Post-treatment conditioning

Han Zhang

Outline

Two representations of causal analysis

- Counterfactual: y^1 and y^0
- DAG: graphical representations of the *theorized* data-generating process.

Elements

- Node: random variables.
  - Can be observed or unobserved. Just variables that the theory believe to be relevant.
  - rectangle means unobserved.
- Edge (arrow): directly causal effect between two variables.
  - Only direct relations. E.g., the effect of X on Y is through T.
  - Missing edges between nodes encode exclusion restrictions.

$$X \longrightarrow T \longrightarrow Y$$

$$\boxed{X} \longrightarrow T \longrightarrow Y$$

Elements

- Path: a sequence of edges connecting two variables (Do not consider direction)
- Causal path: following the directions and from treatment to outcome. $T \rightarrow Y$ and $T \rightarrow C \rightarrow Y$.
- Noncausal path: E.g., $T \rightarrow C \leftarrow U \rightarrow Y$. Brings spurious associations between treatment and outcomes.

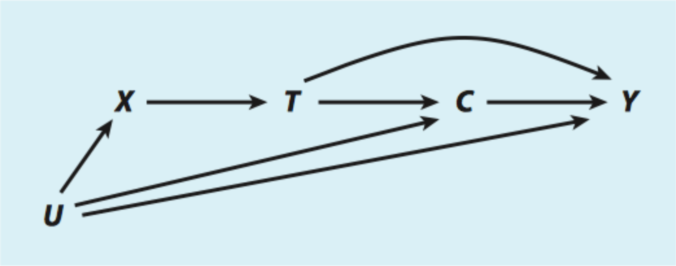


Figure 1

History

- Path analysis
- Often assume linear relationships
- Hard to represent interaction

DAG (Directed Acyclic Graph)

- DAG encodes all marginal (unconditional) and conditional independence relations
- There is no parametric assumptions now.

Identification

- DAG helps to make sure whether the causal relations can be identified.
- identification: given ideal data (non response errors etc.), whether the causal relationship can be recovered in a DAG; if so, under what condition.
- Estimation: choose real statistical models (regression, matching ...).

Simple case

Examine whether $T \perp\!\!\!\perp Y$

$$T \longrightarrow Y$$

Mutual Dependence

- Confounding bias
- Condition on C eliminates confounding bias
- E.g., $A \not\perp\!\!\!\perp B$, but $A \perp\!\!\!\perp B \mid C$.

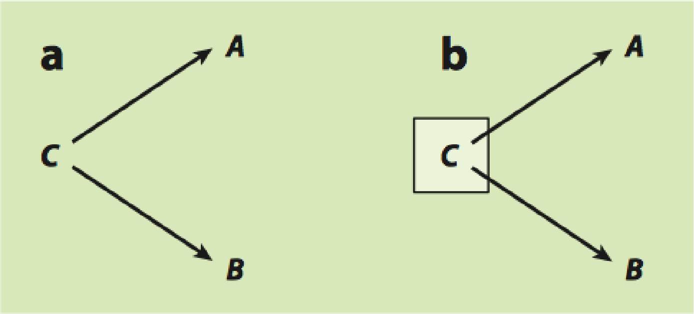
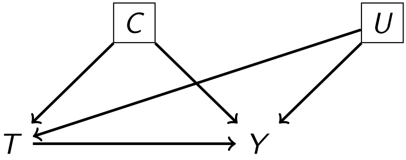


Figure 3

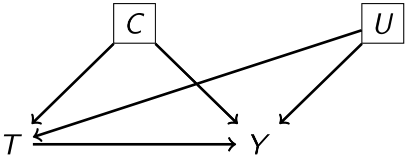
Mutual Dependence

When you have unobserved variables U, $T \rightarrow Y$ remain unidentified.



What does experiments do

- Randomization breaks the links from other variables to treatment/outcomes.



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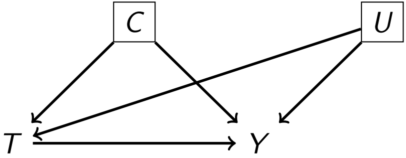

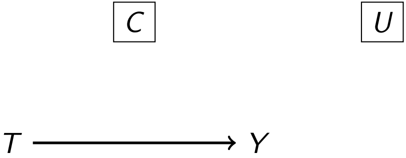

Mutual Causation

- Endogenous selection bias
- Condition on C create endogenous selection bias.

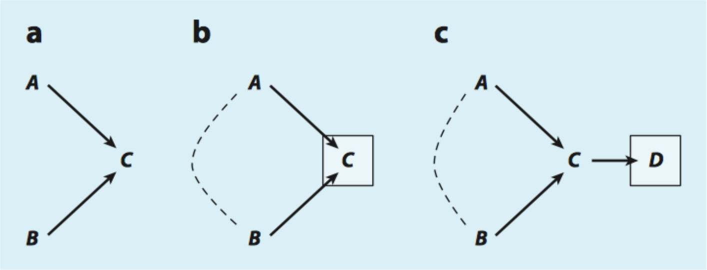

Figure 4

Mutual Causation

- $A \perp\!\!\!\perp B$, $p(A, B) = p(A)p(B)$.
- $A \not\perp\!\!\!\perp B \mid C \iff p(A, B|C) \neq p(A|C)p(B|C)$, or
- $p(A|B, C) \neq p(A|C)$ (giving B as information influence conditional probability of A given C)
- Conditioning on C gives wrong implication that A, B are causally related.

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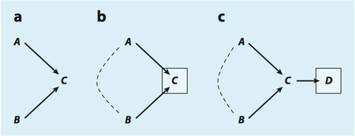

Figure 4

- Example: A is exam difficulty; B is student intelligence; C is test scores.
- Knowing low test score, we can infer A from $p(A|C)$ that the exam will possibly be hard; but with further information of student intelligence, $p(A|B, C) \neq p(A|C)$ typically.

Mediation

- Overcontrol bias
  - $A \not\perp\!\!\!\perp B$, $p(A, B) \neq p(A)p(B)$.
  - $A \perp\!\!\!\perp B \mid C$, or
  - But this only means that the direct effect of A on B is zero; the total effect is not zero.
- Condition on C create overcontrol bias.
- More about interpretation of total/direct effect.

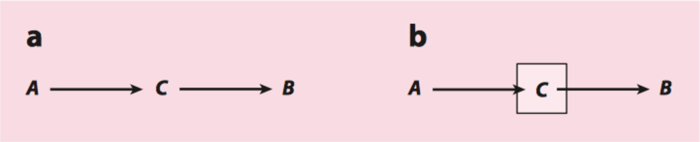

Figure 2

(*a*) *A* and *B* are associated by causation. The marginal association between *A* and *B* identifies the causal effect of *A* on *B*. (*b*) *A* and *B* are conditionally independent given *C*. The conditional association between *A* and *B* given *C* does not identify the causal effect of *A* on *B* (overcontrol bias).

Identification in General DAGs

A path between two variables, A and B, does not transmit association and is said to be blocked (closed, or d-separated) if

- the path contains a noncollider, C, that has been conditioned on
  - $A \rightarrow \boxed{C} \rightarrow B$ (mediation)
  - $A \leftarrow \boxed{C} \rightarrow B$ (confounding)

  or if
- the path contains a collider, C, and neither the collider nor any of its descendants have been conditioned on.

Goal: try to find an observed set of variables as conditions that:

- block all noncausal paths between treatment and outcome.
- do not block any causal paths between treatment and outcome.

Sample truncation bias

Effect of education on income (truncated to only contain low earners); U are other observed errors (embedded in conventional control variables).

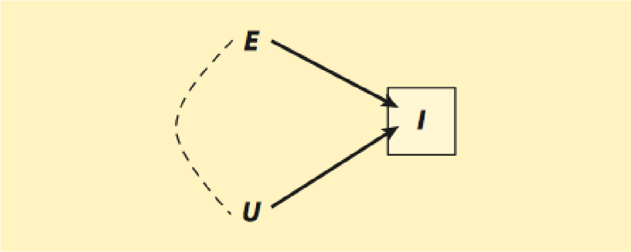

Figure 5

Other examples

- social movement: only sample protests that have 1,000 participants.

Nonresponse bias

A divorced father's income, I, and the amount of child support he pays, P, both influence whether a father responds to the study, R.

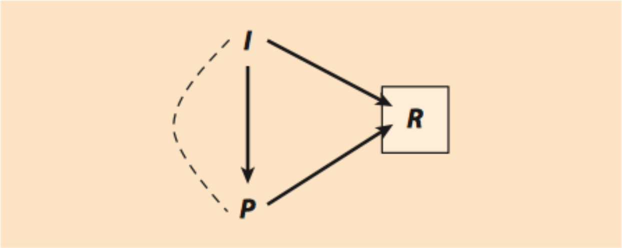

Figure 6

- Nonresponse is causally determined by both treatment and outcome.
- List deletions of non-responding subjects implies conditioning on R.
- Does multiple imputation also introduce nonresponse bias?

Sample selection bias

B, topping the Billboard charts (treatment); R, inclusion in the Rolling Stone 500 (outcome); S, sample selection]

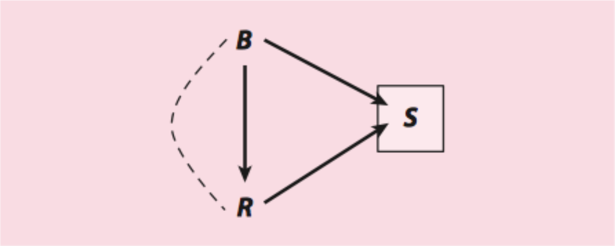

Figure 7

Heckman selection bias

M, motherhood (treatment); W_R, unobserved reservation wage; W_O , offer wage (outcome); E, employment; ϵ, error term on offer wage]

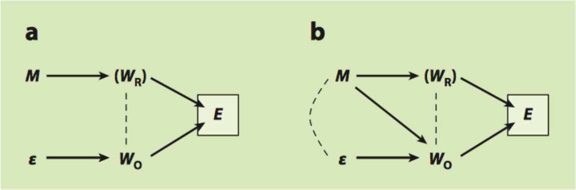

Figure 8

- (a): many dataset restricted attention to employed women (conditioning on colliders): an association between motherhood and wages even if the causal effect of motherhood on wages is in fact zero.
- (b): motherhood may indeed have an effect on offer wages (e.g., because of mothers' differential productivity compared with childless women or because of employer discrimination)
- Even if W_R is measured, there is still a collider W_O.

Censoring

- Censoring: only dependent variables are observed
- (compare with) Truncated: no information at all is available for the non-selected observations.
- Choices
  - handle censoring using duration models (survival analysis, event history models).
  - disgard

- P: poverty (treatment)
- D: divorce rates (outcome)
- C: censoring/attrition.
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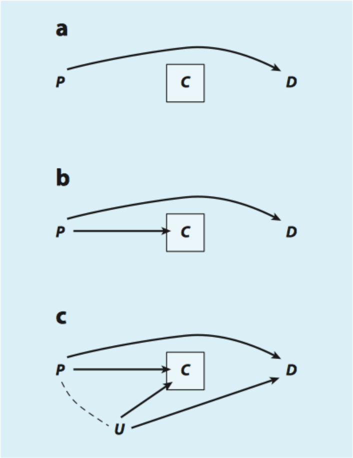

Figure 9

- S: schooling (treatment)
- W: wages (outcome)
- U: ability (unobserved)
- Q: measure test score of ability (such as IQ), as a proxy for U.

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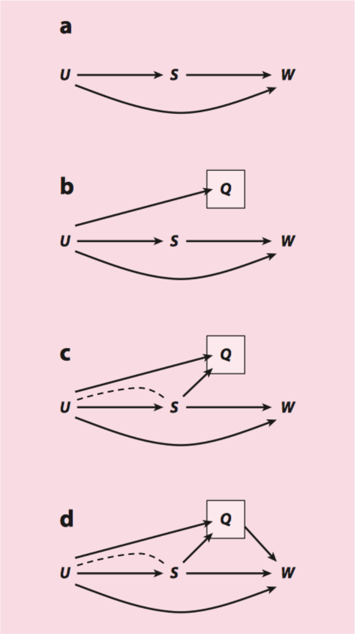

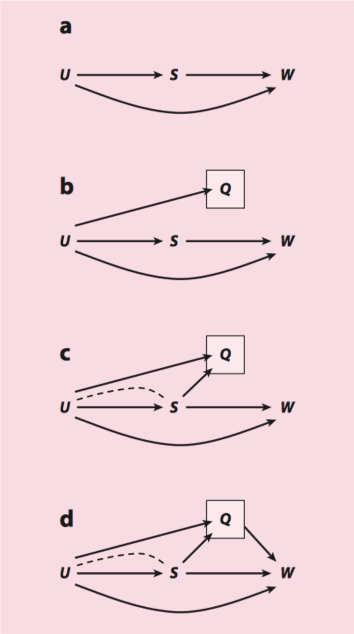
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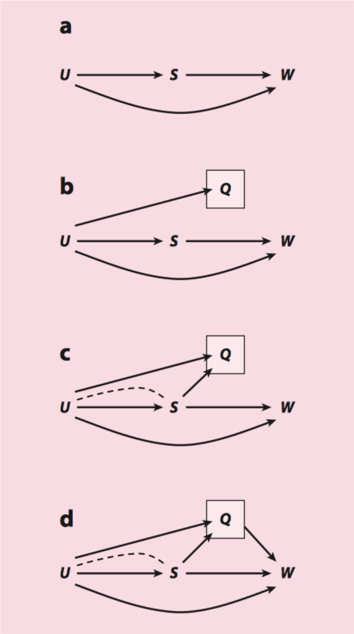

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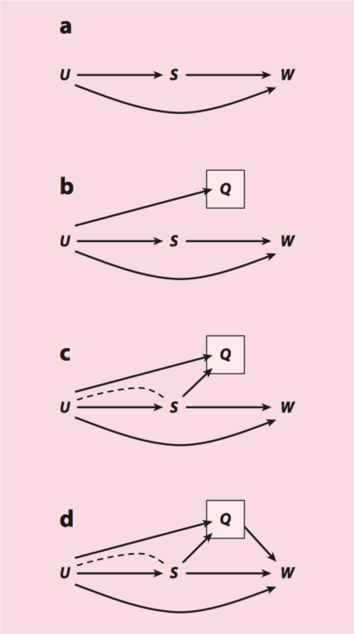

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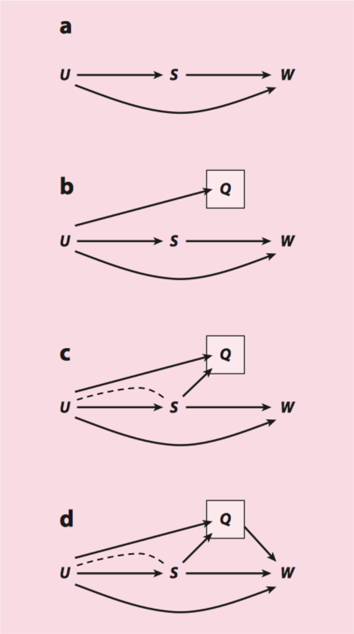

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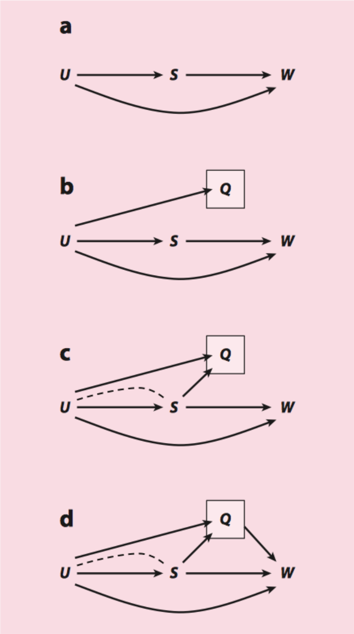

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- (b): Q is an imperfect measure;
- (c): Q is a collider; exogenous selection bias
- (d): Q has a causal impact on outcome W. Say employers use Q as hiring criteria; overcontrol bias.

Mediation analysis

Randomized class-size experiment:

- T: class size in first grade
- Y: high school graduation
- M: boosting student achievement in third grade
- U: unobserved causes of mediator variables

Treatment is randomized but the mediation is not.

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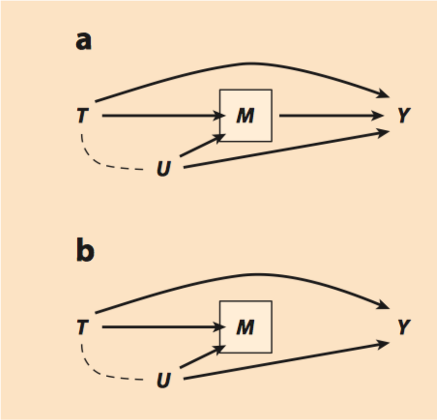

Figure 11

Mediation analysis

Randomized class-size experiment:

- (a): total effect between T and Y is unidentified if condition on M.
- (b):
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  - total effect = direct + indirect effects
  - directed effect (effect of T on Y net of other factors) is unidentified (due to collider).
  - Wrongly claim that there is indirect effect.

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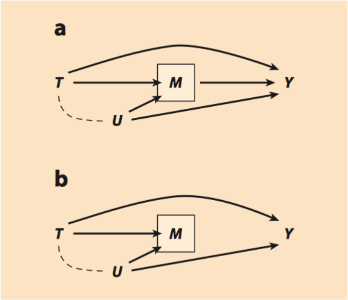

Figure 11

Pre-treatment Collider

- Spread of behaviors
- $Y_{i,t}$ is civic engagement of individual i at time t. Can be extended to other things (say, musical taste, smoking behaviors..)
- pre-treatment collider $F_{i,j}$: existence of friendship ties.
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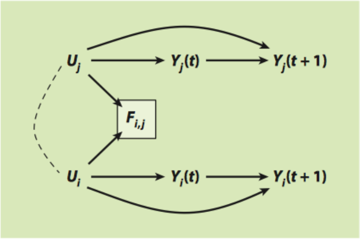
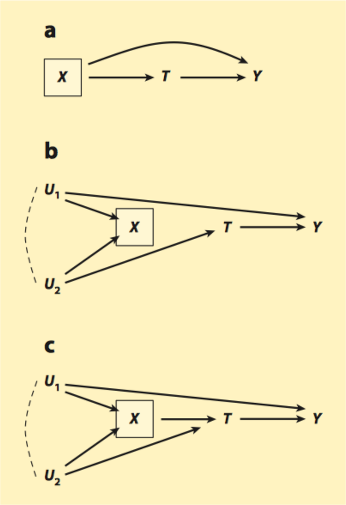


Figure 12

Pre-treatment Collider

- Without a priori theory, hard to distinguish colliders from confounding variables.
- X: 1) pre=treatment and 2) associated with both treatment and outcome.



summary

- Do not condition on post-treatment variables (colliders and intermediates)
- pre-treatment colliders
  - do not condition when it is not a confounder.