Dynamic Committee Decision-Making*

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Abstract

Political science is largely the study of how groups make decisions. But for all of the effort spent to study how actors make a decision, often missing from the analysis is usually many decisions are made in sequence as a part of a larger agenda, and decisions made earlier affect choices later. As a discipline, we know a lot about how legislators vote on a single proposal or the Court decides one case, but we have ignored the effects of an individual decision on the larger agenda. Legislatures expend scarce floor time on some bills at the expense of others as they work through a broader legislative agenda. The Supreme Court docket includes eighty or so cases each term even as they turn away thousands more. Executives hire many employees, but eventually fill all of the jobs (or not).

We begin to fill that gap with a focus on committee decision-making in a dynamic context. We present a model that demonstrates how committee members trade off between certainty in the moment against expectations about the future to fill up a set number of slots from a fixed number of draws. We show that when a decision is made affects what decision is made. Sometimes players take losses instead of a zero payout in the present in an effort to prevent a worse loss in the future, something we call a sacrifice region. Members of a committee that lack to votes to block some outcome may still be able to prevent it by supporting other proposals that may push the least-favored option off of the agenda. Our model has implications for agenda-setting and committee decision-making more broadly. We apply our model to a court, a legislature, and a hiring committee.

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Introduction

Politics, almost by definition, is about groups making constrained decisions over time. Yet many if not most models of politics focus on a single group decision made in isolation and set aside the dynamic nature of political interaction. Careful attention to a particular decision has paid off handsomely, as it allows social scientists to think carefully along a particular margin. It has yielded canonical models that underlie our studies of political institutions. As a discipline, we have a good understanding of monopoly agenda setters pass a proposal (Romer and Rosenthal (1979)), how to divide a single pie in alternating offer games (Rubinstein (1982), Baron and Ferejohn (1987), Baron and Ferejohn (1989)), and conditions under which a proposal survives institutional hurdles to become a law, (Krehbiel (1998), Cameron (2000)). This is not to say that political science has entirely ignored they dynamic nature of politics. Indeed much of the early formal work on committee decision-making focused on the amendment process, which is inherently dynamic.\(^1\) In these models, the choices faced by members of a committee are pairwise and sequential: keep the present bill or vote for the amendment. The sequence of amendments generates a dynamic flavor and distinguishes these models from more static versions. Nonetheless, these are models of how a particular decision is made, i.e. how one bill passes a committee.

This paper in contrast, steps back and considers not just a single constrained decision, but explores the larger picture of committees making a series of decisions over time. We move away from pair-wise comparisons of status quo contra proposal or even tournament-style models and focus instead on the temporal tradeoffs of a constrained committee. Put differently, we consider not just how an executive staffs an administration, not just on how it makes a single hire. We broaden our focus from a single piece of legislation to the larger legislative agenda. We examine the Court’s docket, not a particular case.

Our model looks at just these types of situations where a committee may fill a finite number of slots from a series of sequential draws. These could be applicants for a political appointment, cases appealed to the Supreme Court, or perhaps pieces of legislation that might be passed before the close of a legislative session. We show that in all such instances, players balance the (relatively) certain payoff now against the change in continuation values when they have choose to fill an available slot or leave it open.

Generally, we show that even when players have fixed preferences, full information about other players’ preferences, no private information about the distribution of draws, no change to the rules, and due solely to the progression of play, players may choose different actions when faced with the same case. Indeed, our results do not even rely on uncertainty about future draws, because we show they hold even if the sequence of future draws is known. Focusing on this dynamic nature of certain collective decision-making processes allows us to make several contributions. Specifically, we provide a clear explanation for the existence of a gridlock region where cases or candidates will be rejected even though they provide a positive return to a majority or other selectorate. This resembles the gridlock region familiar to congressional scholars, but arises for different reasons. We also show that at other points in the game, these cases or candidates will be accepted by those same players. Finally, we show that sometimes players actually vote to accept a case or candidate that causes them to suffer a loss, even though the alternative payout is zero. The explanation is simple and intuitive: players are willing to take a small loss now to block a worse loss from filling

the same slot later.

The paper proceeds as follows. After a brief review of the relevant literature, we provide a simple example where a small committee may fill one slot from two draws. We then apply that simplified version to three different political processes across the three familiar branches of government: staffing an administration, agenda setting at the Supreme Court when cases arise randomly, and agenda setting in the legislature when there is monopoly control of the agenda. We then describe the general model for arbitrary numbers of draws and slots, prove equilibrium results, and describe interesting features of that equilibrium related to the sacrifice region. After a discussion of the larger model, we conclude.

Related Literature

Since at least Black (1948) and Downs (1957), committee decision-making has been a foundational study in political science. The median voter theorem has been a workhorse for the study of political institutions ever since. But majority rule is only one way that committees make decisions. Further, committees often make more than one decision, and strategic committee members look ahead to see how decisions in one period affects future rounds. We explore decision-making under different decision rules and in a dynamic context.

Our paper joins recent and ongoing work that looks at the ability of a minority on a committee to influence the outcome of committee decision-making. Chen and Eraslan (2015) shows that parties out of power may strategically manipulate the agenda by making use of checks and balances. Krehbiel, Meirowitz, and Wiseman (2015) shows that the minority’s ability to offer amendments and to expend resources to build coalitions generates more moderate outcomes. Finally, Fong and Krehbiel (working paper) show that when floor time is scarce, the Senate minority can use the stick of obstructing tactics and the carrot of unanimous consent agreements to influence what issues the majority places on the agenda.

Thus, a clear application of our model, as seen in the opening Senate example, is agenda-setting. Our model shows that the order in which cases come before the committee can dramatically affect how players vote. This is clearly related to a longstanding literature that shows that the order in which alternatives are compared matters greatly, that includes Black et al. (1958), McKelvey (1976), Gerber, Barber`a et al. (2016). But as our paper is not focused on pair-wise comparisons, this paper more closely relates to Iaryczower (2008), exploring decision making on sequential committees, Lizzeri and Yariv (2013), focusing on information gathering, and Feddersen and Pesendorfer (1996), focusing on juror welfare under uncertainty.

There is also a close relationship between our inquiry and previous studies of jury selection. See Brams and Davis (1978); DeGroot and Kadane (1980); Alpern and Gal (2009); Alpern, Gal, and Solan (2010). The jury selection literature shares our interest in filling up a fixed number of slots from sequential draws from a known distribution. Seen in one light, our model is a generalization of the jury selection game. The main stylistic difference is the traditional jury selection game relies on vetoes rather than selections, but this difference can be fit into our model without difficulty.

Our work has immediate application to political institutions such as the United States Supreme Court and various state supreme courts that follow a minority rule for case selection. There is an

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2See Bernheim, Rangel, and Rayo (2006); Bernheim and Slavov (2009) for a review of this literature.
extensive literature on Supreme Court agenda setting. Perry (2009); Boucher and Segal (1995); Caldeira and Wright (1988); Ulmer (1984); Epstein and Knight (1997). Within this vein, our paper is most closely related to work by Lax (2003) and Godefroy and Perez-Richet (2013).

Lax shows that if monitoring lower courts requires paying a fixed cost, the Rule of Four is median enhancing, because lower courts know that the four most extreme members of the Court (on either side) will take cases where the lower court deviates too far from the median’s ideal policy, the Rule of Four helps keep lower courts in line. One interesting feature of Lax’s model is that as the cost of taking cases increases, there is a growing range of decisions that will not be reviewed even though the lower court is not implementing the median’s preferred policy. We similarly derive this gridlock region, but in our model this zone expands and contracts throughout the game and flows not from a fixed cost assumption but emerges endogenously due to a constraint on the size of the Court’s docket.

Godefroy and Perez-Richet examine the Rule of Four as a part of a more general study of the effects of different selection rules when a committee first decides whether or not to decide a particular case. In their model, members of a committee individually receive signals about a case. The committee first votes on whether or not to accept the case using a selection rule. If the case is selected, the committee decides the case based on a (possibly different) decision rule. They show that when the selection rule becomes more stringent, players become more conservative in the selection phase and send fewer cases on to the decision stage. Their model shows that justices’ strategies are affected by uncertainty about the decision that will follow a selection: an uncertainty that flows from their private value model.

We remove that uncertainty and allow players to be perfectly aware of other players’ preferences and beliefs. This allows case outcomes to be common knowledge even at the selection stage. What drives our model is instead expected gains or losses in future cases. Thus strategies are not only affected by preferences over the current case but also by the stage of the game. We show that some cases that would be selected in one round are not selected in another and that there are conditions under which justices would vote to take a case even when they fully expect to lose the case in the decision round. Our model also differs from theirs in that it extends beyond the context of agenda-setting. Our model applies to dynamic decision-making in committees even when there is only one stage in the process. That is, whereas our model, like theirs deals with committees who first select and then decide, our model also addresses committees for which the selection is the decision.

Finally, our model is an extension of the “secretary problem” (Ferguson, 1989, 2005; Eriksson, Sjöstrand, and Strimling, 2007). In the traditional formulation, a single player tries to optimize the rank of quality. In our model, individual committee members try to maximize quality while acting as part of committee in a game theoretic context. Similarly, our model extends that offered by Cox and McCubbins (1993) which applies a variant on the multi-armed bandit problem to agenda setting decision in the United States House of Representatives.

**A Two Period Example**

To further motivate our model, consider the following two-period model of a three member committee. Suppose three officials at the National Institutes of Health have enough money to fund one more cancer study. They have a stack of applications that propose to develop new cancer treat-
ments. Studies may be related to chemo drugs, acupuncture, or use embryonic stem cells in their research. We assume that all of the applications are of equal quality, so that the only difference between them is the type of treatment to be studied. The applications come before the committee sequentially and randomly with replacement. Upon reviewing the application, each committee member chooses individually whether to support or to oppose the grant. If a majority of the committee supports the grant, then the study is funded. If less than two members of the committee support the grant, the committee moves onto the next application. If the committee fails to fund a study, the money is returned to the Treasury.

Suppose the utilities the three players receive from the different types of treatments are as given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Chemo</th>
<th>Acupuncture</th>
<th>Stem Cell</th>
<th>Treasury</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.5</td>
<td>0.4</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>P2</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>P3</td>
<td>1</td>
<td>-\frac{1}{3}</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

We want to demonstrate the following things:

- For the first draw, there is a therapy that a majority prefers to returning the money to the Treasury, but it will not gain a majority.
- What therapies a person is willing to support at depends on whether it is the first or second draw.
- In the first round, a player may choose to support a study even if she would prefer to return the money to the Treasury. That is, it gives a negative payoff.

We look for weakly nondominated, subgame perfect equilibria and proceed by backward induction. Clearly, if the group rejects the first draw, then in equilibrium, the committee will accept the second draw, since at least two people prefer every treatment to returning the money. Since every treatment is just as likely to arise in period two, the expected utility for each player from the second draw is $v = \{0.8, 0.3, -\frac{4}{9}\}$.

Notice that Players 1 and 2 prefer chemo to Treasury, but neither will not select chemo in the first round because they expect to get a payout of 1 and .3 in round two, respectively, and chemo provides each less than that reserve utility. Since players 2 and 3 each receive sufficiently low payouts from stem cells, they will not want support such an application at the first draw either. With chemo and stem cells we see an example of gridlock. Even though an option provides a sufficient selectorate with a positive payoff, it is not selected.

See now that Player 3 is willing to sacrifice during the first round. Player 3 prefers to return the money and take zero utility to looking into acupuncture, which gives him negative utility. But because Player 3 realizes that if stem cells come up in the final round Players 1 and 2 will support that proposal, Player 3 will grab at the chance for acupuncture in the first round. He is willing to take a smaller loss now to head off the risk of a larger loss later. This sacrifice makes acupuncture the only option that can prevail in round one.
Finally, notice that while Player 1 will not go for chemo or acupuncture in the first round, she will in the second. Similarly, while Player 3 will go for acupuncture in round one, he will choose to oppose acupuncture in the second round. And Player 2 will oppose chemo and stem cells in the first round but not in the second. Neither the payoffs nor the preferences changed for any players, but they decide differently on the same types of applications depending only on the stage of the game.

Three Political Applications

We now turn to three instances of such committee decision-making across the familiar three branches of government in the United States. We focus on the end of the process where there are two draws and one slot available. Two of these examples show how our model applies in the context of agenda setting. The Supreme Court of the United States—as do the supreme courts of many individual states—has control over its docket. The justices examine cases appealed from lower courts and select which ones will be reviewed on the merits. The Court cannot control the stream of cases it chooses from, and so we model draws as coming randomly, as in the restaurant example. In contrast, we model the Senate as having a monopolist agenda setter. The majority leader can choose what bills come to the floor, and so the assumption of randomness is not justified. Our third example is staffing a new presidential administration. Here, there is no agenda setting stage. While the applications appear in some (possibly random) order, the decision to hire is the only decision.

The first two examples demonstrate the gridlock and sacrifice regions separately. The third shows them both together. Similarly, the first two demonstrate the usefulness of our model in the context of agenda setting, while the third example demonstrates that its applicability extends beyond that particular application to dynamic committee decision-making more broadly. Finally, as our model will be agnostic as to the size of the selectorate, we choose these examples to demonstrate that flexibility. The Supreme Court follows a minority rule for agenda-setting, the Senate follows a supermajority rule for the filibuster, and we assume that the hiring committee operates under majority rule.

The Supreme Court

Our model demonstrates the divergent preferences of “winners” and “losers.” Winners are players who often get what they want; losers usually do not. For instance, in the context of the Supreme Court, the most liberal member of a conservative-dominated Court often “loses.” That is, many of her preferred precedents are shifted to the right by a conservative majority. The median justice is almost always a “winner” in that she can (if the conditions of the median voter theorem hold) move policy closer to her ideal point. Winners want to be selective in cases to get the most bang for the buck. They pass on cases that do not give sufficiently high enough payoffs, hoping that the next case that comes along will be a bigger win.

Our first example is a reduced form of the Supreme Court agenda setting process. The Supreme Court actually has a few different minority voting rules. Revesz and Karlan (1988). The most famous and important of these is the so-called Rule of Four. When cases are appealed to the Court, any combination of four of the nine justices may vote to grant the writ of certiorari and accept the
case. Once accepted, the nine justices decide the case on the merits by majority rule. Perry (2009) remains the best qualitative account of the process. Lax (2003) offers a model that proposes the Rule is actually median enhancing. As a historical matter, the Rule of Four was how the Court addressed congressional concerns that if the Court gained control of its docket, justices would stop taking cases. To remain faithful to the actual institutional practice, we use the minority voting rule, but we stress and will highlight that our results are not contingent on removing the majority decision rule.

To see how our model operates in the context of the Court, consider a simple example of a three-member court that follows a Rule of One. That is, a court that fills a discretionary docket by voting over candidate cases according to a $Q$-rule with $Q = 1$. Suppose there is a single moderate justice, a liberal judge, and a conservative judge with ideal points, $\theta^L \leq \theta^M \leq \theta^R$, respectively in the unit interval. In each of the two rounds, the court randomly draws a case, $x_n$ that implies a status quo policy, $x$, from the uniform distribution on the unit interval policy space. Each judge chooses whether to accept or reject the case as it is observed and prior to observing any subsequent draw. If any judge votes to accept, then the case proceeds to a hearing. The court will sequentially draw $N = 2$ cases and may accept no more than one, so $K = 1$.

If a case with a status quo, $x$, is drawn and accepted for a hearing, the court decides the policy outcome by simple majority rule, adopting the median justice’s ideal point, $\theta^M$, at which point each judge, $i$, earns utility,

$$V^i(x) = |\theta^i - x| - |\theta^i - \theta^M|$$

Figure 1 shows the extensive form of this game.

![Figure 1: Game form with two draws and one slot.](image)

We begin our analysis of this model by examining player strategies at each node. Since rejecting both draws results in a current payoff of zero, at $(1, 1)$ players will take any case that provides a weakly positive payoff. That is, players will follow some individualized cutpoint rule, $\alpha^i_{1,1} = 0$. This implies that $L$ will take any case $x \geq \theta^M$ and $R$ will take any case $x \leq \theta^M$. Since the end players will take any case on the opposite side of the median, in equilibrium, the court will take
any draw at \((1, 1)\). This means that the slot will be filled in equilibrium. Importantly, this is not a requirement of the game but is, instead, induced by equilibrium behaviors. We also stress that even under a majority rule, every case would be accepted in the final round. We can assume that \(M\) weakly prefers to take a case drawn at her ideal point and are certain that she strictly prefers to take a case not at her ideal point. So in fact, every case will get two votes at \((1, 1)\).

If we now define justice \(i\)'s expected value at this last draw to be \(\mu_i\), then a player will only want to take a case at \((2, 1)\) if it offers at least \(\mu_i\). There is no requirement that \(\mu_i = 0\), that is, there is no reason to believe that players all expect to receive zero payouts from the final draw. As such, it is entirely possible—indeed likely—that player strategies vary when \(n\) and \(k\) change. What this means is that some draws will not be taken at \((2, 1)\) but would be taken at \((1, 1)\) or \((2, 2)\). Conversely, some players may be willing to take draws at \((2, 1)\) that they would not take at other times.

Figure 2 shows this model graphically. The blue diagonal lines represent the payoffs to judge \(L\) for a case drawn at a particular location. For instance, if the Court takes a case with a status quo position at \(L\)'s ideal point and then moves the policy to the median justice’s ideal, that is the worst possible outcome for \(L\), and so it is the lowest point along the blue lines. If the Court accepts a case at \(M\)’s ideal point and in deciding the case keeps the policy there, \(L\) receives zero utility as the policy has not moved. Hence the blue line passes through the axis at \(M\). The red diagonal lines show the same payoff structure for justice \(R\). The payoffs for justice \(M\) are along the red line going up and to the left from \(M\) on the axis, and along the blue line up and to the right. Judge \(M\) gets no utility from taking and deciding a case at her ideal point, since there is no policy change, but draws to the right or to the left yield policy payoffs as the case outcome would pull the decision back to \(M\)’s ideal point.

The dashed lines represent each judge’s is expected payoff at \((1, 1)\). Notice that since \(M\) always gets her ideal policy in the end, she expects the highest payout. Judge \(L\), on the other hand, recognizes that \(M\) is to the right of center and expects to lose out in the final round. Equivalently, these are the justices’ cutpoint strategies at \((2, 1)\). The gray region is the gridlock interval. Cases in this region will be rejected at \((2, 1)\) but accepted at \((1, 1)\). Judge \(R\) wants to take any case to the left of the gray region. Judge \(L\) wants to take any case to the right of the gray region. The green region is the sacrifice region for judge \(L\). If the court draws a case in this region on the first draw, \(L\) will take it even though it provides a negative payoff as the Court moves the law to the right when they take the case.

Finally, we note that if the Court took cases under a majority rule, the individual justice strategies would not change in this game. Justice \(L\) still prefers to sacrifice, and \(R\) still wants to take cases immediately to the left of the gray region. What does change is the set of cases the Court would take, since \(M\) is still the median voter. Under a majority rule, the gray region would widen in both directions to the point where the dashed line \(\alpha^M\) representing \(M\)'s reserve utility crosses the relevant utility curve. As seen in figure 4, the gridlock region is much wider. Thus the Rule of Four does seem to expand the set of cases the Court will take just as the justices promised Congress it would.

The Senate

The Court example demonstrates the more general version of the model where draws come randomly from a distribution. But that need not be the case. For instance, one may suppose that in the
Figure 2: Three Judges Court: Rule of One

$V_R(x|a)$

$V_L(x|a)$

$\alpha^M$

$\alpha^R$

$\alpha^L$

$x$

Gridlock

Sacrifice
Senate, the majority leader has near monopoly power over the agenda. Accordingly, he can choose the sequence of draws that maximizes her payoffs. In that instance, the sequence of draws is fixed and contrasts with the random flow of cases in the previous example.

To motivate the Senate example we note that in the early 1960s, Senate Republicans had to balance their relative concerns for business and civil rights. Republicans controlled the filibuster pivot in the 89th Senate for civil rights issues, but Democrats had a filibuster proof majority on labor issues. For Republicans, who were largely answerable to business interests, this posed a problem; they essentially could not stop any Democratic labor bill that made it to the floor. Facing this dilemma, our model offers some opportunity for the Republicans to stave off at least some labor legislation by being accommodating on civil rights issues.

Suppose the Senate has time to pass one more bill before the close of the session. There are two possible bills remaining: a civil rights bill and a labor bill, each existing on a separate dimension as seen in figure 5. The blue circle represents the Senate majority leader—a monopolist agenda setter. For purposes of this example, we assume that she can pass legislation at her ideal point on labor issues without help from the minority leader, represented by the red square. On civil rights issues, the majority leader needs the minority leader to proved the necessary votes to invoke cloture for a proposal at her ideal labor policy.

Suppose the status quo on a particular labor issue is at $Q_L$ as shown, likewise the status quo on civil rights is at $Q_C$. The majority leader would much prefer to pass the civil rights bill, but she cannot without support from the minority leader. From the standpoint of a traditional static model (e.g. the spatial framework underlying Pivotal Politics) the minority leader will not support either proposal. Since the minority leader will not support the civil rights bill, the majority leader will not bring it to the floor. The labor bill can pass without support from the minority leader, so the majority leader brings it to a vote, and it passes. Thus, according to standard models, the majority leader will have to accept the lesser victory and the minority leader take the greater loss. But this cannot be an equilibrium in the dynamic system because both sides are better off if the minority leader agrees to support cloture on the civil rights bill.
Staffing the Executive Branch

When a new President takes office, there are roughly 4,000 political appointees that need to be hired. Obviously some are sui generis (e.g. agency heads, chief-of-staff, etc.), but many are roughly interchangeable across departments. Beyond the political domain, journals must select a finite number of articles, admissions committees can admit only so many graduate students, and corporations need to hire so many engineers or programmers. Many, if not most, of these decisions are made by committees.³

Again we focus on the end of the hiring process, where there are two candidates remaining and only one job left. Suppose that we have three committee members: L, M, R with player i having an ideal point θi in the same [0, 1] space. Two applicants are drawn from the uniform distribution. As before, each applicant (draw) is some x ∈ [0, 1]. As with the previous models, we will assume linear utilities. But as nothing is pushed to the median in this model, players receive utilities based only upon their distance from any drawn and accepted applicant. We say that the payoffs to player i from accepting an applicant at x is

\[ V^i(x) = \pi^i - 2|\theta^i - x| \]  

where \( \pi^i \) is the value player i receives from taking an applicant at her ideal point.

In the previous model, the court took all cases at (1, 1) because R would like to take any case to the left of M and move it to the right, and L has corresponding preferences to the other side. But in this model, if there is no additional constraint that requires the committee to fill up the slots, it is possible for all players to pass on a draw x at (1, 1) if \( V^i(x) < 0 \). That is, there can be a failed search.

Figure 7 shows the utility functions and strategies for each player at (2, 1). As before, M’s cutpoint is higher than threshold’s for L and R. Before this followed from M getting her way via the median voter theorem in the disposition phase of the court’s decision-making process. In this extension, the result follows because each of the extreme players suffers greatly when the committee draws and the other extreme joins the median to accept an applicant on the other side of the interval. The median is still required for any accepting coalition.

As in the Court example, we observe a gridlock region. Panel (b) shows, in grey, three gridlock regions. More extreme candidates cannot get median support, but there is a central region where neither L nor R is willing to join M to accept a centrist candidate. Notice, M cannot get the committee to accept a candidate at her ideal point on the first draw.

Panel (c) highlights that Player R has a small sacrifice region in the area denoted by red stripes. These are candidates that R would not accept in the final round, but is willing to accept on the first draw to head off the risk of L joining with M to take a more leftist candidate in the next round.

³Note that our model also applies to a single decision-maker. In that instance, the committee simply has one member with a unanimity rule.
(a) Policy space with three voters and preferences decreasing in distance from ideal points. Dashed lines are the expected value of the final draw.

(b) Green regions indicate cases accepted when there are two draws remaining and one slot to fill. The gray region is the gridlock region.

(c) Color-coded cross hatches indicate areas where at least one player votes to defensively accept cases that yield negative value. These are so-called sacrifice regions. The projection of these regions is visible on the lower boundary of the figure.

Figure 6: Utility increasing in distance from ideal points.
(a) Policy space with three voters and preferences decreasing in distance from ideal points. Dashed lines are the expected value of the final draw.

(b) Green regions indicate cases accepted when there are two draws remaining and one slot to fill. The gray region is the gridlock region.

(c) Color-coded cross hatches indicate areas where at least one player votes to defensively accept cases that yield negative value. These are so-called sacrifice regions. The projection of these regions is visible on the lower boundary of the figure.

Figure 7: Utility increasing in distance from ideal points.
The General Model

Having shown the existence of a sacrifice region, gridlock, and mobile cutpoints in a two-period game, it remains to show that these phenomena—especially the sacrifice region—remain in a larger game where the number of draws and slots increase. The concern may be that while these effects are clearly of substantive interest in a wide range of situations, if they are merely artifacts of the imminent conclusion of the decision-making process, they are less important than they would be if they were present throughout the game. With that in mind, we need to show the insights from the toy examples above and show that our results hold in larger games.

In addition to allowing an arbitrary number of draws and slots, we also generalize away from the class of games that follow a spatial representation. The three examples in the preceding section all rely on spatial preferences, and our model plainly applies to such models. But the initial example about a committee choosing among applications for cancer treatments does not have an obvious spatial representation. Therefore we describe and solve the larger game in more general terms so that our results are robust to scenarios that are or are not represented by spatial preferences.

There is a committee of \( C \) members. Players are members of the committee \( i \in \{1, 2, \ldots, C\} \).

Nature presents the committee with \( N \) sequential draws \( \{x_N, x_{N-1}, \ldots, x_1\} \) from the Baire space, \( \mathcal{N} \). From these draws, the committee may select up to \( K \). After any draw \( x_n \), when there are \( 0 < k \leq K \) slots remaining, players individually and simultaneously choose an action \( a^i \in \{\text{take, pass}\} \). If at least \( Q \) members of the committee choose to take the draw, the committee accepts the draw, and the number of available slots decreases by one. If fewer than \( Q \) players choose to take the draw, the committee passes. In our baseline version, no player has a veto.\(^4\) Then nature reveals the next draw.

Figure 8 shows the extensive form of the two period game when there are two draws and one slot available. Nature draws \( x_1 \) and then the committee chooses whether to accept or reject it. If accepted, the players get payoffs and the game ends. If the committee rejects the draw, then nature picks again. The committee then accepts or rejects that case.

The game continues until either there are no draws or no slots remaining. If the game concludes with the exhaustion of draws, each remaining slot pays out zero to each player. Once the game ends, payoffs are realized for all accepted draws. Our initial assumption is that the draws are independent, so the value to any player for taking any draw \( j \) does not depend on whether or not some \( j' \) was also accepted.\(^5\)

Each draw \( x_n \) is drawn from a compact set \( \mathcal{X} \) according to a commonly known distribution with cumulative density, \( F(\cdot) \). While this assumption is not strictly necessary for our results, we maintain it for two reasons. First, it simplifies the notation. Secondly, by eliminating the possibility of private information, we highlight that our results do not depend on information asymmetries.

When the committee accepts the draw, players receive (possibly) unique current payoffs. We define \( V^i(x) \) where \( V^i : \mathcal{X} \to \mathbb{R} \) represents a bounded function that maps cases to payoffs for each member of the committee such that \( V^i(x_n) \) is the payoff of draw \( x_n \) to player \( i \) in the event that the draw is accepted.

Figure 9 shows the larger game where there are an arbitrary number of draws and slots. Note that a node is indexed by \( \{n, k\} \) where \( n \) is the number of draws remaining and \( k \) is the number of

\(^4\)This is one key difference between our model and jury selection models like ?.

\(^5\)We argue later that this assumption is not essential to any results, but it is useful to focus attention on the core tradeoff in the model.
slots the committee has left to fill. When the committee accepts a draw, the game proceeds to the node where there are \( n - 1 \) draws and \( k - 1 \) slots remaining. If the committee passes on the draw, the game proceeds down to the right with \( n - 1 \) draws but \( k \) slots remaining.

Let \( c_{n,k}(x) = \{ i \mid a^i = \text{take}, x, n, k \} \) and \( A^Q_{n,k} = \{ x \mid \#(c_{n,k}(x) \geq Q) \} \). In words, \( c_{n,k}(x) \) is the set of players who will vote to take a draw \( x \) when there are \( n \) draws and \( k \) slots remaining. The set \( A^Q_{n,k} \) is the set of draws taken at \((n, k)\) by at least \( Q \) players. Define \( p_{n,k} = Pr\left(x \notin A^Q_{n,k}\right) \) as the probability that the committee will reject a draw, \( x \), when there are \( n \) draws and \( k \) slots remaining. Further, let \( \nu^i_{n,k} = E\left[V^i(x) \mid x \in A^Q_{n,k}\right] \) be the expected value of a draw to player \( i \) conditional on the draw being accepted. Strategies map from the draw and the number of slots and draws remaining to an action. So player \( i \)'s strategy is a mapping \( \sigma^i : X \times n \times k \rightarrow \{\text{take, pass}\} \). We define \( \Sigma = \{\sigma^1, \ldots, \sigma^C\} \).

Let \( x^j \) denote the case accepted in the \( j \)th slot and define

\[
u^i_{n,k}(x^j) = E \left[ \sum_{j=1}^{k} V^i(x^j) \mid n, k, \Sigma \right] \]

where for any slot \( z \) left unfilled, \( V^i(x^j) = 0 \). This simply means that at the end of the game, the players only receive payoffs from the cases they take. If they do not take as many cases as they could, then they forego the payoffs they might have had from those empty slots.

We now want to determine the expected value of a particular draw. Equation 4 defines player \( i \)'s expected utility when there are \( n \) draws and \( k \) slots remaining. That expected value is the weighted sum of two components. The first is the sum of the expected value of a current payoff from the
Figure 9: Game Tree

next draw conditional on it being taken plus the continuation value of the game when there is one less draw and one less slot. The second is the continuation value of the game when there is one less draw but the same number of slots. These terms are weighted by the probability that the committee accepts the next draw. We now write

\[ u_{n,k}^i = (1 - p_{n,k}) \left( \nu_{n,k}^i + u_{n-1,k-1}^i \right) + p_{n,k} u_{n-1,k}^i \]

(4)

where \( \alpha_{n,k}^i = u_{n-1,k-1}^i - u_{n-1,k}^i \). Finally, since the game ends when there are no draws or slots remaining, we define \( u_{n,0}^i = u_{0,k}^i = 0 \) for all \( n, k \).

Equilibrium

We assume players’ strategies do not depend on other players’ strategies\(^6\) in previous rounds and look for weakly nondominated, subgame perfect equilibria. Each draw represents a new subgame. Once players choose to accept or reject the draw, a new subgame begins with the subsequent draw.

\(^6\)As with the common distribution assumption, this assumption is not strictly necessary. Essentially all of our core results hold if strategies depend on histories. We maintain this assumption for notational ease and to highlight that our results do not depend on insincere voting by players.
In this next subgame, the parameters depend on what happened in the previous round. A cutpoint strategy is some set, $x^*_i = \{x^*_{n,k}\}$, such that player $i$ will accept any draw, $x$, at $(n, k)$ if and only if $V^i(x) \geq V^i_i(x^*_{n,k})$. We will focus on these strategies.

At the final draw, if there is a slot remaining, each player faces a well-defined optimal decision for any possible draw. In particular, since leaving the slot unfilled yields a payoff of zero, each player optimally sets her cutpoint at zero. By backward induction, the players can make optimal decisions under the assumption that all players will act optimally in the subsequent periods. The collection of optimal decisions by a player at each point is, by construction, a weakly undominated subgame perfect strategy. We show that they are also cutpoint strategies. The set of such strategies forms an equilibrium strategy profile, $\Sigma^* = \{\sigma^1, \ldots, \sigma^C\}$, with $\sigma^i = A^i = \{\alpha^i_{n,k} \forall n, k\}$ denoting the cutpoint strategies such that player $i$ will accept any draw $x$ at $(n, k)$ when $V^i(x) \geq \alpha^i_{n,k}$.

**Theorem 1: Equilibrium**

**Equilibrium**

1. Define $\mu_i = u^i_{1,1}$. Then $u^i_{n,n} = n\mu_i$ for all $i$, and $\alpha^i_{n,k} = \alpha^i_{1,1} = 0$ for all $i$ whenever $k \geq n$.
2. $\alpha^i_{2,1} = \mu_i$.
3. Player $i$ has a unique weakly undominated, subgame perfect equilibrium strategy $\sigma^i$, and $\Sigma^* = \{\sigma^1, \ldots, \sigma^C\}$ is the corresponding equilibrium. Specifically, players play cutpoint strategies of $\sigma^i = A^i = \{\alpha^i_{n,k} \forall n, k\}$ and accept any draw $x$ at $(n, k)$ when $V^i(x) \geq \alpha^i_{n,k}$.

All proofs are in the appendix. Theorem 1 says that players take any draw that returns at least as much as the difference between the continuation values from passing on and accepting the draw. Notice that the cutpoints here are reservation utilities. In a model with spatial utilities, this would translate directly to cutpoints in the relevant spaces. Further, when the number of slots equals or exceeds the number of draws, players’ strategies lock and expected payoffs are fixed. So players use the same cutpoints at $(1, 1)$, $(2, 2)$, and $(z, z)$, and since we normalize all players’ payoffs for a lost slot to zero, every player uses a cutpoint of zero in these scenarios. More generally, for any $n \leq k$, the players use the same strategy, $\sigma^i_{n,k} = 0$, and $u^i_{n,k} = n\mu_i$.

The following Lemma follows directly from the definition of $\alpha^i_{n,k}$.

**Lemma 1.1: Cutpoint Summation**

$$u^i_{n,k} = \sum_{0 \leq z < k} \alpha^i_{n+1,k-z}$$

Lemma 1.1 tells us something about the relationship between strategies and utilities in the previous period. Consider the extended game tree in Figure 9. Note that any horizontal row has a common number of draws remaining, $n$. From any node $(n, k)$, rejecting the draw moves the game down and to the right and accepting the draw down and left. Lemma 1.1 says that the expected
utility at node \((n, k)\) is equal to the sum of the optimal cutline strategies on the row above when there are \(k\) or fewer draws remaining.

It also leads to the following theorem.

**Theorem 2: Row Cutpoints**

For any \(n \leq N\),

\[
\sum_{k<n} \alpha_{n,k}^i = (n - 1)\mu^i
\]

Equivalently, \(\alpha_{n,k}^i = \frac{1}{n-1} \sum_{k<n} \alpha_{n,k}^i = \mu^i\) for any node \((n, k)\) where \(n > k\).

Theorem 2 follows from Lemma 1.1 and Theorem 1.1. It says that the sum of the optimal cutlines in the row where all nodes have \(n\) draws remaining is \((n - 1)\mu^i\). That is, the sum is equal to the number of draws remaining across the row minus one times the expected value of the last draw when there is at least one slot remaining. Equivalently, the average cutpoint value for any node in any row—save the nodes along the ray where \(n = k\)—is \(\mu^i\). Since it is true for every row, it follows that the average value for all cutpoints at nodes where \(n > k\) is \(\mu^i\).

However, things may look a bit different when the number of draws gets arbitrarily large. When the number of draws is practically infinite and the number of slots is finite, then players no longer care about the diminishing number of slots or draws. Essentially the draws all begin to look the same. Theorem 3 formalizes this result.

**Theorem 3: Large-\(n\)**

As \(n \to \infty\), there exists a \(\nu^i_\infty\) such that \(\alpha^i \to \nu^i_\infty\) for all finite \(k\).

The full proof is in the Appendix, but the intuition is straightforward. From the definition of \(u_{n,k}^i\) in equation 4, we have

\[
u_{n,k}^i = (1 - p_{n,k}) \left( \nu_{n,k}^i + u_{n-1,k-1}^i \right) + p_{n,k} u_{n-1,k}^i
\]

But since \(n\) is arbitrarily large we have \(n \approx n - 1\), which allows us to drop the \(n\) subscripts. Simplification yields

\[
u_k^i - \nu_{k-1}^i = \nu_k^i
\]

But notice that the left-hand side of that equation is equal to \(\alpha_k^i\). Further, \(\nu_k^i\) is a function of \(\alpha_k^i\), and so the index, \(k\), only appears on the variables we are solving for in each period. That is, the solution does not depend on \(k\), and we are left with \(\alpha^i = \nu^i\).

Further, we do know several things about optimal strategies at many points where \(n\) is finite. First, since \(\mathcal{X}\) is bounded, \(\alpha_{n,k}^i\) is bounded. This prevents the cutpoints from spinning off to plus or minus infinity. We can bound cut-lines even more strongly with the following lemma.

**Lemma 3.1: Bounded \(\alpha^i\)**

\(\alpha_{n,k}^i\) is bounded such that

\[
\min\{\alpha_{n-1,k}^i, \alpha_{n-1,k-1}^i\} \leq \alpha_{n,k}^i \leq \max\{\alpha_{n-1,k}^i, \alpha_{n-1,k-1}^i\}
\]

for \(k \geq 3\).
Lemma 3.1 says that the cutpoint for any node \((n, k)\) in the tree where \(k \geq 3\) is bounded by the cutpoints connected to it below.

**Implications**

We focus on two particular implications of our model. First, there are instances where a player will choose to accept a draw even if it means taking a negative current payoff instead of a zero payoff. These sets of cases we call *sacrifice regions*. Second, we examine the set of draws the committee will not take and call it a *gridlock region*.

**Definition 1: Sacrifice Region**

A sacrifice region is a set, 

\[
S_{n,k}^i = \{ x \mid \alpha_{n,k}^i \leq V^i(x) < 0 \}
\]

That is, it is a player-specific collection of draws that \(i\) will vote to take, even though \(i\) receives a negative payoff. A sufficient condition for a sacrifice region is \(\mu^i < 0\).

Definition 1 follows directly from the definition of \(\alpha_{n,k}^i\) and Theorem 1. It states that if a player expects to suffer a utility loss at \((1, 1)\), she is willing to play a strategy in an earlier round that takes a small negative payout to ward off the risk of a very negative payoff in a future round. Such a player takes a negative current payoff in lieu of a zero payoff, simply to defend against the expected downside from a subsequent draw.

A sacrifice region is a strategy for a player who recognizes that the best defense is an aggressive offense. Rather than sitting back and hoping that bad draws do not come her way, a player that expects to lose big in the next round acts aggressively in this round to take a small loss instead. Notice that there is no risk-aversion driving this result. Risk neutral players with cutpoint strategies less than zero simply follow this logic to its inevitable conclusion. Filling up the slot with a small loss is better than leaving the slot open when the player expects a big loss in the following round.

While operating at this level of generality makes it impossible to specify exactly where a sacrifice region will occur in a given game, the following lemma follows from Theorem 2 and Lemma 3.1.

**Lemma 3.2: Sacrifice Regions**

1. If \(\mu^i \leq 0\), then for any \(n > 2\) there is some \(k > 0\) such that \(\alpha_{n,k}^i \leq 0\), and the last inequality is strict if the first is also strict.

2. If for some node \((\bar{n}, \bar{k})\) where \(\bar{k} > 2\) we have \(\alpha_{\bar{n},\bar{k}}^i \leq 0\), then there is a path of connected nodes from \((\bar{n}, \bar{k})\) to some node \((\tilde{n}, 1)\) such that \(\alpha_{n,k}^i \leq 0\) for all \(\tilde{n} < n < \bar{n}\) and \(\bar{k} < k < 1\).

The first part of Lemma 3.2 says that if Player \(i\) follows a sacrifice region in the two period game, then if we expand to the arbitrary game shown in Figure 9, then no matter how many extra draws we add, there is always some number of slots such that the player will also follow a sacrifice strategy. That is, if there is a sacrifice strategy in the two-period game, that strategy will also exist in the larger game with more draws, so our results from the two-period model extend well-beyond the two-period setting.
The second part of the lemma says that if while playing the game Player $i$ follows a sacrifice strategy at some point when there are more than two slots remaining, then there is a sequence of draws for which Player $i$ would continue to play a sacrificing strategy at least until there is only one slot remaining. Together, Lemma 3.2 shows that the presence of a sacrifice region is guaranteed up the game tree if there is a sacrifice strategy in the two-period game, and if there is a sacrifice strategy in the general game where $k > 2$, then the sacrifice strategy flows down the tree until hitting the left diagonal where $k = 1$ in Figure 9, even if there is no sacrifice region in the two-period game.

**Definition 2: Gridlock Regions**

A gridlock region is a set, $\Gamma_{n,k} = \{x \mid x \notin A_{n,k}^Q\}$. It is the set of draws that the committee will not take under a $Q$-rule at $n, k$.

Our concept of a gridlock region is similar to the gridlock interval well known to legislative scholars as the region in a policy space between pivotal players in the legislative process (Krehbiel (1998), Cameron (2000), Krehbiel (2010). In the traditional, legislative formulation, the gridlock region exists because lawmaking requires a coalition that must include certain players: median member, filibuster pivot, president or veto override pivot, etc. If at least one of these players prefers the status quo to the proposal, the proposed policy fails.

In our model, the gridlock interval exists because no coalition of $Q$ players finds it valuable enough to accept any draws in that region. Moreover, this is closely related to the sacrifice region described above in that those policies that fall in a pivotal player’s sacrifice region would otherwise be absorbed into the gridlock region, although there may also be policies in the gridlock region which yield a positive payoff for a majority of players provided that those policies are not sufficiently valuable to induce selection according to the equilibrium cutpoint strategies played by the committee members.

**An Extended Example**

The values of particular equilibrium cutpoints are not so easily discovered. This follows from understanding that the $\nu_{n,k}^i$ value in equation 4, the expected value of a draw conditional on the draw being taken, depends not only on player $i$’s strategy but on all players’ strategies, $\Sigma^*$. To motivate our exploration of the larger game and the sacrifice region in particular, recall the example of NIH committee selecting a study to fund under majority rule.

<table>
<thead>
<tr>
<th>Table 2: NIH Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Chemo    Acupuncture Stem Cell Treasury</td>
</tr>
<tr>
<td>P1    0.5 0.4 1 0</td>
</tr>
<tr>
<td>P2    0.1 1 0.1 0</td>
</tr>
<tr>
<td>P3    1 $-\frac{1}{3}$ -2 0</td>
</tr>
</tbody>
</table>

When there are two draws, Player 3 is forced to sacrifice when $n = 2$ and to support the acupuncture study if offered so as to head off the risk of a stem cell study in the final round. But
things change if we add additional draws to the game. Table 3 shows the first ten periods of the game.

Column two in table 3 shows the probability that the committee will reject a drawn application. The three columns on the right show the expected value to each player at the point where there are \( n \) draws remaining to fill one slot. Notice that if there are six or fewer draws, then Player 3 plays a sacrifice strategy.\(^7\) But when there are six draws remaining, Player 1 will vote for chemo, which greatly improves the outlook for Player 3.

Table 3: Ten Draws and One Slot

<table>
<thead>
<tr>
<th>( n )</th>
<th>( p_{n,1} )</th>
<th>( u_{n,1}^1 )</th>
<th>( u_{n,1}^2 )</th>
<th>( u_{n,1}^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.80</td>
<td>0.40</td>
<td>-0.44</td>
</tr>
<tr>
<td>2</td>
<td>0.667</td>
<td>0.67</td>
<td>0.60</td>
<td>-0.41</td>
</tr>
<tr>
<td>3</td>
<td>0.667</td>
<td>0.58</td>
<td>0.73</td>
<td>-0.38</td>
</tr>
<tr>
<td>4</td>
<td>0.667</td>
<td>0.52</td>
<td>0.82</td>
<td>-0.37</td>
</tr>
<tr>
<td>5</td>
<td>0.667</td>
<td>0.48</td>
<td>0.88</td>
<td>-0.36</td>
</tr>
<tr>
<td>6</td>
<td>0.333</td>
<td>0.46</td>
<td>0.66</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.667</td>
<td>0.47</td>
<td>0.47</td>
<td>0.40</td>
</tr>
<tr>
<td>8</td>
<td>0.667</td>
<td>0.48</td>
<td>0.35</td>
<td>0.60</td>
</tr>
<tr>
<td>9</td>
<td>0.667</td>
<td>0.49</td>
<td>0.27</td>
<td>0.73</td>
</tr>
<tr>
<td>10</td>
<td>0.667</td>
<td>0.49</td>
<td>0.21</td>
<td>0.82</td>
</tr>
</tbody>
</table>

What is happening is that when there are sufficiently many draws, Player 3 aligns with Player 1 to form a chemo faction. But as the number of draws shrinks, Player 1 leaves the coalition hoping for a lucky sequence of draws that will lead to stem cells. Having lost his coalition partner, Player 3 has to shift strategies as well to defend against stem cells by voting for acupuncture.

Notice that if the friends followed a rule of one (like the Supreme Court example), then Player 1 is always holding out for stem cells, so Player 3 would have to play a sacrificing strategy for any \( n \).\(^8\) But under the majority rule, if there are sufficiently many draws, then Player 3 can hope for chemo.

Player 1 is willing to join the chemo coalition because she prefers the chemo study to acupuncture. If she enjoyed them both equally (e.g. if P1 received a .4 payout from chemo and acupuncture), then something surprising happens. Since she is indifferent as to the two, Player 1 has no reason to join a coalition to take chemo over acupuncture. In this case, Player 1 simply holds out hoping for a sequence of draws leading to a stem cell study in the final round. Because Player 1 will never join the chemo coalition and because the loss from stem cells overwhelms the potential gain from chemo in the final round, Player 3 sacrifices and supports acupuncture from the start. Thus a change in Player 1’s payoffs induces a change in Player 3’s strategy.

This shows that as the game expands, what drives behavior is how aligned or opposed preferences are between sufficiently large coalitions. A player can expect a sufficient selectorate to form around a favorable draw if the draw is sufficiently likely and gives a relatively large payoff to

\(^7\)Recall that the cutline when \( n = 6 \) is equal to the expected value if the node where \( n = 5 \).

\(^8\)Of course, in this example, Player 2 would also take acupuncture, but we could alter P2’s payoffs so that she would not choose acupuncture, and then P3’s sacrificing strategy would be pivotal.
enough players. In such a situation, it makes sense to wait for such a nice draw. This is the steak coalition for $n > 6$. But if there is a coalition waiting in the wings that will impose a big loss on a player, she may be willing to form a coalition that leads to a smaller loss for sure to head off that worse coalition later. That is the sacrifice strategy of Player 3 in rounds where $n \leq 6$.

Discussion

Generalizability

In the presented model, we made several simplifying assumptions to focus attention on the key tradeoff in our model between certainty now and expectations in the future. Specifically, we assumed draws players have perfect information about other players’ payoffs and $F(\cdot)$, which we assumed to be random, that players could not bank a draw for later consideration, and that players’ payoffs from accepting a draw did not depend on accepting or denying some other draw. We now argue that our model is robust to relaxing these assumptions.

Perfect Foreknowledge

If the potential current payoffs and order of all subsequent draws are common knowledge (for example because the order is set by a monopolist agenda setter), the model works just as before. The only difference is that instead of drawing each case from $\mathcal{X}$, each case is drawn from a singleton set, $\mathcal{X}_{n,k} \subset \mathcal{X}$. All that changes in this extension is that players have better information about future cases. This does not change their desire to maximize their payouts.

Consider a two-period game where players only have one slot to fill. Suppose player $i$ can vote to accept a draw now that will offer her a payoff of one. But because all players know the following sequence of draws, she knows that if the first draw is declined, the payoff in the next and final round will be worth two. Clearly, she would want to wait. But suppose that player $i$ only knows that the payoff from the final draw will be distributed $\mathcal{N}(0, 1)$. Then she will want to accept the first draw, since in expectation, waiting will only offer a payoff of zero. The difference between the two scenarios is the information available. But from the player’s perspective, all she wants to do is maximize her payoff subject to the information available. The informational environment clearly influences the equilibrium cutlines, but providing better information does not alter the fundamental structure of equilibrium decision-making.

Differing Beliefs

It may be the case that some players pay more attention to the mechanism that governs the drawing process. For instance, suppose in the context of the Supreme Court, some justices are watching the lower courts more closely than others, and such justices have a better sense of the distribution of cases likely to come up on appeal. Similarly, justices may have different priors about the true distribution of draws, and they may update differently based on the sequence of observed draws.

Define $F^i(\cdot)$ be justice $i$’s beliefs about the distribution of draws and $V^i(x) : \mathcal{X} \to \mathbb{R}^C$ be the justice’s belief as to the payoff function that allocates finite payoffs to each player for each draw. Justice $i$ is now able to form beliefs about other players’ strategies, $\sigma^{-i}$. At this point the game follows in exactly the same fashion as the preceding analysis with complete information. While the
realized payoffs may change as a result of uncertainty and differing beliefs, the decision-making processes of the individual players are identical.

If the players’ beliefs are weak, they may also update those beliefs as the game unfolds. Denote the set of draws already observed as $\hat{x}_n = \{x_m | m > n\}$ and the set of votes on those draws as $\hat{c}_n(\hat{x}_n) = \{c_m(x_m) | m > n\}$ where $c_n(x)$ is the observed distribution of votes over whether to accept or reject $x_n$. We can now write the players’ updated beliefs as a function of the prior beliefs, $F^i(\cdot)$ and $V^i(\cdot)$, as well as the observed pattern of observations and votes so that we have $\{F^i_n \times V^i_n\} : \{F^i_m | m > n, j \in C\} \times \{V^i_m | m > n, j \in C\} \times \hat{c}_n \times \hat{n} \rightarrow \mathbb{R}^{C+1}_+$. That is, the players’ beliefs about the distribution of draws and other players’ preferences are determined as a function of the observed histories of the game. Still, though, the period decision-making process remains unchanged and responds to these uncertain beliefs about the state of the world as they would respond in the complete information environment presented above.

Nuance arises in this state, however, as $F^i_n$ and $V^i_n$ are not uniquely determined in this scenario. In particular, consider an example in which beliefs about the distribution are relatively strong but players are completely uninformed about the preferences of the other players. In one case, the first draw such that it is unanimously accepted so that in the subsequent period, players still have uniform beliefs over the preferences of their peers. However in a second case, the draw is only marginally selected so that there is now a clear dichotomy differentiating the preferences of the players supporting the draw and those opposing it. As a result, despite being at the same node in the game, the beliefs of the players about each others’ preferences in the two cases will differ.

**Independence and Replacement**

The equilibrium is also robust to systems in which the draws during each period may not be independent, either because the draws themselves are not independent of each other within a single distribution of possibilities or because the choice of whether to accept or reject a single draw affect the distribution that future draws are pulled from. In the first case, it may simply be that similar draws come up together, such as similar economic matters during an economic crisis. The result of each draw then changes the players’ beliefs about the subsequent draws in the same matter as in the preceding discussion without changing their approach to the equilibrium. In the second case, the players’ decision also affects the distribution of draws that may arise at a later point, which will affect the total payoff induced by the choice and thus the position of each potential draw relative to their cutpoint at any given stage, but again, the equilibrium concept remains the same.

Further, in some instances, it may be possible for players to reserve judgment on a particular draw until a later stage of the game. The draw may return for consideration at either a known point in the game—say after the next draw is observed—or it may simply return with some positive probability at one or more future points. In either case, the result is similar to the perfect foreknowledge case considered earlier. Setting the case aside provides more information about future draws.

Suppose at node $(\hat{n}, \hat{k})$ the committee draws $\bar{x}$ and sets it aside. When the committee makes another draw, there are still $\hat{n}$ draws remaining, but the committee now has information about one of them, since it will consider $\bar{x}$ again. If there is some commonly known, positive probability $\tau$ at node $(n, k)$ that $\bar{x}$ will be considered, then the draw at $(n, k)$ must come from a commonly known distribution with cdf $F^*(\cdot)$ which is a weighted average of a draw from $F(\cdot)$ and $\bar{x}$. Again, the change in information will influence the realization of a particular equilibrium, but it will not alter
the fundamentals of the game.

Substantive Implications

We believe that this larger model is useful in a wide range of political environments and has implications for existing studies. In particular, the model has immediate implications for scaling models. These models universally almost always assume that players vote sincerely for the proposal or status quo based on expressive or policy payoffs determined by the spatial distance between the two options and the player’s ideal point. The idea is that if the proposal $P$ is farther from player $i$’s ideal point $\theta_i$ than the status quo $Q$ is, then player $i$ will vote against the proposal. Our model shows that when placed in a dynamic environment, players have to account for not only the current payoffs due to spatial policy concerns, but also the effect on the larger agenda. Thus a senator may oppose a particular policy but still vote for cloture so that the Senate will spend time on what is from that senator’s perspective the lesser of two evils.

Our model also demonstrates how it is possible for a monopolist agenda setter like the Senate majority leader or the Speaker of the House to choose his or her optimal agenda. Suppose the Senate majority leader has access to a range of potential bills in $X$ and can propose $N$ of them but the legislature only has time to pass $K$. From the perspective of the other senators, each bill is drawn according to some distribution $F(\cdot)$. But now this distribution is not randomly generated by nature but is the strategic selection of the agenda-setter. However, so long as $F(\cdot)$ is a cdf, the model is unchanged. From Theorem 1 and subsequent discussion, we know that each legislator will follow a cut-point strategy based on his belief about $F(\cdot)$ and other players’ strategies. Any agenda, $A_j$ selected by the majority leader will induce outcomes consistent with the equilibrium behavior of the rest of the legislators. Theorem 4 shows that there is always at least one optimal agenda for the agenda-setter that maximizes her payoffs.

**Theorem 4: Optimal Monopoly Agenda**

Let $A_N$ be the set of all possible agendas of size $N$, where any element, $A_j \in A_N$, is an ordered subset of $X$. If the committee can accept $K$ draws and player $i$ has monopoly agenda-setting power, there exists some nonempty subset $A_N^* \subset A_N$ where every agenda in $A_N^*$ maximizes the setter’s total utility.

Conclusion

We present a dynamic model of committee decision-making. Players tradeoff certain payoffs in the present period against prospective payoffs in future periods. As such, whether players accept a draw depends on what stage the game is in. What they are willing to take changes as the game goes on. At each stage, players will accept draws if it exceeds an equilibrium threshold value.

In our two-period examples, we demonstrate that players who expect to win are patient in that they turn down small gains to protect the ability to get a big payoff later. Conversely, players that expect to lose get aggressive and accept draws with lower, and sometimes negative, payoffs. We also show the possibility of a gridlock region within which players will not accept a draw. This is a cousin of the legislative gridlock interval, but it differs in several ways. Our gridlock interval is
not necessarily connected nor is it the result of institutionally empowered specific pivots. These results extend to the larger game where \( n \) and \( k \) are arbitrary. We show that, on average, players always play the same cut-point as the game expands.

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References


Chen, Ying and Hulya Eraslan. 2015. “Dynamic agenda setting.” *Available at SSRN 2615877* .


A Proofs

Proof of Theorem 1: Equilibrium

Check mixed strategies

1. Define $\mu^i = u^i_{1,1}$. Then $u^i_{n,n} = n\mu^i$ for all $i$, and $\alpha^i_{n,k} = \alpha^i_{1,1} = 0$ for all $i$ whenever $k \leq n$.

2. $\alpha^i_{2,1} = \mu^i$.

3. Player $i$ has a unique weakly undominated, subgame perfect equilibrium strategy $\sigma^i*$, and $\Sigma^*$ is the corresponding equilibrium. Specifically, players play cutpoint strategies of $\sigma^i* = A^i = \{\alpha^i_{n,k} \forall n, k\}$ and accept any draw $x$ at $(n, k)$ when $V^i(x) \geq \alpha^i_{n,k}$.

We proceed by backward induction and begin by proving parts (1) and (2). Recall that the value of an unfilled slot at the end of the game is zero. If she is pivotal, player $i$ votes to take a draw $x_n$ if $V^i(x_n) + u^i_{n-1,k-1} \geq u^i_{n-1,k}$. That is, she votes to take a draw if the current payoff plus the continuation value of the next draw when there is one less slot is greater than the continuation value of the next draw when the court does not fill a slot in this period. If she plays any strategy in which she accepts some draw, $x'_n$, such that $V^i(x_n) + u^i_{n-1,k-1} < u^i_{n-1,k}$ or rejects some $x''_n$ such that $V^i(x_n) + u^i_{n-1,k-1} \geq u^i_{n-1,k}$, she can improve her expected payoff by deviating to reject $x'_n$ or accepting $x''_n$, respectively. Thus, at $(1, 1)$, player $i$ plays the strategy, $\sigma^i_{1,1} = 0$ yielding an expected payoff of $\mu^i$ whenever she is pivotal. If she chooses some alternative, $\sigma^i_{1,1} > 0$, her expected payoff when pivotal is

$$E[U^i(\sigma^i_{1,1})] = \int_{\alpha^i_{1,1}}^{\infty} p(x) V^i(x) dx \leq \int_{0}^{\infty} p(x) V^i(x) dx = \mu^i$$

(7)

Since $\int_{0}^{\infty} p(x) V^i(x) dx \geq 0$. Thus, $\sigma^i_{1,1} > 0$ cannot yield a profitable deviation. Con-
versely, if she chooses some $\sigma_{1,1}^i < 0$, her expected payoff in $(1, 1)$ is

$$E[U^i(\sigma_{1,1}^i)] = \int_{\alpha_{1,1}^i}^{\infty} p(x)V^i(x)dx \leq \int_0^{\infty} p(x)V^i(x)dx = \mu^i$$

(8)

Since $\int_{\alpha_{1,1}^i}^0 V^i(x)p(x)dx \leq 0$. Thus, $\sigma_{1,1}^i < 0$ cannot yield a profitable deviation. Therefore, $\sigma_{1,1}^i = 0$ is an equilibrium strategy for player $i$ when she is pivotal. If she is not pivotal, her actions do not affect the outcome of the game and there cannot be any strictly profitable deviation, so the equilibrium is weakly undominated regardless of other players strategies. For the remainder of the proof, we likewise consider weakly undominated strategies

Moreover, for any proposed equilibrium, $\sigma_{1,1}^{i''} \neq 0$,

$$E[U^i(\sigma_{1,1}^{i''})] = \int_{\alpha_{1,1}^{i''}}^{\infty} p(x)V^i(x)dx \leq \int_0^{\infty} p(x)V^i(x)dx = \mu^i$$

(9)

so that there is a weakly profitable deviation that is strict whenever $V^i(x)$ has continuous support over some neighborhood around zero.

Now consider the strategy at $(1, 2)$, $\sigma_{1,2}^i$. In this case, the committee can select at most one case, $x_1$, and the remaining slot will necessarily remain unfilled, yielding a payoff of zero. Therefore, this strategy is identical to that in the state, $(1, 1)$, and $\sigma_{1,2}^i = \sigma_{1,1}^i = 0$.

Finally, consider the general strategy in $(n, k)$ for $n \leq k$, $\sigma_{n,k}^i$. By induction from the base cases of $(1, 1)$ and $(1, 2)$, if she accepts the case, $x_n$ when pivotal, player $i$ expects to earn $\mu^i$ in the subsequent state, $(n - 1, k - 1)$. If she rejects, she expects $\mu^i$ in the subsequent state, $(n - 1, k)$. Thus, her strategy in this period does not affect her payoff in the remaining periods, and she need simply maximize the single-period payoff. Following the same logic as above, this implies $\sigma_{n,k\geq n}^i = \sigma_{1,1}^i = 0$. This concludes the proof of part (1).

For part (2), consider the payoff of player $i$ playing strategy, $\sigma_{2,1}^i$. If player $i$ rejects $x_2$ when pivotal, the case is rejected and she subsequently expects $u^i_{1,1} = \mu^i$. If she accepts the case, she earns the payoff from that case and a payoff from subsequent draws that is equivalent to
Thus for a strategy, \( \sigma_{2,1}^i \), she earns the expected payoff,

\[
E[U^i(\sigma_{2,1}^i)] = \int_{\alpha_{2,1}^i}^{\infty} p(x) V^i(x) dx + \int_{-\infty}^{\alpha_{2,1}^i} p(x) \mu^* dx
\]

(10)

Following the same logic as in part (1), the unique optimal strategy in equilibrium is \( \sigma_{2,1}^i = \mu^* \). Any deviation, higher or lower, will result in a lower payoff. This completes part (2).

Applying the logic of parts (1) and (2) to each player, the full equilibrium strategy profile, \( \Sigma^* \) is a unique, subgame perfect equilibrium, completing the proof.

Proof of Lemma 1.1: Cutpoint Summation

\[
u_{i,n,k}^i = \sum_{0 \leq z < k} \alpha_{n+1,k-z}^i
\]

(11)

Proof of Theorem 2: Row Cutpoints

Consider again the definition of \( \alpha_{n,k}^i \). Recall that

\[
\alpha_{n,k}^i = u_{n-1,k}^i - u_{n-1,k-1}^i
\]

(12)

We can rearrange terms easily enough.

\[
u_{n-1,k}^i = u_{n-1,k-1}^i + \alpha_{n,k}^i
\]

(13)

But this in turn implies that

\[
u_{n-1,k-1}^i = u_{n-1,k-2}^i + \alpha_{n,k-1}^i
\]

(14)
And thus we can rewrite Equation 13 recursively as

$$u_{n-1,k}^i = \sum_{0 \leq z < k} \alpha_{n,k-z}^i$$ (15)

From Theorem 1.1, we know that the expected utility for each node along the right diagonal where \( n = k \) is \( n\mu \) and at such nodes, \( \alpha_{n,k}^i \) is zero. Since we know the value of \( \alpha_{n,k}^i = 0 \) when \( n = k \), we are interested in the value of \( \alpha_{n,k}^i \) when \( n > k \). Equation 15 tells us that those cut-points—along the row where there are \( n \) draws remaining but \( n > k \)—sum to \( (n-1)\mu^i \). This in turn implies that the average value of these \( n - 1 \) cut-points, which we denote \( \bar{\alpha}_{n}^i \), is \( \mu^i \).

$$\bar{\alpha}_{n}^i = \frac{1}{n-1} \sum_{1 \leq z \leq n} \alpha_{n,z}^i = \frac{1}{n-1} u_{n-1,n-1}^i = \frac{1}{n-1} n\mu^i = \mu^i$$ (16)

**Proof of Theorem 3: Large-\( N \) Limit**

Suppose \( n = \infty \) so that \( n = n - 1 \) and \( i \) is pivotal at \((n, k)\). The equilibrium now reduces to

$$\alpha_{n,k}^i + u_{n,k-1}^i = u_{n,k}^i$$

$$= (1 - p_{n,k}) \left( \nu_{n,k}^i + u_{n,k-1}^i \right) + p_{n,k} u_{n,k}^i$$

which, dropping the \( n \) subscript, reduces to

$$\alpha_k^i = (1 - p_k) \nu_k^i + p_k \left( u_k^i - u_{k-1}^i \right)$$

$$= (1 - p_{n,k}) \nu_k^i + p_{n,k} V^i(i + \delta_{i,k})$$

$$= \nu_k^i$$ (18)

Note that \( \nu_k^i \) is simply a function of \( \alpha_k \), so that the index, \( k \), only appears on the variables we are solving for in each period. That is, the solution does not depend on \( k \), and we are left with \( \alpha^i = \nu^i_\infty \).
Proof of Lemma 3.1: Bounded $\alpha^i$

$\alpha^i_{n,k}$ is bounded such that

$$\min\{\alpha^i_{n-1,k}, \alpha^i_{n-1,k-1}\} \leq \alpha^i_{n,k} \leq \max\{\alpha^i_{n-1,k}, \alpha^i_{n-1,k-1}\} \quad (19)$$

for $n > k > 2$.

Consider two draws beginning at an arbitrary node $(n+1, k)$ where $n \geq 2$ and $2 \leq k \leq n-1$. We want to show that $\alpha^i_{n+1,k}$ is weakly bounded by $\{\alpha^i_{n,k}, \alpha^i_{n,k-1}\}$. There are two cases:

1. $\alpha^i_{n,k-1} \geq \alpha^i_{n,k}$
2. $\alpha^i_{n,k-1} \leq \alpha^i_{n,k}$

We prove the Lemma for Case 1. The proof for the second case is symmetric.

Suppose Case 1, the optimal cutpoint $\alpha^i_{n+1,k} > \alpha^i_{n,k-1}$. Then there exists some draw, $x_{n+1}$, such that $V^i(x_{n+1}) > \alpha^i_{n,k-1}$ such that player $i$ is better off passing on $x_{n+1}$ than she would be by taking $x_{n+1}$. Now suppose the next draw, $x_n$ yields a payoff of $V^i(x_n)$, and consider three instances:

1. $V^i(x_n) > \alpha^i_{n,k-1}$
2. $V^i(x_n) < \alpha^i_{n,k}$
3. $\alpha^i_{n,k} \leq V^i(x_n) \leq \alpha^i_{n,k-1}$

If $V^i(x_n) \geq \alpha^i_{n,k-1}$, then regardless of whether or not player $i$ takes $x_{n+1}$, she will accept $x_n$. So the choice is between $\{\text{take, take}\}$ and $\{\text{pass, take}\}$. The former yields $V^i(x_{n+1}) + V^i(x_n) + u^i_{n-1,k-2}$ the latter gives $V^i(x_n) + u^i_{n-1,k-1}$. Recall that $u^i_{n-1,k-1} = u^i_{n-1,k-2} + \alpha^i_{n,k-1}$ by definition and $V^i(x_{n+1}) > \alpha^i_{n,k-1}$ by assumption. Thus $V^i(x_{n+1}) + V^i(x_n) + u^i_{n-1,k-2} > V^i(x_n) + u^i_{n-1,k-2} + \alpha^i_{n,k-1}$.

If $V^i(x_n) < \alpha^i_{n,k}$, then the choice is between $\{\text{take, pass}\}$ and $\{\text{pass, pass}\}$. Taking $x_{n+1}$ then would provide a total payoff of $V^i(x_{n+1}) + u^i_{n-1,k-1}$ whereas passing on both draws
leaves \( u_{n-1,k}^i \). Again, by definition, \( u_{n-1,k}^i = u_{n-1,k-1}^i + \alpha_{n,k}^i \) and by assumption, \( \alpha_{n,k}^i \leq \alpha_{n,k-1}^i < V^i(x_{n+1}) \). And so \( V^i(x_{n+1}) + u_{n-1,k-1}^i > u_{n-1,k}^i \).

If \( \alpha_{n,k}^i \leq V^i(x_n) < \alpha_{n,k-1}^i \), the choice is between \( \{ \text{take, pass} \} \) and \( \{ \text{pass, take} \} \). The former offers \( V^i(x_{n+1}) + u_{n-1,k-1}^i \) the latter yields \( V^i(x_n) + u_{n-1,k-1}^i \). But since by assumption \( V^i(x_{n+1}) > \alpha_{n,k-1}^i > V^i(x_n) \), we have \( V^i(x_{n+1}) + u_{n-1,k-1}^i > V^i(x_n) + u_{n-1,k-1}^i \).

These arguments show that regardless of the value of the following draw, player \( i \) would be better off taking a draw that pays \( V^i(x_{n+1}) \) if \( \alpha_{n,k-1}^i < V^i(x_{n+1}) \leq \alpha_{n+1,k}^i \).

Now suppose \( \alpha_{n+1,k}^i < \alpha_{n,k}^i \). Then there would be some draw \( V^i(x_{n+1}) < \alpha_{n,k}^i \) such that player \( i \) is better off taking \( x_{n+1} \) than she would be by passing on it. We again consider the three possible cases for the next draw, \( x_n \).

If \( V^i(x_n) \geq \alpha_{n,k-1}^i \), the choice is again between \( \{ \text{take, take} \} \) and \( \{ \text{pass, take} \} \). The former yields \( V^i(x_{n+1}) + V^i(x_n) + u_{n-1,k-2}^i \) the latter gives \( V^i(x_n) + u_{n-1,k-1}^i \). Recall that \( u_{n-1,k-1}^i = u_{n-1,k-2}^i + \alpha_{n,k-1}^i \) by definition, but now \( V^i(x_{n+1}) < \alpha_{n,k}^i \leq \alpha_{n,k-1}^i \) by assumption. Thus \( V^i(x_{n+1}) + V^i(x_n) + u_{n-1,k-2}^i < V^i(x_n) + u_{n-1,k-2}^i + \alpha_{n,k-1}^i \).

If \( V^i(x_n) < \alpha_{n,k}^i \), then the choice is between \( \{ \text{take, pass} \} \) and \( \{ \text{pass, pass} \} \). Taking \( x_{n+1} \) then would provide a total payoff of \( V^i(x_{n+1}) + u_{n-1,k-1}^i \) whereas passing on both draws leaves \( u_{n-1,k}^i \). Again, by definition, \( u_{n-1,k}^i = u_{n-1,k-1}^i + \alpha_{n,k}^i \), and by assumption, \( V^i(x_{n+1}) < \alpha_{n,k}^i \). So \( V^i(x_{n+1}) + u_{n-1,k-1}^i < u_{n-1,k}^i \).

If \( \alpha_{n,k}^i \leq V^i(x_n) < \alpha_{n,k-1}^i \), the choice is between \( \{ \text{take, pass} \} \) and \( \{ \text{pass, take} \} \). The former offers \( V^i(x_{n+1}) + u_{n-1,k-1}^i \) the latter yields \( V^i(x_n) + u_{n-1,k-1}^i \). But since by assumption \( V^i(x_{n+1}) < \alpha_{n,k}^i < V^i(x_n) \), we have \( V^i(x_{n+1}) + u_{n-1,k-1}^i < V^i(x_n) + u_{n-1,k-1}^i \).

These show that there is no draw \( V^i(x_{n+1}) < \alpha_{n,k}^i \) that player \( i \) should take. Together these arguments show that the optimal cut-point \( \alpha_{n+1,k}^i \) must be situated between \( \alpha_{n,k-1}^i \) and \( \alpha_{n,k}^i \).
Proof of Lemma 3.2: *Sacrifice Regions*

1. If \( \mu^i \leq 0 \), then for any \( n > 2 \) there is some \( k > 0 \) such that \( \alpha^i_{n,k} \leq 0 \), and the last inequality is strict if the first is also strict.

   Following Theorem 2, the mean value of \( \alpha^i_n, \bar{\alpha}^i_n \), must be equal to \( \mu^i \) for every \( n > 2 \).

   Lemma 3.1,

2. If for some node \( (\bar{n}, \bar{k}) \) where \( \bar{k} > 2 \) we have \( \alpha^i_{\bar{n},\bar{k}} \leq 0 \), then there is a path of connected nodes from \( (\bar{n}, \bar{k}) \) to some node \( (\tilde{n}, 1) \) such that \( \alpha^i_{n,k} \leq 0 \) for all \( \tilde{n} < n < \bar{n} \) and \( \bar{k} < k < 1 \).

1. If \( \mu^i < 0 \), then for any node \( (n, k) \) with \( n - k \leq 4 \), \( \alpha_{n,k} \leq 0 \):

   Consider a case where we have, \( \mu^i < 0 \), \( \alpha^i_{2,1} = \mu^i \), and \( u^i_{m,m} = m\mu^i \). Recalling Lemma 3.1, this implies \( \alpha^i_{3,2} < 0 \), and by induction, that \( \alpha^i_{n,n-1} \leq 0 \) for all \( n > 1 \).

   Furthermore, using Theorem 2, we must have \( \alpha^i_{3,1} < 0 \), so again following the same inductive logic, we have

   \[
   \alpha^i_{n,n-2} \leq \alpha^i_{n-1,n-2} \leq 0 \quad \forall \ n > 2
   \]  

   (20)

   In turn, this implies that

   \[
   \max \{\alpha^i_{n,n-2} + \alpha^i_{n-1,n-2}\} \geq 2\mu^i
   \]  

   (21)

   So that applying the constraint,

   \[
   \alpha^i_{4,1} + \alpha^i_{4,2} + \alpha^i_{4,3} = 3\mu^i
   \]  

   (22)
we have $\alpha_{4,1}^i \leq \mu_i$ and $\alpha_{n,n-3}^i \leq 0$ for all $n > 3$. Finally, consider the constraint,

$$\min\{\alpha_{n,k}^i, \alpha_{n,k+1}^i\} \leq \alpha_{n+1,k+1}^i \leq \max\{\alpha_{n,k}^i, \alpha_{n,k+1}^i\} \quad \forall n > 2, k \in \{2, \ldots, n-1\}$$

(23)

Applying this to the nodes in which $n - k \leq 2$, the minimum derived in Equation (21) is always $2\mu_i$, so that $\alpha_{5,3}^i + \alpha_{5,4}^i \geq 2\mu_i$. Meanwhile, the minimum value of $\alpha_{5,2}^i$ is $\min\{\alpha_{4,1}^i, \alpha_{4,2}^i\}$. Minimizing this value subject to $\alpha_{5,3}^i + \alpha_{5,4}^i = 2\mu_i$ yields the resulting values,

$$\begin{align*}
\alpha_{5,1}^i &= 0 & \alpha_{4,1}^i &= \mu_i \\
\alpha_{5,2}^i &= 2\mu_i & \alpha_{4,2}^i &= 2\mu_i \\
\alpha_{5,3}^i &= 2\mu_i & \alpha_{4,3}^i &= 0 \\
\alpha_{5,4}^i &= 0 & \alpha_{4,4}^i &= 0 \\
\alpha_{5,5}^i &= 0
\end{align*}$$

(24)

which represents an absolute minimum on the sum,

$$\alpha_{5,2}^i + \alpha_{5,3}^i + \alpha_{5,4}^i = 2\mu_i$$

(25)

and correspondingly the maximum, $\alpha_{5,1}^i = 0$. Thus, $\alpha_{n,n-4}^i \leq 0$ for all $n > 4$.

2. If $\mu_i < 0$ and there exists an integer $m > 1$ such that $\alpha_{m,1}^i > 0$, then there also exists an integer, $m^*$ such that $\alpha_{m^*,1}^i < \mu_i$:

Suppose $\mu_i < 0$. From Theorems 1 and 2, we must have $\alpha_{2,1}^i = \mu_i$ and

$$u_{2,2}^i = 2\mu_i = \sum_{0<z<3} \alpha_{3,z}^i$$

(26)

so by taking the minimal value of the cutpoint bounded by $\alpha_{2,1}^i$ and $\alpha_{2,2}^i$, we have $\alpha_{3,2}^i = \mu_i$ and the maximal value of the cutpoint in period $\{3, 1\}$ is $\alpha_{3,1}^i = \mu_i$. Letting
this be our base case \((n = 3)\), assume that for any \(n \geq 3\), we have

\[
\max\{\alpha^i_{n,1}\} = \mu^i \quad (27a)
\]
\[
\min\{\alpha^i_{m,j}\} = \mu^i \forall m, j < n \quad (27b)
\]

Now consider the case for \(n + 1\). In order to satisfy Equation (27), it must be that

\[
\alpha^i_{n,j} = \mu^i \forall j < n \quad (28)
\]

As a result, from Lemma 3.1, we must have

\[
\alpha^i_{n+1,j} = \mu^i \forall j \in \{2, \ldots, n - 1\} \quad (29)
\]

which leaves \(\alpha^i_{n+1,1} + \alpha^i_{n+1,n} = 2\mu^i\), with \(\alpha^i_{n+1,n} \in [\mu^i, 0]\) so that \(\alpha^i_{n+1,1}\) is maximized as \(\alpha^i_{n+1,1} = \mu^i\) and we are left with

\[
\alpha^i_{n+1,k} = \mu^i \forall k < n + 1 \quad (30)
\]

This satisfies Equation (27) and so by induction, there cannot be any cutpoint, \(\alpha^i_{n,1} > \mu^i\) in the absence of some integer \(m < n\) such that \(\alpha^i_{m,1} < \mu^i\).

Proof of Theorem 4: Optimal Monopoly Agenda

For some ordering of \(A_N\) indexed by \(m\), define

\[
A^M_N = \left\{ A_N \Big| A_N \in \arg \max_{A^m_N} U_{setter}(A^m_N), m < M \right\} \quad (31)
\]

Trivially, \(A^1_N = \{A^1_N\}\). Now assume that the set, \(A^M_N\) is nonempty for \(M\). At \(M + 1\), we
must have

\[
A_{N}^{M+1} = \begin{cases} 
\{A_{N}^M\} & \text{if } U_{\text{setter}}(A_{N}^{M+1}) > U_{\text{setter}}(A_{N}^m) \forall m \in A_{N}^M \\
\{A_{N}^M\} \cup A_{N}^M & \text{if } U_{\text{setter}}(A_{N}^{M+1}) = U_{\text{setter}}(A_{N}^m) \forall m \in A_{N}^M \\
A_{N}^M & \text{if } U_{\text{setter}}(A_{N}^{M+1}) < U_{\text{setter}}(A_{N}^m) \forall m \in A_{N}^M 
\end{cases}
\] (32)

Since this holds for \( M = 1 \), it holds for all \( M \geq 1 \). Thus, there is a nonempty set of agendas, \( A_{N}^{\star} = A_{N}^{||A_{N}||} \) which maximize the agenda setter’s utility.