Soc500: Applied Social Statistics
Week 1: Introduction and Probability

Brandon Stewart\(^1\)

Princeton

September 12, 2018

\(^1\)These slides are heavily influenced by Matt Blackwell and Adam Glynn with contributions from Justin Grimmer and Matt Salganik. Illustrations by Shay O'Brien.
Where We’ve Been and Where We’re Going...

- **Last Week**
  - methods camp
  - pre-grad school life

- **This Week**
  - Wednesday
    - welcome
    - basics of probability

- **Next Week**
  - random variables
  - joint distributions

- **Long Run**
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

Questions?
Welcome and Introductions

I am an Assistant Professor in Sociology.
I am trained in political science and statistics.
I do research in methods and statistical text analysis.
I love doing collaborative research.
I talk very quickly.

Your Preceptors
Sage guides of all things
Shay O'Brien (Soc500)
Alex Kindel (Soc400)
Ziyao Tian (Soc400)

Stewart (Princeton)
Week 1: Introduction and Probability
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Introduction to Probability

- What is Probability?
- Sample Spaces and Events
- Probability Functions
- Marginal, Joint and Conditional Probability
- Bayes’ Rule
- Independence
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Goal: train you in statistical thinking

First in a two course sequence ⇝ replication project and longer arc

Difficult course but with many resources to support you.

When we are done you will be able to teach yourself many things

Syllabus is a useful resource including philosophy of the class.
Goal: train you in statistical thinking
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Specific Goals

▶ critically read, interpret and replicate the quantitative content of many articles in the quantitative social sciences
▶ conduct, interpret, and communicate results from analysis using multiple regression
▶ explain the limitations of observational data for making causal claims
▶ write clean, reusable, and reliable R code.
▶ feel empowered working with data
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- For the semester
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- It is free and **open source**
- It is the *de facto standard* in many applied statistical fields
Why RMarkdown?  
What you’ve done before

Image Credit: Baumer et al (2014)
Why RMarkdown?

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Welcome

Goals

Ways to Learn

Core Ideas

Introduction to Probability
  - What is Probability?
  - Sample Spaces and Events
  - Probability Functions
  - Marginal, Joint and Conditional Probability
  - Bayes’ Rule
  - Independence
Welcome

Goals

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Mathematical Prerequisites

No formal pre-requisites

Balancing rigor and intuition

▶ no rigor for rigor’s sake
▶ we will tell you why you need the math, but also feel free to ask
▶ course focus on how to reason about statistics, not just memorize guidelines

We will teach you any math you need as we go along

Crucially though—this class is not about innate statistical aptitude, it is about effort

We all come from very different backgrounds. Please have patience with yourself and with others.
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Ways to Learn

Lecture: learn broad topics
Precept: learn data analysis skills, get targeted help on assignments
Readings: support materials for lecture and precept
Problem Sets: reinforce understanding of material, practice
Piazza: ask questions of us and your classmates
Office Hours: ask even more questions.
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Reading

Think of the lecture slides as primary reading.

How to think about the reading?

Key Books:
- Aronow and Miller. Forthcoming. Foundations of Agnostic Statistics (not yet available)
- Blitzstein and Hwang. 2014. Introduction to Probability (available online through the library)

Optional Books:
- Imai (2017) A First Course in Quantitative Social Science

Why so many books?
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Your Job: work hard and get help when you need it!
Can we go over Bayes' Rule again?
How to Get Help

1. Class and Precept
2. Daily Feedback
3. Readings and Slides
4. Piazza
5. Preceptor Office Hours
6. Instructor Office Hours
7. Final Exam Prep
8. External Consulting
9. Individual and Group Tutoring

Read the syllabus for more details.
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Advice from Prior Generations

- Definitely take it! And be prepared to set aside a lot of time.
- Ask questions if you don't know what's going on!
- Study hard, work hard, review the slides.
- Investing a considerable amount of time in getting familiar with R and its various tools will pay off in the long run!
- Go over the lecture slides each week. This can be hard when you feel like you're treading water and just staying afloat, but I wish I had done this regularly.
- It's challenging but very doable and rewarding if you put the time in. There are plenty of resources to take advantage of for help.
- This course is very challenging but greatly contributed to my understanding of social statistics. If you're truly invested in the subject and willing to put in the work (more than you expect possibly), it will be one of the best courses you've taken.
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Outline of Topics

Outline in reverse order:

- Causal Inference: assess the effect of a counterfactual intervention using observed associations.
- Regression: measure the association (expectation of a variable given a number of others).
- Inference: learn about things we don't know from the things we do know.
- Probability: learn what data we would expect if we did know the truth.
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\text{Probability} \rightarrow \text{Inference} \rightarrow \text{Regression} \rightarrow \text{Causal Inference}
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This Class

Any questions about this class?
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Let’s get started
1 Welcome
2 Goals
3 Ways to Learn
4 Core Ideas
5 Introduction to Probability
   • What is Probability?
   • Sample Spaces and Events
   • Probability Functions
   • Marginal, Joint and Conditional Probability
   • Bayes’ Rule
   • Independence
Welcome

Goals

Ways to Learn

Core Ideas

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- What is Probability?
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- Provides a way of making principled guesses based on stated assumptions.
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- Relatively recent field (started at the very end of the 19th century)
- Provides a way of making principled guesses based on stated assumptions.
- In practice, an essential part of research, policy making, political campaigns, selling people things...
Why study probability?
Why study probability?

It enables inference
In Picture Form

Stewart (Princeton)  Week 1: Introduction and Probability  September 12, 2018  26 / 60
In Picture Form

Data generating process

Probability

Inference

Observed data
In Picture Form

Data generating process

probability

Observed data

inference
Statistical Thought Experiments

Start with probability. Allows us to contemplate the world under hypothetical scenarios. Hypotheticals let us ask: is the observed relationship happening by chance or is it systematic? It tells us what the world would look like under a certain assumption.

We will review probability today, but feel free to ask questions as needed throughout the semester.
Statistical Thought Experiments

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Example: Fisher’s Lady Tasting Tea

The Story Setup
(lady discerning about tea)

The Experiment
(perform a taste test)

The Hypothetical
(count possibilities)

The Result
(boom she was right)

This became the Fisher Exact Test.
Example: Fisher’s Lady Tasting Tea

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<tr>
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<tbody>
<tr>
<td>0</td>
<td>oooo</td>
<td>1 x 1 = 1</td>
</tr>
<tr>
<td>1</td>
<td>ooox, ooxo, oxxo, xooo</td>
<td>4 x 4 = 16</td>
</tr>
<tr>
<td>2</td>
<td>ooxo, oxox, oxoo, xoxo, xxoo, xoox</td>
<td>6 x 6 = 36</td>
</tr>
<tr>
<td>3</td>
<td>oxxx, xxxx, xxxo, xxxx</td>
<td>4 x 4 = 16</td>
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<td><strong>Total</strong></td>
<td></td>
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1 Welcome

2 Goals

3 Ways to Learn

4 Core Ideas

5 Introduction to Probability
   - What is Probability?
   - Sample Spaces and Events
   - Probability Functions
   - Marginal, Joint and Conditional Probability
   - Bayes’ Rule
   - Independence
Introduction to Probability

- What is Probability?
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- Independence
Can we make this more precise?
Why Probability?

Helps us envision hypotheticals
Describes uncertainty in how the data is generated
Data Analysis: estimate probability that something will happen
Thus: we need to know how probability gives rise to data
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- Helps us envision **hypotheticals**
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- Thus: we need to know how **probability** gives rise to **data**
Intuitive Definition of Probability

While there are several interpretations of what probability is, most modern (post 1935 or so) researchers agree on an axiomatic definition of probability.

3 Axioms (Intuitive Version):

1. The probability of any particular event must be non-negative.
2. The probability of anything occurring among all possible events must be 1.
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All the rules of probability can be derived from these axioms.
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All the rules of probability can be derived from these axioms. (we will return to these in a minute)
Sample Spaces

To define probability we need to define the set of possible outcomes. The sample space is the set of all possible outcomes, and is often written as $S$ or $\Omega$.

For example, if we flip a coin twice, there are four possible outcomes, $S = \{\{\text{heads}, \text{heads}\}, \{\text{heads}, \text{tails}\}, \{\text{tails}, \text{heads}\}, \{\text{tails}, \text{tails}\}\}$.

Thus the table in Lady Tasting Tea was defining the sample space. (Note we defined illogical guesses to be prob = 0)
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A Running Visual Metaphor

Imagine that we sample an apple from a bag. Looking in the bag we see:

The sample space is:
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The sample space is:

\[ \Omega = S = \{ \text{apple, apple, apple, apple, apple} \} \]
Events

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For Example, if

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**Events**

Events are subsets of the sample space.

For Example, if

\[ \cap = S = \{ \text{apple, apple, apple, apple} \} \]

then

\[ \{ \text{apple, apple, apple} \} \]

and

\[ \{ \text{apple} \} \]

are both events.
Events Are a Kind of Set

Sets are collections of things, in this case collections of outcomes. One way to define an event is to describe the common property that all of the outcomes share. We write this as \( \{ \omega | \omega \text{ satisfies } P \} \), where \( P \) is the property that they all share.
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A complement of event $A$ is a set: $A^c$, is collection of all of the outcomes not in $A$. That is, it is “everything else” in the sample space.
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Important complement: $\Omega^c = \emptyset$, where $\emptyset$ is the empty set—it’s just the event that nothing happens.
Unions and intersections (Operations on events)

The union of two events, $A$ and $B$, is the event that $A$ or $B$ occurs:

$$A \cup B = \{ \omega \mid \omega \in A \text{ or } \omega \in B \}.$$

The intersection of two events, $A$ and $B$, is the event that both $A$ and $B$ occur:

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Operations on Events

We say that two events $A$ and $B$ are **disjoint** or **mutually exclusive** if they don’t share any elements or that $A \cap B = \emptyset$. 

An event and its complement $A$ and $\overline{A}$ are disjoint.

Sample spaces can have infinite events $A_1, A_2, \ldots$. 
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\[ U \cap I = \emptyset \]

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Probability Function

A probability function $P(\cdot)$ is a function defined over all subsets of a sample space $S$ that satisfies the following three axioms:

1. **Nonnegativity**: $P(A) \geq 0$ for all $A$ in the set of all events.
2. **Normalization**: $P(S) = 1$.
3. **Additivity**: If events $A_1, A_2, \ldots$ are mutually exclusive then $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$.

All the rules of probability can be derived from these axioms. (See Blitzstein & Hwang, Def 1.6.1.)
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\[
\begin{align*}
1. & \quad P(\text{apple}) = -.5 \\
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A Brief Word on Interpretation

Massive debate on interpretation:

Subjective Interpretation
▶ Example: The probability of drawing 5 red cards out of 10 drawn from a deck of cards is whatever you want it to be.

But...
▶ If you don’t follow the axioms, a bookie can beat you
▶ There is a correct way to update your beliefs with data.

Frequency Interpretation
▶ Probability is the relative frequency with which an event would occur if the process were repeated a large number of times under similar conditions.

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Three Big Ideas

Marginal, joint, and conditional probabilities
Bayes' rule
Independence
Three Big Ideas

Marginal, joint, and conditional probabilities
Three Big Ideas

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Marginal and Joint Probability

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Suppose we are now in a situation where we would like to express the probability that an event $A$ and an event $B$ occur. This quantity is written as $P(A \cap B)$, $P(B \cap A)$, $P(A, B)$, or $P(B, A)$ and is the joint probability of $A$ and $B$.

$$P(\text{🍎, 🍏}) = P(\text{🍎}) = P(\text{🍏, 🍏})$$
\[ P(\text{Not leaf}) = ? \]
\[ P(\text{Not leaf, apple}) = ? \]
Conditional Probability

The "soul of statistics"

If \( P(A) > 0 \) then the probability of \( B \) conditional on \( A \) can be written as

\[
P(B | A) = \frac{P(A, B)}{P(A)}
\]

This implies that

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P(A, B) = P(A) \times P(B | A)
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Conditional Probability: A Visual Example

\[ P(\text{\textbullet} | \text{\textbullet}) = \frac{P(\text{\textbullet}, \text{\textbullet})}{P(\text{\textbullet})} \]
Conditional Probability: A Visual Example

\[ P(\text{\textbullet} \mid \text{apple}) = \frac{P(\text{\textbullet}, \text{apple})}{P(\text{apple})} \]
Conditional Probability: A Visual Example

\[ P(\text{red} | \text{apple}) = \frac{P(\text{red, apple})}{P(\text{apple})} \]
A Card Player’s Example

If we randomly draw two cards from a standard 52 card deck and define the events
\[ A = \{\text{King on Draw 1}\} \text{ and } B = \{\text{King on Draw 2}\}, \text{ then} \]
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- \( P(A) = \)

\[ P(A,B) = P(A) \times P(B|A) = \frac{4}{52} \times \frac{3}{51} \approx 0.0045 \]
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Stewart (Princeton)
A Card Player’s Example

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Law of Total Probability (LTP)

With 2 Events:

\[
P(B) = P(B, A) + P(B, A^c)
= P(B|A) \times P(A) + P(B|A^c) \times P(A^c)
\]

\[
P(\text{🍎}) = P(\text{🍎}, \text{🍏}) + P(\text{🍎}, \text{🍋})
= P(\text{🍎}|\text{🍏}) \times P(\text{🍏}) + P(\text{🍎}|\text{🍋}) \times P(\text{🍋})
\]
Recall, if we randomly draw two cards from a standard 52 card deck and define the events $A = \{\text{King on Draw 1}\}$ and $B = \{\text{King on Draw 2}\}$, then

- $P(A) = 4/52$
- $P(B|A) = 3/51$
- $P(A, B) = P(A) \times P(B|A) = 4/52 \times 3/51$

Question: $P(B) = ?$
Confirming Intuition with the LTP

\[ P(B) = P(B|A) + P(B|A^c) \]

\[ = P(B|A) \times P(A) + P(B|A^c) \times P(A^c) \]

\[ = \frac{3}{51} \times \frac{1}{13} + \frac{4}{51} \times \frac{12}{13} = \frac{111}{663} = \frac{4}{52} \]
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\[ = \frac{3 + 48}{51 \times 13} = \frac{1}{13} = \frac{4}{52} \]
Example: Voter Mobilization

Suppose that we have put together a voter mobilization campaign and we want to know what the probability of voting is after the campaign: \( \Pr[\text{vote}] \).

We know the following:

\[
\Pr(\text{vote} | \text{mobilized}) = 0.75
\]
\[
\Pr(\text{vote} | \text{not mobilized}) = 0.15
\]

\[
\Pr(\text{mobilized}) = 0.6 \quad \text{and so} \quad \Pr(\text{not mobilized}) = 0.4
\]

Note that mobilization partitions the data. Everyone is either mobilized or not.

Thus, we can apply the LTP:

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\Pr(\text{vote}) = \Pr(\text{vote} | \text{mobilized}) \Pr(\text{mobilized}) + \Pr(\text{vote} | \text{not mobilized}) \Pr(\text{not mobilized})
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Bayes’ Rule

Often we have information about $\Pr(B|A)$, but require $\Pr(A|B)$ instead. When this happens, always think: Bayes’ rule:

Bayes’ rule: if $\Pr(B)>0$, then:

$$\Pr(A|B) = \frac{\Pr(B|A) \Pr(A)}{\Pr(B)}$$

Proof: combine the multiplication rule $\Pr(B|A) \Pr(A) = \Pr(A \cap B)$, and the definition of conditional probability.
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- Bayes’ rule: if $\Pr(B) > 0$, then:

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Bayes’ Rule Mechanics

\[ P(\text{bitten} | \text{red}) = \frac{P(\text{bitten} | \text{green}) P(\text{green})}{P(\text{red})} \]
Bayes’ Rule Mechanics

\[ P(\text{\textbullet\textbullet\textbullet\textbullet\textbullet} | \text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet}) = \frac{P(\text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet} | \text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet}) P(\text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet})}{P(\text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet})} \]
Bayes’ Rule Mechanics

\[ P(\text{bit} | \text{red}) = \frac{P(\text{red} | \text{bit}) P(\text{bit})}{P(\text{red})} \]
Bayes’ Rule Mechanics

\[ P(\text{\textcolor{green}{\textbf{apple}}}|\text{\textcolor{red}{\textbf{apple}}}) = \frac{P(\text{\textcolor{red}{\textbf{apple}}}|\text{\textcolor{green}{\textbf{apple}}}) \cdot P(\text{\textcolor{red}{\textbf{apple}}})}{P(\text{\textcolor{red}{\textbf{apple}}})} \]
Bayes’ Rule Example

**U.S. Billionaires, 2014**

- 76.5% of female billionaires inherited their fortunes, compared to 24.5% of male billionaires

- So is $P(\text{woman} | \text{inherited billions})$ greater than $P(\text{man} | \text{inherited billions})$?
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\[
P(W | I) = \frac{P(I | W) P(W)}{P(I)}
\]

\[
= 0.765 \left( \frac{82}{568 + 82} \right)
\]

\[
= \frac{0.765 \times 82 + 0.245 \times 568}{568 + 82}
\]

\[
= 0.31
\]

Data source: Billionaires characteristics database
Example: Race and Names

Enos (2015): how do we identify a person's race from their name?

First, note that the Census collects information on the distribution of names by race. For example, Washington is the most common last name among African-Americans in America:

- \( \Pr(AfAm) = 0.132 \)
- \( \Pr(\text{not } AfAm) = 1 - \Pr(AfAm) = 0.868 \)
- \( \Pr(\text{Washington} | AfAm) = 0.00378 \)
- \( \Pr(\text{Washington} | \text{not } AfAm) = 0.000061 \)

We can now use Bayes' Rule

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\Pr(AfAm | \text{Wash}) = \frac{\Pr(\text{Wash} | AfAm) \Pr(AfAm)}{\Pr(\text{Wash})}
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$$
\Pr(AfAm|Wash) = \frac{\Pr(Wash|AfAm) \Pr(AfAm)}{\Pr(Wash)}
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Example: Race and Names

Note we don't have the probability of the name Washington. Remember that we can calculate it from the LTP since the sets African-American and not African-American partition the sample space:

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\Pr(\text{AfAm}|\text{Wash}) = \frac{\Pr(\text{Wash}|\text{AfAm}) \Pr(\text{AfAm})}{\Pr(\text{Wash}|\text{AfAm}) \Pr(\text{AfAm}) + \Pr(\text{Wash}|\text{not AfAm}) \Pr(\text{not AfAm})} = 0.
\]

\[
0.132 \times 0.00378 + 0.868 \times 0.000061 \approx 0.09.
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Independence

Events $A$ and $B$ are independent if knowing whether $A$ occurred provides no information about whether $B$ occurred.

Formal Definition

$$P(A, B) = P(A) \cdot P(B) \Rightarrow A \perp \perp B$$

With all the usual $> 0$ restrictions, this implies

$$P(A|B) = P(A) \quad P(B|A) = P(B)$$

Independence is a massively important concept in statistics.
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Intuitive Definition
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- A word from your preceptors
References