Aiyagari (1995) shows that a tax on capital earnings can address this efficiency problem
  - This decreases the net interest rate received by agents

Government debt does not work “perfectly”
  - No finite amount of government debt will achieve \( r = \rho \)
Bewley-Aiyagari economies result in competitive allocations that are not only Pareto inefficient, but are also \textit{constrained} Pareto inefficient.

- Social planner can achieve a Pareto superior outcome even facing same market incompleteness.

This result can be attributed to \textit{pecuniary externalities}.

- In competitive equilibrium, agents take prices as given whereas a social planner can induce wealth transfers by affecting relative prices.

- Stiglitz (1982), Geanakoplos-Polemarcharkis (1986)
Davila, Hong, Krusell, Rios-Rull (2005) consider welfare increasing changes in Aiyagari setting.

Higher level capital leads to higher wages and lower interest rates:
- Higher wage amplifies contemporaneous effect of labor endowment shock.
- Lower interest rate dampens impact of endowment shock in following periods.
CI: Amplification

- Two period setting with $t \in \{0,1\}$
  - Initial wealth $y$
  - Labor endowment $e \in \{e_1, e_2\}$ (i.i.d)
  - Aggregate labor: $L = \pi e_1 + (1 - \pi) e_2$
  - Production function $f(K, L)$

- Agent consumption plan given by $\{c_0, c_1, c_2\}$
  - $c_i \leq e_i w + K(1 + r)$
  - $\frac{dU}{dK} = \{-u'(c_0) + \beta (1 + r)[\pi u'(c_1) + (1 - \pi)u'(c_2)]\} + \\
    \beta [\pi u'(c_1)K + (1 - \pi)u'(c_2)K] \frac{dr}{dK} + \\
    \beta [\pi u'(c_1)e_1 + (1 - \pi)u'(c_2)e_2] \frac{dw}{dK}$
**Cl: Amplification**

- The first expression is zero from agent’s FOC
  - Agents take prices as given, i.e. assume \( \frac{dw}{dK} = \frac{dr}{dK} = 0 \)

- In a competitive equilibrium \( \frac{dr}{dK} = f_{KK} \) and \( \frac{dw}{dK} = f_{KL} \)
  - \( f \) linearly homogeneous implies \( Kf_{KK} + Lf_{KL} = 0 \)

- This provides:
  - \( \frac{dU}{dK} = \beta \pi (1 - \pi) \frac{Kf_{KK}}{L} (u'(c_1) - u'(c_2))(e_2 - e_1) < 0 \)
  - Reducing level of capital improves ex-ante utility
Consider addition of third period $t = 2$

- Same labor endowment $e \in \{e_1, e_2\}$

Effect of change in capital level at $t = 1$ depends on realization of labor endowment

\[ \Delta = \beta \pi (1 - \pi) \frac{K f_{KK}}{L} (u'(c_1) - u'(c_2))(e_2 - e_1) < 0 \]

\[ \frac{dU_i}{dK} = \beta [\Delta + \beta (\pi u'(c_{i1})) + (1 - \pi)u'(c_{i2})](K_i - K)f_{KK} ] \]

Second term is positive if and only if $K_i < K$

- Increasing capital more desirable for low endowment agents and less desirable for high endowment agents
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Aggregate Risk

- Krusell, Smith (1998) introduce aggregate risk into the Aiyagari framework
  - Aggregate productivity shock that follows a Markov process $z_t$ and $Y_t = z_tF(K_t, L_t)$
- Aggregate capital stock determines equilibrium prices $r_t, w_t$
  - However, the evolution of aggregate stock is affected by the distribution of wealth since poor agents may have a much higher propensity to save
  - Tracking whole distribution is practically impossible
AR: Bounded Rationality

- Krusell, Smith assume agents are boundedly rational and approximate the distribution of capital by a finite set of moments $M$
  - Regression $R^2$ is relatively high even if $#M = 1$
- This result is strongly dependent on low risk aversion and low persistence of labor shocks
  - Weak precautionary savings motive except for poorest agents
  - Since wealth-weighted averages are relevant, this has a negligible effect on aggregate quantities
Constantinides & Duffie (1996) highlight importance of persistent income shocks
  - Any price process can be replicated (in a non-trading environment)

With non-stationary and heteroskedastic individual income processes, self-insurance through precautionary saving is far less effective
  - Any shock to agent’s income permanently affects expected share of future aggregate income
  - Wealth heterogeneity is significant
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- Amplification Revisited
Financial instability arises from the fragility of liquidity.

**Technological liquidity**
- Reversibility of investment

**Market liquidity**
- Specificity of capital
- Price impact of capital sale

**Funding liquidity**
- Maturity structure of debt
  - Can’t roll over short term debt
- Sensitivity of margins
  - Margin-funding is recalled

*Liquidity mismatch* determines severity of amplification.
Amplification Revisited

- Investment possibility shocks
  - Production possibilities: Scheinkman & Weiss (1986)
  - Investment possibilities: Kiyotaki & Moore (2008)

- Interim liquidity shocks
  - Endogenous shock: Shleifer & Vishny (1997)

- Preference shocks
  - No aggregate risk: Diamond & Dybvig (1983)
  - Aggregate risk: Allen & Gale (1994)
Two types of agents with perfectly negatively correlated idiosyncratic shocks
- No aggregate risk, but key difference is that labor supply is now elastic

Productivity reflects *technological liquidity*
- Productivity switches according to a Poisson process
- Productive agents can produce consumption goods

No capital in the economy
- Can only save by holding money (fixed supply)
- Productive agents exchange consumption goods for money from unproductive agents
SW: Aggregate Dynamics

- Aggregate fluctuations due to elastic labor supply
- Price level is determined in equilibrium
  - As productive agents accumulate money, wealth effect induces lower labor supply
  - Aggregate output declines and price level increases
- Effect of changes in money supply depends on distribution of money between agent types
  - Increase in money supply will reduce (increase) aggregate output when productive agents hold less (more) than half the money supply, i.e. when output is high (low)
Two types of agents, entrepreneurs and households

- Entrepreneurs can invest, but only when they have an investment opportunity
- Opportunities correspond to technological liquidity

Investment opportunities arrive i.i.d. and cannot be insured against

- Entrepreneur can invest with probability $\pi$

Agents can hold equity or fiat money
KM: Financing

- Entrepreneurs have access to 3 sources of capital
  - New equity claims, but a fraction $1 - \theta$ must be held by entrepreneur for at least one period
  - Existing equity claims, but only a fraction $\phi_t$ of these can be sold right away
  - Money holdings, with no frictions
- Capital frictions represent *illiquidity*
KM: Entrepreneurs

- **Budget constraint:**
  - \[ c_t + i_t + q_t(n_{t+1} - i_t) + p_t(m_{t+1} - m_t) = r_t n_t + q_t(1 - \delta)n_t \]
  - Equity holdings net of investment \( n_{t+1} - i_t \)
  - Price of equity/capital \( q_t \) can be greater than 1 due to limited investment opportunities

- **Liquidity constraint:**
  - \[ n_{t+1} \geq (1 - \theta)i_t + (1 - \phi_t)(1 - \delta)n_t \]
  - Limits on selling new and existing equity place lower bound on future equity holdings
For low $\theta$, $\phi_t$, liquidity constraints are binding and money has value.

An entrepreneur with an investment opportunity will spend all of his money holding

- Budget constraint can be rewritten as $c_t^i + q_t^R n_{t+1} = r_t n_t + (\phi_t q_t + (1 - \phi_t)q^R_t)(1 - \delta)n_t + p_t m_t$
- Replacement cost of capital: $q_t^R \equiv \frac{1-\theta q_t}{1-\theta}$
- Can create new equity holdings at cost $q_t^R < q_t$, but this reduces value of remaining unsold holdings.
KM: No Investment Opportunity

- Entrepreneur without investment opportunity decides on allocation between equity (depends on opportunity at $t + 1$) and money
  - Return to money: $R^m_{t+1} \equiv \frac{p_{t+1}}{p_t}$
  - No opportunity: $R^s_{t+1} \equiv \frac{r_{t+1} + qt_{t+1}(1-\delta)}{qt}$
  - Opportunity: $R^i_{t+1} \equiv \frac{r_{t+1} + (\phi_{t+1}qt_{t+1} + (1-\phi_{t+1})q_{t+1}^R(1-\delta))}{qt}$
KM: Logarithmic Utility

- Under logarithmic utility, entrepreneurs will consume $1 - \beta$ fraction of wealth
- Around steady-state, aggregate level of capital is smaller than in first-best economy, i.e. $K_{t+1} < K^*$
  - Expected return on capital $E_t[f'(K_{t+1}) - \delta] > \rho$
- Conditional liquidity premium arises since $E_t[R_{t+1}^m] < E_t[R_{t+1}^s] < 1 + \rho$
  - Unconditional liquidity premium may also arise (but is smaller) since $E_t[R_{t+1}^i] < E_t[R_{t+1}^m]$
KM: Real Effects

- Negative shocks to market liquidity $\phi_t$ of equity have aggregate effects
  - Shift away from equity into money
  - Drop in price $q_t$ and increase in $p_t$
  - Decrease in investment and capital

- Shock to financing conditions feeds back to real economy as a reduction in output
  - KM find that government can counteract effects by buying equity and issuing new money (upward pressure on $q_t$ and downward pressure on $p_t$)
Three period model with \( t \in \{0,1,2\} \)

Entrepreneurs with initial wealth \( A \)
- Investment of \( I \) returns \( RI \) in \( t = 2 \) with probability \( p \)
- Interim funding requirement \( \rho I \) at \( t = 1 \) with \( \rho \sim G \)
- Extreme *technological illiquidity*, as investment is worthless if interim funding is not provided

Moral hazard problem
- Efficiency requires \( \rho \leq \rho_1 \equiv pR \Rightarrow \) continuation
- Only \( \rho \leq \rho_0 < \rho_1 \) of funding can be raised at \( t = 1 \) due to manager’s private benefit, i.e. principal-agent conflict
Optimal contract specifies:
- Investment size $I$
- Continuation cutoff $\hat{\rho}$
- Division of returns contingent on realized $\rho$

Entrepreneurs maximize expected surplus, i.e.

$$\max_{I, \hat{\rho}} \left\{ I \int_{0}^{\hat{\rho}} (\rho_1 - \rho) dG(\rho) - I \right\}$$

Households can only be promised $\rho_0$ at $t = 1$
- Breakeven condition: $I \int_{0}^{\hat{\rho}} (\rho_0 - \rho) dG(\rho) = I - A$

Solution provides cutoff $\hat{\rho} \in [\rho_0, \rho_1]$
Without a storage technology, liquidity must come from financial claims on real assets

- *Market liquidity* of claims becomes crucial

If there is no aggregate uncertainty, the optimal contract can be implemented:

- Sell equity
- Hold part of market portfolio
- Any surplus is paid to shareholders as dividends
HT: Aggregate Risk

- With aggregate risk, optimal contract may not be implementable
  - Market liquidity of equity is affected by aggregate state
- Consider perfectly correlated projects
  - Liquidity is low when it is needed (bad aggregate state)
  - Liquidity is high when it is not needed (good state)
- This introduces a role for government to provide a store of wealth
Fund managers choose how aggressively to exploit an arbitrage opportunity

Mispricing can widen in interim period
- Investors question investment and withdraw funds
- Managers must unwind position when mispricing is largest, i.e. most profitable
- Low market liquidity due to limited knowledge of opportunity

Fund managers predict this effect, and thus limit arbitrage activity
- Keep buffer of liquid assets to fund withdrawals
Three period model with $t \in \{0,1,2\}$

Continuum of ex-ante identical agents
- Endowment of 1 in $t = 0$
- Idiosyncratic preference shock, i.e. probability $\lambda$ that agent consumes in $t = 1$ and probability $1 - \lambda$ that agent consumes in $t = 2$

Preference shock is not observable to outsiders
- Not insurable, i.e. incomplete markets
**DD: Investment**

- **Good can be stored without cost**
  - Payoff of 1 in any period
- **Long term investment project**
  - Payoff of $R > 1$ in $t = 2$
  - Salvage value of $r \leq 1$ if liquidated early in $t = 1$
  - Market for claims to long-term project at price $p$
- **Trade-off between return and liquidity**
  - Investment is subject to *technological illiquidity*, i.e. $r \leq 1$
  - Market liquidity is represented by interim price $p$
DD: Consumption

- Investing $x$ induces contingent consumption plan:
  - $c_1 = px + (1 - x)$
  - $c_2 = Rx + \frac{R(1-x)}{p}$

- In equilibrium, we require $p = 1$
  - If $p < 1$, then agents would store the asset and purchase project at $t = 1$
  - If $p > 1$, then agents would invest and sell project at $t = 1$
With interim markets, any investment plan leads to $c_1 = 1, c_2 = R$

- If $r < 1$, fraction $1 - \lambda$ of aggregate wealth must be invested in project (market clearing)
- Since $p > r$, then asset’s *market liquidity* is greater than its *technological liquidity*

This outcome is clearly superior to autarky, with $c_1' = r, c_2' = R$ or $c_1'' = c_2'' = 1$

Optimality:
- For log utility market outcome is optimal
- If customers are more risk averse banks dominate
AG extend DD framework by adding aggregate risk

- Here, $\lambda = \lambda_H$ with probability $\pi$ and $\lambda = \lambda_L < \lambda_H$ with probability $1 - \pi$

Agents observe realization of aggregate state and idiosyncratic preference shock at $t = 1$

- After resolution of uncertainty, agents can trade claims to long-term project at $p_s \in \{p_H, p_L\}$
- Asset's *market liquidity* will vary across states

For simplicity, assume $r = 0$
Market clearing requires $p_s \leq R$

- Late consumers stored goods: $(1 - \lambda_s)(1 - x)$
- Early consumers invested goods: $\lambda_s x$

Cash-in-the-market pricing

- $p_s = \min\left\{ R, \frac{(1-\lambda_s)(1-x)}{\lambda_s x} \right\}$

This implies that $p_H \leq p_L$, i.e. market liquidity is weaker when there are a large proportion of early consumers

Despite deterministic project payoffs, there is volatility in prices
Overview

- Persistence
- Dynamic Amplification
  - Technological illiquidity BGG
  - Market illiquidity KM97
- Instability, Volatility Dynamics, Volatility Paradox
- Volatility and Credit Rationing/Margins/Leverage
- Demand for Liquid Assets
- Financial Intermediation
Creating Info-Insensitive Securities

- Debt contract payoff – prior distribution of cash flow

- Asymmetric info (lemons’) problem kicks in
  - No more rollover

- Maturity choice:
  - Short-term debt: distribution shrinks (less info-sensitivity)
Creating Info-Insensitive Securities

- Debt contract payoff

- Informational value of signal is extremely low (in flat part of contract payoff)
Creating Info-Insensitive Securities

- Increasing the information sensitivity of debt
  - bad signal  good signal

- Now signal is very valuable

- Asymmetric info (lemons’) problem kicks in
  - No more rollover

- Maturity choice:
  - Short-term debt: distribution shrinks (less info-sensitivity)
Repo market

- Repurchase agreement
  - Borrow: sell assets with an agreement to repurchase it in one day/months
  - Repo types:
    - General collateral (GC) repos
      collateral are treasuries, agency papers
    - MBS repos
      collateral are mortgage backed securities
  - Outside of bankruptcy protection (in US not in UK)

- Repo haircuts widened sharply