CoVaR

A Systemic Contribution Risk Measure

Tobias Adrian and Markus K. Brunnermeier
Current financial regulation

1. Risk of each bank in isolation → Value at Risk
   - Capital requirements
   - Haircuts/margins
   - Ratings

2. Procyclical of capital requirements, haircuts, ratings

3. Focus on asset side of the balance sheet
   Liability side – maturity mismatch gets little attention
   - Maturity rat race
   - Implicit subsidies for short-term funding

4. Focus on banks – shadow banking system gets little attention
Three challenges....

1. Focus on externalities – systemic risk contribution
   - What are the externalities?
     ▪ Regulate based on externalities (functional criterion)
   - How to measure externalities (contribution to systemic risk)?
     ▪ CoVaR

2. Countercyclical regulation
   - Avoid procyclicality
     ▪ leverage, maturity mismatch,... predict future CoVaR

3. Incorporate funding structure
   asset-liability interaction, debt maturity, liquidity risk
1. Externalities

“stability is a public good”

1. Fire-sale externality
   - Maturity mismatch + Leverage
     - Raise new funds
     - Sell off assets
       (at fire sale prices due to crowded trades)

2. Hoarding externality
   - micro-prudent response: Hoard funds/reduce lending
   - ... but not necessarily macro-prudent
   - Systemic risk is endogenous (multiple equil)

3. Runs – dynamic co-opetition

4. Network Externality
   - Hiding own’s commitment uncertainty for counterparties

2. Procyclicality due to Liquidity spirals

- **Loss spiral**
  - same leverage
  - mark-to-market

- **Margin/haircut spiral**
  - Margin/haircut max leverage
  - The more short-term, the lower margin/haircut
  - *delever!
  - mark-to-model

- Mark-to-funding

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Brunnermeier-Pedersen (2009)
Margins/haircut increase in times of crisis → delever margin = f(risk measure)

Three reasons:
1. Backward-looking estimation of risk measure
   - Use forward looking measures
   - Use long enough data series
2. Fundamental volatility increases
3. Adverse selection
   - Debt becomes more information sensitive

Credit bubbles
- whose bursting undermines financial system

Great moderation ≠ great complacency

Countercyclical regulation
Margin/haircut spiral - Procyclicality

- Margins/haircut increase in times of crisis → delever
  margin = f(risk measure)

- Three reasons:
  1. Backward-looking estimation of risk measure
     - Use forward looking measures
     - Use long enough data series
  2. Fundamental volatility increases
  3. Adverse selection
     - Debt becomes more information sensitive (not so much out of the money anymore)

- Credit bubbles
  - whose bursting undermines financial system

Countercyclical regulation
Credit/Leverage Bubble

- Why did nobody delever/act against it earlier?
  - “dance as long as the music plays”
  - Lack of coordination when to go against the bubble
    - Not riding a bubble for too long is ... can cost you your shirt
    - Even if one identify bubbles, predicting the time of its bursting is infinitely more difficult
  - Investors/institutions ride the bubble which allows it to persist
    - Little heterogeneity

- Credit bubble led to housing bubble
  - Note similarity to Nordic countries, Japan,...
    (foreign capital, agency problems were less of an issue there)
1. **Externality:**
   - Measure contribution of institution to systemic risk: CoVaR
   - Response to current regulation
     - “hang on to others and take positions that drag others down when you are in trouble”
     - (maximize bailout probability)
     - become big
     - hold similar position (be in trouble when others are)
     - become interconnected

2. **Procyclicality:**
   - Lean against “credit bubbles” – laddered response
     - Bubble + maturity mismatch impair financial system (vs. NASDAQ bubble)
   - Impose Capital requirements/Pigouvian tax/Private insurance scheme
     - *not directly* on $\Delta\text{CoVaR}$, but on
     - frequently observed factors, like maturity mismatch, leverage, B/M, *crowdedness* of trades/credit, ...

3. **Funding:** **Asset-Liability Maturity Match**
### Who should be regulated?

<table>
<thead>
<tr>
<th>group</th>
<th>examples</th>
<th>macro-prudential</th>
<th>micro-prudential</th>
</tr>
</thead>
<tbody>
<tr>
<td>“individually systemic”</td>
<td>International banks (national champions)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>“systemic as part of a herd”</td>
<td>Leveraged hedge funds</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>non-systemic large</td>
<td>Pension funds</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>“tinies”</td>
<td>unlevered</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

- **Micro**: based on risk in isolation
- **Macro**: Classification on systemic risk contribution measure, e.g. CoVaR
- **Annual list** (not publicized)
CoVaR

- CoVaR_{q,i} is implicitly defined as quantile
  \[ \Pr(X_i \leq VaR_{q,i}) = q \]

- CoVaR_{q,j|i} is the VaR conditional on institute \( i \) (index) is in distress (at its VaR level)
  \[ \Pr(X_j \leq CoVaR_{q,j|i} \mid X_i = VaR_{q,i}) = q \]

- \( \Delta CoVaR_{q,j|i} = CoVaR_{q,j|i} - VaR_{q,j} \)

Various conditioning possibilities? (direction matters!)

**Contribution \( \Delta CoVaR \)**
- **Q1:** Which institutions contribute (in a non-causal sense)
- \( VaR_{\text{system}} \mid \) institution \( i \) in distress

**Exposure \( \Delta CoVaR \)**
- **Q2:** Which institutions are most exposed if there is a systemic crisis?
- \( VaR_i \mid \) system in distress

**Network \( \Delta CoVaR \)**
- \( VaR \) of institution \( j \) conditional on \( i \)

Can be extended to Co-Expected Shortfall!
Network CoVaR

- conditional on origin of arrow
Overview

- Challenges
- Measuring Systemic Risk Spillover/Externalities
- One Method: Quantile Regressions
- CoVaR vs. VaR
- Addressing Procyclicality
  - Predict using institutions’ characteristics
    - Balance sheet variables
    - Market variables (CDS, implied vol.,...)

...
Quantile Regressions: A Refresher

- **OLS Regression**: \( \min \text{ sum of squared residuals} \)
  \[
  \beta^{OLS} = \arg \min_{\beta} \sum_t \left( y_t - \alpha - \beta x_t \right)^2
  \]
  - **Predicted value**: \( E[y \mid x] = \alpha + \beta x \)

- **Quantile Regression**: \( \min \text{ weighted absolute values} \)
  \[
  \beta^q = \arg \min_{\beta} \sum_t \left\{ \begin{array}{ll}
  q |y_t - \alpha - \beta x_t| & \text{if } y_t - \alpha - \beta x_t \geq 0 \\
  1-q |y_t - \alpha - \beta x_t| & \text{if } y_t - \alpha - \beta x_t < 0
  \end{array} \right.
  \]
  - **Predicted value**: \( \text{VaR}_q \mid x = F^{-1}_y(q \mid x) = \alpha_q + \beta_q x \)

Note out (non-traditional) sign convention!
Financial Intermediary Data

- Publicly traded financial intermediaries 1986-2008
  - Commercial bank, security broker-dealers, insurance companies, real estate companies, etc.
  - Weekly market equity data from CRSP
  - Quarterly balance sheet data from COMPUSTAT

- CDS and option data of top 10 US banks, daily 2004-2008
Overview

- Measuring Systemic Risk Contribution
- One Method: Quantile Regressions
- CoVaR vs. VaR
- Addressing Procyclicality
  - Time-varying CoVaR/VaR
  - Predict using institutions’ characteristics
    - Balance sheet variables
    - Market variables (CDS, implied vol.,...
$\Delta \text{CoVaR}$ vs. $\text{VaR}$

- $\text{VaR}$ and $\Delta \text{CoVaR}$ relationship is very weak
- Data up to 12/06
Overview

- Challenges
- Measuring Systemic Risk Contribution
- One Method: Quantile Regressions
- CoVaR vs. VaR
- Addressing Procyclicality
  - Step 1: Time-varying CoVaRs
  - Step 2: Predict CoVaR using institution characteristics
    - Balance sheet variables (leverage, maturity mismatch, + interdependence, ...)
    - Market variables (CDS, implied vol., ...)

28
Step 1: Time-varying CoVaR

- Relate to \textit{macro factors}, $M_t$
  - VIX Level
  - 3 month yield
  - Repo – 3 month Treasury
  - Moody’s BAA – 10 year Treasury
  - 10Year – 3 month Treasury
  - Real estate index
  - Equity market risk

**interpretation**
- “Volatility”
- “Flight to Liquidity”
- “Credit indicator”
- “Business Cycle”
- “Housing”

Obtain Panel data of CoVaR
- Next step: Relate to institution specific (panel) data
**Step 1: Time-varying ΔCoVaR**

- **Derive time-varying VaR** \( \text{VaR}_t \)
  - For institution \( i \):
    \[
    X_t^i = \alpha_q^i + \beta_q^i M_t + \epsilon_t^i
    \]
  - For financial system:
    \[
    X_t^{\text{system}} = \alpha_q^{\text{system}} + \beta_q^{\text{system}} M_t + \epsilon_t^{\text{system}}
    \]

- **Derive time-varying CoVaR** \( \text{CoVaR}_t \)
  \[
  X_t^{\text{system}} = \alpha_q^{\text{system}i} + \beta_q^{\text{system}i} M_t + \gamma X_t^i + \epsilon_t^{\text{system}i}
  \]

- **ΔCoVaR** \( t = \text{CoVaR}_t - \text{VaR}_t \)
## Table 2: Average Exposures to Risk Factors

| INSTITUTIONS | VaR_{system} | VaR_i | CoVaR_{system|i} |
|--------------|--------------|-------|-----------------|
| Repo spread (lag) | -1163*** | -0.60 | -877.94*** |
| Credit spread (lag) | -107.75 | -0.47 | -226.75** |
| Term spread (lag) | 128.71 | 0.64 | 18.80 |
| VIX (lag) | -68.97*** | -0.16*** | -43.35*** |
| 3 Month Yield (lag) | 118.73 | 0.42 | 15.95* |
| Market Return (lag) | 242.74*** | 0.50*** | 196.00*** |
| Housing (lag) | 5.63 | 0.03 | 5.17 |

*** p< 0.01  
** p< 0.05  
* p< 0.1
Time-varying VaR
Time-varying VaR and ΔCoVaR
Step 2a: Portfolios Sorted on Characteristics

- Institutional characteristics matter
- ... but individual financial institutions have changed the nature of their business over time
- Form decile portfolios, each quarter, according to previous quarter’s data:
  1. Leverage
  2. Maturity mismatch
  3. Size
  4. Book-to-Market
- Add 4 industry portfolios
  1. Banks
  2. Security broker-dealers
  3. Insurance companies
  4. Real estate companies
<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>2 Years</th>
<th>1 Year</th>
<th>1 Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \text{CoVaR}$ (lagged)</td>
<td>0.71***</td>
<td>0.80***</td>
<td>0.94***</td>
</tr>
<tr>
<td>VaR (lagged)</td>
<td>-1.99***</td>
<td>-2.27***</td>
<td>-0.47***</td>
</tr>
<tr>
<td>Leverage (lagged)</td>
<td>-9.43***</td>
<td>-10.73***</td>
<td>-2.53**</td>
</tr>
<tr>
<td>Maturity mismatch (lagged)</td>
<td>-0.89***</td>
<td>-0.30</td>
<td>-0.14</td>
</tr>
<tr>
<td>Relative Size (lagged)</td>
<td>170.84***</td>
<td>-161.99***</td>
<td>-38.58***</td>
</tr>
<tr>
<td>Book-to-Market (lagged)</td>
<td>85.24***</td>
<td>87.65***</td>
<td>31.03**</td>
</tr>
<tr>
<td>Constant</td>
<td>-40.92**</td>
<td>-50.04**</td>
<td>-19.93*</td>
</tr>
<tr>
<td>Observations</td>
<td>3627</td>
<td>3805</td>
<td>3939</td>
</tr>
<tr>
<td>R²</td>
<td>0.62</td>
<td>0.69</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Table 3B: ΔCoVaR Forecasts by Characteristics
Cross-section, 2 years

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔCoVaR (lagged)</td>
<td>0.71***</td>
<td>0.63***</td>
<td>0.70***</td>
</tr>
<tr>
<td>VaR (lagged)</td>
<td>-1.99***</td>
<td>-1.86***</td>
<td>-1.38***</td>
</tr>
<tr>
<td>Leverage (lagged)</td>
<td>-9.43***</td>
<td>-5.08***</td>
<td>-4.23**</td>
</tr>
<tr>
<td>Maturity mismatch (lagged)</td>
<td>-0.89***</td>
<td>-0.51***</td>
<td>0.10</td>
</tr>
<tr>
<td>Relative Size (lagged)</td>
<td>170.84***</td>
<td>-105.62***</td>
<td>-86.84***</td>
</tr>
<tr>
<td>Book-to-Market (lagged)</td>
<td>85.24***</td>
<td>26.95***</td>
<td>-14.77**</td>
</tr>
<tr>
<td>Constant</td>
<td>-40.92**</td>
<td>-14.70*</td>
<td>36.88***</td>
</tr>
<tr>
<td>Observations</td>
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<td>R²</td>
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<td>0.70</td>
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Table 4: ΔCoVaR Forecasts by Characteristics
Time Series/Cross Section, Portfolios, 1%

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
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<th>1 Year</th>
<th>1 Quarter</th>
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<tbody>
<tr>
<td>ΔCoVaR (lagged)</td>
<td>0.41***</td>
<td>0.58***</td>
<td>0.86***</td>
</tr>
<tr>
<td>VaR (lagged)</td>
<td>-1.30***</td>
<td>-1.74***</td>
<td>0.06</td>
</tr>
<tr>
<td>Leverage (lagged)</td>
<td>0.92</td>
<td>-8.10***</td>
<td>-1.64</td>
</tr>
<tr>
<td>Maturity mismatch (lagged)</td>
<td>-0.31</td>
<td>-0.53</td>
<td>-0.33</td>
</tr>
<tr>
<td>Relative Size (lagged)</td>
<td>-230***</td>
<td>-229***</td>
<td>-56***</td>
</tr>
<tr>
<td>Book-to-Market (lagged)</td>
<td>29.25</td>
<td>42.69</td>
<td>31.03**</td>
</tr>
<tr>
<td>Constant</td>
<td>332.58***</td>
<td>-239.05***</td>
<td>-96.84***</td>
</tr>
<tr>
<td>Observations</td>
<td>3627</td>
<td>3805</td>
<td>3939</td>
</tr>
<tr>
<td>R²</td>
<td>0.69</td>
<td>0.73</td>
<td>0.89</td>
</tr>
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Timing of tail risk is harder to forecast than cross-section contribution.
Step 2b: Forecasting with Market Variables

- CDS spread and equity implied volatility for 10 largest US commercial and investment banks (from Bloomberg)

- Betas:
  - Extract principal component from CDS spread changes/implied vol changes within each quarter from daily data
  - Regress each CDS spread change/implied vol change on first principal component
Table 6: ΔCoVaR Forecasts by Market Variables Cross Section, Portfolios, 1%

<table>
<thead>
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<th>1 Quarter</th>
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<tbody>
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<td>ΔCoVaR (lagged)</td>
<td>0.60***</td>
<td>0.79***</td>
<td>0.94***</td>
</tr>
<tr>
<td>VaR (lagged)</td>
<td>-1.84</td>
<td>0.05</td>
<td>-0.08</td>
</tr>
<tr>
<td>CDS beta (lagged)</td>
<td>-1.727**</td>
<td>787.92</td>
<td>95.37</td>
</tr>
<tr>
<td>CDS (lagged)</td>
<td>1.320</td>
<td>-2.211</td>
<td>-40.26</td>
</tr>
<tr>
<td>Implied Vol beta (lagged)</td>
<td>-8.30</td>
<td>-590.28**</td>
<td>-85.78</td>
</tr>
<tr>
<td>Implied Vol (lagged)</td>
<td>-144.60</td>
<td>111.02</td>
<td>234.56***</td>
</tr>
<tr>
<td>Constant</td>
<td>-335.30</td>
<td>-147.72</td>
<td>-114.07*</td>
</tr>
<tr>
<td>Observations</td>
<td>114</td>
<td>154</td>
<td>184</td>
</tr>
<tr>
<td>R²</td>
<td>0.36</td>
<td>0.57</td>
<td>0.77</td>
</tr>
</tbody>
</table>

short data-span (2004-2008)!
Extension to our Analysis

- Co-Expected Shortfall ("Co-ES")
  - Advantage: coherent risk measure
  - Disadvantage: any estimate "in" the tail is very noise

- Inclusion of additional information
  - derivative positions
  - off-balance sheet exposure
  - Crowdedness measure
  - Interdependence measures
  - Bank supervision information
Countercyclical Regulation

- **When market is relaxed**
  Strict Laddered Response
  - **Step 1:** supervision enhanced
  - **Step 2:** forbidden to pay out dividends
    - See connection to debt-overhang problem
  - **Step 3:** No Bonus for CEOs
  - **Step 4:** Recapitalization within two months + debt/equity swap

- **When market is strict**
  Relax regulatory requirement
What type of charge?

- Capital charge
  - Strictly binding
  - Might stifle competition

- Pigouvian tax + government insurance
  - Generates revenue
  - In times of crisis it is cheap to issue government debt
  - Very salient

- Private insurance scheme
  - (Kashap, Rajan & Stein, 2008 + NYU report)
  - Requires lots of regulation
### Conclusion

- **Macro-prudential regulation**
  - Focus on externalities
  - Measure for systemic risk is needed, e.g. CoVaR
  - Maturity mismatch (+ Leverage) – encourage long-term funding

- **Countercyclical regulation**
  - Find variables that predict average future CoVaR
  - Forward-looking measures, spreads, ...

- **Also,**
  - VaR measure is not sufficient – incorrect focus
  - Quantile regressions are simple and efficient way to calculate CoVaR