# A NOTE ON OPTIMA WITH NEGISHI WEIGHTS

FRANCIS DENNIG AND JOHANNES EMMERLING

Abstract. The use of time-varying weights in the objectives of multi-region integrated assessment models is common practice, but their effects on the resulting optimum are not well understood. In this paper we discuss the implications of the use of such weights on inter-temporal preferences and on distributional preferences. For the special case of log utility we show analytically that these weights distort regional inter-temporal preferences affecting the implied discount rate and the resulting savings rates in undesirable ways. High growth economies over save and low growth economies under save relative to what they would do if the savings rate was determined by a regionally specific representative agent. The introduced distortions are numerically important, and can imply differences in optimal saving rates of 10 percentage points for high growth countries such as China. Our numerical results hold for utility specification other than the log. On the choice of optimal mitigation – which unlike savings has an inter-temporal effect across regions – we report the previously existing result stating that the optimal choice with the weights is equal to the optimal choice in a model where consumption in all regions is taken to be the global average consumption. We compare this to an alternative optimisation, without weights, but with a constraint against equalisation of consumption. The comparative statics depend on the distributional features of the model.

Keywords: Negishi weights, Integrated assessment models, discounting, inequality

## 1. Introduction

The use of time-varying Negishi weights is common practice in economic and integrated assessment modeling (IAM) with multiple countries or region. These models include large-scale Integrated Assessment Models such as MERGE (Manne et al., 1995),RICE (Nordhaus and Yang, 1996), ReMIND (Bauer et al., 2008)or WITCH (Bosetti et al., 2009). Their use is based loosely on the theorem in Negishi

(FD) Yale-NUS College, 12 College Ave W, #01-201, Singapore 138610
(JE) Fondazione Eni Enrico Mattei and Centro-Euro Mediterraneo per i Cambiamenti Climatici, Corso Magenta, 63, 20123 Milano, Italy
*E-mail addresses*: fdennig@yale-nus.edu.sg, johannes.emmerling@feem.it.
*Date*: Sunday 9th April, 2017.
Preliminary draft, comments are welcome.

(1972) which states that in a static general equilibrium framework one can solve for a market equilibrium by optimising the appropriately weighted objective function. In Manne and Rutherford (1994) and Nordhaus and Yang (1996, 747) the authors apply modified time-varying weights to a dynamic setting, showing that the modified weights restrict inter-regional transfers and ensure that the optimal marginal cost of abatement is the same across regions. An optimum with such weights yields "...a competitive equilibrium under the assumption that the preferences or technological constraints limit the international flows of capital". These seem desirable properties for multi-region IAMs because the common assumption of decreasing marginal utility of consumption leads the unweighted optima to unrealistically equalise consumption across regions. For these reasons their use has been well established in the multi-region IAM literature.

However, for all their desirable properties, the time-varying weights do not have the properties of Negishi's original weights, and so the original theoretical results don't apply. In fact, it is common for modellers to apply the time-varying weights without knowing exactly all the ways in which they affect the results. In addition to restricting transfers and equalising optimal marginal abatement costs, they also distort the savings rates and the evaluation of costs and benefits in undesirable ways, as we show below.

We first analyze the effect of these weights on savings rates in an economy without externality. In order to get analytical results we first consider a simplified model in which the correct equilibrium savings rates depend only on the taste and capital share parameters, and are thus the same for all regions and time periods. We show that the use of time-varying Negishi weights leads to severely distorted savings rates if the regions have sufficiently different growth rates. Based on a simple calibration exercise, we find that the difference for the savings rates can be to up to ten percentage points relative to the optimum without weights. For a country with higher growth rate projections such as China, the use of time-varying Negishi weights implies that the savings rate increases from about 25 to 35 percent. This error is due to the fact that the weights affect the regional discount rates, distorting them by the difference in consumption growth across regions. Given that the discount rate is crucial for implementing the intertemporal welfare weighting across generations in the context of climate change, this error could be significant, quantitatively important and at odds with the normative foundations for discounting future impacts.

We then add a stock externality such as climate change to the model, in which past actions in any region affect current output in all periods. We reproduce a result from Anthoff (2009a), showing that for this simple model, with logarithmic utility, the Negishi weighted optimum yields the same optimal mitigation as the globally aggregated model in which all regions consume the global average consumption. That is to say, if all the modeller is interested in is the optimal abatement path, the multi-region disaggregation is not necessary if one intends to optimise with time-varying Negishi weights. The aggregate model yields the same (inequality ignorant) results. We compare this optimum with two alternatives. In both alternatives we constrain against any direct interregional transfers. In one case we allow for different regional marginal costs of mitigation at the optimum, and in the other we constrain the planner to equalise the marginal costs of mitigation across regions. We argue that the second approach is a viable alternative to optimising with Negishi weights if what the modeller is interested in is an optimum that takes into account the incidence of costs and benefits across regions with different marginal utilities, while at the same time ensuring that no transfers take place.

The rest of the paper is structured as follows. In section 2 we define the time-varying Negishi weights. In section 3 we illustrate the effect of these weights on the savings rates and in section 4 we illustrate the effect on the optimal emission of a stock externality in a simple two-region model.

## 2. Time-varying Negishi weights

For the case of isoelastic utility functions the time-varying Negishi weighted objective is

$$W^N = \sum_{t=1}^{T} \sum_{r=1}^{R} L_{rt}\omega_{rt} \frac{c_{rt}^{1-\eta} - 1}{1-\eta}(1+\delta)^{-t} \tag{1}$$

where $r$ and $t$ are indices for regions and time, $L_{rt}$ is population, $c_{rt}$ is regional per-capita consumption, $\delta$ is the pure rate of time preference, $\eta$ is the elasticity of marginal utility, and $\omega_{rt}$ are the time-varying Negishi weights.[1] For a general utility function $U$ the weights are defined as the inverse of marginal utility of consumption normalized so that the weights sum to one[2], see Nordhaus and Yang (1996):

$$\omega_{rt} = \frac{\frac{1}{U'(c_{rt})}}{\sum_{j=1}^{R} \frac{1}{U'(c_{jt})}}. \tag{2}$$

For the isoelastic case, this implies that $\omega_{rt}$ can be written as

$$\omega_{rt} = \frac{\frac{1}{c_{rt}^{-\eta}}}{\sum_{j=1}^{R} \frac{1}{c_{jt}^{-\eta}}} = \frac{c_{rt}^{\eta}}{\sum_{j=1}^{R} c_{jt}^{\eta}}.$$

[1]For the special case of $\eta = 1$ the isolelastic utility is logarithmic, $lim_{\eta\to 1}\frac{x^{1-\eta}-1}{1-\eta} = ln(x)$ by l'Hopital's rule.

[2]Some models such as MERGE include relative population shares in the denominator.

If we compute the marginal change in welfare with respect of any region's per capita consumption level, we obtain $\frac{\partial W^R}{\partial c_{rt}} = L_{rt}\left(\sum_{j=1}^{R}\frac{1}{c_{jt}^{-\eta}}\right)^{-1}(1+\delta)^{-t}$ which is time-varying, but constant across regions in per capita terms. This is the desirable feature of these weights that ensures the objective has not redistributive motive and therefore equalises marginal mitigation costs at the optimum. As we shall see below, this is also the feature that yields some of the undesirable features of such an objective.

## 3. Time preference, and savings rates

We use a simplified Ramsey optimal growth model with multiple regions and without trade. For simplicity we consider a version with zero population growth and with 100% depreciation rate. At a 10 year time step as in the RICE model the arithmetic accumulation of the widely assumed 10% per annum depreciation yields this. If we also assume a Cobb-Douglass production function $Y_{rt} = A_{rt}K_{rt}^{\alpha}L_{rt}^{\gamma}E_{rt}^{1-\alpha-\gamma}$ there is a closed form solution to the optimal savings problem of a representative agent with logarithmic utility

$$U_r = \sum_{t=0}^{T} log(c_{rt})\beta^t$$

With full depreciation the regional capital stock in period $t+1$ is simply the regional investment in period $t$: $K_{rt+1} = I_{rt}$. Total consumption is output minus investment, $C_{rt} = Y_{rt} - I_{rt}$. The Euler equation of this optimisation problem yields[3]

$$\frac{1}{C_{rt}} = \beta\frac{1}{C_{t+1}}\alpha\frac{Y_{rt+1}}{K_{rt+1}}.$$

It turns out that a constant savings rate satisfies the this Euler equation with

$$s_{rt} \equiv \frac{K_{rt+1}}{Y_{rt}} = \beta\alpha \tag{3}$$

In practice, with a capital share $\alpha = 0.3$ and a pure rate of time preference of $\delta = 1.5\%$ per annum we have $\beta = (1/(1+\delta))^{10}$ and a savings rate of $s_{rt} = 25.9\%$. Note that if different regions have the same capital share and representative agent they must have the same savings rate.

Notice that a social planner choosing the savings rates in all period for all regions with the aim of maximising the objective

$$W = \sum_{r=1}^{R} U_r = \sum_{r=1}^{r}\sum_{t=0}^{T} log(c_{rt})\beta^t$$

[3]See Brock and Mirman (1972), Engstrm (2012), or Golosov et al. (2014) for details.

would also choose the savings rate computed above. This must be the case because in absence of interactions across regions through prices, trade, or the maximisation of $W$ is equivalent to $R$ separate maximisations of the $U_r$.

This is *not* true of the Negishi weighted objective (1). In that case (still with logarithmic utility) the first order condition of the Lagrangian with respect to investment is given by

$$\mathcal{L}_{K_{t+1}} = \omega_{rt}\beta^t \frac{1}{C_{rt}} - \omega_{rt+1}\beta^{t+1}\frac{1}{C_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}} = 0. \tag{4}$$

which condition can be rewritten as

$$K_{rt+1} = \beta\alpha\frac{\omega_{r,t+1}}{\omega_{r,t}}\frac{C_{rt}}{C_{rt+1}}\frac{Y_{rt+1}}{Y_{rt}}Y_{rt}. \tag{5}$$

Noting that $C_{rt} = (1 - s_{rt})Y_{rt}$ the condition yields the savings rate as

$$s_{rt}^{Neg} = \beta\alpha\frac{\omega_{r,t+1}}{\omega_{r,t}}\frac{1 - s_{rt}}{1 - s_{rt+1}}. \tag{6}$$

Immediately we observe that in general the savings rate is not longer constant, neither across regions nor across time. Notably it will be higher for regions where the Negishi weights increase over time – regions that become relatively richer in the future – and lower where the Negishi weights decrease over time. We can see this by rewriting

$$\frac{\omega_{r,t+1}}{\omega_{r,t}} = \frac{\frac{c_{rt+1}}{c_{rt}}}{\frac{\sum_{j=1}^{R} c_{jt+1}}{\sum_{j=1}^{R} c_{jt}}} \tag{7}$$

Notice that, even if consumption in some region $r$ were to be constant in the steady state, the savings rate would be different due to the normalization changing over time unless all regions' consumption were to be constant. This insight can be generalized to any constant steady state growth rate: if all regions were to grow at the same rate every any point in time we would recover the constant optimal savings rate from the unweighted solution. In general, however, the Negishi weights will change over time under heterogeneity of regional growth rates and will thereby will lead to different savings rates.

We can describe such savings rates by rewriting (6). Substituting for the definition of the weights the condition for the savings rates becomes

$$s_{rt}^{Neg} = \beta\alpha\frac{\sum_{j=1}^{R}(1 - s_{jt})\frac{Y_{jt}}{Y_{rt}}}{\sum_{j=1}^{R}(1 - s_{jt+1})\frac{Y_{jt+1}}{Y_{rt+1}}} \tag{8}$$

Denoting the growth rate in region $j$ between dates $t$ and $t+1$ by $g_j$.[4] (8) becomes

$$s_{rt}^{Neg} = \beta\alpha \frac{(1+g_r)}{\sum_{j=1}^{R} \frac{C_{jt}}{\sum_{k=1}^{R} C_{kt}}(1+g_j)}. \tag{9}$$

Recalling from (3) that the optimal savings rate for region $r$ in period $t$ is $s_{rt} = \alpha\beta$, this shows that the use of Negishi weights distorts the savings rates. Since the denominator on the right hand side is the same for every region this shows that regions with greater growth rates have higher savings rates when computed with Negishi weights.

We can interpret the distortion of the savings rate as resulting from a distortion of the regional time preference. Recall that if $\delta$ is the annual pure rate of time preference the decadal discount factor becomes $\beta = 1/(1+\delta)^10$. If we denote by $g_r^a$ the annualised growth rate in region $r$ and by $\bar{g}^a = \sum_{r=1}^{R} \frac{C_{rt}}{\sum_{j=1}^{R} C_{jt}} g_r^a$ the annualised global average growth we can approximate (9) by

$$s_{rt}^{Neg,approx} = \left(\frac{1}{1+\delta-(g_r-\overline{g})}\right)^{\Delta t} \alpha \tag{10}$$

Comparing the two savings rates (3) and (10), one observes that using Negishi weights distorts the pure rate of time preference by approximately the difference in consumption growth between a region and the global average growth rate $\overline{g}$. To see the quantitative impact of the distortion, consider the specification as above now with a global average growth rate of $\overline{g} = 2\%$ and focus on one region $r$, an emerging economy, with an average growth rate of $g_r = 5\%$. Based on a capital share $\alpha = 0.3$ and an annual pure rate of time preference of $\delta = 1.5\%$ one gets an optimal savings rate of about 25.9% and a distorted savings rate in the high growth region $r$ of 34.8%, which is about 9 percentage points higher.

These exact results are for the specific case of logarithmic utility and full depreciation of capital, but they illustrate a distortion that persists with more general specifications. To understand the exact extent of the distortion in a specific model a similar exercise is easily done numerically.

## 4. A simple two-period model

Consider as before $R$ regions, in two periods $t = \{1,2\}$. Let $Y_{rt}$ be the output in region $r$ and period $t$. In the first period local output depends negatively on the local abatement level, $a_r$, and in the second period outputs depend positively on the

[4]We drop the time subscript for readability, but all the following results hold also for non-constant regional growth rates.

total past abatement $A \equiv \sum_r a_r$. We have that

$$Y_{r1}(a_r); \quad Y'_{r1}(\alpha_r) < 0 \tag{11}$$

and

$$Y_{r2}(A); \quad Y'_{r2} > 0 \tag{12}$$

The representative agent in each region and period gets utility from consuming $c_{rt}$. To simplify notation and without loss of generality, we normalize the population in all regions and across time to unity, i.e., $L_{rt} = 1$. If no transfers occur across regions and there is no transformation of output across time, in the absence of abatement we thus have that $c_{rt} = Y_{rt} \quad \forall t, r$.

We assume logarithmic utility and again we consider two objectives: the the utilitarian and time-varying Negishi weighted welfare functions

$$\mathcal{W} = \sum_r \log(c_{r1}) + \beta \sum_r \log(c_{r2}) \tag{13}$$

and

$$\mathcal{N} = \sum_r \omega_{r1} \log(c_{r1}) + \beta \sum_r \omega_{r2} \log(c_{r2}) \tag{14}$$

where $\omega_{rt}$ are time varying Negishi weights

$$\omega_{rt} = \frac{c_{rt}}{\sum_j c_{jt}} \tag{15}$$

We will consider four cases:

(1) The social planner maximises the Utilitarian objective $\mathcal{W}$ and can freely choose the levels of $a_r$, as well as allocate consumption across regions within a period.
(2) The social planner maximises Negishi weighted objective $\mathcal{N}$ and can freely choose the levels of $a_r$, as well as allocate consumption across regions within a period.
(3) The social planner maximises $\mathcal{W}$, and cannot freely allocate consumption across regions: no redistribution.
(4) The social planner maximises $\mathcal{W}$, cannot freely allocate consumption across regions, and has the further restriction that marginal abatement costs are equalized.

We show, as Anthoff (2009b) shows in a more general model, that the first two cases yield the same optimal abatement measures. This is problematic for the use of time-varying Negishi weights, because it highlights that, these weights do not just restrict against transfers and different marginal abatement costs, but in fact also result in a valuation of climate damages that ignores the different marginal utilities of the regions that incur these costs.

We compare these cases with the utilitarian optimum with no transfers. A case studied in Chichilnisky and Heal (1994). This values climate damages as the sum of the marginal costs they generate across the different regions. The common objection to this solution is that it results in implicit redistribution via the differential abatement effort.

For the modellers who oppose the first two solutions on account of the damage valuation being incorrect, but are also opposed to the implicit redistribution of the Chichilnisky and Heal (1994) solution, we propose the fourth case in which there is a constraint against such a solution. As we show below the marginal abatement cost is set equal to an adjusted marginal damage, where the adjustment depends on the second derivative of the abatement cost curves.

4.1. **The unconstrained Utilitarian case.** The Lagrangian of the problem is

$$\mathcal{L}_1 = \mathcal{W} - \lambda_1 \left[ \sum_r c_{r1} - \sum_r Y_{r1} \right] - \lambda_2 \left[ \sum_r c_{r2} - \sum_r Y_{r2} \right] \tag{16}$$

The first order conditions for consumption yield the immediate equalization of consumption across regions in each period:

$$\lambda_1 = \frac{1}{c_{r1}} \quad \forall r \text{ and } \lambda_2 = \frac{\beta}{c_{r2}} \quad \forall r \tag{17}$$

and the first order conditions for abatement yield

$$-\lambda_1 Y'_{r1}(a_r) = \lambda_2 \left[ \sum_j Y'_{j2}(A) \right] \quad \forall r. \tag{18}$$

By substituting the ratio of Lagrange multipliers from (17) into (18), this results in the well known first-best outcome of equal consumption across regions and the following condition on the abatement levels:

$$-Y'_{r1}(a_r) = \frac{\beta}{G} \left[ \sum_r Y'_{r2}(A) \right] \quad \forall r \tag{19}$$

Where $G \equiv [\sum_r Y_{r2}] / [\sum_r Y_{r1}]$ denotes the total growth factor of the economy. That is, in this case consumption and marginal abatement costs are equalized at any point in time across regions. That is, the solution assumes that there is a cost-less transfer scheme available to the social planner to implement an equalization of consumption across regions in every period.

4.2. **The Negishi solution.** The Lagrangian of the problem is

$$\mathcal{L}_2 = \mathcal{N} - \lambda_1 \left[ \sum_r c_{r1} - \sum_r Y_{r1} \right] - \lambda_2 \left[ \sum_r c_{r2} - \sum_r Y_{r2} \right] \tag{20}$$

The first order conditions are now

$$\lambda_1 = \frac{1}{\sum_r c_{r1}} \text{ and } \lambda_2 = \frac{\beta}{\sum_r c_{r2}} \tag{21}$$

implying that the distributions of consumption across regions within a period are now not determined due to the Negishi weights. However, the condition for abatement levels is identical to the unconstrained Utilitarian case:

$$-Y'_{r1}(a_r) = \frac{\beta}{G} \left[ \sum_r Y'_{r2}(A) \right] \quad \forall r \tag{22}$$

That is, the optimal level of abatement is leading to exactly the same amount of abatement as in the first case. Given that the consumption distribution is not determined, the solution could be simply in maintaining the initial allocation of consumption across regions thus not requiring a transfer possibility and maintaining the initial distribution across regions.

4.3. **The Utilitarian case without transfers.** The Lagrangian of the problem is

$$\mathcal{L}_3 = \mathcal{W} - \sum_r \lambda_{r1} \left[ c_{r1} - Y_{r1} \right] - \sum_r \lambda_{r2} \left[ c_{r2} - Y_{r2} \right] \tag{23}$$

and we get that

$$\lambda_{r1} = \frac{1}{c_{r1}} \quad \forall r \text{ and } \lambda_{r2} = \frac{\beta}{c_{r2}} \quad \forall r \tag{24}$$

and

$$-\lambda_{r1} Y'_{r1}(a_r) = \sum_j \lambda_{j2} Y'_{j2}(A) \quad \forall r \tag{25}$$

and thus, that

$$-\frac{Y'_{r1}(a_r)}{Y_{r1}} = \beta \left[ \sum_j \frac{Y'_{j2}(A)}{Y_{j2}} \right] \equiv MB. \tag{26}$$

That is, now marginal abatement costs are not anymore equalized across countries unless output $Y_{r1}$ is equal across regions. Moreover, consumption is different across regions. Denote the right hand side of ((26)) as the marginal benefit from abatement: $MB \equiv \beta \left[ \sum_r \frac{Y'_{r2}(A)}{Y_{r2}} \right]$, which for a more general utility function $U(c_{rt})$ would be written as $MB \equiv \beta \left[\sum_r \frac{\partial U(c_{rt})}{\partial A}\right]$. That is, the first-best result of the Samuelson rule that marginal costs are equated to the sum of marginal benefits still holds in this case.

This case is the one discussed in Chichilnisky and Heal (1994).

4.4. **The Utilitarian case with equalized MACs.** Adding the constraint of equalized marginal abatement costs (MAC)[5], the Lagrangian of the problem becomes now

$$\mathcal{L}_4 = \mathcal{W} - \sum_r \lambda_{r1}\left[c_{r1} - Y_{r1}\right] - \sum_r \lambda_{r2}\left[c_{r2} - Y_{r2}\right] - \sum_r \mu_r \left[Y'_{\rho 1}(a_\rho) - Y'_{r1}(a_r)\right] \tag{27}$$

In addition to equations (24), we now get

$$-\lambda_{r1} Y'_{r1}(a_1) = \sum_j \lambda_{j2} Y'_{j2}(A) + \mu Y''_{r1}(a_r) \tag{28}$$

Based on the definition of $MB$, we have for each region $r$ that

$$-Y'_{r1} = Y_{\rho 1} MB \left[1 - \frac{Y''_{r1}(a_r)\left[Y_{r1}(a_r) - Y_{\rho 1}(a_\rho)\right]}{\sum_j Y_{j1}(\alpha_j) Y''_{j1}(a_j)}\right] \tag{29}$$

For the case of only two regions $r \in \{A, B\}$, this simplifies so that we get for both regions $r \in \{A, B\}$ that:

$$-Y'_{r1} = Y_{A1} MB \left[1 - \frac{Y''_{A1}\left[Y_{A1} - Y_{B1}\right]}{Y_{A1} Y''_{A1} + Y_{B1} Y''_{B1}}\right]. \tag{30}$$

Comparing this condition to (26), one observes that the additional constraint of equalizing marginal abatement costs distorts the marginal benefits by the social planner $MB$ by the factor in parenthesis. This factor is equal to unity only if marginal costs are constant or output is equal across both countries

## 5. Conclusion

The use of Negishi weights is common practice in economic and integrated assessment modeling with multiple countries or region. While its original conceptualization in a general equilibrium framework provides a useful concept to obtain efficient allocations, its application in particular in intertemporal models lead to undesirable artifacts from an inter-temporal and international equity perspective. In this note we showed how it distorts time preferences and hence optimal saving rates in a simple model, a feature that will inevitably also occur in larger numerical applied models

[5]Here referring to any region $\rho$.

such as IAMs. Based on some back of the envelope calculations we find that the implications can lead to differences in saving rates of up to ten percentage points and thus are significant. Therefore, using Negishi weights in intertemporal models should be seen with caution in particular when considering a long time horizon, and alternatives should be considered.

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