Post-treatment conditioning

Han Zhang
1 Intro to DAG

2 Identification in DAG

3 Endogenous selection bias ((Post-) Outcome Collider)

4 Overcontrol bias

5 Condition on pre-treatment colliders
Two representations of causal analysis

- Counterfactual: \( y^1 \) and \( y^0 \)
- DAG: graphical representations of the theorized data-generating process.
Elements

- **Node**: random variables.
  - Can be observed or unobserved. Just variables that the theory believe to be relevant.
  - Rectangle means unobserved.
- **Edge (arrow)**: directly causal effect between two variables.
  - Only direct relations. E.g., the effect of $X$ on $Y$ is through $T$.
  - Missing edges between nodes encode exclusion restrictions.

\[
\begin{align*}
X & \rightarrow T \rightarrow Y \\
\text{rectangle } X & \rightarrow T \rightarrow Y
\end{align*}
\]
Elements

- **Path**: a sequence of edges connecting two variables (Do not consider direction)
- **Causal path**: following the directions and from treatment to outcome. $T \rightarrow Y$ and $T \rightarrow C \rightarrow Y$.
- **Noncausal path**: E.g., $T \rightarrow C \leftarrow U \rightarrow Y$. Brings spurious associations between treatment and outcomes.

![Diagram](image-url)
History

- Path analysis
- Often assume linear relationships
- Hard to represent interaction
DAG (Directed Acyclic Graph)

- DAG encodes all marginal (unconditional) and conditional independence relations
- There is no parametric assumptions now.
DAG helps to make sure whether the causal relations can be identified.

identification: given ideal data (non response errors etc.), whether the causal relationship can be recovered in a DAG; if so, under what condition.

Estimation: choose real statistical models (regression, matching ...).
Examine whether $T \perp Y$
Confounding bias
Condition on $C$ eliminates confounding bias
E.g., $A \perp\!\!\!\!\!\!\!\!\!\perp B$, but $A \perp\!\!\!\!\!\!\!\!\!\perp B \mid C$.

![Diagram showing mutual dependence and conditional independence](image)
When you have unobserved variables $U$, $T \rightarrow Y$ remain unidentified.
What does experiments do

- Randomization breaks the links from other variables to treatment/outcomes.

![Diagram]

- C
- U
- T
- Y
Randomization breaks the links from other variables to treatment/outcomes.
Mutual Causation

- Endogenous selection bias
- Condition on \( C \) create endogenous selection bias.

![Diagram showing mutual causation](image)

**Figure 4**
Mutual Causation

- \( A \perp B, \quad p(A, B) = p(A)p(B) \).

- \( A \not\perp B \mid C \iff p(A, B \mid C) \neq p(A \mid C)p(B \mid C), \) or

- \( p(A \mid B, C) \neq p(A \mid C) \) (giving \( B \) as information influence conditional probability of \( A \) given \( C \))

- Conditioning on \( C \) gives wrong implication that \( A, B \) are causally related.
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- Example: \( A \) is exam difficulty; \( B \) is student intelligence; \( C \) is test scores.

- Knowing low test score, we can infer \( A \) from \( p(A \mid C) \) that the exam will possibly be hard; but with further information of student intelligence, \( p(A \mid B, C) \neq p(A \mid C) \) typically.
Mediation

- Overcontrol bias
  - $A \not\perp B$, $p(A, B) \neq p(A)p(B)$.
  - $A \perp B \mid C$, or
  - But this only means that the direct effect of $A$ on $B$ is zero; the total effect is not zero.

- Condition on $C$ create overcontrol bias.

- More about interpretation of total/direct effect.

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Figure 2

(a) $A$ and $B$ are associated by causation. The marginal association between $A$ and $B$ identifies the causal effect of $A$ on $B$. (b) $A$ and $B$ are conditionally independent given $C$. The conditional association between $A$ and $B$ given $C$ does not identify the causal effect of $A$ on $B$ (overcontrol bias).
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A path between two variables, $A$ and $B$, does not transmit association and is said to be blocked (closed, or d-separated) if

- the path contains a noncollider, $C$, that has been conditioned on
  - $A \to C \to B$ (mediation)
  - $A \leftarrow C \to B$ (confounding)

or if

- the path contains a collider, $C$, and neither the collider nor any of its descendants have been conditioned on.

Goal: try to find an observed set of variables as conditions that:

- block all noncausal paths between treatment and outcome.
- do not block any causal paths between treatment and outcome.
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Sample truncation bias

Effect of education on income (truncated to only contain low earners); $U$ are other observed errors (embedded in conventional control variables).

Other examples

- social movement: only sample protests that have 1,000 participants.
Nonresponse bias

A divorced father’s income, $I$, and the amount of child support he pays, $P$, both influence whether a father responds to the study, $R$.

![Diagram showing causal relationships between income, child support, and response]

**Figure 6**

- Nonresponse is causally determined by both treatment and outcome.
- List deletions of non-responding subjects implies conditioning on $R$.
- Does multiple imputation also introduce nonresponse bias?
Sample selection bias

$B$, topping the Billboard charts (treatment); $R$, inclusion in the Rolling Stone 500 (outcome); $S$, sample selection

Figure 7
Heckman selection bias

\( M \), motherhood (treatment); \( W_R \), unobserved reservation wage; \( W_O \), offer wage (outcome); \( E \), employment; \( \epsilon \), error term on offer wage

(a): many dataset restricted attention to employed women (conditioning on colliders): an association between motherhood and wages even if the causal effect of motherhood on wages is in fact zero.

(b): motherhood may indeed have an effect on offer wages (e.g., because of mothers’ differential productivity compared with childless women or because of employer discrimination)

Even if \( W_R \) is measured, there is still a collider \( W_O \).
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Censoring: only dependent variables are observed

(compare with ) Truncated: no information at all is available for the non-selected observations.

Choices

- handle censoring using duration models (survival analysis, event history models).
- discard
- $P$: poverty (treatment)

- $D$: divorce rates (outcome)

- $C$: censoring/attrition.

- $U$: unmeasured factors, such as marital distress.
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Figure 9
- $S$: schooling (treatment)

- $W$: wages (outcome)

- $U$: ability (unobserved)

- $Q$: measure test score of ability (such as IQ), as a proxy for $U$. 
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- $Q$: measure test score of ability (such as IQ), as a proxy for $U$. 
(a) $U$ is unobserved; the causal path is not blocked and the effect of $S$ on $W$ is unidentified.

(b) $Q$ is an imperfect measure;

(c) $Q$ is a collider; exogenous selection bias

(d) $Q$ has a causal impact on outcome $W$. Say employers use $Q$ as hiring criteria; overcontrol bias.
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Mediation analysis

Randomized class-size experiment:

- $T$: class size in first grade
- $Y$: high school graduation
- $M$: boosting student achievement in third grade
- $U$: unobserved causes of mediator variables

Treatment is randomized but the mediation is not.
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**Figure 11**
Mediation analysis

Randomized class-size experiment:

(a): total effect between $T$ and $Y$ is unidentified if condition on $M$.

(b):
- total effect is identified
- total effect = direct + indirect effects
- directed effect (effect of $T$ on $Y$ net of other factors) is unidentified (due to collider).
- Wrongly claim that there is indirect effect.

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Pre-treatment Collider

- Spread of behaviors
- $Y_{i,t}$ is civic engagement of individual $i$ at time $t$. Can be extended to other things (say, musical taste, smoking behaviors..)
- pre-treatment collider $F_{i,j}$: existence of friendship ties.
- $U_i, U_j$ are individual attributes that shapes friendship ties (homophily) as well as outcome.
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Figure 12
Pre-treatment Collider

- Without a priori theory, hard to distinguish colliders from confounding variables.

- $X$: 1) pre-treatment and 2) associated with both treatment and outcome.
summary

- Do not condition on post-treatment variables (colliders and intermediates)
- pre-treatment colliders
  - do not condition when it is not a confounder.