Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

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Abstract: In a textbook New Keynesian model extended to allow for uninsurable household income risk, any aggregate allocation implementable via nominal interest rate policy is also implementable by adjusting uniform lump-sum transfer payments, and vice-versa. The equivalence fails only in the limit cases of perfectly Ricardian and perfectly hand-to-mouth households. It follows that conventional fiscal policy using stimulus checks can substitute for monetary policy when interest rates are constrained by an effective lower bound. In a tractable special case of the model, the mapping between the equivalent fiscal and monetary policies is fully characterized by a small number of readily measurable sufficient statistics. This closed-form characterization remains nearly exact even in a Heterogeneous Agent (HANK) model. However, while macro-equivalent, checks and rate cuts have different distributional effects in the cross-section of households.

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1 Introduction

The prescription of standard New Keynesian theory is to conduct stabilization policy through changes in short-term nominal interest rates: by the “divine coincidence”, cutting nominal interest rates in the face of deficient demand is enough to fully stabilize output and inflation. Recently, however, with rates close to an effective lower bound (ELB), the policy space for such monetary stimulus has narrowed. A natural question, then, is how to replicate the effects of standard monetary policy through alternative tools. Previous work has identified tax policy – often labeled unconventional fiscal policy – as an attractive option (Farhi & Werning, 2007; Correia et al., 2013): time-varying tax rates manipulate intertemporal prices just like monetary policy, and thus can replicate the desired monetary allocation. This influential “wedge” approach, however, faces two challenges. First, it matches substitution effects, and so fails in models in which income effects matter (e.g., incomplete-markets models). Second, the required business-cycle fine-tuning of taxes may not be practically feasible.

In this paper I focus on a different fiscal policy instrument: uniform lump-sum payments (“stimulus checks”) sent to households, as seen in recent U.S. recessions. The setting for my analysis is a standard New Keynesian model without capital, extended to feature uninsurable household income risk. I present three main results. First, I give conditions under which any sequence of macroeconomic aggregates that is implementable via conventional interest rate policy is also implementable by only adjusting uniform lump-sum transfers, and vice-versa. This equivalence fails in the limit cases of perfectly Ricardian and perfectly hand-to-mouth households, but holds otherwise. It follows that, generically, a binding effective lower bound on interest rates does not constrain the policymaker’s ability to close output and inflation gaps given deficient demand. Second, I study the mapping from (infeasible) nominal interest rate policy to macro-equivalent transfer stimulus. In an analytically tractable model variant, this mapping is fully characterized by a small number of already measured sufficient statistics; in a richer Heterogeneous Agent New Keynesian (HANK) model, my closed-form sufficient statistics characterization remains almost exact. And third, I use the rich HANK model to illustrate that, while macro-equivalent, transfer stimulus and interest rate cuts have very different distributional effects in the cross-section of households. In richer models with investment, all of those results continue to apply without change as a way for the policymaker to perfectly replicate the consumption channel of monetary policy transmission.

My analysis of policy equivalence builds on a variant of the canonical textbook New Keynesian model (Galí, 2015), augmented to allow for general forms of uninsurable household
income risk and liquidity constraints. In this environment, I characterize and compare the (linearized) perfect foresight transition paths induced by arbitrary nominal interest rate and uniform lump-sum transfer policies. My results on policy equivalence leverage properties of the two linear maps $C_\tau$ and $C_{ib}$, whose $(t,s)$th entries are, respectively, the partial derivatives of aggregate consumption demand at time $t$ with respect to (i) a uniform lump-sum transfer paid out at time $s$ and (ii) a change in the time-$s$ nominal rate of interest on bonds. Note that $C_\tau$ is a matrix of intertemporal marginal propensities to consume (iMPCs), as studied first in Auclert et al. (2018).

My first main result is to show that, if the map $C_\tau$ is invertible – a condition that I will refer to as strong Ricardian non-equivalence –, then any sequence of macroeconomic aggregates that can be attained via interest rate policy is also implementable by only adjusting the time profile of uniform lump-sum transfers. In particular, if aggregate output and inflation gaps can in principle be closed without the ELB, then they can also be closed given a binding ELB. To see the logic, consider an arbitrary path of nominal interest rates. Through the consumption-savings problem of households, that sequence of nominal interest rates induces some path of net excess consumption demand. If $C_\tau$ is invertible, then there must exist some path of uniform lump-sum transfers (or taxes) that – by moving households away from or towards borrowing constraints – perturbs consumption demand in exactly the same way. The proof is completed by showing that, in my environment, interest rate and transfer policies that generate the same partial equilibrium net excess consumption demand paths must be accommodated in general equilibrium through the exact same market-clearing adjustments in prices and quantities. As part of the proof, I also show that the two policies are revenue-equivalent: they have the exact same net present value, and so can be financed using identical increases or reductions in future taxes. Finally, I note that the proof also works just as well in the other direction; overall, interest rate and transfer policies implement the same sets of aggregate allocations if and only if the two maps $C_\tau$ and $C_{ib}$ share the same image.\footnote{Formally, the requirement is that any zero net present value spending path in the image of $C_\tau$ is also in the image of $C_{ib}$, and vice-versa.}

I illustrate the policy equivalence result through a series of simple models in which the two linear maps $C_\tau$ and $C_{ib}$ are available in closed-form. In the canonical permanent-income consumption-savings problem, households only care about the present value of their lifetime income; $C_\tau$ is thus of rank 1, and transfers today financed with taxes in the future leave consumer spending unchanged. In contrast, through their presence in the household Euler equation, arbitrary paths of nominal rates can generate arbitrary (zero net present value)
paths of net excess consumption demand. The baseline Ricardian model thus features the familiar *pecking order* of policies: transfers have no effects, while interest rate policy can offset any possible path of deficient excess demand. The opposite extreme is that of purely hand-to-mouth households: here interest rate changes have no effect, while transfer stimulus immediately feeds through to household spending. Analytically tractable models of partially binding liquidity constraints—e.g., the spender-saver model of Campbell & Mankiw (1989) or the reduced-form approach of including bonds in household utility (Michaillat & Saez, 2018)—are generically in between these two extremes: strong Ricardian non-equivalence holds, an ELB on nominal rates does not at all constrain the space of implementable allocations, and nominal interest rate policy and transfer stimulus are macro-equivalent.

In the second part of the paper I explicitly characterize the mapping between the macro-equivalent nominal interest rate and uniform transfer policies. This step is essential because my equivalence result only asserts the existence of a transfer policy that replicates any given interest rate policy; in principle, the mapping between the two could be ill-behaved, limiting the practical relevance of the equivalence result. To make analytical progress, I first restrict attention to the tractable special case of a simple hybrid model with spenders and bonds in the household utility function. I choose this model because it is rich enough to match existing evidence on household MPCs and so the map $C_r$ (e.g., Parker et al., 2013; Fagereng et al., 2018), yet simple enough to remain tractable. I show that, in this special case, the mapping between the equivalent policies is fully characterized by a small number of estimable iMPCs together with the elasticity of intertemporal substitution. At standard empirical estimates of those, the model is far away from the boundary cases of perfectly Ricardian or perfectly hand-to-mouth households, so policy equivalence holds, and the mapping from infeasible interest rate policy to equivalent transfer stimulus is well-behaved.

My analytical results on the space of implementable allocations and the shape of the equivalence mapping generalize with little change to a richer “HANK” version of my model (Kaplan et al., 2018). Following Auclert et al. (2018), I parameterize the incomplete-markets consumption block of the model to ensure consistency with empirical evidence on iMPCs. As expected, I find that $C_r$ and $C_n$ are such that policy equivalence holds. Furthermore I document that, for a range of (infeasible) monetary stimuli, the approximate equivalent transfer policy based on my analytical formula from the hybrid model is close to the actual HANK model-implied equivalent policy. Through the lens of the model, a transfer stimulus like that of 2008—i.e., staggered, deficit-financed checks of around $600—is roughly equivalent to a transitory 170 basis point nominal interest rate cut.
Finally, I use the quantitative HANK model to move beyond macro equivalence and compare policy propagation in the cross-section of households. By construction, two macro-equivalent interest rate and transfer policies necessarily induce the exact same general equilibrium movements of wages, hours worked, inflation, and dividend payouts. However, they very much differ in their direct effects on household spending: nominal rate cuts mostly act by directly stimulating consumption at the top of the liquid wealth distribution, whereas the macro-equivalent transfer stimulus almost exclusively acts at the bottom.

My approach to policy equivalence also extends naturally to a model with investment. Generally, as soon as productive capital is introduced, monetary policy transmits to macroeconomic aggregates by stimulating both consumption and investment demand. In this setting, my results on transfer stimulus continue to apply unchanged as a way for the policymaker to replicate the consumption channel of monetary transmission. To keep the entire set of implementable aggregate allocations unchanged, the policymaker then needs to complement lump-sum transfer payments with a second instrument. Proceeding exactly as in my baseline model, it is straightforward to establish that any policy instrument $q$ inducing an invertible investment demand map $I_q$ would suffice. In standard models of investment, bonus depreciation stimulus – a policy observed in each of the recent U.S. recessions (Zwick & Mahon, 2017) – satisfies this invertibility assumption; in models of investment breaking the Modigliani–Miller theorem, cash transfers to firms would also work, perfectly analogous to my discussion of Ricardian non-equivalence on the consumption side.

LITERATURE. The paper relates and contributes to several strands of literature.

First, the analysis is motivated by the recent experience of limits to conventional monetary policy space, caused in particular by: a binding ELB on nominal rates due to arbitrage between bonds and money; adverse effects of further rate cuts on bank profitability (Brunnermeier & Koby, 2018); rates on most outstanding mortgages being close to the ELB (Berger et al., 2018); and durables spending adjustments already having been pulled forward in time (McKay & Wieland, 2019). I blend theory and empirical evidence to show how a conventional policy instrument – lump-sum stimulus payments – can overcome those limits.

Second, my proof of policy equivalence relies heavily on equilibrium characterizations in sequence space (Boppart et al., 2018; Auclert et al., 2019). So far, the sequence-space set-up has been used to analytically characterize general equilibrium effects (Auclert & Rognlie, 2018; Auclert et al., 2018) or to construct general equilibrium counterfactuals for unobserved shocks (Wolf, 2020). I instead use the same observations to sidestep constraints on policy
space. Similar to classical general equilibrium theory (Arrow & Debreu, 1954), the sequence-space perspective reveals that two policies are equivalent if they induce the same net excess demand paths. As such, my equivalence results are conceptually distinct from Correia et al. (2013) and Farhi et al. (2014): there, equivalence comes from identical wedges in optimality conditions. The two drawbacks of this “wedge” approach are that (i) it fails in models in which income effects matter (like my HANK model), and (ii) it leverages policy instruments that are arguably harder to vary at business-cycle frequencies.

Third, my characterization of equivalent transfer policies builds on recent work that uses micro regressions to inform macro questions. I show that, to map a given interest rate policy into an equivalent transfer stimulus, it suffices to learn about the derivatives of an aggregate consumption function (i.e., the two linear maps $C_\tau$ and $C_{ib}$). Cross-sectional regressions of consumer spending on household-level shocks (as in Parker et al., 2013; Fagereng et al., 2018) promise to recover precisely those direct consumption spending responses (Auclert et al., 2018; Wolf, 2020). With extant empirical evidence however insufficient to characterize the entire map, I use theory-based cross-column restrictions to arrive at my characterizations of equivalent policies.

**OUTLOOK.** The rest of the paper proceeds as follows. Section 2 sets up the baseline model. Section 3 presents the main result on the set of implementable allocations, and Section 4 uses an empirically relevant but analytically tractable special case to characterize the mapping between equivalent policies. Section 5 shows how my analytical results extend to a quantitative HANK model, and then uses that model to study non-equivalence in the cross-section of households. Section 6 generalizes the results to a model with investment, Section 7 discusses the relation of my analysis to other perspectives on policy equivalence, and finally Section 8 concludes. Proofs and supplementary results are collected in several appendices.

## 2 Environment

This section presents the environment. I begin with a model outline in Section 2.1, and then in Section 2.2 define the two key consumption maps $C_\tau$ and $C_{ib}$.

### 2.1 Model outline

Time is discrete and runs forever, $t = 0, 1, \ldots$. The model economy is populated by households, unions, firms, and a government, and is initially at its deterministic steady state. At
time $t = 0$, households are subject to an unexpected shock to their patience; as usual, this shock should be interpreted as a reduced-form stand-in for more plausibly structural shocks to household consumption spending (e.g., changes in borrowing constraints). In this perfect-foresight environment I study the set of allocations implementable by the policymaker, and in particular discuss the extent to which her ability to offset the impatience shocks varies with the policy toolkit. The overall model set-up is kept deliberately close to the textbook New Keynesian business-cycle framework (Galí, 2015; Woodford, 2011).²

The realization of a variable $x$ at time $t$ along the equilibrium perfect foresight transition path will be denoted $x_t$, while the entire time path will be denoted $x = \{x_t\}_{t=0}^{\infty}$. Hats denote deviations from the deterministic steady state, bars denote steady-state values, and tildes denote logs.

**HOUSEHOLDS.** There is a unit continuum of households $i$. Households consume a final consumption good $c_{it}$ and provide hours worked $\ell_{it}$, leading to lifetime utility of

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left\{ u(c_{it}) - v(\ell_{it}) \right\} \right] \tag{1}$$

where $\zeta_t$ is the impatience (demand) shock. Household income consists of: labor earnings $(1 - \tau_t)w_t e_{it} \ell_{it}$, where $w_t$ is the wage, $e_{it}$ denotes a household’s (time-varying) productivity and $\tau_t$ is the (fixed) labor tax rate; uniform lump-sum transfer receipts $\tau_t$; and dividends $d_{it}$. Households can self-insure by investing in liquid nominal bonds $b_{it}$ with nominal returns $i_{b,t}$, but borrowing in bonds is subject to a borrowing constraint $b$. The real return to saving is affected by the inflation rate $\pi_t$. The period-$t$ budget constraint of household $i$ is thus

$$c_{it} + b_{it} = (1 - \tau_t)w_t e_{it} \ell_{it} + \frac{1 + i_{b,t-1}}{1 + \pi_t} b_{it-1} + \tau_t + d_{it} \tag{2}$$

Household $i$ maximizes (1) by choosing the path of consumption $c_{it}$ and wealth $b_{it}$ subject to (2) as well as the borrowing constraint $b_{it} \geq b$. Hours worked $\ell_{it}$ are – due to frictions in the labor market – taken as given by households and set by optimizing labor unions.

**UNIONS & FIRMS.** I summarize the production and wage bargaining block through three key relations. First, a unit continuum of firms produces the final output good using a labor-

²The equilibrium transition paths characterized here are thus equivalent to the first-order perturbation solution of an analogous model with aggregate risk (e.g. Boppart et al., 2018).
only production technology. Imposing that \( \int_0^1 e_u \, du = 1 \) for all \( t \), we can write
\[
y_t = y(\ell_t)
\]
Price-setting is subject to the usual Rotemberg adjustment costs, giving a standard New Keynesian Phillips Curve (NKPC) in prices:
\[
\pi_t = \pi(\ell_t, w_t; \pi_{t+1})
\]
I will consider a zero-inflation steady state, i.e. \( \bar{\pi} = 0 \). Producer dividends are paid out to households; as in McKay et al. (2016), I assume that the dividend payments to household \( i \) are set as a function of its productivity \( e_{it} \). Finally, wage bargaining is subject to similar Rotemberg adjustment costs and so induces a wage-NKPC, linking current and future wage inflation to total household consumption and hours worked:
\[
\pi^w_t = \pi^w(\ell_t, c_t; \pi^w_{t+1})
\]
where \( \pi^w_t = \frac{w_t}{\bar{w}_{t-1}} \pi_t \) denotes wage inflation. I assume that the union demands the same hours worked from all households, so \( \ell_{it} = \ell_t \) for all \( i \in [0, 1] \). Detailed derivations of the structural relations (3) and (4) are presented in Appendix A.1.3

**Policy.** The government flow budget constraint is
\[
\frac{1 + \tilde{b}_{t-1}}{1 + \pi_t} b_{t-1} + \tau_t = \tau_t w_t \ell_t + b_t
\]
The policymaker sets nominal interest rates \( \tilde{i}_{b,t} \) and uniform lump-sum transfers \( \tau_t \) subject to the flow budget constraint (5) and the requirement that \( \lim_{t \to \infty} \hat{b}_t = 0 \).

The remainder of this paper will study the implications of constraints on this policy toolkit. I will focus on two particular kinds of restrictions: transfer-only policies and interest rate-only policies.

**Definition 1.** A transfer-only policy is a policy that sets \( \tilde{i}_{b,t} = \bar{i}_b \) for all \( t \).

In a transfer-only policy, the policymaker is forced to keep nominal interest rates fixed,
perhaps due to a binding effective lower bound on nominal rates. In light of the ELB’s empirical relevance, most discussion in this paper will center on the extent to which a restriction to transfer-only policies meaningfully constrains the policymaker.

Definition 2. An interest rate-only policy is a policy that sets, for all $t = 0, 1, \ldots$, 

$$\tau_t = \tau_t w_t \ell_t + \left(1 - \frac{1 + i_{b,t-1}}{1 + \pi_t}\right)b_t.$$  

(6)

Definition 2 gives the natural opposite to a transfer-only policy: in an interest rate-only policy, the policymaker is completely free to adjust the path of nominal interest rates, but forced to passively adjust transfers to balance the budget period-by-period. This policy thus operates by manipulating intertemporal prices, without any time variation in the total amount of government debt in the hands of households.

Equilibrium. I am now in a position to formally define a perfect foresight transition equilibrium in this baseline economy.

Definition 3. Given a path of shocks $\{\zeta_t\}_{t=0}^{\infty}$, an equilibrium is a set of government policies $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$ and a set of macroeconomic aggregates $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ such that:

1. Aggregate consumption is given as

$$c = \int_0^1 c_i di$$

where the individual consumption paths $c_i$ solve the household consumption-savings problem of maximizing (1) subject to (2) and $b_t \geq b$.

2. Wage inflation $\pi_t^w = \frac{w_t}{w_{t-1}}(1 + \pi_t)$ together with $\{\ell_t, c_t\}_{t=0}^{\infty}$ are consistent with the aggregate wage-NKPC (4).

3. The paths $\{\pi_t, w_t, y_t\}_{t=0}^{\infty}$ are consistent with the aggregate price-NKPC (3), and dividends are given as $d_t = y_t - w_t \ell_t$.

4. The output market clears: $y_t = c_t$ for all $t \geq 0$, the government budget constraint (5) holds at all $t$, and $\lim_{t \to \infty} \hat{b}_t = 0$. The bond market then clears automatically by Walras’ law.

Given $\{\zeta_t\}_{t=0}^{\infty}$, an allocation of macroeconomic aggregates $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ is said to be implementable if it can be supported as an equilibrium sequence.
Nested models & tractable special cases. My environment nests two important families of structural models popular in the study of monetary and fiscal policy. First, without uninsurable idiosyncratic productivity risk \((e_{it} = 1\) for all \(i)\) and with a loose borrowing constraint \(b\), the household problem is simply the familiar consumption-savings problem at the heart of standard small-scale New Keynesian models (Galí, 2015). In fact, with flexible wages, my model exactly reduces to the familiar three-equation model of monetary policy transmission. Second, keeping uninsurable idiosyncratic income risk, the setting becomes a simple labor-only HANK model, as in McKay et al. (2016) or Auclert et al. (2018).

To build intuition, I will find it useful to go beyond the baseline Ricardian and HANK models, and also consider several other intermediate, yet analytically tractable variants of the household consumption-savings problem (1). Those include: the addition of spenders to the baseline permanent-income representative household (Campbell & Mankiw, 1989); bonds in the household utility function (Michaillat & Saez, 2018); and a hybrid of the two. As is well-known, these tractable models can plausibly mimic important features of household behavior in richer HANK models (Auclert et al., 2018; Kaplan et al., 2018), and thus provide a useful bridge between the two model extremes. A detailed discussion of each of these modifications of the baseline consumption-savings problem (1) is provided in Section 3.3.

2.2 Household consumption behavior

In my environment, policy equivalence – or the lack thereof – is all about the properties of household consumption behavior. To make progress, I aggregate the solution to the consumption-savings problem across all households \(i \in [0, 1]\), thus obtaining an aggregate consumption function \(C(\bullet)\):\(^4\)

\[
c = C(\underbrace{w, \ell, \pi, d}_{\text{eq'm aggregates}}; \underbrace{\tau, i_b}_{\text{policy}}; \underbrace{\zeta}_{\text{shock}}) \quad (7)
\]

Given paths of the aggregate demand shock \(\zeta\), the policy instruments \(\{\tau, i_b\}\), and the endogenous equilibrium aggregates \(\{w, \ell, \pi, d\}\), the aggregate consumption function (7) gives the path of household consumption consistent with the solution of each individual household’s consumption-savings problem. By definition of the deterministic steady state, this aggregate

\(^4\)My assumptions on labor supply and dividend payouts allow me to aggregate household behavior in this way. Note that the consumption function is necessarily indexed by the initial distribution of households over wealth holdings and idiosyncratic productivity. I provide details in Appendix A.2.
consumption function satisfies
\[ \bar{c} = C(\bar{w}, \bar{\ell}, \bar{\pi}, \bar{\nu}, \bar{\imath}_b, 0) \]

In my linearized environment, policy equivalence will be fully governed by the properties of 
\( C(\bullet) \) around the deterministic steady state. The first key object is the map \( C_\tau \), defined as
\[ C_\tau \equiv \frac{\partial C(\bullet)}{\partial \tau}, \tag{8} \]
where the derivative is evaluated at the deterministic steady state. The \((t, s)\)th entry of this infinite-dimensional linear map is the derivative of aggregate consumption demand at time \( t \) with respect to a uniform lump-sum transfer \( \tau \) at time \( s \). This linear map will play a central role in characterizing the space of implementable allocations using transfer-only policies. The second key object is the map \( C_{i_b} \), defined as
\[ C_{i_b} \equiv \frac{\partial C(\bullet)}{\partial i_b} - \bar{b} \times C_{\tau, •, -1}, \tag{9} \]
where again the derivative is evaluated at the steady state, and where \( C_{\tau, •, -1} \) is the map \( C_\tau \) from the second column onwards. The \((t, s)\)th entry of this infinite-dimensional linear map is the derivative of aggregate consumption demand at time \( t \) with respect to the nominal interest rate \( i_b \) at time \( s \), with transfers adjusting as prescribed by (6) to ensure budget balance. This linear map will play a central role in characterizing the space of implementable allocations using interest rate-only policies.

To build intuition for the abstract definitions in (8) and (9), I will derive the two key linear maps in simple, familiar models of the household consumption-savings problem. For a fully insured, permanent-income household with conventional isoelastic consumption utility (with elasticity of intertemporal substitution \( 1/\gamma \)), and assuming a steady-state real rate of interest \( \bar{r}_b = \frac{1}{\beta} - 1 \), it is straightforward to establish that
\[ C^R_\tau = \begin{pmatrix} \frac{\bar{r}_b}{1+\bar{r}_b} & \frac{\bar{r}_b}{(1+\bar{r}_b)^2} & \frac{\bar{r}_b}{(1+\bar{r}_b)^3} & \cdots \\ \frac{\bar{r}_b}{1+\bar{r}_b} & \frac{\bar{r}_b}{(1+\bar{r}_b)^2} & \frac{\bar{r}_b}{(1+\bar{r}_b)^3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad C^R_{i_b} = \frac{-1}{\gamma} (I - C^R_\tau) \begin{pmatrix} 1 & 1 & 1 & \ldots \\ 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{10} \]

Following receipt of lump-sum income, permanent-income households consume the annuity
value of that receipt in each period, giving the expression for $C^R_{\tau}$. In particular, any sequence of transfers with zero net present value does not affect household consumption at all. Current and future (expected) movements in real interest rates, in contrast, tilt household consumption expenditure via the usual intertemporal substitution motive. The upper-triangular matrix in the expression for $C^R_{\tau}$ reflects the absence of Euler equation discounting, familiar from the literature on forward guidance (e.g. McKay et al., 2016).

As a second example, consider a fully hand-to-mouth household (i.e., $\beta = 0$ in (1)). Here we find

$$C^H_{\tau} = I, \quad C^H_{ib} = 0$$

This household immediately consumes any transfer receipt, and does not react to changes in interest rates. The two familiar extreme cases in (10) and (11) will play a special role in communicating the intuition for the policy equivalence result in Section 3.

**Strong Ricardian non-equivalence.** For future reference, I will give a name to a particular feature of the linear consumption derivative map $C_{\tau}$:

**Definition 4.** A consumption function $C(\bullet)$ exhibits strong Ricardian non-equivalence if the induced linear map $C_{\tau}$ is invertible. I denote its inverse by $C_{\tau}^{-1}$.

Under the Barro (1974) definition of Ricardian equivalence, the time path of (lump-sum) taxes used to finance any given fiscal expenditure is irrelevant for consumption – only the present value matters. Mathematically, we see this in the expression for $C^R_{\tau}$ in (10): $C^R_{\tau}$ is rank-1, and so in particular it is not invertible. $C^H_{\tau}$, in contrast, clearly satisfies the notion of strong Ricardian non-equivalence introduced here.

**A review of empirical evidence.** Finally, and as a backdrop for the theoretical analysis in Sections 3 to 5, I here briefly review the state of empirical evidence on the maps $C_{\tau}$ and $C_{ib}$. The remainder of this paper will pay particular attention to models that are consistent with those empirical estimates.

The most robust empirical evidence relates to the first column of $C_{\tau}$ – that is, the response of household spending today and in the future to a lump-sum income receipt today. Here, the literature has so far presented three main findings. First, average impact MPCs (i.e., entry $C_{\tau}(1,1)$) are robustly estimated to be high, around $0.2-0.25$ quarterly for non-durable consumption spending out of an income receipt of around $500$. Second, households spread out their consumption of the lump-sum windfall over time, with iMPCs declining from period
to period (i.e., \( C_\tau(t+1,1) < C_\tau(t,1) \)). And third, the rate of decay is largest between impact and the following period (i.e., \( C_\tau(2,1) \) vs. \( C_\tau(1,1) \)), and then roughly constant at a smaller rate thereafter. These findings suggest a simple three-parameter function for the vector of quarterly iMPCs following a one-off lump-sum transfer receipt: the impact MPC \( \omega \), a rate of decay \( \theta \), and an additional downward scaling after impact of \( \xi \), giving a response vector \((\omega, \omega \xi \theta, \omega \xi \theta^2, \ldots)\). A simple interpolation based on the annual estimates in Fagereng et al. (2018) would roughly suggest \{\( \omega = 0.25, \theta = 0.84, \xi = 0.40 \)\}.

### 3 Policy equivalence

This section presents the policy equivalence result. I first in Section 3.1 state the conditions on \( C_\tau \) and \( C_{ib} \) needed for equivalence of the set of implementable allocations. Section 3.2 uses the Ricardian and hand-to-mouth models to provide some basic intuition. Section 3.3 then goes beyond those knife-edge boundary cases and establishes policy equivalence in empirically realistic, yet analytically tractable models with partially binding liquidity constraints.

#### 3.1 A general equivalence result

I begin by stating the main equivalence result: under suitable restrictions on the images of the two linear maps \( C_\tau \) and \( C_{ib} \), transfer-only and interest rate-only policies are equivalent, in the sense that any allocation of macroeconomic aggregates implementable with one policy is also implementable with the other.\(^5\)

**Theorem 1.** Consider the model of Section 2.1, and let \( \widehat{c} \) be a path of household consumption with zero net present value, i.e., \( \sum_{t=0}^{\infty} \left( \frac{1}{1+i_b} \right)^t \widehat{c}_t = 0 \). Suppose that

\[
\widehat{c} \in \text{image}(C_\tau) \iff \widehat{c} \in \text{image}(C_{ib})
\]

(12)

with the maps \( C_\tau \) and \( C_{ib} \) as defined in Section 2.2. Then the two policy instruments \( \tau \) and \( i_b \) are macro-equivalent: any aggregate allocation that is implementable with transfer-only policy is also implementable with interest rate-only policy, and vice-versa.

\(^5\)Note that, at the level of abstraction of Theorem 1, my results apply to any model that admits an aggregate consumption function (7) and the corresponding linear maps \{\( C_\tau, C_{ib} \)\}. Thus, the conclusions presented here in particular also apply to general behavioral consumption models, e.g. with hyperbolic discounting (present bias), as studied in Laibson (1997) and Laibson et al. (2020).
An easy-to-interpret sufficient condition ensuring the direction “⇐” in the equivalence condition (12) is the notion of strong Ricardian non-equivalence: if \( C_r \) is invertible, then the space of aggregate allocations implementable using transfer-only policies is at least as large that implementable using interest rate-only policies.

**Proof sketch.** Key to the proof of Theorem 1 is the insight that both interest rate as well as transfer policies only directly perturb the model’s equilibrium conditions in two places: first, the output market-clearing condition

\[
C(\bullet) = y(\ell)
\]

and second, the sequence of government budget constraints (5).

A monetary policy is simply a perturbation of relative intertemporal prices that, prior to general equilibrium price and quantity adjustments, induces some zero net present value path of net excess consumption demand

\[
\hat{c}_{PPE}^{i_b} \equiv C_{i_b} \times \hat{i}_b
\]

By (12), we can find *some* transfer sequence \( \hat{r}(\hat{i}_b) \) that induces the exact same perturbation of net excess demand. Intuitively, this equivalent transfer-only policy twists the household intertemporal spending profile by changing the amount of government bonds held by households, thereby affecting the severity of liquidity constraints. That same argument also works in reverse: any transfer-only policy that is consistent with the government budget constraint (5) (i.e., \( \lim_{t \to \infty} \hat{b}_t = 0 \)) induces a perturbation of consumption demand with zero net present value, and so by (12) we can always find an equivalent interest rate-only policy. Intuitively, this first step of the proof works because both policies affect the economy through the same lever: the consumption-savings decision of households.

Given that the transfer and rate policies perturb the output market-clearing and government budget equations by exactly the same amounts, it follows via the implicit function theorem that they must also induce the same general equilibrium paths of inflation, hours worked, wages, and so on.\(^6\) This proof strategy is a simple application of the Arrow &

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\(^6\)Note that this equivalence proof does not presume the absence of a labor supply channel. As households adjust their consumption demand, labor supply may change through the aggregate wage-NKPC (4). By construction, however, two policies that induce the same paths of net excess consumption demand will also induce the same shift in aggregate labor supply. This argument would fail if unions cared not only about aggregate consumption, but also about its cross-sectional distribution. I discuss this case in Appendix A.1.
Debreu (1954) approach to general equilibrium characterization using state-by-state (here \( t \)-by-\( t \)) net excess demand functions. Importantly, since households and the government borrow and lend at identical interest rates, interest rate and transfer policies that lead to identical zero net present value perturbations of household spending – and so identical general equilibrium movements of labor tax revenue and debt servicing costs – also give the same surplus (in net present value terms) in the government budget constraint. The two policies are thus revenue-equivalent: they induce the same general equilibrium adjustments in taxes, analogous to the discussion of revenue equivalence for interest rate policy and distortionary taxation (i.e., unconventional fiscal policy) in Correia et al. (2013).

**Policy at the ELB.** Theorem 1 is directly informative about possible policy options with a (binding) effective lower bound on nominal interest rates.

**Corollary 1.** Suppose that (12) holds, and that conventional interest rate policy is subject to an ELB constraint,

\[ i_{b,t} \geq i^b, \quad \forall t. \tag{13} \]

If, given an exogenous aggregate demand shock path \( \{\zeta_t\}_{t=0}^{\infty} \), the allocation

\[ x_t = \bar{x} \quad \forall t, \quad x \in \{c, \ell, y, w, \pi, d\} \tag{14} \]

is implementable without the ELB constraint (13), then it is also implementable with the ELB constraint.

Corollary 1 reveals that, with policy equivalence, the ELB does not constrain the policymaker: if a zero aggregate output and inflation gap is implementable with the usual interest rate policy, then it remains implementable even when nominal rates are constrained. It follows in particular that, if strong Ricardian non-equivalence holds, then the ELB does not at all constrain the space of implementable allocations.

**Aggregate vs. cross-sectional outcomes.** It is important to note that the equivalence results in Theorem 1 and Corollary 1 concern aggregate outcomes: under the stated assumptions, any sequence of aggregates that is implementable via one policy tool is also implementable via the other one. The results are silent, however, on the potentially differing cross-sectional implications of the two policies. For an individual household \( i \), we have

\[ c_i = C_i(w, \ell, \pi, d; \tau, i_b; \zeta) \]
where $C_i(\cdot)$ is the solution to the individual consumption-savings problem, indexed by that individual’s initial asset holdings and productivity. By construction, equivalent interest rate and transfer policies induce the same general equilibrium paths of wages, hours worked, inflation, and dividend receipts. However, the common aggregate partial equilibrium net excess consumption demand paths $\hat{C}^{PE}_b = \hat{C}^{PE}_r$ may be attained by stimulating the individual consumption demand of different households $i$; if so, the actual general equilibrium consumption path of household $i$, $c_i$, may well differ across the two equilibria. I leave an analysis of these heterogeneous cross-sectional effects to the quantitative HANK model of Section 5.

Outlook. Sections 3.2 and 3.3 will consider particular, analytically tractable variants of the household consumption-savings problem, allowing me to relate the conditions of strong Ricardian non-equivalence and equality of images (12) to (measurable) primitives of the environment. Section 5 will do the same in a rich, quantitative HANK model.

3.2 Non-equivalence in limit cases

I begin with the two illustrative special cases considered in Section 2.2: the baseline Ricardian model, and an economy populated only by hand-to-mouth households.

It is straightforward to see that, in both cases, the equivalence condition (12) is violated, implying a clear pecking order of stabilization policies. For the Ricardian model, (10) reveals that $C^R_\tau$ is a rank-1 matrix: any path of transfers invariably leads to a flat consumption path, with the level of the response equal to the annuity value of the transfer stream. In equilibrium, any such path must have zero net present value (by the government budget constraint), so time-varying transfer policy will have no real effects – the Ricardian equivalence result of Barro (1974). Conversely, as long as $\gamma > 0$, the image of $C^R_\tau$ is the space of all possible consumption sequences with zero net present value, as shown in Appendix B.2. Thus, in the Ricardian model, a policymaker with access to standard interest rate policy can perfectly stabilize output and inflation given any possible path of demand shocks $\{\zeta_t\}_{t=0}^\infty$, while a policymaker with access only to transfer-only policies is powerless. The situation is reversed with hand-to-mouth households: $C^H_\tau$ is now the identity map and so invertible (i.e., strong Ricardian non-equivalence), while $C^H_b$ is a map of zeros, as hand-to-mouth households do not respond to budget-neutral changes in interest rates. (12) thus again fails, with transfer stimulus now a powerful demand management tool, and interest rate policy powerless.

While instructive, both of these cases are clearly at odds with the empirical evidence reviewed in Section 2.2: average MPCs are neither close to zero (as predicted by the Ricardian
model), nor equal to one (as in the hand-to-mouth model). The next section thus turns to
models that are consistent with existing evidence on household consumption behavior, and
then uses those to revisit strong Ricardian non-equivalence and the general condition (12).

3.3 Equivalence in tractable, empirically relevant models

In the remainder of this paper I will argue that, away from the knife-edge cases of Section 3.2,
policy equivalence holds generically. This section makes the point in a series of analytically
tractable models, designed explicitly to speak to the empirical evidence on household con-
sumption behavior reviewed in Section 2.2. For each model I will proceed in two steps: first
I establish that strong Ricardian non-equivalence holds (so the ELB is never a constraint),
and then I discuss the condition on matrix images (12) required for full policy equivalence.

SPENDER-SAVER: MATCHING $\omega$. I begin with a very simple two-agent spender-saver struc-
ture (Campbell & Mankiw, 1989; Farhi & Werning, 2017). With a fraction $\mu \in (0,1)$ of
spenders, this model is simply a convex combination of the pure Ricardian and pure hand-
to-mouth models considered before. We first of all have

$$C^T_\tau = \mu \times I + (1 - \mu) \times C^R_\tau$$

To match a given average impact MPC $\omega$, it suffices to set $\mu = \frac{1}{\beta} [\omega - (1 - \beta)] \approx \omega$. Thus, with $\omega = 0.25$, the model is a well-behaved weighted average of the two extreme cases studied in
Section 3.2, and as such satisfies strong Ricardian non-equivalence. Formally, for $\beta = 1$, the
eigenvalues of $C^T_\tau$ are $(1, \omega, \omega, \omega, \ldots)'$, so it follows immediately that any ELB on nominal
rates will not constrain the space of implementable allocations.

Next, we have

$$C^T_{ib} = \frac{-1}{\gamma} (I - C^T_\tau) \begin{pmatrix} 1 & 1 & 1 & \ldots \\ 0 & 1 & 1 & \ldots \\ 0 & 0 & 1 & \ldots \\ \vdots & \vdots & \vdots & \ldots \end{pmatrix}$$

The eigenvalues of $I - C^T_\tau$ are $(0, 1 - \omega, 1 - \omega, 1 - \omega, \ldots)'$, and so interest rate policy can
generically induce any possible (zero net present value) perturbation of consumption demand.
(12) thus holds, and the model features policy equivalence.

Relative to the evidence reviewed in Section 2.2, the main limitation of the spender-saver
model is that it fails to capture the dynamic response pattern of household spending to
lump-sum income receipt. The next two model extensions address this concern, allowing me to arrive at formulas that remain informative even for the HANK model of Section 5.

Bond-in-utility: Matching \( \{\omega, \theta\} \). To match the gradual decay of household MPCs discussed in Section 2.2, I consider the consumption-savings problem of a representative household with separable isoelastic preferences over consumption and wealth holdings, exactly as in Michaillat & Saez (2018) and Kaplan & Violante (2018). As I show formally in Appendix B.3, the linear transfer derivative map \( \mathcal{C}_\tau \) implied by this model admits a simple characterization in the limit \( \beta \to 1 \):

\[
\mathcal{C}_\tau^B \approx \omega \times \begin{pmatrix}
1 & \theta & \theta^2 & \ldots \\
\theta & 1 & \theta & \ldots \\
\theta^2 & \theta & 1 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\] (17)

where the approximate equality indicates asymptotic equality, in the sense of convergence in weak (Hilbert-Schmidt) matrix norm.\(^7\) The expression in (17) reveals that, when calibrated to match empirical evidence on gradually decaying household MPCs, the reduced-form model with bonds in the utility function also has very strong predictions for the remainder of the consumption derivative map \( \mathcal{C}_\tau^B \). To translate those predictions into a discussion of policy equivalence, I consider first the eigenvalue distribution of \( \mathcal{C}_\tau^B \).

**Proposition 1.** Consider the consumption-savings problem with bonds in the utility function, with the transfer derivative matrix \( \mathcal{C}_\tau^B \) defined in (17). The cumulative distribution function of the distribution of eigenvalues of \( \mathcal{C}_\tau^B \) is given as

\[
F^B(\lambda) = 1 - \frac{1}{\pi} \arccos \left( \frac{1 + \theta^2 - \frac{(1-\theta^2)\omega}{\lambda}}{2\theta} \right), \quad \lambda \in [\underline{\lambda}, \bar{\lambda}]
\]

with the bounds of the support \([\underline{\lambda}, \bar{\lambda}]\) given as

\[
\underline{\lambda} \equiv \omega \times \frac{1 - \theta}{1 + \theta}, \quad \bar{\lambda} \equiv \omega \times \frac{1 + \theta}{1 - \theta}
\]

\(^7\)I provide a formal definition of this notion of convergence in Definition B.1, stated in Appendix B.1.

\(^8\)The proof of Proposition 1 leverages the fact that, at least asymptotically, \( \mathcal{C}_\tau^B \) is a Toeplitz matrix. By Szegö’s theorem, the limiting eigenvalue distribution of any such matrix can be derived directly from the Fourier series of its coefficients. I provide details in Appendix B.1.
Figure 1: Eigenvalue distributions of the asymptotic linear maps $C^B_\tau$ and $C^{TB}_\tau$, parameterized to give $\{\omega = 0.22, \theta = 0.73\}$ and $\{\omega = 0.25, \theta = 0.84, \xi = 0.40\}$, respectively.

The blue line in Figure 1 shows the distribution of eigenvalues in a simple bond-in-utility model parameterized to give $\omega = 0.22$ and $\theta = 0.73$, in line with empirical evidence. Given this parameterization, the eigenvalue distribution is bounded below away from zero (with $\lambda = 0.034$), so $C^B_\tau$ is invertible – i.e., strong Ricardian non-equivalence holds. It follows from Corollary 1 that, in this particular model of the consumption-savings problem, a lower bound on nominal rates does not at all constrain the space of implementable allocations.

Finally, for policy equivalence, it remains to evaluate the condition in (12) on the images of the linear maps $C^B_\tau$ and $C^B_{ib}$. As also shown in Appendix B.3, if $\beta(1 + \bar{r}_b) = 1$, then the relation between $C^B_\tau$ and $C^B_{ib}$ is exactly as in the baseline Ricardian and spender-saver models:

$$C^B_{ib} = -\frac{1}{\gamma}(I - C^B_\tau) \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{18}$$

The eigenvalues of $I - C^B_\tau$ are then just 1 minus the eigenvalues of $C^B_\tau$, so $C^B_{ib}$ is generally not invertible. However, for empirically relevant values of $\omega$ and $\theta$, it is straightforward to verify numerically that all consumption sequences with zero net present value lie in the
image of $C^B_T$, ensuring full policy equivalence.\footnote{To be precise, in finite-horizon approximations of the linear map $C^B_{i_b}$, I find that exactly one of its $T$ eigenvalues is 0, where $T$ is the approximation horizon.}

**Hybrid: Matching $\{\omega, \theta, \xi\}$.** For a third illustration, I turn to a hybrid model, featuring both a fringe of hand-to-mouth spenders as well as savers with bonds in their utility function. The principal appeal of this model is that it can match the entire vector $\{\omega, \theta, \xi\}$ of moments characterizing the first column of empirically realistic matrices $C_T$. In fact, as I show in Section 5, this model will in fact turn out to be rich enough to be quantitatively similar to a calibrated HANK model.

Combining (15) and (17), we find the transfer consumption derivative matrix as

$$C^{TB}_T \approx \omega \times \begin{pmatrix} 1 & \xi \theta & \xi \theta^2 & \ldots \\ \xi \theta & 1 & \xi \theta & \ldots \\ \xi \theta^2 & \xi \theta & 1 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$  \hspace{1cm} (19)

I can now proceed as before: first evaluate the eigenvalue distribution of $C^{TB}_T$ to check for strong Ricardian non-equivalence, and then compare the images of $C^{TB}_T$ and $C^{TB}_{i_b}$.

**Proposition 2.** Consider the consumption-savings problem with bonds in the utility function and spender households, with the transfer derivative matrix $C^{TB}_T$ defined in (19). The cumulative distribution function of the distribution of eigenvalues of $C^{TB}_T$ is given as

$$F^{TB}(\lambda) = 1 - \frac{1}{\pi} \arccos \left( \frac{1 + \theta^2 - \frac{\xi(1-\theta^2)}{\xi(1-\xi)}}{2\theta} \right)$$

with the bounds of the support $[\underline{\lambda}, \overline{\lambda}]$ given as

$$\underline{\lambda} \equiv \omega \times \left[ (1 - \xi) + \xi \frac{1 - \theta}{1 + \theta} \right], \quad \overline{\lambda} \equiv \omega \times \left[ (1 - \xi) + \xi \frac{1 + \theta}{1 - \theta} \right]$$

The asymptotic distribution of eigenvalues in a model matching the targets $\{\omega = 0.25, \theta = 0.84, \xi = 0.4\}$ is depicted as the orange line in Figure 1. As before, the distribution is bounded away from zero (with $\underline{\lambda} \approx 0.16$), so strong Ricardian non-equivalence holds, and a lower bound on nominal rates does not at all affect the space of implementable allocations.
The model also yet again robustly features full policy equivalence. For $C_{tb}^{TB}$ we find that, exactly as before,

$$C_{tb}^{TB} = \frac{-1}{\gamma} (I - C_{\tau}^{TB}) \begin{pmatrix} 1 & 1 & 1 & \ldots \\ 0 & 1 & 1 & \ldots \\ 0 & 0 & 1 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$ (20)

The previous discussion still applies: for empirically relevant values of $\{\omega, \theta, \xi\}$, any consumption path with zero net present value lies in the images of $C_{TB}^{TB}$ and $C_{tb}^{TB}$, so transfer-only and interest rate-only policies are macro-equivalent.

**Summary & outlook.** Having established that equivalent transfer stimulus policies robustly exist across a family of empirically relevant yet analytically tractable models, I will in Section 4 use those same models to explicitly characterize the mapping between the equivalent rate and transfer policies. Section 5 will then evaluate the extent to which the lessons on policy equivalence and the equivalence mapping extend to a richer HANK model.

### 4 Characterizing the equivalence mapping

This section provides formulas for the mapping between conventional nominal interest rate policies and macro-equivalent transfer stimulus. This step is essential because the analysis in Section 3 has revealed only that equivalent transfer policies exist; the equivalence mapping, however, may well be ill-behaved, limiting the practical usefulness of my results.

I begin in Section 4.1 with a brief general discussion, and then in Section 4.2 provide explicit formulas that rely only on the empirically measured household MPC statistics $\{\omega, \theta, \xi\}$. All results in this section refer to the three tractable models studied in Section 3.3.

#### 4.1 General expression

Consider an arbitrary net excess demand path $\hat{c}^{PE}$. Under strong Ricardian non-equivalence, the policymaker can induce the same perturbation of private net excess demand by setting

$$\hat{\tau}(\hat{c}^{PE}) \equiv C_{\tau}^{-1} \times \hat{c}^{PE}$$ (21)

It follows that, to learn about the mapping into equivalent transfer stimulus — i.e., the mapping relevant to sidestep ELB constraints —, we need to learn about $C_{\tau}^{-1}$. 21
While (21) provides the mapping from net excess demand space to the equivalent transfer policy, interest may also center on the mapping from interest rate space to transfer space. Here we have

$$\hat{\tau}(\hat{i}_b) \equiv C^{-1}_\tau \times C_{ib} \times \hat{i}_b$$

Using the results in Section 3.3, we can simplify this expression to get

$$\hat{\tau}(\hat{i}_b) \equiv -\frac{1}{\gamma}(C^{-1}_\tau - I) \begin{pmatrix} 1 & 1 & 1 & \ldots \\ 0 & 1 & 1 & \ldots \\ 0 & 0 & 1 & \ldots \\ 0 & 0 & 0 & \ldots \end{pmatrix} \times \hat{i}_b$$

Thus $C_\tau$ and its inverse $C^{-1}_\tau$ – now together with the intertemporal elasticity of substitution $\gamma$ – are again central to the characterization of the equivalent transfer stimulus policy.

### 4.2 A simple sufficient statistics formula

The main result of this section are (approximate) closed-form expressions for $C^{-1}_\tau$ in my three analytical models – spender-saver, bonds in the utility function, and hybrid –, and thus by (21) closed-form expressions for $\hat{\tau}$ as a function solely of the sufficient statistics $\{\omega, \theta, \xi\}$ as well as the target net excess consumption demand path $\hat{c}^{PE}$. These closed-form expressions reveal that equivalent transfer policies are robustly well-behaved, scale in $1$ over the average MPC, and are generally – except for the extreme spender-saver economy – less persistent than the underlying excess demand paths that they are supposed to induce.\(^\text{10}\)

**Spender-Saver: Matching $\omega$.** In the simple spender-saver model, policy characterization is straightforward.

**Proposition 3.** Consider the consumption-savings problem with spenders and savers, and suppose that $\beta \to 1$. Then

$$\left(C^T_\tau\right)^{-1} = \frac{1}{\omega} \times I$$

(22)

Figure 2 illustrates. The grey line in the left panel shows that, to increase net excess demand by 1 per cent at $t = 4$, the policymaker simply needs to pay out a transfer of size $1/\omega$.

\(^{10}\)This section provides the mapping between arbitrary sequences of nominal interest rates and transfers. Appendix C.1 does the same for policy rules.
at exactly that time. Correspondingly, the grey line in the right panel shows that, to engineer a persistent increase in net excess demand, the policymaker needs to increase transfers with exactly the same persistence, throughout scaling up by a factor of $1/\omega$. Evidently, as we approach the Ricardian limit without spenders, the required transfer stimulus becomes arbitrarily large, and so the equivalent policy diverges.

**Bond-in-utility: Matching $\{\omega, \theta\}$.** I next consider the bond-in-utility model. In this model, households spend any lump-sum receipt gradually. Proposition 4 characterizes the effect of this persistence on the shape of $C^{-1}_{\tau}$.

**Proposition 4.** Consider the consumption-savings problem with bonds in the utility function, with the approximate transfer derivative matrix $C^B_{\tau}$ defined in (17). Then

$$
(C^B_{\tau})^{-1} = \frac{1}{\omega} \times \begin{pmatrix}
\frac{1}{1-\theta^2} & -\frac{\theta}{1-\theta^2} & 0 & 0 & \ldots \\
-\frac{\theta}{1-\theta^2} & \frac{1+\theta^2}{1-\theta^2} & -\frac{\theta}{1-\theta^2} & 0 & \ldots \\
0 & -\frac{\theta}{1-\theta^2} & \frac{1+\theta^2}{1-\theta^2} & -\frac{\theta}{1-\theta^2} & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

(23)

The left panel (blue line) again shows the entries of the fifth column of $(C^B_{\tau})^{-1}$, while the

![Figure 2: Left panel: Entries of the fifth column of the (asymptotic) inverse matrices $(C^T_{\tau})^{-1}$, $(C^B_{\tau})^{-1}$ and $(C^{TB}_{\tau})^{-1}$, parameterized to give $\{\omega = 0.25\}, \{\omega = 0.22, \theta = 0.73\}$ and $\{\omega = 0.25, \theta = 0.84, \xi = 0.40\}$, respectively. Right panel: Equivalent transfer policies to mimic a shock inducing excess demand of $\hat{c}_{t}^{PE} = 0.8^t$, same model parameterizations.](image)
right panel depicts the transfer stimulus required to mimic the path \( \hat{c}^{PE} \). Relative to the simple spender-saver model, two observations stand out. First, equivalent stimulus policies are still well-behaved, as expected; in particular, they continue to scale in \( \frac{1}{\omega} \). Second, the endogenous persistence of the household spending response – encapsulated in \( \theta \) – now results in the equivalent transfer policy being strictly less persistent than the targeted net excess demand path. Figure 2 provides two different perspectives on this finding. The left panel reveals that, for a one-off shock at \( t = 4 \), negative transfers (i.e., higher taxes) at \( t = 3 \) and \( t = 5 \) are needed to offset leakage to adjacent periods. In the right panel, the underlying shock is roughly as persistent as the cross-period leakage \( \theta \) (shock persistence of 0.8 vs. \( \theta = 0.73 \)), so transfers need to remain only mildly positive from \( t = 1 \) onwards.\(^{11}\)

**HYBRID: MATCHING \( \{\omega, \theta, \xi\} \).** Finally I turn again to the rich hybrid model, allowing me to match the entire vector of empirical targets \( \{\omega, \theta, \xi\} \). While the previous two models were useful to communicate intuition, this third model will be rich enough to yield formulas with the potential to apply even in the rich HANK model of Section 5.

**Proposition 5.** Consider the consumption-savings problem with bonds in the utility function and spender households, with the transfer derivative matrix \( C^{TB} \) defined in (19). Then the \((i,j)\)th entry of the inverse of \( C^{TB} \) satisfies

\[
(C^{TB})^{-1}(i,j) \approx \frac{1}{\omega} \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(|i-j|\lambda) \frac{1 - 2\theta \cos(\lambda) + \theta^2}{\xi(1 - \theta^2) + (1 - \xi)(1 - 2\theta \cos(\lambda) + \theta^2)} d\lambda \tag{24}
\]

where the approximate equality indicates asymptotic equality.

The orange lines in Figure 2 show that the resulting equivalent transfer policies naturally combine the intuitions from the simpler spender-saver and bond-in-utility models considered before. The policies remain well-behaved, continue to scale in the inverse average MPC \( \omega^{-1} \), and are less persistent than the target excess demand path \( \hat{c}^{PE} \). In particular, the entries of the inverse \((C^{TB})^{-1}\) are simply smoothed, stretched-out versions of the sharp zig-zags in (23). The inverse \((C^{TB})^{-1}\) – and so the equivalent transfer policies that it implies – thus continue to have a simple and intuitive shape.

\(^{11}\)Note that all three of the equivalent transfer stimulus plotted in Figure 2 by construction have the exact same net present value as the path \( \hat{c}^{PE} \), equal to \( \sum_{t=0}^{\infty} \left( \frac{1}{1+\bar{b}} \right)^t 0.8^t \). In all models I set \( \beta = 1 \), but they are allowed to differ in their steady-state real rates of interest.
5 Equivalent policies in a quantitative HANK model

I now study the equivalence of interest rate and transfer policies in a rich heterogeneous-agent business-cycle model with nominal rigidities (HANK).

The purpose of this section is twofold. First, I document that the conclusions of Sections 3.3 and 4.2 extend with very little change to quantitative incomplete-markets models: interest rate and transfer policies are indeed robustly equivalent, and the mapping between them remains well-characterized by the most general of my three sufficient statistics expressions, (24). Second, I leverage the rich cross-sectional dimension of the HANK model to establish that, while equivalent in terms of implementable aggregates, interest rate and transfer policies are likely to have very different effects in the household cross section.

5.1 Model details

I consider a calibrated version of the baseline model of Section 2.1. The environment is thus similar to those in McKay et al. (2016) and Auclert et al. (2018).

Parameterization. I present a brief overview of the model parameterization here, and relegate further details to Appendix A.3.

Households face the same income risk process as in Kaplan et al. (2018), and can self-insure by saving, but not borrowing. I calibrate total bond holdings to the amount of liquid wealth in the U.S. economy; corporate wealth, instead, is perfectly illiquid, with households receiving dividend payments as a function of their labor productivity. The resulting partial equilibrium consumption-savings problem fully characterizes the consumption response matrices $C_{\tau}^H$ and $C_{i_o}^H$, and thus contains all the information needed to evaluate policy equivalence and to construct the mapping from infeasible rate policy to equivalent transfer stimulus.

The remainder of the model then matters only for the mapping from partial equilibrium net excess consumption demand paths – and so in particular equivalent transfer policies – into general equilibrium outcomes. I consider a simple constant-returns-to-scale production function as well as conventional degrees of nominal wage and price stickiness.

Consumption behavior. Figure 3 displays entries of the first column of $C_{\tau}^H$ (grey line). Since income risk is high, but opportunities for self-insurance are limited, the average quarterly MPC out of a one-off, lump-sum income gain of $500 is high, at around 25 per cent quarterly. The iMPC path first drops off sharply, and then decays gradually, consistent with
Figure 3: iMPCs in the HANK model. The grey line shows the actual path of iMPCs for a 500$ income receipt at $t = 0$, while the blue dashed line shows the path for a spender-saver model with bonds in the utility function, parameterized to match the model-implied iMPCs for $t \in \{0, 1, 2\}$.

the evidence reviewed in Section 2.2. Figure 3 also shows the path of iMPCs implied by a simple spender-saver model with bonds in the utility function (blue dashed line), calibrated to match the HANK iMPCs at $t = 0$, $t = 1$ and $t = 2$. The key take-away from the figure is that the two lines remain almost indistinguishable even for $t > 2$.

In the next subsection I investigate whether the similarity extends beyond the first column of $C_\tau$, and so whether (i) policy equivalence still holds, and (ii) equivalent transfer policies can still – at least approximately – be recovered from a small number of measurable iMPCs.

5.2 Macro policy equivalence in HANK

By the results in Section 3.1, we know that transfer-only and interest rate-only policies are macro-equivalent as long as any zero net present value path of consumption in the image of one map is also in the image of the other one. Computing these two linear maps – and so their images – is straightforward using either the simple brute-force approach of Boppart et al. (2018) or the more efficient algorithm of Auclert et al. (2019).

I find that $C^H_\tau$ is invertible, so strong Ricardian non-equivalence holds, and any possible lower bound on nominal interest rates does not at all constrain the space of implementable allocations; furthermore, the image of $C^H_{ib}$ is the space of all consumption sequences with zero
net present value, so the two instruments are macro-equivalent. Thus, just like the analytical models of Section 3.3, the quantitative HANK model is very clearly situated in between the two extreme boundary cases of perfectly Ricardian and perfectly hand-to-mouth households.

**The equivalence mapping.** Next, I study the properties of the equivalence mapping from infeasible interest rate policy to equivalent transfer stimulus.

In keeping with the literature on optimal monetary policy in business-cycle models with nominal rigidities (Galí, 2015; Woodford, 2011), I subject the economy to a one-off, unexpected demand shock path \( \{ \zeta_t \}_t=0^\infty \). Following this shock, without policy intervention, output and consumption will necessarily deviate from their steady-state values, at least as long as prices and wages are not perfectly flexible. By the usual divine coincidence logic, however, nominal interest rates can in principle be set to perfectly offset this shock, thus keeping the output and inflation gaps at zero forever. For the remainder of this section, I will assume that this conventional policy prescription is unattainable because of a binding ELB on nominal interest rates, and instead study the equivalent transfer stimulus. Letting \( \tilde{c}_{\zeta}^{PE} \) denote the shock-induced partial equilibrium net excess consumption demand path, this equivalent transfer stimulus satisfies

\[
\hat{\tau}(\zeta) = (C_t^H)^{-1} \times (-\tilde{c}_{\zeta}^{PE})
\]

To illustrate the properties of the HANK-implied equivalence mapping (25), I will focus on two particular sequences of shocks \( \{ \zeta_t \}_t=0^\infty \): two AR(1) sequences, one transitory (with persistence \( \rho_{\zeta} = 0.2 \)) and one persistent (with persistence \( \rho_{\zeta} = 0.7 \)), and both normalized to depress partial equilibrium consumption demand by one per cent on impact.

Figure 4 reveals that the conclusions of Section 4.2 for my analytical models extend with little change to the richer quantitative HANK model. To begin, the green lines show the net excess demand paths needed to perfectly offset a given negative demand shock sequence \( \{ \zeta_t \}_t=0^\infty \). With steady-state consumption of around $12,000 per quarter for each household, a one per cent excess demand increase corresponds to around $120. The grey lines in Figure 4 then show the exact model-implied equivalent transfer stimulus policies \( \hat{\tau}(\zeta) \), on the left for a persistent demand shock, and on the right for a transitory demand shock. Exactly as in Section 4.2, the required transfer scales in the inverse average MPC of the economy, and is less persistent than the excess demand path that it tries to engineer. Finally, the blue lines show the equivalent transfer stimulus policies computed using the sufficient statistics formula in (24), with \( \{ \omega, \theta, \xi \} \) set to their values in the HANK model. The figure reveals that, for both persistent as well as transitory shocks, the simple sufficient
The green lines show the household spending net excess demand path (in $) needed to offset impatience (i.e., natural rate) shocks $\zeta$ with persistence $\rho = 0.7$ (left panel) and $\rho = 0.2$ (right panel), with both shocks normalized to move partial equilibrium consumption demand by one per cent on impact. The grey lines show the exact equivalent transfer policies, while the blue lines show the approximate policies computed using the sufficient statistics approximation. The statistics expression provides an excellent approximation of the actual equivalent transfer policy, with the approximation error at all times very small relative to the impact size of the policy.\(^{12}\) Intuitively, the approximation is good because both households with bonds in the utility function and households close to (but not at) the borrowing constraint $b$ tend to gradually spend down lump-sum income receipts.

**Interpreting past policy stimulus.** The results displayed in Figure 4 also help to shed new light on past U.S. policy practice. In 2008, as part of the Economic Stimulus Act, U.S. fiscal authorities sent out a large number of checks, staggered over two quarters. By the grey and blue lines in the left panel of Figure 4, a staggered stimulus policy of this sort closely approximates the transfer stimulus needed to offset a shock to the equilibrium real rate with a half-life of around six months. In particular, re-scaling the size of the initial check to around $600 per household (as in 2008), the stimulus would offset a shock to the real rate that contracts partial equilibrium consumption demand by around 1.4 per cent on impact. Using the model-implied map $C^H_{ib}$, we can translate this equivalence statement from

\(^{12}\)The largest error is equal to around $30 for the persistent shock, and $50 for the transitory shock.
Aggregate Consumption Response to Transfer Stimulus

Figure 5: Response of aggregate consumption to the exogenous demand shock $\zeta$ with persistence $\rho_\zeta = 0.7$ (green) vs. demand shock plus: (i) the sufficient statistics approximation to the equivalent transfer (blue), (ii) a simple “Helicopter money” policy in which the sufficient statistics approximation is truncated to be weakly positive (orange).

net excess demand space back to interest rate space: in the model, given a unit intertemporal elasticity of substitution, a 1.4 per cent demand stimulus corresponds to a transitory nominal interest rate cut of around 170 basis points.

General equilibrium outcomes. Finally, I use the non-consumption blocks of the model to map the partial equilibrium net excess demand paths displayed in Figure 4 into general equilibrium counterfactuals. Figure 5 shows results for three different exercises.

The main take-away from Figure 5 is that simple, practically feasible policies constructed using the sufficient statistics formula (24) do a good job of almost perfectly closing the aggregate output gap. To begin, the green line shows the aggregate effects of an unaccommodated persistent negative demand shock (i.e., $\rho_\zeta = 0.7$). Due to a combination of high MPCs and nominal rigidities, equilibrium consumption drops by even more than the direct shock to spending (which was normalized to one per cent). By construction, the exact stabilization policy from (25) (not shown) would perfectly offset this output contraction, keeping consumption, output and inflation at steady state forever. The blue line instead shows what happens after the transfer policy characterized by my simpler sufficient statistics approxi-
mation. Since that approximate policy is close to the actual model-implied equivalent policy (recall the left panel of Figure 4), it is unsurprising that the implied equilibrium path of consumption is close to zero throughout, with a maximal deviation from steady-state consumption of only 0.09 per cent. Finally, the orange line truncates the sufficient statistics policy to be weakly positive at all times (i.e., a simple “helicopter money” policy), and then finances this transfer outlay with a very delayed increase in taxes – a policy close to that of 2008.\textsuperscript{13} Unsurprisingly, since this helicopter money stimulus only involves a very delayed increase in taxes, the implied general equilibrium consumption path now more than offsets the initial shock, but the difference remains very small throughout.

5.3 Non-equivalence at the household level

So far, the analysis in this section has merely confirmed that the lessons from simple analytical models of the consumption-savings problem extend with little change to a quantitative incomplete-markets model. An important added benefit of this richer model, however, is that it allows us to go beyond the macro equivalence of the two stimulus policies and gauge the extent of micro non-equivalence – i.e., heterogeneity in the transmission of interest rate cuts and transfer stimulus in the cross section of households.

I compare the distributional effects of the macro-equivalent persistent nominal interest rate and transfer policies studied in Section 5.2 (i.e., the policies offsetting the demand shock with $\rho_\zeta = 0.7$). Since both policies by construction induce the same general equilibrium responses of wages, hours worked, inflation and dividends, I will pay particular attention to the direct effects of the two policies, defined as the response of optimal household consumption demand to the two policies alone, fixing all non-policy variables at their steady state values forever. To represent the policies’ distributional implications, I aggregate individual-specific consumption responses separately by household liquid wealth percentile. Figure 6 illustrates, giving the cross-section of impact consumption responses by wealth percentile for the two policies, separated into direct and general equilibrium effects.

While macro-equivalent, the interest rate and transfer policies turn out to have vastly different effects in the cross-section of households. To engineer the required one per cent increase in household consumption demand on impact, interest rate cuts work mostly by directly stimulating the consumption of rich households. This is not surprising: wealthy

\textsuperscript{13} Specifically, I use the financing rule (A.4). Since the financing is so delayed, results at short horizons look very similar if the financing relied on distortionary labor taxes instead of uniform lump-sum taxes.
Figure 6: Left panel: The green line shows the time-0 direct household spending response by liquid wealth percentile to the nominal interest rate policy that perfectly offsets the persistent negative demand shock $\zeta$ of Section 5.2. The shaded purple area gives the corresponding indirect general equilibrium effects. Right panel: Analogous figure for the equivalent uniform transfer stimulus.

Households substitute intertemporally, while poor households are close to their borrowing constraint and so do not. Thus, at the low end of the liquid wealth distribution, monetary policy operates largely through indirect effects (Kaplan et al., 2018). Conversely, the equivalent lump-sum transfer policy – which has zero net present value, simply because the demand shortfall $\hat{c}_P$ necessarily has zero net present value – barely moves consumption of the rich. At the same time, it relaxes borrowing constraints of the poor, and so directly stimulates their consumption, with this direct effect actually dwarfing indirect general equilibrium effects.

The results presented here may inform the conduct of optimal stabilization policy. For a policymaker concerned only with aggregate output and inflation, interest rate policy and transfer stimulus are equivalent. However, while interest rate policy works through intertemporal substitution, the equivalent transfer has the added benefit of providing cross-sectional consumption insurance. I further elaborate on this discussion in Appendix C.2.

6 Extension to investment

I now extend the policy equivalence result to an economy with investment. Generally, in such an environment, monetary policy affects real outcomes through shifts in both consumption
and investment behavior. I first show that, in this case, transfer stimulus can still perfectly replicate the direct consumption channel of monetary transmission. I then proceed to discuss other (fiscal) instruments that allow us to also replicate the investment channel and so again circumvent any constraints on nominal interest rates, as in Corollary 1.

6.1 Extended model

I augment the model of Section 2.1 to allow for productive capital. The firm block in this extended environment closely follows the tradition of standard business-cycle modeling (e.g. Smets & Wouters, 2007; Justiniano et al., 2010). I provide a brief sketch of this familiar model here, and relegate further details to Appendix A.4.

**Production.** A unit continuum of identical, perfectly competitive firms \( j \in [0, 1] \) produces a homogeneous intermediate good, sold at real relative price \( p^I_t \). The problem of firm \( j \) along the perfect foresight transition path is to

\[
\max_{\{d_{jt}, \ell_{jt}, k_{jt}, b^f_{jt}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left( \prod_{q=0}^{t-1} \frac{1 + \pi_q - 1}{1 + i_{b,q}} \right) e^{\zeta t} d_{jt}
\]

subject to the flow budget constraint

\[
d_{jt} = p^I_t y(\ell_{jt}, k_{jt-1}) - w_t \ell_{jt} - (1 - \delta)k_{jt-1} + \tau_{f,t}(\{i_{jt-q}\}_{q=0}^{t}) - b^f_{jt} + \frac{1 + i_{b,t-1}}{1 + \pi_t} b^f_{jt-1} \tag{26}
\]

as well as constraints on equity and debt issuance

\[
d_{jt} \geq d, \quad b^f_{jt} \geq b^f
\]

To summarize, intermediate goods producers hire labor on spot markets, invest, pay out dividends, and save in liquid bonds, perhaps subject to financing constraints.\(^{14}\) My only twist to this entirely familiar model block is that I allow for a general fiscal investment stimulus policy \( \tau_{f,t}(\bullet) \), mapping investment today into future payments to the firm. Importantly, this general set-up nests the popular bonus depreciation stimulus policy, in which investment today reduces tax liabilities in the future (Zwick & Mahon, 2017; Koby & Wolf, 2020). I will index time-\( t \) investment stimulus policies by a single parameter \( \tau_{f,t} \).

\(^{14}\) Capital or investment adjustment costs could be added without affecting any of the subsequent results.
Proceeding exactly as in Section 2.2, we can define an aggregate investment function \( I(\bullet) \); I relegate a discussion of the arguments of this function to Appendix A.4, as it is not essential here. We can then as before consider a fiscal stimulus map

\[
\mathcal{I}_{rf} \equiv \frac{\partial I(\bullet)}{\partial \tau_f}
\]

Statements about the degree to which an ELB on nominal interest rates can be circumvented will again be statements about the properties of \( \mathcal{I}_{rf} \) (and \( C_{\tau} \), as before).

Rest of the economy. The intermediate good is sold to monopolistically competitive retailers subject to nominal rigidities, summarized again with a general price-NKPC:

\[
\pi_t = \pi(p^I_t; \pi_{t+1})
\]  

(27)

The remainder of the model is unchanged. The extension of the equilibrium definition in Definition 3 is then straightforward, and provided in Appendix A.4.

6.2 Policy equivalence

In this extended model, monetary policy operates through two channels: first, as before, it affects household consumption demand, and second, it changes firm investment and so labor hiring as well as intermediate goods production. Thus, transfer stimulus policy alone is now insufficient to replicate the effects of (infeasible) conventional monetary policy – exactly as in Correia et al. (2013), an additional policy instrument is needed. In a straightforward generalization of Theorem 1 and Corollary 1, Theorem 2 shows that invertibility of \( C_{\tau} \) and \( \mathcal{I}_{rf} \) is sufficient to leave the space of implementable allocations unchanged.

**Theorem 2.** Consider the extended model of Section 6.1. Suppose that \( C_{\tau} \) and \( \mathcal{I}_{rf} \) are both invertible, and that conventional interest rate policy is subject to an ELB constraint (13). If, given an exogenous aggregate demand shock path \( \{\zeta_t\}_{t=0}^\infty \), the allocation

\[
x_t = \bar{x} \quad \forall t, \quad x \in \{c, \ell, y, i, k, w, \pi\}
\]

is implementable without the ELB constraint (13), then it is also implementable with a binding ELB constraint.
Transfer stimulus now only replicates the consumption channel of monetary policy transmission. If additionally $\mathcal{I}_{\tau_f}$ is invertible, then the general form of investment stimulus considered in (26) suffices to replicate the investment channel, and so leave the set of implementable allocations unchanged.\footnote{Unlike the case studied in Theorem 1, the space of allocations implementable via equivalent fiscal stimulus is now strictly larger than the space implementable via conventional monetary policy: with its two separate levers, fiscal policy can differentially move the investment and consumption channels of net excess demand.} The remainder of this section gives examples of equivalent investment policies under different possible assumptions on production side primitives.

Suppose first that intermediate goods firms are not subject to any financial frictions, as in most conventional business-cycle models. Then, as is well-known, bonus depreciation is equivalent to a standard investment subsidy, and so the firm budget constraint becomes\footnote{Firm bond holdings are now indeterminate, so I set them to zero at all times.}

$$d_{jt} = p_t y(\ell_{jt}, k_{jt}) - w_t \ell_{jt} - [k_{jt} - (1 - \delta)k_{jt-1}] + \tau_{f,t}i_t$$

Given this equivalence of bonus depreciation and investment subsidies, the argument of Correia et al. (2013) applies unchanged: interest rates $i_{b,t}$ and the subsidy $\tau_{f,t}$ enter investment optimality conditions symmetrically, so the investment channel of monetary policy can be replicated by matching wedges. Combining this observation with the results of Section 3 we can conclude that, in a medium-scale HANK model with investment, an entirely conventional mix of fiscal instruments – transfer stimulus payments together with bonus depreciation tax stimulus – suffice to replicate any monetary allocation.

Second, with binding financial frictions, investment responds even to unconditional cash transfers to firms, i.e., to a subsidy schedule $\tau_{f,t}(\{i_{qjt}\}_{q=0}^t)$ that is independent of the level of investment. Analogously to my earlier discussion of Ricardian equivalence, this violation of Modigliani-Miller may in fact result in an invertible matrix $\mathcal{I}_{\tau_f}$, thus again ensuring policy equivalence. Contrary to $C_\tau$, however, empirical evidence on entries of $\mathcal{I}_{\tau_f}$ is more limited, so I leave a discussion of the practical implications of this observation to future work.\footnote{Examples of papers estimating parts of this $\mathcal{I}_{\tau_f}$ include Melcangi (2020) and Lian & Ma (2021).}

7 A comparison of policy equivalence results

The objective of this paper is identical to that of Correia et al. (2013) and Farhi et al. (2014): I characterize (feasible) policies that can, under plausible assumptions on model primitives,
replicate the effects of other conventional yet in some cases infeasible policies. The underlying conceptual approach, however, is different. In this section I briefly discuss those differences and then draw some conclusions for the scope and limitations of each approach.

**Conceptual differences.** The policy equivalence results in the influential earlier work of Farhi & Werning, Correia et al. and Farhi et al. all leverage household and firm first-order conditions: if two policies induce the same sequence of wedges in all equations characterizing a model’s equilibrium, then the policies must also have the same aggregate effects. The equivalence results in Theorem 1, Corollary 1 and Theorem 2 instead rely on an equilibrium representation in net excess demand space, as in the traditional Arrow & Debreu approach to general equilibrium analysis.

It is of course immediate that, if two policies induce the same wedges in all equilibrium conditions, then they also generate the same paths of net excess demand, ensuring equivalence. This paper builds on the observation that the opposite is not true: two policies can generate the same net excess demand paths – and so be equivalent in general equilibrium – even if they do not enter optimality conditions symmetrically.

**Scope & limitations.** The net excess demand approach is appealing because it is broadly applicable. First, it works in a larger set of structural models. For example, in the HANK model of Sections 2 and 5, equivalence results based on tax wedges in first-order conditions fail because they do not mimic the income effects of nominal interest rate policy. Generally, as soon as policies work through both substitution and income effects, the wedge approach – which is designed to replicate substitution effects – fails. Second, it allows us to identify a larger family of candidate equivalent policies than the wedge approach. For example, in the context of the ELB constraint on monetary policy, this broadening is attractive because the equivalent unconventional fiscal policies discussed in Correia et al. (2013) may well be hard to implement in practice.

The cost of this generality is that the characterization of equivalent policies is both harder and less robust to variations in the economic environment. As emphasized in particular in Farhi et al. (2014), policies that induce identical wedges in first-order conditions are robust to quite general model perturbations; for example, to mimic a 100 basis point wedge in

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18The problem is that consumption taxes and nominal interest rates do not enter in exactly the same way in the household budget constraint (2). In models with Ricardian equivalence, this is irrelevant – it only matters that both policies enter the household Euler equation symmetrically. Without Ricardian equivalence, however, the distributional income effects matter.
household Euler equations, a 100 basis point consumption tax will always work, independent of the details of household preferences. Equivalence via net excess demand paths is instead invariably sensitive to all model details that affect the relevant derivative matrices (e.g., $C_\tau$). However, since the entries of such derivative matrices can, under some conditions, be estimated consistently through household- or firm-level regressions (Wolf, 2020), this challenge can in principle be addressed through sufficiently rich microeconometric evidence. In this paper I show that, in the context of the transfer spending response matrix $C_\tau$, extant microeconometric evidence in conjunction with some consumer theory is indeed sufficient to arrive at sharp and model-robust policy characterizations.

8 Conclusion

In recent years, the policy space for conventional interest rate-based stimulus has narrowed. In this paper I have shown that, in a standard business-cycle model with nominal rigidities and liquidity constraints, uniform lump-sum transfers to households are enough to circumvent such constraints and leave the space of implementable aggregate allocations unchanged. The second main result is an explicit characterization of the transfer policy needed to replicate any given (but infeasible) monetary easing; in particular I show that, even in a relatively rich quantitative HANK model, a small number of already measured statistics suffice to fully characterize the transfer policy needed to mimic any given monetary stimulus.

I leave several important extensions for future work. First, simulations in my quantitative HANK model suggest that, while macro-equivalent, interest rate and transfer stimulus have very different implications in the household cross-section. It would be interesting to study the implications of this observation for the design of optimal stabilization policy (e.g., using the tools of Bhandari et al. (2018) or Acharya et al. (2020)). Second, to the extent that the two linear maps $C_\tau$ and $C_{i_0}$ change over the business cycle, equivalent transfer policies will also depend on the aggregate state of the economy. Future empirical work should try to better measure that state dependence. And third, the general equivalence logic presented here applies not only to uniform transfer stimulus; for example, analogous equivalence results could also be established for policies that redistribute across households or through transfers targeted at sub-populations. If large swings in the amount of outstanding government debt are intrinsically undesirable, then such alternative equivalent policies become attractive.
References


A Model details

This appendix contains supplementary model details. I begin in Appendix A.1 by completing the specification of the model of Section 2. Appendix A.2 then provides some additional discussion on the aggregate consumption function $C(\bullet)$. Appendix A.3 presents the particular quantitative HANK model used for all simulations in Section 5, and finally, in Appendix A.4, I provide the missing details and equilibrium definition for the extended model with investment considered in Section 6.

A.1 Rest of the economy

It only remains to derive the price- and wage-NKPCs in (3) and (4). For the price-NKPC (3), let $1 - \alpha$ denote the elasticity of output with respect to total labor input at the steady state, $1 - \theta_p \in (0, 1)$ denote the probability of a price re-set, and $\epsilon_p$ the substitutability between different retail varieties in aggregation to the final good. We can then follow the derivations in Galí (2015) to arrive at the following log-linearized price-NKPC:

$$
\hat{\pi}_t = \frac{(1 - \theta_p)(1 - \theta_p)}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon_p} \left( \hat{w}_t - \alpha \hat{\ell}_t \right) + \frac{1}{1 + \bar{r}_b} \hat{\pi}_{t+1}
$$

(A.1)

(3) is a generalized version of (A.1), postulating an arbitrary link between $\{\pi_t, w_t, \ell_t, \pi_{t+1}\}$.

For the wage-NKPC (4), I consider the same union structure as in Auclert et al. (2018) and Wolf (2020). To derive a wage-NKPC from first principles, I need to specify a union objective function; for concreteness, and in keeping with the model simulation in Section 5, I will assume that household preferences take the particular form

$$
E_0 \left[ \sum_{t=0}^{\infty} \beta^t e^\xi \left\{ \frac{1}{1 - \gamma} - 1 \right\} \left( \frac{1}{1 - \gamma} - \chi \frac{1}{1 + \frac{1}{\varphi}} \right) \right]
$$

If all households are identical (e.g., as in the Ricardian model), then we can follow the same steps as in Erceg et al. (2000) to arrive at a standard log-linearized wage-NKPC:

$$
\hat{\pi}_t^w = \kappa_w \times \left[ \frac{1}{\varphi} \hat{\ell}_t - (\hat{w}_t - \gamma \hat{c}_t) \right] + \frac{1}{1 + \bar{r}_b} \hat{\pi}_{t+1}^w
$$

(A.2)
where \( \kappa_w \) is a function of model primitives, satisfying

\[
\kappa_w = \frac{(1 - \frac{1}{1 + \bar{r}_b} \phi_w)(1 - \phi_w)}{\phi_w (\varepsilon_w \frac{1}{\varphi} + 1)},
\]

with \( \phi_w \) indicating the degree of wage stickiness, and \( \varepsilon_w \) indicating the elasticity of substitution between different types of labor. As in the price-NKPC case, (4) is a generalized version of (A.2), here now postulating a link between \( \{w_{t-1}, \pi_t, w_t, \ell_t, c_t, w_{t+1}, \pi_{t+1}\} \).

Note that, with ex-post household heterogeneity (e.g., due to uninsurable income risk), a weighted average of household marginal consumption utilities – rather than marginal utility evaluated at aggregate consumption – would instead enter (A.2). The model would thus be inconsistent with a wage-NKPC of the form (4). However, since (i) average marginal utility and marginal utility at the average rarely deviate much, and (ii) these deviations anyway do not matter much with sufficiently sticky wages, the equivalence results presented in this paper continue to go through almost exactly in a variant of my baseline HANK model with (A.2) replaced by

\[
\hat{\pi}_w^w = \kappa_w \times \left[ \frac{1}{\varphi} \hat{\ell}_t - (\hat{w}_t - \gamma \hat{c}_t^*) \right] + \frac{1}{1 + \bar{r}_b} \hat{\pi}_{t+1}^w
\]

where

\[
c_t^* \equiv \left[ \int_0^1 e_it_c_{it}^{-\gamma} dt \right]^{-\frac{1}{\gamma}}
\]

These results are available upon request.

### A.2 The aggregate consumption function

Since all households supply the same amount of hours worked \( \ell_{it} = \ell_t \), and since dividend receipts \( d_{it} \) depend only on the productivity type \( e \), the path of aggregate consumption \( c \) can be obtained solely as a function of (i) the initial distribution of households over asset holdings \( b \) and productivity \( e \) and (ii) the input sequences \( \{w, \ell, \pi, \tau, i_b, \zeta\} \). This construction of an aggregate consumption function closely follows the analysis of Auclert et al. (2018).

I assume that the consumption function \( C(\bullet) \) is well-defined for bounded sequences of inputs. Since \( C(\bullet) \) is required to be consistent with the aggregated household budget constraint

\[
c_t + b_t = (1 - \tau_t)w_t \ell_t + \frac{1 + i_t b_{t-1}}{1 + \pi_t} b_t - \gamma c_t + d_t
\]

the borrowing constraint \( b_t \geq b \) and the condition \( \lim_{t\to\infty} (\prod_{s=0}^t \frac{1 + \pi_s}{1 + b_{s-1}}) b_t = 0 \), it follows
that the resulting path \( c = C(\cdot) \) is also bounded. \( C(\cdot) \) is thus a mapping from sequences in \( \ell^\infty \) to \( \ell^\infty \), i.e., the vector space of bounded sequences with the supremum norm.

For the definitions in (8) and (9), I assume that the limits

\[
\lim_{||h|| \to 0} \frac{||C(\bar{w}, \bar{\ell}, \bar{\pi}, \bar{d}; \bar{r}, \bar{i}_b, 0) - \bar{c} - C_{\tau}h||}{||h||} = 0
\]

and

\[
\lim_{||h|| \to 0} \frac{||C(\bar{w}, \bar{\ell}, \bar{\pi}, \bar{d}; \bar{r}, \bar{i}_b + h, 0) - \bar{c} - \tilde{C}_{ib}h||}{||h||} = 0
\]

exist, and use them to define the linear maps \( C_{\tau} \) and \( \tilde{C}_{ib} = \tilde{C}_{ib} - \bar{b} \times C_{\tau, i} \). That is, at the deterministic steady state, \( C(\cdot) \) is assumed to be Fréchet-differentiable in the lump-sum transfer and nominal interest rate paths. Under those assumptions, \( C_{\tau} \) and \( \tilde{C}_{ib} \) are well-defined linear maps from \( \ell^\infty \) to \( \ell^\infty \). The inverse of \( C_{\tau} \), denoted \( C_{\tau}^{-1} \), is then simply a linear map such that \( C_{\tau}^{-1}C_{\tau} = I \) and \( C_{\tau}C_{\tau}^{-1} = I \) where \( I \) is the identity map on \( \ell^\infty \).

### A.3 The quantitative HANK model

I first discuss the parameterization of the model’s steady state, and so in particular the induced aggregate consumption function \( C(\cdot) \).

The values of all parameters relevant for the model’s deterministic steady state are displayed in Table A.1. Preference parameters \( \{\gamma, \varphi, \varrho\} \) as well as the labor substitutability \( \varepsilon_w \) are set to standard values. The average return on (liquid) assets is set in line with standard calibrations of business-cycle models, and the discount rate is then disciplined through the total amount of liquid wealth. As in McKay et al. (2016), I assume that households cannot borrow in the liquid asset. Next, for income risk, I adopt the 33-state specification of Kaplan & Violante (2018), ported to discrete time. For share endowments, I assume that \( d_{it} = \begin{cases} 0 & \text{if } e_{it}^p \leq \xi^p \\ \chi_0(e_{it}^p - \xi^p)^{\chi_1} \times d_t & \text{otherwise} \end{cases} \)

where \( e_{it}^p \) is the permanent component of household \( i \)'s labor productivity. I set the parameters \( \{\xi^p, \chi_0, \chi_1\} \) as in Wolf (2020). On the firm side, I assume constant returns to scale in production, and set the substitutability between goods to a standard value. Finally, the average government tax take, transfers, and debt issuance are all set in line with direct empirical evidence. Relative to Definition 3, I thus slightly generalize the model to allow for
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</tr>
<tr>
<td>$\tau_l$</td>
<td>Labor Tax</td>
<td>0.3</td>
<td>Average Labor Tax</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$\tau/Y$</td>
<td>Transfer Share</td>
<td>0.05</td>
<td>Transfer Share</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$B/Y$</td>
<td>Liquid Wealth Supply</td>
<td>1.04</td>
<td>Government Debt/Y</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>

**Table A.1:** HANK model, steady-state calibration.

non-zero government spending, giving the new market-clearing condition

$$y_t = c_t + g_t$$ (A.3)

In the second step I set the remaining model parameters (which exclusively govern dynamics around the deterministic steady state). Before presenting the model parameterization, however, I first need to complete the model specification by explicitly setting fiscal and monetary feedback rules.\(^{19}\) I assume that nominal interest rates are set as

$$\hat{i}_{b,t} = \rho_m \hat{i}_{b,t-1} + (1 - \rho_m) \left( \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_{dy} \hat{y}_{t-1} \right)$$

\(^{19}\)These results are used to compute the general equilibrium consumption paths under imperfect shock accommodation in Figure 5.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_p$</td>
<td>Price Calvo Parameter</td>
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</tr>
<tr>
<td>$\phi_p$</td>
<td>Wage Calvo Parameter</td>
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</tr>
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<td>$\rho_m$</td>
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<td>$\phi_x$</td>
<td>Taylor Rule Inflation</td>
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</tr>
<tr>
<td>$\phi_y$</td>
<td>Taylor Rule Output</td>
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</tr>
<tr>
<td>$\phi_{dy}$</td>
<td>Taylor Rule Output Growth</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table A.2: HANK model, parameters governing dynamics, set as in Wolf (2020).

This rule is chosen to ensure equilibrium determinacy. On the fiscal side, I assume that lump-sum taxes respond with a delay to increases in debt:

$$\tilde{\tau}_t = \tilde{\tau}_x^x - (1 - \rho_\tau) \times \tilde{b}_{t-1}$$ (A.4)

where $\tilde{\tau}_x^x$ is the exogenous “policy” component of the transfer path (see the proof of Theorem 1). I set $\rho_\tau = 0.95$ to minimize the effects of tax financing on the experiments of Section 5.2. All other parameters are set to the posterior mode estimates in Wolf (2020).

A.4 Adding investment

All firms are identical, so I drop the $j$ subscript. Analogously to the discussion in Section 2.1, we can summarize the solution to the firm problem with an investment demand function,

$$i = I (w, p^I, \pi; \tau_f, i_b; \zeta)$$ (A.5)

a production function,

$$y = Y (w, p^I, \pi; \tau_f, i_b; \zeta)$$ (A.6)

a labor demand function,

$$\ell = L (w, p^I, \pi; \tau_f, i_b; \zeta)$$ (A.7)

and a dividend function

$$d = D (w, p^I, \pi; \tau_f, i_b; \zeta)$$ (A.8)

where the dividend function aggregates over both intermediate goods producers and sticky-price retailers. Note that, since intermediate goods firms hire labor on a competitive spot..
market, and since productivities are equalized across firms \( j \), two sequences \( \zeta \) and \( \tau_f \) that induce the same paths of investment also invariably induce the same paths of output and labor hiring. However, since subsidies enter the budget constraint while the shock \( \zeta \) does not, the implied dividend paths may be different.

Given investment subsidies to firms, the government budget constraint is adjusted to give

\[
\frac{1 + i_{b,t-1} b_t}{1 + \pi_t} + \tau_t + \tau_{f,t}(\{i_{t-q}\}_{q=0}^t) = \tau_t w_t \ell_t + b_t \tag{A.9}
\]

All other parts of the model are unchanged relative to Section 2.1. We thus arrive at the following equilibrium definition:

**Definition A.1.** Given a path of shocks \( \{\zeta_t\}_{t=0}^\infty \), an equilibrium is a set of government policies \( \{\iota_{b,t}, \tau_t, \tau_{f,t}, b_t\}_{t=0}^\infty \) and a set of aggregates \( \{c_t, \ell_t, y_t, i_t, k_t, w_t, \pi_t, d_t\}_{t=0}^\infty \) such that:

1. Aggregate consumption is given as

\[
c = \int_0^1 c_i di
\]

where the individual consumption paths \( c_i \) solve the household consumption-savings problem of maximizing (1) subject to (2) and \( b_i \geq b \).

2. Aggregate investment, output, hours worked and dividends satisfy

\[
i = \mathcal{I}(w, p^f, \pi; \tau_f, i_b; \zeta) \\
y = \mathcal{Y}(w, p^f, \pi; \tau_f, i_b; \zeta) \\
\ell = \mathcal{L}(w, p^f, \pi; \tau_f, i_b; \zeta) \\
d = \mathcal{D}(w, p^f, \pi; \tau_f, i_b; \zeta)
\]

3. Wage inflation \( \pi_t^w \equiv \frac{w_t}{w_{t-1}}(1 + \pi_t) \) together with \( \{\ell_t, c_t\}_{t=0}^\infty \) are consistent with the aggregate wage-NKPC (4).

4. The paths \( \{\pi_t, w_t, \ell_t\}_{t=0}^\infty \) are consistent with the aggregate price-NKPC (27).

5. The output market clears: \( y_t = c_t + i_t \) for all \( t \geq 0 \), the government budget constraint (A.9) holds at all \( t \), and \( \lim_{t \to \infty} \hat{b}_t = 0 \). The bond market then clears automatically by Walras’ law.
\section*{B \ $C_\tau$ and $C_{ib}$ in simple models}

This section provides further details on my characterizations of the consumption derivative matrices $C_\tau$ and $C_{ib}$ in simple spender-saver, bond-in-utility and hybrid models. I begin in Appendix B.1 with a review of the necessary mathematical background. Appendices B.2 to B.4 then provide detailed discussions of my three analytically tractable model variants.

\subsection*{B.1 Mathematical background}

A Toeplitz matrix is an $n \times n$ matrix of the form

\begin{equation}
T_n = \begin{pmatrix}
t_0 & t_{-1} & t_{-2} & \cdots & t_{-(n-1)} \\
t_1 & t_0 & t_{-1} & & \\
t_2 & t_1 & t_0 & \cdots & \\
& \ddots & \ddots & \ddots & \\
t_{n-1} & \cdots & t_0
\end{pmatrix}
\end{equation}

My results on eigenvalue distributions and inverses leverage well-known theorems on the limiting behavior of Toeplitz matrices induced by an infinite sequence $\{t_k\}_{k=-\infty}^{\infty}$, with the sequence restricted to be absolutely summable.\footnote{Formally, all results stated below require the Toeplitz matrix to fall into the Wiener class, which is easily verified for all matrices considered here.} I briefly review those results here, with a detailed discussion provided for example in Gray (2006). Central to all expressions is the Fourier series with coefficients $t_k$:

\begin{equation}
f(\lambda) \equiv \sum_{k=-\infty}^{\infty} t_k e^{ik\lambda}
\end{equation}

The inverse mapping is given as

\begin{equation}
t_k = \frac{1}{2\pi} \int_{0}^{2\pi} f(\lambda)e^{-ik\lambda}d\lambda
\end{equation}

and I will use the notation $T_n(f)$ to indicate a matrix with Fourier series $f$. 

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Definition B.1. Two sequences of \( n \times n \) matrices \( \{ A_n \} \) and \( \{ B_n \} \) are said to be asymptotically equivalent if

1. \( A_n \) and \( B_n \) are uniformly bounded in strong norm:

\[
\| A_n \|, \| B_n \| \leq M < \infty, \quad n = 1, 2, \ldots
\]

2. \( D_n \equiv A_n - B_n \) goes to zero in weak norm as \( n \to \infty \):

\[
\lim_{n \to \infty} |A_n - B_n| = \lim_{n \to \infty} |D_n| = 0
\]

Here the strong (or operator) norm is defined as

\[
\| A \| = \max_x \left( \frac{x^*(A^*A)x}{x^*x} \right)^{\frac{1}{2}}
\]

where \( * \) denotes conjugate transpose, while the weak norm (or Hilbert-Schmidt norm) is defined as

\[
|A| = \left( \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{n} |a_{k,j}|^2 \right)^{\frac{1}{2}}
\]

Eigenvalue distribution. Let \( \tau_{n,k} \) denote the \( k \)-th eigenvalue of a given \( n \)-dimensional Toeplitz matrix, and define the eigenvalue distribution function \( D_n(x) = \frac{\sum_{k=1}^{n} 1_{\tau_{n,k} \leq x}}{n} \). Then

\[
D(x) = \lim_{n \to \infty} D_n(x) = \frac{1}{2\pi} \int_{f(\lambda) \leq x} d\lambda
\]

This general result is known as the fundamental eigenvalue distribution theorem of Szegö.

Inverse. If \( f(\lambda) > 0 \) for all \( \lambda \in [-\pi, \pi] \) then the Toeplitz matrix \( T_n(f) \) is invertible, and its inverse is asymptotically equivalent – in the sense of Definition B.1 – to a Toeplitz matrix with sequence \( \{ t_k^{-1} \}_{k=-\infty}^{\infty} \) where

\[
t_k^{-1} = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{e^{-ik\lambda}}{f(\lambda)} d\lambda
\]
B.2 $C_{ib}$ for Ricardian households

In Sections 3.2 and 3.3 I claimed that $C^R_{ib}$ and $C^T_{ib}$ are such that interest rate policy can engineer any consumption path $\hat{c}^{PE}$ with zero net present value. I here provide the missing details for that argument. Since $C^T_{ib} = (1 - \mu)C^R_{ib}$ I can, for $\mu < 1$, equivalently prove all results by just considering $C^R_{ib}$.

Recall that

$$C^R_{ib} = -\frac{1}{\gamma} \times (I - C^R_T) \times U$$

where $U$ is shorthand for the upper-triangular matrix in (10). Now let $\hat{c}^{PE}$ be any consumption path with zero net present value, and consider setting

$$\hat{i}_b = (-\frac{1}{\gamma} \times U)^{-1} \times \hat{c}^{PE}$$

where I have used the fact that $U$ is invertible. But then we have

$$C^R_{ib} \times \hat{i}_b = \hat{c}^{PE} - C^R_T \times \hat{c}^{PE} = \hat{c}^{PE}$$

where the last equality follows because, by assumption, $\hat{c}^{PE}$ has zero net present value.

B.3 Bonds in the utility function

Section 3.3 without proof claimed: first, that the transfer derivative matrix $C_T$ in the bond-in-utility model takes the (asymptotic) shape (17); and second, that the interest rate derivative matrix satisfies (18). I here provide the arguments for both.

TRANSFER RESPONSE $C_T$. To characterize the model-implied $C_T$ matrix, I consider the problem of maximizing

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1 - \gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1 - \eta} \right\} \quad \text{(B.6)}$$

subject to the budget constraint

$$c_t + b_t = y_t + (1 + \bar{r}_b) b_{t-1} \quad \text{(B.7)}$$

where $y_t$ denotes an exogenous stream of income, and $\bar{r}_b$ is the steady-state real rate of interest. Linearizing around a deterministic steady state with $y_t = \bar{y}$, $b_t = \bar{b}$ and $c_t = \bar{c}$
for all \( t \), we find that the consumption and saving paths are characterized by the following system of equations:

\[
\begin{align*}
\hat{c}_t + \hat{b}_t - (1 + \bar{r}_b) \hat{b}_{t-1} &= \hat{y}_t \\
\hat{c}_t - \beta (1 + \bar{r}_b) \hat{c}_{t+1} - \frac{\eta}{\gamma} (1 - \beta (1 + \bar{r}_b)) \hat{b}_t &= 0
\end{align*}
\]

Now let \( \hat{c}_{t,H} \times \hat{y}_H \) denote the impulse response of consumption at time \( t \) to an income change \( \hat{y}_H \) at time \( H \), so that \( \hat{c}_t \times \hat{y}_t \) is the \( t \)th column of \( C_{\tau} \). We arrive at the following result:

**Proposition B.1.** Consider the consumption-savings problem (B.6) of a household with bonds in the utility function, and suppose that \( \beta = 1 \). Then, for any \( \ell > 0 \) the impulse response path \( \hat{c}_{H} \) to an income shock at time \( H \) satisfies

\[
\lim_{H \to \infty} \hat{c}_{H,H} = \frac{\hat{y}}{\hat{c}} \times \omega, \quad \lim_{H \to \infty} \frac{\hat{c}_{H+\ell,H}}{\hat{c}_{H+\ell-1,H}} = \lim_{H \to \infty} \frac{\hat{c}_{H-\ell-1,H}}{\hat{c}_{H-\ell,H}} = \theta \tag{B.8}
\]

where the coefficients \( \{\omega, \theta\} \) are functions of model primitives.

Proposition B.1 states that, at least asymptotically, iMPCs for an income shock at time \( H \) increase at a geometric rate \( \theta \) up to time \( H \), equal some time-invariant constant \( \omega \) at \( H \), and decrease at that same geometric rate \( \theta \) after time \( H \).\(^{21}\) The resulting asymptotic version of \( C_{\tau}^B \) is evidently a particular Toeplitz matrix, with

\[
f(\lambda) = \omega \sum_{k=-\infty}^{\infty} \theta^{|k|} e^{ik\lambda} = \omega \frac{1 - \theta^2}{1 - 2\theta \cos(\lambda) + \theta^2} \tag{B.9}
\]

**Interest rate response \( C_{ib} \).** With a time-varying rate of interest and transfers set as in (6) to offset income effects, the solution to the consumption-savings problem is characterized by the pair of equations

\[
\begin{align*}
\hat{c}_t + \hat{b}_t - (1 + \bar{r}_b) \hat{b}_{t-1} &= \hat{y}_t \\
\hat{c}_t - \beta (1 + \bar{r}_b) \hat{c}_{t+1} - \frac{\eta}{\gamma} (1 - \beta (1 + \bar{r}_b)) \hat{b}_t &= -\frac{1}{\gamma} \beta (1 + \bar{r}_b) \hat{r}_t
\end{align*}
\]

Note that interest rate changes have no income effects here, and purely act through intertemporal substitution. I will first of all consider time-0 income and interest rate shocks with

\(^{21}\)For \( \beta < 1 \), iMPCs before time \( H \) increase at a rate \( \frac{\beta}{\eta} \).
impulse responses $\hat{c}_y$ and $\hat{c}_r$, respectively, and show that

$$\hat{c}_r = -\frac{1}{\gamma} \beta (1 + \bar{r}_b) (e_1 - \bar{c} \hat{c}_y)$$

where $e_1 = (1, 0, 0, \ldots)'$. To see this, let

$$\hat{c}_t^* = \hat{c}_t - \bar{y} \hat{y}_t$$

We can then re-write the system for income shocks as

$$\bar{c} \hat{c}_t^* + \bar{b} \bar{b}_t - (1 + \bar{r}_b) \bar{b} \bar{b}_{t-1} = 0$$

$$(\bar{c}_t^* + \bar{y} \hat{y}_t) - \beta (1 + \bar{r}_b) (\hat{c}_{t+1}^* + \bar{y} \hat{y}_{t+1}) - \frac{\eta}{\gamma} (1 - \beta (1 + \bar{r}_b)) \hat{b}_t = 0$$

Since $\hat{y}_{t+1} = 0$ this is identical to a system with a time-0 interest rate wedge of $\frac{\bar{y}}{\bar{c}} \frac{\gamma}{\beta (1 + \bar{r}_b)} \hat{y}_t$, so we have

$$\hat{c}_y^* = \frac{\gamma}{\beta (1 + \bar{r}_b)} \hat{c}_r$$

Re-arranging this:

$$\hat{c}_r = \frac{1}{\gamma} \beta (1 + \bar{r}_b) \hat{c}_y^*$$

But by construction

$$\hat{c}_y^* = \hat{c}_y - \bar{y} e_1$$

Translating $\hat{c}_y$ to MPC space (i.e., the first column of $C_r$), the claim follows. Proceeding similarly for future interest rate changes, we find that

$$C_{ib} = -\frac{1}{\gamma} (I - C_r) \begin{pmatrix} [\beta (1 + \bar{r}_b)] & [\beta (1 + \bar{r}_b)]^2 & [\beta (1 + \bar{r}_b)]^3 & \ldots \\ 0 & [\beta (1 + \bar{r}_b)] & [\beta (1 + \bar{r}_b)]^2 & \ldots \\ 0 & 0 & [\beta (1 + \bar{r}_b)] & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Thus, with $\beta (1 + \bar{r}_b) = 1$, we get (18).
B.4 Hybrid model

With a fringe $\mu$ of spenders, we instead get the asymptotic transfer consumption derivative matrix in (19). This matrix is again a Toeplitz matrix, now with

$$f(\lambda) = \omega \frac{\xi - \frac{1 - \theta^2}{1 - 2\theta \cos(\lambda) + \theta^2} + (1 - \xi)}{\xi - \frac{1 - \theta^2}{1 - 2\theta \cos(\lambda) + \theta^2} + (1 - \xi)}$$  \hspace{1cm} (B.10)

The interest rate derivative matrix (20) then follows by simply combining the previous conclusions for the spender-saver and bond-in-utility models.

Figure B.1 illustrates Proposition B.1 in the context of the empirically relevant hybrid model. For large enough horizons $H$, the matrix columns $C_{\tau, H-\ell}$, $C_{\tau, H}$ and $C_{\tau, H+\ell}$ are simply left- and right-translations of each other, with a common MPC $\omega$ and constant decay rates away from the main diagonal. The grey line reveals that, even for impact transfer receipt, the approximation in (19) is quite accurate.

![Figure B.1: Entries of the consumption derivative matrix for the calibrated hybrid model, for transfer receipts at time periods $\{0, 5, 10, 15\}$.](image)
C Implications for policy design

This section discusses two implications of the equivalence result for policy design: first the equivalence mapping between given policy rules, and then optimal policymaking.

C.1 Equivalence mapping between policy rules

A policy rule is a mapping from the current policymaker information set $\Omega_t$ into current and future expected paths of policy instruments. In particular, an interest rate-only rule is a mapping from $\Omega_t$ to current and future expected paths of nominal rates, with taxes and transfers set as in (6). For a familiar example, consider a simple Taylor rule

$$\hat{i}_{b,t} = \phi_{\pi} \hat{\pi}_t + \phi_y \hat{y}_t,$$

postulating that the current nominal rate of interest can be obtained solely as a function of inflation and output today. Given today’s information set $\Omega_t$ (which includes $\pi_t$ and $y_t$), the implied expected interest rate path then satisfies

$$E[\hat{i}_{b,t+\ell} | \Omega_t] = \phi_{\pi} E[\hat{\pi}_{t+\ell} | \Omega_t] + \phi_y E[\hat{y}_{t+\ell} | \Omega_t], \quad \ell = 0, 1, 2, \ldots$$

It follows immediately from Theorem 1 that, if the economy in Section 2.1 were closed with any possible such interest rate-only rule, the $\Omega_t$-measurable transfer rule

$$E[\hat{\tau}_{t+\ell} | \Omega_t] = E[\hat{\tau}_{t+\ell} | \Omega_{t-1}] + \sum_{s=0}^{\infty} w_{\ell,s} \left\{ E[\hat{i}_{b,t+s} | \Omega_t] - E[\hat{i}_{b,t+s} | \Omega_{t-1}] \right\}, \quad \ell = 0, 1, 2, \ldots$$

would implement the exact same sequences of aggregate outcomes for any possible realized sequence of shocks $\{\zeta_t\}_{t=0}^{\infty}$, where $w_{\ell,s}$ is the $(\ell, s)$th entry of the linear map

$$W \equiv C_{\pi}^{-1} \times C_{ib}$$

For example, to replicate a given Taylor rule, the time-$t$ revision of current and future output

---

22By the equivalence of first-order perturbations and perfect foresight transition paths, this equivalence extends to versions of the baseline economy in Section 2.1 with arbitrary sources of aggregate risk, solved using first-order perturbation methods. Note, however, that this says nothing about equilibrium uniqueness: the equilibrium allocation may be unique given the interest rate-only rule, yet it is only guaranteed to be supported as one possible equilibrium given the transfer-only rule.
and inflation expectations would translate into a revision of current and the planned path of future transfer payments.

### C.2 Optimal policy

Most standard treatments of optimal policymaking over the business cycle (e.g. Galí, 2015; Woodford, 2011) consider policymaker loss functions of the form

\[
\mathcal{L} = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \tilde{\pi}_t + \lambda \tilde{y}_t \right) \right]
\]  

(C.1)

It follows immediately from Theorem 1 that, for any policymaker with a loss function like (C.1), an ELB on nominal rates has no effect on the attainable loss: any loss attainable with unconstrained interest rate-only policy is also attainable with transfer-only policy, and thus in particular attainable at the ELB.

However, as discussed in Section 5.3, interest rate and transfer policy generally differ at the micro level, in the cross section of households. It follows that, for richer policymaker loss functions, the two instruments are not equivalent. Previous work on optimal monetary policy in models with household heterogeneity has emphasized the insurance benefits of general equilibrium stabilization of wage income (e.g. Bhandari et al., 2018; Acharya et al., 2020). My analysis reveals that the equivalent transfer policy would have the exact same general equilibrium insurance benefits, but provide even further insurance through its direct effect on consumption expenditures. A detailed characterization of the implied optimal mix of interest rate and transfer policies is left to future work.
D Proofs and auxiliary lemmas

D.1 Proof of Theorem 1

Linearizing the government budget constraint (5), we find

\[
\dot{b}_{t-1} - (1 + \delta_b) \ddot{b}_t + (1 + \delta_b) \dddot{b}_{t-1} + \dddot{t}_t = \tau_t \dddot{w} \dddot{e} (\dddot{w}_t + \dddot{\ell}_t) + \dddot{b}_t
\]  

(D.1)

Using (D.1), I decompose total transfers into two parts: an endogenous “general equilibrium" component related to labor tax revenue and inflation debt servicing costs,

\[
\dddot{t}_t^e \equiv \tau_t \dddot{w} \dddot{e} (\dddot{w}_t + \dddot{\ell}_t) + (1 + \delta_b) \dddot{b}_t
\]  

(D.2)

and an exogenous “policy" component

\[
\dddot{t}_t^x \equiv \dddot{t}_t - \dddot{t}_t^e
\]  

(D.3)

I now present a constructive proof of Theorem 1: leveraging (12) I will show how to construct a transfer-only policy replicating any interest rate-only policy, and vice-versa. The decomposition in (D.2) and (D.3) will prove useful in this constructive proof. Also note that the proofs use the map \(\tilde{C}_i\) defined in Appendix A.2.

1. An interest rate-only policy is a tuple \(\{\delta_b, \tau_t\}_{t=0}^\infty\) with

\[
\dddot{t}_t^* = \dddot{t}_t^e - \dddot{b}_{t-1}
\]

so that \(\dddot{t}_t^* = -\dddot{b}_{t-1}\). By (12), there exists a path of transfers \(\{\tau_t^*\}_{t=0}^\infty\) such that

\[
C_\tau \times \dddot{t}^* = \tilde{C}_i \times \delta_b + C_\tau \times \dddot{t}^*
\]  

(D.4)

and where

\[
\sum_{t=0}^\infty \left( \frac{1}{1 + \delta_b} \right)^t \dddot{t}_t^* = 0
\]  

(D.5)

Now consider the transfer-only policy tuple \(\{\tilde{\delta}_b, \tilde{\tau} + \dddot{t}_t^e + \dddot{t}_t^*\}_{t=0}^\infty\}. I will verify that, at the initial \(\{\zeta_t\}_{t=0}^\infty\) and \(\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty\), all markets still clear and all agents still behave optimally. First, by (D.5) and the definition of \(\dddot{t}_t^*\), we still have that \(\lim_{t \to \infty} \dddot{b}_t = 0\). Second, by construction of \(\dddot{t}_t^*\) in (D.4), the path \(\dddot{c}\) is still consistent with optimal household
behavior given \{w_t, \ell_t, \pi_t, d_t; \bar{i}_b, \bar{\tau} + \hat{\tau}^e + \hat{\tau}^*_t; \zeta_t\}_{t=0}^\infty}. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

2. A transfer-only policy is a tuple \{\bar{i}_b, \tau_t\}_{t=0}^\infty with

\[ \sum_{t=0}^\infty \left( \frac{1}{1+i_b} \right)^t \hat{\tau}_t^* = 0 \]

By (12), there exists a path of interest rates \{i_{b,t}^*\}_{t=0}^\infty with \hat{\tau}_t^* = -\hat{\tau}^e_{b,t-1} such that

\[ C_{\tau} \times \hat{\tau}^e = \tilde{C}_{i_b} \times \hat{i}_b + C_{\tau} \times \hat{\tau}^* \]

and where of course by construction

\[ \sum_{t=0}^\infty \left( \frac{1}{1+i_b} \right)^t \hat{\tau}_t^* + \sum_{t=0}^\infty \left( \frac{1}{1+i_b} \right)^t \hat{\tau}^e_{b,t-1} = 0 \]

Now consider the interest rate-only policy tuple \{i_{b,t}^*, \bar{\tau} + \hat{\tau}^e + \hat{\tau}^*_t\}_{t=0}^\infty. As before I will verify that, at the initial \{\zeta_t\}_{t=0}^\infty and \{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty, all markets still clear and all agents still behave optimally. First, by (D.7) and the definition of \hat{\tau}^e_t, we still have that \hat{\ell}_t = 0 for all \ell, so indeed the policy is a valid interest rate-only policy. Second, by construction of \hat{\tau}^*_t in (D.6), the path \hat{c} is still consistent with optimal household behavior given \{w_t, \ell_t, \pi_t, d_t; i_{b,t}^*, \bar{\tau} + \hat{\tau}^e + \hat{\tau}^*_t; \zeta_t\}_{t=0}^\infty. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

\[ \square \]

D.2 Proof of Corollary 1

By assumption, the allocation (14) is implementable without the ELB constraint (13), with some policy tuple \{i_{b,t}, \pi_t\}_{t=0}^\infty. Note that by construction we must have

\[ \sum_{t=0}^\infty \left( \frac{1}{1+i_b} \right)^t \hat{\pi}_t + \sum_{t=0}^\infty \left( \frac{1}{1+i_b} \right)^t \hat{\ell}_{b,t-1} = 0 \]

since \pi_t = \bar{\pi}, w_t = \bar{w} and \ell_t = \bar{\ell} for all \ell. Now consider the alternative policy tuple \{\hat{i}_b, \tau_t^*\}_{t=0}^\infty where

\[ C_{\tau} \times \hat{\tau}^* = \tilde{C}_{i_b} \times \hat{i}_b + C_{\tau} \times \hat{\tau} \]

(D.8)
The existence of such a $\hat{\tau}^*$ is assured by (12). But then we can conclude, by exactly the same argument as in the proof of Theorem 1, that all markets still clear and all agents still behave optimally with equilibrium aggregates as in (14). However, the ELB constraint (13) is not violated by construction, completing the argument.

\[ \Box \]

D.3 Proof of Proposition 1

Plugging the expression for the Fourier series (B.9) into the general eigenvalue distribution expression (B.4), straightforward re-arrangement gives the desired expression. We can then recover $\lambda$ and $\bar{\lambda}$ by solving the equations $F^B(\lambda) = 0$ and $F^B(\bar{\lambda}) = 1$.

\[ \Box \]

D.4 Proof of Proposition 2

The proof is exactly analogous to that of Proposition 1: First plug the expression for the Fourier series (B.10) into the general eigenvalue distribution expression (B.4) to recover the desired expression for the cdf, and then find $\lambda$ and $\bar{\lambda}$ by solving the equations $F^{TB}(\lambda) = 0$ and $F^{TB}(\bar{\lambda}) = 1$.

\[ \Box \]

D.5 Proof of Proposition 3

Fix an arbitrary partial equilibrium net excess demand path $\hat{c}^{PE}$, and consider setting

\[ \hat{\tau}_t = \frac{1}{\mu} \left( \hat{c}_t^{PE} - (1 - \mu)(1 - \beta) \sum_{s=0}^{\infty} \beta^s \hat{c}_s^{PE} \right) \]

Collect the entire sequence of transfers in the vector $\hat{\tau}$. It is then straightforward to verify that indeed

\[ C^T \hat{\tau} = \hat{c}^{PE} \]

Thus the desired matrix inverse $(C^T)^{-1}$ satisfies

\[ (C^T)^{-1} = \frac{1}{\mu} \left[ I - (1 - \mu)C^R \right] \]

Since $\omega \equiv \mu + (1 - \mu)(1 - \beta)$, the desired expression follows from straightforward substitution for the limit case $\beta \to 1$.

\[ \Box \]
D.6 Proof of Proposition 4

The expression for the inverse in (23) is easily verified through direct computation, revealing that indeed

\[ C^B_{\tau}(C^B_{\tau})^{-1} = (C^B_{\tau})^{-1}C^B_{\tau} = I \]

as required. For completeness, I also present the computations for the asymptotic inverse. By (B.5) and (B.9), we have

\[ t_k^{-1} = \frac{1}{2\pi \omega} \int_0^{2\pi} e^{-i\lambda} \frac{1 - 2\theta \cos(\lambda) + \theta^2}{1 - \theta^2} d\lambda \]

Solving:

\[ t_k^{-1} = \begin{cases} 
\frac{1+\theta^2}{1-\theta^2} & \text{if } k = 0 \\
-\frac{\theta}{1-\theta^2} & \text{if } k = 1 \text{ or } k = -1 \\
0 & \text{otherwise}
\end{cases} \]

These entries correctly characterize \((C^B_{\tau})^{-1}\) from the second row and column onwards.

D.7 Proof of Proposition 5

By (B.5) and (B.10), we have

\[ t_k^{-1} = \frac{1}{2\pi \omega} \int_0^{2\pi} e^{-i\lambda} \frac{1 - 2\theta \cos(\lambda) + \theta^2}{\xi(1 - \theta^2) + (1 - \xi)(1 - 2\theta \cos(\lambda) + \theta^2)} d\lambda \]

This yields (24).

D.8 Proof of Theorem 2

By assumption, the allocation (2) is implementable without the ELB constraint (13), with a policy tuple \(\{i_{b,t}, \tau_t, \tau_{f,t}\}\)\(^\infty\). Now consider the alternative policy tuple \(\{\hat{i}_b, \tau_t + \hat{\tau}^*_t, \tau_{f,t} + \hat{\tau}^*_f\}\)\(^\infty\) where

\[ \hat{\tau}^*_f = \mathcal{T}^{-1}_{\tau_f} \mathcal{T}_{i_b} \hat{i}_b \]  

(D.9)

and

\[ \hat{\tau}^* = C^{-1}_\tau \left[ C_{i_b} \hat{i}_b + C_d \left( D_{i_b} \hat{i}_b - D_{\tau_f} \hat{\tau}^*_f \right) \right] \]  

(D.10)
I now claim this policy tuple similarly engineers the allocation (2). First, with \( \hat{\tau}_f \) set as in (D.9), the investment, output and labor demand paths are unchanged; however, as remarked in Appendix A.4, the dividend paths may be different. The transfer path \( \hat{\tau}^* \) is constructed to offset both the missing monetary stimulus as well as neutralize any potential dividend-related effects: to see this, note that we have

\[
\hat{c} = C_t(\hat{\tau} + \hat{\tau}^*) + C_dD_{\tau}(\hat{\tau}_f + \hat{\tau}^*_f) + \text{non-policy terms}
\]

where the last equality follows because \( \hat{\tau} = 0 \) in the initial equilibrium. Next note that, since they induce the same paths of consumption, investment, hours worked and production, and since they leave wages unchanged, the initial policy \( \{i_{b,t}, \tau_t, \tau_{f,t}\}_{t=0}^{\infty} \) and the new policy \( \{\tilde{i}_{b,t}, \tilde{\tau}_t, \tilde{\tau}_{f,t} + \tilde{\tau}^*_t\}_{t=0}^{\infty} \) must have the same present value in the aggregated household-plus-firm budget constraint

\[
c_t + b_t + i_t = y_t + (1 + i_{b,t-1})b_{t-1} + \tau_t + b_t - \tau_{f,t}w_t + \tau_{f,t}\{i_{t-q}\}_{q=0}^{t}
\]

But then, since households, firms and the government borrow and lend at the same rate, the two policies must also have the same (zero) present value in the government budget constraint (5). With \( \lim_{t \to \infty} \hat{b}_t = 0 \) in the initial equilibrium, it then follows that we must also have \( \lim_{t \to \infty} \hat{b}_t = 0 \) in the new one, as required. \( \square \)

### D.9 Auxiliary lemmas for the proof of Proposition B.1

Consider an income shock at time \( H \geq 0 \), and suppose that \( \beta = 1 \). We then get the following simplified equilibrium system:

\[
\hat{c}_t + \hat{b}_t - (1 + \hat{\tau}_b)\hat{b}_{t-1} = \hat{y}_{t-H}
\]

\[
\hat{c}_t - (1 + \hat{\tau}_b)\hat{c}_{t+1} + \frac{\eta}{\gamma}\hat{\tau}_b\hat{b}_t = 0
\]

I will guess and verify that lagged wealth is the only endogenous state variable. Plugging in this guess the equilibrium system becomes

\[
c(\theta_{cb}\hat{b}_{t-1} + \sum_{h=0}^{H} \theta_{cyh}\hat{y}_{t-h}) + \hat{b}(\theta_{bb}\hat{b}_{t-1} + \sum_{h=0}^{H} \theta_{byh}\hat{y}_{t-h}) - (1 + \hat{\tau}_b)\hat{b}_{t-1} = \hat{y}_{t-H}
\]

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Matching coefficients, we find that the equilibrium is fully characterized by the following system of $2H + 4$ equations in $2H + 4$ unknowns:

\[\begin{align*}
\bar{c} \theta_{cb} + \bar{b} \theta_{bb} & - (1 + \bar{r}_b) \bar{b} = 0 \quad (D.11) \\
\bar{c} \theta_{c_yh} + \bar{b} \theta_{byh} & = 0, \quad h = 0, 1, \ldots, H - 1 \quad (D.12) \\
\bar{c} \theta_{c_yH} + \bar{b} \theta_{byH} & = \bar{y} \quad (D.13) \\
\theta_{cb} - (1 + \bar{r}_b) \theta_{cb} + \frac{\eta}{\gamma} \bar{r}_b \theta_{bb} & = 0 \quad (D.14) \\
\theta_{c_yh} - (1 + \bar{r}_b) [\theta_{cb} \theta_{byh} + \theta_{c_yh+1}] + \frac{\eta}{\gamma} \bar{r}_b \theta_{byh} & = 0, \quad h = 0, 1, \ldots, H - 1 \quad (D.15) \\
\theta_{c_yH} - (1 + \bar{r}_b) \theta_{cb} \theta_{byH} + \frac{\eta}{\gamma} \bar{r}_b \theta_{byH} & = 0 \quad (D.16)
\end{align*}\]

I now state and prove two useful properties of this system.

**Lemma D.1.** Consider the system (D.11) - (D.16). The solution satisfies

\[\frac{\theta_{cb}}{\theta_{bb}} \theta_{byH} = \theta_{c_yH} \quad (D.17)\]

**Proof.** By (D.14):

\[\frac{\theta_{cb}}{\theta_{bb}} = (1 + \bar{r}_b) \theta_{cb} - \frac{\eta}{\gamma} \bar{r}_b\]

By (D.16):

\[\frac{\theta_{c_yH}}{\theta_{byH}} = (1 + \bar{r}_b) \theta_{cb} - \frac{\eta}{\gamma} \bar{r}_b\]

Combining the two, we arrive at the stated result.

**Lemma D.2.** Consider the system (D.11) - (D.16). The solution satisfies

\[\frac{\theta_{c_yh}}{\theta_{c_yh+1}} = \theta_{bb}, \quad h = 0, 1, \ldots, H - 1 \quad (D.18)\]
and

\[
\frac{\theta_{byh}}{\theta_{byh+1}} = \theta_{bb}, \quad h = 0, 1, \ldots, H - 2
\] (D.19)

**Proof.** Use (D.12) to re-arrange (D.15) to get

\[
\theta_{cyh} = \frac{1 + \bar{r}_b}{1 + (1 + \bar{r}_b) \frac{\eta}{\gamma} \bar{c} \bar{b} \theta_{cyh+1}} \theta_{cyh+1}
\]

We need to show that the first term on the right-hand side is equal to \(\theta_{bb}\). To see this, note that (D.11) and (D.14) together imply that

\[
\theta_{cb} - (1 + \bar{r}_b) \theta_{cb} \theta_{bb} + \frac{\eta}{\gamma} \bar{r}_b \theta_{bb} = \frac{\bar{b}}{\bar{c}} \theta_{bb} - \frac{\bar{b}}{\bar{c}} (1 + \bar{r}_b) + \theta_{cb}
\]

But this condition can be re-arranged to give

\[
\theta_{bb} = \frac{1 + \bar{r}_b}{1 + (1 + \bar{r}_b) \frac{\eta}{\gamma} \bar{c} \bar{b} \theta_{cyh+1}} \theta_{cyh+1}
\]

as claimed. This proves (D.18). For (D.19) simply note that, by (D.12), we for all \(h = 0, 1, \ldots, H - 2\) have that

\[
\frac{\theta_{cyh}}{\theta_{cyh+1}} = \frac{\theta_{byh}}{\theta_{byh+1}}
\]

\(\Box\)

**D.10 Proof of Proposition B.1**

The proof of Proposition B.1 leverages Lemmas D.1 and D.2.

The impulse response of wealth holdings is given as

\[
\hat{b}_{t, H} = \sum_{\ell=0}^{t} \theta_{bb}^\ell \theta_{byt-\ell}
\]

The consumption impulse responses around horizon \(H\) are thus given as

\[
\hat{c}_{H-1, H} = \theta_{cb} \sum_{\ell=0}^{H-2} \theta_{bb}^\ell \theta_{byH-\ell-2} + \theta_{cyH-1}
\] (D.20)

\[
\hat{c}_{H, H} = \theta_{cb} \sum_{\ell=0}^{H-1} \theta_{bb}^\ell \theta_{byH-\ell-1} + \theta_{cyH}
\] (D.21)

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\[ \hat{c}_{H+1,H} = \theta_{cb} \sum_{\ell=0}^{H} \theta_{bb}^{\ell} \theta_{byH-\ell} \]  

(D.22)

I begin with the MPC at time \( H \). By Lemma D.2 we find that

\[ \lim_{H \to \infty} \hat{c}_{H,H} = \frac{\theta_{cb}}{1 - \theta_{bb}^2} \theta_{byH-1} + \theta_{cyH} \]

But by (D.12) - (D.13) and (D.15) - (D.16), \( \theta_{byH-1} \) and \( \theta_{cyH} \) are actually independent of \( H \), so \( \hat{c}_{H,H} \) converges to some fixed limit \( \frac{\theta_{cb}}{1} \times \omega \).

Next, consider moving down below the main diagonal, i.e., comparing (D.21) and (D.22). Note that we have

\[ \frac{1}{\theta_{bb}} \hat{c}_{H+1,H} = \frac{\theta_{cb}}{\theta_{bb}} (\theta_{bb} \hat{c}_{H-1,H} + \theta_{byH}) \]

\[ = \frac{\theta_{cb}}{\theta_{bb}} \hat{c}_{H-1,H} + \frac{\theta_{cb}}{\theta_{bb}} \theta_{byH} \]

By Lemma D.1, this is equal to \( \hat{c}_{H,H} \), proving the claim for \( \theta = \theta_{bb} \). For \( H + 2 \), we have

\[ \frac{1}{\theta_{bb}} \hat{c}_{H+2,H} = \frac{\theta_{cb}}{\theta_{bb}} \theta_{byH+1,H} = \frac{\theta_{cb}}{\theta_{bb}} \theta_{bb} \theta_{byH} = \hat{c}_{H+1,H} \]

The argument applies unchanged for any \( H + \ell \) with \( \ell \geq 2 \), proving the half of (B.8) below the main diagonal. The proof reveals that the result holds for any \( H \), not just \( H \to \infty \).

Now consider moving up above the main diagonal, i.e., comparing (D.21) and (D.20). Here we have

\[ \frac{1}{\theta_{bb}} \hat{c}_{H-1,H} = \frac{1}{\theta_{bb}} \left\{ \theta_{cb} \sum_{\ell=0}^{H-2} \theta_{bb}^{\ell} \theta_{byH-\ell-2} + \theta_{cyH-1} \right\} \]

\[ = \theta_{cb} \sum_{\ell=0}^{H-2} \theta_{bb}^{\ell} \theta_{byH-\ell-1} + \theta_{cyH} \]

where the second line has used Lemma D.2. We can thus write

\[ \frac{1}{\theta_{bb}} \hat{c}_{H-1,H} = \hat{c}_{H,H} - \theta_{cb} \theta_{bb}^{H-1} \theta_{by0} \]

But with \( \theta_{bb} \) inside the unit circle – the stable solution of the system (D.11) and (D.14) – it follows that second term converges to zero for \( H \to \infty \), establishing the claim with \( \theta = \theta_{bb} \),
the same constant of proportionality as below the main diagonal.\textsuperscript{23} Proceeding analogously for any $\ell > 0$, we have

$$
\frac{1}{\theta_{bb}} \tilde{c}_{H-h,H} = \tilde{c}_{H-h+1,H} - \theta_{cb} \theta_{bb}^{H-h} \theta_{by0}
$$

thus completing the proof of the half of (B.8) above the main diagonal. \hfill \Box

\textsuperscript{23}This argument leverages the fact that $\theta_{by0} \to 0$, which follows from Lemma D.2 together with the fact $\theta_{bb}$ is inside the unit circle.