Abstract. I develop and estimate a model of export dynamics featuring self-discovery. The estimated model accounts well for the relationship between growth, survival, and tenure in the export market that is observed in the data for new exporters: (a) continuation rates are increasing with export tenure, and (b) growth rates of export sales are decreasing with export tenure. I use the estimated model to quantify the role of learning in shaping the firm level export decision and to undertake counterfactuals for the effects of export promotion on aggregate trade. I find that: (i) first-time exporters expect to incur losses by serving the foreign market, but the option value generated by the acquisition of more precise information regarding export profitability compensates inexperienced exporters for these losses; (ii) the initial period serving the foreign market provides a crucial learning experience for new exporters, but the discovery stage extends beyond the first year: the value of learning remains positive for the first four years of tenure in the export market. The probability of exiting the export market decreases with tenure and after the discovery stage is only 5% higher than the exit probability of well established exporters; the cutoff for exporting experiences 90% of its long-term adjustment over the same period; (iii) firms that continuously export over a period of six years observe a 137% increase in their (ex-ante) probability of serving the foreign market and a 900% increase in their (average) export premia, and (iv) temporary shocks to the profitability of serving the export market can have permanent consequences on aggregate trade volumes. In particular, export promotion policies that temporarily subsidize the fixed costs of maintaining a presence in the foreign market can result in permanent increases in aggregate trade volumes. The impact of these types of policies crucially depends on the speed at which firms are able to uncover their export profitability.

1. Introduction

What explains the internationalization process of new exporters? The extensive margin of entrants into export markets plays an important role in export booms (see Roberts and Tybout [1997]). New export entrants exhibit a Darwinian process of
selection with high rates of exit and rapid growth conditional on survival.\(^1\) Conditional on survival, exporters undergo a period of adjustment in their foreign market presence lasting several years as they transition from new to mature exporters. Export supply responsiveness is of central importance to policymakers who often tie the success of structural adjustment programs to the extent to which strong export responses follow these reforms. Structural models of export supply with microeconomic foundations provide a useful framework to study the dynamics of firm level trade and to evaluate the impact of trade policy on aggregate trade volumes.

The dynamics of new export entrants in structural models of export supply a la Melitz such as Das et al. [2007] and Ruhl and Willis [2008], that focus on the role of sunk entry costs, producer heterogeneity and exchange rate movements on the dynamics of exporting firms, are at odds with the dynamics of new exporters observed in the data. In these models exporters grow too large too quickly and survive for too long.\(^2\) By generating new exporters that live too long and export too much, these class of models provide an inaccurate depiction of the importance of the contribution of the extensive margin of entrants in aggregate trade growth and fail to capture the dynamics of internationalization that new exporters go through in the process of establishing a secure foreign market presence. As such, a deeper understanding of the microeconomic foundations of export supply is needed to understand the dynamics of firm level trade and to properly assess the contribution of export entrants in aggregate export growth.

In this paper I develop a model of export supply based on Melitz [2003], but draw from Jovanovic [1982] to develop a quantitative model of these new exporter dynamics based on self-discovery. Self-discovery in the export market will lead to a model with “noisy” selection into exporting and where a firm’s tenure in the export market is the firm characteristic determining growth and survival in the foreign market. I structurally estimate the model using transaction-level customs data for Mexican exporters and show that the estimated model accounts both qualitatively and quantitatively for the observed patterns of export dynamics of new exporters observed in the data.

\(^1\)Besedes and Prusa [2011] find that 70% of new export relationships fail within the first two years.

\(^2\)In this class of models the interaction between sunk entry costs and other factors determining the profitability of the firm are key in understanding the failure of the model to account for high failure rates and gradual expansion of new exporters. The failure to generate a gradual increase in exports stems from that fact that, upon entry, a new exporter immediately adjusts its exports to the optimal level since there are no other barriers to exporting (i.e. once the plant pays the sunk entry costs it exports as much as it can). On the other hand, the models failure to deliver high exit rates early on in the tenure of a new exporter results from the interaction between sunk entry costs and the persistent nature of productivity and exchange rate shocks: sunk entry costs generate an option value to exporting, thus selection is based not only on current profitability, but also on future profitability. When firms face a positive shock to their profitability, the persistent nature of these shocks implies that the firms that enter the export market will be the most profitable and the least likely to exit in the subsequent periods (see Ruhl and Willis [2014] for details).
I use the estimated model to quantify the role of learning in shaping the dynamics of export supply. Little is known about the time span of foreign market unfamiliarity. When export entrants perceive their own lack of foreign-market knowledge, how long does it take them to remedy this situation? The structural model allows me to address questions such as: How fast do firms learn their way out of the uncertainty they face in the foreign market? How does the option value generated by self-discovery shape the export supply decision of firms relative to a purely static model of export supply such as Melitz [2003]? Additionally, the model also provides a framework that can be used to undertake counterfactuals for the effects of export promotion on aggregate trade.

In contrast to models which highlight sunk entry costs and production heterogeneity (see Das et al. [2007]), this model gives prominence to self-discovery as a key determinant of the export supply responsiveness of new exporters. Export promotion agencies (EPAs) often argue that limited information about foreign markets represents an important barrier to the internationalization process of new export entrants. Survey evidence supports this view. In a survey of non-exporting members of the Turkish Chamber of Commerce, Karakaya and Harcar [1999] found that “lack of information about foreign markets” was the most important external barrier to exporting perceived by respondents. Jalali [2012] and Kneller and Pisu [2011] find similar results in surveys of Greek and UK firms, respectively. Interestingly, the latter study finds that after two years of experience in the export market half of the responding firms no longer perceived “lack of information about foreign markets” as a barrier to their export activities.

For domestic markets, recent studies have found that unfamiliarity with demand conditions can account for the observed differences between firms of different ages. For U.S. manufacturing plants Foster et al. [2012] argue that the observed size differences between young and old plants are unlikely to be the results of productivity differences. These authors find that physical TFP levels of new plants are slightly higher than those of incumbents and that TFP differences vanish by the time plants are five years old. On the other hand, they document important differences in the idiosyncratic demands faced by plants: at the same price a new plant will sell only 58% of the output of a plant in the same industry that is more than 15 years old. These authors find evidence which lends support to a model featuring dynamic demand-side forces that lead to the accumulation of relationship capital along buyer-supplier links that leads to the gradual growth of entrants (conditional on survival). Furthermore, the uncertainties tied to such processes create an option value of waiting to expand until further information about demand is revealed. A model of “learning”

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3For example, the UK Department for International Development writes

“Entering a new market is an inherently more risky and uncertain process for a firm than operating in a market in which it is already established...qualitative research suggests that prior to investigating overseas markets some firms underestimate the potential demand for their product or services in overseas markets.”
about demand would be consistent with these findings. The dynamics that differentiate young and old plants may also apply to the dynamics that distinguish new and established exporters. Thus, “learning” about conditions in the foreign market could help understand the gradual adjustment in the foreign market presence of new exporters. In the international context, Artopoulos et al. [2013] conducted a study of Argentinian exporters in four selected industries which experienced episodes of export emergence and found that foreign market knowledge was a critical constraint to achieving consistent exports. In fact, these authors find that it is a lack of foreign market knowledge rather than a lack of production knowledge which inhibits firms from developing an established export presence in foreign markets.

The model developed in this paper features self-discovery as the driving force shaping the dynamic behavior of export entrants. The evolution of a firm’s beliefs regarding its “export profitability” is the key determinant of the firm’s expansion in foreign markets. Expectations concerning export profitability will affect a firm’s calculations regarding whether future export profits will cover the costs of maintaining a foreign market presence or not. In fact, all the dynamics in the model will be driven by the learning process that firms undergo and the state dependence that this process generates through the firm’s information sets. However, this force shaping export dynamics will be decreasing in importance with export tenure as firms uncover their true export profitability.

The main results that I obtain from the estimated model and counterfactuals are: (i) first-time exporters expect to incur losses by serving the foreign market, but the option value generated by the acquisition of more precise information regarding export profitability compensates inexperienced exporters for these losses; (ii) the initial period serving the foreign market provides a crucial learning experience for new exporters, but the discovery stage extends beyond the first year: the value of learning remains positive for the first four years of tenure in the export market. The probability of exiting the export market decreases with tenure and after the discovery stage is only 5% higher than the exit probability of well established exporters; the cutoff for exporting experiences 90% of its long-term adjustment over the same period; (iii) firms that continuously export over a period of six years observe a 137% increase in their (ex ante) probability of serving the foreign market and a 900% increase in their (average) export premia, and (iv) temporary shocks to the profitability of serving the export market can have permanent consequences on aggregate trade volumes. In particular, export promotion policies that temporarily subsidize the fixed costs of maintaining a presence in the foreign market can result in permanent increases in aggregate trade volumes. The impact of these types of policies crucially depends on the speed at which firms are able to uncover their export profitability.

My work is related to a recent literature that has exploited firm- and plant-level data to uncover a set of stylized facts for exporters (see, for example, Eaton et al. [2011] and Bernard et al. [2012]). This paper is related to the work of Arkolakis [2010], Ruhl and Willis [2008], and Alessandria et al. [2013] which study the dynamics of
new export entrants. However, rather than focusing on the cost structure faced by firms as these authors have done, I focus on the role of demand side uncertainties on export decisions. Akhmetova and Mitaritonna [2013] also study the effects of demand side uncertainties on exporter behavior, but their focus is on the choice of technology used to serve the foreign market while the focus here is on the observed relationship between growth, survival and export tenure for new export entrants.

This paper is closely related to a research program advocated by Arkolakis and Papageorgiou [2009] and to the work of Albornoz et al. [2012]. The latter provides reduced form evidence in support of the claim that when firms face ex-ante uncertainty regarding the profitability of serving the export market, shortcomings at the discovery stage are an important explanation for the limited export success of some developing countries. In contrast to these authors my approach is structural and allows me to quantify the role played by self-discovery in shaping the export supply decision of new exporters.

This model studied here is also related to the work of Arkolakis [2013] who studies the dynamics of selection and growth in a general equilibrium model of international trade. Here, however, given that self-discovery is the driving force behind firm dynamics in the foreign market it is export tenure rather than size which determines the opportunities for growth and survival. Finally, my work also relates to the dynamic structural models of export supply of Das et al. [2007] and Morales et al. [2014] which structurally estimate micro-founded models of export dynamics. Das et al. focus on the firm level dynamics implied by sunk entry costs and production heterogeneity and their consequences for aggregate trade in response to devaluations and export subsidies. Morales et al. focus on the dynamics of the extensive margin of destinations served and study “extended gravity” forces that lead exporting firms to enter foreign markets which are similar to markets previously served. In contrast, the focus here is on the firm level dynamics implied by demand-side uncertainties and the adjustment of firms along the intensive margin of trade.

The rest of the paper is organized as follows. Section 2 documents the dynamics of new export entrants that motivate the rest of the paper using Mexican micro data. Section 3 develops a model of export-supply featuring self-discovery, and Section 4 describes the estimation approach and presents the results from estimation. Section 5 uses the estimated model to quantify the role of self-discovery in the export supply decisions of new exporters, and Section 6 uses the model to perform counterfactuals regarding export promotion and the speed of learning. Section 7 concludes.

2. Data and Empirical Regularities

Micro-level trade data reveal that new exporters experience a substantial amount of adjustment that continues after entry into export markets. In this section I document the dynamics of new exporters as they transition from new to experienced exporters which is the focus of this paper. The data that I employ is transaction-level customs
data for Mexican exporters for the period 2000-2007. The sources for this data are detailed in the Annex of Cebeci et al. [2012]. The cross-sectional features of the Mexican transactions-level trade data is consistent with the stylized facts that have informed trade models over the last decade which emphasize the importance of selection into exporting to account for the patterns observed in transaction-level trade data. Tables 1 and 2 show that, consistent with models of selection into exporting, Mexican exports are driven by a relatively narrow set of “superstar” exporters who sell multiple products in multiple destinations. The majority of exporters sell only one product in one destination, but they command a very low share of aggregate trade volumes. Further details regarding the cross-sectional features of the Mexican trade data can be found in Cebreros [2014].

<table>
<thead>
<tr>
<th>No. destinations</th>
<th>No. products</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5+</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44.3</td>
<td>1.8</td>
<td>0.5</td>
<td>0.3</td>
<td>0.4</td>
<td>47.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10.7</td>
<td>2.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.4</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.3</td>
<td>1.4</td>
<td>0.5</td>
<td>0.3</td>
<td>0.4</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.3</td>
<td>1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.4</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>5+</td>
<td>12.9</td>
<td>3.5</td>
<td>2.1</td>
<td>1.4</td>
<td>5.6</td>
<td>25.5</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>76.5</td>
<td>9.9</td>
<td>4</td>
<td>2.4</td>
<td>7.2</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Distribution by Share of Exporting Firms (2004)

<table>
<thead>
<tr>
<th>No. destinations</th>
<th>No. products</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5+</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>0.2</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>0.3</td>
<td>0.1</td>
<td>0</td>
<td>0.2</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>5+</td>
<td>24</td>
<td>8.4</td>
<td>3.2</td>
<td>3.5</td>
<td>54.6</td>
<td>93.7</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>27.8</td>
<td>9.3</td>
<td>3.5</td>
<td>3.8</td>
<td>55.6</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Distribution by Share of Export Value (2004)

2.1. New Exporter Dynamics. I concentrate my attention on the cohort of exporters whose first period reporting positive exports is 2001 (i.e. “new” exporters) and track the outcomes of these firms over the 2001-2007 period. Figure 2.1 documents two prominent features of new exporter dynamics present in the Mexican firm-level trade data: panel (a) documents that continuation rates are increasing with tenure, while panel (b) documents the decreasing average growth of export.

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4This data was collected by the Trade and Integration Unit of the World Bank Research Department as part of their efforts to build the Exporters Dynamics Database.

Table 3 presents summary statistics for the evolution of the 2001 cohort of exporters over the sample period. Column 1 presents the fraction of the cohort that is active. By focusing on the subset of exporters who maintained a continued export presence throughout the sample period we can see the effect of tenure on average growth absent selection effects.

The exit hazard is mechanically related to the exporter continuation rate.
within a given year. What immediately stands out is the sharp drop in export participation that occurs in the first two years of the cohort’s life: after the first year only 35 percent of the cohort will continue to serve the foreign market and after two years only 26 percent of the cohort will export. Columns 2 and 3 display the mean number of products exported and mean number of destinations served by the cohort, respectively. It can be appreciated that the foreign market presence of new export entrants is established over a number of years. Conditional on survival, firms will gradually expand in foreign markets by reaching more destinations and doing so with more products.

<table>
<thead>
<tr>
<th>Year</th>
<th>% of Cohort Active</th>
<th>Mean No. of Products</th>
<th>Mean No. of Destinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>100</td>
<td>1.7</td>
<td>1.2</td>
</tr>
<tr>
<td>2002</td>
<td>35.24</td>
<td>2.9</td>
<td>1.5</td>
</tr>
<tr>
<td>2003</td>
<td>26.23</td>
<td>3.2</td>
<td>1.6</td>
</tr>
<tr>
<td>2004</td>
<td>23.75</td>
<td>5.3</td>
<td>1.8</td>
</tr>
<tr>
<td>2005</td>
<td>21.83</td>
<td>6.8</td>
<td>1.9</td>
</tr>
<tr>
<td>2006</td>
<td>18.96</td>
<td>6.6</td>
<td>2.0</td>
</tr>
<tr>
<td>2007</td>
<td>17.13</td>
<td>7.3</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Table 3. Evolution of Cohort of New Exporters (2001 cohort)

To isolate the dynamics of firm-level adjustment in export presence conditional on survival, I turn attention to the subset of firms that started exporting in 2001 and maintained a continued export presence throughout the sample. I will refer to these firms as “long term survivors”. By focusing on this subset of firms I can isolate the effects of firm-level adjustment from selection effects that can also affect aggregate cohort dynamics.\(^7\) A better understanding of the forces behind the dynamics observed in panel (b) of Figure 1 can be gained by expressing total firm exports as \(X_{ft} = D_{ft}M_{ft}x_{ft}\), where \(D_f\) is the number of destinations served, \(M_f\) is the number of products exported, and \(x_f\) is the firm’s intensive margin (i.e. average sales per product per destination). This expression serves as the basis for the following decomposition:

\[
\frac{1}{N} \sum_{f=1}^{N} \ln \left( \frac{X_{ft+1}}{X_{ft}} \right) = \frac{1}{N} \sum_{f=1}^{N} \ln \left( \frac{D_{ft+1}}{D_{ft}} \right) + \frac{1}{N} \sum_{f=1}^{N} \ln \left( \frac{M_{ft+1}}{M_{ft}} \right) + \frac{1}{N} \sum_{f=1}^{N} \ln \left( \frac{x_{ft+1}}{x_{ft}} \right),
\]

where

\[
\begin{align*}
(1) & = \text{ Average growth rate of exports} \\
(2) & = \text{ Average growth rate of no. of destinations served} \\
(3) & = \text{ Average growth rate of no. of products exported} \\
(4) & = \text{ Average growth rate of intensive margin.}
\end{align*}
\]

\(^7\)Though, in general equilibrium, the dynamics for survivors will be influenced by selection.
Table 4 reports the result for this decomposition. Column 2 reports the average growth rate of exports, while columns 3-5 report the share of export growth explained by each margin. From Table 4 we can see that adjustments along the intensive margin play the most important role in accounting for the growth in export sales of new export entrants. This finding is consistent with the results of Besedes and Prusa [2011] who find that the deepening of export relationships (exporting more products more intensively to a given market) is the key driving force behind export growth. Albornoz et al. [2012] also find that the intensive margin of firm adjustment is economically sizable for a sample of Argentinian exporters. These authors find that unconditional intensive-margin growth is 20% in their sample, while the intensive-margin growth of new exporters is 23 percentage points higher in a firm’s initial period of export activity in a market, and this effect is larger if the market is the firm’s first. Given the significance of the intensive margin for new exporter growth, in section 5 I use my estimated model to assess the role played by self-discovery in shaping the adjustment of exporters along the intensive margin.

<table>
<thead>
<tr>
<th>$t + 1$</th>
<th>Growth</th>
<th>Destinations</th>
<th>Products</th>
<th>Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>84.60</td>
<td>12.21</td>
<td>34.51</td>
<td>53.28</td>
</tr>
<tr>
<td>2003</td>
<td>22.26</td>
<td>9.14</td>
<td>3.20</td>
<td>87.66</td>
</tr>
<tr>
<td>2004</td>
<td>13.09</td>
<td>76.12</td>
<td>267.36</td>
<td>-243.48</td>
</tr>
<tr>
<td>2005</td>
<td>11.65</td>
<td>38.22</td>
<td>112.78</td>
<td>-51</td>
</tr>
<tr>
<td>2006</td>
<td>4.69</td>
<td>17.57</td>
<td>-36.75</td>
<td>119.18</td>
</tr>
<tr>
<td>2007</td>
<td>-18.93</td>
<td>2.81</td>
<td>4.21</td>
<td>92.99</td>
</tr>
</tbody>
</table>

Table 4. Margins of Adjustment for New Exporters

Figures 2.2-2.3 present the evolution of the distribution of number of products exported and number of foreign destinations served for long-term survivors. Figure 2.2 shows that, to a first approximation, the distribution of number of export destinations in $t + 1$ first-order stochastically dominates the distribution of number of export destinations in $t$. The same is true for the distribution of number of products presented in Figure 2.3. That is, as time goes by, it becomes increasingly likely for long-term survivors to reach more destinations and to do so with more products. These figures suggest that new exporters not only experience substantial adjustment after export entry, but that the magnitude of this adjustment decreases gradually.

8In 2004 Mexican exporting activity experienced an upsurge that affected incumbent and new exporters alike. This upsurge in exporting activity occurred mostly along the extensive margins of trade as documented in Cebreros [2014]. This can be seen in Table 4 where for 2004 and 2005 growth in export sales was driven by adjustments along the extensive margin. Subsequently, the intensive margin of adjustment for new exporters regains its prominence in explaining changes in firm export sales.

9Figures 2.2 and 2.3 present the counter-cumulative distribution functions constructed along the lines suggested in Gabaix and Ibragimov [2009]. For clarity I only present the distributions for 2001, 2004 and 2007. The distributions for the missing years respect the pattern suggested by these figures.
over time. That is, the magnitude of adjustment in the extensive margins of number of products and number of destinations is inversely related to tenure in the export market.

Figure 2.2. Distribution of Number of Destinations for 2001 Long-Term Survivors

Figure 2.3. Distribution of Number of Products for 2001 Long-Term Survivors

The results presented here show that new exporters experience substantial adjustment after export entry, and that the magnitude of these adjustments decreases over time as firms transition from inexperienced to well established exporters. Figures 2.1-2.3 suggest that that the internationalization process of new exporters is a slow
process with substantial risk of failure early in the firm’s export tenure. In what follows I show how introducing self-discovery into an otherwise standard model of export supply goes a long way in explaining the features of the data presented in Figure 2.1, and use the model to quantify the importance of learning in the export supply decisions of new exporters.

3. An Empirical Model of Export Supply with Self-Discovery

In this section I present a model of export supply that introduces firm learning into an otherwise standard trade model in a way that is both tractable and amenable to estimation. As such, the model will rely heavily on several key assumptions. First, I start with what has become by now a standard framework for studying the export supply decision (see Melitz [2003] and Bernard et al. [2011]). The domestic and export markets are assumed to be segmented and monopolistically competitive. Specifically, I assume that the demand side of the economy is described by underlying CES preferences that lead to the following revenue functions for the firm:

\[
\begin{align*}
\text{(Domestic Revenues)} & : r(q_t) = q_t^{\frac{\sigma-1}{\sigma}} \\
\text{(Export Revenues)} & : r^*(q_t^*) = h(\theta_t)q_t^{\frac{\sigma-1}{\sigma}}.
\end{align*}
\]

For simplicity I assume that in the domestic market firms face no aggregate or idiosyncratic uncertainty and normalize the demand shifter to unity. In the export market all the uncertainty faced by the firm is idiosyncratic uncertainty regarding its export profitability which is captured through the term \(\theta_t\) which is unknown to the firm. That is, firms face uncertainty regarding the strength of demand for their product in the foreign market. The function \(h : \mathbb{R} \rightarrow \mathbb{R}_+\) is assumed to be continuous and bounded.

The analysis here focuses on the microeconomic foundations of the dynamic behavior of firm level export supply decisions. Thus, I abstract from general equilibrium effects by focusing on the firm level export supply decision taking the demand shifter in

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10 A firm is a producer of one of the differentiated varieties available for consumption in the economy.

11 This simplification highlights that the focus here is on domestically established firms which have the potential to export, but have not yet started to do so.

12 I refer to “export profitability” as the firm’s potential to earn revenues in the foreign market.

13 The representation of the firm’s dynamic optimization problem through the functional equation defined by the Bellman operator necessitates the period return function to be bounded in the firm’s state variables. Restricting \(h(\cdot)\) to be bounded guarantees that the firm’s period return function is bounded in the relevant state variables.
the foreign market as given. Implicitly, the demand shifter $h(\cdot)$ is absorbing the
aggregate demand shifter in the foreign market.\footnote{These firms are measure zero and take demand shifters in the home and foreign market as given. The demand shifter is normalized to unity in the domestic market. In general equilibrium the demand shifter in the foreign market would be determined by: (a) aggregate expenditures on foreign goods and (b) price indices. The function $h(\cdot)$ can be time varying, $h_t(\cdot)$, the important thing is that the firm knows the exact form of the demand shifter in the foreign market, but not the realization of $\theta$.}

I assume that
\[
\theta_t = \theta + \varepsilon_t,
\]
where $\varepsilon_t \sim N(0, \nu^{\star})$ are firm specific shocks, independent over time and across firms. $\theta$ is the firm’s “fundamental export profitability”, a persistent component affecting foreign market revenues. I specify the stochastic process for $\{\theta_t\}$ in this way because this specification provides a tractable way to introduce transitory and permanent, but unknown, components affecting firm revenues into a standard trade model.

The distribution of “fundamental export profitabilities” among the potential entrants is known to all (common prior), but no firm knows what its true export profitability is. That is, an export entrant only knows that $\theta$ is a random draw from a normal distribution with mean $\mu_{\theta}$ and precision $\nu_{\theta}$.\footnote{For convenience when studying the firm’s signal extraction problem I parametrize $\varepsilon$ and $\theta$ in terms of their “precision” rather than their standard deviation. That is, $\nu_\varepsilon = 1/\sigma_\varepsilon^2$ and $\nu_{\theta} = 1/\sigma_{\theta}^2$, where $\sigma_\varepsilon^2$ is the standard deviation for $\varepsilon$ and $\sigma_{\theta}^2$ is the standard deviation of the distribution characterizing the (common) prior beliefs of firms.} A firm also knows the variance of $\varepsilon$, as well as the exact functional form of $h(\cdot)$ so that this “prior” distribution is updated as evidence comes in.

3.1. Firm’s Static Profit Maximization Problem. In this section I describe the firm’s static profit maximization problem. Conditional on $\theta_t$ and the firm’s export status, total firm revenues are given by
\[
    r_t = y_t^{\sigma-1} + d_t h(\theta_t) y_t^{\sigma-1},
\]
where
\[
d_t = \begin{cases} 
    1 & \text{if the firm exports in period } t \\
    0 & \text{otherwise,}
\end{cases}
\]
and $y_t$ and $y_t^{\sigma}$ are the quantities supplied (and sold) by the firm in the domestic and foreign markets, respectively.

Conditional on export status, profit maximizing firms will equate marginal revenues at home and abroad:
\[
y_t^{\sigma} = d_t [h(\theta_t)]^\sigma y_t.
\]
I define $\tilde{y}_t = y_t + y_t^*$, the firm’s total output. Then, total output can be expressed as $\tilde{y}_t = [1 + d_t (h(\theta_t))^\sigma] y_t$, and I can write the firm’s revenues in terms of it’s scale of operation as

$$r_t = (1 + d_t (h(\theta_t))^\sigma)^{\frac{1}{\sigma}} \tilde{y}_t^\frac{\sigma - 1}{\sigma}.$$

Let $f$ denote the fixed costs of production (paid in units of domestic output) and let $f_x$ denote the fixed costs of exporting. Conditional on $\theta_t$ and the firm’s export status, firm’s choose their optimal scale of operation to maximize profits:

$$\max_{\tilde{y}_t} \left\{ (1 + d_t (h(\theta_t))^\sigma)^{\frac{1}{\sigma}} \tilde{y}_t^\frac{\sigma - 1}{\sigma} - (f + d_t f_x + \tilde{y}_t) \right\}.$$

The CES assumption on the demand side allows me to assume that there are no productivity differences between firms and that all heterogeneity is captured through heterogeneity in the underlying “export profitability” of firms (i.e. heterogeneity in $\theta$, under the CES assumption, is isomorphic to productivity heterogeneity). 16

Firms face a constant marginal cost of production (normalized to unity so that the numeraire is the cost of one unit of output), which implies that the decision to serve each market is separable on the cost side. Therefore, the firm’s profit maximizing scale of operation, conditional on $\theta_t$ and the firm’s export status, is given by

$$\tilde{y}_t = \left(\frac{\sigma - 1}{\sigma}\right)^\sigma (1 + d_t (h(\theta_t))^\sigma).$$

Using this expression for the optimal scale of operation I can express firm profits, conditional on $\theta_t$ and export status, as

$$\Pi(d_t|\theta_t) = \left(\frac{1}{\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma - 1} - f\right) + d_t \left(\frac{1}{\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma - 1} [h(\theta_t)]^\sigma - f_x\right).$$

Taking the expectation over the conditional distribution of $\theta_t$, the firm’s expected profits are given by

$$\Pi(d_t) = \left(\frac{1}{\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma - 1} - f\right) + d_t \left(\frac{1}{\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma - 1} \mathbb{E}[(h(\theta_t))^\sigma | \mathcal{I}_t] - f_x\right),$$

where $\mathcal{I}_t$ denotes the firm’s information set at the outset of period $t$.

In what follows it will proof useful to define $A_t := (\mathbb{E}[(h(\theta_t))^\sigma | \mathcal{I}_t])^{\frac{1}{\sigma}}$. Using this notation it can be shown that a modified certainty-equivalence result holds: the firm’s optimal export status decision and optimal scale of operation is the same as

16 However, one key feature of demand as opposed to productivity is that it is likely to be market specific. This is relevant here as I am looking at the decision of domestic firms to enter a second market (the export market).
that of a firm which replaces the unknown \( h(\theta_t) \) by its risk-adjusted expected value \( A_t \) and then proceeds as if there were no uncertainty.\(^{17}\)

With this notation I can express the firm’s optimal scale of operation in the export market as

\[
y_t^* = d_t \left( \frac{\sigma - 1}{\sigma} \right)^\sigma A_t^\sigma,
\]

which displays how both the firm’s optimal export status decision and its scale of operation in the foreign market depends on its beliefs regarding export profitability.

### 3.2. Exporting and Self-Discovery

In this section I describe the self-discovery process of firms. The timing of events is as follows: at the beginning of the period the firm makes a quantity decision based on the information it has accumulated up to that point. After the firm makes its quantity decision demand uncertainty is realized (i.e. \( \theta_t \) is realized). The market clearing price for the firm’s output provides a signal that can be used by the firm to update its beliefs. That is, a firm’s revision of its export profitability depends on how realized revenues \( r_t \) compare to expected revenues \( r_t^e \):

\[
r_t - r_t^e = (h(\theta_t) - A_t) y_t^{e-1}.
\]

If a firm’s revenues at \( t \) are large compared to what it expected, it means that \( \theta_t \) was unusually high and this induces in an upward revision of “export profitability”. Notice that today’s high revenues are transformed into growth as firms use this newly acquired information next period to increase their scale of operation in the foreign market.

Firms use these signals to update their beliefs in a Bayesian manner. Notice, crucially, that the signals \( \theta_t \) are revealed only after the firm has made the decision to export. That is, exporting is a pure “experience good”. I could consider situations in which signals regarding export profitability are realized before the firm decides its export status. For example, exogenous signals, such as the export success of other exporters, could represent a secondary source of information regarding export profitability that is available to the firm before it decides whether to export or not (i.e. exporting could also be an “inspection good”, whose quality can be learned merely by inspecting it). However, as long as potential exporters cannot learn everything they need to know through these other external sources of information, there would still be a role for self-discovery through exporting. For the sake of simplicity here I abstract from such secondary sources of learning for the firm.

\(^{17}\)I refer to \( A_t \) as the risk-adjusted expected value in analogy to its usage in non-expected utility theory where utility \( U_t \) is defined recursively as the solution to the non-linear stochastic difference equation

\[
U_t = \left[ (1 - \beta) c_t^{\pi} + \beta \left( \frac{E_t U_{t+1}^{\alpha}}{1 + \rho} \right)^{\frac{\pi}{\rho}} \right]^{\frac{1}{\pi}},
\]

and \( [E_t U_{t+1}^{\alpha}]^{\frac{\pi}{\rho}} \) represents the risk-adjusted continuation value.
I study the firm’s signal extraction problem and Bayesian updating by utilizing the Kalman Filter.\textsuperscript{18} Let $z_t = \theta$, which I interpret as the hidden value of “export profitability”. Then, the firm’s learning problem can be posed in the state-space representation of the Kalman Filter:

\begin{align*}
\text{(Evolution of Unobserved State)} & : z_{t+1} = z_t \\
\text{(Observation Equation)} & : \theta_t = z_t + \varepsilon_t; \varepsilon_t \sim \text{i.i.d.} \mathcal{N} (0, \nu_\varepsilon) \\
\end{align*}

where $\nu_\varepsilon = 1/\sigma_\varepsilon^2$ and $\nu_\theta = 1/\sigma_\theta^2$.

It will be convenient to define $\mu_t \equiv \mathbb{E} [\theta | \theta^{t-1}]$ and $\sigma_t^2 = \mathbb{E} [(\theta - \mu_t)^2 | \theta^{t-1}]$, which capture the firm’s current beliefs about its true “export profitability” $\theta$. Then, the Kalman Filter implies that $\mu_t$ and $\nu_t = 1/\sigma_t^2$ evolve according to a controlled first-order Markov process, with transition equations for the mean and precision given by

\begin{align*}
\mu_{t+1} &= \mu_t + d_t \left( \frac{\nu_\varepsilon}{\nu_t + \nu_\varepsilon} \right) (\theta_t - \mu_t) \\
\nu_{t+1} &= \nu_t + d_t \nu_\varepsilon \\
\mu_0 &= \mu_\theta, \ \nu_0 = \nu_\theta \text{ given}. \\
\end{align*}

Additionally, the Kalman Filter implies the following conditional distributions

\begin{align*}
\mu_{t+1} | \theta^{t-1} & \sim \mathcal{N} \left( \mu_t, \frac{\nu_\varepsilon}{\nu_t (\nu_t + \nu_\varepsilon)} \right) \\
\theta_t | \theta^{t-1} & \sim \mathcal{N} \left( \mu_t, \frac{1}{\nu_t + \nu_\varepsilon} \right). \\
\end{align*}

The firm’s level of uncertainty, as captured by the precision $\nu_t$, evolves independently of the realization of signals: it only depends on the fact that a signal was received. This will offer a key simplification to the solution of the firm’s dynamic optimization problem. On the other hand, the evolution of the prior mean $\mu_t$ will depend on the realization of signals since the new information revealed through observation of the signal, $(\theta_t - \mu_t)$, will determine the direction in which the firm updates it’s beliefs regarding the mean of “export profitability”.

The pair $(\mu, \nu)$ are sufficient statistics for the firm’s information (i.e. beliefs regarding export profitability). Since $\nu_t$ evolves deterministically, the transition equation for $\nu$ readily implies that

$\nu_t = \nu_\theta + n_t \nu_\varepsilon \ \forall t \geq 0$.

\textsuperscript{18}DeGroot [1970] and Ljungqvist and Sargent [2012] provide a comprehensive discussion of the theoretical relationship between Bayesian updating and the use of the Kalman filter as a device for signal extraction.
where \( n_t = \sum_{t=0}^{t-1} d_t \) is equal to the total number of periods on which the firm has decided to export before period \( t \). That is, tenure in the export market is a sufficient statistic for the precision of the firm’s beliefs regarding its export profitability.

Because I am interested in the relationship between export tenure, growth and survival as new exporters enter and exit from the foreign market, it proves useful to replace \( \nu_t \) with \( n_t \), the firm’s “export tenure”, as a state variable, with \( n \) evolving according to \( n_{t+1} = n_t + d_t \). This implies that the risk adjusted value of \( h(\theta_t) \) is a function of \( (\mu, n) \) alone: \( A_t = A(\mu_t, n_t) \). Therefore, I may write the (expected) export profits as \( \pi(d, \mu, n) = d \tilde{\pi}(\mu, n) \), where

\[
\tilde{\pi}(\mu, n) = \left( \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right)^{-1} \left[ A(\mu, n)^{\sigma} - f_x \right] \right).
\]

With \( h(\cdot) \) bounded, \( A(\cdot, \cdot) \) is also bounded and so are per period (expected) export profits \( \tilde{\pi}(\mu, n) \) for any \( (\mu, n) \in \mathbb{R} \times \mathbb{R}_+ \).

Per-period expected export profits are non-decreasing in \( \mu \) and non-increasing in \( n \). To see why, other things equal, expected export profits are non-increasing with export tenure consider two firms \( i \) and \( j \) with the same beliefs regarding mean profitability \( \mu_i = \mu_j \), but with different tenures \( n_i \neq n_j \). Suppose that \( n_i > n_j \), then the conditional distribution for \( \theta_t \) for both firms is centered about the same value, but the firm with a longer export tenure has more precise information and thus its distribution is more compressed about the common mean. With a more compressed distribution, higher values for \( h(\theta_t) \) are assessed as less likely by the firm which results in calculating a lower risk-adjusted value for \( h(\theta_t) \). This in turn leads to a lower optimal scale of operation, which results in lower expected export profits: \( \tilde{\pi}(\mu, n_i) \leq \tilde{\pi}(\mu, n_j) \).

Firm uncertainty regarding export profitability means that past experience in the export market will affect a firm’s information set, which in turn will affect their current choices. The dependence of information sets on export tenure will generate state dependence. The state dependence generated through the process of self-discovery gives the model an interesting dynamic component with firms adjusting their presence in the foreign market gradually as information comes in and in which export tenure is an important determinant of firm growth in the foreign market.

3.3. The Export Market Participation Rule: Firm’s Dynamic Optimization Problem. Firms will optimally choose to serve the export market depending on: (a) their beliefs regarding their export profitability, and (b) the fixed costs associated with maintaining a presence in foreign markets. Absent any additional sources of uncertainty the firm’s problem would be an optimal stopping problem: given current state variables, if the firm decided to stop exporting it would never re-enter the
export market. The *stopping property* results from the fact that, without any additional sources of uncertainty, there is no reason for the firm to re-enter the foreign market once it has decided to exit.\(^{19}\)

In the data firms are constantly observed to be coming in and out of exporting. To address the model’s capability to rationalize the entry-exit behavior of exporters observed in the data, I assume that fixed costs of exporting at time \(t\) are given by

\[
f_{xt} = f_x + \zeta \epsilon_t,
\]

where \(\zeta > 0\), and where \(f_x\) denotes the observable component of fixed costs and \(\epsilon_t = \epsilon_{1t} - \epsilon_{0t}\) denotes unobserved (by the econometrician) state variables that may affect the decision to export. These fixed costs of serving the foreign market are faced every year and are independent of previous exporting history. I assume that \(\epsilon_{1t}\) are i.i.d. Extreme Value with shape parameter \(\gamma\) equal to the Euler-Mascheroni constant (this implies \(E[\epsilon_{1t}] = 0\)), and independent of the other state variables \((\mu_t, n_t)\). The distribution of \(\epsilon_t\) is approximately Normal, but modeling this unobserved state variable as the difference of Extreme Value distributions offers important computational advantages in terms of solving the firm’s dynamic optimization problem.

Prior to making the export decision, firms observe the current realization of \(\epsilon_t\). Thus, the firm’s state vector is given by \(s_t = (\mu_t, n_t, \epsilon_t)\). Let \(\vartheta\) denote the vector of parameters of the model. The dynamic programming problem characterizing the firm’s optimal export participation choice is given by

\[
V_{\vartheta}(\mu, n, \epsilon) = \max_{d \in D} \left\{ d \left( \tilde{\pi}(\mu, n; \vartheta \epsilon) + \zeta \epsilon \right) + \beta E[V_{\vartheta}(\mu', n', \epsilon') | \mu, n, d] \right\},
\]

subject to the constraints on the evolution of the state variables given in section 3.2. Further details regarding the firm’s dynamic optimization problem can be found in the appendix.

Firms solve a dynamic program with discrete controls: the decision to export or not. Since firms are assumed to be forward-looking, firms make decisions today not only looking at current period payoffs, but also on the effect that choices today have on tomorrow’s information set. Recall that the focus here is on domestically established firms that have the potential to export, but have not yet done so, and their dynamics after export entry. Thus, by the way in which the firm’s value function \(V_{\vartheta}\) is defined it can be interpreted as the value to the firm of having the option to serve the foreign market.

It will be convenient to define

\[
W_0(n, \mu; \vartheta) \equiv \beta E[V_{\vartheta}(n', \mu', \epsilon') | n, \mu, d = 0]
\]

\[
W_1(n, \mu; \vartheta) \equiv \tilde{\pi}(n, \mu; \vartheta) + \beta E[V_{\vartheta}(n', \mu', \epsilon') | n, \mu, d = 1],
\]

\(^{19}\)Unless there was a secular change in the fundamentals of the foreign market which would change the profitability of exporting, such as changes in market size or trade costs.
which are commonly referred to as the “alternative specific” value functions in the discrete choice literature.

Then, I can write the firm’s dynamic optimization problem more compactly as

$$V_{\theta} (n, \mu, \epsilon) = \max_{d \in D} \{ W_d (n, \mu; \vartheta) + \zeta \epsilon_d \} .$$

I define the “exporter premia” as the difference between the alternative specific value functions:

$$\delta (n, \mu; \vartheta) \equiv W_1 (n, \mu; \vartheta) - W_0 (n, \mu; \vartheta) .$$

With this notation I can write the optimal policy rule for the firm as

$$d_t^* = d (n_t, \mu_t, \epsilon_t; \vartheta) = I [\delta (n_t, \mu_t; \vartheta) + \zeta \epsilon_t > 0] ,$$

where $I [\cdot]$ is an indicator function.

Using the definitions for $W_0$ and $W_1$, the exporter premia is given by

$$\delta (n, \mu; \vartheta) = \bar{\pi} (n, \mu; \vartheta) + \beta \left[ \mathbb{E} \left[ V_{\theta} \left( n', \mu', \epsilon' \right) \mid n, \mu, d = 1 \right] - \mathbb{E} \left[ V_{\theta} \left( n', \mu', \epsilon' \right) \mid n, \mu, d = 0 \right] \right] .$$

This model based definition of the exporter premia differs from that commonly estimated in reduced form regressions. In particular, this definition of the exporter premia crucially includes the option value created for the firm from the advantage that additional information can have on deciding tomorrow’s optimal scale of operation in the foreign market and optimal export market participation decision. Thus, the premium to becoming an exporter is composed of two terms: (i) $\bar{\pi} (n, \mu; \vartheta)$ the current period (expected) payoff from serving the foreign market; and (ii) the “gains from trial”:

$$G (n, \mu; \vartheta) \equiv \beta \left[ \mathbb{E} \left[ V_{\theta} \left( n', \mu', \epsilon' \right) \mid n, \mu, d = 1 \right] - \mathbb{E} \left[ V_{\theta} \left( n', \mu', \epsilon' \right) \mid n, \mu, d = 0 \right] \right] .$$

The “gains from trial” arise from the fact that by exporting the firm receives information that allows it to decrease the amount of uncertainty regarding export profitability. This option value arises from the forward-looking nature of the firm’s optimal export status decision and the state dependence that self-discovery induces in the firm’s information set. This results in a key difference in relation to static models of export supply a la Melitz [2003]. The “gains from trial” are akin to the option value of exporting that arises in models with sunk entry costs (see, for example, Das et al. [2007]): by not exporting the firm forgoes a (possibly positive) stream of profits in the foreign market. However, by exporting today, even possibly at a loss, the firm acquires the option to not export tomorrow based on more precise information regarding the payoffs from serving the export market.

The “gains from trial” are approximately given by

$$G (n, \mu; \vartheta) \simeq W_0 (n + 1, \mu; \vartheta) - W_0 (n, \mu; \vartheta) ,$$

the change in the value of not exporting when this decision is made with more precise information regarding the firms true export profitability (see the appendix for details).
In this section I describe the parametrization and estimation of the model outlined in section 3. The structural parameters are estimated using simulation methods. Within the estimation procedure, the dynamic programming problem defining the firm’s optimal policy rule is solved for each guess of the parameter vector. Using this parameter vector and corresponding policy rule, an artificial dataset is simulated from which moments are computed for a moment matching exercise. The following sections discuss these points in detail.

4.1. Parametrization. For the purposes of estimation I need to specify a functional form for the function $h(z)$. Here I assume that $h(z) = \kappa \exp(-\lambda \exp(-gz))$, where $\kappa > 0$ and $\lambda, g > 0$. Under this functional form assumption:

1. $h(z) \geq 0$ for all $z \geq 0$.
2. $h(\cdot)$ is continuous, differentiable, and monotone increasing.
3. $h(\cdot)$ satisfies
   \[
   \lim_{z \to -\infty} h(z) = 0.
   \]

This functional form assumption imposes the boundedness condition assumed in section 3, while allowing for flexibility in the shape that $h(\cdot)$ can take on its domain.\(^{20}\) The parameter $\kappa$ controls the upper bound for $h(\cdot)$, while $\lambda$ and $g$ affect the growth rate of $h(\cdot)$. Observe that $\kappa$ plays an interesting role: if $\kappa \leq 1$ then, for the same scale of operation, revenues in the foreign market are strictly lower than domestic revenues. On the other hand, if $\kappa > 1$ then, for the same scale of operation, the firm can (potentially) earn larger revenues in the export market than it can in the home market. In the model, the maximum foreign market revenues attainable for the firm are entirely determined by $\kappa$ and $\sigma$:

\[
\max_{r_t} = \kappa^\sigma \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1}.\]

\(^{20}\)Because this is a bounded, positive and monotone function, $h(\cdot)$ must be “S-shaped” on this domain. However, the parameters $\lambda$ and $g$ provide flexibility in terms of the displacement along the $x-axis$ and growth rate, respectively.

\(^{21}\)In the model, observed export revenues are given by $r_t = h(\theta_t) y_t^{\frac{\sigma-1}{\sigma}}$, where $y_t^\sigma$ is the optimal scale of operation. Given the assumed functional form assumption for $h(\cdot)$, the risk-adjusted expected value of $h$ is given by

\[
A_t = \mathbb{E}[(h(\theta_t))^{\sigma} | \mathcal{F}_t]^{\frac{1}{\sigma}} = \kappa \mathbb{E}[\exp(-\lambda \exp(-g\theta_t)) | \mathcal{F}_t]^{\frac{1}{\sigma}},
\]

where $0 \leq \mathbb{E}[\exp(-\lambda \exp(-g\theta_t)) | \mathcal{F}_t]^{\frac{1}{\sigma}} \leq 1$. Thus, export revenues can be written as

\[
r_t = \kappa^\sigma \exp(-\lambda \exp(-g\theta_t)) \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \mathbb{E}[\exp(-\lambda \exp(-g\theta_t)) | \mathcal{F}_t],
\]

which immediately implies the given upper bound for export sales.
I assume that $\beta$, the time discount factor, and $\sigma$, the CES elasticity of substitution, are known and set $\beta = 0.96$ and $\sigma = 5$. This value for the time discount factor is standard in the literature and it is the one used by Alessandria et al. [2013]. The choice for $\sigma$ draws on various sources. Alessandria et al. set $\sigma = 5$ to generate a 25% markup for firms; Broda and Weinstein report that for the period 1990-2001 the average elasticity of substitution was 8 for 10-digit (HTS) goods and 4 within 3-digit goods. Furthermore, Lai and Trefler (2002) report an estimated elasticity of substitution of approximately 5 for various econometric specifications considered by the authors. In particular, their maximum likelihood estimate of $\sigma$ is 5.25. Based on these disparate sources of evidence I set $\sigma = 5$.

4.2. Estimation Procedure. Since the model outlined in section 3 involves unobserved state variables, I estimate the remaining parameters $\vartheta = (\nu_0, \mu_0, \nu_0, f_x, \lambda, g, \kappa, \zeta)$ using indirect inference methods as discussed in Gouriéroux and Monfort [2002]. In particular, I use the moment-matching simulation estimator:

$$\hat{\vartheta} (\Omega) = \arg\min_{\vartheta} (\hat{m}_d - \hat{m} (\vartheta))^\top \Omega (\hat{m}_d - \hat{m} (\vartheta)),$$

were $\hat{m}_d$ is a vector of data moments, $\hat{m} (\vartheta)$ are the corresponding simulated moments for parameter vector $\vartheta$, and $\Omega$ the weighting matrix defining a metric for the distance between the data and the simulated moments.\footnote{We do not estimate the model via maximum likelihood because constructing the likelihood for this model imposes a greater computational burden than simulating moments. In particular, the probability of observing a particular export history $d = (d_1, \ldots, d_T)^\top$, which is required to evaluate the likelihood, must be constructed by integration over all histories $\mu$ that are consistent with $d$ since $\{\mu_t\}_{t=1}^T$ is an unobserved state variable. This high-dimensional integral does not have a closed form solution and must be approximated by simulation. These high dimensional integrals have to be approximated for all unique export histories that are observed in the data. Doing so to evaluate the likelihood increases the computational burden relative to the moment matching approach taken here.}

The estimate $\hat{\vartheta}$ is the result of an iterative procedure: for an initial guess $\hat{\vartheta}_1$ I calculate the optimal weighting matrix $\hat{\Omega}_1$ and use this to calculate $\hat{\vartheta}_2 = \hat{\vartheta} (\hat{\Omega}_1)$. This process is repeated until the estimates for $\hat{\vartheta}_j$ converge, yielding the moment-matching simulation estimator $\hat{\vartheta}$. Details of the estimation procedure can be found in the appendix.

4.3. Specifying Moments. For a candidate value $\vartheta$, I simulate the export sales and dynamics of 20,000 firms using the model outlined in section 3. Out of these 20,000 firms I choose the subset of firms which exported in the initial period and track the outcomes of these firms over time in analogy to the cohort of exporters analyzed in section 2. In the data, the 2001 cohort of Mexican exporters is comprised of approximately 13,000 firms.\footnote{Exporting cohorts for 2002-2007 are of a similar size.} By simulating 20,000 firms I obtain new exporting cohorts of roughly the same size as those seen in the data. For the artificial data
I compute a vector of moments $\hat{m}(\vartheta)$ analogous to particular moments $\hat{m}_d$ in the data. The set of moments that I use for estimation are:

1. Mean log Sales (conditional on exporting) for the first three period of the cohort.\(^{24}\)
2. Continuations rates for $n = 0, 1, \ldots, 5$, where $n$ denotes years since export entry.
3. Average export tenure.

In total I use 10 moments to identify 8 parameters. The first-year mean log sales will contain information about the initial scale of operation of firms, and thus about the initial beliefs regarding export profitability $\mu_\theta$ and $\nu_\theta$. Together, the set of moments concerning mean log sales will also provide information relating to the revenue function parameters $\kappa, \lambda, g$. The continuation rates and average export tenure will provide information about the entry-exit behavior of firms which will be informative about the parameters that affect the optimal export status decision of firms such as the fixed costs $f_x$, the rate of learning $\nu_z$, and the size of the idiosyncratic shocks to fixed costs $\zeta$.

4.4. Estimation Results and In-Sample Model Performance. The best fit is achieved at the parameter values reported in Table 5 below. Table 6 reports the data moments used in estimation and their counterparts in the model for the estimated parameter values. Compared to the data, in the model firms live (on average) for slightly longer and start out smaller. The fact that firms start smaller in the model but in their second year reach export sales similar to those observed in the data means that the model will over-predict the first year average growth rate of firms (conditional on survival).

In order to generate the large attrition rate of firms after the first year that is observed in the data the model needs to generate a large mass of firms with relatively low export sales (which is the signal that would tell firms that they are unprofitable exporters) which drags down the mean export sales of the first year in the model. In simulation exercises, attempts to push these two simulated moments closer to their empirical counterparts resulted in a higher discrepancy between the data and simulated moments for second and third year mean log sales. The first-year continuation rate and the first year mean log sales cannot be simultaneously pushed closer to their data counterparts without affecting the value of other matched moments because all parameters jointly determine all moments.

Figures 4.1 and 4.3 serve as a check for over-identification as they compare the predictions of the estimated model for some non-targeted data moments. Figure 4.1 presents the distribution of export tenures (no. of years as an exporter). The

\(^{24}\)Due to partial year effects (see Bernard et al. [2014]), I make an adjustment to the data moment corresponding to the first year mean log sales by assuming that export entrants and their revenues are uniformly distributed over the calendar year.
Table 5. Parameter Estimates

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<th>Parameter</th>
<th>Estimate</th>
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<td>$\nu_c$</td>
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</tr>
<tr>
<td>$\zeta$</td>
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</table>

* hundreds of thousands of dollars

Table 6. Matched Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Export Tenure</td>
<td>2.4</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Continuation Rates

- $n = 0$: 0.36 0.30
- $n = 1$: 0.58 0.50
- $n = 2$: 0.71 0.71
- $n = 3$: 0.80 0.83
- $n = 4$: 0.84 0.88
- $n = 5$: 0.88 0.91

Mean log Sales*

- Year 1: -5.70 -7.2
- Year 2: -5.29 -5.25
- Year 3: -5.06 -5.16

* Mean log Sales are in tens of millions of U.S. dollars.

The model does a good job matching the tail of this distribution: for 4 or more years as an exporter, the data and model frequencies are a good match. However, the model implies that after the large attrition rate of the first year firms are subsequently more likely to re-enter the export market than what is observed in the data. This implies that the model over-predicts the likelihood of 2 and 3 year tenures and under-predicts the likelihood of single-year exporters. That is, in the model, even after receiving a very bad signal about export profitability in the initial period, firms are likely to re-enter the export market after receiving a good enough idiosyncratic shock to their fixed costs of serving the foreign market.

Figure 4.2 graphically depicts the continuation rates presented in Table 6 to more clearly show that the estimated model is able to provide a good fit to continuation rates that are increasing with export tenure as observed in the data. Figure 4.3 presents the growth dynamics of “long-term” survivors. As mentioned above, the large first year attrition rate is generated by the model at the cost of relatively low (average) first year export sales, which results in over-predicting the first-year growth rate. This stands in contrast to models of exporter dynamics based on financial frictions such as Kohn et al. [2014] where first-year growth rates are under-predicted. However, the model with self-discovery does generate the growth dynamics observed...
in the data: very strong average sales growth in the first year, followed by rapidly decaying growth rates.\footnote{In the midpoint of the sample, 2004, there appears to be a generalized increase in all export activity from Mexican exporters (see Cebreros [2014]). Exports as a share of GDP averaged 24.5\% between 2001 and 2004, and increased to an average of 27.6\% between 2005 and 2009. This increase in the share of exports in GDP coincides with a 7\% reduction in the weighted average tariff index for industrial production and an elimination of effectively applied tariffs with Canada and a gradual elimination of this same tariffs with the USA, Mexico’s two largest trading partners (see the World Bank’s World Integrated Trade Solution http://wits.worldbank.org/default.aspx). This suggests that the growth dynamics of long-term survivors observed in the data decayed more slowly than they otherwise would have as firms adjusted to this trade liberalization. Additionally, in the data there is a large drop in export sales towards the end of the sample due to the 2007 financial crisis.}

The evidence presented here shows that the estimated model gives rise to entry-exit behavior and growth that is consistent with the data. In particular, Figures 4.2 and 4.3 show that the model with self-discovery can \textit{qualitatively} and \textit{quantitatively} succeed in explaining the gradual adjustment of new exporters observed in the data, a feat not achieved by the standard sunk entry cost model with productivity heterogeneity (see Ruhl and Willis [2008]).

5. Implications of Self-Discovery for Export Supply: Profits, Option Values, and the Effects of Tenure

In this section I use the parameter estimates of section 4 to calculate option values, probabilities, and scales of operation. These objects will be useful to understand the dynamic adjustment of export supply as firms transition from new to mature exporters. In particular, I will be interested in quantifying the role that self-discovery...
plays in the export supply decisions of new exporters and the length of time in the export market necessary for firms to uncover their true export profitability.

5.1. **Option Values: Quantifying the Gains from Trial.** In section 3 it was argued that the “gains from trial” represent a crucial component of the exporter premia which shapes the export supply decision of firms. I use the estimated model to quantify the importance of this option value for the dynamics of new exporters and to show how the dynamic model differs from a static model of export supply.
To gain further insights into how the exporter premia and the “gains from trial” evolve as a cohort of new exporters matures I will define the “term structure” of the “gains from trial”, conditional on survival. Let $\delta_t$ and $G_t$ be the average export premia and average gains from trial, where the average is taken over the set of firms that export in both $t$ and $t+1$ (i.e. the “continuers”). Figure 5.1 presents the evolution of the share of the “gains from trial” $G_t$ in the exporter premia $\delta_t$:

$$s_{G_t} = \frac{G_t}{G_t + \delta_t}.$$ 

If $s_{G_t} > 1$, then the export premia $\delta_t$ is negative and since $G_t$ is non-negative this implies that expected export profits must be negative. Similarly to Alessandria et al. [2013], I find that new exporters will, on average, earn negative profits on entry. For first time exporters the value they attach to the information gained through serving the export market is the most important component to the value from serving the foreign market. With no previous export experience, the “gains from trial” compensate new entrants for their expected losses to the point of leaving them indifferent between entering the export market or not. Entry of new exporters is driven by temporary below average fixed costs of entering the export market.

Figure 5.1 shows that the initial export period provides a crucial learning experience for first-time exporters and that following the initial participation in the foreign market there there is a very quick and sharp drop in the contribution of the “gains from trial” in the exporter premia. However, Figure 5.1 also shows that there is a positive value to learning over the first four years of the firm’s tenure in the export market. That is, export profitability is not entirely uncovered by the firm in its first year serving the export market. It is only after the discovery stage that the export premia is entirely comprised of expected export profits and learning about the foreign market ceases to have any value for the firm.

To further understand the role of the gains from trial in shaping the export supply decision of firms it is also interesting to understand how the forward-looking behavior of firms affects entry-exit decisions. To do so I simulate a myopic version of the model ($\beta = 0$) and compare this to the forward-looking model ($\beta > 0$). Myopic firms will learn their export profitability in the same way that forward-looking firms do, the only difference is that the export supply decision of myopic firms is entirely shaped by the expected profits in the foreign market. That is, when making export supply decisions myopic firms do not place any value on how serving the foreign market can affect their information sets. Figure 5.2 depicts the difference in continuation values between the forward-looking and myopic models. We can see that in the first three years of tenure in the export market the “gains from trial” has a non negligible effect on the export supply decision of firms, resulting in higher continuation rates for forward-looking firms relative to their myopic counterparts. After this discovery stage, the difference in continuation rates is negligible or non-existing since firms have mostly uncovered their true export profitability and thus the gains from trial play an inconsequential role in determining the firm’s export supply decision.
5.2. The Effects of Tenure on Export Status. How does the probability of serving the foreign market change with export tenure? In section 3 it was shown that tenure is a sufficient statistic for the precision of the firm’s information. Here I quantify how tenure affects the decision to serve the export market. Given the distributional assumptions of section 3, the ex-ante probability of exporting\(^{26}\) given

---

\(^{26}\)By ex-ante probability of exporting I mean the probability of serving the foreign market before the firm observes the idiosyncratic shock to its fixed export costs.
state variables \((\mu, n)\) is given by

\[
\Pr (d = 1|\mu, n) = \left(1 + \exp \left(-\frac{\delta(\mu, n; \hat{\varphi})}{\xi}\right)\right)^{-1}
\]

Since this probability depends on the unobserved state variable \(\mu\), I define

\[
P_t := \left(1 + \exp \left(-\frac{\tilde{\delta}_t}{\xi}\right)\right)^{-1},
\]

where \(\tilde{\delta}_t := H_t^{-1} \sum_{h=1}^{H_t} \delta \left(n^b_t, \mu^b_t; \hat{\varphi}\right)\). Here \(H_t\) denotes the number of firms that have exported every period through \(t - 1\) (i.e., conditional on survival, \(H_t\) is the set of potential exporters in period \(t\)).

Table 7 presents the effects of tenure on the probability of being an exporter. The first row shows how this probability evolves, while the second row shows the evolution of this probability relative to the probability of serving the foreign market for a firm with no previous experience in the export market. To better understand these results, the third row of Table 7 shows the evolution in the (average) export premia of potential exporters. Changes in these rewards to exporting are the driving force behind changes in the likelihood of serving the foreign market. The first thing that can be gleaned from Table 7 is that after the first year there is a large drop in the likelihood of serving the export market. The reason behind this result is a powerful selection effect that affects first-time exporters. Recall that \(H_t\) is the set of, conditional on survival, potential exporters at time \(t\). By definition of the exporting cohort and of \(H_t\), the set of exporters in \(t = 0\) and of potential exporters at \(t = 1\) is the same. The initial exporting period reveals a lot of information to export entrants, and during their first venture into the export market many members of the initial cohort of exporters will receive unfavorable information regarding their export profitability. The third row of Table 7 demonstrates how the revelation of unfavorable information regarding export profitability that drives the high first-year exit rate entails a drop in the average exporter premia for the set of potential second-year exporters. After the sharp attrition rate that occurs during the first year, this selection is dominated by the increase in the export premia of continuing firms and we observe that the exporter premia of the average potential exporter gradually increases giving rise to a positive, but diminishing, effect of tenure on the probability of being an exporter.

<table>
<thead>
<tr>
<th>(t)</th>
<th>(t = 0)</th>
<th>(t = 1)</th>
<th>(t = 2)</th>
<th>(t = 3)</th>
<th>(t = 4)</th>
<th>(t = 5)</th>
<th>(t = 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_t)</td>
<td>0.41</td>
<td>0.28</td>
<td>0.58</td>
<td>0.86</td>
<td>0.94</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>(P_t/P_0)</td>
<td>1</td>
<td>1.06</td>
<td>1.41</td>
<td>2.10</td>
<td>2.29</td>
<td>2.34</td>
<td>2.37</td>
</tr>
<tr>
<td>(\tilde{\delta}_t)</td>
<td>-0.01</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
</tr>
</tbody>
</table>

**Table 7.** Effect of Tenure on the (ex-ante) Probability of Exporting
After a firm has maintained a continuous export presence for 7 years, the (ex-ante) probability that it will serve the foreign market in the current period increases by 137%. During this same time span, “long-term” survivors see their export premia grow by approximately 900% as they develop from new to established exporters. Conditional on survival, the increase in the ex-ante probability of serving the foreign market is concentrated in the first four years of tenure: after the 4th year the ex-ante probability of serving the foreign market has already experienced 95% of its long-term adjustment. These numbers reveal that valuable rewards are available to those firms lucky enough to discover that they can profitably serve the foreign market.

The model of export supply with self-discovery leads to a theory of “noisy” selection in which exporters, through a bit of luck, are able to gradually learn their true export profitability. This process of noisy selection can account for the gradual thinning of active firms in the export market that is observed in the data, and continuation rates that are increasing with export tenure. To further understand how tenure and selection work in the model, I define “export cutoff” \( \mu^* = \mu(n) \) by

\[
\mu(n) = \inf \left\{ \mu : \delta \left( \mu, n; \hat{\theta} \right) > 0 \right\}.
\]

Figure 5.3 plots the evolution of the cutoff for export entry \( \mu^* \). In contrast to the static model of Melitz [2003], where the cutoff for export entry is fixed and only responds to changes in fixed costs and/or changes in the distribution of production heterogeneity, here the cutoff for export entry changes with tenure. At any finite \( t \), the threshold for exporting is more lax than the zero-profit cutoff “at infinity.” As was discussed in section 5.1, early in the firm’s tenure the entry decision is mostly driven by the gains from trial so firms are willing to export even at an expected loss because of the value they attach to gathering information. As information comes in which allows exporters to decrease the amount of uncertainty regarding their true profitability in the export market, firms are able to set export cutoffs more accurately.

The cutoffs for export entry converge from below to the zero-profit cutoff “at infinity” as the value to gathering information decreases over time. Notice, specially, that after the initial year there is a substantial adjustment in the cutoff for export entry. This sharp increase in the cutoff for export entry after the initial year accounts for the large attrition rate of first-time exporters. Figure 5.3 also shows that not all adjustment occurs after the first period: export cutoffs continue to adjust after the first year of tenure in the export market, with 90 percent of the adjustment occurring in the first four years of tenure in the foreign market.

5.3. Implications for the Intensive Margin. Turning to the adjustment of the foreign market presence of new exporters, the results presented in section 2 showed that the deepening of export relationships, by exporting more products and doing so more intensively, is key in understanding the export growth of new exporters. In
particular, the firm’s intensive margin was shown to be the most important channel through which long-term survivors expand their foreign market presence as they mature into well established exporters. I use the estimated model to investigate how self-discovery affects the adjustment of the firm’s optimal scale of operation in the foreign market.

In section 3 it was shown that the firm’s optimal scale of operation in the foreign market was given by

$$y_t^* = \left(\frac{\sigma - 1}{\sigma}\right)^\sigma A_t^\sigma,$$

where $A_t$ was the risk-adjusted expected value of $h(\theta_t)$. Here I write $A_t^\sigma = I\left(\mu_t, n_t; \theta_t\right)$, and decompose the adjustment in the firm’s scale of operation into the effect of receiving more (less) favorable information and the effect of obtaining more precise information as

$$\log y_{i,t+1}^* - \log y_{i,t}^* = \log I\left(\mu_{i,t+1}, n_{i,t+1}; \theta_t\right) - \log I\left(\mu_{i,t}, n_{i,t}; \theta_t\right)$$

$$= \log I\left(\mu_{i,t+1}, n_{i,t+1}; \theta_t\right) - \log I\left(\mu_{i,t}, n_{i,t}; \theta_t\right)$$

$$+ \log I\left(\mu_{i,t}, n_{i,t+1}; \theta_t\right) - \log I\left(\mu_{i,t}, n_{i,t}; \theta_t\right).$$

The first column presents the (average) growth in the intensive margin, while columns two and three decompose this growth into the effect of a change in the beliefs about mean export profitability and the effects of more precise information, respectively.
The first three years are particularly meaningful since mean log sales for the first three years of the cohort were part of the targeted moments used for estimation in Section 4. Table 8 shows that the growth in the foreign market presence of long-term survivors is driven by the effect of the change in beliefs regarding the mean of export profitability: positive information regarding export profitability translates into growth as newly acquired information is used to adjust the optimal scale of operation in the foreign market.

On the other hand, Table 8 reports that the effect of more precise information is to contract the firm’s foreign market presence. From section 3 we know that, conditional on the firm’s information set, the distribution for the revenue shock faced by the firm is given by

$$
\theta_t | \mathcal{I}_t \sim N \left( \mu_t, \frac{1}{\nu_0 + \eta_t \nu_\varepsilon} + \frac{1}{\nu_\varepsilon} \right).
$$

Thus, when the firm receives information that does not lead to a change in its beliefs regarding mean export profitability, the only effect of this additional information is to compress the conditional distribution of \( \theta_t \) about its current mean. This compression implies, in particular, that the firm perceives a decreased likelihood for very high values of the demand shifter which in turn results in a downsizing of the firm’s scale of operation in the foreign market. The results in Table 8 also show that, conditional on survival, the first-year of tenure in the export market reveals a large amount of information to firms which results in high first-year growth rates for continuing firms. Table 8 also shows that during the first four years of tenure in the export market there are non-negligible adjustments along the intensive margin: full adjustment in the firm’s foreign market presence is not attained immediately after surviving the first period.

<table>
<thead>
<tr>
<th></th>
<th>Growth</th>
<th>( \Delta ) Beliefs Mean</th>
<th>( \uparrow ) Precision of Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t + 1 )</td>
<td>179.00</td>
<td>499.36</td>
<td>-320.36</td>
</tr>
<tr>
<td>( t + 2 )</td>
<td>16.53</td>
<td>17.41</td>
<td>-0.88</td>
</tr>
<tr>
<td>( t + 3 )</td>
<td>3.79</td>
<td>3.84</td>
<td>-0.05</td>
</tr>
<tr>
<td>( t + 4 )</td>
<td>0.90</td>
<td>1.22</td>
<td>-0.32</td>
</tr>
<tr>
<td>( t + 5 )</td>
<td>0.05</td>
<td>0.42</td>
<td>-0.37</td>
</tr>
<tr>
<td>( t + 6 )</td>
<td>-0.04</td>
<td>0.26</td>
<td>-0.30</td>
</tr>
</tbody>
</table>

Table 8. Decomposing the Intensive Margin of Firm Adjustment: Long-Term Survivors

To summarize, I have shown that: (i) first-time exporters expect to incur losses by serving the foreign market; the option value generated by the acquisition of more precise information regarding export profitability compensates inexperienced exporters for their losses and entry is driven by below average costs of serving the foreign market; (ii) the “gains from trial” as a share of the export premium remains positive for the first four years of tenure in the export market, after this initial discovery stage the export premia is entirely comprise of expected export profits;
(iii) long-term survivors observe a 137% increase in their ex ante probability of serving the foreign market and a 900% increase in their (average) export premia as they transition from new to mature exporters; 95% of the long-term adjustment in the ex ante probability of being an exporter is attained during the first four years of tenure in the export market; (iv) self-discovery leads to a theory of noisy selection with the cutoff for serving the foreign market converging from below to that static full-information export cutoff; 90% of the adjustment in the cutoff for exporting is realized in the first four years of export tenure, and (v) the growth in the foreign market presence of long-term survivors is led by growth in the intensive margin, where most growth occurs after the initial revelation of information regarding export profitability. However, full adjustment is not attained after surviving the first year, adjustments along the intensive margin continues during the first four years of tenure in the export market. More precise information regarding export profitability leads to a downsizing in the scale of operation of firms. Positive growth for long-term survivors is the result of above expected performance in the export market which is a source of positive information regarding the mean of export profitability.

Using the estimated model I find that, while the first year of tenure in the export market provides a crucial learning experience for firms, export profitability is not uncovered after the first year serving the foreign market. The results of this section suggest that the discovery stage last approximately four years. This result contrasts with the reduced formed evidence presented by Albornoz et al. [2012], who find that uncovering export profitability is attained in the firm’s first year as an exporter. The fact that export profitability is not fully uncovered in the first year implies that, conditional on survival, firm’s will not fully adjust their foreign market presence immediately. Adjustment continues for a number of periods as firms gradually uncover export profitability, and it is this gradual learning that leads to the firm dynamics that is observed in the data concerning growth and survival of new exporters.

6. COUNTERFACTUAL ANALYSIS: THE SPEED OF LEARNING, EXPORT PROMOTION AND IMPLICATIONS FOR AGGREGATE TRADE

How does the speed of learning affect export dynamics? How effective are export promotion policies? In this section I use the estimated model to assess the export supply consequences of the learning environment faced by firms and to study the effectiveness of export promotion policies.

6.1. The Speed of Learning. In section 3 it was shown that the firm’s signal extraction problem and Bayesian updating implies that the precision of the firm’s beliefs regarding its unknown export profitability evolve as

$$\nu_{t+1} = \nu_t + d_t \nu_e$$

where \(\nu_e\) is the precision of the revenue (demand) shocks faced by firms in the foreign market. The rate at which firms increase the precision of their information (i.e. the
speed of learning) is entirely determined by the parameter $\nu$: the more variability there is in the demand shocks faced by the firm, the less information it can extract from its signals.

Firms may face different learning environments if, for example, learning is destination and/or industry specific. On this latter point it is often argued that entrant firms may differ significantly on product characteristics and that producers of customized products are involved in more extended learning processes than are producers of standardized products (see Pedersen and Petersen [2003]). Waller et al. [2001] document that demand variability ranges widely across product categories/industries: basic consumer products exhibit low demand variability, while more differentiated products such as electronics exhibit significantly higher demand variabilities due to their short product life cycles. In the context of the present model, these differences in demand variability across industries can be interpreted as differences in the parameter $\nu$ and thus, as differences in the learning environment faced by firms in different industries.

In this section, I study the export supply consequences of the learning environment faced by firms by considering a counterfactual environment where learning happens more slowly by reducing $\nu$ to 25% of its benchmark value. Figure 6.1 shows the evolution of the gains from trial as a share of the exporter premia for both the benchmark and slow learning case. As might be anticipated, in the slow learning environment the value that firms attach to the option value of making future choice using more precise information is greater as reflected by the higher participation of the gains from trial in the exporter premia. It can also be seen that the value of learning remains positive for approximately 50% longer in the slow learning environment relative to the benchmark. This speaks to the point mentioned above that producers of customized products (where demand is more volatile) undergo learning processes that are more prolonged than those experienced by producers of standardized products.

Figures 6.2 present the effect of a slower learning environment on continuation rates. As might be expected, when it takes more time for firms to uncover their export profitability they are less likely to continue serving the export market. Figure 6.2 shows that in the slow learning environment continuation rates are uniformly lower than in the benchmark case: even when firms receive a very positive signal $\theta$, the amount of information they are able to extract about $\theta$, their “fundamental export profitability”, is small since the signal contains a lot of noise. Thus, firm’s beliefs regarding their export profitability adjust slowly which results in less firms deciding to continue serving the export market.

Evidence presented by Sabuhoro et al. [2006] for Canadian exporters shows that exporters in service providing industries such are business services, construction, and finance (which are more customized) are more likely to exit export markets than firms in good providing industries such as fishing and trapping and logging and
forestry (which are more standardized). This pattern of results is consistent with the findings shown in Figure 6.2.

Sectoral differences in continuation rates can be accounted for by the learning model of section 3 if demand volatility varies by sector since this would imply that producers in different sectors face different learning environments. Also, exporter continuation rates that differ across destinations, as documented by Besedes and Prusa [2011], can be accounted for by the learning model if demand volatility varies across destinations.
6.2. Export Promotion and Aggregate Trade. Over the last two decades national export promotion agencies (EPAs) have tripled and have had a strong and statistically significant impact on aggregate export volumes (see Ledermand et al. [2010]). The case for export promotion is, however, contentious (see Grossman [1998]). Nevertheless, given the popularity of export promotion policies in developing nations and the prominence given to these by policymakers as an integral part of a nation’s development strategy (see Bhagwati [1988]) it is of interest to investigate the impact of these policies on export volumes. As in Jovanovic [1982], the equilibrium is efficient as there are no externalities or market imperfections that warrant intervention. Here the focus is not normative, it is merely a positive evaluation of the type of export promotion policies typically carried out by policymakers and EPA’s (see, for example, OECD [2009]). The objective is as in Roberts and Tybout [1997] and Das et al. [2007]: to understand how micro-founded firm level export dynamics affect aggregate exports in response to changes in the economic environment that affect the profitability of serving the foreign market.  

28

The type of export promotion policy I consider here are direct subsidies to the fixed costs associated with maintaining a foreign market presence. 29 Policy makers justify these type of export assistance programs under the guise that there are exporting firms that would increase their foreign market presence and non-exporters that would start to export, but do not do so because they lack crucial information about foreign markets (see Pursell [2000]). In the current setup, export promotion policies would help firms overcome the key piece of information they are missing: knowledge about persistent demand components in the foreign market.

I simulate the effects of a temporary export subsidy to the fixed costs of exporting of 50, 75 and 100 percent of the benchmark value. 30 From the date at which the EPA makes the subsidy available it lasts for three years (i.e. if the EPA announces the subsidy program at \( t \) the subsidy is available until \( t + 2 \)). I also consider the impact of these trade policies in a counterfactually slow learning environment.

As can be seen from Figure 6.3, temporary export subsidies can have permanent consequences on aggregate trade. The temporarily low cost of serving the export market implies that some unprofitable exporters will remain in the export market.

---

28 Roberts and Tybout (1997) write “Export supply responsiveness is of central concern to the World Bank and its client countries. The success of structural programs has depended significantly on the extent to which strong export responses have followed commercial policy reforms and devaluation...Strong export responses have also enabled countries to quickly reap the efficiency gains associated with larger trade volumes. Unfortunately, export supply responses are not well understood...Seemingly similar reform packages have generated a large range of export responses in different countries and time periods. Policymakers have faced substantial uncertainty whether a given reform package will, for their country, generate the needed response.”

29 For example, in Australia the Export Market Development Grants scheme reimburses up to 50% of eligible export promotion expenses which are above a given threshold.

30 A 100 percent subsidy implies that the EPA completely funds the fixed cost of serving the foreign market.
longer than they should, but it also means that profitable exporters who are unlucky at the outset of their export tenure can remain in the export market long enough to uncover that they can profitably serve the export market. It is precisely these firms which account for the long term increase in trade volumes in response to temporary subsidies.

Figure 6.3. Temporary Export Subsidies and Aggregate Trade

Figure 6.4 shows the effects of the speed of learning on the impact of export promotion. The simulation results presented in figure 6.4 suggest that in the long-run there are no consequences for aggregate trade volumes: in response to a temporary subsidy to the fixed costs of exporting, aggregate trade volumes converge to the same value regardless of the speed of learning. As is clear from this figure, it is during the transition that the speed of learning can affect the influence of export promotion. Over a 15 year horizon, the net present value of the trade that is “lost” under the counterfactually slower learning environment is in the order of 13.6 billion U.S. dollars for a policy which temporarily subsidizes 75% of the fixed costs of exporting.\textsuperscript{31}

Thus, the effectiveness of temporary export subsidies, in terms of engineering aggregate trade growth, is critically affected by the speed at which firms are able to learn their way out of the uncertainty they face in the foreign market.

7. Conclusions

I have developed and estimated a quantitative model of export dynamics featuring self-discovery. The estimated model accounts well for the pattern of export dynamics of new exporters that is observed in the data. In particular, the model is able to

\textsuperscript{31}I use the same discount factor used by firms in section 3, which is equivalent to discounting at a 4% annual real rate of interest.
qualitatively and quantitatively account for the relationship between growth, survival, and tenure in the export market that is observed in the data: (a) continuation rates which are increasing with export tenure, and (b) high initial and subsequent gradual growth of export sales of new exporters.

The model provides a framework that can be used to quantify the role of learning dynamics in shaping the firm level export decision. Additionally, the model can be used to undertake counterfactuals for the effects of trade liberalization on micro and macro export growth. The main results that I obtain from the estimated model and counterfactuals are: (i) first-time exporters expect to incur losses by serving the foreign market; the option value generated by the acquisition of more precise information regarding export profitability compensates inexperienced exporters for their losses and entry is driven by below average costs of serving the foreign market; (ii) while the first-year serving the foreign market provides a crucial learning experience for new exporters, the discovery stage is more prolonged: the value of learning remains positive for the first four years of tenure in the export market. During the discovery stage, the export cutoff experiences 90% of its long-term adjustment and the (ex-ante) probability of serving the foreign market for long-term survivors realizes 95% of its long-term adjustment; (iii) in the transition from new to established exporters, long-term survivors observe a 137% increase in their ex-ante probability of serving the foreign market and a 900% increase in their (average) export premia, and (iv) temporary shocks to the profitability of serving the export market can have permanent consequences on aggregate trade volumes. In particular, export promotion policies that temporarily subsidize the fixed costs of maintaining a presence in the foreign market can result in permanent increases in aggregate trade volumes. However, the impact of these types of policies crucially depends on the speed at which firms are able to uncover their export profitability.
In contrast to the evidence on learning and export dynamics afforded by reduced form specifications such as those considered by Albornoz et al. [2012], by developing and estimating a structural model of export supply featuring self-discovery I was able to quantify the role of learning in shaping the export supply decision of firms. By doing so I found that export profitability is not fully uncovered in the first year as suggested by these authors: the discovery stage lasts for approximately four years. Conditional on survival, firm’s will not fully adjust their foreign market presence immediately. Adjustment continues for a number of periods as firms gradually uncover export profitability, and it is this gradual learning that leads to the firm level export dynamics that is observed in the data concerning the growth and survival of new exporters.

In order to highlight the role that self-discovery plays in explaining the relationship between export tenure, growth and survival, the model has abstracted from certain aspects that are important in shaping the internationalization process of new exporters. The focus here has been one of partial equilibrium which allowed me to concentrate my attention on the firm level dynamics induced by self-discovery. It would be interesting to embed this mechanism in a general equilibrium model which would provide the framework for welfare analysis. In particular, a framework for welfare analysis would be necessary if secondary sources of learning, which here have been neglected, are incorporated into the discussion. There is some evidence that incumbent exporters provide informational spillovers for new export entrants (see Roberts and Tybout [1997] and Cadot et al. [2013]) and it would be interesting to include such secondary sources of firm learning as these informational spillovers would warrant a normative analysis since there is a case for policy interventions which compensate exporters for the information externalities they generate.

Other extensions of the basic setup considered here would also be of interest to further understand the dynamics of firm level exports and the internationalization process of new exporters. First, while the decision to acquire information is endogenous in the model presented here, the amount of information acquired is not: all firms learn at the same rate. It would be interesting to incorporate self-discovery into the market penetration cost framework of Arkolakis [2010]. There, the endogenous choice of number of consumers reached by the firm can be linked to the amount of information acquired by the firm if it is assumed that each consumer provides an independent signal regarding the firm’s export profitability (see Akhmetova and Mitaritonna [2013] for an approach along these lines). Second, the extensive margin of number of destinations served is abstracted from. When export profitability is a persistent component that is global in scope, self-discovery could lead to a pattern of sequential expansion in export markets where the magnitude of first-year growth in export sales in a given destination depends on the time in the firm’s export tenure when that market was reached for the first time: first-year growth in export sales is stronger in destinations which are reached earlier on in the firm’s tenure in the export market. This pattern of sequential exporting is discussed and documented.

8. Appendix

8.1. Solving the Firm’s Dynamic Optimization Problem. In this section I provide a more thorough characterization of the firm’s dynamic optimization problem presented in section 3.3. It will be useful to work with a scaled version of the dynamic programming problem that defines the firm’s optimal policy. To that end, I define $v_\theta := \frac{1}{\zeta} V_\theta$ and $w_\theta := \frac{1}{\zeta} W_\theta$ and study the dynamic programming problem

$$v_\theta (n, \mu, \epsilon) = \max_{d \in D} \left\{ w_\theta (n, \mu, \vartheta) + \epsilon_d \right\}.$$ 

Under the assumptions presented in section 3 this dynamic optimization problem satisfies all of the assumptions of Theorem 3.1 in Rust [1988] (also see Rust [1994]), so the value function exists and is unique and the firm’s optimal policy can be determined from the Bellman equation representing the firm’s problem.

The assumptions made in section 3 allow for a more detailed characterization of the solution to the firm’s dynamic programming problem. Under the assumption that the unobserved state variables $\epsilon$ are independent of the other state variables, the expected value function can be written as

$$E \left[ v_\theta (n', \mu', \epsilon') | n, \mu, d \right] = E_{\mu'} \left[ E_{\epsilon'} \left[ v_\theta (n + d, \mu, \vartheta) \right] | n, \mu, d \right].$$

I define $W_\theta (n', \mu') \equiv E_{\epsilon'} \left[ v_\theta (n', \mu', \epsilon') \right]$, which allows me to write the expected value function as

$$E \left[ v_\theta (n', \mu', \epsilon') | n, \mu, d \right] = E_{\mu'} \left[ W_\theta (n + d, \mu') | n, \mu, d \right].$$

Claim. Under the distributional assumption for $\epsilon$, the expected value function $W_\theta (n, \mu)$ can be expressed as

$$W_\theta (n, \mu) = \ln \left[ \exp \left( w_0 (n, \mu; \vartheta) \right) + \exp \left( w_1 (n, \mu; \vartheta) \right) \right],$$

where $w_0$ and $w_1$ are the alternative specific value functions.

Proof. I prove the claim in two steps. First, I prove that if $\epsilon_i \sim F_{EV} (\cdot; \gamma)$, where $F_{EV} (x; \gamma) = \exp \left\{ - \exp \left\{ - (x + \gamma) \right\} \right\}$ is the CDF of an extreme value distribution with parameter $\gamma$ equal to the Euler-Mascheroni constant ($\simeq 0.577$), and $v_i$ are constants, then

$$\max_i \{ v_i + \epsilon_i \} \sim F_{EV} \left( \cdot; \gamma - \log \left[ \sum_i \exp (v_i) \right] \right),$$

with $E \left[ \max_i \{ v_i + \epsilon_i \} \right] = \log \left[ \sum_i \exp (v_i) \right]$. 


Notice that
\[
\Pr\left(\max_i \{v_i + \epsilon_i\} \leq x\right) = \Pr\left(v_1 + \epsilon_1 \leq x, \ldots, v_I + \epsilon_I \leq x\right)
\]
\[
= \prod_i \Pr\left(v_i + \epsilon_i \leq x\right) \quad \text{(by independence)}
\]
\[
= \prod_i \exp\left\{-\exp\left\{-\left(x + \gamma - v_i\right)\right\}\right\}
\]
\[
= \exp\left\{-\sum_i \exp\left\{-\left(x + \gamma - v_i\right)\right\}\right\}
\]
\[
= \exp\left\{-\exp\left[-(x + \gamma)\right]\exp\left[\log \sum_i e^{v_i}\right]\right\}
\]
\[
= \exp\left\{-\exp\left[-(x + \xi)\right]\right\},
\]
where \(\xi = \gamma - \log \sum_i e^{v_i}\). The last line is just the CDF for an extreme value distribution with parameter \(\xi\).

Now, if \(x \sim F_{EV}(\cdot; \gamma)\), then \(\mathbb{E}[x] = \delta - \gamma\), where \(\delta\) is the Euler-Mascheroni constant (thus, when \(\gamma\) is equal to the Euler-Mascheroni constant \(x\) has an expected value of zero). Applying this result to the random variable \(\max_i \{v_i + \epsilon_i\}\) we have that
\[
\mathbb{E}\left[\max_i \{v_i + \epsilon_i\}\right] = \delta - \xi = (\delta - \gamma) + \log \left[\sum_i \exp(v_i)\right] = \log \left[\sum_i \exp(v_i)\right],
\]
since \(\gamma\) is assumed to be equal to \(\delta\).

Finally, recall from section 3.3 that \(V_{\vartheta}(n, \mu, \epsilon) = \max_{d \in D}\{w_d(n, \mu; \vartheta) + \epsilon_d\}\), so that applying this last result we have that
\[
W_{\vartheta}(n, \mu) = \mathbb{E}_{\epsilon}[v_{\vartheta}(n, \mu, \epsilon)]
\]
\[
= \mathbb{E}_{\epsilon}\left[\max_{d \in D}\{w_d(n, \mu; \vartheta) + \epsilon_d\}\right]
\]
\[
= \ln \left[\exp\left(w_0(n, \mu; \vartheta)\right) + \exp\left(w_1(n, \mu; \vartheta)\right)\right].
\]

\[\square\]

If the firm decides to not serve the foreign market, then its state variables will remain at their current levels. That is, if \(d = 0\), then \(n' = n\) and \(\mu' = \mu\). Therefore, in the case in which the firm decides not to export, the expected value function is given by
\[
\mathbb{E}\left[v_{\vartheta}(n', \mu', \epsilon') | n, \mu, d = 0\right] = W_{\vartheta}(n, \mu),
\]
which from the previous claim and the definition of the alternative specific value functions implies that the value of not exporting is given by

$$W_0(n, \mu; \vartheta) = \beta \ln \{ \exp (w_0(n, \mu; \vartheta)) + \exp (w_1(n, \mu; \vartheta)) \},$$

which is just the discounted expected value of $\max_{d \in D} \{w_d(n, \mu; \vartheta) + \epsilon_d\}$.

For the alternative in which the firm chooses to export, we have that the output from the Kalman Filter implies the following conditional distribution:

$$\mu' \sim N \left( \mu, \frac{\nu_{\theta}}{(\nu_{\theta} + n\nu_{\varepsilon})(\nu_{\theta} + (n + 1)\nu_{\varepsilon})} \right).$$

Thus, in the case $d = 1$ the expected value function is given by

$$\mathbb{E}[\nu_0(n', \mu', \varepsilon') | n, \mu, d = 1] = \mathbb{E}_{\nu_0'} [W_0(n + d, \mu') | n, \mu, d = 1]$$

$$= \int_{-\infty}^{\infty} \ln \{ \exp (w_0(n + 1, \mu'; \vartheta)) + \exp (w_1(n + 1, \mu'; \vartheta)) \} f_{\theta} (\mu' | n, \mu) d\mu',$$

where $f_{\theta} (\mu' | n, \mu)$ is the density of a Gaussian distribution with mean and standard deviation given as above.

From the definition of the alternative specific value functions, we have that the value of choosing to serve the foreign market is given by

$$w_1(n, \mu; \vartheta) = \zeta^{-1} \tilde{\pi} (n, \mu; \vartheta) + \beta \int_{-\infty}^{\infty} \ln \{ \exp (w_0(n + 1, \mu'; \vartheta)) + \exp (w_1(n + 1, \mu'; \vartheta)) \} f_{\theta} (\mu' | n, \mu) d\mu',$$

the sum of expected current export profits and the discounted expected continuation value.

Therefore, the alternative specific value functions are the solution to the functional equations ($FE$):

$$w_0(n, \mu; \vartheta) = \beta \ln \{ \exp (w_0(n, \mu; \vartheta)) + \exp (w_1(n, \mu; \vartheta)) \}$$

$$w_1(n, \mu; \vartheta) = \zeta^{-1} \tilde{\pi} (n, \mu; \vartheta) + \beta \int_{-\infty}^{\infty} \ln \{ \exp (w_0(n + 1, \mu'; \vartheta)) + \exp (w_1(n + 1, \mu'; \vartheta)) \} f_{\theta} (\mu' | n, \mu) d\mu'.$$

These functional equations define a contraction mapping which possess a unique fixed point for $(w_0, w_1)$ as shown in Rust [1994]. From section 3.3, we know that the firm’s optimal policy is given by

$$d^* = \mathbb{I} \left[ \delta (n, \mu; \vartheta) + \zeta \epsilon > 0 \right],$$

where $\delta (n, \mu; \vartheta) = W_1(n, \mu; \vartheta) - W_0(n, \mu; \vartheta)$. Thus, to solve for the optimal policy function all that is required is to solve the above functional equations for the alternative specific functions $(w_0, w_1)$.

In section 3.3 it was shown that a crucial distinction between this dynamic model and static models of export supply is that the “exporter premia” in the dynamic model includes the “gains from trial”: the value that the firm attaches to gaining more precise information about its true profitability in the export market (information that
can only be acquired by exporting). To further our intuition regarding the “gains from trial” or the option value of exporting, recall that for $d = 0$ the alternative specific value function was given by $w_0 (n, \mu; \vartheta) = \beta \ln [\exp (w_0 (n, \mu; \vartheta)) + \exp (w_1 (n, \mu; \vartheta))]$, thus I can re-write the above expression for $w_1$ as

$$w_1 (n, \mu; \vartheta) = \zeta^{-1} \tilde{\pi} (n, \mu; \vartheta) + \int_{-\infty}^{\infty} w_0 (n + 1, \mu'; \vartheta) f_{\vartheta} (\mu'|n, \mu) \, d\mu',$$

which in turn implies that

$$w_1 (n, \mu; \vartheta) - w_0 (n, \mu; \vartheta) = \zeta^{-1} \tilde{\pi} (n, \mu; \vartheta) + \int_{-\infty}^{\infty} [w_0 (n + 1, \mu'; \vartheta) - w_0 (n, \mu; \vartheta)] f_{\vartheta} (\mu'|n, \mu) \, d\mu'.$$

Thus, the “exporter premia” can be expressed as

$$\delta (n, \mu; \vartheta) \equiv \tilde{\pi} (n, \mu; \vartheta) + \int_{-\infty}^{\infty} [W_0 (n + 1, \mu'; \vartheta) - W_0 (n, \mu; \vartheta)] f_{\vartheta} (\mu'|n, \mu) \, d\mu',$$

from which we readily see that the “gains from trial” are given by

$$G (n, \mu; \vartheta) = \int_{-\infty}^{\infty} [W_0 (n + 1, \mu'; \vartheta) - W_0 (n, \mu; \vartheta)] f_{\vartheta} (\mu'|n, \mu) \, d\mu'.$$

By Taylor’s Theorem there exists $R (\cdot)$, a real-valued function, such that

$$W_0 (n + 1, \mu'; \vartheta) = W_0 (n + 1, \mu; \vartheta) + W_{0,\mu} (n + 1, \mu; \vartheta) (\mu' - \mu) + R (|\mu' - \mu|),$$

where

$$\lim_{\mu' \to \mu} R (|\mu' - \mu|) = 0.$$

Therefore, I can re-write the “gains from trial” as

$$G (n, \mu; \vartheta) = \int_{-\infty}^{\infty} [W_0 (n + 1, \mu; \vartheta) + W_{0,\mu} (n + 1, \mu; \vartheta) (\mu' - \mu) + R (|\mu' - \mu|) - W_0 (n, \mu; \vartheta)] f_{\vartheta} (\mu'|n, \mu) \, d\mu'$$

$$+ \left[ W_0 (n + 1, \mu; \vartheta) - W_0 (n, \mu; \vartheta) \right] + W_{0,\mu} (n + 1, \mu; \vartheta) \mathbb{E}_{f_{\vartheta}} [\mu' - \mu] + \mathbb{E}_{f_{\vartheta}} [R (|\mu' - \mu|)],$$

where $\mathbb{E}_{f_{\vartheta}} [\cdot]$ denotes the expectation taken with respect to the density $f_{\vartheta} (\mu'|n, \mu)$.

Since $\mu'$ has mean $\mu$ under $f_{\vartheta} (\mu'|n, \mu)$, the third term in the above expression drops out, and $\mathbb{E}_{f_{\vartheta}} [R (|\mu' - \mu|)]$ is “small” since most of the mass of $f_{\vartheta} (\mu'|n, \mu)$ is concentrated around $\mu$ and in that neighborhood $R (|\mu' - \mu|)$ is close to zero. Thus, the “gains from trial” are approximately given by

$$G (n, \mu; \vartheta) \simeq W_0 (n + 1, \mu; \vartheta) - W_0 (n, \mu; \vartheta).$$

This is the expression presented in section 3.3.
8.1.1. Numerical Solution to the Firm’s Dynamic Programming Problem. To characterize the optimal policy rule of the firm I need to solve for the alternative specific value functions \( w_0 \) and \( w_1 \), which are the solution to the functional equations:

\[
\begin{align*}
\mu \land n, \mu \land \mu & = \beta \ln \left( \exp (w_0(n, \mu \land \mu)) + \exp (w_1(n, \mu \land \mu)) \right)
\end{align*}
\]

\[
\begin{align*}
w_1(n, \mu \land \mu) & = \zeta^{-1} \tilde{\pi}_n(n, \mu \land \mu) + \beta \mathbb{E} \left[ \ln \left( \exp (w_0(n + 1, \mu \land \mu)) + \exp (w_1(n + 1, \mu \land \mu)) \right) \big| n, \mu, d = 1 \right],
\end{align*}
\]

where \( \mathbb{E} \left[ \cdot | n, \mu, d = 1 \right] \) denotes the expectation taken with respect to the density for \( \theta \sim \mathcal{N}(\theta \land n + \zeta \xi, \theta \land (n + 1) \xi) \),

which is the conditional distribution resulting from the signal extraction problem defined by the Kalman filter.

Observe that the functional equations defining \( w_0 \) and \( w_1 \) involve \( w \) at both \( n \) and \( n + 1 \). Given that to solve for the exporter premium I will work with \( N < \infty \), I need to make an assumption about the exporter premium at \( n + 1 \). Since the underlying learning process implies that exporting will eventually learn their true export profitability, I impose that for sufficiently large \( N \): \( w(N, \mu; \theta) \approx w(N + 1, \mu; \theta) \).

That is, I assume that for sufficiently large \( N \) exporters have gathered enough information such that an additional export episode does not affect their perceived premium for exporting. Assumptions such as this are commonly used in the numerical solution of dynamic programming problems with unbounded state variables whose transition implies that the state variable must be non-decreasing. In practice I choose \( N = 25 \) to solve for the exporter premia.\(^{32}\) The results are not significantly different for \( N = 20 \) or \( N = 30 \).

I solve for the exporter premia numerically as follows: Let \( N = \{0, 1, \ldots, N\} \subset \mathbb{N} \) and \( G_\mu = \{-M, \ldots, -1, \mu_0, \mu_1, \ldots, M\} \subset \mathbb{R} \), where \( \mu_0 = \mu_0 \land \theta \) and \( M = \mu_0 + 2.5\sigma_\theta \). The grid for the unobserved state variable \( \mu \) defined by \( G_\mu \) is such that I cover 99\% of the mass for the initial prior distribution for the unknown export profitability \( \theta \).

I discretize the distributions implied by the Kalman filter over the grid \( G_\mu \) to define transition matrices as follows:

\[
\begin{align*}
\Pr \left( \mu' = \mu_j | \mu = \mu_k, n, d = 0 \right) & = \mathbb{I} \{ j = k \}
\end{align*}
\]

\[
\begin{align*}
\Pr \left( \mu' = \mu_j | \mu = \mu_k, n, d = 1 \right) & = \zeta \left[ \Phi \left( \frac{\Delta_{jk} + 0.5\Delta_j}{\sigma_n} \right) - \Phi \left( \frac{\Delta_{jk} - 0.5\Delta_{j-1}}{\sigma_n} \right) \right],
\end{align*}
\]

where

\[
\begin{align*}
\Delta_{jk} & \equiv \mu_j - \mu_k \\
\Delta_j & \equiv \mu_{j+1} - \mu_j \\
\sigma_n & \equiv \sqrt{\frac{\nu_\xi}{(\nu_\theta + n\nu_\xi)(\nu_\theta + (n + 1)\nu_\xi)}}
\end{align*}
\]

\(^{32}\)What is important is that \( N \gg T \), where \( T \) is the number of time periods for which data is available in the sample.
and \( \zeta \) is a normalizing constant such that \( \sum_j P_{kj}^n = 1 \).

Let \( J = |G_\mu| \), the number of grid points on the grid for the state variable \( \mu \), and let \( \pi (\theta) \) be an \( N \times J \) matrix with typical element

\[
\pi_{nj} (\theta) = \zeta^{-1} \pi (n-1, \mu_j; \theta) .
\]

The following algorithm solves numerically for the exporter premia:

Step 1 - Select an accuracy level \( \varepsilon > 0 \) and an initial guess \((w_0^0 (\theta), w_1^0 (\theta))\) which are \((N+1) \times J\) matrices.

Step 2 - Functional equation step : use the functional equations defined above to solve for \((w_0^{m+1} (\theta), w_1^{m+1} (\theta))\).

For \( k = 1, \ldots, J \):

For \( n = 1, \ldots, N \):

\[
\begin{align*}
    w_{0,nk}^{m+1} &= \beta \ln \left[ \exp (w_{0,nk}^m) + \exp (w_{1,nk}^m) \right] \\
    w_{1,nk}^{m+1} &= \pi_{nk} (\theta) + \beta \sum_j P_{kj}^n \ln \left[ \exp (w_{0,n+1j}^m) + \exp (w_{1,n+1j}^m) \right] \\
    w_{0,N+1k}^{m+1} &= w_{0,Nk}^{m+1} \\
    w_{1,N+1k}^{m+1} &= w_{1,Nk}^{m+1}.
\end{align*}
\]

Step 3 - End of iteration: If

\[
\max \left\{ \max \left| w_{0,nj}^{m+1} (\theta) - w_{0,nj}^m (\theta) \right|, \max \left| w_{1,nj}^{m+1} (\theta) - w_{1,nj}^m (\theta) \right| \right\} \leq \varepsilon
\]

stop; else, increment \( m \) by 1 and return to step 2.

\( P^n \) is the \( J \times J \) transition matrix with typical element

\[
P_{kj}^n = \Pr \left( \mu' = \mu_j | \mu = \mu_k, n-1, d = 1 \right) = \zeta \left[ \Phi \left( \frac{\Delta_{jk} + 0.5\Delta_j}{\sigma_{n-1}} \right) - \Phi \left( \frac{\Delta_{jk} - 0.5\Delta_{j-1}}{\sigma_{n-1}} \right) \right],
\]

as outlined above.

Let \((\tilde{w}_0 (\theta), \tilde{w}_1 (\theta))\) denote the result from the this algorithm. I use these matrices to construct the exporter premium by: \( \delta (\theta) = [\delta_{nj}] = [\zeta (\tilde{w}_{1,nj} - \tilde{w}_{0,nj})] \).

---

33To calculate the per period profits of the firm I need to evaluate the integral \( \mathbb{E} [ (h (\theta_1))^\sigma | \mu, n] \) which defines the risk-adjusted value of \( h (\cdot) \). I calculate this integral using the Gauss-Hermite quadrature method (see Judd [1999] for details).
8.2. Estimation Procedure: Indirect Inference and Moment Matching. I estimate the model’s parameters using indirect inference as described in Gouriéroux and Monfort (2002). I use the iterative procedure described in Dejong and Dave (2007) which proceeds as follows:

Step 1 - Select an accuracy level $\varepsilon > 0$ and an initial guess $\hat{\theta}_0$.

Step 2 - Weighting matrix step: Use $\hat{\theta}_j$ to construct

$$
\Sigma_j = \frac{1}{S} \sum_{s=1}^{S} \left( m_s(\hat{\theta}_j) - \frac{1}{S} \sum_{s=1}^{S} m_s(\hat{\theta}_j) \right) \left( m_s(\hat{\theta}_j) - \frac{1}{S} \sum_{s=1}^{S} m_s(\hat{\theta}_j) \right)'
$$

$$
\Omega_j = \Sigma_j^{-1},
$$

where $m_i(\hat{\theta}_j)$ is the $ith$ of $S$ realizations of model moments under the parameter vector $\hat{\theta}_j$. The matrix $\Omega_j$ is a symmetric non-negative matrix.

Step 3 - Minimization step: Find $\hat{\theta}_{j+1}$ as

$$
\hat{\theta}_{j+1} = \arg \min_{\hat{\theta}} (\hat{m}_d - \hat{m}(\hat{\theta}))' \Omega_j (\hat{m}_d - \hat{m}(\hat{\theta})).
$$

Step 4 - End of iteration: If $\|\hat{\theta}_{j+1} - \hat{\theta}_j\| < \varepsilon$, stop and set $\hat{\theta} = \hat{\theta}_{j+1}$; else, increment $j$ by 1 and return to step 2.

For the minimization step I use a simulated annealing algorithm.

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Princeton University

E-mail address: cebreros@princeton.edu