A Welfare Criterion for Models with Distorted Beliefs
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Welfare Analysis for Behavioral Models

• Vast evidence on people holding wrong beliefs and making inefficient decisions.
  – e.g., reviews of Hirshleifer (2001), Barberis and Thaler (2003), Della Vigna (2009)

• For normative analysis, a welfare criterion is needed.

• BF literature commonly assumes a true belief and the planner knows the true belief
  – e.g., Gabaix and Laibson (2006), Weyl (2007), Spinnewijn (2010), and Gennaioli, Shleifer, Vishny (2011)

• Whose belief is wrong? And which belief should a planner use?
  – One cannot liberally overturn revealed beliefs.

• This question becomes even more serious for models with heterogeneous beliefs
  – Harrison and Kreps (1978), Detemple and Murthy (1994), Scheinkman and Xiong (2003), Geanakoplos (2009), and others
A Belief-Neutral Criterion

• This paper provides a belief-neutral welfare criterion.
  – The planner is aware of presence of distorted beliefs but not sure of the true belief.

• Negative or positive sums often appear in models with heterogeneously distorted beliefs.
  – One can evaluate welfare without taking a stand on whose belief is right or wrong.

• Negative-sum speculation in macro and finance models
  1. Over-investment in bubble models
  2. Bankruptcy costs in leverage cycle models
  3. Excessive risk taking in speculative trading models
  4. Consumption-savings distortions in macro models

• Positive-sum speculation
  5. Overcoming market breakdown in Lemons models
An Example

Joe Stiglitz:  
With 90% chance it is made of cotton

Bob Wilson:  
With 90% chance it is made of polyester

Joe and Bob took a bet:  
• If it is made of cotton, Bob pays Joe $100; otherwise, Joe pays Bob $100. 
They had to cut the pillow open to find out its content  
• It cost $50 to replace the pillow, which is paid by the winner.
An Example

Both Joe and Bob found the bet desirable

- The bet is Pareto efficient!

Expected return from the bet:

\[ 90\% \times (\$100 - \$50) - 10\% \times \$100 = \$35 \]
An Example

- The bet induces a wealth transfer between them, but a perfect pillow is destroyed!

$100
Negative Externality

• The bet is a **negative-sum** game!
  – Socially inefficient regardless of whose belief is right or wrong.

• Externality driven by conflicting beliefs
  – Joe believes that he will win and thus the cost of replacing the pillow goes to Bob
  – Bob believes that he will win and thus the cost of replacing the pillow goes to Joe
  – The presence of the negative externality holds in both Joe and Bob’s beliefs.
A Belief-Neutral Welfare Criterion

We propose a belief-neutral welfare criterion.

- A set of reasonable beliefs, spanned by convex combinations of agents’ beliefs.
  - The objective measure lies between agents’ beliefs.
  - Can be further expanded in some settings.

- A choice $x$ is efficient (inefficient) if the planner finds it efficient (inefficient) by using every reasonable belief as the common measure to evaluate all agents’ welfare.

- Two ways to implement
  - Social welfare function
  - Pareto efficiency
Welfare Analysis with Conflicting Beliefs


• Spurious unanimity problem of Pareto criterion, e.g., Mongin (1997) and Gilboa, Samuelson, and Schmeidler (2012).

Sources of heterogeneous beliefs

• Subjective beliefs
  – Savage’s view: beliefs are part of their preferences under uncertainty.
  – Beliefs may reflect state-dependent preferences.
  – The bet did not help Bob and Joe hedge their state-dependent risk, rather each believed he would win and the other would lose.

• Distorted beliefs
  – Mounting evidence that biases, like overconfidence, representativeness, etc., can distort people’s beliefs.
  – Then, social planner needs to use a common, objective measure to evaluate agents’ welfare on their behalves.
  – Our framework allows state-dependent utility functions for subjective beliefs.
  – Our criterion requires only presence of belief distortion but not precise identification of whose beliefs are distorted, complementary to Bernheim and Rangel (2009).
Model Setting

- Consider a generic setting with $T$ periods: $t = 0, 1, \ldots, T$.
  - The state follows a binomial tree.

- $N$ agents holding different beliefs
  - $\Pi^i = \{\pi^i_{t,s}\}, i \in \{1, \ldots, N\}$
  - $\pi^i_{t,s} > 0$

- A social choice:
  - $x = \{x^i_T(s_T)\}$

- State-dependent utility
  - $u_i[s_T, x^i_T(s_T)]$
  - Capturing state-dependent preferences and subjective priors

- Set of reasonable beliefs:
  - any convex combination of agents’ beliefs: $\Pi^h = \sum_i h^i \Pi^i$, where $h^i \geq 0$ and $\sum_i h^i = 1$.
  - Includes all extreme beliefs that are present in the system.
Implementation by a Social Welfare Function

• For a given social welfare function
  – Bergsonian social welfare function: \( W(u^1, u^2, ..., u_N) = \sum_i \lambda_i u_i \), where \( \{\lambda_i\} \) are non-negative weights
  • Varying the weights gives Pareto frontier
  – Utilitarian social welfare function: \( W(u^1, u^2, ..., u_N) = \sum_i u_i \)

• Allocation \( x \) is **belief-neutral superior to** \( y \) if \( \forall \Pi^h \), the planner finds that

\[
W(\mathbb{E}_0^h [u^1(s_T, x_1^T(s_T))], ..., \mathbb{E}_0^h [u_N(s_T, x_N^T(s_T))]) \geq W(\mathbb{E}_0^h [u^1(s_T, y_1^T(s_T))], ..., \mathbb{E}_0^h [u_N(s_T, y_N^T(s_T))])
\]
Implementation by a Social Welfare Function

• Back to the bet between Joe and Bob.
• Suppose that both of them are risk neutral and that the planner uses a utilitarian welfare function:
  \[ W(u_{Joe}, u_{Bob}) = u_{Joe} + u_{Bob} = w_{Joe} + w_{Bob} \]
  – Social welfare is equivalent to expected social wealth.

• The bet generates a wealth transfer and a pillow being destroyed.
  – The destroyed pillow leads to a negative sum, which is independent of the beliefs used by the planner.

• What if Bob and Joe have unequal weights?
  – The bet can transfer wealth from the low-weight person to the other.
Implementation by Pareto Dominance

• An allocation $x$ is called belief-neutral Pareto efficient if under any measure $\Pi^h$ there does not exist another allocation $x'$ such that it improves some agents’ expected utilities without reducing anyone’s, i.e.,

$$\forall i, E^h_0 [u_i(s_T, x^i_T(s_T))] \leq E^h_0 [u_i(s_T, x'^i_T(s_T))].$$

  – Different from standard Pareto dominance, the planner uses a common measure to evaluate all agents’ welfare, instead of their own.
  – The standard economic theory: for a given, common belief measure, each allocation on the Pareto frontier maximizes a linear social welfare function with a certain set of Pareto weights.
Implementation by Pareto Dominance

• Back to the bet between Joe and Bob.

• Suppose that the planner uses Joe’s beliefs.
  – The bet leads to an expected gain of $35 to Joe and an expected loss of $85 to Bob.
  – An alternative by transferring $35 to Joe from Bob makes Joe indifferent and improves Bob’s welfare by $50.

• Suppose that the planner uses any convex combination of their beliefs, say with weight $h \in (0,1)$ to Joe.
  – A higher $h$ means a larger expected gain to Joe from the bet under the measure.
  – Still, an appropriate transfer from Bob to Joe can make Joe indifferent and save Bob some money.

• Thus, the bet is belief-neutral Pareto inefficient.
  – The belief-neutral inefficiency of the bet does not rely on any particular welfare function.
  – The bet is belief-neutral inefficient, even though which allocation dominates the bet may depend on the belief measure or welfare function.
Generalize the Bet

State-dependent replacement cost:
• It costs $50 if it is made of cotton but $20 if it is made of polyester.

• The externality is still belief neutral negative

• Under Joe’s belief:
  – His expected return is $90\% \cdot ($100 - $50) - 10\% \cdot $100 = $35;
  – while expected return to Bob is
    $- 90\% \cdot $100 + 10\% \cdot ($100 - $20) = -$82.$

• Under Bob’s belief:
  – His expected return is $90\% \cdot ($100 - $20) - 10\% \cdot $100 = $62;
  – while expected return to Joe is
    $- 90\% \cdot $100 + 10\% \cdot ($100 - $50) = -$85.
Generalize the Bet

What if there is benefit from knowing the pillow’s content?

• The pillow is either made of cotton or a poisonous material.
  – In the first case, the winner pays $50 to replace the pillow;
  – in the latter, the winner gets another reward of $100 from turning in the poisonous pillow.

• The externality is not belief-neutral negative

• Under Joe’s belief:
  – Expected return to himself is $90\% \cdot ($100 − $50) − 10\% \cdot $100 = $35;
  – Expected return to Bob is $−90\% \cdot $100 + 10\% \cdot ($100 + $100) = −$70.
  – A transfer of $35 from Bob to Joe dominates the bet.

• Under Bob’s belief:
  – Expected return to himself is $90\% \cdot ($100 + $100) − 10\% \cdot $100 = $170;
  – Expected return to Joe is $−90\% \cdot $100 + 10\% \cdot ($100 − $50) = −$85.
  – No transfer can improve one’s welfare without hurting the other.
Applications

• The belief-neutral criterion is incomplete

• Nevertheless, useful for spotting negative-sum & positive-sum speculation induced by heterogeneously distorted beliefs.

• These applications are simplified versions of prominent economic models.
  – We aim to demonstrate the relevance of the belief-neutral criterion rather than to advocate for any specific policy recommendation.
Application 1: Over-investment in Bubble Models


- Decreasing return to scale and invest $n$ units at $t = 0$.
  - Firm objective: $\max_n n \cdot p_0$

  Market setting:
  
  $p_0 = 57.5 - n$
  
  $\max_n n \cdot (57.5 - n) \Rightarrow n_* = \frac{57.5}{2}$.

  If the planner adopts A’s beliefs:
  
  $p_0 = E^A_0 [\bar{R}] = 50 - n$
  
  $\max_n n \cdot (50 - n) \Rightarrow n_* = 25$.

  If the planner adopts B’s beliefs:
  
  $p_0 = E^B_0 [\bar{R}] = 50 - n$
  
  $\max_n n \cdot (50 - n) \Rightarrow n_* = 25$.

  Over-investment from both A and B’s beliefs!
Application 2: Benefits of Speculation in Lemons Model

• Speculation caused by heterogeneous beliefs can be beneficial in lemons model, a la Akerlof (1970).

• We adopt a simple version of Tirole (2012):
  – A seller needs to liquidate a legacy asset to finance a profitable investment opportunity.
  – A lemons problem arises as the seller knows more about the quality of the legacy asset than potential buyers.
  – Speculation induced by the heterogeneousy distorted beliefs of potential buyers can lead to positive externality as it overcomes the adverse selection problem.
  – Belief-neutral efficient outcome!
Application 3: Bankruptcy Costs in Leverage Cycle Models

- Cash-constrained optimists tend to use collateralized debt to finance their investments, which fuels initial price boom and later price bust.
  - Geanakoplos (2003, 2009), Fostel and Geanakoplos (2008), Simsek (2010), and He and Xiong (2012).

- A is always more optimistic than B, both risk neutral.
  - At \( t = 0 \), A is endowed with $20 and no asset.
  - Owner incurs a cost of \( \alpha = $20 \) to liquidate per unit of asset.

\[ E_0^A[R] = 96.8 \]
\[ E_0^B[R] = 48.8 \]

- A uses one-period debt with promise 36.
- \( p_0 = 20 + 36 = 56 \).
- In state \( d \), A has to promise 36 to rollover his debt, which exposes him to bankruptcy risk if fundamental ends in 20.

\[ \pi_u^A = 0.8, \pi_u^B = 0.2 \]
\[ \pi_d^A = 0.8, \pi_d^B = 0.2 \]

- The bankruptcy cost \( \alpha \) induces a welfare loss under any reasonable beliefs.
- Ex ante, A believes the asset is so cheap that he has a good deal despite the cost.
Application 4: Excessive Risk Taking in Speculative Trading Models

- Many general equilibrium models with heterogeneous beliefs:
  - Speculation between optimists and pessimists lead to endogenous risk and amplified price volatility

- When agents are risk averse, trading makes each agent’s consumption more volatile without changing the aggregate wealth.
  - negative-sum game in utility terms if speculation is induced by belief distortions rather than differences in preferences.
Application 5: Consumption-Savings Distortions in Macro Models

- In macro models with investment, speculation induced by belief disagreements can also distort savings and thus investments.
  - Speculation not only makes their consumption more volatile but also distorts the aggregate consumption:

- Sims (2008)
  - Two types of agents disagree about future inflation.
  - Inflation optimists prefer to borrow nominal from pessimists.
    - Substitution effect: speculation motivates both types to save
    - Wealth effect: expectations of speculation gains induce both to consume more
    - Depending on their rate of relative risk aversion, substitution effect dominates wealth effect or vice versa, and thus leads to over- or under-investment.

- Our criterion can also detect belief-neutral inefficiency of such distortions.
Comparing to Gilboa, Samuelson, and Schmeidler (2012)

• Gilboa, Samuelson, and Schmeidler (2012) also recognize that the standard Pareto criterion can be spurious in the presence of conflicting beliefs.

• They propose to weaken the criterion: Allocation $f$ no-betting Pareto dominates $g$ (i.e., $f \succ_{NBP} g$) if
  1. $\forall i, f \succeq_i g; \exists j, f \succeq_j g$
  2. There exists at least one measure $p_0$ such that, for all $i$,
     $$\int u_i(f(s))dp_0 > \int u_i(g(s))dp_0$$

• The non-betting Pareto criterion rules the bet between Bob and Joe as neither efficient nor inefficient.
  – The additional requirement makes the criterion more restrictive and thus more incomplete than the standard Pareto criterion.

• Our criterion let the planner use a common belief to evaluate each agent’s welfare but let the common belief to vary across a large set.
  – The key premise is that the planner is sure of the presence of distorted beliefs.
  – Our criterion gives more clear-cut ranking.
Conclusion

• A belief-neutral welfare criterion for behavioral models

• Opens normative analysis for financial regulation
  – Avoid negative-sum speculation and facilitate positive-sum one

• Separate “preferences” from “belief distortions”
  – Only require presence of belief distortions
  – Don’t need to know the truth

• Negative externality
  – Over-investment in bubble models
  – Bankruptcy costs in leverage cycle models
  – Excessive risk taking in speculative trading models
  – Consumption-savings distortions in macro models

• Positive externality
  – Overcoming market breakdown in Lemons models