# Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

Christian K. Wolf†
MIT & NBER

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**Abstract:** In a textbook New Keynesian model extended to allow for uninsurable household income risk, any path of inflation and output implementable via interest rate policy is similarly implementable through uniform lump-sum transfers ("stimulus checks"). A dual-mandate policymaker can thus use checks to perfectly substitute for conventional monetary policy when rates are constrained by a lower bound. In a quantitative heterogeneous-agent (HANK) model, the stimulus check policy that implements a given monetary allocation is well-characterized by a small number of measurable sufficient statistics. In the household cross-section, the transfer policy is associated with lower consumption inequality than the equivalent rate cut.

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# 1 Introduction

The prescription of standard New Keynesian theory is to conduct stabilization policy through changes in short-term interest rates. Recently, however, with rates close to a zero or effective lower bound (ELB), the space for such monetary stimulus has narrowed. A natural question, then, is how to replicate the effects of standard monetary policy through alternative tools. Previous work has identified tax policy – often labeled unconventional fiscal policy – as an attractive option (Correia et al., 2008, 2013): time-varying tax rates manipulate intertemporal prices just like monetary policy, and thus can replicate the desired monetary allocation. In practice, however, implementing such policies is challenging: the policymaker would need to jointly fine-tune several different taxes and subsidies at business-cycle frequencies.

In this paper I develop a different conceptual approach to policy equivalence – one based on equilibrium characterization via net excess demand functions – and apply it to a different policy instrument – lump-sum payments ("stimulus checks") sent to households, a tool used in all recent U.S. recessions. The setting for my analysis is a standard New Keynesian model without capital, extended to feature uninsurable household income risk. I establish three main results. *First*, I give conditions under which any sequence of macroeconomic aggregates – including in particular output and inflation – that is implementable via interest rate policy is also implementable by only adjusting uniform lump-sum taxes and transfers. The core intuition is that, with Ricardian equivalence broken, time-varying paths of transfers can re-shuffle consumer "demand" over time just as flexibly as nominal interest rate policy. Thus, for a dual mandate policymaker, conventional monetary policy and stimulus checks are perfect substitutes, so the latter can replace the former at the ELB. *Second*, I characterize the transfer stimulus policy that replicates any given monetary allocation. I find that, even in a rich heterogeneous-agent (HANK) model, the equivalent transfer policy can be summarized well with a small number of (empirically measurable) sufficient statistics. *Third*, I use the HANK model to compare macro-equivalent interest rate and stimulus check policies in the household cross-section. I find that, since checks directly boost consumption of the asset-poor, they compress consumption inequality relative to the equivalent rate cut.

My model economy features a policymaker with access to two instruments: nominal interest rates and uniform, lump-sum taxes and transfers. The objective is to characterize the space of allocations implementable through manipulation of these two instruments. All results apply to (linearized) perfect foresight transition paths, or equivalently to the first-order perturbation solution of the model. Key to my analysis are properties of the two linear maps $\mathcal{C}_{i_b}$ and $\mathcal{C}_\tau$, whose $(t, s)$th entries are, respectively, the derivatives of partial equilibrium consumption demand at time $t$ with respect to (a) a change in the time-$s$ rate of interest on bonds and (b) a uniform lump-sum transfer paid out at time $s$. Note that $\mathcal{C}_\tau$ is a matrix of intertemporal marginal propensities to consume (iMPCs), as studied in Auclert et al. (2018).

My first result is to show that, if $\mathcal{C}_\tau$ is invertible – a condition that I will refer to as *strong Ricardian non-equivalence* –, then any sequence of output and inflation that can be attained via interest rate policy is also (uniquely) implementable by *only* adjusting the time profile of uniform lump-sum transfers. The proof begins with the household consumption-savings problem. A feasible monetary policy is a path of nominal rates together with a path of lump-sum taxes or transfers that ensures a balanced government budget. Through the household problem, this policy induces some path of net excess consumption demand. Can a transfer-only policy – i.e., a policy that only changes the time profile of taxes and transfers, again subject to budget balance – engineer the same path of net excess demand? For Ricardian households, the answer is no: for them, only the *net present value* of transfers matters, so any budget-feasible transfer policy leaves spending unchanged. Mathematically, this is reflected in $\mathcal{C}_\tau$ being rank-1. If instead the *timing* of transfers matters (i.e., $\mathcal{C}_\tau$ is invertible), then there does exist some path of uniform lump-sum transfers and taxes alone that – by moving households away from or towards borrowing constraints – perturbs consumption demand in exactly the same way as the baseline monetary policy. Since this monetary policy was by assumption budget-balanced, the equivalent transfer policy is feasible as well. The argument is then completed by showing that, in my environment, two policies that generate the same partial equilibrium net excess consumption demand paths must be accommodated in general equilibrium through the same market-clearing adjustments in prices (inflation, wages, ...) and quantities (output, hours worked, ...).[1] Though revenue-equivalent in net present value terms, the two policies do invariably induce different short-run government debt dynamics: while interest rate policy can in principle have aggregate effects even if outstanding debt is fixed, stimulus checks work *only* because they change the time path of government bonds held by the private sector. Finally, applying the same argument in reverse, I conclude that the two instruments in fact implement the exact same space of aggregate allocations.[2]

[1] This part of the proof relies on homogeneous labor supply wealth effects in the cross-section of households. Sufficient conditions are either the absence of such wealth effects or a sticky-wage union bargaining protocol as in Erceg et al. (2000). In line with the arguments in Auclert et al. (2020) I adopt the second approach.

[2] The formal statement is that interest rate and transfer policies implement the same sets of aggregate allocations if and only if any *zero net present value* spending path in the image of $\mathcal{C}_\tau$ is also in the image of $\mathcal{C}_{i_b}$, and vice-versa.

Under the conditions of the equivalence result, interest rate and transfer stimulus policies are perfect substitutes for a dual mandate policymaker. In particular, a binding ELB on nominal rates does not at all constrain the space of implementable output-inflation allocations, independent of the menu of non-policy shocks hitting the economy. The key question, of course, is whether the required condition of *strong Ricardian non-equivalence* is actually empirically plausible. Furthermore, even if this condition holds and $\mathcal{C}_\tau$ is strictly speaking invertible, it may be sufficiently close to being non-invertible to imply that the mapping from interest rate to equivalent transfer policy is ill-behaved, requiring large and erratic – and so practically infeasible – fluctuations in transfers, taxes, and government debt.

In the second part of the paper I characterize the mapping between the equivalent stimulus policies using a quantitative, calibrated version of my HANK environment. Guided by results from a simpler, analytically tractable model of partially binding liquidity constraints, I am able to show that – even in this HANK setting – the inverse of $\mathcal{C}_\tau$ and so the transfer policy that implements a given monetary allocation are well-described by a small number of empirically measurable "sufficient statistics." Unsurprisingly, a key statistic for this mapping is the economy's average MPC, with the equivalent transfer stimulus – and so the government debt dynamics that it induces – scaling in the inverse of that average MPC. As we approach the Ricardian limit (and thus as the average MPC converges to zero), the required transfers and so the implied movements in government debt diverge. For empirically relevant levels of household MPCs, in contrast, even moderate increases in transfers and in debt have substantial effects on short-run spending; for example, to close a transitory demand shortfall of one per cent, the policymaker could equivalently (i) cut interest rates by a cumulative 150 basis points or (ii) increase transfers by around \$600 per household, somewhat similar in scope to the stimulus check policy enacted as part of the Economic Stimulus Act of 2008. Since the equivalent transfer policy in (ii) is quickly financed with higher taxes, the induced debt fluctuations are relatively transitory, with a half life of around two years.

Finally, I use the calibrated quantitative HANK model to move beyond macro equivalence and compare policy propagation in the cross-section of households. By the second step of my equivalence proof, two macro-equivalent interest rate and transfer policies necessarily induce the exact same general equilibrium movements of wages, hours worked, inflation, and dividend payouts. However, they very much differ in their *direct* effects on household spending: nominal interest rate cuts mostly act by directly stimulating consumption at the top of the liquid wealth distribution (via intertemporal substitution), whereas the macro-equivalent transfer stimulus almost exclusively acts at the bottom (by relaxing private borrowing constraints). Thus, compared to a given interest rate policy, the macro-equivalent transfer delivers the same macroeconomic stimulus at smaller cross-sectional consumption dispersion. In this paper I only establish this positive observation, and leave a discussion of its normative implications to McKay & Wolf (2021).

I conclude with an extension of my policy equivalence results to a model with investment. Generally, as soon as productive capital is introduced, monetary policy transmits to macroeconomic aggregates by stimulating both consumption *and* investment demand. In this setting, my results on transfer stimulus continue to apply unchanged as a way for the policymaker to replicate the *consumption channel* of monetary transmission. To keep the entire set of implementable aggregate allocations unaffected by the ELB, the policymaker needs to complement transfer payments with a second instrument. Proceeding exactly as in my baseline model, it is straightforward to establish that any policy instrument $q$ inducing an invertible investment demand map $\mathcal{I}_q$ would suffice. In standard models of investment, bonus depreciation stimulus – a policy observed in each of the recent U.S. recessions (Zwick & Mahon, 2017) – satisfies this invertibility assumption; in models of investment breaking the Modigliani–Miller theorem, cash transfers to firms would also work, perfectly analogous to my discussion of Ricardian non-equivalence on the consumption side.

Literature. The paper relates and contributes to several strands of literature.

First, the analysis is motivated by the recent experience of limits to conventional monetary policy space, caused in particular by: a binding ELB on nominal rates due to arbitrage between bonds and money; adverse effects of further rate cuts on bank profitability (Brunnermeier & Koby, 2018); rates on most outstanding mortgages being close to the ELB (Berger et al., 2018); and durables spending adjustments already having been pulled forward in time (McKay & Wieland, 2019). I blend theory and empirical evidence to show how a conventional policy instrument – lump-sum stimulus payments – can overcome those limits.

Second, I build on and extend existing work on monetary-fiscal interactions in incomplete-markets settings. Woodford (1990) and Bilbiie et al. (2013) both emphasize that, with private borrowing constraints, public debt can stimulate spending through its role as private liquidity. Bilbiie et al. (2021) furthermore show that redistribution from savers to spenders in a two-type model can perfectly mimic monetary demand stimulus. Relative to this line of work, my contribution is to: (i) identify conditions under which uniform stimulus checks (and so pure public debt management) are enough to sidestep the ELB in a general HANK environment; (ii) characterize the mapping from monetary allocation to equivalent transfer as a function of empirically measurable sufficient statistics; and (iii) compare the distributional implications of macro-equivalent interest rate vs. (uniform) stimulus check policies.

Third, my proof of policy equivalence relies heavily on equilibrium characterizations in sequence space (Boppart et al., 2018; Auclert et al., 2019). So far, the sequence-space set-up has been used to analytically characterize general equilibrium effects (Auclert & Rognlie, 2018; Auclert et al., 2018) or to construct general equilibrium counterfactuals for unobserved shocks (Wolf, 2020). I instead use the same observations to sidestep constraints on policy space. Similar to classical general equilibrium theory (Arrow & Debreu, 1954), the sequence-space perspective reveals that two policies are equivalent if they induce the same net excess demand paths. As such, my equivalence results are conceptually distinct from Correia et al. (2013) and Farhi et al. (2014): there, equivalence comes from identical wedges in optimality conditions. The key drawback of this "price wedge" approach is that it leverages policy instruments that are arguably harder to vary at business-cycle frequencies.

Outlook. The rest of the paper proceeds as follows. Section 2 sets up the baseline model, and Section 3 presents the main theoretical result on the set of implementable allocations. Section 4 then uses a quantitative HANK model to (i) characterize the mapping between equivalent policies and (ii) study their heterogeneous effects in the cross-section of households. Section 5 generalizes the results to a model with investment, and Section 6 concludes.

# 2 Environment

This section presents the environment. I begin with a model outline in Section 2.1, and then in Section 2.2 define the two key consumption maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$.

## 2.1 Model outline

Time is discrete and runs forever, $t = 0, 1, \ldots$. The model economy is populated by households, unions, firms, and a government, and is initially at its deterministic steady state. I study linearized perfect foresight transition paths.[3] The overall set-up is kept deliberately close to the textbook New Keynesian business-cycle framework (Galí, 2015).

At time $t = 0$, the policymaker announces paths for her policy instruments. My objective

[3] My results can thus equivalently be interpreted as applying to the first-order perturbation solution of an analogous model with aggregate risk (e.g. Boppart et al., 2018).

is to characterize the set of allocations that she can (uniquely) implement. For illustrative purposes, I additionally allow for households to be subject to an unexpected time-0 shock to their patience; as usual, this shock should be interpreted as a reduced-form stand-in for more plausibly structural shocks to consumer spending (e.g., changes in borrowing constraints). I will pay particular attention to the extent to which the policymaker's ability to stabilize aggregate output and inflation in the face of this consumer demand shock varies with constraints on her policy toolkit.

The realization of a variable $x$ at time $t$ along the equilibrium perfect foresight transition path will be denoted $x_t$, while the entire time path will be denoted $\mathbf{x} = \{x_t\}_{t=0}^{\infty}$. Hats denote (log-)deviations from the deterministic steady state and bars denote steady-state values.

Households. There is a unit continuum of households $i$. Households consume a final consumption good $c_{it}$ and provide hours worked $\ell_{it}$, leading to lifetime utility of

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left\{ u(c_{it}) - v(\ell_{it}) \right\} \right] \tag{1}$$

where $\zeta_t$ is the impatience (demand) shock. Household income consists of: labor earnings $(1-\tau_\ell)w_t e_{it}\ell_{it}$, where $w_t$ is the wage, $e_{it}$ denotes a household's (time-varying) productivity and $\tau_\ell$ is the (fixed) labor tax rate; uniform lump-sum transfer receipts $\tau_t$; and dividends $d_{it}$. Households can self-insure by investing in liquid nominal bonds $b_{it}$ with nominal returns $i_{b,t}$, but borrowing in bonds is subject to a borrowing constraint $\underline{b}$. The real return to saving is affected by the inflation rate $\pi_t$. The period-$t$ budget constraint of household $i$ is thus

$$c_{it} + b_{it} = (1-\tau_\ell)w_t e_{it}\ell_{it} + \frac{1+i_{b,t-1}}{1+\pi_t} b_{it-1} + \tau_t + d_{it} \tag{2}$$

Household $i$ maximizes (1) by choosing the path of consumption $c_{it}$ and wealth $b_{it}$ subject to (2) as well as the borrowing constraint $b_{it} \geq \underline{b}$. Hours worked $\ell_{it}$ are – due to frictions in the labor market – taken as given by households and set by optimizing labor unions.

Unions & Firms. I summarize the production and wage bargaining block through three key relations. First, a unit continuum of firms produces the final output good using a labor-only production technology. Imposing that $\int_0^1 e_{it} di = 1$ for all $t$, we can write

$$y_t = y(\ell_t)$$

Price-setting is subject to the usual Rotemberg adjustment costs, giving a standard New Keynesian Phillips Curve (NKPC) in prices:

$$\pi_t \quad = \quad \kappa_p \times \left(\frac{y'(\ell_t)}{\mu_p^{-1} w_t} - 1\right) + \beta \pi_{t+1} \tag{3}$$

where $\mu_p$ is the steady-state mark-up and $\kappa_p$ is a function of model primitives. Producer dividends are paid out to households; as in McKay et al. (2016), I assume that the dividend payments to household $i$ are set as a function of its productivity $e_{it}$. Finally, wage bargaining is subject to similar Rotemberg adjustment costs and so induces a wage-NKPC, linking current and future wage inflation to total household consumption and hours worked:

$$\pi_t^w \quad = \quad \kappa_w \times \left(\frac{v'(\ell_t)}{\mu_w^{-1} w_t u'(c_t)} - 1\right) + \beta \pi_{t+1}^w \tag{4}$$

where $\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t$ denotes wage inflation, $\mu_w$ is the steady-state wage mark-up and $\kappa_w$ is a function of model primitives. I assume that the union demands the same hours worked from all households, so that $\ell_{it} = \ell_t$ for all $i \in [0, 1]$. For my purposes, the crucial feature of (4) is that it implies homogeneous wealth effects in the cross-section of households: the response of labor supply to changes in consumption depends only on (mean) aggregate consumption $c_t$, not its distribution $\{c_{it}\}$. I will return to the importance of this assumption (and discuss alternatives) when proving the equivalence result in Section 3.1.

Derivations of the structural relations (3) and (4) are presented in Appendix A.1.

Policy. The government flow budget constraint is

$$\frac{1 + i_{b,t-1}}{1 + \pi_t} b_{t-1} + \tau_t = \tau_\ell w_t \ell_t + b_t \tag{5}$$

The policymaker sets nominal interest rates $i_{b,t}$ and uniform lump-sum taxes and transfers $\tau_t$ subject to the flow budget constraint (5) and the requirement that $\lim_{t \to \infty} \widehat{b}_t = 0$.

The remainder of this paper will study the implications of constraints on this policy toolkit. I will focus on two particular kinds of restrictions: *transfer-only* policies and *interest rate-only* policies.

**Definition 1.** *A* transfer-only *policy is a policy that sets $i_{b,t} = \bar{i}_b$ for all $t$.*

In a transfer-only policy, the policymaker is forced to keep nominal interest rates fixed, perhaps due to a binding effective lower bound on nominal rates. In light of the ELB's empirical relevance, most discussion in this paper will center on the extent to which a restriction to transfer-only policies meaningfully constrains the policymaker.

**Definition 2.** *An* interest rate-only *policy is a policy that sets, for all* $t = 0, 1, \dots,$

$$\tau_t = \tau_\ell w_t \ell_t + \left(1 - \frac{1 + i_{b,t-1}}{1 + \pi_t}\right)\bar{b}. \tag{6}$$

Definition 2 gives the natural opposite to a transfer-only policy: in an interest rate-only policy, the policymaker is completely free to adjust the path of nominal interest rates, but forced to passively adjust lump-sum transfers/taxes to balance the budget period-by-period. This policy thus operates by manipulating intertemporal prices, without any time variation in the total amount of government debt in the hands of households.[4]

Equilibrium. I am now in a position to define a perfect foresight transition equilibrium in this economy. Throughout, I restrict attention to equilibria in which all sequences of policies and macroeconomic aggregates are bounded.

**Definition 3.** *Given a path of shocks* $\{\zeta_t\}_{t=0}^{\infty}$*, an equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$ *and a set of macroeconomic aggregates* $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ *such that:*

1. *Aggregate consumption is given as*

$$\mathbf{c} = \int_0^1 \mathbf{c}_i di$$

   *where the individual consumption paths* $\mathbf{c}_i$ *solve the household consumption-savings problem of maximizing* (1) *subject to* (2) *and* $b_{it} \geq \underline{b}$.

2. *Wage inflation* $\pi_t^w \equiv \frac{w_t}{w_{t-1}}(1+\pi_t)$ *together with* $\{\ell_t, c_t\}_{t=0}^{\infty}$ *are consistent with the aggregate wage-NKPC* (4).

3. *The paths* $\{\pi_t, w_t, y_t\}_{t=0}^{\infty}$ *are consistent with the aggregate price-NKPC* (3)*, and dividends are given as* $d_t = y_t - w_t \ell_t$.

4. *The output market clears:* $y_t = c_t$ *for all* $t \geq 0$*, the government budget constraint* (5) *holds at all* $t$*, and* $\lim_{t\to\infty} \widehat{b}_t = 0$*. The bond market then clears by Walras' law.*

[4] More generally, an interest rate policy is a policy that freely sets nominal rates $i_{b,t}$ and then ensures budget balance in net present value terms (i.e., $\lim_{t\to\infty} \widehat{b}_t = 0$) through some fixed tax-and-transfer adjustment rule. (6) is simply a particularly transparent example of such a financing rule.

*Given $\{\zeta_t\}_{t=0}^{\infty}$, an allocation of macroeconomic aggregates $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ is said to be implementable if it can be supported as an equilibrium sequence.*

Note that Definition 3 specifies policy directly as a path of policy instruments $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$. As is well-known, policies of this sort generically do not induce unique equilibria (Sargent & Wallace, 1975). To address this challenge, I will later also discuss equilibria induced by policy *rules* for interest rates and transfers. I call an allocation $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ uniquely implementable if it is the only equilibrium sequence consistent with those rules.

## 2.2 Household consumption behavior

In my environment, policy equivalence – or the lack thereof – is all about the properties of household consumption behavior. To make progress, I aggregate the solution to the consumption-savings problem across all households $i \in [0,1]$, thus obtaining an aggregate consumption function $\mathcal{C}(\bullet)$:[5]

$$\mathbf{c} \quad = \quad \mathcal{C}(\underbrace{\mathbf{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \mathbf{d}}_{\text{eq'm aggregates}}; \underbrace{\boldsymbol{\tau}, \mathbf{i}_b}_{\text{policy}}; \underbrace{\boldsymbol{\zeta}}_{\text{shock}}) \tag{7}$$

Given paths of the aggregate demand shock $\zeta$, the policy instruments $\{\tau, i_b\}$, and the endogenous equilibrium aggregates $\{w, \ell, \pi, d\}$, the aggregate consumption function (7) gives the path of household consumption consistent with the solution of each individual household's consumption-savings problem. By definition of the deterministic steady state, this aggregate consumption function satisfies

$$\bar{\mathbf{c}} \quad = \quad \mathcal{C}\left(\bar{\mathbf{w}}, \bar{\boldsymbol{\ell}}, \bar{\boldsymbol{\pi}}, \bar{\mathbf{d}}; \bar{\boldsymbol{\tau}}, \bar{\mathbf{i}}_b; \mathbf{0}\right)$$

In my linearized environment, policy equivalence will be fully governed by the properties of $\mathcal{C}(\bullet)$ around the deterministic steady state. The first key object is the map $\mathcal{C}_\tau$, defined as

$$\mathcal{C}_\tau \quad \equiv \quad \frac{\partial \mathcal{C}(\bullet)}{\partial \boldsymbol{\tau}}, \tag{8}$$

where the derivative is evaluated at the deterministic steady state. The $(t,s)$th entry of this infinite-dimensional linear map is the derivative of aggregate consumption demand at

[5] My assumptions on labor supply and dividend payouts allow me to aggregate household behavior in this way. Note that the consumption function is necessarily indexed by the initial distribution of households over wealth holdings and idiosyncratic productivity. I provide details in Appendix A.2.

time $t$ with respect to a uniform lump-sum transfer $\tau$ at time $s$. This linear map will play a central role in characterizing the space of implementable allocations using *transfer-only* policies. The second key object is the map $\mathcal{C}_{i_b}$, defined as

$$\mathcal{C}_{i_b} \quad \equiv \quad \frac{\partial \mathcal{C}(\bullet)}{\partial \boldsymbol{i_b}} \quad - \quad \bar{b} \times \mathcal{C}_{\tau,\bullet,-1}, \tag{9}$$

where again the derivative is evaluated at the steady state, and where $\mathcal{C}_{\tau,\bullet,-1}$ is the map $\mathcal{C}_\tau$ from the second column onwards. The $(t,s)$th entry of this infinite-dimensional linear map is the derivative of aggregate consumption demand at time $t$ with respect to the nominal interest rate $i_b$ at time $s$, with transfers adjusting as prescribed by (6) to ensure budget balance. This linear map will play a central role in characterizing the space of implementable allocations using *interest rate-only* policies.

Examples. To build some intuition for the abstract definitions in (8) and (9), I will derive the two maps in simple, familiar models of the household consumption-savings problem. For a fully insured, permanent-income household with conventional isoelastic consumption utility (with elasticity of intertemporal substitution $1/\gamma$), and assuming a steady-state real rate of interest $\bar{r}_b = \frac{1}{\beta} - 1$, it is straightforward to establish that

$$\mathcal{C}_\tau^R \quad = \quad \begin{pmatrix} \frac{\bar{r}_b}{1+\bar{r}_b} & \frac{\bar{r}_b}{(1+\bar{r}_b)^2} & \frac{\bar{r}_b}{(1+\bar{r}_b)^3} & \cdots \\ \frac{\bar{r}_b}{1+\bar{r}_b} & \frac{\bar{r}_b}{(1+\bar{r}_b)^2} & \frac{\bar{r}_b}{(1+\bar{r}_b)^3} & \cdots \\ \frac{\bar{r}_b}{1+\bar{r}_b} & \frac{\bar{r}_b}{(1+\bar{r}_b)^2} & \frac{\bar{r}_b}{(1+\bar{r}_b)^3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathcal{C}_{i_b}^R \quad = \quad -\frac{1}{\gamma}(I - \mathcal{C}_\tau^R) \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{10}$$

Following receipt of lump-sum income, permanent-income households consume the annuity value of that receipt in each period, giving the expression for $\mathcal{C}_\tau^R$. In particular, any sequence of transfers with zero net present value does not affect household consumption at all. Current and future (expected) movements in real interest rates, in contrast, tilt household consumption expenditure via the usual intertemporal substitution motive. The upper-triangular matrix in the expression for $\mathcal{C}_{i_b}^R$ reflects the absence of Euler equation discounting, familiar from the literature on forward guidance (e.g. McKay et al., 2016).

As a second example, consider a fully hand-to-mouth household (i.e., $\beta = 0$ in (1)) with zero wealth holdings. Here we find

$$\mathcal{C}_\tau^H \quad = \quad I, \quad \mathcal{C}_{i_b}^H \quad = \quad 0 \tag{11}$$

This household immediately consumes any transfer receipt, and does not react to changes in interest rates. The two familiar extreme cases in (10) and (11) will play a special role in communicating the intuition for the policy equivalence result in Section 3.

Strong Ricardian non-equivalence. For future reference, I will give a name to a particular feature of the linear consumption derivative map $\mathcal{C}_\tau$:

**Definition 4.** *A consumption function $\mathcal{C}(\bullet)$ exhibits strong Ricardian non-equivalence if the induced linear map $\mathcal{C}_\tau$ is invertible. I denote its inverse by $\mathcal{C}_\tau^{-1}$.*

Under the Barro (1974) definition of Ricardian equivalence, the time path of (lump-sum) taxes used to finance any given fiscal expenditure is completely irrelevant for consumption – only the present value matters. Mathematically, we see this in the expression for $\mathcal{C}_\tau^R$ in (10): $\mathcal{C}_\tau^R$ is rank-1, and so in particular it is not invertible. With binding liquidity constraints, the timing of transfer receipts starts to matter and so the rank of $\mathcal{C}_\tau$ increases, with strong Ricardian equivalence referring to the limit case of an invertible (full-rank) map. Note that the spender $\mathcal{C}_\tau^H$ trivially satisfies this notion of strong Ricardian non-equivalence.

# 3 Interest rate cuts vs. stimulus checks

This section presents the policy equivalence result. I first in Section 3.1 discuss the conditions on $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ needed for equivalence of the set of implementable aggregate allocations. I then in Section 3.2 show that the equivalent aggregate allocations do not generally correspond to equivalent household-level allocations.

## 3.1 Aggregate policy equivalence

I begin by stating the main equivalence result: under suitable restrictions on the images of the two linear maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$, transfer-only and interest rate-only policies are equivalent, in the sense that any allocation of macroeconomic aggregates implementable with one policy is also implementable with the other.

**Theorem 1.** *Consider the model of Section 2.1, and let $\widehat{\mathbf{c}}$ be a path of household consumption with zero net present value, i.e., $\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{i}_b}\right)^t \widehat{c}_t = 0$. Suppose that*

$$\widehat{\mathbf{c}} \;\in\; image(\mathcal{C}_\tau) \quad \Leftrightarrow \quad \widehat{\mathbf{c}} \;\in\; image(\mathcal{C}_{i_b}) \tag{12}$$

*with the maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ as defined in Section 2.2. Then the two policy instruments $\tau$ and $i_b$ are macro-equivalent: any aggregate allocation that is implementable with transfer-only policy is also implementable with interest rate-only policy, and vice-versa.*

An easy-to-interpret sufficient condition ensuring the direction "$\Leftarrow$" in the equivalence condition (12) is the notion of strong Ricardian non-equivalence: if $\mathcal{C}_\tau$ is invertible, then the space of aggregate allocations implementable using transfer-only policies is at least as large that implementable using interest rate-only policies.

PROOF SKETCH. Key to the proof of Theorem 1 is the insight that both interest rate as well as transfer policies only directly perturb the model's equilibrium conditions in two places: first, the left-hand side of the output market-clearing condition $\mathcal{C}(\bullet) = \mathbf{y}(\boldsymbol{\ell})$, and second, the sequence of government budget constraints (5).

In partial equilibrium – i.e., prior to general equilibrium price and quantity adjustments – a feasible monetary policy is simply a budget-neutral perturbation of relative intertemporal prices, inducing a path of net excess consumption demand

$$\widehat{\mathbf{c}}_{i_b}^{PE} \quad \equiv \quad \mathcal{C}_{i_b} \times \widehat{\mathbf{i}}_b$$

Note that, since my definition of an interest rate policy includes its financing, this demand path necessarily has zero net present value. By (12), we can find *some* transfer sequence $\widehat{\boldsymbol{\tau}}(\widehat{\mathbf{i}}_b)$ that induces the exact same perturbation of net excess demand. Intuitively, the equivalent transfer-only policy $\widehat{\boldsymbol{\tau}}(\widehat{\mathbf{i}}_b)$ twists the household intertemporal spending profile by changing the amount of government bonds held by households, thereby affecting the severity of liquidity constraints. Furthermore, since the initial monetary policy was consistent with fiscal budget balance, and so since $\widehat{\mathbf{c}}_{i_b}^{PE}$ has zero net present value, it follows from the household lifetime budget constraint that the equivalent transfer also necessarily has zero net present value,

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{i}_b}\right)^t \widehat{\tau}_t(\widehat{\mathbf{i}}_b) \quad = \quad \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{i}_b}\right)^t \widehat{\mathbf{c}}_{i_b}^{PE} \quad = \quad 0$$

Thus, prior to any general equilibrium feedback, $\widehat{\boldsymbol{\tau}}(\widehat{\mathbf{i}}_b)$ is also consistent with budget balance, in the sense that $\lim_{t\to\infty} \widehat{b}_t = 0$. Note that the exact same argument also works in reverse: any transfer-only policy that is consistent with the government budget constraint (5) (i.e., $\lim_{t\to\infty} \widehat{b}_t = 0$) necessarily induces a perturbation of consumption demand with zero net present value, and so by (12) we can always find an equivalent interest rate-only policy. To summarize, this first step of the proof leverages the fact that both policies equally flexibly manipulate the same lever: the consumption-savings decision of households.

Given that the transfer and nominal interest rate policies are both budget-feasible and both perturb the output market-clearing condition by exactly the same amounts time period by time period, it then follows via the implicit function theorem that they must also induce the same general equilibrium paths of inflation, hours worked, wages, and dividends. The intuition is simple: if, for example, the excess demand path induced by some monetary policy is accommodated in general equilibrium through increases in inflation and hours worked, then the same inflation and hours worked paths are also consistent with agent optimality and market-clearing after the equivalent transfer policy. This proof strategy is a simple application of the Arrow & Debreu (1954) approach to general equilibrium characterization using state-by-state (here $t$-by-$t$) net excess demand functions. Finally, the proof also reveals that the two policies are revenue-equivalent in general equilibrium: if say increases in economic activity and inflation lead to an additional budget surplus after a rate cut, then the exact same budget surplus also opens up after the equivalent transfer stimulus.[6]

Discussion of assumptions. The first step of the argument relied on two key assumptions: (i) the existence of an aggregate consumption function $\mathcal{C}(\bullet)$, and (ii) the restriction that both policies operate *only* by manipulating that function. The equivalence result thus actually goes beyond the incomplete-markets consumption-savings problem considered in Section 2.1: rather, it applies to *any* model of the consumption-savings problem admitting a solution of the form (7), including for example models in which Ricardian equivalence fails due to behavioral biases (e.g., Laibson et al. (2020)). By (ii), however, the result does not immediately extend to models in which monetary policy acts through multiple levers, e.g. firm investment. I discuss the extension to models with investment in Section 5.

The second (general equilibrium) step requires that partial equilibrium net excess demand is a sufficient statistic for general equilibrium effects. My union bargaining protocol plays an important role in ensuring that this is the case: in my set-up, labor supply responses depend only on the *average* level of consumption, and not its distribution. Alternative sufficient conditions would be either the absence of wealth effects in labor supply (Greenwood et al., 1988) or the homogeneity of such wealth effects in the cross-section of households. For moderately large stimulus checks – like those in my numerical exercise in Section 4 – these

[6]This notion of revenue equivalence is perfectly analogous to the discussion of interest rate policy and distortionary taxation (i.e., unconventional fiscal policy) in Correia et al. (2013).

sufficient conditions appear consistent with available empirical evidence (e.g. Cesarini et al., 2017). Appendix B.1 provides a more detailed discussion of why, in light of the available empirical evidence, the labor supply channel is unlikely to result in quantitatively meaningful departures from policy equivalence.

Examples. To further illustrate the logic underlying Theorem 1 I return to the two simple examples for $\{\mathcal{C}_\tau, \mathcal{C}_{i_b}\}$ presented in Section 2.2. For Ricardian households, it is straightforward to see from (10) that interest rate policy can induce any possible zero net present value path of net excess consumer demand; formally, the image of $\mathcal{C}_{i_b}^R$ is the space of all 0-NPV sequences. $\mathcal{C}_\tau^R$ is instead rank-1, so the conditions of Theorem 1 are violated.

However, if a positive mass $\mu \in (0,1)$ of hand-to-mouth households is added, we get

$$\mathcal{C}_\tau \;=\; (1-\mu) \times \mathcal{C}_\tau^R \;+\; \mu \times I,$$

so $\mathcal{C}_\tau$ is invertible, strong Ricardian non-equivalence holds, and the two policy instruments are equivalent. Intuitively, through the presence of the hand-to-mouth households, uniform transfers and taxes become an equally viable tool to re-shuffle consumer spending over time.[7]

The sharp dichotomy of Ricardian savers and fully myopic spenders, while useful to illustrate the workings of Theorem 1, is of course not particularly realistic empirically. Section 4 will thus study the shape of the linear maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ in much richer environments that are actually consistent with empirical evidence on consumer spending behavior.

Unique implementation. Theorem 1 asserts the equivalence of implementable allocations. A natural follow-up question concerns *unique* implementability: are the two policy tools also equivalent in their ability to uniquely induce the same sets of allocations?

My analysis is in keeping with discussions of equilibrium existence and uniqueness in the standard New Keynesian business-cycle literature (Woodford, 2011; Cochrane, 2011): I focus on equilibria that are uniquely implementable with interest rate rules of the form

$$\widehat{\boldsymbol{i}}_b \;=\; \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} + \boldsymbol{b}_{i_b} \tag{13}$$

$$\widehat{\boldsymbol{\tau}} \;=\; \tau_\ell \bar{w} \bar{\ell} (\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) - \bar{b} \widehat{\boldsymbol{i}}_{b,-1} + (1 + \bar{i}_b) \bar{b} \widehat{\boldsymbol{\pi}} \tag{14}$$

Here (13) is the rule for the nominal rate of interest (e.g., a simple Taylor rule), and (14) specifies how the policymaker ensures budget balance. I then ask what – if any – transfer-only

[7] See Bilbiie et al. (2013) for the corresponding spender-saver Euler equation representation.

rule uniquely implements the same sequence of macroeconomic aggregates.[8]

**Theorem 2.** *Consider the model of Section 2.1, and suppose that strong Ricardian non-equivalence holds. Consider an interest rate-only policy rule (13)-(14) that uniquely implements the aggregate allocation $\{c_t^*, \ell_t^*, y_t^*, w_t^*, \pi_t^*, d_t^*\}_{t=0}^{\infty}$. Then the transfer-only policy rule*

$$\begin{aligned} \widehat{\boldsymbol{\tau}} &= \tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) + (1+\bar{i}_b)\bar{b}\widehat{\boldsymbol{\pi}} + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} + \boldsymbol{b}_{i_b}\right) && (16) \\ \widehat{\boldsymbol{i}}_b &= 0 && (17) \end{aligned}$$

*uniquely implements the same aggregate allocation.*

The logic underlying Theorem 2 is as follows. Conventional interest rate-only policy rules of the form (13)-(14) are well-known to induce equilibrium uniqueness by inducing unstable dynamics away from the desired unique equilibrium path (e.g. Cochrane, 2011; Atkeson et al., 2010). Since interest rates only enter the equilibrium system in the household consumption-savings problem, it follows that they must do so through their effects on consumer spending. Under the conditions of Theorem 1, however, suitably chosen paths of lump-sum taxes and transfers can mimic that effect on consumer demand. Intuitively, by the equivalence of the images of $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$, we can mimic the explosive off-equilibrium paths induced by a particular interest rate rule by appropriately adjusting the time path of taxes and transfers. The second part of the transfer rule in (16) gives precisely this mapping from nominal interest rate rule space to transfer space.[9]

Implications for dual-mandate policymakers. The previous results imply that, for a conventional inflation-output dual mandate policymaker, interest rate and transfer stimulus policy are perfect substitutes. Corollary 1 summarizes the implications of this insight for policy with a (binding) effective lower bound on nominal interest rates.

**Corollary 1.** *Suppose that strong Ricardian non-equivalence holds, and that conventional*

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[8] Note that my focus on rules of the sort (13)-(14) is without loss of generality: to implement an equilibrium with paths $\boldsymbol{i}_b^*$ and $\boldsymbol{\pi}^*$, we can set $\mathcal{B}_\pi = \phi_\pi \times I$, $\mathcal{B}_y = 0$ and $\boldsymbol{b}_{i_b} = \boldsymbol{i}_b^* - \phi_\pi \boldsymbol{\pi}^*$ to get the King-type rule

$$i_{b,t} = i_{b,t}^* + \phi_\pi(\pi_t - \pi_t^*) \qquad (15)$$

Such rules are well-known to uniquely implement the desired equilibrium for large enough $\phi_\pi$.

[9] Of course this implies that the usual qualifiers for uniqueness through rate policy also apply to uniqueness via transfer policy: both work by "blowing up the economy" off the equilibrium path (Cochrane, 2011).

*interest rate policy is subject to an ELB constraint,*

$$i_{b,t} \geq \underline{i}_b, \quad \forall t. \tag{18}$$

*If the output-inflation allocation $\{\pi_t^*, y_t^*\}_{t=0}^{\infty}$ is uniquely implementable without the ELB constraint (18) through an interest rate-only rule of the form (13)-(14), then that same allocation is also uniquely implementable with the ELB constraint.*

Thus, under strong Ricardian non-equivalence, an ELB on interest rates is of no concern to a dual-mandate policymaker: any paths of inflation and output that were attainable to her without the ELB constraint remain uniquely implementable even with the ELB constraint. For example, if given the demand shock path $\{\zeta_t\}_{t=0}^{\infty}$ the allocation $\pi_t^* = \bar{\pi}$ and $y_t^* = \bar{y}$ – i.e., perfect dual mandate stabilization – is attainable using nominal interest rate policy ("divine coincidence"), then it can also be achieved through a suitable transfer policy. More generally, for any menu of aggregate (non-policy) shocks added to the economy, any aggregate allocation that is implementable using an interest rate rule can equivalently be implemented using a transfer-only rule. Thus, in the equivalent linearized economy with aggregate risk, the second-moment properties of output and inflation under a given interest rate rule can be replicated perfectly with an analogous transfer rule (see Appendix B.3).

## 3.2 Distributional outcomes

The equivalence results in Section 3.1 concern aggregate outcomes. At the level of an individual household $i$, however, the macro-equivalent nominal interest rate and transfer policies may have very different effects.

Along a generic perfect foresight transition path, consumption of an individual household $i$ is given as

$$\mathbf{c}_i \quad = \quad \mathcal{C}_i(\mathbf{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \mathbf{d}; \boldsymbol{\tau}, \mathbf{i}_b; \boldsymbol{\zeta})$$

where $\mathcal{C}_i(\bullet)$ is the solution to the individual consumption-savings problem, indexed by that individual's initial asset holdings and productivity. Now consider two macro-equivalent policies $\widehat{\boldsymbol{i}}_b$ and $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$. Since by construction both induce the same sequences of wages, hours worked, inflation and dividends, we find that

$$\Delta_i(\widehat{\boldsymbol{i}}_b) \;\; \equiv \;\; \mathbf{c}_{i,i_b} - \mathbf{c}_{i,\tau(i_b)} \;\; = \;\; \mathcal{C}_{i,i_b}\widehat{\boldsymbol{i}}_b \; - \; \mathcal{C}_{i,\tau}\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$$

where $i_b$ and $\tau$ subscripts indicate transition paths corresponding to interest rate and transfer policy, respectively, and the linear maps $\mathcal{C}_{i,i_b}$ and $\mathcal{C}_{i,\tau}$ are defined as in (8) - (9). We know that, since by construction $\widehat{\boldsymbol{i}}_b$ and $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ are such that $\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b) = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b$, we must have

$$\int_0^1 \Delta_i(\widehat{\boldsymbol{i}}_b) di \quad = \quad \mathbf{0}$$

For individual households, however, we may well have $\Delta_i(\widehat{\boldsymbol{i}}_b) \neq \mathbf{0}$. Intuitively, while both policies by construction induce the exact same aggregate demand stimulus, they may do so by affecting consumption at different points in the cross-section of households.

A simple spender-saver model again provides a transparent illustration: there, conventional nominal interest rate policy works exclusively through intertemporal substitution of the savers, while the equivalent transfer stimulus works through spending of the borrowing-constrained, with no direct effect on savers. Later, in Section 4.3, I will study the extent to which this sharp dichotomy of the spender-saver model survives in shaping the cross-sectional distribution of $\Delta_i(\widehat{\boldsymbol{i}}_b)$ in a richer HANK model.

# 4 Equivalent policies in a quantitative model

In Section 3 I have identified general conditions ensuring the aggregate equivalence of interest rate and transfer policies. This analysis, however, has left several questions unanswered: Are the high-level conditions on the linear maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ required for policy equivalence likely to be satisfied in practice? In particular, even if $\mathcal{C}_\tau$ is technically invertible, could simple interest rate cuts perhaps correspond to ill-behaved sequences of stimulus checks and taxes, together with large fluctuations in government debt? And finally, how should we expect the propagation of macro-equivalent stimulus policies to differ in the cross section of households?

In this section I answer these questions using a calibrated heterogeneous-agent (HANK) incomplete-markets model. Section 4.1 describes the model and its calibration, Section 4.2 characterizes the mapping between macro-equivalent policies, and Section 4.3 analyzes distributional implications.

## 4.1 Model details

I consider a calibrated version of the model of Section 2.1. In my discussion I pay particular attention to the model's implications for consumer spending responses to the receipt of one-off lump-sum transfers, and then compare those predictions with existing empirical evidence.

Parameterization. I only present a very brief overview of the (standard) model parameterization here, and relegate further details to Appendix A.3.

Households face the same income process as in Kaplan et al. (2018), and can self-insure by saving, but not borrowing. I calibrate total bond holdings to the amount of liquid wealth in the U.S. economy; corporate wealth, instead, is perfectly illiquid, with households receiving dividend payments as a function of their labor productivity. The economy is closed with a simple constant-returns-to-scale production function as well as conventional degrees of nominal wage and price stickiness.

Policy experiments. I fix a given interest rate-only policy, and then use the constructive proof of Theorem 1 to recover the equivalent transfer-only policy. In a slight generalization of Definition 2, I assume that the baseline interest rate-only policy is not financed through taxes and transfers adjusting period-by-period (as in (6)), but instead consider a more general fiscal financing rule of the form

$$\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \left[\widehat{\tau}_t + (1+\bar{i}_b)\bar{b}(\widehat{i}_{b,t-1} - \widehat{\pi}_t) - \tau_\ell(\bar{w}\widehat{\ell}_t + \bar{\ell}\widehat{\ell}_t)\right] \tag{19}$$

Under this fiscal rule, for $\rho_b > 0$, an interest rate cut at time $t$ only feeds through to lower taxes/higher transfers with a delay. While the financing rule in Definition 2 was conceptually simpler, the alternative rule (19) has the advantage that interest rate movements are not accompanied by (counterfactual) large contemporaneous changes in transfers.

Consumption behavior. The distinguishing feature of incomplete-markets models of consumer spending is that they break Ricardian equivalence. By my results in Section 3, the key question for the purposes of this paper is whether my notion of *strong* Ricardian non-equivalence in fact holds, in the sense that the inverse map $\mathcal{C}_\tau^{-1}$ is well-defined (and well-behaved). For my calibrated HANK model to give a convincing answer to this question, it should be consistent with existing empirical evidence on consumer spending in response to (uniform) lump-sum transfer receipts. I here first briefly review that empirical evidence, and then compare it to consumer behavior in the model.

The most robust available evidence relates to the first column of $\mathcal{C}_\tau$ – that is, the response of household spending today and in the future to a lump-sum income receipt today. Here, the literature has so far presented three main findings. First, average impact MPCs (i.e., entry $\mathcal{C}_\tau(1,1)$) are robustly estimated to be high, around $0.2 - 0.3$ quarterly for non-durable consumption spending out of an income receipt of around \$500. Second, households spread

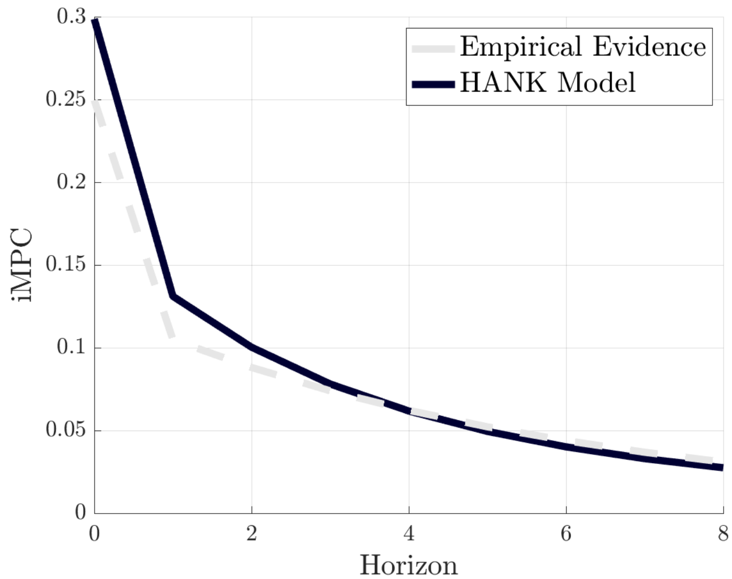


**Figure 1:** iMPCs in the HANK model. The blue line shows the actual model-implied path of iMPCs for income receipt at $t = 0$, while the grey dashed line plots the iMPC path corresponding to my review of the empirical evidence, based on Fagereng et al. (2018).

out their consumption of the lump-sum windfall over time, with iMPCs declining from period to period (i.e., $\mathcal{C}_\tau(t+1,1) < \mathcal{C}_\tau(t,1)$). And third, the rate of decay is largest between impact and the following period (i.e., $\mathcal{C}_\tau(2,1)$ vs. $\mathcal{C}_\tau(1,1)$), and then roughly constant at a smaller rate thereafter. These findings suggest a simple three-parameter function for the vector of quarterly iMPCs following a one-off lump-sum transfer receipt: the impact MPC $\omega$, a rate of decay $\theta$, and an additional downward scaling after impact of $\xi$, giving a response vector $(\omega, \omega\xi\theta, \omega\xi\theta^2, \dots)'$. An interpolation based on the annual estimates in Fagereng et al. (2018) would for example roughly suggest the parameterization $\{\omega = 0.25, \theta = 0.84, \xi = 0.50\}$. The dashed grey line in Figure 1 plots this path of average household MPCs.

Next, I solve for the first column of $\mathcal{C}_\tau$ in the calibrated HANK model, displayed as the dark blue line in Figure 1. Since household income risk is high, but opportunities for self-insurance are limited, the average quarterly MPC out of a one-off lump-sum income gain is high, somewhat above 25 per cent quarterly. The iMPC path first drops off sharply, and then decays gradually, very much consistent with the empirical evidence reviewed above. The remaining question, then, is what the HANK model implies for the other columns of $\mathcal{C}_\tau$ – columns that clearly matter for my equivalence mapping, but for which we currently have little empirical evidence.

## 4.2 The policy equivalence mapping

Under strong Ricardian non-equivalence, we can construct the mapping from a given nominal interest rate policy into the corresponding macro-equivalent transfer stimulus as

$$\widehat{\boldsymbol{\tau}}(\widehat{\mathbf{i}}_b) \quad \equiv \quad \mathcal{C}_\tau^{-1} \quad \times \quad \underbrace{\mathcal{C}_{i_b} \times \widehat{\mathbf{i}}_b}_{\widehat{\boldsymbol{c}}_{i_b}^{PE}} \tag{20}$$

The objective of this section is to learn about $\mathcal{C}_\tau^{-1}$: this map characterizes, for *any* given net excess demand target $\widehat{\boldsymbol{c}}^{PE}$, the path of stimulus checks required to engineer it.[10]

The remainder of this section proceeds in two steps. First, I solve the quantitative HANK model numerically and present the macro-equivalent transfer stimulus policy corresponding to a given short-lived nominal interest rate cut. Second, I use a tractable incomplete-markets model to better understand the nature of this mapping and summarize it through a sufficient statistics formula that relates back to the empirical evidence reviewed in Section 4.1.

A SHORT-RUN STIMULUS EXPERIMENT. Figure 2 shows the paths of interest rates, lump-sum transfers, aggregate output and government debt following (a) a baseline interest rate policy and (b) the macro-equivalent stimulus check policy. Consider first the green line in the top right panel: this line gives $\widehat{\boldsymbol{c}}_{i_b}^{PE} = \widehat{\boldsymbol{c}}_{\tau}^{PE}$, the direct change of net excess consumer demand induced by the two policies. I consider policies that briefly push up net excess demand, for a total of \$150 on impact per household, or around one per cent of steady-state consumption. The grey line in the top left panel indicates that this spending increase corresponds to a relatively short-lived nominal interest rate cut with a cumulative total of around 150 basis points. The bottom left panel shows that, once general equilibrium effects are taken into account, total consumption actually increases by more than one per cent – a Keynesian multiplier in excess of one, reflecting the model's elevated average MPC. The orange lines instead indicate what happens under the equivalent transfer stimulus: nominal interest rates are fixed (by construction); transfers have to initially go up significantly before then being cut (ensuring an overall net present value of zero); aggregate consumption changes exactly

[10] $\mathcal{C}_\tau^{-1}$ characterizes the mapping from net excess demand target to equivalent transfer. In Appendix C.4 I characterize instead the mapping from interest rate target to equivalent transfer, giving conditions under which $\mathcal{C}_{i_b}$ is a function only of (i) $\mathcal{C}_\tau$ and (ii) the elasticity of intertemporal substitution. In the main text I focus on $\mathcal{C}_\tau^{-1}$ since, in practice, $\mathcal{C}_{i_b}$ is likely to be shaped by features outside of my model, e.g. mortgage refinancing. I emphasize that, whatever the effects of such additional model features, invertibility of $\mathcal{C}_\tau$ is sufficient to ensure that *any* monetary policy demand path can be synthesized via time-varying transfers.

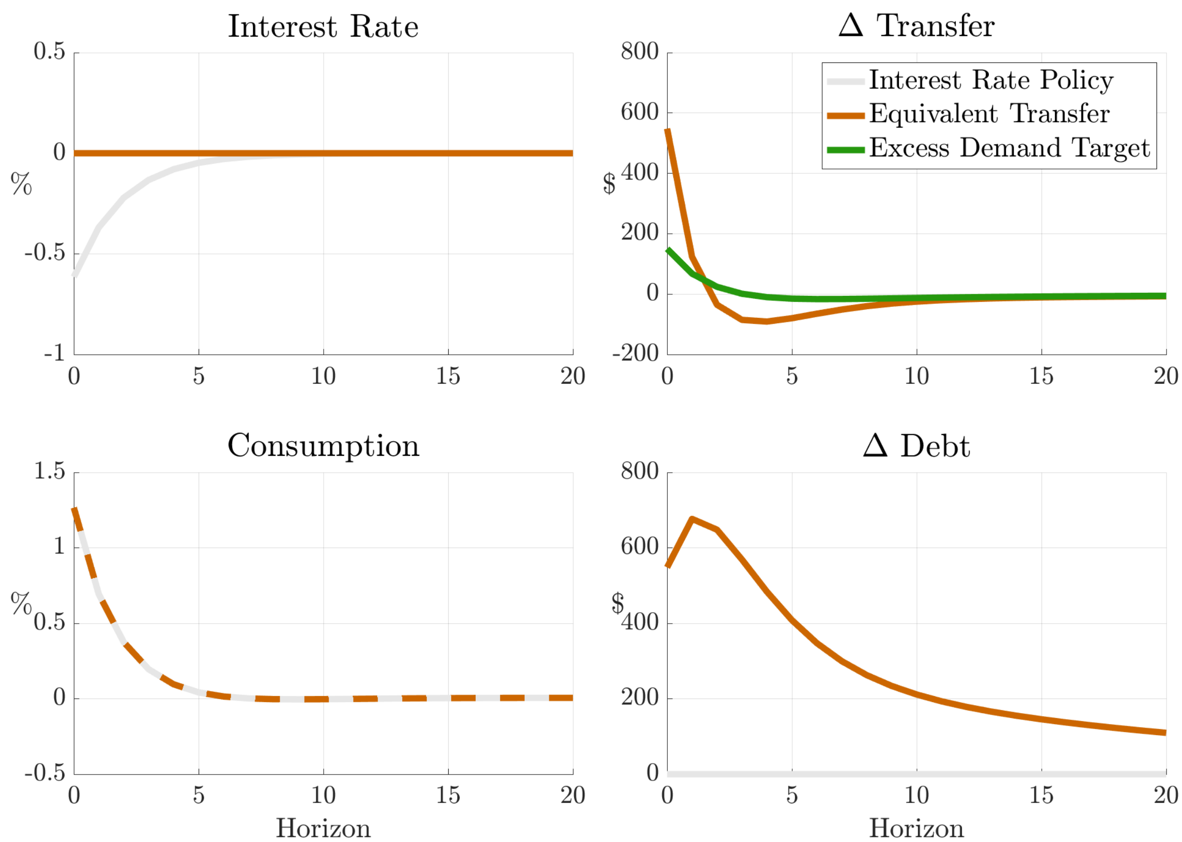


**Figure 2:** Impulse responses of nominal interest rates, transfers, consumption, and debt under the macro-equivalent interest rate and transfer policies in the calibrated HANK model. For transfers and debt I show the differences between their paths under the two policies.

as it did under the interest rate policy; and the total amount of outstanding debt jumps up initially to finance the transfer, before then gradually returning to baseline.

For the purposes of a policymaker contemplating the use of transfer stimulus payments to circumvent a binding lower bound on nominal rates, the two panels on the right of Figure 2 are key: they indicate that, to engineer a short-lived demand stimulus with a peak effect of one per cent of steady-state consumption, transfers need to increase moderately for a couple of quarters, with taxes then subsequently increasing to finance the deficit.[11] The implied fluctuations in debt are moderate in size and persistence, with a half life of around two years.

[11] Note that this hypothetical stimulus policy relatively closely approximates parts of the 2008 Economic Stimulus Act – checks totaling around \$600 per individual, and sent out in a staggered fashion. My results suggest that a fiscal policy of this sort closely approximates the transfer stimulus needed to offset a cumulative nominal interest rate cut of around 150 basis points.

The takeaway from this numerical analysis is that, at least in my calibrated HANK model and for the particular interest rate policy and corresponding demand target considered here, the policy mapping governed by the inverse map $\mathcal{C}_\tau^{-1}$ appears to be well-behaved. The key question, of course, is what properties of the model govern this mapping, and in particular whether my finding of a smooth equivalent transfer path and moderate debt fluctuations is likely to be robust across models and policy experiments.

A SUFFICIENT STATISTICS INTERPRETATION. To shed light on these questions, I study the map $\mathcal{C}_\tau$ in an analytically tractable incomplete-markets model of the consumption-savings problem: a hybrid model with (i) bonds in the household utility function (Michaillat & Saez, 2018) and (ii) a fixed margin of spenders (Campbell & Mankiw, 1989). The details for this simple consumption-savings problem are relegated to Appendix C.

It is well-known that this hybrid model can, if suitably parameterized, perfectly match the three-parameter impulse response of consumer spending to a lump-sum transfer receipt displayed in Figure 1 (e.g. Auclert et al., 2018; Bilbiie, 2021). As it turns out, this model furthermore implies very strong cross-column restrictions on the rest of $\mathcal{C}_\tau$, allowing us to (approximately) pin down the entire map $\mathcal{C}_\tau$ as a function only of its first column and so the three MPC coefficients $\{\omega, \theta, \xi\}$. I first in Proposition 1 formalize this claim, and then ask whether the same cross-column restrictions also hold in my calibrated HANK model.

**Proposition 1.** *Consider the consumption-savings problem with bonds in the utility function in Appendix C.4, parameterized so that the first three entries of the first column of the implied transfer map* $\mathcal{C}_\tau^{TB}$ *are given as* $\{\omega, \omega\xi\theta, \omega\xi\theta^2\}$. $\mathcal{C}_\tau^{TB}$ *then has the following properties:*

*1.* $\mathcal{C}_\tau^{TB}$ *is given as*

$$\mathcal{C}_\tau^{TB} \approx \omega \times \begin{pmatrix} 1 & \xi\theta & \xi\theta^2 & \xi\theta^3 & \dots \\ \xi\theta & 1 & \xi\theta & \xi\theta^2 & \dots \\ \xi\theta^2 & \xi\theta & 1 & \xi\theta & \dots \\ \xi\theta^3 & \xi\theta^2 & \xi\theta & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{21}$$

*where the approximate equality indicates* asymptotic *equality, in the sense of convergence in weak (Hilbert-Schmidt) matrix norm.*[12]

[12] I provide a formal definition of this notion of convergence in Definition C.1, stated in Appendix C.1.

*2. The cumulative distribution function of the distribution of eigenvalues of $\mathcal{C}_\tau^{TB}$ is given as*

$$F^{TB}(\lambda) = 1 - \frac{1}{\pi} arccos\left(\frac{1+\theta^2 - \frac{\xi(1-\theta^2)}{\frac{\lambda}{\omega}-(1-\xi)}}{2\theta}\right) \tag{22}$$

*with the bounds of the support $[\underline{\lambda}, \bar{\lambda}]$ given as*

$$\underline{\lambda} \equiv \omega \times \left[(1-\xi) + \xi\frac{1-\theta}{1+\theta}\right], \quad \bar{\lambda} \equiv \omega \times \left[(1-\xi) + \xi\frac{1+\theta}{1-\theta}\right]$$

*3. The $(i,j)$th entry of the inverse of $\mathcal{C}_\tau^{TB}$ satisfies*

$$(\mathcal{C}_\tau^{TB})^{-1}(i,j) \approx \frac{1}{\omega}\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(|i-j|\lambda)\frac{1-2\theta\cos(\lambda)+\theta^2}{\xi(1-\theta^2)+(1-\xi)(1-2\theta\cos(\lambda)+\theta^2)}d\lambda \tag{23}$$

*where the approximate equality indicates asymptotic equality.*

Proposition 1 provides a sharp characterization of the policy equivalence mapping as a function only of the three empirical moments $\{\omega, \theta, \xi\}$. First, (21) shows that – at least approximately – the three moments characterize all of $\mathcal{C}_\tau$. The key cross-column restriction implied by the model is thus that the gradually decaying spending response to transfer receipt today is also highly informative about the strength of anticipation effects tomorrow. Second, (2) presents the distribution of eigenvalues of this linear map. The key take-away here is that, for empirically plausible values of $\{\omega, \theta, \xi\}$, this distribution has support bounded away from zero, with $\underline{\lambda} \approx 0.14$. $\mathcal{C}_\tau$ is thus invertible and strong Ricardian non-equivalence holds. Third, (23) provides an expression for the entries of that inverse. The expression is slightly unwieldy, so I provide a visual representation in Figure 3. If bonds were unvalued by households (which can be shown to imply that $\xi = 0$), then (23) would simplify to

$$\mathcal{C}_\tau^{-1} = \frac{1}{\omega} \times I$$

Thus, exactly as expected given the spender-saver discussion in Section 3.1, the equivalent transfer would simply scale in one over the average MPC $\omega$. The grey line in the top left panel of Figure 3 illustrates, showing the entries of the fifth column of $\mathcal{C}_\tau^{-1}$. For a different perspective on the same logic, the top right panel uses the spender-saver $\mathcal{C}_\tau^{-1}$ to map the net

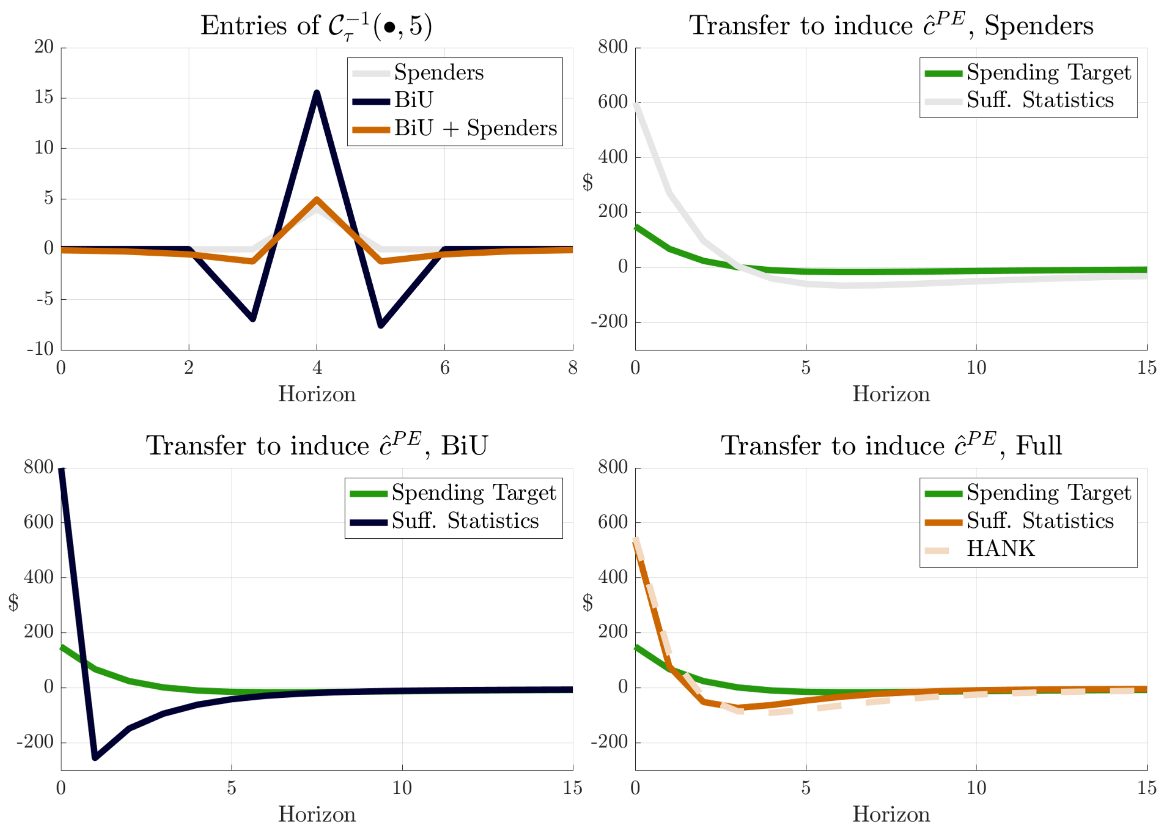


**Figure 3:** Top left panel: The fifth column of $\mathcal{C}_\tau^{-1}$ for (i) spender-saver, (ii) bond-in-utility and (iii) hybrid model. Other panels: Spending target and exact equivalent transfer in models (i) - (iii), based on the sufficient statistics approximation from Proposition 1. The bottom right panel additionally shows the exact equivalent transfer from the calibrated HANK model.

excess demand path $\widehat{\boldsymbol{c}}_{i_b}^{PE}$ from Figure 2 into the corresponding equivalent transfer, giving

$$\widehat{\boldsymbol{\tau}}(\widehat{\mathbf{i}}_b) \equiv \frac{1}{\omega} \times \widehat{\boldsymbol{c}}_{i_b}^{PE}$$

If instead bonds were valued but there are no spenders (which can be shown to imply that $\xi = 1$), then (23) would instead simplify to

$$\mathcal{C}_\tau^{-1} = \frac{1}{\omega} \times \begin{pmatrix} \frac{1}{1-\theta^2} & -\frac{\theta}{1-\theta^2} & 0 & 0 & \cdots \\ -\frac{\theta}{1-\theta^2} & \frac{1+\theta^2}{1-\theta^2} & -\frac{\theta}{1-\theta^2} & 0 & \cdots \\ 0 & -\frac{\theta}{1-\theta^2} & \frac{1+\theta^2}{1-\theta^2} & -\frac{\theta}{1-\theta^2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

The argument becomes slightly more involved: households now spend any lump-sum transfer receipt gradually, so the equivalent transfer has to be less persistent than the targeted net excess demand path. Thus, to engineer one dollar of excess consumer spending at some time period $t$, the policymaker would need to increase transfers at $t$ and lower them at $t-1$ and $t+1$, offsetting the cross-period leakage, as seen in the dark blue line in the top left panel of Figure 3. Similarly, if the policymaker wants to engineer a persistent increase in spending, then a transitory transfer increase suffices, as illustrated in the bottom left panel. Finally, the most general expression in (23) just combines the previous two: the transfer scales in one over the MPC, but is somewhat less persistent than the spending target, as illustrated by the dark orange lines in the top left and bottom right panels of Figure 3.

Proposition 1 has shown that, in a particular analytical model that is consistent with the evidence in Figure 1, the transfer equivalence mapping takes a very simple form and is robustly well-behaved. These conclusions, however, are only useful to the extent that they help to inform our understanding of richer models of the consumption-savings problem. The bottom right panel of Figure 3 reveals that, at least for the HANK model considered in my numerical analysis, this is indeed the case. Here, the green and dotted pale orange lines are copies of those in the top right panel of Figure 2. Recall that the solid orange line was instead constructed by fixing the vector $\{\omega, \theta, \xi\}$ and then using the results of Proposition 1 to map $\widehat{\boldsymbol{c}}_{i_b}^{PE}$ into $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$. Crucially, the dark and pale orange lines are very close to each other, so we can conclude that – even in HANK – a relatively small number of measurable sufficient statistics is highly informative about the policy equivalence mapping. In other words, the cross-column restrictions of the simple model embedded in (21) continue to hold at least approximately in my richer incomplete-markets setting.[13]

Summary. The analysis of this section has revealed that my notion of strong Ricardian non-equivalence (and so the policy equivalence that it implies) is not a mere theoretical curiosity – rather, it is a robust feature of incomplete-markets models with empirically realistic consumer spending behavior.

In the context of Figure 2, the transfer stimulus that replicates my given (sizable) interest rate cut is neither implausibly large nor erratic, nor does it lead to excessive fluctuations in outstanding government debt. More generally, my results imply that familiar interest rate

[13] While $\{\omega, \theta, \xi\}$ thus pin down anticipation effects in bond-in-utility and richer incomplete-markets models alike, this feature of models is of course not necessarily a feature of reality. In Appendix B.2, I present an alternative sufficient statistics formula that allows for anticipation effects to be parameterized more flexibly.

rules – such as simple Taylor rules – have robustly well-behaved analogues in transfer space, allowing a dual mandate policymaker to leverage transfers in order to completely sidestep interest rate lower bound constraints. I further elaborate on those implications for policy rules as well as (optimal) dual mandate policy in Appendices B.3 and B.4.

### 4.3 Who increases consumption?

In addition to gauging the empirical plausibility of policy equivalence, a second benefit of the quantitative HANK model is that it allows us to understand the heterogeneous distributional implications of macro-equivalent interest rate and transfer stimulus policies.

Since the ability of households to self-insure is governed by their wealth holdings, I summarize the heterogeneous distributional implications of the two policies by sorting households along the liquid wealth dimension. Formally, letting $\bar{\lambda}(b,e)$ denote the steady-state distribution of households across their idiosyncratic state space, my object of interest is

$$\Delta_b(\widehat{\boldsymbol{i}}_b) \;\equiv\; \int_e \left(\widehat{\mathbf{c}}_{(b,e),i_b} - \widehat{\mathbf{c}}_{(b,e),\tau(i_b)}\right) d\bar{\lambda}(b,e) \;=\; \int_e \left(\mathcal{C}_{(b,e),i_b}\widehat{\boldsymbol{i}}_b \;-\; \mathcal{C}_{(b,e),\tau}\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)\right) d\bar{\lambda}(b,e)$$

where $\mathcal{C}_{(b,e),i_b}$ and $\mathcal{C}_{(b,e),\tau}$ are defined analogously to $\mathcal{C}_{i_b}$ and $\mathcal{C}_\tau$, but conditioning on households with initial state $(b,e)$. In words, I sort households by their ex-ante liquid wealth position, and then compute the differential evolution of consumption along the wealth distribution in response to the two macro-equivalent policies. Note that the second line of the equality follows because both policies by construction induce the same general equilibrium effects, so they differ only in their direct effects on consumer spending.

I find that, while macro-equivalent, the interest rate and transfer stimulus policies have vastly different effects in the cross-section of households. Figure 4 presents the results for the two macro-equivalent stimulus policies considered in Figure 2, splitting the impact consumption response by liquid wealth percentile into direct effects – defined as the response of consumption demand to the two policies alone, fixing all non-policy variables at their steady state values forever – and the residual indirect effects. By construction, the two policies induce the exact same indirect general equilibrium effects household-by-household. The direct effects, on the other hand, are only guaranteed to integrate to the same number across the entire population, but do not need to agree household-by-household. Figure 4 reveals that these direct effects are indeed very heterogeneous, resulting in large *overall* differences in consumption along the liquid wealth distribution, $\Delta_b(\widehat{\boldsymbol{i}}_b)$. On the one hand, to engineer the required one per cent increase in household consumption demand on impact, interest rate

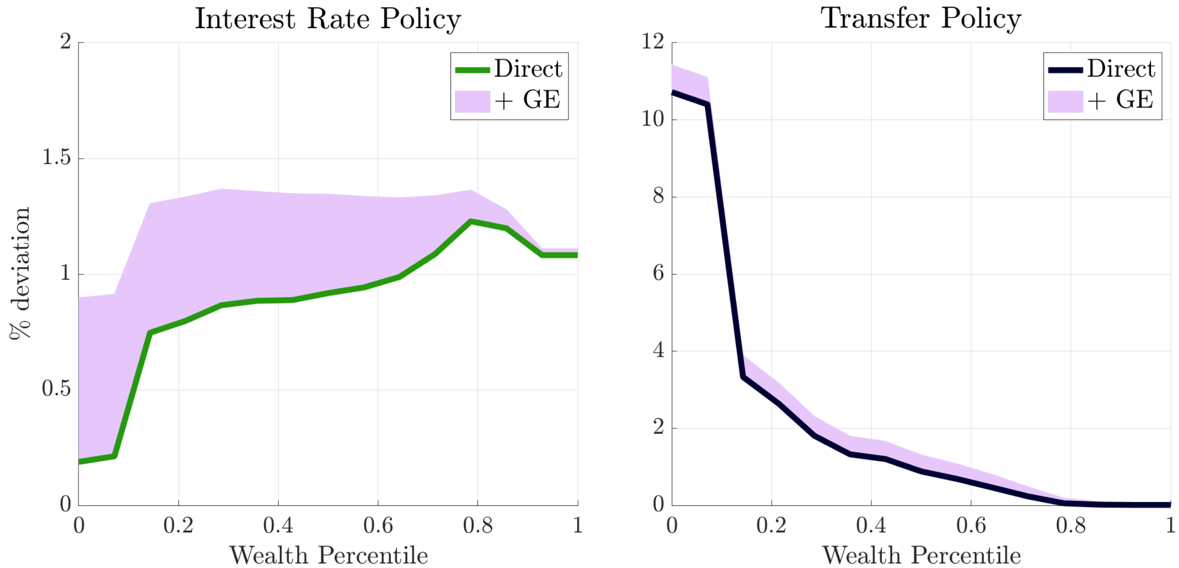


**Figure 4:** Left panel: The green line shows the time-0 *direct* household spending response by liquid wealth percentile to the nominal interest rate policy of Figure 2. The shaded purple area gives the corresponding *indirect* general equilibrium effects. Right panel: Analogous figure for the equivalent uniform transfer stimulus.

cuts work mostly by directly stimulating the consumption of the rich. This is not surprising: wealthy households substitute intertemporally, while poor households are close to their borrowing constraint and so do not. Thus, at the low end of the liquid wealth distribution, monetary policy operates largely through indirect effects (Kaplan et al., 2018).[14] On the other hand, the equivalent lump-sum transfer policy – which by construction has zero net present value – barely moves consumption of the rich. At the same time, it relaxes borrowing constraints of the poor, and so directly stimulates their consumption, with this direct effect actually dwarfing indirect general equilibrium effects.

My results suggest that, relative to a given interest rate cut, stimulus checks that deliver the exact same aggregate stabilization do so at strictly smaller cross-sectional consumption dispersion. In Figure 4, monetary stimulus – once all general equilibrium effects are taken into account – is roughly distributionally neutral, while transfer stimulus leads to a substantial compression of cross-sectional consumption inequality. As is well-known, these *absolute*

[14]There is, of course, a second effect: lower interest rates redistribute from bondholders (rich households) to the government. Since taxes and transfers are uniform, the lower interest rate expenses are then passed on to all households. These redistributive effects, however, only matter at longer horizons; in the short term, consumption of rich households increases because of intertemporal substitution, and lower government interest rate expenses are largely absorbed by a reduction in outstanding debt (see (19)).

responses depend sensitively on many model details, including the stickiness of prices, the cyclicality of dividend payments, and the incidence of short-run fluctuations in labor demand.[15] In contrast, my conclusions on *relative* effects are likely to go beyond the particular HANK model considered here: transfer stimulus is always likely to have first-round effects concentrated on the (borrowing-constrained) poor, while monetary policy – acting through intertemporal substitution and (in richer environments) mortgage refinancing and stock market wealth – directly acts through consumption of the asset-rich. I leave a more complete quantification of these differential direct effects to future work, and instead only emphasize here that the *qualitative* pattern in Figure 4 is likely to be robust. McKay & Wolf (2021) build on these positive observations and investigate normative implications for optimal aggregate stabilization policy.

# 5 Extension to investment

The equivalence result extends to an economy with investment. Generally, in such an environment, monetary policy affects real outcomes through shifts in both consumption and investment behavior. I first show that, in this case, transfer stimulus can still perfectly replicate the *consumption channel* of monetary transmission. I then proceed to discuss other (fiscal) instruments that allow the policymaker to also replicate the *investment channel* and so again circumvent any constraints on nominal interest rates, as in Corollary 1.

## 5.1 Extended model

I augment the model of Section 2.1 to allow for productive capital. The firm block in this extended environment closely follows the tradition of standard business-cycle modeling (e.g. Smets & Wouters, 2007; Justiniano et al., 2010). I provide a brief sketch of this familiar model here, and relegate further details to Appendix A.4.

PRODUCTION. A unit continuum of identical, perfectly competitive firms $j \in [0, 1]$ produces a homogeneous intermediate good, sold at real relative price $p_t^I$. The problem of firm $j$ along

[15] Consistent with these theoretical ambiguities, empirical work has also not reached any definitive conclusions on the implications of monetary stimulus for inequality: in some studies inequality goes down (Coibion et al., 2017), while in others it increases (Andersen et al., 2021).

the perfect foresight transition path is to

$$\max_{\{d_{jt},\ell_{jt},k_{jt},b^f_{jt}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty}\left(\prod_{q=0}^{t-1}\frac{1+\pi_{q-1}}{1+i_{b,q}}\right)e^{\zeta_t}d_{jt}$$

subject to the flow budget constraint

$$d_{jt} = p^I_t y(\ell_{jt},k_{jt-1}) - w_t\ell_{jt} - \underbrace{[k_{jt}-(1-\delta)k_{jt-1}]}_{\equiv i_{jt}} + \tau_{f,t}(\{i_{jt-q}\}_{q=0}^{t}) - b^f_{jt} + \frac{1+i_{b,t-1}}{1+\pi_t}b^f_{jt-1} \quad (24)$$

as well as constraints on equity and debt issuance

$$d_{jt} \geq \underline{d}, \quad b^f_{jt} \geq \underline{b}^f$$

To summarize, intermediate goods producers hire labor on spot markets, invest, pay out dividends, and save in liquid bonds, perhaps subject to financing constraints.[16] My only twist to this entirely familiar model block is that I allow for a general fiscal investment stimulus policy $\boldsymbol{\tau}_f(\bullet)$, mapping investment today into future payments to the firm. Importantly, this general set-up nests the popular bonus depreciation stimulus policy, in which investment today reduces tax liabilities in the future (Zwick & Mahon, 2017; Koby & Wolf, 2020). I will index time-$t$ investment stimulus policies by a single parameter $\tau_{f,t}$.

Proceeding exactly as in Section 2.2, we can define an aggregate investment function $\mathcal{I}(\bullet)$; I relegate a discussion of the arguments of this function to Appendix A.4, as it is not essential here. We can then as before consider a fiscal stimulus stimulus map

$$\mathcal{I}_{\tau_f} \quad \equiv \quad \frac{\partial\mathcal{I}(\bullet)}{\partial\boldsymbol{\tau}_f}$$

Statements about the degree to which an ELB on nominal interest rates can be circumvented will again be statements about the properties of $\mathcal{I}_{\tau_f}$ (and $\mathcal{C}_\tau$, as before).

Rest of the economy. The intermediate good is sold to monopolistically competitive retailers subject to nominal rigidities, summarized again with a general price-NKPC:

$$\pi_t \quad = \quad \kappa_p \times \left(\frac{1}{\mu_p^{-1}p^I_t}-1\right) + \beta\pi_{t+1} \quad (25)$$

[16] Capital or investment adjustment costs could be added without affecting any of the subsequent results.

The remainder of the model is unchanged. The extension of the equilibrium definition in Definition 3 is then straightforward, and provided in Appendix A.4.

## 5.2 Policy equivalence

In this extended model, monetary policy operates through two channels: first, as before, it affects household consumption demand, and second, it changes firm investment and so labor hiring as well as intermediate goods production. Thus, transfer stimulus policy alone is now insufficient to replicate the effects of (infeasible) conventional monetary policy – exactly as in Correia et al. (2013), an additional instrument is needed. In a straightforward generalization of Theorem 1 and Corollary 1, Theorem 3 shows that invertibility of $\mathcal{C}_\tau$ and $\mathcal{I}_{\tau_f}$ is sufficient to leave the space of implementable output-inflation allocations unchanged.

**Theorem 3.** *Consider the extended model of Section 5.1. Suppose that $\mathcal{C}_\tau$ and $\mathcal{I}_{\tau_f}$ are both invertible, and that conventional interest rate policy is subject to an ELB constraint* (18)*. If the allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$ is implementable without the ELB constraint* (18)*, then it is also implementable with a binding ELB constraint.*

Transfer stimulus now only replicates the consumption channel of monetary policy transmission. If additionally $\mathcal{I}_{\tau_f}$ is invertible, then the general form of investment stimulus considered in (24) suffices to replicate the investment channel, and so leave the set of implementable aggregate allocations unchanged.[17] The remainder of this section gives examples of equivalent investment policies under different possible assumptions on production side primitives.

Examples. Suppose first that intermediate goods firms are not subject to any financial frictions, as in most conventional business-cycle models. Then bonus depreciation is equivalent to a standard investment subsidy (e.g. see the discussion in Winberry, 2021), and so the firm budget constraint becomes[18]

$$d_{jt} = p_t^I y(\ell_{jt}, k_{jt}) - w_t \ell_{jt} - \underbrace{[k_{jt} - (1-\delta)k_{jt-1}]}_{\equiv i_{jt}} + \tau_{f,t} i_t$$

[17] Unlike the case studied in Theorem 1, the space of allocations implementable via equivalent fiscal stimulus is now strictly larger than the space implementable via conventional monetary policy: with its two separate levers, fiscal policy can differentially move the investment and consumption channels of net excess demand.

[18] Firm bond holdings are now indeterminate, so I set them to zero at all times.

Given this equivalence of bonus depreciation and investment subsidies, the argument of Correia et al. (2013) applies unchanged: interest rates $i_{b,t}$ and the subsidy $\tau_{f,t}$ enter investment optimality conditions symmetrically, so the investment channel of monetary policy can be replicated by matching wedges. Combining this observation with the results of Section 4 we can conclude that, in a medium-scale HANK model with investment, an entirely conventional mix of fiscal instruments – transfer stimulus payments together with bonus depreciation tax stimulus – suffice to replicate any monetary allocation.

Second, with binding financial frictions, investment responds even to unconditional cash transfers to firms, i.e., to a subsidy schedule $\tau_{f,t}(\{i_{jt-q}\}_{q=0}^{t})$ that is independent of the level of investment. Analogously to my earlier discussion of Ricardian equivalence, this violation of Modigliani-Miller may in fact result in an invertible matrix $\mathcal{I}_{\tau_f}$, thus again ensuring policy equivalence. Contrary to $\mathcal{C}_\tau$, however, empirical evidence on entries of $\mathcal{I}_{\tau_f}$ is more limited, and so I leave a discussion of the practical implications of this observation to future work.[19]

# 6 Conclusion

In recent years, the policy space for conventional interest rate-based stimulus has narrowed.

In this paper I have shown that, in a standard business-cycle model with nominal rigidities and liquidity constraints, uniform lump-sum transfers to households are enough to circumvent such constraints and leave the space of implementable aggregate allocations unchanged. The core insight is a formalization of the notion that nominal interest rate and stimulus check policies can manipulate consumer demand "equally flexibly." This model property – together with a net excess demand approach to equilibrium characterization à la Arrow & Debreu – establishes policy equivalence. My second main result is an explicit characterization of the transfer policy needed to replicate any given (but infeasible) monetary easing; in particular I show that, even in a relatively rich quantitative HANK model, a small number of already measured statistics suffice to fully characterize the mapping to the equivalent transfer policy. Finally I have argued that, while macro-equivalent, interest rate and transfer stimulus policies have very different implications in the household cross-section, with transfers delivering a given aggregate stimulus at lower cross-sectional consumption dispersion.

I leave several important extensions for future work. First, it would be interesting to compare interest rate and stimulus policies in a full Ramsey problem. Steps in this direction

[19] Examples of papers estimating parts of this $\mathcal{I}_{\tau_f}$ include Melcangi (2020) and Lian & Ma (2021).

are taken in McKay & Wolf (2021). Second, to the extent that the two linear maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ change over the business cycle, equivalent transfer policies will also depend on the aggregate state of the economy. Future empirical work should try to better measure that state dependence. And third, the general equivalence logic presented here applies not only to uniform transfer stimulus; for example, analogous equivalence results could also be established for policies that redistribute across households or through transfers targeted at sub-populations. If (large) swings in the amount of outstanding government debt are intrinsically undesirable, then such alternative equivalent, budget-neutral policies become attractive.

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# A Model details

This appendix contains supplementary model details. I begin in Appendix A.1 by completing the specification of the model of Section 2. Appendix A.2 then provides some additional discussion on the aggregate consumption function $\mathcal{C}(\bullet)$. Appendix A.3 presents the particular quantitative HANK model used for all simulations in Section 4, and finally, in Appendix A.4, I provide the missing details and equilibrium definition for the extended model with investment considered in Section 5.

## A.1 Rest of the economy

It only remains to derive the price- and wage-NKPCs in (3) and (4). For the price-NKPC (3), let $1-\alpha$ denote the elasticity of output with respect to total labor input at the steady state, $1-\theta_p \in (0,1)$ denote the probability of a price re-set, and $\epsilon_p$ the substitutability between different retail varieties in aggregation to the final good. We can then follow the derivations in Galí (2015) to arrive at the following log-linearized price-NKPC:

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}_b})}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \left(\widehat{w}_t - \alpha\widehat{\ell}_t\right) + \beta\widehat{\pi}_{t+1} \tag{A.1}$$

(A.1) is the log-linearized version of (3).

For the wage-NKPC (4), I consider the same union structure as in Auclert et al. (2018) and Wolf (2020). To derive a wage-NKPC from first principles, I need to specify a union objective function; I assume that unions order aggregate consumption and employment streams according to

$$\sum_{t=0}^{\infty} \beta^t \left\{u(c_t) - v(\ell_t)\right\} \tag{A.2}$$

We can then follow the same steps as in Erceg et al. (2000) to arrive at a standard log-linearized wage-NKPC:

$$\widehat{\pi}^w_t = \kappa_w \times \left[v''(\bar{\ell})\bar{\ell}\widehat{\ell}_t - \bar{w}u'(\bar{c})(\widehat{w}_t + \frac{u''(\bar{c})}{u'(\bar{c})}\bar{c}\widehat{c}_t)\right] + \beta\widehat{\pi}^w_{t+1} \tag{A.3}$$

where $\kappa_w$ is a function of model primitives, satisfying

$$\kappa_w = \frac{(1-\frac{1}{1+\bar{r}_b}\phi_w)(1-\phi_w)}{\phi_w(\varepsilon_w\frac{1}{\varphi}+1)},$$

with $\phi_w$ indicating the degree of wage stickiness, and $\varepsilon_w$ indicating the elasticity of substitution between different types of labor. As in the price-NKPC case, (A.3) is the log-linearized version of (4).

Note that, if unions instead maximized an equal-weighted average of household utility (as in Auclert et al., 2018),

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 \{u(c_{it}) - v(\ell_t)\} \, di \tag{A.4}$$

then a weighted average of household marginal consumption utilities – rather than marginal consumption utility evaluated at the aggregate consumption level $c_t$ – would enter (A.3). The model would thus be inconsistent with a wage-NKPC of the form (4). However, since (i) average marginal utility and marginal utility at the average rarely differ much, and (ii) these deviations anyway do not matter much with sufficiently sticky wages, the equivalence results presented in this paper are essentially unaffected by such an alternative set-up. I discuss this case further in Appendix B.1.

## A.2 The aggregate consumption function

Since all households supply the same amount of hours worked $\ell_{it} = \ell_t$, and since dividend receipts $d_{it}$ depend only on the productivity type $e$, the path of aggregate consumption $\mathbf{c}$ can be obtained solely as a function of (i) the initial distribution of households over asset holdings $b$ and productivity $e$ and (ii) the input sequences $\{\mathbf{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \mathbf{d}, \boldsymbol{\tau}, \mathbf{i}_b, \boldsymbol{\zeta}\}$. Similar aggregate consumption and asset supply functions are constructed in Auclert et al. (2018) and Aguiar et al. (2021).

I assume that the consumption function $\mathcal{C}(\bullet)$ is well-defined for bounded sequences of inputs. Since $\mathcal{C}(\bullet)$ is required to be consistent with the aggregated household budget constraint

$$c_t + b_t = (1 - \tau_\ell) w_t \ell_t + \frac{1 + i_{t-1}^b}{1 + \pi_t} b_{t-1} + \tau_t + d_t$$

the borrowing constraint $b_t \geq \underline{b}$ and the condition $\lim_{t\to\infty} \left( \prod_{s=0}^{t} \frac{1+\pi_s}{1+i_{b,s-1}} \right) b_t = 0$, it follows that the resulting path $\mathbf{c} = \mathcal{C}(\bullet)$ is also bounded. $\mathcal{C}(\bullet)$ is thus a mapping from sequences in $\ell^\infty$ to $\ell^\infty$, i.e., the vector space of bounded sequences with the supremum norm.

For the definitions in (8) and (9), I assume that the limits

$$\lim_{||\mathbf{h}||\to 0} \frac{||\mathcal{C}\left(\bar{\mathbf{w}}, \bar{\boldsymbol{\ell}}, \bar{\boldsymbol{\pi}}, \bar{\mathbf{d}}; \bar{\boldsymbol{\tau}} + \mathbf{h}, \bar{\mathbf{i}}_b; \mathbf{0}\right) - \bar{\mathbf{c}} - \mathcal{C}_\tau \mathbf{h}||}{||\mathbf{h}||} = 0$$

and

$$\lim_{||\mathbf{h}||\to 0} \frac{||\mathcal{C}\left(\bar{\mathbf{w}}, \bar{\boldsymbol{\ell}}, \bar{\boldsymbol{\pi}}, \bar{\mathbf{d}}; \bar{\boldsymbol{\tau}}, \bar{\mathbf{i}}_b + \mathbf{h}; \mathbf{0}\right) - \bar{\mathbf{c}} - \tilde{\mathcal{C}}_{i_b}\mathbf{h}||}{||\mathbf{h}||} = 0$$

exist, and use them to define the linear maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b} = \tilde{\mathcal{C}}_{i_b} - \bar{b} \times \mathcal{C}_{\tau,\bullet,-1}$. That is, at the deterministic steady state, $\mathcal{C}(\bullet)$ is assumed to be Fréchet-differentiable in the lump-sum transfer and nominal interest rate paths. Under those assumptions, $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ are well-defined linear maps from $\ell^\infty$ to $\ell^\infty$. The inverse of $\mathcal{C}_\tau$, denoted $\mathcal{C}_\tau^{-1}$, is then simply a linear map such that $\mathcal{C}_\tau^{-1}\mathcal{C}_\tau = I$ and $\mathcal{C}_\tau\mathcal{C}_\tau^{-1} = I$ where $I$ is the identity map on $\ell^\infty$.

## A.3 The quantitative HANK model

I first discuss the parameterization of the model's steady state, and so in particular the induced aggregate consumption function $\mathcal{C}(\bullet)$.

The values of all parameters relevant for the model's deterministic steady state are displayed in Table A.1. I assume that household preferences take the particular form

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left\{\frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \chi \frac{\ell_{it}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right\}\right]$$

I slightly enrich the baseline model to allow for exogenous household death at rate $\varrho$. Preference parameters $\{\gamma, \varphi, \varrho\}$ as well as the labor substitutability $\varepsilon_w$ are set to standard values. The average return on (liquid) assets is set in line with standard calibrations of business-cycle models, and the discount rate is then disciplined through the total amount of liquid wealth. As in McKay et al. (2016), I assume that households cannot borrow in the liquid asset. Next, for income risk, I adopt the 33-state specification of Kaplan et al. (2018), ported to discrete time. For share endowments, I assume that

$$d_{it} = \begin{cases} 0 & \text{if } e_{it}^p \leq \underline{e}^p \\ \chi_0(e_{it}^p - \underline{e}^p)^{\chi_1} \times d_t & \text{otherwise} \end{cases}$$

where $e_{it}^p$ is the permanent component of household $i$'s labor productivity. I set the parameters $\{\underline{e}^p, \chi_0, \chi_1\}$ as in Wolf (2020). On the firm side, I assume constant returns to scale in production, and set the substitutability between goods to a standard value. Finally, the average government tax take, transfers, and debt issuance are all set in line with direct empirical evidence. Relative to Definition 3, I slightly generalize the model to allow for non-zero

| Parameter | Description | Value | Target | Value |
|---|---|---|---|---|
| *Households* | | | | |
| $\rho_e$, $\sigma_e$ | Income Risk | - | Kaplan & Violante (2018) | - |
| $\underline{e}^p$, $\chi_0$, $\chi_1$ | Dividend Endowment | - | Illiquid Wealth Shares | - |
| $\beta$ | Discount Rate | 0.97 | B/Y | 1.5 |
| $i_b$ | Average Return | 0.01 | Annual Rate | 0.04 |
| $\varrho$ | Death Rate | 1/180 | Average Age | 45 |
| $\gamma$ | Preference Curvature | 1 | Standard | |
| $\varphi$ | Labor Supply Elasticity | 0.5 | Standard | |
| $\varepsilon_w$ | Labor Substitutability | 10 | Standard | |
| $\underline{b}$ | Borrowing Limit | 0 | McKay et al. (2016) | |
| *Firms* | | | | |
| $1-\alpha$ | Returns to Scale | 1 | Standard | |
| $\varepsilon_p$ | Goods Substitutability | 16.67 | Profit Share | 0.06 |
| *Government* | | | | |
| $\tau_\ell$ | Labor Tax | 0.3 | Average Labor Tax | 0.30 |
| $\tau/Y$ | Transfer Share | 0.05 | Transfer Share | 0.05 |

**Table A.1:** HANK model, steady-state calibration.

government spending, giving the new market-clearing condition

$$y_t = c_t + g_t \tag{A.5}$$

Note that, for all experiments, I keep government expenditure fixed at $g_t = \bar{g}$, so its presence only matters for the steady-state fiscal tax-and-transfer system, and does not directly show up anywhere in equilibrium dynamics.

In the second step I set the remaining model parameters (which exclusively govern dynamics around the deterministic steady state). For the baseline interest rate-only policy considered in Section 4.2, I consider an interest rate rule of the form

$$\widehat{i}_{b,t} = \phi_\pi \widehat{\pi}_t + m_t$$

where $m_t$ is the monetary shock, set to give the gradually decaying path of nominal rates

| Parameter | Description | Value |
|---|---|---|
| $\phi_p$ | Price Calvo Parameter | 0.85 |
| $\phi_w$ | Wage Calvo Parameter | 0.70 |
| $\phi_\pi$ | Taylor Rule Inflation | 1 |
| $\rho_b$ | Financing Rule Persistence | 0.95 |

**Table A.2:** HANK model, parameters governing dynamics.

displayed in Figure 2. Together with the fiscal financing rule (19), this completes the specification of policy for the baseline monetary experiment. I present the rule parameterizations as well as all other model parameters in Table A.2.

## A.4 Adding investment

All firms are identical, so I drop the $j$ subscript. Analogously to the discussion in Section 2.1, we can summarize the solution to the firm problem with an investment demand function,

$$\mathbf{i} = \mathcal{I}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \tag{A.6}$$

a production function,

$$\mathbf{y} = \mathcal{Y}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \tag{A.7}$$

a labor demand function,

$$\boldsymbol{\ell} = \mathcal{L}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \tag{A.8}$$

and a dividend function

$$\mathbf{d} = \mathcal{D}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \tag{A.9}$$

where the dividend function aggregates over both intermediate goods producers and sticky-price retailers. Note that, since intermediate goods firms hire labor on a competitive spot market, and since productivities are equalized across firms $j$, two sequences $\boldsymbol{\zeta}$ and $\boldsymbol{\tau}_f$ that induce the same paths of investment also invariably induce the same paths of output and labor hiring. However, since subsidies enter the budget constraint while the shock $\boldsymbol{\zeta}$ does not, the implied dividend paths may be different.

Given investment subsidies to firms, the government budget constraint is adjusted to give

$$\frac{1+i_{b,t-1}}{1+\pi_t} b_{t-1} + \tau_t + \tau_{f,t}(\{i_{t-q}\}_{q=0}^{t}) = \tau_\ell w_t \ell_t + b_t \tag{A.10}$$

All other parts of the model are unchanged relative to Section 2.1. We thus arrive at the following equilibrium definition:

**Definition A.1.** *Given a path of shocks* $\{\zeta_t\}_{t=0}^{\infty}$*, an equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, \tau_{f,t}, b_t\}_{t=0}^{\infty}$ *and a set of aggregates* $\{c_t, \ell_t, y_t, i_t, k_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ *such that:*

1. *Aggregate consumption is given as*

$$\mathbf{c} = \int_0^1 \mathbf{c}_i di$$

   *where the individual consumption paths* $\mathbf{c}_i$ *solve the household consumption-savings problem of maximizing* (1) *subject to* (2) *and* $b_{it} \geq \underline{b}$.

2. *Aggregate investment, output, hours worked and dividends satisfy*

$$\begin{aligned}
\mathbf{i} &= \mathcal{I}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \\
\mathbf{y} &= \mathcal{Y}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \\
\boldsymbol{\ell} &= \mathcal{L}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right) \\
\mathbf{d} &= \mathcal{D}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b; \boldsymbol{\zeta}\right)
\end{aligned}$$

3. *Wage inflation* $\pi_t^w \equiv \frac{w_t}{w_{t-1}}(1+\pi_t)$ *together with* $\{\ell_t, c_t\}_{t=0}^{\infty}$ *are consistent with the aggregate wage-NKPC* (4).

4. *The paths* $\{\pi_t, w_t, \ell_t\}_{t=0}^{\infty}$ *are consistent with the aggregate price-NKPC* (25).

5. *The output market clears:* $y_t = c_t + i_t$ *for all* $t \geq 0$*, the government budget constraint* (A.10) *holds at all* $t$*, and* $\lim_{t\to\infty} \widehat{b}_t = 0$*. The bond market then clears by Walras' law.*

# B Supplementary theoretical results

This section presents several supplementary theoretical results. I discuss alternative assumptions on wealth effects in labor supply in Appendix B.1, provide an extended sufficient statistics characterization allowing for more general anticipation effects in Appendix B.2, and consider implications of my results for (optimal) policy rules in Appendices B.3 and B.4.

## B.1 Wealth effects in labor supply

Theorem 1 requires homogeneous wealth effects in labor supply in the cross section of households. As discussed in Section 3.1, sufficient conditions for this include: preferences implying the absence of wealth effects (Greenwood et al., 1988); fully rigid wages; or a union bargaining protocol that responds only to *average* consumption, not its distribution (as in (4)). These assumptions receive some empirical support: wealth effects in labor supply – in particular for moderate wealth gains like for example the \$600 in Figure 2 – are generally estimated to be small *and* homogeneous in the household cross section (Cesarini et al., 2017). Furthermore empirical evidence suggests at least partially sticky nominal wages (Grigsby et al., 2019).

To get a sense of the quantitative magnitudes of non-equivalence implied by a violation of these sufficient conditions, I solve a variant of my baseline HANK model in which the wage-NKPC (A.3) is replaced by

$$\widehat{\pi}_t^w = \kappa_w \times \left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma\widehat{c}_t^*)\right] + \frac{1}{1+\bar{r}_b}\widehat{\pi}_{t+1}^w \tag{B.1}$$

where

$$c_t^* \equiv \left[\int_0^1 e_{it} c_{it}^{-\gamma} di\right]^{-\frac{1}{\gamma}}$$

This specification is derived from (A.4), assuming the particular household preferences considered in my calibrated HANK model. Unions bargain based on the average marginal utility of consumption (rather than marginal utility at the average), so distributional incidence matters for equilibrium wage dynamics, violating the conditions of Theorem 1. This specification is exactly the same as that in Auclert et al. (2018); as discussed there, it is consistent with small wealth effects, but does not impose their cross-sectional homogeneity.

My main finding is that, even in this extended model, equivalence still holds *nearly* exactly, as illustrated in Figure B.1. The figure shows the path of aggregate consumption in response to the two stimulus policies plotted in Figure 2. Since exact equivalence fails,

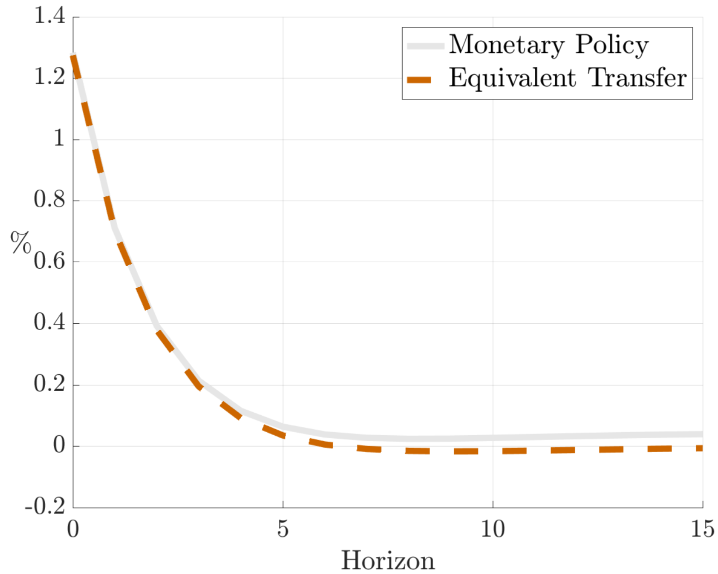


**Figure B.1:** See the caption of Figure 2, for the bottom left panel (aggregate consumption). The only difference is that now I consider a variant of my quantitative HANK model with the wage-NKPC (B.1).

the two lines now are not on top of each other, unlike in the lower left panel of Figure 2. However, they remain nearly identical. This result is reminiscent of the discussion in Wolf (2020) where I show that, even with moderately sticky wages, short-run wealth effects in labor supply do not matter much for aggregate general equilibrium dynamics. The result here in fact is even stronger, since departures from exact equivalence are not driven by the presence of wealth effects *per se*, but are related to their *heterogeneity* in the cross section of households.

## B.2 Extended sufficient statistics formula

I provide an extension of the sufficient statistics formula in Proposition 1 to an environment with household cognitive discounting, as in Gabaix (2020). I summarize the informational effects of cognitive discounting in the linear map $\mathcal{E}$, given as

$$\mathcal{E} = \begin{pmatrix} 1 & \psi & \psi^2 & \dots \\ 1 & 1 & \psi & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where the $(i,j)$th entry of $\mathcal{E}$ indicates the fraction of households at time $i$ that are aware of transfers paid out at time $j$. In words, all households are aware of transfers after they have been paid out, and they anticipate transfers at a constant rate $\psi$. Given this informational structure, we can proceed as in Auclert et al. (2019) to recover the adjusted consumption derivative matrix $\mathcal{C}_\tau^\dagger$ from the full-information analogue $\mathcal{C}_\tau$ as

$$\mathcal{C}_\tau^\dagger(t,s) = \sum_{q=1}^{\min(t,s)} (\mathcal{E}_{q,s} - \mathcal{E}_{q-1,s})\mathcal{C}_\tau(t-q+1, s-q+1)$$

In particular, for the analytical hybrid model of Proposition 1, we find that

$$\mathcal{C}_\tau^{\dagger,TB} \approx \omega \times \begin{pmatrix} 1 & \xi\theta\psi & \xi(\theta\psi)^2 & \xi(\theta\psi)^3 & \dots \\ \xi\theta & 1 & \xi\theta\psi & \xi(\theta\psi)^2 & \dots \\ \xi\theta^2 & \xi\theta & 1 & \xi\theta\psi & \dots \\ \xi\theta^3 & \xi\theta^2 & \xi\theta & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

The expression is intuitive: below the main diagonal everything is as in Proposition 1, while above the main diagonal there is constant discounting at $\psi$. We now find the adjusted sufficient statistics formula

$$\begin{aligned} (\mathcal{C}_\tau^{\dagger,TB})^{-1}(i,j) &\approx \frac{1}{\omega}\frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-ik\lambda}}{(1-\xi)+\xi\frac{1-\theta^2\psi}{(1-\theta\psi e^{-i\lambda})(1-\theta e^{i\lambda})}} d\lambda \\ &\approx \frac{1}{\omega}\frac{1}{2\pi}\int_0^{2\pi} e^{-ik\lambda}\frac{(1-\theta\psi e^{-i\lambda})(1-\theta e^{i\lambda})}{\xi(1-\theta^2\psi)+(1-\xi)(1-\theta\psi e^{-i\lambda})(1-\theta e^{i\lambda})} \end{aligned}$$

Note that, for fully myopic households ($\psi = 0$), the expression simplifies to

$$(\mathcal{C}_\tau^{\dagger,TB})^{-1} = \frac{1}{\omega} \times \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ -\xi\theta & 1 & 0 & 0 & \dots \\ -\xi\theta^2(1-\xi) & -\xi\theta & 1 & 0 & \dots \\ -\xi\theta^3(1-\xi)^2 & -\xi\theta^2(1-\xi) & -\xi\theta & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

## B.3 Equivalence mapping between policy rules

The equivalence results apply without change to policy *rules*. If, for example, the economy is closed with a conventional Taylor-type interest rate-only rule of the form

$$\widehat{\boldsymbol{i}}_b = \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} \tag{B.2}$$

$$\widehat{\boldsymbol{\tau}} = \tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) - \bar{b}\widehat{\boldsymbol{i}}_{b,-1} + (1+\bar{i}_b)\bar{b}\widehat{\boldsymbol{\pi}}, \tag{B.3}$$

then an alternative transfer-only rule of the form

$$\widehat{\boldsymbol{\tau}} = \tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) + (1+\bar{i}_b)\bar{b}\widehat{\boldsymbol{\pi}} + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}\right) \tag{B.4}$$

$$\widehat{\boldsymbol{i}}_b = 0 \tag{B.5}$$

implies the exact same sequence of macroeconomic aggregates $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$.

Note that this result in particular also applies to the equivalent linearized economy with aggregate risk. In that economy, whatever the menu of (non-policy) shocks, the two policy rules (B.2) - (B.3) and (B.4) - (B.5) induce exactly the same realizations of macroeconomic aggregates at all points in time, for all states of the world. It follows that the second-moment properties of all macroeconomic aggregates – including in particular output and inflation – are the same under the two policy regimes.

## B.4 Optimal policy

Most standard treatments of optimal policymaking over the business cycle (e.g. Galí, 2015; Woodford, 2011) consider policymaker loss functions of the form

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{\lambda_\pi \widehat{\pi}_t + \lambda_y \widehat{y}_t\right\} \tag{B.6}$$

It follows immediately from Theorem 1 that, for any policymaker with a loss function like (B.6), an ELB on nominal rates has no effect on the attainable loss: any loss attainable with unconstrained interest rate-only policy is also attainable with transfer-only policy, and thus in particular attainable at the ELB. In particular, standard optimal implicit targeting rules of the form (see Woodford, 2011)

$$\widehat{\pi}_t + \frac{\lambda_y}{\lambda_\pi \kappa_p} \times (\widehat{y}_t - \widehat{y}_{t-1}) = 0$$

are uniquely implementable using either interest rate-only or transfer-only policies. It follows that, for such a policymaker, the optimal mix of nominal interest rate and transfer policies is in fact indeterminate (except at the ELB).

However, as discussed in Section 4.3, interest rate and transfer policy generally differ at the micro level, in the cross section of households. It follows that, for richer policymaker loss functions, the two instruments are not equivalent. Previous work on optimal monetary policy in models with household heterogeneity has emphasized the insurance benefits of general equilibrium stabilization of wage income (e.g. Bhandari et al., 2018). My analysis reveals that the equivalent transfer policy would have the exact same general equilibrium insurance benefits, but provide even further insurance through its *direct* effect on consumption expenditures. A detailed characterization of the implied optimal mix of interest rate and transfer policies is left to future work (McKay & Wolf, 2021).

# C $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ in simple models

This section provides further details on my characterizations of the consumption derivative matrices $\mathcal{C}_\tau$ (and also $\mathcal{C}_{i_b}$) in simple analytically tractable models of the consumption-savings problem. I begin in Appendix C.1 with a review of the necessary mathematical background. Appendices C.2 to C.4 then build up to the expressions in Proposition 1.

## C.1 Mathematical background

A Toeplitz matrix is an $n \times n$ matrix of the form

$$T_n = \begin{pmatrix} t_0 & t_{-1} & t_{-2} & \dots & t_{-(n-1)} \\ t_1 & t_0 & t_{-1} & & \\ t_2 & t_1 & t_0 & & \vdots \\ \vdots & & & \ddots & \\ t_{n-1} & & & \dots & t_0 \end{pmatrix} \tag{C.1}$$

My results on eigenvalue distributions and inverses leverage well-known theorems on the limiting behavior of Toeplitz matrices induced by an infinite sequence $\{t_k\}_{k=-\infty}^{\infty}$, with the sequence restricted to be absolutely summable.[20] I briefly review those results here, with a detailed discussion provided for example in Gray (2006). Central to all expressions is the Fourier series with coefficients $t_k$:

$$f(\lambda) \equiv \sum_{k=-\infty}^{\infty} t_k e^{ik\lambda} \tag{C.2}$$

The inverse mapping is given as

$$t_k = \frac{1}{2\pi} \int_0^{2\pi} f(\lambda) e^{-ik\lambda} d\lambda \tag{C.3}$$

and I will use the notation $T_n(f)$ to indicate a matrix with Fourier series $f$.

[20]Formally, all results stated below require the Toeplitz matrix to fall into the Wiener class, which is easily verified for all matrices considered here.

Asymptotic equivalence. I begin by defining the notion of asymptotic equivalence of matrices used throughout this paper.

**Definition C.1.** *Two sequences of $n \times n$ matrices $\{A_n\}$ and $\{B_n\}$ are said to be asymptotically equivalent if*

*1. $A_n$ and $B_n$ are uniformly bounded in strong norm:*

$$||A_n||, ||B_n|| \leq M < \infty, \quad n = 1, 2, \ldots$$

*2. $D_n \equiv A_n - B_n$ goes to zero in weak norm as $n \to \infty$:*

$$\lim_{n \to \infty} |A_n - B_n| = \lim_{n \to \infty} |D_n| = 0$$

Here the strong (or operator) norm is defined as

$$||A|| = \max_x \left( \frac{x^*(A^*A)x}{x^*x} \right)^{\frac{1}{2}}$$

where $*$ denotes conjugate transpose, while the weak norm (or Hilbert-Schmidt norm) is defined as

$$|A| = \left( \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{n} |a_{k,j}|^2 \right)^{\frac{1}{2}}$$

Eigenvalue distribution. Let $\tau_{n,k}$ denote the $k$-th eigenvalue of a given $n$-dimensional Toeplitz matrix, and define the eigenvalue distribution function $D_n(x) = \frac{\sum_{k=1}^{n} \mathbf{1}_{\tau_{n,k} \leq x}}{n}$. Then

$$D(x) = \lim_{n \to \infty} D_n(x) = \frac{1}{2\pi} \int_{f(\lambda) \leq x} d\lambda \tag{C.4}$$

This general result is known as the fundamental eigenvalue distribution theorem of Szegö.

Inverse. If $f(\lambda) > 0$ for all $\lambda \in [-\pi, \pi]$ then the Toeplitz matrix $T_n(f)$ is invertible, and its inverse is asymptotically equivalent – in the sense of Definition C.1 – to a Toeplitz matrix with sequence $\{t_k^{-1}\}_{k=-\infty}^{\infty}$ where

$$t_k^{-1} = \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-ik\lambda}}{f(\lambda)} d\lambda \tag{C.5}$$

## C.2 $\mathcal{C}_{i_b}$ for Ricardian households

In Section 3.1 I claimed that $\mathcal{C}_{i_b}^R$ is such that interest rate policy can engineer any consumption path $\widehat{\mathbf{c}}^{PE}$ with zero net present value. I here provide the missing details for that argument.

Recall that

$$\mathcal{C}_{i_b}^R = -\frac{1}{\gamma} \times (I - \mathcal{C}_\tau^R) \times \mathcal{U}$$

where $\mathcal{U}$ is shorthand for the upper-triangular matrix in (10). Now let $\widehat{\mathbf{c}}^{PE}$ be any consumption path with zero net present value, and consider setting

$$\widehat{\mathbf{i}}_b = (-\frac{1}{\gamma} \times \mathcal{U})^{-1} \times \widehat{\mathbf{c}}^{PE}$$

where I have used the fact that $\mathcal{U}$ is invertible. But then we have

$$\mathcal{C}_{i_b}^R \times \widehat{\mathbf{i}}_b = \widehat{\mathbf{c}}^{PE} - \mathcal{C}_\tau^R \times \widehat{\mathbf{c}}^{PE} = \widehat{\mathbf{c}}^{PE}$$

where the last equality follows because by Ricardian equivalence together with the fact that, by assumption, $\widehat{\mathbf{c}}^{PE}$ has zero net present value, so $\mathcal{C}_\tau^R \times \widehat{\mathbf{c}}^{PE} = \mathbf{0}$.

## C.3 Bonds in the utility function

This section derives the linear maps $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$ for a consumption-savings problem of a representative household with bonds in the utility function, denoted $\mathcal{C}_\tau^B$ and $\mathcal{C}_{i_b}^B$.

Transfer response $\mathcal{C}_\tau$. To characterize the model-implied $\mathcal{C}_\tau$ matrix, I consider the problem of maximizing

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} \right\} \tag{C.6}$$

subject to the budget constraint

$$c_t + b_t = y_t + (1 + \bar{r}_b) b_{t-1} \tag{C.7}$$

where $y_t$ denotes an exogenous stream of income, and $\bar{r}_b$ is the steady-state real rate of interest. Linearizing around a deterministic steady state with $y_t = \bar{y}$, $b_t = \bar{b}$ and $c_t = \bar{c}$ for all $t$, we find that the consumption and saving paths are characterized by the following system of equations:

$$\begin{aligned} \bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t - (1+\bar{r}_b)\bar{b}\widehat{b}_{t-1} &= \bar{y}\widehat{y}_t \\ \widehat{c}_t - \beta(1+\bar{r}_b)\widehat{c}_{t+1} - \frac{\eta}{\gamma}(1-\beta(1+\bar{r}_b))\widehat{b}_t &= 0 \end{aligned}$$

Now let $\widehat{c}_{t,H} \times \widehat{y}_H$ denote the impulse response of consumption at time $t$ to an income change $\widehat{y}_H$ at time $H$, so that $\frac{\bar{c}}{\bar{y}} \times \widehat{\mathbf{c}}_H$ is the $H$th column of $\mathcal{C}_\tau$. We arrive at the following result:

**Proposition C.1.** *Consider the consumption-savings problem (C.6) of a household with bonds in the utility function, and suppose that $\beta = 1$. Then, for any $\ell > 0$ the impulse response path $\widehat{\mathbf{c}}_H$ to an income shock at time $H$ satisfies*

$$\lim_{H\to\infty} \widehat{c}_{H,H} = \frac{\bar{y}}{\bar{c}} \times \omega, \quad \lim_{H\to\infty} \frac{\widehat{c}_{H+\ell,H}}{\widehat{c}_{H+\ell-1,H}} = \lim_{H\to\infty} \frac{\widehat{c}_{H-\ell-1,H}}{\widehat{c}_{H-\ell,H}} = \theta \tag{C.8}$$

*where the coefficients $\{\omega, \theta\}$ are functions of model primitives.*

Proposition C.1 states that, at least asymptotically, iMPCs for an income shock at time $H$ increase at a geometric rate $\theta$ up to time $H$, equal some time-invariant constant $\omega$ at $H$, and decrease at that same geometric rate $\theta$ after time $H$.[21] The resulting asymptotic version of $\mathcal{C}_\tau^B$ is evidently a particular Toeplitz matrix, with

$$f(\lambda) = \omega \sum_{k=-\infty}^{\infty} \theta^{|k|} e^{ik\lambda} = \omega \frac{1-\theta^2}{1-2\theta\cos(\lambda)+\theta^2} \tag{C.9}$$

Interest rate response $\mathcal{C}_{i_b}$. With a time-varying rate of interest and transfers set as in (6) to offset income effects, the solution to the consumption-savings problem is characterized by the pair of equations

$$\begin{aligned} \bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t - (1+\bar{r}_b)\bar{b}\widehat{b}_{t-1} &= \bar{y}\widehat{y}_t \\ \widehat{c}_t - \beta(1+\bar{r}_b)\widehat{c}_{t+1} - \frac{\eta}{\gamma}(1-\beta(1+\bar{r}_b))\widehat{b}_t &= -\frac{1}{\gamma}\beta(1+\bar{r}_b)\widehat{r}_t \end{aligned}$$

Note that interest rate changes have no income effects here, and purely act through intertemporal substitution. I will first of all consider time-0 income and interest rate shocks with

[21] For $\beta < 1$, iMPCs before time $H$ increase at a rate $\frac{1}{\beta\theta}$.

impulse responses $\widehat{\mathbf{c}}_y$ and $\widehat{\mathbf{c}}_r$, respectively, and show that

$$\widehat{\mathbf{c}}_r = -\frac{1}{\gamma}\beta(1+\bar{r}_b)(e_1 - \frac{\bar{c}}{\bar{y}}\widehat{\mathbf{c}}_y)$$

where $e_1 = (1,0,0,\dots)'$. To see this, let

$$\widehat{c}_t^* = \widehat{c}_t - \frac{\bar{y}}{\bar{c}}\widehat{y}_t$$

We can then re-write the system for income shocks as

$$\begin{aligned} \bar{c}\widehat{c}_t^* + \bar{b}\widehat{b}_t - (1+\bar{r}_b)\bar{b}\widehat{b}_{t-1} &= 0 \\ (\widehat{c}_t^* + \frac{\bar{y}}{\bar{c}}\widehat{y}_t) - \beta(1+\bar{r}_b)(\widehat{c}_{t+1}^* + \frac{\bar{y}}{\bar{c}}\widehat{y}_{t+1}) - \frac{\eta}{\gamma}(1-\beta(1+\bar{r}_b))\widehat{b}_t &= 0 \end{aligned}$$

Since $\widehat{y}_{t+1} = 0$ this is identical to a system with a time-0 interest rate wedge of $\frac{\bar{y}}{\bar{c}}\frac{\gamma}{\beta(1+\bar{r}_b)}\widehat{y}_t$, so we have

$$\widehat{\mathbf{c}}_y^* = \frac{\bar{y}}{\bar{c}}\frac{\gamma}{\beta(1+\bar{r}_b)}\widehat{\mathbf{c}}_r$$

Re-arranging this:

$$\widehat{\mathbf{c}}_r = \frac{1}{\gamma}\beta(1+\bar{r}_b)\frac{\bar{c}}{\bar{y}}\widehat{\mathbf{c}}_y^*$$

But by construction

$$\widehat{\mathbf{c}}_y^* = \widehat{\mathbf{c}}_y - \frac{\bar{y}}{\bar{c}}e_1$$

Translating $\widehat{\mathbf{c}}_y$ to MPC space (i.e., the first column of $\mathcal{C}_\tau$), the claim follows. Proceeding similarly for future interest rate changes, we find that

$$\mathcal{C}_{i_b}^B = -\frac{1}{\gamma}(I - \mathcal{C}_\tau^B)\begin{pmatrix} [\beta(1+\bar{r}_b)] & [\beta(1+\bar{r}_b)]^2 & [\beta(1+\bar{r}_b)]^3 & \dots \\ 0 & [\beta(1+\bar{r}_b)] & [\beta(1+\bar{r}_b)]^2 & \dots \\ 0 & 0 & [\beta(1+\bar{r}_b)] & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

In particular, for the special case $\beta(1+\bar{r}_b) = 1$, this becomes

$$\mathcal{C}_{i_b}^B = -\frac{1}{\gamma}(I - \mathcal{C}_\tau^B)\begin{pmatrix} 1 & 1 & 1 & \dots \\ 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

## C.4 Hybrid model

With a fringe $\mu$ of spenders, we instead get the asymptotic transfer consumption derivative matrix in (21). This matrix is again a Toeplitz matrix, now with

$$f(\lambda) = \omega\left\{\xi\frac{1-\theta^2}{1-2\theta\cos(\lambda)+\theta^2} + (1-\xi)\right\} \tag{C.10}$$

Combining the previous conclusions for the spender-saver and bond-in-utility models, we furthermore get that

$$\mathcal{C}^{TB}_{i_b} = -\frac{1}{\gamma}(I - \mathcal{C}^{TB}_{\tau})\begin{pmatrix} 1 & 1 & 1 & \dots \\ 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.11}$$

Figure C.1 illustrates Proposition C.1 in the context of the empirically relevant hybrid model. For large enough horizons $H$, the matrix columns $\mathcal{C}_{\tau,\bullet,H-\ell}$, $\mathcal{C}_{\tau,\bullet,H}$ and $\mathcal{C}_{\tau,\bullet,H+\ell}$ are simply left- and right-translations of each other, with a common MPC $\omega$ and constant decay rates away from the main diagonal. The grey line reveals that, even for impact transfer receipt, the approximation in (21) is quite accurate.

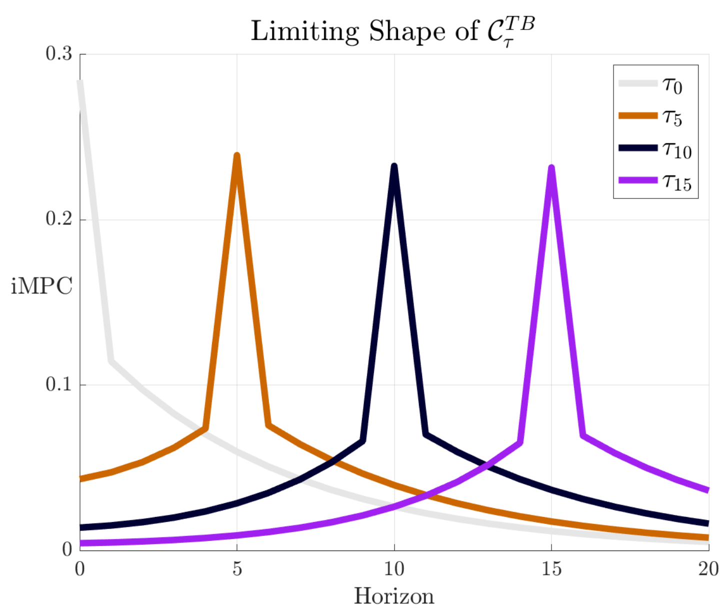


**Figure C.1:** Entries of the consumption derivative matrix for the calibrated hybrid model, for transfer receipts at time periods $\{0, 5, 10, 15\}$.

From interest rate space to transfer space. (C.11) implies that, at least in this simple two-type model, the mapping from interest rate space to transfer space – rather than from net excess demand space to transfer space, as in my main analysis – is fully characterized by (i) $\mathcal{C}_\tau$ and (ii) the inverse elasticity of substitution $\gamma$, as claimed in Footnote 10. The intuition is straightforward: as in Werning (2016), the presence of borrowing-constrained households simply one-to-one weakens intertemporal substitution effects.

In my quantitative HANK model, $\mathcal{C}_{i_b}$ is further shaped by uneven distributional incidence: an interest rate cut lowers government debt payments and so (ultimately) increases transfers, redistributing from the asset-rich to the poor. In practice, $\mathcal{C}_{i_b}$ is also likely to reflect income effects associated with mortgage refinancing (see Di Maggio et al., 2017). In its focus on $\mathcal{C}_\tau^{-1}$, my analysis is robust to all such extensions – whatever the demand implications of the interest rate policy, they can be mimicked in transfer space as long as $\mathcal{C}_\tau$ is invertible.

# D Proofs and auxiliary lemmas

## D.1 Proof of Theorem 1

Linearizing the government budget constraint (5), we find

$$\bar{b}\widehat{i}_{b,t-1} - (1+\bar{i}_b)\bar{b}\widehat{\pi}_t + (1+\bar{i}_b)\widehat{b}_{t-1} + \widehat{\tau}_t = \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + \widehat{b}_t \tag{D.1}$$

Using (D.1), I decompose total transfers into two parts: an endogenous "general equilibrium" component related to labor tax revenue and inflation debt servicing costs,

$$\widehat{\tau}^e_t \equiv \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{i}_b)\bar{b}\widehat{\pi}_t \tag{D.2}$$

and an exogenous "policy" component

$$\widehat{\tau}^x_t \equiv \widehat{\tau}_t - \widehat{\tau}^e_t \tag{D.3}$$

I now present a constructive proof of Theorem 1: leveraging (12) I will show how to construct a transfer-only policy replicating any interest rate-only policy, and vice-versa. The decomposition in (D.2) and (D.3) will prove useful in this constructive proof. Also note that the proofs use the map $\tilde{\mathcal{C}}_{i_b}$ defined in Appendix A.2.

1. An interest rate-only policy is a tuple $\{i_{b,t}, \tau_t\}_{t=0}^{\infty}$ with

   $$\widehat{\tau}_t = \widehat{\tau}^e_t - \bar{b}\widehat{i}_{b,t-1}$$

   so that $\widehat{\tau}^x_t = -\bar{b}\widehat{i}_{b,t-1}$. By (12), there exists a path of transfers $\{\tau^*_t\}_{t=0}^{\infty}$ such that

   $$\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^* = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b = \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b + \mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x \tag{D.4}$$

   Since the interest rate-only policy by construction has zero net present value, it follows that the transfer-only policy $\widehat{\boldsymbol{\tau}}^*$ – which induces the exact same (zero-NPV) consumption sequence – also has zero NPV:

   $$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{i}_b}\right)^t \widehat{\tau}^*_t = 0 \tag{D.5}$$

   Now consider the transfer-only policy tuple $\{\bar{i}_b, \bar{\tau} + \widehat{\tau}^e_t + \widehat{\tau}^*_t\}_{t=0}^{\infty}$. I will verify that, at the initial $\{\zeta_t\}_{t=0}^{\infty}$ and $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$, all markets still clear and all agents still behave optimally. First, by (D.5) and the definition of $\widehat{\tau}_t^e$, we still have that $\lim_{t\to\infty} \widehat{b}_t = 0$. Second, by construction of $\widehat{\tau}_t^*$ in (D.4), the path $\widehat{\mathbf{c}}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; \bar{i}_b, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*; \zeta_t\}_{t=0}^{\infty}$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

2. A transfer-only policy is a tuple $\{\bar{i}_b, \tau_t\}_{t=0}^{\infty}$ with

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{i}_b}\right)^t \widehat{\tau}_t^x = 0$$

By (12), there exists a path of interest rates $\{i_{b,t}^*\}_{t=0}^{\infty}$ with $\widehat{\tau}_t^* = -\bar{b}\widehat{i}_{b,t-1}^*$ such that

$$\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b^* = \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b^* + \mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^* \tag{D.6}$$

Since the transfer-only policy has zero net present value, it follows that the interest rate-only policy $\{\widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$ – which induces the exact same (zero-NPV) consumption sequence – also has zero NPV:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{i}_b}\right)^t \widehat{\tau}_t^* + \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{i}_b}\right)^t \bar{b}\widehat{i}_{b,t-1}^* = 0 \tag{D.7}$$

Now consider the interest rate-only policy tuple $\{i_{b,t}^*, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. As before I will verify that, at the initial $\{\zeta_t\}_{t=0}^{\infty}$ and $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$, all markets still clear and all agents still behave optimally. First, by (D.7) and the definition of $\widehat{\tau}_t^e$, we have that $\widehat{b}_t = 0$ for all $t$, so indeed the policy is a valid interest rate-only policy. Second, by construction of $\widehat{i}_{b,t}^*$ in (D.6), the path $\widehat{\mathbf{c}}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; i_{b,t}^*, \bar{\tau}+\widehat{\tau}_t^e+\widehat{\tau}_t^*; \zeta_t\}_{t=0}^{\infty}$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

□

## D.2 Proof of Theorem 2

The strategy of the proof is as follows. First, I write the equilibrium as a linear system of equations in $\{\boldsymbol{y}, \boldsymbol{\pi}, \boldsymbol{i}_b, \boldsymbol{\tau}\}$. Second, I show that this system has a unique bounded solution for both policy rules.

I begin with some preliminary simplifications. The non-policy block of the economy is summarized by the following system of equations, now written in compact sequence-space notation (as in Auclert et al., 2019). First, the price-NKPC,

$$\widehat{\boldsymbol{\pi}} = \Pi_w \widehat{\boldsymbol{w}} + \Pi_\ell \widehat{\boldsymbol{\ell}} + \beta \widehat{\boldsymbol{\pi}}_{+1}$$

Second, the production function

$$\widehat{\boldsymbol{y}} = \mathcal{Y}_\ell \widehat{\boldsymbol{\ell}}$$

Third, firm dividends,

$$\widehat{\boldsymbol{d}} = \mathcal{D}_y \widehat{\boldsymbol{y}} + \mathcal{D}_w \widehat{\boldsymbol{w}} + \mathcal{D}_\ell \widehat{\boldsymbol{\ell}}$$

Fourth, consumer demand

$$\widehat{\boldsymbol{c}} = \mathcal{C}_w \widehat{\boldsymbol{w}} + \mathcal{C}_\ell \widehat{\boldsymbol{\ell}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{C}_d \widehat{\boldsymbol{d}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}} + \tilde{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b + \mathcal{C}_\zeta \widehat{\boldsymbol{\zeta}}$$

where the derivative maps $\mathcal{C}_\bullet$ are defined as in Appendix A.2. Fifth, the wage-NKPC,

$$\widehat{\boldsymbol{\ell}} = \mathcal{L}_w \widehat{\boldsymbol{w}} + \mathcal{L}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{L}_c \widehat{\boldsymbol{c}}$$

And sixth, the output market-clearing condition

$$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{c}}$$

Using the price-NKPC, the production function, the equation for firm dividends, and the output market-clearing condition, we can substitute out $\{\boldsymbol{c}, \boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{d}\}$ in the consumer demand relation. This gives

$$\bar{\mathcal{C}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b + \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}} + \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \tag{D.8}$$

where $\bar{\mathcal{C}}_{i_b} = \tilde{\mathcal{C}}_{i_b}$, $\bar{\mathcal{C}}_\tau = \mathcal{C}_\tau$, $\bar{\mathcal{C}}_\zeta = \mathcal{C}_\zeta$ and $\{\bar{\mathcal{C}}_y, \bar{\mathcal{C}}_\pi\}$ are functions of model primitives. Similarly, using the price-NKPC as well as the production function and the output market-clearing condition, we can substitute out $\{\boldsymbol{\ell}, \boldsymbol{w}, \boldsymbol{c}\}$ in the wage-NKPC to write it as

$$\bar{\mathcal{L}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{L}}_\pi \widehat{\boldsymbol{\pi}} = \boldsymbol{0} \tag{D.9}$$

where $\{\bar{\mathcal{L}}_y, \bar{\mathcal{L}}_\pi\}$ are functions of model primitives.

Now consider first the equilibrium system under the interest rate-only rule (13) - (14). Plugging the price-NKPC and the production function into the financing rule (14), we can write the financing rule compactly as

$$\widehat{\boldsymbol{\tau}} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}} + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b \tag{D.10}$$

where $\{\bar{\mathcal{T}}_y, \bar{\mathcal{T}}_\pi, \bar{\mathcal{T}}_{i_b}\}$ are functions of the financing rule (14) and model primitives. Using the simplifications in (D.8), (D.9) and (D.10), we can write the equilibrium system as the following compact linear system:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ -\mathcal{B}_y & -\mathcal{B}_\pi & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \\ \mathbf{0} \\ \boldsymbol{b}_{i_b} \\ \mathbf{0} \end{pmatrix} \tag{D.11}$$

By assumption, the system (D.11) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the equilibrium system corresponding to the proposed transfer-only rule (16) - (17). Using the simplifications from above, we can write that system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y - \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_y & -\bar{\mathcal{T}}_\pi - \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \\ \mathbf{0} \\ \mathbf{0} \\ \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\boldsymbol{b}_{i_b} \end{pmatrix} \tag{D.12}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (D.12). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where

$$\widehat{\boldsymbol{\tau}}^{**} = (\bar{\mathcal{T}}_y + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_y)\widehat{\boldsymbol{y}}^* + (\bar{\mathcal{T}}_\pi + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_\pi)\widehat{\boldsymbol{\pi}}^* + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\boldsymbol{b}_{i_b}$$

Plugging the candidate solution into the consumer demand function (D.8), we get

$$\begin{aligned} & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b}\mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \\ \Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^* - \mathcal{C}_{i_b}\left(\mathcal{B}_y \widehat{\boldsymbol{y}}^* + \mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^* + \boldsymbol{b}_{i_b}\right) = \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \\ \Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau(\bar{\mathcal{T}}_{i_b}\widehat{\boldsymbol{i}}_b^* + \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^*) = \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \\ \Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_\zeta \boldsymbol{\zeta} \end{aligned}$$

Thus (D.8) still holds. It is immediate that all other relations in (D.12) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (D.12). Finally, by the properties of $\mathcal{C}_\tau$ and $\mathcal{C}_{i_b}$, $\widehat{\boldsymbol{\tau}}^{**}$ is bounded, and so the overall solution is bounded.

To show uniqueness, suppose for a contraction that (D.12) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. Now consider the tuple $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\widehat{\boldsymbol{i}}_b^\dagger = \mathcal{B}_y \widehat{\boldsymbol{y}}^\dagger + \mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \boldsymbol{b}_{i_b}$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^\dagger$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (D.11). Contradiction. □

## D.3 Proof of Corollary 1

The conditions of Theorem 1 and Theorem 2 are satisfied, so it follows that the allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$ is uniquely implementable using a transfer-only rule. But since $i_{b,t} = \bar{i}_b \geq \underline{i}_b$ the statement follows. □

## D.4 Proof of Proposition 1

I prove each of the three parts of the proposition in turn.

1. By Proposition C.1 we have that, for savers with bonds in the utility function, the consumption transfer derivative map satisfies

$$\mathcal{C}_\tau^B \;\approx\; \tilde{\omega} \;\times\; \begin{pmatrix} 1 & \theta & \theta^2 & \dots \\ \theta & 1 & \theta & \dots \\ \theta^2 & \theta & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Since for spenders the derivative map is just the identify matrix, we have

$$\mathcal{C}_\tau^{TB} \;\approx\; (1-\mu) \;\times\; \tilde{\omega} \;\times\; \begin{pmatrix} 1 & \theta & \theta^2 & \dots \\ \theta & 1 & \theta & \dots \\ \theta^2 & \theta & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \;+\; \mu \;\times\; I$$

where $\mu$ is the share of spenders. (21) then follows with $\omega = (1-\mu)\tilde{\omega}+\mu$ and $\xi = (1-\mu)\frac{\tilde{\omega}}{\omega}$.

2. Plugging the expression for the Fourier series (C.10) into the eigenvalue distribution expression (C.4), straightforward re-arrangement gives the eigenvalue distribution as

$$F^{TB}(\lambda) = 1 - \frac{1}{\pi}\text{arccos}\left(\frac{1+\theta^2 - \frac{\xi(1-\theta^2)}{\frac{\lambda}{\omega}-(1-\xi)}}{2\theta}\right)$$

We can recover the bounds $\underline{\lambda}$ and $\bar{\lambda}$ by solving $F^{TB}(\underline{\lambda}) = 0$ and $F^{TB}(\bar{\lambda}) = 1$. □

3. By (C.5) and (C.10), we have

$$t_k^{-1} = \frac{1}{2\pi}\frac{1}{\omega}\int_0^{2\pi} e^{-ik\lambda}\frac{1-2\theta\cos(\lambda)+\theta^2}{\xi(1-\theta^2)+(1-\xi)(1-2\theta\cos(\lambda)+\theta^2)}d\lambda$$

This yields (23). □

## D.5 Proof of Theorem 3

By assumption, the allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$ is implementable without the ELB constraint (18), with a policy tuple $\{i_{b,t}^*, \tau_t^*, \tau_{f,t}^*\}_{t=0}^\infty$. Now consider the alternative policy tuple $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \tau_{f,t}^* + \widehat{\tau}_{f,t}^\dagger\}_{t=0}^\infty$ where

$$\widehat{\boldsymbol{\tau}}_f^\dagger = \mathcal{I}_{\tau_f}^{-1}\mathcal{I}_{i_b}\widehat{\mathbf{i}}_b^* \tag{D.13}$$

and

$$\widehat{\boldsymbol{\tau}}^\dagger = \mathcal{C}_\tau^{-1}\left[\tilde{\mathcal{C}}_{i_b}\widehat{\mathbf{i}}_b^* + \mathcal{C}_d\left(\mathcal{D}_{i_b}\widehat{\mathbf{i}}_b^* - \mathcal{D}_{\tau_f}\widehat{\boldsymbol{\tau}}_f^\dagger\right)\right] \tag{D.14}$$

I now claim that this policy tuple similarly engineers the allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$. First, with $\widehat{\boldsymbol{\tau}}_f^\dagger$ set as in (D.13), the investment, output and labor demand paths are unchanged; however, as remarked in Appendix A.4, the dividend paths may be different. The transfer path $\widehat{\boldsymbol{\tau}}^\dagger$ is constructed to offset both the missing monetary stimulus as well as neutralize any potential dividend-related effects: to see this, note that we have

$$\begin{aligned}
\widehat{\mathbf{c}}^\dagger &= \mathcal{C}_\tau(\widehat{\boldsymbol{\tau}}^* + \widehat{\boldsymbol{\tau}}^\dagger) + \mathcal{C}_d\mathcal{D}_{\tau_f}(\widehat{\boldsymbol{\tau}}_f^* + \widehat{\boldsymbol{\tau}}_f^\dagger) + \text{non-policy terms} \\
&= \mathcal{C}_\tau\widehat{\boldsymbol{\tau}}^* + \tilde{\mathcal{C}}_{i_b}\widehat{\mathbf{i}}_b^* + \mathcal{C}_d\mathcal{D}_{i_b}\widehat{\mathbf{i}}_b^* + \mathcal{C}_d\mathcal{D}_{\tau_f}\widehat{\boldsymbol{\tau}}_f^* + \text{non-policy terms} = \widehat{\mathbf{c}}^*
\end{aligned}$$

Next note that, since they induce the same paths of consumption, investment, hours worked and production, and since by assumption wages are unchanged, the initial policy $\{i_{b,t}^*, \tau_t^*, \tau_{f,t}^*\}_{t=0}^\infty$

and the new policy $\{\bar{i}_b, \tau_t^*+\widehat{\tau}_t^\dagger, \tau_{f,t}^*+\widehat{\tau}_{f,t}^\dagger\}_{t=0}^\infty$ have the same present value in the augmented government budget constraint (A.10), exactly as in the proof of Theorem 1. With $\lim_{t\to\infty}\widehat{b}_t = 0$ in the initial equilibrium, it then follows that we must also have $\lim_{t\to\infty}\widehat{b}_t = 0$ in the new one, as required. All other model equations are unaffected, so the guess is verified. □

## D.6 Auxiliary lemmas for the proof of Proposition C.1

Consider an income shock at time $H \geq 0$, and suppose that $\beta = 1$. We then get the following simplified equilibrium system:

$$\begin{aligned} \bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t - (1+\bar{r}_b)\bar{b}\widehat{b}_{t-1} &= \bar{y}\widehat{y}_{t-H} \\ \widehat{c}_t - (1+\bar{r}_b)\widehat{c}_{t+1} + \frac{\eta}{\gamma}\bar{r}_b\widehat{b}_t &= 0 \end{aligned}$$

I will guess and verify that lagged wealth is the only endogenous state variable. Plugging in this guess the equilibrium system becomes

$$\begin{aligned} \bar{c}(\theta_{cb}\widehat{b}_{t-1} + \sum_{h=0}^{H}\theta_{cyh}\widehat{y}_{t-h}) + \bar{b}(\theta_{bb}\widehat{b}_{t-1} + \sum_{h=0}^{H}\theta_{byh}\widehat{y}_{t-h}) - (1+\bar{r}_b)\bar{b}\widehat{b}_{t-1} &= \bar{y}\widehat{y}_{t-h} \\ \theta_{cb}\widehat{b}_{t-1} + \sum_{h=0}^{H}\theta_{cyh}\widehat{y}_{t-h} - (1+\bar{r}_b)[\theta_{cb}(\theta_{bb}\widehat{b}_{t-1} + \sum_{h=0}^{H}\theta_{byh}\widehat{y}_{t-h}) + \sum_{h=1}^{H}\theta_{cyh}\widehat{y}_{t+1-h}] & \\ + \frac{\eta}{\gamma}\bar{r}_b(\theta_{bb}\widehat{b}_{t-1} + \sum_{h=0}^{H}\theta_{byh}\widehat{y}_{t-h}) &= 0 \end{aligned}$$

Matching coefficients, we find that the equilibrium is fully characterized by the following system of $2H+4$ equations in $2H+4$ unknowns:

$$\bar{c}\theta_{cb} + \bar{b}\theta_{bb} - (1+\bar{r}_b)\bar{b} = 0 \tag{D.15}$$

$$\bar{c}\theta_{cyh} + \bar{b}\theta_{byh} = 0, \quad h = 0,1,\ldots,H-1 \tag{D.16}$$

$$\bar{c}\theta_{cyH} + \bar{b}\theta_{byH} = \bar{y} \tag{D.17}$$

$$\theta_{cb} - (1+\bar{r}_b)\theta_{cb}\theta_{bb} + \frac{\eta}{\gamma}\bar{r}_b\theta_{bb} = 0 \tag{D.18}$$

$$\theta_{cyh} - (1+\bar{r}_b)[\theta_{cb}\theta_{byh} + \theta_{cyh+1}] + \frac{\eta}{\gamma}\bar{r}_b\theta_{byh} = 0, \quad h = 0,1,\ldots,H-1 \tag{D.19}$$

$$\theta_{cyH} - (1+\bar{r}_b)\theta_{cb}\theta_{byH} + \frac{\eta}{\gamma}\bar{r}_b\theta_{byH} = 0 \tag{D.20}$$

I now state and prove two useful properties of this system.

**Lemma D.1.** *Consider the system (D.15) - (D.20). The solution satisfies*

$$\frac{\theta_{cb}}{\theta_{bb}}\theta_{byH} = \theta_{cyH} \tag{D.21}$$

*Proof.* By (D.18):

$$\frac{\theta_{cb}}{\theta_{bb}} = (1+\bar{r}_b)\theta_{cb} - \frac{\eta}{\gamma}\bar{r}_b$$

By (D.20):

$$\frac{\theta_{cyH}}{\theta_{byH}} = (1+\bar{r}_b)\theta_{cb} - \frac{\eta}{\gamma}\bar{r}_b$$

Combining the two, we arrive at the stated result.

□

**Lemma D.2.** *Consider the system (D.15) - (D.20). The solution satisfies*

$$\frac{\theta_{cyh}}{\theta_{cyh+1}} = \theta_{bb}, \quad h = 0, 1, \ldots, H-1 \tag{D.22}$$

*and*

$$\frac{\theta_{byh}}{\theta_{byh+1}} = \theta_{bb}, \quad h = 0, 1, \ldots, H-2 \tag{D.23}$$

*Proof.* Use (D.16) to re-arrange (D.19) to get

$$\theta_{cyh} = \frac{1+\bar{r}_b}{1+(1+\bar{r}_b)\frac{\bar{c}}{\bar{b}}\theta_{cb} - \frac{\eta}{\gamma}\bar{r}_b\frac{\bar{c}}{\bar{b}}}\theta_{cyh+1}$$

We need to show that the first term on the right-hand side is equal to $\theta_{bb}$. To see this, note that (D.15) and (D.18) together imply that

$$\theta_{cb} - (1+\bar{r}_b)\theta_{cb}\theta_{bb} + \frac{\eta}{\gamma}\bar{r}_b\theta_{bb} = \frac{\bar{b}}{\bar{c}}\theta_{bb} - \frac{\bar{b}}{\bar{c}}(1+\bar{r}_b) + \theta_{cb}$$

But this condition can be re-arranged to give

$$\theta_{bb} = \frac{1+\bar{r}_b}{1+(1+\bar{r}_b)\frac{\bar{c}}{\bar{b}}\theta_{cb} - \frac{\eta}{\gamma}\bar{r}_b\frac{\bar{c}}{\bar{b}}}$$

as claimed. This proves (D.22). For (D.23) simply note that, by (D.16), we for all $h = 0, 1, \ldots, H-2$ have that

$$\frac{\theta_{cyh}}{\theta_{cyh+1}} = \frac{\theta_{byh}}{\theta_{byh+1}}$$

□

## D.7 Proof of Proposition C.1

The proof of Proposition C.1 leverages Lemmas D.1 and D.2.

The impulse response of wealth holdings is given as

$$\widehat{b}_{t,H} = \sum_{\ell=0}^{t} \theta_{bb}^{\ell} \theta_{byt-\ell}$$

The consumption impulse responses around horizon $H$ are thus given as

$$\widehat{c}_{H-1,H} = \theta_{cb} \sum_{\ell=0}^{H-2} \theta_{bb}^{\ell} \theta_{byH-\ell-2} + \theta_{cyH-1} \tag{D.24}$$

$$\widehat{c}_{H,H} = \theta_{cb} \sum_{\ell=0}^{H-1} \theta_{bb}^{\ell} \theta_{byH-\ell-1} + \theta_{cyH} \tag{D.25}$$

$$\widehat{c}_{H+1,H} = \theta_{cb} \sum_{\ell=0}^{H} \theta_{bb}^{\ell} \theta_{byH-\ell} \tag{D.26}$$

I begin with the MPC at time $H$. By Lemma D.2 we find that

$$\lim_{H\to\infty} \widehat{c}_{H,H} = \frac{\theta_{cb}}{1-\theta_{bb}^2} \theta_{byH-1} + \theta_{cyH}$$

But by (D.16) - (D.17) and (D.19) - (D.20), $\theta_{byH-1}$ and $\theta_{cyH}$ are actually independent of $H$, so $\widehat{c}_{H,H}$ converges to some fixed limit $\frac{\bar{y}}{\bar{c}} \times \omega$.

Next, consider moving down below the main diagonal, i.e., comparing (D.25) and (D.26). Note that we have

$$\begin{aligned} \frac{1}{\theta_{bb}} \widehat{c}_{H+1,H} &= \frac{\theta_{cb}}{\theta_{bb}} (\theta_{bb} \widehat{b}_{H-1,H} + \theta_{byH}) \\ &= \theta_{cb} \widehat{b}_{H-1,H} + \frac{\theta_{cb}}{\theta_{bb}} \theta_{byH} \end{aligned}$$

By Lemma D.1, this is equal to $\widehat{c}_{H,H}$, proving the claim for $\theta = \theta_{bb}$. For $H+2$, we have

$$\frac{1}{\theta_{bb}} c_{H+2,H} = \frac{\theta_{cb}}{\theta_{bb}} b_{H+1,H} = \frac{\theta_{cb}}{\theta_{bb}} \theta_{bb} \widehat{b}_{H,H} = \widehat{c}_{H+1,H}$$

The argument applies unchanged for any $H+\ell$ with $\ell \geq 2$, proving the half of (C.8) below the main diagonal. The proof reveals that the result holds for any $H$, not just $H \to \infty$.

Now consider moving up above the main diagonal, i.e., comparing (D.25) and (D.24). Here we have

$$\begin{aligned}\frac{1}{\theta_{bb}}\widehat{c}_{H-1,H} &= \frac{1}{\theta_{bb}}\left\{\theta_{cb}\sum_{\ell=0}^{H-2}\theta_{bb}^{\ell}\theta_{byH-\ell-2}+\theta_{cyH-1}\right\} \\ &= \theta_{cb}\sum_{\ell=0}^{H-2}\theta_{bb}^{\ell}\theta_{byH-\ell-1}+\theta_{cyH}\end{aligned}$$

where the second line has used Lemma D.2. We can thus write

$$\frac{1}{\theta_{bb}}\widehat{c}_{H-1,H} = \widehat{c}_{H,H} - \theta_{cb}\theta_{bb}^{H-1}\theta_{by0}$$

But with $\theta_{bb}$ inside the unit circle – the stable solution of the system (D.15) and (D.18) – it follows that second term converges to zero for $H \to \infty$, establishing the claim with $\theta = \theta_{bb}$, the same constant of proportionality as below the main diagonal.[22] Proceeding analogously for any $\ell > 0$, we have

$$\frac{1}{\theta_{bb}}\widehat{c}_{H-h,H} = \widehat{c}_{H-h+1,H} - \theta_{cb}\theta_{bb}^{H-h}\theta_{by0}$$

thus completing the proof of the half of (C.8) above the main diagonal. □

[22] This argument leverages the fact that $\theta_{by0} \to 0$, which follows from Lemma D.2 together with the fact $\theta_{bb}$ is inside the unit circle.