The Political Economy of Sovereign Defaults

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Abstract

This paper studies how the income distribution and the tax system affect sovereign borrowing and default decisions. Does a more regressive tax system or a more unequal income distribution make a government more prone to default? Do they affect the amount of sovereign borrowing? We analyze these issues taking into account the political restrictions that governments face when raising funds to repay the debt. We perform our analysis by introducing in a standard DSGE model with endogenous sovereign default risk, two novel features: income and tax heterogeneity across households, and a political constraint. The political constraint requires the government to obtain a favourable vote to pass any fiscal package aimed at repaying the debt. We show that inequality makes defaults more likely and increase sovereign spreads; that political constraints sometimes force a government to default in equilibrium even if, in their absence, it might be optimal to repay; and that this effect becomes stronger when there is more income inequality. These results are a novel contribution to the literature on the political economy of sovereign defaults.

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1 Introduction

Since the onset of the Greek sovereign debt crisis, in 2009, the Greek government implemented eight consecutive fiscal austerity packages, all of which were met by increasing social unrest and discontent. In July 2015, Greek Prime Minister Alexis Tsipras, probably fearing a domestic political backlash over a new fiscal austerity package with the Troika, called a referendum about it before continuing the negotiations. The result of the referendum was a massive rejection to the proposal, with the strongest rejection manifested by the most economically vulnerable groups.\(^1\) Social unrest triggered by fiscal adjustment needed to avoid default also arose in Argentina a few years earlier. In the months prior to Argentina’s 2001 sovereign default, the government tried to lower public wages and pensions to be able to repay the debt. This plan met strong political opposition and, in December 2001, in the midst of a popular revolt, President De la Rúa resigned and the country defaulted on its debt. This same story repeated itself in Portugal. In March 2011, the Portuguese government proposed a package of austerity measures to restore fiscal balance and debt sustainability. However, opposition parties refused to back the proposal which led to the resignation of the Prime Minister and prompted the need for a European Union - International Monetary Fund rescue package.

These examples highlight two issues. First, the distribution of the burden of debt repayment across society may involve conflict. Second, in the build-up of sovereign defaults, governments face strong political constraints when they try to raise revenues to repay the debt.

The way that the burden of public debt repayment is shared across society is the result of the interaction between the tax system and income distribution. Thus, it is natural to wonder about its effects on the probability of sovereign default. A few papers provide a partial answer to this question. They show empirically that higher income inequality is associated with a higher default risk.\(^2\) However, we still lack an understanding of the

\(^1\)See Arnett and Galatsidas (2015) for evidence on this regard. Tsipras finally reached an agreement with the Troika over a new fiscal austerity package.

\(^2\)Berg and Sachs (1988) find that higher income inequality is a significant predictor of a higher probability of debt rescheduling, and they attribute this to the difficulties of political management in economies with
mechanisms through which this happens. Existing models of sovereign debt are ill-suited to study these issues, as agents are typically assumed to be homogeneous and, as a result, there is no role for distributional issues.

The role of political constraints restricting the ability of governments to raise resources for public debt repayment has not received much attention in the sovereign debt literature either. Indeed, in standard sovereign debt models the government has unrestricted access to the resources needed to repay debt.

Our paper does away with the representative agent assumption by introducing in a standard dynamic stochastic general equilibrium (DSGE) model with endogenous sovereign default risk, heterogeneity in terms of both the income distribution and the taxes that the agents have to pay. This change makes the model suitable to study the effect of income distribution and tax systems on sovereign borrowing and default decisions. In addition to the heterogeneity, we introduce a voting process in order to capture the potential conflicts of interest that arise around the repayment/default decision. Analyzing the interaction between political constraints and the distribution of the repayment burden across society, allows us to make progress towards understanding the political economy of sovereign defaults.

The basic structure of the model is the following. There is a small open economy inhabited by a benevolent government and a continuum of heterogeneous households. Households differ in the share they receive from the stochastic aggregate income and in the way the fiscal policy affects them. This heterogeneity generates different opinions regarding the convenience of repaying and contracting sovereign debt.

The government borrows from foreign creditors using non-contingent bonds with the objective of smoothing households’ consumption paths. The non-contingent nature of the debt contract captures the actual terms of international financial markets for sovereign debt. If the government wants to repay, it proposes a fiscal program to raise the necessary funds. The fiscal program must achieve an exogenous minimum level of political support from the households in order to be implemented. If the minimum level of political support is not achieved, the government is forced to default, triggering a temporary exclusion from international financial markets and direct output costs.\(^3\) The political constraint, captures extreme inequality. In a more recent paper, Aizenman and Jinjarak (2012) find that a one standard deviation increase of the Gini coefficient is associated with a rise of around 500 to 700 basis points of the sovereign spread.\(^3\)

\(^3\) This approval can be thought of as a referendum as happened in Greece in July 2015 and in Iceland in 2010 and 2011, a formal requirement associated with passing legislation through Congress, or an informal
the fact that governments do not have unlimited access to the country’s resources.

We calibrate the model to the Argentine economy to analyze the effects among the interactions between income distribution, the tax system, and the political constraints on the sovereign borrowing and default decisions.\footnote{We choose Argentina in order to make our results easier to compare with those of the existing literature. See for example Aguiar and Gopinath (2006), Arellano (2008) or more recently Padilla (2015).} We find three main results:

- Inequality is bad for sovereign borrowing: more regressive tax system and more unequal income distributions increase default probability and sovereign spreads. If those with lower income disproportionally shoulder the burden of sovereign debt repayment, the incentives of a social-welfare-maximizing government are tilted towards default.

- Political constraints matter: a proportion of the observed defaults in equilibrium do capture situations in which the government is \textit{unable} to repay because it cannot obtain sufficient political support for the proposed fiscal plan.

- Inequality potentiates political constraints: political constraints have a stronger impact the more unequal is the sharing of sovereign repayment. In the extreme case in which all households are homogeneous, the households and the government have the same preferences, and political constraints play no role. However, as the income distribution and the tax system become more unequal, households diverge in their preferences, making political support a potential issue for debt repayment.

The sovereign default literature that deals with distributional issues has focused mostly on the analysis of the relative power of domestic bond holders and the agents that bear the burden of repayment.\footnote{See, for example, Tabellini (1991), Aghion and Bolton (1990) and Dixit and Londregan (2000) or, more recently, Guembel and Sussman (2009) and D’Erasmo and Mendoza (2015).} Unlike these papers, our mechanism does not rely on assuming that a fraction of the sovereign debt is in the hands of domestic agents. In a recent paper, Azzimonti et al. (2014), analyze the effect of income inequality on the optimal level of sovereign debt in a model without defaults. Our model differs, in that the sovereign default decision is a key element and we have sovereign defaults in equilibrium.

By incorporating political considerations as a potential cause for sovereign default, our model speaks to a large literature on endogenous sovereign defaults. Typically, this literature focuses on the role of political turnover or different government types in triggering approval required to avoid protests, demonstrations and riots.
sovereign defaults. Unlike this literature, our paper has neither governments alternating in power nor different types of governments, it has only one benevolent government that needs political support to repay the debt. The political considerations arise from the agents’ heterogeneity and the need for voters’ support to implement a given tax policy.

The paper is organized as follows: Section II presents the theoretical model, Section III characterizes the equilibrium, Section IV presents the main numerical results, and Section V concludes.

2 The model

Consider a small open economy inhabited by a continuum of households on the unit interval and a benevolent government. Households are risk-averse and have the same preferences. Each household’s income is given by:

\[ y_i' = \alpha_i y, \]  

with \( \int_0^1 \alpha_i \, di = 1 \), where \( \alpha_i \) is the constant share of the aggregate endowment \( y \) that household \( i \) receives. The aggregate endowment follows a Markov process with transition density \( f(y', y) \) defined on the compact subset \( Y \subset \mathbb{R}_+ \). Households derive utility from consumption:

\[ U(c_i) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}), \]

where \( u(c) \) is strictly increasing and concave, and \( \beta \in (0,1) \) refers to the discount factor.

The government maximizes a standard utilitarian social welfare function in which the utility of each household type \( i \) is weighted according to the parameter \( \omega_i \):

\[ W = \int_0^1 U(c_i) \omega_i \, di. \]

The government is the only agent within the small economy that has access to international credit markets. In each period, the government issues one-period zero-coupon bonds and sells them to foreign lenders. The government can be either a debtor or a creditor in

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international credit markets depending on its foreign asset position $B$: if $B < 0$, the government has accumulated external debt and hence it is a debtor; otherwise, the government holds assets and it is a creditor.

A debtor government willing to repay is subject to constraints. First, the fiscal plan proposed by the government has to be resource-feasible--i.e., it has to satisfy the government’s budget constraint:

$$\tau - B' q(B', y) \geq -B, \quad (2)$$

where $B'$ denotes the amount of debt that the government issues in the current period and that promises a payment of $B'$ units of consumption in the following period. When the government issues debt, it obtains $B' q(B', y)$ units of current consumption, where $q(B', y)$ refers to the endogenously-determined unitary price of sovereign bonds. Sovereign bonds are assumed to be non-collateralized and defaultable. The term $\tau$ represents the transfers from the government to the agents, which can be positive (taxes) or negative (subsidies). Transfers can differ across household types in the following way:

$$\tau_i = \alpha_i \chi(\alpha_i) \tau, \quad (3)$$

where the function $\chi(\alpha_i)$ measures the divergence of the tax system from a perfectly proportional system. Note that if $\chi = 1$, taxes are proportional whereas if $\chi$ is increasing (decreasing) on $\alpha$, the tax system is progressive (regressive) We assume $\chi$ to be exogenously given.\footnote{This assumption tries to capture the rigidities that governments usually face in terms of allocating the tax burden across different groups of the population. According to Vegh and Vuletin (2011) tax rates change, on average, about every 5 years for corporate and personal income taxes and every 8 years for value-added taxes. However, we also introduce some flexibility to this assumption by comparing the results under alternative calibrations for $\chi_i$ (see Section 4.2).}

Second, the fiscal program has to be politically-feasible--i.e., it has to satisfy the political support constraint:

$$p(B', \tau; y) \equiv \int_0^1 p_i(B', \tau; y) \, di, \geq p^r. \quad (4)$$

The political support constraint above captures the notion that the government must garner sufficient support from individual households for a fiscal plan to be implementable. The function $p_i(B', \tau; y)$ collects the voting decisions of households $i$, assigning a value of 1 to an approval and a value of 0 to a rejection. The political support function $p(B', \tau; y)$
aggregates individual voting decisions. The parameter \( p \in [0, 1] \) captures the political discretion of the government in terms of the set of policies it can implement to raise funds. A higher \( p \) means that the government has less political discretion, as it requires the approval of a higher share of households to implement a fiscal plan. A literal interpretation of our political constraint is in line with the 2015 Greek referendum or the 2010/2011 Icelandic default, the latter being triggered by a popular binding referendum. An alternative interpretation of this political constraint is to consider the Parliament or Congress as intermediating this vote, as is typically the case with fiscal policy issues.

A debtor government is unable to repay when no fiscal program simultaneously satisfies the resource constraint, (2), and the political constraint, (4). If no fiscal program satisfies the resource constraint, the government lacks enough fiscal capacity to implement the repayment. Alternatively, if no fiscal program satisfies the political constraint but some of them do satisfy the resource constraint, the government lacks the political support required to implement the repayment even though it has access to enough fiscal resource to repay its debt. In either case, the government is forced to default.

A debtor government able to repay still might choose not to repay its debt.

If the government defaults, regardless of the cause, it is temporarily excluded from international credit markets. We take the exclusion period to be exogenous and stochastic. Specifically, the reentry time follows an exogenous Poisson process with flow probability equal to \( \theta \). Once the economy randomly regains market access, we assume that it does so with zero debt. While in autarky, the economy suffers an output loss in its aggregate endowment, and households consume their individual financial autarky endowments, \( y^d_i \), defined as:

\[
y^d_i = c_i h(y),
\]

where \( h(y) \) stands for the output loss function.

Foreign lenders have risk-neutral preferences, behave competitively and can trade both the sovereign bond and a risk-free asset that yields \( r > 0 \). Consequently, they are willing to lend to the government as long as they break even in expected value. Foreign lenders are fully aware of the resource and the political economy constraints that the government

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\( ^8 \)In 2010 and 2011, Icelanders were asked to vote on a binding referendum regarding the repayment of the sovereign obligations that the government had incurred in order to rescue the domestic financial system. While the Icelandic government was in favor of repaying and had proposed a fiscal program to implement the repayment, the people expressed through the referendum their preferences against repayment, leading the country into default.
faces. They also recognize the government’s incentives to default on the sovereign bonds. Then, in equilibrium, the sovereign bond price, \( q(B', y) \), perfectly captures the sovereign default risk prevailing in the economy and is given by:

\[
q(B', y) = \frac{1 - \Pr \left[ D(B') | Y = y \right]}{1 + r},
\]

(6)

where \( D(B') \) is the government’s default set.

3 Value Functions and Recursive Equilibrium

Before analyzing the equilibrium of the model it is useful to clarify the timing of events in the economy. At the beginning of each period, the current aggregate endowment, \( y \), is observed and, given the amount of sovereign debt, \( B \), the government proposes a fiscal program, \( (B', \tau) \), or declares a default. If the government proposes a fiscal program, each household then decides whether to approve or reject the proposal.\(^9\) Households’ individual responses are aggregated by the political support function, \( p(B', \tau; y) \). If the political support exceeds the threshold \( p^r \) and the fiscal program raises at least \( B \), the government can implement the proposal and repay the debt. Otherwise, the government is forced to default. Finally, consumption takes place.

3.1 Government’s problem

In every period in which the government is current on its debt, it chooses whether to repay or default on the sovereign debt:

\[
v^0_g(B, y) = \max_{\{r, d\}} \left\{ v^r_g(B, y), v^d_g(y) \right\},
\]

(7)

where \( v^r_g(B, y) \) is the value associated with repayment and \( v^d_g(y) \) is the value associated with default.

If the government chooses not to default, it proposes a fiscal program, \( (B', \tau) \), to fund

\(^9\)We assume that households cannot enter into cooperative arrangements and that the government cannot commit to ex-post transfers to compensate households.
the repayment, subject to the feasibility and political constraints:

\[ v^r_g (B, y) = \max_{(B', \tau)} \int_0^1 u (y_i^r - \tau_i) \omega_i d\bar{i} + \beta \int_Y v^0_g (B', y') f (y', y) dy' \]

subject to (2) and (4).

The value function of default is given by:

\[ v^d_g (y) = \int_0^1 u (y_i^d) \omega_i d\bar{i} + \beta \int_Y [\theta v^0_g (0, y') + (1 - \theta) v^d_g (y^d')] f (y', y) dy' \]

From the government’s problem we can characterize the default set \( D (B) \) and repayment set \( R (B) \) as:

\[ D (B) = \left\{ y \in Y : \begin{array}{l}
\text{if } \frac{\partial}{\partial \tau} (B', \tau) : (2) \text{ and } (4) \text{ hold} \\
\text{or } v^r_g (B, y) < v^d_g (y)
\end{array} \right\}, \]

where the first line represents the cases when the government is unable to repay, due to either insufficient resources or insufficient political support, and the second line represents the cases when the government is able to repay but still chooses not to do it. The repayment set \( R (B) \) is the complement of the default set.

When repaying, the proposed fiscal program \( (B' (B, y), \tau (B, y)) \) is the one that solves problem (8).

### 3.2 Households’ problem

Households maximize their utility by choosing whether to approve or reject the fiscal program proposed by the government. The utility of household \( i \) under repayment by implementing a fiscal plan \( (B', \tau), v^r_i (B', \tau; y) \), is given by:

\[ v^r_i (B', \tau; y) = u (y_i^r - \tau_i) + \beta \int_Y v^0_i (B', y') f (y', y) dy' \]

While the utility of household \( i \) under default, \( v^d_i (y) \), is given by:

\[ v^d_i (y) = u (y_i^d) + \beta \int_Y [\theta v^0_i (0, y') + (1 - \theta) v^d_i (y')] f (y', y) dy' \]

(10)
The continuation utility of the households is determined by the anticipation of the government’s behavior, which ultimately defines households’ future levels of consumption. Then, $v_0^i (B, y)$ is:

$$v_0^i (B, y) = \begin{cases} v_r^i (B', B, y; \tau (B, y); y) & \text{if } y \in R (B) \\ v_d^i (y) & \text{if } y \in D (B) \end{cases}$$

We define the optimal voting decision for household $i$, given current aggregate output $y$ and the government’s fiscal program $(B', \tau)$ as follows:

$$p_i (B', \tau; y) = \begin{cases} 1 & \text{if } v_r^i (B', \tau; y) \geq v_d^i (y) \\ 0 & \text{if } v_r^i (B', \tau; y) < v_d^i (y) \end{cases}$$

where 1 stands for voting in favor and 0 for voting against the fiscal program.

### 3.3 Recursive Equilibrium

A Recursive Equilibrium for this economy is: i) a government’s policy set, 
$(B' (B, y), \tau (B, y)); D(B))$; ii) a household’s voting strategy, $p_i (B', \tau; y)$; iii) a sovereign bond price function, $q (B', y)$; and iv) a political support function, $p (B', \tau; y)$, such that:

1. Given the sovereign bond price function $q (B', y)$ and the political support function $p (B', B, y)$, the government’s policy set $(B' (B, y), \tau (B, y)); D(B))$ satisfies the government’s optimization problem.

2. Given the government’s policy set $(B' (B, y), \tau (B, y)); D(B))$, the household’s voting strategy $p_i (B', \tau; y)$ satisfies the household’s optimization problem.

3. The sovereign bond price function $q (B', y)$ reflects the government’s default probability and satisfies the foreign lenders’ break-even condition.

4. The political support function $p (B', \tau; y)$ is consistent with households’ voting strategies.

### 3.4 A simplified static model

Before moving to the numerical solution of the full model described above, we analyze a simplified version of the model to gain some intuition on the main mechanisms at work.
The main advantage of this simplified version is that it allows us to characterize several properties of the model analytically.

In this simplified version time is infinite but there is a single decision period, \( t = 0 \), where the government decides whether to repay or default. The initial amount of debt, \( B < 0 \), is exogenously given. If the government defaults, the economy is permanently excluded from the international credit markets. On the other hand, if the government repays, the economy can swap its endowment process for the endowment’s expected value—i.e., the economy can perfectly insure against future aggregate shocks. The government cannot issue new debt at \( t = 0 \), so fiscal plans are determined by taxes alone. The rest of the model remains as before.

To derive closed-form solutions, we assume that aggregate output is i.i.d. across time and that \( \alpha \) and \( \chi \) are bivariate log-normally distributed: both centered around 1, with dispersion coefficients \( \sigma_\alpha \) and \( \sigma_\chi \), respectively, and with correlation coefficient \( \rho_{\alpha\chi} \). In this specification, \( \sigma_\alpha = 0 \) essentially means that household receive the same share of aggregate output whereas \( \sigma_\alpha > 0 \) means that households’ shares differ. Additionally, \( \sigma_\chi = 0 \) means that the tax system is proportional whereas \( \sigma_\chi > 0 \) indicates deviations from proportionality: If \( \rho_{\alpha\chi} > 0 \) the tax system is progressive while if \( \rho_{\alpha\chi} < 0 \) the tax system is regressive. To easy the exposition, we set \( \omega = 1 \) and we restrict our analysis to CRRA preferences:

\[
\begin{align*}
    u(c) &= \frac{c^{1-\sigma} - 1}{1-\sigma},
\end{align*}
\]

with \( \sigma = 2 \).

In what follows, we derive the value functions and then we compute the government’s default set.

### 3.4.1 Value functions revised

The households’ value of defaulting and repaying at \( t = 0 \) are:

\[
\begin{align*}
    V_d(y; \alpha) &= \frac{(\alpha y)^{1-\sigma} - 1}{1-\sigma} + \frac{\beta}{1-\beta} \int_{\gamma} \frac{(\alpha \tilde{y})^{1-\sigma} - 1}{1-\sigma} dF(\tilde{y}), \quad \text{and} \\
    V_r(B, y; \alpha, \chi) &= \frac{(\alpha y + \alpha \chi B)^{1-\sigma} - 1}{1-\sigma} + \frac{\beta}{1-\beta} \left[ \int_{\gamma} \alpha \tilde{y} dF(\tilde{y}) \right]^{1-\sigma} - 1,
\end{align*}
\]
where $F$ is the cumulative probability distribution of aggregate output. In this notation, we index households by their individual type $(\alpha, \chi)$.

Let $\bar{y} (\alpha, \chi)$ denote the aggregate output threshold that leaves a household $(\alpha, \chi)$ indifferent between defaulting and repaying. The threshold $\bar{y} (\alpha, \chi)$ satisfies that:

$$V_d [\bar{y} (\alpha, \chi); \alpha] = V_r [B, \bar{y} (\alpha, \chi); \alpha, \chi].$$

This threshold does not depend on $\alpha$ due to the CRRA preference specification but it is strictly increasing on $\chi$ as households who shoulder a higher burden of the debt repayment relative to their income are less willing to repay the public debt. For levels of aggregate output higher (lower) than $\bar{y} (\alpha, \chi)$, a household $(\alpha, \chi)$ is willing to repay (default).

Similarly, let $\bar{\chi}$ denote the threshold household who is indifferent between defaulting and repaying. The threshold household $\bar{\chi}$ satisfies that $\bar{y} (\alpha, \bar{\chi}) = y$. Households with $\chi > \bar{\chi}$ favor default whereas households with $\chi < \bar{\chi}$ favor repayment. The political support function then satisfies that:

$$p (B, y) = \Pr (\chi < \bar{\chi}) = \Phi \left[ \frac{\ln (1 + \beta \Delta y) + \ln (y + B) - \ln y - \sigma^2 \Delta y^2 / 2}{\sigma \chi} \right],$$

where $\Phi$ is the normal cumulative distribution function and $\Delta > 0$ is the certainty equivalent of the process of aggregate output—i.e. the utility difference between consuming the mean of aggregate output forever after and the process of aggregate output itself.\textsuperscript{10} The parameter $\Delta$ can also be interpreted as measuring the cost of defaulting in terms of future consumption-based utility losses.

The government’s value of defaulting and that of repaying at $t = 0$ are:

$$V_d^0 (y) = \int V_d (y; \alpha) dH (\alpha, \chi), \quad \text{and} \quad V_r^0 (B, y) = \int V_r (B, y; \alpha, \chi) dH (\alpha, \chi),$$

where $H$ is the distribution of households with respect to the pair $(\alpha, \chi)$. The aggregate output $\bar{y}$ is the aggregate expected value of $y$ with respect to the distribution of households. Formally, the certainty equivalent $\Delta$ satisfies that:

$$\Delta = \frac{1}{1 - \beta} \left[ u \left( \int \bar{y} dF (\bar{y}) \right) - \int u (\bar{y}) dF (\bar{y}) \right].$$

\textsuperscript{10} For example, the certainty equivalent $\Delta$ satisfies that:

\begin{align*}
\Delta &= \frac{1}{1 - \beta} \left[ u \left( \int \bar{y} dF (\bar{y}) \right) - \int u (\bar{y}) dF (\bar{y}) \right].
\end{align*}
output threshold \( \bar{y}_g \) that leaves the government indifferent between defaulting and repaying, assuming no political constraints, is such that:

\[
V^g_d (\bar{y}_g) = V^g_r (B, \bar{y}_g).
\]

Then, the government is willing to repay when aggregate output is higher than \( \bar{y}_g \) and vice versa.

### 3.4.2 Default set revised

The key technical advantage of the simplified model is that we can compute the government’s default set in closed-form solution. In particular, the default set is given by:

\[
D (B) = \{ y \in Y : y \leq \max \{ \bar{y}_p, \bar{y}_g \} \}
\]

where \( \bar{y}_p \) is the aggregate output threshold that satisfies the political constraint with equality. The thresholds \( \bar{y}_j \), for \( j = g, p \), in turn satisfy that:

\[
\bar{y}_j = \frac{1}{\beta} \left[ \Omega_j - B \beta \Delta - 1 \right] + \sqrt{\left[ \frac{1}{\beta} \left[ \Omega_j - B \beta \Delta - 1 \right] \right]^2 - \frac{4B}{\beta^2}}
\]

where \( \Omega_g = \exp \left\{ \sigma^2 - 2 \rho \chi \right\} \) represents the effects of the distributional aspects on the government’s willingness to repay and \( \Omega_p = \exp \left\{ \sigma \left[ \Phi^{-1} (p_r) + \sigma / 2 \right] \right\} \) the effects of the political constraint on the government’s ability to repay. Notice that if \( \bar{y}_g < y < \bar{y}_p \) the government is willing to repay but politically unable whereas if \( \bar{y}_p < y < \bar{y}_g \) the government is unwilling to repay even though it is politically able to do it. Furthermore, notice that the political constraint restricts the government’s repayment decision iff:

\[
\Omega_p > \Omega_g \iff \Phi^{-1} (p_r) > \frac{1}{2} \sigma \chi - 2 \frac{\rho \chi \alpha}{\sigma \chi}
\]

The analysis of the two thresholds and the inequality above shows that:

- When tax systems move away from proportionality (i.e., higher \( \sigma \chi \)) there is an increase in both the government and the political output threshold that lead to higher default probability. The interpretation of this result is the following. A higher \( \sigma \chi \) means that some agents pay a lot in terms of their income while others pay very
little. Since a benevolent government cares about all agents equally, this discrepancy reduces aggregate welfare under repayment increasing default probability. The effect on the political constraint is straightforward, a higher $\sigma_X$ means more disagreement across different types of agents, reducing the political support.

- A more regressive tax system (i.e., lower $\rho_{\chi\alpha}$) also increases the default probability by raising the government output threshold, $\bar{y}_g$. The intuition works as follows. A regressive tax system ($\rho_{\chi\alpha} < 0$) implies that those agents with lower income shoulder a disproportionate share of the sovereign debt burden. Since these agents are the ones with the larger marginal utility, repaying becomes very costly for the benevolent government and it defaults more often.

- A lower political discretion of the government (i.e., a higher $p_r$) makes the political constraint more binding as the government requires the approval of a higher share of household to implement the repayment.

Also, as it is standard in the literature, higher levels of public debt tightens the political constraint and makes the government less willing to repay while a riskier income process increases repayment incentives. Finally, if taxes are proportional, i.e. $\sigma_X = 0$, the model converges to a representative agent model.\(^{11}\)

A shortcoming of the simplified model is that we cannot study borrowing decisions because the stock of public debt in this simple version of the model is exogenous. To fully investigate the dynamics of sovereign borrowing and default decisions we proceed to study the richer yet more complex dynamic model.

### 4 Quantitative analysis of the full model

In this section we present the main results of the dynamic model by solving it numerically. We calibrate the model to the Argentine economy in order to make our results more easily comparable with those of previous papers in the literature. In our baseline scenario, we calibrate agent heterogeneity to reflect the actual situation in Argentina in terms of income distribution and tax regime. Then, we analyze in detail the interaction among income distribution, tax regimes and political constraints by assuming alternative values

\(^{11}\)Notice that $\sigma_X = 0$ automatically implies that $\rho_{\chi\alpha} = 0$ as well.
for the relevant parameters. This allows us to understand how the above affect sovereign borrowing decisions, default probability and bond prices.

4.1 Calibration

As is standard in sovereign default studies, we choose a CRRA utility function in the numerical simulations:

$$u(c) = c^{1-\sigma} - 1 \over 1 - \sigma,$$

with a coefficient of relative risk aversion $\sigma$ equal to 2.

We set the model at the quarterly frequency and we assume the aggregate output to follow an AR(1) stochastic process: $\ln y_t = \rho_y \ln y_{t-1} + \varepsilon_t$, with $|\rho_y| < 1$ and $\varepsilon_t \sim N(0, \sigma_{ye}^2)$. To estimate these parameters, we use quarterly GDP data taken from the Argentine Ministry of Finance, ranging from the first quarter of 1980 to the second quarter of 2001. Our estimates of $\rho_y$ and $\sigma_{ye}$ are 0.945 and 0.025, respectively. Following Arellano (2008), we choose an asymmetric output loss function: $h(y) = \min\{y, (1 - \lambda)E(Y)\}$, where $E(Y)$ stands for the aggregate output unconditional mean and $\lambda$ represents the aggregate output loss during a sovereign default episode.

We follow Arellano (2008) for comparability in setting the rest of the parameters. We set $\theta = 0.282$, consistent with the empirical findings of Gelos et al. (2011), the subjective discount factor $\beta = 0.953$ and the aggregate output loss $\lambda = 4\%$. Finally, the risk-free interest rate $r$ is set to 1.7\%, to equal the average quarterly interest rate of a five-year U.S. treasury bond from the first quarter of 1980 to the second quarter of 2001. Table I summarizes the parameter values:

<table>
<thead>
<tr>
<th>Table I. Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

We consider five types of households of equal size to reflect each quintile. We calibrate each quintile income to the actual income distribution in Argentina in 1998 using data from the Center for Distributive, Labor and Social Studies (CEDLAS):

---

12 Arellano (2008) sets this parameters to target the following moments: a default probability of 3\%, an average debt service to GDP ratio of 5.53\%, and the standard deviation of the trade balance.
Table II. Income distribution (1998)

<table>
<thead>
<tr>
<th></th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>(\alpha_3)</th>
<th>(\alpha_4)</th>
<th>(\alpha_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.6%</td>
<td>8.1%</td>
<td>12.2%</td>
<td>20.1%</td>
<td>56.0%</td>
</tr>
</tbody>
</table>

where \(\alpha_1\) represents the poorest quintile. For the tax burden calibration, we use the estimates of Gasparini and Cruces (2010). The following table summarizes them:

Table III. Tax share per quintile

<table>
<thead>
<tr>
<th></th>
<th>(\alpha_1\chi_1)</th>
<th>(\alpha_2\chi_2)</th>
<th>(\alpha_3\chi_3)</th>
<th>(\alpha_4\chi_4)</th>
<th>(\alpha_5\chi_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.1%</td>
<td>10.7%</td>
<td>14.9%</td>
<td>20.1%</td>
<td>47.2%</td>
</tr>
</tbody>
</table>

Regarding the government welfare function, we assume an utilitarian social welfare function, in which the government assigns weights equal to the relative size of each type with respect to the aggregate population—i.e., \(\omega_i = 0.2, \forall i\).

In terms of the political support threshold \(p^r\), we consider three possible values: \(p^r = (0, \frac{2}{3}, 1)\). While the first case represents the standard scenario in the literature, in which there are no political constraints, the second case corresponds to a qualified majority voting process and the third one to a situation that requires unanimous consensus.

As Table IV shows, the model does a proper job of matching the real business cycle frequencies of the 2001 Argentine crisis.\(^{13}\) However, our focus in this paper is to achieve a better understanding of the role of the tax system, income distribution and political constraints in sovereign borrowing and default decisions.

\(^{13}\)In particular, output and aggregate consumption volatilities are as much as 75% and 71% of actual volatilities, respectively; aggregate consumption is more volatile than aggregate output and strongly procyclical; and the interest rate is countercyclical. In other dimensions, our model displays some mismatches with data (the average debt to GDP ratio and the average spread rate predicted in our model accounts for only 8% and 36% of those documented in the data, respectively). Failure to match these dimensions of the data is a feature shared by most sovereign debt models in the literature (Aguiar and Gopinath, 2006 and Arellano, 2008). Including long-term debt usually improves the matching of this type of models. We chose not to do it to keep the baseline model as simple as possible.
Table IV. Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Argentina (2001)</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std(GDP)</td>
<td>7.81</td>
<td>5.86</td>
</tr>
<tr>
<td>Std(Consumption)</td>
<td>8.60</td>
<td>6.11</td>
</tr>
<tr>
<td>Std(Spread)</td>
<td>5.58</td>
<td>6.47</td>
</tr>
<tr>
<td>Corr(GDP, Cons)</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Corr(SR, Cons)</td>
<td>-0.89</td>
<td>-0.25</td>
</tr>
<tr>
<td>GDP Decline</td>
<td>-16.01</td>
<td>-8.42</td>
</tr>
<tr>
<td>Mean Debt/GDP</td>
<td>-43.30</td>
<td>-3.42</td>
</tr>
<tr>
<td>Mean Spread Rate</td>
<td>10.35</td>
<td>3.69</td>
</tr>
<tr>
<td>Default Probability</td>
<td>3.00</td>
<td>3.00</td>
</tr>
</tbody>
</table>

4.2 The effect of alternative tax systems

In this section, we explore in detail the effect of alternative tax systems on sovereign borrowing, defaults and bond prices. In order to isolate this effect, we assume, for the time being, that the income distribution is given and that there are no political constraints.\textsuperscript{14}

We evaluate the impact of three alternative tax systems:

- Argentina’s tax system: as described in Table III.
- Proportional taxes: $\chi_i = 1, \forall i$.
- Uniform taxes: $\alpha_i\chi_i = \frac{1}{n}, \forall i$, with $n = 5$.

Table V translates the alternative tax systems under consideration into the relevant parameters identified in the solution of our static model: $\chi_i$, $\sigma^2_\chi$, and $\rho_{\chi\alpha}$. Since changes in the tax system typically affect both $\sigma^2_\chi$ and $\rho_{\chi\alpha}$, it is important to notice that it is the relation between the two, particularly through $\Omega_g = \exp\{\sigma^2_\chi - 2\rho_{\chi\alpha}\}$ what eventually affects the output thresholds. Specifically, according to $\bar{g}_g$, higher values of $\Omega_g$ lead to higher thresholds therefore increasing default probability.

\textsuperscript{14}We reintroduce the political constraints below.
Table V. Tax proportionality per quintile

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_X^2$</th>
<th>$\rho_{\alpha X}$</th>
<th>$\sigma_X^2 - 2\rho_{\alpha X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.18</td>
<td>-0.74</td>
<td>1.66</td>
</tr>
<tr>
<td>Proportional T.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Uniform T.</td>
<td>4.12</td>
<td>-0.70</td>
<td>5.52</td>
</tr>
</tbody>
</table>

Figure I shows the default sets under each alternative tax system. The default sets, which consist of combinations of debt and GDP (both normalized by expected GDP) such that the government prefers to default than to repay are the areas below each line in Figure I. As we can see in Figure I, the results of the simulation corroborate the analytical insights: proportional taxes reduce the default set while uniform taxes, which are more regressive than the prevalent tax system in Argentina, increase the default set. That is, governments are more likely to default for a given level of debt when the tax system is more regressive.
A complementary way to analyze this effect is to look at the sovereign bond price schedule, $q(B', y)$. As we can observe in Figure II, for all tax systems, the sovereign bond price decreases when the country becomes more indebted. This is a consequence of foreign lenders correctly anticipating that the default probability increases with the ratio of debt over GDP. Additionally, more regressive tax systems are associated with lower bond prices and higher sovereign spreads, all else equal. This is consistent with regressive tax systems generating larger default sets and increasing default probability.
The intuition behind these results is the following. Heterogeneous agents with differing income and taxes typically differ in their marginal utilities. In particular, in an economy characterized by an unequal income distribution and a regressive tax system, those in the lower quintiles tend to have a higher marginal utility. Furthermore, the regressive tax system implies that in the event of a sovereign debt repayment, the lower quintiles shoulder a disproportionate share of the sovereign debt burden. Since these quintiles are the ones with the larger marginal utility, repaying becomes very costly for the benevolent government. As a result, default becomes more likely for any given level of debt. On the other hand, tax systems that are relatively more proportional distribute the burden of sovereign debt repayment more evenly across society. In doing so, they increase sovereign debt sustainability by reducing the probability of default. As a result, the equilibrium ratio of sovereign debt to GDP goes up, as can be observed in Table VI, which summarizes the
main RBC statistics for the three scenarios under study.

Table VI. Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Baseline (Argentina)</th>
<th>Proportional Taxes</th>
<th>Uniform Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std(GDP)</td>
<td>5.86</td>
<td>5.91</td>
<td>6.44</td>
</tr>
<tr>
<td>Std(Consumption)</td>
<td>6.11</td>
<td>10.22</td>
<td>6.39</td>
</tr>
<tr>
<td>Std(Spread)</td>
<td>6.47</td>
<td>2.13</td>
<td>21.33</td>
</tr>
<tr>
<td>Corr(GDP, Cons)</td>
<td>0.98</td>
<td>0.61</td>
<td>0.99</td>
</tr>
<tr>
<td>Corr(SR, Cons)</td>
<td>-0.25</td>
<td>-0.45</td>
<td>-0.42</td>
</tr>
<tr>
<td>GDP Decline</td>
<td>-8.42</td>
<td>-6.72</td>
<td>-18.8</td>
</tr>
<tr>
<td>Mean Debt/GDP</td>
<td>-3.42</td>
<td>-4.52</td>
<td>-0.00</td>
</tr>
<tr>
<td>Mean Spread Rate</td>
<td>3.69</td>
<td>1.28</td>
<td>4.93</td>
</tr>
<tr>
<td>Default Probability</td>
<td>3.00</td>
<td>1.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>

4.3 The effect of income inequality

In this section, we explore in detail the effect of alternative income distributions on sovereign borrowing, defaults and bond prices. In order to isolate this effect, we take as given the tax system and assume that there are no political constraints. We evaluate the impact of three alternative income distributions:

- Argentina’s income distribution: as described in Table II.
- Higher Gini (higher inequality): 1 percentage point of income of each of the two lowest quintiles goes to the two top quintiles.
- Lower Gini (lower inequality): 1 percentage point of income of each of the two top quintiles goes to the two lowest quintiles.

Table VII presents the implications of the alternative income distribution under consideration in terms of the relevant parameters. As expected, changing the income distribution while maintaining the tax system affects the proportionality of the latter. In particular, higher inequality makes the tax system more regressive and vice versa. Again, as in the
previous scenario, higher values of $\sigma_X^2 - 2\rho_{\alpha X}$ increase the minimum level of output such that the government prefers to repay, consequently increasing the default probability.

Table VII. Tax proportionality per quintile

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_X^2$</th>
<th>$\rho_{\alpha X}$</th>
<th>$\sigma_X^2 - 2\rho_{\alpha X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.18</td>
<td>-0.74</td>
<td>1.66</td>
</tr>
<tr>
<td>Higher Gini</td>
<td>0.58</td>
<td>-0.86</td>
<td>2.30</td>
</tr>
<tr>
<td>Lower Gini</td>
<td>0.06</td>
<td>-0.83</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Figures IV and V show the results in terms of default sets and bond prices for these scenarios. Consistent with our previous analysis, for a given ratio of debt to GDP, the default set is larger and bond prices lower when the income distribution is more unequal, and vice versa. The intuition for these results is analogous to the case of different tax systems.
Figure III
Table VIII shows that, in equilibrium, the ratio of sovereign debt to GDP is also affected. The economy with a more equal income distribution is able to sustain larger levels of sovereign debt, and vice versa. To understand the higher spread of the low-Gini scenario versus the lower spread of the high-Gini scenario we need to take into account the equilibrium level of debt to GDP. For a given level of sovereign debt, the low-Gini economy faces lower default probabilities and spreads. However, if the equilibrium level of sovereign debt in the low-Gini economy is sufficiently larger than that of the high-Gini economy, the default probability and the spread faced by the low-Gini economy can be higher, as Table VIII shows.
Table VIII. Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Baseline (Argentina)</th>
<th>Higher Gini</th>
<th>Lower Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std(GDP)</td>
<td>5.86</td>
<td>5.82</td>
<td>5.81</td>
</tr>
<tr>
<td>Std(Consumption)</td>
<td>6.11</td>
<td>5.99</td>
<td>6.12</td>
</tr>
<tr>
<td>Std(Spread)</td>
<td>6.47</td>
<td>6.10</td>
<td>7.31</td>
</tr>
<tr>
<td>Corr(GDP, Cons)</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Corr(SR, Cons)</td>
<td>-0.25</td>
<td>-0.24</td>
<td>-0.41</td>
</tr>
<tr>
<td>GDP Decline</td>
<td>-8.42</td>
<td>-8.57</td>
<td>-10.12</td>
</tr>
<tr>
<td>Mean Debt/GDP</td>
<td>-3.42</td>
<td>-2.09</td>
<td>-4.11</td>
</tr>
<tr>
<td>Mean Spread Rate</td>
<td>3.69</td>
<td>3.57</td>
<td>4.82</td>
</tr>
<tr>
<td>Default Probability</td>
<td>3.00</td>
<td>2.89</td>
<td>4.12</td>
</tr>
</tbody>
</table>

4.4 The effect of political constraints

Standard sovereign default models have focused on default episodes in which the government is unwilling to repay. In those models, the government has full access to the resources of the economy, which are assumed to be sufficient to repay the debt. Then, a sovereign default can arise only if the government prefers to default rather than repay—i.e., if the government is unwilling to repay its debts. The real-world sovereign default universe is richer than its traditional theoretical depiction. In particular, as discussed in the introduction, a distinctive feature of many sovereign defaults is that they are not the result of the government being unwilling to repay but of political constraints that governments sometimes face when trying to raise funds to repay.

The presence of the political constraint can give rise to a new type of default triggered by the government’s inability to find a fiscal plan \((B', \tau)\) that raises enough funds while garnering sufficient political support. As a result, the government has no option but to default. Formally:

\[
\hat{\beta} (B', \tau) \text{ with } \tau \leq y'_{\min} : \tau - q (B', y) B' \geq -B \wedge p (B', \tau; y) \geq p'.
\]

Given the tax system and the income distribution in Argentina, in the next paragraphs, we evaluate the impact of three alternative political scenarios:
• No political constraint: \( p^r = 0 \).

• Qualified Majority: \( p^r = \frac{2}{3} \).

• Consensus: \( p^r = 1 \).

Figures VI and VII present the default sets and the bond prices for the three alternative political constraints. The reason for the increased default sets and lower bond prices is the uneven distribution of the sovereign debt repayment burden across the groups of households. Under CRRA preferences, if the repayment burden were distributed proportionally across households, the households’ preferences between repayment and default would coincide. Then, the economy would aggregate to a representative agent. However, as taxation in Argentina is regressive, households differ in their default/repayment preferences. Households in the lower quintiles have relatively higher preferences for default than those in the higher quintiles. The benevolent government aggregates the households’ preferences according to its welfare function, which constitutes a weighted average of the preferences of all households. Once the political constraint is introduced, the equilibrium of the economy changes as long as the pivotal voter’s preferences are relatively more tilted towards default than those of the government. In the case of Argentina, where relatively poorer households are shouldering a disproportionate amount of the burden, when given the chance, these households veto the government’s proposed repayment program. As the political constraint becomes more stringent, the effect on the default set and bond prices is heightened.
Figure V
In this case again, as depicted in Table IX, the equilibrium ratio of debt to GDP is affected, with the ratio decreasing as the political constraint becomes more stringent. As before, the equilibrium levels of default probability and spread rate adjust in response to both the political constraint and the equilibrium ratio of debt to GDP.
Table IX. Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Qualified Majority</th>
<th>Consensus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std(GDP)</td>
<td>5.86</td>
<td>5.70</td>
<td>5.84</td>
</tr>
<tr>
<td>Std(Consumption)</td>
<td>6.11</td>
<td>5.80</td>
<td>5.95</td>
</tr>
<tr>
<td>Std(Spread)</td>
<td>6.47</td>
<td>10.40</td>
<td>6.03</td>
</tr>
<tr>
<td>Corr(GDP, Cons)</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Corr(SR, Cons)</td>
<td>-0.25</td>
<td>-0.39</td>
<td>-0.37</td>
</tr>
<tr>
<td>GDP Decline</td>
<td>-8.42</td>
<td>-9.74</td>
<td>-11.55</td>
</tr>
<tr>
<td>Mean Debt/GDP</td>
<td>-3.42</td>
<td>-2.14</td>
<td>-1.81</td>
</tr>
<tr>
<td>Mean Spread Rate</td>
<td>3.69</td>
<td>4.38</td>
<td>2.27</td>
</tr>
<tr>
<td>Default Probability</td>
<td>3.00</td>
<td>3.00</td>
<td>1.61</td>
</tr>
</tbody>
</table>

4.4.1 The interaction of political constraints and tax systems

After analyzing the behavior of each of the novel ingredients of the model, we now interact them. In particular, in this subsection, we evaluate the impact of different political constraints under alternative tax systems.\(^\text{15}\)

Figure VII presents the default sets for the cases of no political constraints, qualified majority and full consensus under two alternative tax regimes: the case of Argentina and uniform taxation. We do not include the case with proportional taxation since, under CRRA preferences, the economy aggregates to a representative agent, leaving no role for political constraints. As the figure shows, political constraints become more relevant when the tax system becomes more regressive. This happens because more regressive tax systems make households diverge more in their opinions regarding the convenience of repaying. On one extreme, with linear taxes, all households have the same opinion regarding the convenience of repaying and, as a result, the political constraint becomes irrelevant. On the other extreme, with uniform taxes, the political constraint becomes very relevant as households’ opinions diverge significantly.

\(^\text{15}\)For reasons of space we discuss just the interaction between political constraints and the tax system. We get similar results when we change the income distribution instead of the tax system, or both at the same time.
5 Conclusion

This paper fills a gap in the literature by incorporating political economy considerations in the sovereign borrowing and default decisions. In particular, we analyze how income distribution, the tax system and the political constraints faced by the government when raising revenues for debt repayment affect these decisions. We do so by introducing two novel features: heterogeneous agents and political constraints in an otherwise standard DSGE model of sovereign debt.

We find that income inequality and regressive tax systems increase the probability of default and that political constraints can force a government to default even if it would
prefer to repay. We also show that political constraints become more relevant when the income distribution is more unequal or the tax system more regressive.

An additional contribution of the paper is the development of a new typology of sovereign default events related to whether a government is forced to default due to political constraint or chooses to do so. This typology is novel in the theoretical literature (though it has been present in more politically-oriented analyses) and constitutes an issue that merits further research, as it could help identify different types of real-world defaults.

We believe that understanding the political economy of sovereign debt crises is crucial to devising better ways to deal with them. Our work should be seen as a first step in this direction.

**References**


6 Appendix

In this Appendix we solve for the simplified model. To do so, we proceed by steps.

First, we compute the aggregate output threshold \( \bar{y}(\alpha, \chi) \) of household \((\alpha, \chi)\). The threshold \( \bar{y}(\alpha, \chi) \) satisfies that:

\[
V_d [\bar{y}(\alpha, \chi); \alpha] = V_r [B, \bar{y}(\alpha, \chi); \alpha, \chi].
\]

Whence

\[
\frac{[\bar{y}(\alpha, \chi)]^{1-\sigma} - 1}{1-\sigma} + \frac{\beta}{1-\beta} \int_Y \frac{\bar{y}^{1-\sigma} - 1}{1-\sigma} dF(\bar{y}) = \frac{([\bar{y}(\alpha, \chi) + \chi B])^{1-\sigma} - 1}{1-\sigma} + \frac{\beta}{1-\beta} \left[ \int_Y \bar{y} dF(\bar{y}) \right]^{1-\sigma} - 1.
\]

Let define \((1 - \beta) \Delta > 0\) as the per-period certainty equivalent of aggregate output. That is:

\[
\Delta (1 - \beta) = \left[ \int_Y \bar{y} dF(\bar{y}) \right]^{1-\sigma} - 1 - \int_Y \frac{\bar{y}^{1-\sigma} - 1}{1-\sigma} dF(\bar{y}).
\]

Notice that \(\Delta > 0\) due to the Jensen inequality. We replace the expression for \(\Delta\) and \(\sigma = 2\) in the equality above. After some algebraic steps, we obtain that \(\bar{y}(\alpha, \chi)\):

\[
\bar{y}(\alpha, \chi) = \frac{-\chi B + \sqrt{(\chi B)^2 - 4\chi B \Delta}}{2}.
\]

Second, we compute the political support function. The political support function satisfies that:

\[
p(y, B) = \int 1[\bar{y}(\alpha, \chi) \geq y] dH(\alpha, \chi).
\]

After some algebraic manipulation we obtain:

\[
p(B, y) = \int 1 \left[ \chi \leq \frac{\beta \Delta y^2}{-B(1 + \beta \Delta y)} \right] dH(\alpha, \chi).
\]
In line with the main section of the paper, we assume that:

\[ \alpha = e^x, \quad \chi = -\frac{y - e^{-Y}}{B}, \]

where \((X, Y)\) are bivariate normal with:

\[
\begin{pmatrix}
X \\
Y
\end{pmatrix}
\sim N
\left(
\begin{pmatrix}
-\frac{\sigma^2}{2} \\
\frac{\sigma^2}{2} \ln (y + B)
\end{pmatrix},
\begin{pmatrix}
\sigma^2 & \rho_{\alpha\chi} \\
\rho_{\alpha\chi} & \sigma^2
\end{pmatrix}
\right).
\]

The political support function then reduces to:

\[
p(B, y) = \Phi \left[ \frac{\ln (1 + \beta \Delta y) + \ln (y + B) - \ln y - \sigma^2/2}{\sigma} \right].
\]

Third, we compute the aggregate output threshold \(\bar{y}_g\). The output threshold \(\bar{y}_g\) satisfies that:

\[ V_d^g (\bar{y}_g) = V_r^g (B, \bar{y}_g). \]

Whence:

\[
\int \alpha^{1-\sigma} dH (\alpha, \chi) \left[ \frac{(\bar{y}_g)^{1-\sigma} - 1}{1-\sigma} + \frac{\beta}{1-\beta} \int_Y \frac{\bar{y}^{1-\sigma} - 1}{1-\sigma} dF (\bar{y}) \right] =
\int \alpha^{1-\sigma} \left[ \frac{(\bar{y}_g + \chi B)^{1-\sigma} - 1}{1-\sigma} + \frac{\beta}{1-\beta} \left[ \int_Y \bar{y} dF (\bar{y}) \right]^{1-\sigma} - 1 \right] dH (\alpha, \chi),
\]
or

\[
\int \alpha^{1-\sigma} dH (\alpha, \chi) \left[ \frac{(\bar{y}_g)^{1-\sigma} - 1}{1-\sigma} + \beta \Delta \right] = \int \alpha^{1-\sigma} \frac{(\bar{y}_g + \chi B)^{1-\sigma} - 1}{1-\sigma} dH (\alpha, \chi).
\]

Notice that when \(\sigma = 2\):

\[
\int \alpha^{1-\sigma} dH (\alpha, \chi) = 1, \quad \text{and}
\]

\[
\int \alpha^{1-\sigma} (\bar{y}_g + \chi B)^{1-\sigma} = \frac{1}{\bar{y}_g - B} \exp \{ \sigma^2 - 2\rho_{\alpha\chi} \}.
\]
We replace these latter expression on the equation above and obtain:

\[- \frac{1}{y_g + B} \exp \{ \sigma^2 - 2\rho \} + \beta \Delta = - \frac{1}{y_g} \, .\]

The output threshold \( \tilde{y}_g \) is:

\[ \tilde{y}_g = \frac{1}{\beta \Delta} \left[ \Omega_g - B \beta \Delta - 1 \right] + \sqrt{\left( \frac{1}{\beta \Delta} \left[ \Omega_g - B \beta \Delta - 1 \right] \right)^2 - 4 \frac{B}{\beta \Delta}} \, , \]

where \( \Omega_g = \exp \{ \sigma^2 - 2\rho \} \).

Fourth, we compute the aggregate output threshold \( \tilde{y}_p \). The output threshold \( \tilde{y}_p \) satisfies that:

\[ \Phi \left[ \frac{\ln (1 + \beta \Delta y) + \ln (y + B) - \ln y - \sigma^2 / 2}{\sigma} \right] = p_r. \]

Whence

\[ \tilde{y}_p = \frac{1}{\beta \Delta} \left[ \Omega_p - \beta \Delta B - 1 \right] + \sqrt{\left( \frac{1}{\beta \Delta} \left[ \Omega_p - \beta \Delta B - 1 \right] \right)^2 - 4 \frac{B}{\beta \Delta}} \, , \]

where \( \Omega_p = \exp \{ \sigma \left[ \Phi^{-1}(p_r) + \sigma / 2 \right] \} \).