SEQUENTIAL VETO BARGAINING

Charles M. Cameron
Department of Political Science
Columbia University
New York, N.Y. 10027

Susan Elmes
Department of Economics
Columbia University
New York, N.Y. 10027

Preliminary Version 1.0
April 13, 1994

Please send comments to Cameron at cmc1@Columbia.edu
This research was supported by National Science Foundation grant SES-9223396.
ABSTRACT

We study lawmaking by a legislature and an executive, which we model as a process of sequential bargaining under incomplete information. The model provides an explanation for well-known patterns in the use of the veto. It also provides an explanation for previously unexplained patterns and generates new hypotheses about presidential-congressional relations. The model allows us to evaluate the institutional stakes in veto bargaining in the American system of separated powers and in other systems of presidential government. This class of bargaining problem appears to be new in the literature and would seem to have wider application in political settings.
INTRODUCTION

We model lawmaking by a legislature and an executive as a process of bargaining under incomplete information. The focus of the paper is sequential veto bargaining. By sequential veto bargaining, we mean a process in which a single veto may be but one step in a back-and-forth sequence of bills and vetoes. In sequential veto bargaining the president acts "not as the executive but as a third branch of the legislature," in Woodrow Wilson's phrase. In this capacity the president can veto a bill and, perhaps only implicitly, insist Congress pass a more acceptable one. By adamantly rejecting repeated offers, the executive may wring large concessions from the legislature. It is this dynamic process of bargaining and concessions that we study.

Sequential veto bargaining is frequent and substantively important in American politics (). When a public bill is vetoed, often (about 30%) of the time it is re-passed by Congress. If the bill is non-trivial, this percentage rises to about 50%. If the bill is important enough to warrant an override attempt, the re-passage rate rises to about 75%. When a bill is vetoed and re-passed, in most cases (about 83%) the president ultimately accepts some version of the bill.

We know of no studies of sequential veto bargaining other than in American national politics. However, the almost ubiquitous provision of the executive veto in presidential systems of government suggests that probing the analytical foundations of veto power has application beyond American politics (see Shugart and Carey 1992, Table 8.2).

The model we study features one-sided incomplete information: the president has private information about his policy preferences. Although Congress has some idea what policies the president prefers, it cannot be sure what he really will accept. We assume Congress and the president bargain in the shadow of a deadline, trying to enact policy before the next election, so the bargaining period is of fixed length. We also assume some time must elapse between congressional policy offers since Congress must re-pass a bill if
it is vetoed. We assume that each time the president vetoes a bill there is a possibility the bargaining process breaks down; if it does, Congress fails to return with another offer before the election. This assumption reflects the inherent uncertainty of legislative politics, where "policy windows" often open and close somewhat unexpectedly (Kingdon 1984). We extend the model to include the possibility of veto over-rides.

The model belongs to the "screening" class of incomplete information bargaining models, prototypically represented by Sobel and Takahashi 1983 and Fudenberg and Tirole 1983. This class of model was developed to study economic phenomena such as strikes, monopoly pricing by the producer of a durable good, and settlement in negligence suits. Typical equilibria in such models involve a succession of offers by the uninformed party (e.g., a labor union or, here, Congress), who begins with tough offers but gradually softens its demands as bargaining unfolds. Refusing an offer imposes on the informed party (the firm or the president) a cost, such as the risk of an unattractive status quo if bargaining breaks down. The informed party balances the gains from holding out for a more attractive offer against the risk of waiting. The size of the risk from waiting varies according to the quality that is private information for the informed party (e.g., the profitability of the firm or the true policy preferences of the president). Hence, the succession of offers allows the uninformed party to discriminate among ("screen") different types of the informed party and adjust its subsequent offers accordingly. Refusals in early rounds involve bluffing or strategic reputation building by the informed party.

Our model incorporates this logic of screening, bluffing, and reputation building. However, the model differs in one important way from screening models tailored to economic settings: the bargainers in our model have political preferences, i.e., single peaked utility functions defined over a policy space. Although this seems like a small difference it leads to large qualitative differences in bargaining behavior. In particular, the model does not display the celebrated Coase property. The Coase property is an
important element of screening models in economic settings. Consider, for example, a seller and a buyer haggling over a price. Suppose price offers from the seller come very quickly rather than slowly; then the Coase property indicates the initial price falls to zero and is accepted immediately. This property is important because it suggests that apparently minor variations in the procedures determining the speed of offers can dramatically alter the relative bargaining power of the two parties. In addition, the Coase property implies that screening models cannot explain failures to reach immediate agreement absent (unmodeled) restrictions on the speed of offers. In our model, in contrast, as the number of possible offers increases the initial offer does not rise to the largest doctrine that Congress would ever be willing to offer (the equivalent of a zero price offer in the economic context). Instead, the initial offer asymptotically approaches a fixed value whose level depends on various exogenous parameters, and this offer may indeed be vetoed. In addition, the model displays a pronounced deadline effect in which the probability of striking an agreement increases toward the end of the bargaining period. Deadline effects are a relatively unusual feature among incomplete information bargaining models.²

The model directs attention to novel political phenomena, especially legislative concession schedules, settlement rates, failure rates, and the duration of bargaining. Consider a sequence of repeated offers, which we call a veto chain. Legislative concession schedules indicate the content or spatial location of each bill in the chain, e.g., the location of the initial bill, the location of the second bill conditional on a veto of the first bill, and so on. The slope of the concession schedule indicates the ability of an additional veto to extract a concession. Settlement rates indicate the probability each period that president and Congress agree on a bill. Escalating settlement rates indicate a deadline effect. The failure rate indicates the overall probability that no legislative bargain is struck, so the status quo prevails. The magnitude of failure rates is central to concerns about legislative deadlock and governability, especially during periods of divided
government. The duration of bargaining indicates the length of veto chains and is indicative of the toughness of bargaining and intensity of conflict between the executive and legislature.

An attractive feature of the model is its ability to generate testable hypotheses about concession schedules, failure and settlement rates, and bargaining duration. Particularly striking are differences between significant and less important legislation, the effect of more distant and more proximal status quo's, the timing of the initial offer during the electoral cycle (i.e., the length of the bargaining period), and the effects of unified vs. divided government.

The model allows us to frame more precisely questions about the design of institutions: what effect does veto bargaining have on policy outcomes and how much is Congress advantaged or disadvantaged by the veto? This institutional analysis casts light on the basic contours of executive-legislative relations in the American system of separation of powers.

Although no other models study sequential veto bargaining, two in particular study vetoes under incomplete information. Matthews 1989 analyzes a one-shot model focusing on veto threats. He provides an elegant formulation of the veto problem, which we adopt. McCarty 1993 studies a interesting model in which the legislature makes a series of one-shot offers to the president, each in different policy arenas. The analysis addresses the larger dynamics of reputation building over many policy arenas during the electoral cycle. For example, vetoes in health policy may build a reputation that can be exploited to gain concessions on welfare policy. Our model examines situations in which the legislature may repeatedly pass the same or modified versions of a bill. Strategic maneuvers, including reputation-building, bluffing, and screening, take place within a specific policy arena or legislative context. The three models can be seen as quite complementary.
The following section lays out the model and indicates equilibria. The next section explores legislative concession schedules, settlement rates, failure rates, and the duration of bargaining. We derive testable propositions about sequential veto bargaining and explore the institutional implications of the bargaining process. We then take a closer look at legislative policy making during periods of divided government. The penultimate section discusses some of the implications of the model for institutional design, while the last section concludes.

THE MODEL

Model

There are two players, Congress and the president. There is a unidimensional policy space $X = \mathbb{R}$ with common knowledge status quo policy $x(0) > 0$. In each period $i$ Congress makes a proposal $x(i) \in X$, $i = n, 1 - n, ... 1$, unless a breakdown has occurred previously. Note that offers are denoted by the time remaining in the game so that period $n$ is the initial period in the game and period 1 the last. Following a proposal by Congress, the president may accept the proposal thus ending the game, or veto the proposal. Following a veto, the game ends with common knowledge probability $q$ and continues with probability $1 - q$ (except in the terminal period 1). The players receive payoffs according to the policy in effect when the game ends.

We assume Congress has a most-preferred policy, with the attractiveness of other policies declining linearly with distance, to wit, it's payoffs are

$$U = -|x(k)|$$

where $x(k)$ is the status quo $x(0)$ if the parties fail to reach an agreement before the game terminates, and the agreed-on policy otherwise. Congress's most preferred policy is normalized to $x = 0$.

Payoffs to the president are similar, except that the most preferred policy for the president may differ from that of Congress. A simple form for this utility function would
be a "tent utility" function of the form $v = -|z(t) - x(k)|$, where $z(t)$ is the president's most preferred policy. However, a very convenient normalization of this utility function is

$$V = \frac{|x(0) - t| - |t + x(0) - 2x(k)|}{2}$$

with $t = 2z(t) - x(0)$. As shown in Figure 1, this utility function is normalized so that one side of the function is anchored at $x(0) > 0$ with $V(x(0))$ equal to zero. If $z(t) < x(0)$, the right hand side is anchored at $x(0)$. In this case, as $x$ decreases from $x(0)$ utility rises at the marginal rate of 1 until it reaches a maximum at $z(t)$. Then, as $x$ continues to fall, utility declines at a marginal rate of -1 reaching a value of 0 at $t$. If $z(t) > x(0)$, the left hand side is anchored at $x(0)$. Matthews 1989 provides a set of sufficient conditions on general utility functions so that this normalization retains the preference ordering of the original function; tent utility satisfies these assumptions.

Under this normalization the president's "type" $t$ has the following interpretation. The value $t$ is the point in the policy space utility-equivalent to the status quo, i.e., $V(t) = V(x(0))$. Hence, if $t < x(0)$ a type $t$ president prefers $x'$ to $x(0)$ if and only if $x' \in (t, x(0))$ (or $x' \in (x(0), t)$ if $t > x(0)$). In essence, $t$ is a reservation policy for the president: he will never accept policy proposals more distant from his most preferred policy than his reservation policy.

Incomplete information takes the following form: the type of the president is private information. More specifically, his type $t$ is drawn from the type space $T = [t, \tilde{t}]$ using continuous distribution $F$, with positively and continuously differentiable function $f$. $F$ is common knowledge. We assume $F$ is uniformly distributed on $T$.

In the absence of incomplete information, the model would reduce to repeated play of the standard Romer-Rosenthal monopoly agenda-setter game. In that case, subgame perfection implies that vetoes never occur, the offer and legislative outcome is $x = 0$ if $t$ lies below 0 and $x = t$ if $t$ lies between 0 and $x(0)$, while no agreement is possible if $t$ lies at
or above $x(0)$. If an agreement is possible, it would be reached immediately. In the absence of repeated play but with incomplete information, the model is also straightforward to analyze. For example, it is easy to show that if the support of $T$ lies between 0 and $x(0)$, then $x = \min \left\{ \frac{T}{2} + \frac{x(0)}{2}, x = \min \left\{ \frac{T}{2} + \frac{x(0)}{2} \right\} \right.$.

Given incomplete information and repeated play, the analysis of the game is more complex. A strategy for Congress specifies an offer each turn in an $n$ turn game, given that previous offers were refused. Hence, it is a mapping from the history of past offers and vetoes and Congress’s current belief at each time $i$, into an offer $x(i)$. A system of beliefs for Congress is a mapping from any history of offers and replies into a probability distribution over the unknown value of $t$. A strategy for the president specifies an accept or reject (veto) decision for each possible offer each turn in the game. Hence, it is a mapping from the history of past offers and replies and the current offer, into a veto response.

We focus on perfect Bayesian equilibria in this bargaining game. Informally, an equilibrium has the following form. The president forms a conjecture about the relationship between the current bill and subsequent bills, conditional on a veto. This conjecture allows him to predict the expected value of future bills given the current bill, and thus decide whether to veto or accept. Congress, taking the president’s behavior as given and taking into account the fact that future bills will be conditionally optimal given vetoes in earlier periods, decides which current offer to make. In equilibrium the president will accept in the correct period given his expectations, and his expectations about the course of future bills is actually correct. Congress’s beliefs are given by the strategies and application of Bayes’s Rule. A formal definition of this version of perfect Bayesian equilibrium may be found in Sobel and Takahashi 1983 (pp. 413-414).

Behaviorally, this concept of equilibrium assumes the players understand the bargaining process very well indeed. Of course, presidents and congresses have played the
veto game for more than 200 years and there is a wealth of accumulated experience about the nature of the game.

Cases

The normalization of the president's utility function allows a classification of presidents into three strategically relevant categories (refer to Figure 1). Following Matthews 1989, if \( t \leq 0 \) we say the president is *accommodating* because he would be willing to accept a final offer of \( x = 0 \), Congress's ideal policy. If \( 0 < t < x(0) \) we say the president is *compromising*, because he is willing to accept some final proposals that Congress prefers to the status quo, but he will not accept Congress's ideal policy. If \( t \geq x(0) \) the president is *recalcitrant* because he is unwilling to accept as a final offer any bill that Congress prefers to the status quo.

The strategic character of bargaining is closely related to the distribution of presidential types across the three basic categories. For example, if all presidential types are accommodating (i.e., \( \bar{t} < 0 \)) then Congress will only offer its ideal point, and any attempt at bluffing via a veto is pointless. Conversely, if all presidential types are recalcitrant (i.e., \( t > x(0) \)), the only possible legislative outcome is the status quo.

Combining distributions across the basic categories creates six different cases: three involving just one category of presidential type, two involving two categories of presidential types (i.e., some accommodators and some compromisers, and some compromisers and some recalcitrants), and one involving all three categories (since we assume a continuous distribution of types the combination of just accommodators and recalcitrants is not possible). It is not necessary to analyze each of these cases in depth, however. As already noted, the cases in which the president is definitely an accommodator or definitely recalcitrant are analytically trivial. As will become clearer shortly, the other cases involve simple extensions to the all-compromisers case.
Equilibria

The key to constructing equilibria is the following observation. Given two bills, \( x(i) \) and \( x(i-1) \), with \( x(i) < x(i-1) \), there will be a presidential type \( t(i) \) who is just indifferent between the two offers. For this type,

\[
V(x(i), t(i)) = (1 - q) V(x(i-1), t(i)) + q V(x(0), t(i))
\]

It is easy to see that all types less than \( t(i) \) prefer \( x(i) \) to \( x(i-1) \), while all higher types prefer \( x(i-1) \) to \( x(i) \). This has two implications. First, consider an ascending sequence of offers all less than \( \min \{x(0), \bar{t}\} \) (we need not be concerned with higher offers since Congress would never make such offers). Then the president's strategy takes the form of a simple cut-off rule in all periods except the last: accept if \( t \leq t(i) \), and reject otherwise. In the last period \( i=1 \), the rule is: accept if \( t \leq x(1) \) and reject otherwise, just as in the one-shot game. Second, given this cut-off rule and the sequence of offers, Congress can be certain following a veto that \( t > t(i) \). This fact allows Congress to update its beliefs about the president's type and make subsequent offers contingent on those beliefs. The progressive winnowing down of possible types as offers are rejected gives the model its distinctive flavor of screening.

All-compromisers case. We state our first result.

**Proposition 1** (All compromisers case -- \( t \geq 0, \bar{t} \leq x(0) \)). There are two cases, the unconstrained and the constrained offer cases.

A. Unconstrained offers. Suppose

\[
t \leq \frac{2(\bar{t} - x(0))}{B(1)B(2)\ldots B(n)} + x(0)
\]

where

\[
t(i) = A(i) x(0) + B(i) t(i+1)
\]  

(1)

and \( A(i) + B(i) = 1 \) (\( A(i) \) and \( B(i) \) are defined in the Appendix.) Then unless bargaining has broken down
$x(i) = \alpha(i) \cdot x(0) + \beta(i) \cdot t(i+1)$ (2)

where $\alpha(i) + \beta(i) = 1$ ($\alpha(i)$ and $\beta(i)$ are defined in the Appendix). The president accepts $x(i)$ iff $t \leq t(i)$. At the beginning of each period $i$, Congress's beliefs are uniformly distributed on $[t(i+1), t]$.

B. Constrained offers. Suppose

$t > \frac{2(i - x(0))}{B(1)B(2) \ldots B(j)} + x(0)$

and

$t \leq \frac{2(i - x(0))}{B(1)B(2) \ldots B(j-1)} + x(0)$

where $j \leq n$. Unless bargaining has broken down, for the first $n + 1 - j$ periods

$t(i) = A(i - (n + 1 - j)) x(0) + B(i - (n + 1 - j)) t(i+1)$

and

$x(i) = \alpha(i - (n + 1 - j)) x(0) + \beta(i - (n + 1 - j)) t(i+1)$

Then, from period $n - j$ to the end of the game, $x(i) = \bar{t}$. The president follows the same cut-off strategy as in the unconstrained offers case. Beliefs for Congress are identical to those in the unconstrained case in all rounds before $x(i) = \bar{t}$, and concentrated on $t = \bar{t}$ thereafter.

Proof. See Appendix.

Proposition 1 indicates the trajectory of offers over the course of an $n$ period game. In the unconstrained case, every offer before the last offer is less than $\bar{t}$; the last offer may be less than or equal to $\bar{t}$. In this case, each offer is given by equation (2) and is simply a weighted average of the status quo $x(0)$ and the bottom end of possible presidential types, $t(i+1)$, given vetoes of the prior offers. Congress can infer the lowest possible type based on the type who is exactly indifferent between the current offer and the subsequent offer contingent on a veto. The constrained case arises if equation (2) generates an offer greater than $\bar{t}$. Of course Congress would never make an offer greater
than \( \bar{t} \) since \( x(i) = \bar{t} \) surely would be accepted. If equation (2) generates such an offer, the actual offer (and any subsequent offers, if they were to be made) equals \( \bar{t} \); the president immediately accepts such an offer, ending the game. Part B indicates that prior to a constrained offer, play of the game proceeds exactly as in the unconstrained case. The condition defining the unconstrained case requires the game to be short enough in duration (given \( \xi \)) so that the progressive winnowing down of types cannot eliminate all possible types over the course of the game. An obvious implication of the condition is that in the all-compromisers case the game must always be of finite length even absent a deadline, since eventually the constraint will be reached and the president will end the game by accepting \( \bar{t} \).\(^5\)

Because the president’s strategy space is restricted to "accept" or "veto" at each stage, the issue of updating beliefs about the president’s type off the equilibrium path does not arise except when \( x(i) = \bar{t} \) and the president (contrary to the action prescribed by his strategy) vetoes the bill. We assume Congress continues to believe \( t = \bar{t} \) in this case. Hence, the model avoids the multiplicity of equilibria that typically arise in bargaining under incomplete information when beliefs must be updated at unreached information sets (Sutton (1986)).

*Compromisers and recalcitrants* \( (t \geq 0, \bar{t} > x(0)) \). Proposition 1 easily extends to cover the case when the president may be a compromiser or a recalcitrant. In this case \( \bar{t} \) lies above the status quo \( x(0) \). Of course Congress would never make an offer greater than the status quo. However equation (2) never actually generates an offer greater than the status quo since \( x(i) \) is just a weighted average of \( x(0) \) and \( t(i+1) \). Thus, when the type space includes both compromisers and recalcitrants the path of offers is exactly the same as that given in the unconstrained case of Proposition 1.

*Compromisers and accommodators* \( (t \leq 0, \bar{t} \leq x(0)) \). The case when the type space includes both compromisers and accommodators requires an additional result.
PROPOSITION 2 (accommodators and compromisers -- \( t < 0, \bar{t} \leq x(0) \)). There are two cases depending on the location of \( t \).

A. Few accommodators case. If
\[
t \geq -\frac{\alpha(n)}{\beta(n)} x(0)
\]
equilibrium strategies are identical to those detailed in Proposition 1.

B. Many accommodators case. If
\[
t < -\frac{\alpha(n)}{\beta(n)} x(0)
\]
then \( x(n) = 0 \), \( \tau(n) = \frac{(1-q)(1-\alpha(n-1))}{(1-(1-q)\beta(n-1))} x(0) \), the remaining offers are described by equations (1) and (2), and the president's strategy is as indicated in Proposition 1.

**Proof:** See Appendix.

Proposition 2 indicates the following. If the probability that the president is accommodating is sufficiently low, the equilibrium is just like the all-compromisers case. However, if the probability the president is accommodating is sufficiently high, Congress begins its offers with its own most preferred policy. If the president vetoes this policy, the game continues just like the all-compromisers case, with the new bottom of possible presidential types reflecting the veto of the initial offer.

**Concessions, bargaining failures, and offers.** Propositions 1 and 2 together imply the following:

COROLLARY 1. Throughout the game Congress makes concessions with each succeeding bill; bills are accepted in each period with positive probability; and there is a positive probability that no offer will be accepted (unless the final offer equals \( \bar{t} \)) even if bargaining does not break down. Finally, in all but the last period, some types of president are willing to bluff -- that is, veto a bill they actually prefer to the status quo.

Corollary 1 underscores what is perhaps the outstanding feature of the model: the power of a veto to extract concessions from Congress. This feature is an example of a important monotonicity property that arises in many models of sequential bargaining under incomplete information. For obvious reasons, this property is called the haggling
property. Another important feature of the model is the risk that no bargain may be struck, even after repeated tries. In other words, bargaining can end inefficiently due to incomplete information, not just the vagaries of break downs. Finally, the role of bluffing and strategic maneuvering is striking.

Coase conjecture. An additional feature of the model is sufficiently distinctive to warrant discussion. As mentioned in the introduction, screening models of bargaining typically exhibit the Coase phenomenon: as the time between offers goes to zero, offers fall to zero and are accepted immediately (Gul, Sonnenschein, and Wilson 1985). The intuition for the result is relatively simple: if offers come frequently the cost of rejecting an offer is low, since a better offer will soon follow. The informed party can therefore afford to be patient. Conversely, the uninformed party always prefers a given accepted offer to come earlier rather than later and thus has an incentive to speed up its concessions. The result is, the uninformed party immediately makes all the concessions it is willing to make (Kennan and Wilson 1993).

The Coase phenomena is a distressing feature of screening models, at least if they are to explain disagreement in bargaining. The models imply that, absent exogenous break downs, bargaining cannot end in disagreement if offers come quickly. So, if bargaining does end in disagreement the real reason is the delay between offers, a form of commitment by the uninformed party that is exogenous to and unexplained by the model. For this reason, the existence of the Coase phenomenon is often taken as a fundamental flaw of screening models of strikes and similar phenomena.6

The following proposition addresses this problem:

PROPOSITION 3 (no Coase phenomenon). As $n \to \infty$, $\beta(n) \to \frac{1}{2 - q + \sqrt{q}}$ and $\alpha(n) \to 1 - \beta(n) = \frac{1 - q + \sqrt{q}}{2 - q + \sqrt{q}} \neq 1$. I.e., as the number of possible offers increases the initial offer $x(n)$ does not rise to the status quo $x(0)$. 

13
Proof. See Appendix.

Recall from equation (2) that offers are a weighted average of the status quo, $x(0)$, and the smallest possible presidential type at the beginning of a period, $t(n+1)$. If the Coase phenomenon held in the model, then as the number of possible offers in a game increased the initial weight on the status quo ($\alpha(n)$) would approach one while that on the smallest possible presidential type ($\beta(n)$) would fall to zero in the unconstrained case. This implies that Congress would immediately make as large concessions as it is willing to make. For example, $x(n)$ would equal $\min\{x(0), \bar{r}\}$ if the type space included any compromisers. Proposition 3 therefore indicates that the Coase phenomenon does not hold in the model -- as the number of possible rounds increases the weight on the smallest possible presidential type does not fall to zero but instead asymptotically approaches a particular bound that depends on $q$ (the bound lies between $\frac{1}{3}$ and $\frac{2}{3}$ as $q \in (0,1)$).

In fact, initial offers converge vary rapidly to the bound given in Proposition 3, unless $q$ is close to zero. This means that initial offers will be about the same regardless of the possible length of the game, second offers will be about the same, and so on. Figure 2 illustrates this by showing offers in games of two through five periods in length when $x(0) = 1$ and $q = .3$. As shown, the first offer is always the same regardless whether the game is two periods or five periods in length (hence, no Coase phenomenon). The same is true of every succeeding offer, unless the offer is the last offer in a game. In that case, the ability to make a take-it-or-leave-it offer enables Congress to make a somewhat tougher offer than it would if bargaining could continue.

The intuition behind Proposition 3 is simple: in economic versions of the model, all types of the informed party prefer lower prices (for example) and hence all have an incentive to wait for the next offer. In contrast, all presidential types do not prefer a higher policy: since preferences are single peaked, some presidential types prefer today's lower policy even aside from the possibility of bargaining breaking down tomorrow.
Although Congress continues to prefer a given accepted bill earlier rather than later, the pressure to make all possible concessions is removed.

The absence of the Coase phenomenon means that the balance of bargaining power between president and Congress will not depend on the speed of offers. In other words, the basic pattern of inter branch bargaining is robust to small changes in institutional procedures. In addition, the model can explain vetoes without relying on exogenous delays between congressional offers.

*Veto over-rides.* Up to this point, we have assumed an absolute veto over legislation. Suppose instead the legislature can over-ride a veto with a super majority, say, two-thirds. Let \( \hat{x} \) be the proposal such that two-thirds of the members will vote for \( \hat{x} \) over the status quo \( x(0) \). Obviously \( \hat{x} \in [0, x(0)] \), assuming "Congress" is the median voter in the legislature (Ferejohn and Shaposh 1990). Assume \( \hat{x} \) is common knowledge. Then the legislature need never make an offer greater than \( \hat{x} \), as it can override a veto of \( \hat{x} \). In fact, \( x(i) \leq \min \{\hat{x}, \bar{i}, x(0)\} \) \( \forall i \) given any compromisers (i.e., \( \bar{i} < x(0) \) and \( \bar{i} > 0 \)). Hence, there are two cases. If \( \bar{i} \leq \hat{x} \) bargaining proceeds exactly as given in Propositions 1 and 2. If \( \bar{i} > \hat{x} \) the path of offers follows that specified in Propositions 1 and 2, except the largest offer the legislature will be willing to make is \( \hat{x} \). In short, a provision for a veto over-ride may limit the amount of concessions the legislature must make when it faces an president with extreme preferences. However, the basic logic of sequential veto bargaining remains essentially unaffected.

**Examples**

*Unconstrained offers in the all-compromisers case.* Suppose \( T \) is uniformly distributed on \([0, 1] \), the status quo \( x(0) = 1.25 \), the game has two periods, and \( q = .4 \).

Then (using equations (1) and (2) in the Appendix) the first period offer \( x(2) = .69 \), and the second period offer (following a veto) is \( x(1) = .85 \). The president accepts the first offer if \( t < t(2) = .45 \), vetoes the first and accepts the second offer if \( t(2) < t < x(1) \), and
vetoes both offers if \( t > x(1) \). The probability the first period bill is vetoed is .55, and the probability the second period bill is vetoed (conditional on its being offered) is .27. *Ex ante*, the probability of enacting some form of the bill is .69.

This example illustrates the possibility of bluffing and reputation building. In the first period, there is a group of types -- specifically, those lying between \( t(2) \) and \( x(2) \), i.e., .45 and .69 -- who "bluff" by vetoing. These types actually prefer \( x(2) \) to the status quo but they veto \( x(2) \) in order to build a reputation for extremism and extract the more attractive bill \( x(1) \) from Congress. To see how this could make sense note that the ideal points for the lowest and highest type bluffers are .85 and .97 respectively. The second bill, located at .85, is much more attractive to them than the first bill, located at .69. Of course, by vetoing the first bill they run a 40% chance that bargaining will break down, in which case the status quo prevails. Nonetheless, the second period offer is sufficiently attractive that they are willing to run this risk.

Finally, compare this example with one-shot play. In the one-shot version of this example, Congress's optimal bill is \( x = .625 \). Both offers in the repeated game are higher. However, the probability of reaching an agreement in the one-shot is (of course) .625. In the repeated game, it is .69, 11% higher.

*Constrained offers in the accommodators-compromisers case.* Suppose \( T = [-.5,.5], x(0) = 1, \quad q = .3, \quad \text{and} \quad n = 3 \). In this case the type space includes both accommodators and compromisers. Calculation of the ratio in Proposition 2A indicates the case can be analyzed as if the type space contained only compromisers (the ratio takes the value -1.05, which is less than \( t = -.5 \)). However it is an example of the constrained case, for in the third period the offer generated by equation (2) exceeds .5. The path of offers is actually given by the two period game, with the third offer equaling \( \bar{t} = .5 \). Calculation shows that Congress offers .33 in the initial period, and .49 in the following period. \( t(3) = .125 \), so types between .125 and .33 bluff in order to get the second offer of .49. The probability the first offer is accepted is 63%.
SENSITIVITY ANALYSIS

Equations (1) and (2) define a system of non-linear difference equations that can be solved iteratively to generate exact values for \( x(i) \) and \( t(i) \). This fact allows us to explore the sensitivity of veto bargaining to changes in important parameters.\(^7\) In this section we examine the effects of varying \( q \), \( x(0) \) and \( n \) on offers and concessions, settlement rates and deadline effects, enactment rates, the duration of bargaining, and expected outcomes and utility. We focus on the all-compromisers case so that policy differences between the president and Congress are sufficiently large to ensure that the president is never willing simply to accept Congress's ideal policy. This case can be seen as representative of situations of divided government. We contrast unified and divided government in the following section.

The parameter \( q \), the probability bargaining breaks down, can be viewed as a proxy for the importance or significance of bills. Even though Congress enacts a great many bills that are minor by any conceivable measure of significance, the likelihood a given minor bill makes its way onto the crowded congressional agenda is quite small. If such a bill is vetoed, it is unlikely to force its way onto the agenda a second time -- the policy window will have closed. Hence, for minor bills \( q \) is high. Conversely, Congress is much more likely to devote additional time and effort to important bills. If the president vetoes a critical appropriations bill, for example, Congress may well return with another bill rather than allow the program to terminate or limp along with even lower funding. Therefore, \( q \) is likely to be low for major bills. The parameter \( n \), the maximum number of possible offers, is indicative of the timing of the initial offer during the electoral cycle. Bargaining that begins early in the electoral cycle allows for more rounds of offers and vetoes, so \( n \) is larger. Conversely, bargaining that begins late in the electoral cycle is characterized by lower \( n \).
To better convey the character of bargaining, our analysis extends through the five period game; however, very few veto chains in the post-War period have continued beyond three rounds of bargaining.

**Offers and Concessions**

Figure 3 shows optimal offers in the five period game at different levels of $q$, conveying many of the differences between veto bargaining over legislation of major, moderate, and minor significance. (The figure is drawn for $x(0) = 1.0$ and $T = [0,1]$. ) The initial offer, $x(5)$, lies under the later offers at all levels of $q > 0$. Similarly, the second offer, $x(4)$ lies everywhere under the third offer, and so on. Thus, Congress makes continuous concessions during bargaining, whether for legislation of major, moderate, or minor significance. At high levels of $q$, the final offer converges on the top end of the distribution of presidential types -- Congress writes a concession laden bill that is sure to pass. As $q$ goes to zero, all five offers converge on the best one-shot offer. This makes sense, for in this case reputation building cannot work and the game becomes equivalent to repeating the one-shot game five times.

Because status quo's are rarely observable, empirically estimating the relationships shown in Figure 3 would be difficult. Legislative concession schedules are of greater empirical interest for they show the course of bargaining for a single veto chain (i.e., with a fixed status quo). Figure 4 shows legislative concession schedules for the same example with $q = .1$, .3, and .9. The slope of the concession schedule measures incremental concessions. As shown, at any point in the veto chain the slope of the concession schedule increases as $q$ increases. In other words, the more important the bill, the smaller the concessions at a given round of the bargaining. This important pattern makes intuitive sense: the probability of break down is so high for minor bills that Congress, if it gets a chance to pass the bill again, offers a whopping concession to assure passage of the legislation. Because the probability of breakdown is much lower for major bills, Congress
can afford to be more parsimonious with its concessions. Hence, bargaining over major bills is much "tougher" in character than bargaining over bills of only moderate or minor significance. For a given \( q \), the slope of the concession curve becomes progressively less steep. In other words, concessions in earlier rounds are always larger than concessions in later rounds, regardless of the importance of the bill.

A more distant status quo can move a game from the unconstrained case analyzed in Proposition 1A to the constrained case analyzed in Proposition 1B. This effect is illustrated in Figure 5. In this five period game, \( T = [0,1] \) and \( q = .1 \), but \( x(0) \) increases gradually from 1.0 to 1.9. As the status quo becomes more distant, Congress offers \( \overline{T} \), ensuring agreement, after early offers are rejected. The intuition is clear: Congress capitulates early in order to avoid the risk of a bad outcome in the event bargaining breaks down.

**Settlement Rates, Deadline Effects, and Failure Rates**

The settlement rate is the per period probability a bill will be accepted by the president. Even though Congress makes continuous concessions over the course of bargaining, this need not imply increasing settlement rates, for each offer reflects what Congress has learned about the president's preferences in the earlier rounds of bargaining.

Figure 6 indicates settlement rates in the five period game at three levels of \( q \). As shown, settlement rates are generally flat over the early periods of bargaining, fall in the penultimate round if \( q \) is low (i.e., legislation is important), and then increase in the last period. This increase is a deadline effect, and if the bill is important (low \( q \)) the deadline effect can be dramatic. Behavior in the three and four period games is similar. In the all-compromisers case, increasing the status quo or the length of the game can drive the settlement rate late in the game to certainty, as Congress offers \( \overline{T} \).

Figure 7 shows the probability of bargaining failure, that is, the probability that some form of the bill fails to pass in the five period game. The failure rate takes into
account both the possibility of bargaining breaking down and the possibility that bargaining does not break down but the two parties fail to reach agreement by the deadline. As shown, the failure rate is highly non-monotonic with respect to the significance of legislation (the three period game displays a very similar pattern). Bills that are ultimately most likely to pass in some form are neither the most important nor the least important, but those lying between the extremes. Although we do not provide examples in the preliminary version of the paper, more distant status quo's and lengthier games can reduce the failure rate by assuring that Congress offers $I$ promptly.

**Duration of Bargaining**

The expected length of veto chains reflects the probability of break down and the toughness of bargaining (whether concessions come rapidly or slowly). Not surprisingly, the importance of a bill affects the average length of veto chains. The expected length of veto chains is shorter for less important bills and longer for more significant ones. As Figure 8 shows, the relationship between $q$ and the average length of veto chains is monotonic but quite non-linear (a very similar pattern occurs in shorter game). The non-linearity of the relationship is not obvious, and therefore provides a potential test of the model.

**Expected Outcomes and Institutional Stakes**

Figure 9 shows the expected utility of Congress in the four-round game at various levels of $q$, taking into account the dynamics of offers, the probability of vetoes and breakdowns, and the location of the status quo. Because expected outcomes are simply the negative of the expected utility of Congress, the figure also provides a useful shorthand summary of the policy consequences of veto bargaining. Expected outcomes are closest to Congress at high and low levels of $q$; hence expected utility is highest in
those cases. The expected outcome most distant from Congress occurs at moderate levels of $\theta$ (a similar pattern occurs in games of other lengths).

Figure 10 shows the expected utility of Congress as the game lengthens. As shown, the expected utility of Congress is greatest when Congress can make a single take-it-or-leave-it offer. At most levels of importance, there is a substantial drop in expected utility from a game between the single offer game and games of greater length, but little difference among games of greater length. This is not true for the most important legislation. This pattern suggests that Congress would prefer to present the president with many bills at the end of the legislative session so that each bill becomes a take-it-or-leave-it offer -- a different kind of dead-line effect, in which a huge surge of legislation occurs at the end of the session. Because the drop-off from one offer to multiple offers is greatest for moderately and very important bills, one might expect to see this kind of deadline effect operating more strongly for these types of bills.

**UNIFIED VS. DIVIDED GOVERNMENT**

One of the most striking facts about veto bargaining is that vetoes are much rarer and veto chains notably shorter under unified than divided government (). It would be a serious problem for the model if it could not explain this difference. Fortunately it can.

An essential difference between divided and unified government is the greater convergence of interests between Congress and president when both come from the same party. In other words, under unified government the president will often be an accommodator. Therefore we analyze veto bargaining under unified government by examining the case when the president may be an accommodator or a compromiser. We retain a large degree of uncertainty about the location of president's ideal point. If Congress is much less uncertain about presidential preferences under unified government, then the president's type space would be narrower and the results we report would be strengthened.
First consider the case where the ideal point of Congress and the expected ideal point of the president are identical. Given a uniform distribution over presidential types, the president's true ideal point lies to the left of Congress with probability one-half, and to the right of Congress with probability one-half. This configuration can be seen as a reasonable base-line approximation for unified government. But note the following: all the types whose ideal points lie to the left of \( \frac{x(0)}{2} \) are accommodators (since \( t = 2x(t) - x(0) \)). Only those with ideal points above \( \frac{x(0)}{2} \) (and below \( x(0) \)) are compromisers. Unless the most extreme presidential type to the right is closer to the status quo than to Congress, all the presidential types will be accommodators. In this case, the model predicts no vetoes at all -- a reasonable approximation of unified government.

Suppose the expected difference in presidential and congressional ideal points is zero but there is a substantial chance the president is a compromiser. As an example, suppose the ideal point of the highest presidential type, \( z(t) \), equals \( \frac{1}{2} x(0) \). Then the ideal point of the lowest presidential type, \( z(t) \), must equal \( -\frac{1}{2} x(0) \) and \( T = [-2.5x(0), 5x(0)] \). The bulk of presidential types will be accommodators and Congress may present the president with a bill at its ideal point, which the president is likely to accept (recall Proposition 2B). For instance, suppose \( x(0) = 1 \) in this example. Regardless of the value of \( q \), Congress's initial offer will be zero (using the ratio in Proposition 2A). If \( q = .3 \), this initial offer will be accepted with a 56% probability and if vetoed will screen out types lower than - . 8 . Types between - . 8 and 0 will bluff by vetoing in order to get the next bill, which is located at . 20. Taking into account the probability of break down, there is about a 30% chance the second offer is made (ex ante). If the second offer is rejected the final offer will be . 38. But what is the chance bargaining will actually last as long a three rounds? Taking into account the probability of break down, the answer is only 2% -- there is a 98% chance bargaining ends earlier.

As a final example, consider a case when there is much less convergence of interest between president and Congress. Suppose \( T = [-.5, .5] \), \( x(0) = 1 \), and \( n = 3 \). In this case
the ideal points of all possible presidential types lie to the right of Congress and the expected value of the president's ideal point is .375. This example thus involves considerable tension between president and Congress. Even so, half the possible presidential types remain accommodators. The ratio in Proposition 2A indicates this case is similar to an all-compromisers case, so the first offer is greater than zero. If \( q > .33 \), the second offer equals .5, which would surely be accepted, ending the game. Suppose \( q = .5 \). Then the first offer is .25 (the best one-shot offer); \( t(3) = 0 \) so there is a 50% chance the offer is accepted, while types between 0 and .25 bluff in order to get the second offer, and the ex ante probability of seeing the second offer is .25. In short, one would probably only see a single veto, if any. Even if \( q \) lies between .09 and .33, the third offer hits the constraint of \( T \).

In sort, the model suggests that the most likely configurations of preferences under unified government lead to few or no vetoes, and only very short veto chains, the patterns observed historically.

**CONCLUSION**

It is possible to study the executive veto from at least four distinct theoretical perspectives. The first and most familiar focuses on the blocking power of the veto. In this view, the veto is a legislative bullet that presidents use to kill bills (Ferejohn and Shiplan 1990, Kornell 1991, Matthews 1989, McCarty 1993). The second perspective is closely related and examines what might be called the balancing power of the veto. For example Alexander Hamilton, in his discussion of executive powers in *Federalist 73*, justified the veto as protection for the executive branch from an overbearing legislature. Experience with state governments during the Revolution and under the Articles of Confederation had persuaded many to Hamilton's view (Wood 1969). This perspective suggests that the veto is critical in maintaining the institutional balance in a separation of powers system. A third perspective views vetoes as signals to the electorate about
presidential preferences. Though often discussed informally this perspective has yet to be developed in any depth, either theoretically or empirically. Finally, the veto can be seen as a bargaining tool in executive-legislative relations. This perspective focuses on the ability of vetoes to wrest policy concessions from Congress. It is this perspective we have explored.

Our analysis, employing recent advances in bargaining theory, stresses the importance of incomplete information in executive-legislative relations. Vetoes emerge as a powerful tool for the executive, not simply through the implicit threat to block legislation but through a dynamic bargaining process in which vetoes are actually used. Use of the veto can shift outcomes much more toward the president's most desired policy than one might suspect based on an analysis assuming full information. Because the model does not exhibit the Coase phenomenon, its broad dynamics are robust to exact institutional details, such as the speed with which Congress can enact legislation.

The model has implications that extend beyond the American executive veto. First, the model has broad applicability to presidential systems of government in general, with implications for constitutional design. Second, many other political situations resemble that studied here, for example, bargaining between conference committees and the floor in Congress. Finally, the analysis has implications about the relationship between formal models and empirical tests in political science, a topic that has recently received increased attention. Green and Shapiro (1994) assert that formal models cannot generate testable, non-obvious, true propositions, or at least have not done so. The results reported in the sensitivity analysis are testable, and we look forward to learning whether they are rejected by data or not. Krehbiel 1993 notes the difficulty of testing full information models whose outstanding prediction is that phenomena like vetoes never occur. He goes on to suggest ingenious indirect tests for such models. Our analyses provides an example of the feasibility of the direct approach: explicitly incorporating incomplete information in the analysis and deriving empirically testable results.
APPENDIX

Proof of Proposition 1

Part 1 (derivation of equations (1) and (2)). Congress must solve the dynamic programming problem

$$\max_{x(n), t(n)} -x(n) \left( \frac{t(n) - t(n+1)}{\bar{t} - t(n+1)} \right) + \left( \frac{\bar{t} - t(n)}{\bar{t} - t(n+1)} \right) \left\{ -qx(0) + (1-q)EU(t(n)) \right\}$$ (DP)

subject to

$$x(n) = t(n) + (1-q)(x(0) - x^*(n-1)) \quad (C')$$

$EU(t(n))$ is the expected utility of $t(n)$ with $n-1$ periods left given $t \in [t(n), \bar{t}]$, that is

$$-x^*(n-1) \frac{t^*(n-1) - t(n)}{\bar{t} - t(n)} + \frac{\bar{t} - t^*(n-1)}{\bar{t} - t(n)} \left\{ -qx(0) + (1-q)EU(t(n-1)) \right\}$$

$(C')$ is derived from the indifference constraint given in the text, namely

$$V(x(n), t(n)) = (1-q)V(x(n-1), t(n)) + qV(x(0), t(n)) \quad (C)$$

which implicitly defines $x(n)$. Note that $(C)$ assumes the proposal $x(i)$ lies to the right of the peak of the indifferent type's utility function. The alternative assumption that the proposal lies to the left of the peak is inconsistent, i.e., use of that indifference equation generates solutions to (DP) greater than $\frac{t(n) + x(0)}{2}$.

It is convenient to treat Congress as solving DP by choosing an indifferent type $t(i)$ in each period $i$, which is equivalent to choosing an offer in each period. Straight

forwardly the first order condition for (DP) after substitution is

$$-x(n) - \frac{dx(n)}{dt(n)}(t(n) - t(n+1)) - \left\{ -qx(0) + (1-q)EU(t(n)) \right\} + (\bar{t} - t(n)) \left\{ (1-q) \frac{dEU(t(n))}{dt(n)} \right\} = 0 \quad (FOC)$$

From the definition for $EU(t(n))$ (using the envelope theorem)
\[
\frac{dEU(t(n))}{dt(n)} = \frac{x^*(n-1)}{1-t^*(n)} + \frac{EU(t(n))}{1-t^*(n)}
\]

Substituting into (FOC)

\[
x(n) + \frac{dx(n)}{dt(n)} (t(n) - t(n+1)) = qx(0) + (1-q)x^*(n-1)
\]

(FOC')

(note that \(x^*(n-1)\) is a function of \(t(n)\)). The left-hand side of (FOC') is the marginal utility of lowering the offer today, i.e., \(x(n)\) less the change in \(x(n)\) caused by raising the offer; the right hand side is the marginal cost of lowering the offer today, namely, the foregone policy tomorrow (in expectation). Hence, (FOC') requires choosing \(t(n)\) so that the marginal utility of receiving \(x(n)\) today equals the expected marginal utility of receiving \(x(n-1)\) tomorrow.

From (C')

\[
\frac{dx(n)}{dt(n)} = 1 + (1-q) \left( \frac{dx^*(n-1)}{dt} \bigg|_{t(n)} \right)
\]

Suppose the solution is of the form

\[
x(n) = \alpha(n) x(0) + \beta(n) t(n+1)
\]

(1)

and

\[
t(n) = A(n) x(0) + B(n) t(n+1)
\]

(2)

We can solve for \(\alpha(n)\) as a function of \(\alpha(n-1)\), and \(\beta(n)\), \(A(n)\), and \(B(n)\) similarly.

Some algebra yields

\[
\alpha(n) = \frac{1-(1-q)\beta(n-1)(2-q-(1-q)\alpha(n-1))}{2-3(1-q)\beta(n-1)}
\]

\[
\beta(n) = \frac{(1-(1-q)\beta(n-1))^2}{2-3(1-q)\beta(n-1)}
\]

(3)

\[
A(n) = \frac{2q-1+2(1-q)\alpha(n-1)}{2-3(1-q)\beta(n-1)}
\]
\[ B(n) = \frac{(1-(1-q)\beta(n-1))}{2-3(1-q)\beta(n-1)} \]

with \( \alpha(1) = \beta(1) = \frac{1}{2} \) and \( t(n+1) = t \). \(^9\) Part 2. The condition on \( t \) in Part A of the proposition is derived as follows. If the last offer in a two round game is to be unconstrained it must be that \( t \geq \frac{x(0)+t(2)}{2} \) so \( t(2) \leq 2t - x(0) \) or, using equation (1) and recalling that \( A(n) + B(n) = 1 \) \( t(3) = t = \frac{2(t-x(0))}{B(2)} + x(0) \). The last relationship must also hold in a three round game if the last offer is to be unconstrained; but again using equation (1) this implies \( t(4) = t = \frac{2(t-x(0))}{B(2)B(3)} + x(0) \). The condition in Part A follows from induction. The condition in Part B follows straightforwardly from the condition in Part A. \( \square \)

**Proof of Proposition Two**

Consider an unconstrained initial offer \( x(n) = 0 \). Equation (2) then implies the condition on \( t \). Values of \( t \) less than the critical value imply an initial offer \( x(n) = 0 \) which, from \((C')\), implies the indicated value of \( t(n) \). \( \square \)

**Proof of Proposition Three**

To show: a) \( \beta(n) \) has a fixed point and b) equations 1) and 2) converge to the fixed point.

**Step One.** It is easy to show that if \( \beta(n-1) \leq \frac{1}{2} \) then \( \beta(n) \leq \frac{1}{2} \). **Step Two.** Let \( \beta^* \) be the value of \( \beta \) that solves

\[ \beta(n) = \beta(n-1) \]

bearing in mind that \( \beta(n) \) is a function of \( \beta(n-1) \) (see equation (3)). That is, we seek a fixed point of \( \beta(n) \). Via the quadratic equation

\[ \beta^* = \frac{1}{2-q+\sqrt{q}} \]

(The process cannot converge to the other root given \( \frac{1}{2} \) as the initial condition). **Step Three.** Consider
\[
\beta(n) - \beta^* = \frac{(1-q)^2(2+\sqrt{q-q})b(n)^2 + (1-q)(2q-1-2\sqrt{q})b(n) + \sqrt{q-q}}{(2-3(1-q)b(n-1)(2+\sqrt{q-q})}
\] (4)

The denominator of the RHS of (4) \geq 1 for all relevant values of \(q\) and \(\beta(n)\), i.e., \(q \in [0,1]\) and \(\beta(n) \in [0,\frac{1}{2}]\) (the upper boundary of the latter interval comes from Step 1).

Therefore we need only show the numerator of the RHS of (4) \(\to 0\) as \(n \to \infty\). The numerator is a second order difference equation; hence, the solution to the numerator converges to 0 (locally at least) so long as the derivative at \(\beta(n) = \beta^*\) is less than 1 in absolute value. The derivative of the numerator of (4) with respect to \(n\) is

\[
2(1-q)^2(2+\sqrt{q-q})b(n) + (1-q)(2q-1-2\sqrt{q})
\]

which at \(\beta(n) = \beta^*\) is

\[
(1-q)(1-2\sqrt{q})
\] (D)

There are then two cases: 1) \(0 < \sqrt{q} < \frac{1}{2}\). Then both terms of (D) are positive but less than 1 so \(0 < (D) < 1\). 2) \(\frac{1}{2} < \sqrt{q} < 1\). Then \(-1 < (D) < 0\). Therefore in either case there is local convergence. In fact case 1) displays monotone convergence while case 2) displays a cobweb process. ■

**Mathematica Code**

\[
\begin{align*}
b[i_] &= b[i] = ((1- (1-q)b[i-1])^2)/(2 - 3(1-q) b[i-1]) \\
a[i_] &= a[i] = ((1-(1-q) b[i-1](2 - q -(1-q)a[i-1])))/(2-3(1-q)b[z-1]) \\
t[i_] &= t[i] = ((2q - 1 + 2(1-q)a[i-1])x0)/(2-3(1-q) b[i-1]) + \\
      ((1-(1-q) b[i-1]) t[i+1])/(2-3(1-q) b[i-1]) \\
x[i_] &= x[i] = a[i] x0 + b[i] t[i+1] \\
a[1] &= 1/2 \\
b[1] &= 1/2
\end{align*}
\]

These equations calculate \(t(i)\) and \(x(i)\) (\(i\) denotes periods remaining in the game). First calculate all the \(b's\) and all the \(a\'s\), starting with the last period, i.e., \(i = 1\). Then calculate all the \(t's\), starting with the first period e.g., \(i = 5\) in the five period game. In that case \(t[6]\)
is the initial lower bound on the distribution of types (this needs to be initialized to some value). Then calculate the $x$'s, also starting with the first period. Care must be taken to clear definitions when working a series of examples. Note that $t[1]$ is not defined.

NOTES

1 Kennan and Wilson 1993 provides an accessible overview of this and related classes of models of bargaining under incomplete information.

2 Hart's 1989 model of strike duration displays a deadline effect but the effect vanishes when offers come rapidly, due to the Coase phenomenon. The model in Spier 1992 displays pronounced deadline effects but is quite different from the model studied here, as it features an initial bargaining cost and discounting, with one of the parties preferring agreements to come as late as possible.

3 There is a large literature on one-shot take-it-or-leave-it (TILJ) bargaining games, originating with Romer and Rosenthal (1978), including variants with incomplete information, reviewed in Banks (1991). Recent applications of one-shot complete information TILJ bargaining models to vetoes include Kernell 1991, Ingberman and Yao 1991a and b, and Ferejohn and Shipan 1990.

4 McCarty's model and our model bear almost exactly the same relation as models of the chain store paradox and models of strikes. Thus, McCarty's model can be seen as a variant of Kreps and Wilson 1982 while our model, as noted, is closely related to Sobel and Takahashi 1983.

5 Because $B(i) < 1$ for all $i$ (see Appendix), the right hand side of the condition becomes asymptotically large as $n$ goes to infinity. Note that $\bar{T}$ must be strictly less than $x(0)$.

6 The Coase phenomenon can be invalid if both parties have private information or if the private information concerns the value of the good to both parties (Kennan and Wilson 1993 provides a discussion of the issues).

7 Standard techniques of comparative statics cannot be employed, absent a solution to the system of non-linear difference equations. We proceed somewhat similarly to Kennan and Wilson 1989, relying on
exact computation of results using computer programming. The Appendix includes the critical portion of the Mathematica programs used to analyze the model (Wolfram 1991).

8 We thank Joel Kaji for this observation.

9 Note that $h(i)$ is defined only for $i > 1$. 
REFERENCES


President type $t_1$ is accommodating; $t_2$ is compromising; and $t_3$ is recalcitrant.

Figure 1. The President's Utility Function
Figure 2. The Canonical Path of Offers

T=[0,1], x(0) = 1, q = .3
\( x(0) = 1.0, \ T = [0,1] \)

**Figure 3. Offers at Various Levels of \( q \) in the Five Period Game**
$x(0) = 1.0, \ T = [0,1] \ q = .1,.3,.9$

**Figure 4. Concession Schedules in the Five Period Game**
Figure 5. Offers As the Status Quo Becomes More Distant
$x(0) = 1.0, \ T = [0,1], \ q = .1, .3, .9$

Figure 6. Settlement Rates in the Five Period Game
Figure 7. Failure Rate at Various Levels of $q$ in the Five Period Game
$x(0) = 1.0, \ T = [0, 1]$

**Figure 8. Average Length of Veto Chains at Various Levels of $q$ in the Five Period Game**
\[ x(0) = 1.0, \quad T = [0,1] \]

**Figure 9. Expected Utility of Congress at Various Levels of \( q \) in the Four Period Game**
$x(0) = 1.0, \ T = [0,1]$

**Figure 10. Expected Utility of Congress and the Length of the Game**