Precept 1: Probability, Simulations, Working with Data

Soc 500: Applied Social Statistics

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Princeton University

September 2016
Support Resources

- Office hours
- Math camp materials
- Piazza
- Email (please CC both of us)
- Google is your best friend!
Learning objectives

- Create an R Markdown document
Learning objectives

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- Translate information provided in word problems into probability statements
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Acknowledgements: These slides draw on materials developed by past preceptors, Elisha Cohen and Clark Bernier. Thanks!
**R Markdown**

- `install.packages("knitr")`
- File - New File - R Markdown
- Preferences - Under Sweave set “Weave Rnw files with” to “knitr”
- See 1_Sample Markdown Document.Rmd
Probability from a 2 X 2 Table

- Imagine that someone on GradCafe posts that they have just been admitted to all ten of the top 10 sociology programs. Is this claim plausible?
- Consider the following table of grad school applicants:

<table>
<thead>
<tr>
<th></th>
<th>Princeton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanford?</td>
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Probability from a 2 X 2 Table

What is the sample space here?

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<td></td>
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<td>15</td>
<td>760</td>
<td></td>
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<td>Total</td>
<td>30</td>
<td>770</td>
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Probability from a 2 X 2 Table

What is the sample space here?
Admissions outcomes for people who applied to Princeton and Stanford

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- What is the sample space here?
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- What is the probability that a randomly selected student got into both Stanford and Princeton?

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Probability from a 2 X 2 Table

- What is the sample space here? Admissions outcomes for people who applied to Princeton and Stanford
- What is the probability that a randomly selected student got into both Stanford and Princeton?
  \[ \text{Pr}( P = Y, S = Y) = \frac{15}{800} \]
Probability from a 2 X 2 Table

- What is the sample space here?
  Admissions outcomes for people who applied to Princeton and Stanford

- What is the probability that a randomly selected student got into both Stanford and Princeton?
  \[ Pr( P = Y, S = Y) = \frac{15}{800} \]

- Given that a student got into Princeton, what is the probability that they got into Stanford?

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- What is the probability that a randomly selected student got into both Stanford and Princeton?
  \[ \Pr( P = Y, S = Y) = \frac{15}{800} \]
- Given that a student got into Princeton, what is the probability that they got into Stanford?
  \[ \Pr( S = Y \mid P = Y ) = \frac{15}{30} \]

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Probability from a 2 X 2 Table

Is getting into Stanford independent of getting into Princeton?

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Recall that events A and B are independent if knowing that A occurred provides no information about whether B occurred.

\[
\Pr(A,B) = \Pr(A)\Pr(B) = \Rightarrow A \perp B
\]

\[
\Pr(A|B) = \Pr(A) \text{ and } \Pr(B|A) = \Pr(B)
\]

Applying that here:

\[
\Pr(P=Y, S=Y) = \frac{15}{800} = 0.01875
\]

\[
\Pr(P=Y)\Pr(S=Y) = \frac{30}{800} \times \frac{25}{800} = 0.00117
\]

Getting into Princeton and getting into Stanford are not independent.
Probability from a 2 X 2 Table

- Is getting into Stanford independent of getting into Princeton?
- Recall that events $A$ and $B$ are independent if knowing that $A$ occurred provides no information about whether $B$ occurred

$$\Pr(A,B) = \Pr(A)\Pr(B) \implies A \perp B$$

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Prosecutor’s Fallacy

A woman has been murdered, and her husband is accused of having committed the murder. It is known that the man abused his wife repeatedly in the past, and the prosecution argues that this is important evidence pointing towards the man’s guilt. The defense attorney says that the history of abuse is irrelevant, as only 1 in 1000 women who experience spousal abuse are subsequently murdered.

Assume that the defense attorney’s 1 in 1000 figure is correct, and that half of men who murder their wives previously abused them. Also assume that 20% of murdered women were killed by their husbands, and that if a woman is murdered and the husband is not guilty, then there is only a 10% chance that the husband abused her. What is the probability that the man is guilty? Is the prosecution right that the abuse is important evidence in favor of guilt?
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Prosecutor’s Fallacy

- Let’s define our events
Prosecutor’s Fallacy

Let’s define our events

\[ M \Rightarrow \text{woman is murdered} \]
\[ A \Rightarrow \text{woman has previously experienced abuse} \]
\[ G \Rightarrow \text{woman’s husband is guilty} \]
Prosecutor’s Fallacy

Let’s define our events

- \( M \): woman is murdered
- \( A \): woman has previously experienced abuse
- \( G \): woman’s husband is guilty

What do we know?
Prosecutor’s Fallacy

- Let’s define our events
  \( M \Rightarrow \) woman is murdered
  \( A \Rightarrow \) woman has previously experienced abuse
  \( G \Rightarrow \) woman’s husband is guilty

- What do we know?
  \( P(M|A), P(A|M, G), P(G|M), P(A|G', M) \)
Prosecutor’s Fallacy

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- $M$ => woman is murdered
- $A$ => woman has previously experienced abuse
- $G$ => woman’s husband is guilty

What do we know?

- $P(M|A)$, $P(A|M, G)$, $P(G|M)$, $P(A|G', M)$

What do we want to know?
Prosecutor’s Fallacy

Let’s define our events
- $M$ => woman is murdered
- $A$ => woman has previously experienced abuse
- $G$ => woman’s husband is guilty

What do we know?
- $P(M|A)$, $P(A|M, G)$, $P(G|M)$, $P(A|G', M)$

What do we want to know?
- $P(G|M, A)$
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  \( M \Rightarrow \text{woman is murdered} \)
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- What can we use to get our quantity of interest?
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  \[ P(G|M, A) \]

- What can we use to get our quantity of interest?
  Bayes’ Rule
Bayes’ Rule

- Often we have information about $Pr(B|A)$, but require $Pr(A|B)$ instead.
- When this happens, always think **Bayes’ Rule**
- Bayes’ rule: if $Pr(B) > 0$
  \[
  Pr(A \mid B) = \frac{Pr(B|A)Pr(A)}{Pr(B)}
  \]
Bayes’ Rule

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- When this happens, always think **Bayes’ Rule**
- Bayes’ rule: if \( Pr(B) > 0 \)

\[
Pr(A \mid B) = \frac{Pr(B \mid A)Pr(A)}{Pr(B)}
\]

- Also recall from the definition of conditional probability:

\[
Pr(A, B) = Pr(B \mid A)Pr(A)
\]
Prosecutor’s Fallacy

\[
P(M|A) = \frac{1}{1000}
\]

\[
P(A|G, M) = \frac{1}{2}
\]

\[
P(G|M) = \frac{1}{5}
\]

\[
P(A|G', M) = \frac{1}{10}
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\[
P(G|M, A) = \frac{P(M, A|G)P(G)}{P(M, A)}
\]
Prosecutor’s Fallacy

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P(M|A) = \frac{1}{1000}
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\[
\frac{P(M,A,G)}{P(M,A)}
\]
Prosecutor’s Fallacy

\[
P(M|A) = \frac{1}{1000} \\
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\]

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P(G|M, A) = \frac{P(M, A|G)P(G)}{P(M, A)}
\]

\[
\frac{P(M, A, G)}{P(M, A)} \\
P(A|G, M)P(G|M)P(M)
\]

\[
P(A|M)P(M)
\]
Prosecutor’s Fallacy

\[ P(M|A) = \frac{1}{1000} \]
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\[ \frac{P(M, A, G)}{P(M, A)} \]
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\]

How do we find \(P(A \mid M)\)?

Recall Law of Total Probability:

\[
P(X) = P(X \mid Y)P(Y) + P(X \mid Y')P(Y')
\]
Prosecutor’s Fallacy

\[ P(M|A) = \frac{1}{1000} \]
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How do we find \( P(A | M) \)?

Recall Law of Total Probability:

\[ P(X) = P(X | Y)P(Y) + P(X | Y')P(Y') \]

Applying here:

Prosecutor’s Fallacy

\[ P(M|A) = \frac{1}{1000} \]

\[ P(A|G, M) = \frac{1}{2} \]

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Putting it all together:

\[
P(G|M, A) = \frac{P(A|G, M)P(G|M)}{P(A|G, M)P(G|M) + P(A|G', M)P(G'|M)}
\]
Prosecutor’s Fallacy

\[
P(M|A) = 1/1000 \\
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P(G|M) = 1/5 \\
P(A|G', M) = 1/10
\]

Putting it all together:

\[
P(G|M, A) = \frac{P(A|G, M)P(G|M)}{P(A|G, M)P(G|M) + P(A|G', M)P(G'|M)} \\
= \frac{(0.5)(0.2)}{(0.5)(0.2) + (0.1)(1 - 0.2)}
\]
Prosecutor’s Fallacy

\[
P(M|A) = \frac{1}{1000}
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Putting it all together:

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\]

\[
= \frac{(0.5)(0.2)}{(0.5)(0.2) + (0.1)(1 - 0.2)}
\]

\[
= 0.556
\]
Prosecutor’s Fallacy

What does this mean for our defendant?
Problem: You have a bag of five marbles. Three are red and two are blue. You draw one marble. Without replacing it, you then draw another marble. What is the probability that the two marbles are the same colour?

- We could do this analytically:
Problem: You have a bag of five marbles. Three are red and two are blue. You draw one marble. Without replacing it, you then draw another marble.
What is the probability that the two marbles are the same colour?

- We could do this analytically:
  
P(Same colour)
  =P(D1 = R)P(D2 = R | D1 = R) + P(D1 = B)P(D2 = B | D1 = B)
  = (3/5)(2/4) + (2/5)(1/4)
  = 2/5
Problem by Simulation

Problem: You have a bag of five marbles. Three are red and two are blue. You draw one marble. Without replacing it, you then draw another marble. What is the probability that the two marbles are the same colour?

- We could do this analytically:
  
  \[
  P(\text{Same colour}) = P(D1 = R)P(D2 = R \mid D1 = R) + P(D1 = B)P(D2 = B \mid D1 = B)
  \]
  
  \[
  = \left(\frac{3}{5}\right)\left(\frac{2}{4}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{4}\right)
  \]
  
  \[
  = \frac{2}{5}
  \]

- Or we can run a simulation!

  See 2_Simulation example.R
Writing Functions

- We've already used many built-in R functions: `mean()`, `head()`, etc.
- We can also define our own functions:

Define a function that takes 3 arguments; it will add the first two and divide by the third:

```r
> my.function <- function(x,y,z){
+   out <- (x + y)/z
+   return(out)
+ }
> ## use the function
> my.function(1, 5, 2)
[1] 3
```
Data Manipulation and Tables

See 3_Data Manipulations and Tables.Rmd
Graphics

America’s Most Popular Charts

- Pie: 34%
- Line: 20%
- Bar: 13%
- Histogram: 24%
- Other: 9%

Goals of Data Visualization (i.e. why use graphics?)

- Discovery (exploratory)

---

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- Discovery (exploratory)
  - qualitative overview, looking for patterns, outliers, scale of data

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- Discovery (exploratory)
  - qualitative overview, looking for patterns, outliers, scale of data
- Communication (presentation)

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Goals of Data Visualization (i.e. why use graphics?)

- Discovery (exploratory)
  - qualitative overview, looking for patterns, outliers, scale of data
- Communication (presentation)
  - displaying information from the data in an accessible way

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Goals of Data Visualization (i.e. why use graphics?)

- **Discovery** (exploratory)
  - qualitative overview, looking for patterns, outliers, scale of data
- **Communication** (presentation)
  - displaying information from the data in an accessible way
  - telling a story, reporting results

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Goals of Data Visualization (i.e. why use graphics?)

- **Discovery (exploratory)**
  - qualitative overview, looking for patterns, outliers, scale of data

- **Communication (presentation)**
  - displaying information from the data in an accessible way
  - telling a story, reporting results
  - grab your audience and keep them interested

---

Graphics using `ggplot2()`³

`ggplot2()` conceptually:
- each graphic is made up of different layers of components

³Wickham, Hadley. ggplot2: elegant graphics for data analysis. 2009
Graphics using \texttt{ggplot2()}\textsuperscript{3}

ggplot2() conceptually:
  - each graphic is made up of different layers of components
  - start with layer plotting raw data

\textsuperscript{3}\textit{Wickham, Hadley. ggplot2: elegant graphics for data analysis.} 2009
Graphics using `ggplot2()`

`ggplot2()` conceptually:

- each graphic is made up of different layers of components
  - start with layer plotting raw data
  - add annotations

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Graphics using `ggplot2()`

`ggplot2()` conceptually:

- each graphic is made up of different layers of components
  - start with layer plotting raw data
  - add annotations
  - add statistical summaries

---

Graphics using \texttt{ggplot2()}\textsuperscript{3}

\texttt{ggplot2()} conceptually:

- each graphic is made up of different layers of components
  - start with layer plotting raw data
  - add annotations
  - add statistical summaries
- highly customizable

\textsuperscript{3}Wickham, Hadley. \textit{ggplot2: elegant graphics for data analysis}. 2009
Grammar of `ggplot2()` is composed of:

- **data** that you want to visualize
  - set of aesthetic **mappings**
- **geoms**: geometric shapes – points, lines, polygons, etc.
- **stats**: statistical transformations e.g. binning and counting for histogram
- **scales**: map data values to aesthetical values – color, shape, size, and legend
- **coord**: coordinate system – how data is mapped to coordinate; provides axes and gridlines
- **facet**: how to break up the data into subsets
diamonds data

easy way to start plotting is to use `qplot()`, short for quick plot

Show distribution of 1 variable:

```r
> qplot(carat, data = diamonds, geom = "histogram")
> qplot(carat, data = diamonds, geom = "density")
```
Change `binwidth` argument:

```r
> qplot(carat, data = diamonds, geom = "histogram",
+       binwidth = 1, xlim = c(0,3))
> qplot(carat, data = diamonds, geom = "histogram",
+       binwidth = 0.1, xlim = c(0,3))
```
qplots

To compare different subgroups (diamonds of different color groups) use an aesthetic mapping:

```r
> qplot(carat, data = diamonds, geom = "histogram",
+       fill = color)
```
Reproduce the same histogram using full `ggplot()`

```r
> p <- ggplot(diamonds, aes(x = carat))
> p + geom_histogram()
```
ggplot()

Change binwidth:

```r
> p <- ggplot(diamonds, aes(x = carat))
> p + geom_histogram(binwidth = 0.1)
```
ggplot()

Group counts by diamond color:

```r
> p <- ggplot(diamonds, aes(x = carat))
> p + geom_histogram(aes(fill = color))
```
Plots with more options

More complicated `qplot`:

```r
> qplot(carat, data = diamonds,
+       geom = "histogram",
+       binwidth = 0.1,
+       main = "Histogram for Carat",
+       xlab = "Carat",
+       fill=I("green"),
+       col=I("red"),
+       alpha=I(.2), # transparency
+       xlim=c(0,4))
```
Plots with more options

Same plot, but using ggplot() specification:

```r
> p1 <- ggplot(data = diamonds, aes(x = carat))
> p1 + geom_histogram(binwidth = 0.1,
+                        col = "red",
+                        fill = "green",
+                        alpha = .2) +
+   labs(title = "Histogram for Carat") +
+   labs(x = "Carat", y = "Count") +
+   xlim(c(0,4))
```
Resources

- R Cookbook: http://www.cookbook-r.com/
- Kosuke Imai’s textbook contains lots of sample R code!