Defaultable debt, interest rates and the current account

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Abstract

World capital markets have experienced large scale sovereign defaults on a number of occasions. In this paper we develop a quantitative model of debt and default in a small open economy. We use this model to match four empirical regularities regarding emerging markets: defaults occur in equilibrium, interest rates are countercyclical, net exports are countercyclical, and interest rates and the current account are positively correlated. We highlight the role of the stochastic trend in emerging markets, in an otherwise standard model with endogenous default, to match these facts.

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JEL classification: F3; F4

1. Introduction

World capital markets have experienced large scale sovereign defaults on a number of occasions, the most recent being Argentina’s default in 2002. This latest crisis is the fifth...
Argentine default or restructuring episode in the last 180 years.\footnote{See Reinhart et al. (2003).} While Argentina may be an extreme case, sovereign defaults occur with some frequency in emerging markets. A second set of facts about emerging markets relates to the behavior of the interest rates at which these economies borrow from the rest of the world and their current accounts. Interest rates and the current account are strongly countercyclical and positively correlated to each other. That is, emerging markets tend to borrow more in good times and at lower interest rates as compared to slumps. These features contrast with those observed in developed small open economies.

In this paper we develop a quantitative model of debt and default in a small open economy, which we use to match the above facts. Our approach follows the classic framework of Eaton and Gersovitz (1981) in which risk sharing is limited to one-period bonds and repayment is enforced by the threat of financial autarky. In all other respects the model is a standard small open economy model where the only source of shocks are endowment shocks. In this framework, we show that the model’s ability to match certain features in the data improve substantially when the productivity process is characterized by a volatile stochastic trend as opposed to transitory fluctuations around a stable trend. In a previous paper (Aguiar and Gopinath, 2004b), we document empirically that emerging markets are indeed more appropriately characterized as having a volatile trend. The fraction of variance at business cycle frequencies explained by permanent shocks is shown to be around 50% in a small developed economy (Canada) and more than 80% in an emerging market (Mexico).

To isolate the importance of trend volatility in explaining default, we first consider a standard business cycle model in which shocks represent transitory deviations around a stable trend. We find that default is extremely rare, occurring roughly twice every 2500 years. The weakness of the standard model begins with the fact that autarky is not a severe punishment, even adjusting for the relatively large income volatility observed in emerging markets. The welfare gain of smoothing transitory shocks to consumption around a stable trend is small. This in turn prevents lenders from extending debt, which we demonstrate through a simple calculation à la Lucas (1985). We can support a higher level of debt in equilibrium by assuming an additional loss of output in autarky. However, in a model of purely transitory shocks, this does not lead to default at a rate that resembles those observed in many economies.

The intuition behind why default occurs so rarely in a model with transitory shocks and a stable trend is described in Section 3. The decision to default rests on the difference between the present value of utility (value function) in autarky versus that of financial integration. Quantitatively, the level of default that arises in equilibrium depends on the relative sensitivity of the two value functions to endowment shocks. When the endowment process is close to a random walk there is limited need to save out of additional endowment, leaving little difference between financial autarky and a good credit history, regardless of the realization of income. At the other extreme, if the transitory shock is iid over time, then there is an incentive to borrow and lend, making integration much more valuable than autarky. However, an iid shock has limited impact on the entire present
discounted value of utility, and so the difference between integration and autarky is not sensitive to the particular realization of the iid shock. At either extreme, therefore, the decision to default is not sensitive to the realization of the shock. Consequently, when shocks are transitory, the level of outstanding debt—and not the realization of the stochastic shock—is the primary determinant of default. This is reflected in financial markets by an interest rate schedule that is extremely sensitive to quantity borrowed. Borrowers internalize the steepness of the “loan supply curve” and recognize that an additional unit of debt at the margin will have a large effect on the cost of debt. Agents therefore typically do not borrow to the point where default is probable.

On the other hand, a shock to trend growth has a large impact on the two value functions (because of the shock’s persistence) and on the difference between the two value functions. The latter effect arises because a positive shock to trend implies that income is higher today, but even higher tomorrow, placing a premium on the ability to access capital markets to bring forward anticipated income. In this context, the decision to default is relatively more sensitive to the particular realization of the shock and less sensitive to the amount of debt. Correspondingly, the interest rate is less sensitive to the amount of debt held. Agents are consequently willing to borrow to the point that default is relatively likely. This theme is developed in Section 4.

The next set of facts concerns the phenomenon of countercyclical current accounts and interest rates. In the current framework where all interest rate movements are driven by changes in the default rate, the steepness of the interest rate schedule makes it challenging to even qualitatively match the positive correlation between interest rates and the current account. This is because, on the one hand, an increase in borrowing in good states (countercyclical current account) will, all else equal, imply a movement along the heuristic “loan supply curve” and a sharp rise in the interest rate. On the other hand, if the good state is expected to persist, this lowers the expected probability of default and is associated with a favorable shift in the interest rate schedule. To generate a positive correlation between the current account and interest rates we need the effect of the shift of the curve to dominate the movement along the curve. A stochastic trend is again useful in matching this fact since the interest rate function tends to be less steeply sloped and trend shocks have a significant effect on the probability of default. Accordingly, in our benchmark simulations, a model with trend shocks matches the empirical feature of a positive correlation between the interest rate and the current account. The model with transitory shocks however fails to match this fact. The prediction for which both models perform poorly is in matching the volatility of the interest rate process.2

The model with shocks to trend generates default roughly once every 125 years, which is a 10-fold improvement over the standard model but still shy of the observed pattern for chronic defaulters. We bring the default rate closer to that observed empirically for Latin America by introducing third-party bailouts. Realistic bailouts raise the rate of default dramatically—bailouts up to 18% of GDP lead to defaults once every 27 years. However, the subsidy implied by bailouts breaks the tight linkage between default probability and

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2 The additional observed volatility may be due to a volatile risk premium, as suggested by Broner et al. (2004).
the interest rate. Interest rate volatility is therefore an order of magnitude below that observed empirically.

The business cycle behavior of markets in which agents can choose to default has received increasing attention in the literature. The approach we adopt here is a dynamic stochastic general equilibrium version of Eaton and Gersovitz (1981) and is similar to the formulations in Chatterjee et al. (2002) on household default and Arellano (2004) on emerging market default. Our point of distinction from the previous literature is the emphasis we place on the role of the stochastic trend in driving the income process in emerging markets. We find that the presence of trend shocks substantially improves the ability of the model to generate empirically relevant levels of default. Moreover, we obtain the coincidence of countercyclical net exports, countercyclical interest rates and the positive correlation between interest rates and current account observed in the data. This is distinct from what is obtained in Arellano (2004) and Kehoe and Perri (2002).

In the next section we describe empirical facts for Argentina. Section 2 describes the model environment, parameterization and solution method. Section 3 describes the model with a stable trend and its predictions. Section 4 describes the model with a stochastic trend and performs sensitivity analysis. Section 5 examines the effect of third party bailouts on the default rate and Section 6 concludes.

1.1. Empirical facts

Reinhart et al. (2003) document that among emerging markets with at least one default or restructuring episode between 1824 and 1999, the average country experienced roughly 3 crises every 100 years. The same study documents that the external debt to GDP ratio at the time of default or restructuring averaged 71%. A goal of any quantitative model of emerging market default is to generate a fairly high frequency of default coinciding with an equilibrium that sustains a large debt to GDP ratio.

<table>
<thead>
<tr>
<th>Data</th>
<th>HP</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>4.08</td>
<td>(0.52)</td>
</tr>
<tr>
<td>$\sigma(R_s)$</td>
<td>3.17</td>
<td>(0.54)</td>
</tr>
<tr>
<td>$\sigma(TB/Y)$</td>
<td>1.36</td>
<td>(0.24)</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>1.19</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$\rho(Y)$</td>
<td>0.85</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\rho(R_s, Y)$</td>
<td>$-0.59$</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$\rho(TB/Y, Y)$</td>
<td>$-0.89$</td>
<td>(0.10)</td>
</tr>
<tr>
<td>$\rho(R_s, TB/Y)$</td>
<td>0.68</td>
<td>(0.13)</td>
</tr>
<tr>
<td>$\rho(C, Y)$</td>
<td>0.96</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

The series were deseasonalized if a significant seasonal component was identified. We log the income, consumption and investment series and compute the ratio of the trade balance (TB) to GDP ($Y$) and the interest rate spread ($R_s$). $R_s$ refers to the difference between Argentina dollar interest rates and US 3-month treasury bond rate (annualized numbers). All series were then HP filtered with a smoothing parameter of 1600. GMM estimated standard errors are reported in parenthesis under column SE. The standard deviations ($Y, R_s, TB/Y$) are reported in percentage terms.
Table 1 documents business cycle features for Argentina over the period 1983.1 to 2000.2 using an HP filter with a smoothing parameter of 1600 for quarterly frequencies. A striking feature of the business cycle is the strong countercyclicality of net exports (−0.89) and interest rates spreads (−0.59). Interest rates and the current account are also strongly positively correlated (0.68). These features regarding interest rates, in addition to the high level of volatility of these rates, have been documented by Neumeyer and Perri (2004) to be true for several other emerging market economies and to contrast with the business cycle features of Canada, a developed small open economy. Aguiar and Gopinath (2004b) document evidence of the stronger countercyclicality of the current account for emerging markets relative to developed small open economies. In the model we emphasize the distinction between shocks to the stochastic trend and transitory shocks. This is motivated by previous research (Aguiar and Gopinath (2004b)) that documents that emerging markets are subject to more volatile shifts in stochastic trend as compared to a developed small open economy.

2. Model environment

To model default we adopt the standard framework of Eaton and Gersovitz (1981). Specifically, we assume that international assets are limited to one period bonds. If the economy refuses to pay any part of the debt that comes due, we say the economy is in default. Once in default, the economy is forced into financial autarky for a period of time as punishment. This is similar to the framework adopted in Chatterjee et al. (2002) and Arellano (2004).

We begin our analysis with a standard model of a small open economy that receives a stochastic endowment stream, $y_t$. (We discuss a production economy in Section 4.1.) The economy trades a single good and single asset, a one period bond, with the rest of the world. The representative agent has CRRA preferences over consumption of the good:

$$u = \frac{c^{1-\gamma}}{1-\gamma}.$$  \hfill (1)

The endowment $y_t$ is composed of a transitory component $z_t$ and a trend $\Gamma_t$:

$$y_t = e^{zt} \Gamma_t.$$  \hfill (2)

The transitory shock, $z_t$, follows an AR(1) around a long run mean $\mu_z$

$$z_t = \mu_z(1 - \rho_z) + \rho_z z_{t-1} + \epsilon_t^z$$  \hfill (3)

$|\rho_z| < 1$, $\epsilon_t^z \sim N(0, \sigma_z^2)$, and the trend follows

$$\Gamma_t = g_t \Gamma_{t-1}$$  \hfill (4)

$$\ln(g_t) = (1 - \rho_g) (\ln(\mu_g) - c) + \rho_g \ln(g_{t-1}) + \epsilon_t^g$$  \hfill (5)

$|\rho_g| < 1$, $\epsilon_t^g \sim N(0, \sigma_g^2)$, and $c = \frac{1}{2} \frac{\sigma_z^2}{1 - \rho_z^2}$. 

We denote the growth rate of trend income as \( g_t \), which has a long run mean \( \mu_g \). The log growth rate follows an AR(1) process with AR coefficient \(|\rho_g|<1\). Note that a positive shock \( e^g \) implies a permanently higher level of output, and to the extent that \( \rho_g>0 \), a positive shock today implies that the growth of output will continue to be higher beyond the current period. We assume that \( E\{\lim_{t\to\infty} B_t(\Gamma_t)^{1-\gamma}\}=0 \) to ensure a well defined problem, where \( 0<\beta<1 \) denotes the agent’s discount rate.

Let \( a_t \) denote the net foreign assets of the agent at time \( t \). Each bond delivers one unit of the good next period for a price of \( q \) this period. We will see below that in equilibrium \( q \) depends on \( a_t \) and the state of the economy. We denote the value function of an economy with assets at and access to international credit as \( V(a_t, z_t, \Gamma_t) \). At the start of the period, the agent decides whether to default or not. Let \( V^B \) denote the value function of the agent once it defaults. The superscript \( B \) refers to the fact that the economy has a bad credit history and therefore cannot transact with international capital markets (i.e., reverts to financial autarky). Let \( V^G \) denote the value function given that the agent decides to maintain a good credit history this period. The value function of being in good credit standing at the start of period \( t \) with net assets can then be defined as \( V(a_t, z_t, \Gamma_t) = \max( V^G(a_t, z_t, \Gamma_t), V^B(a_t, z_t, \Gamma_t) ) \). At the start of period \( t \), an economy in good credit standing and net assets \( a_t \) will default only if \( V^B(z_t, \Gamma_t) > V^G(a_t, z_t, \Gamma_t) \).

An economy with a bad credit rating must consume its endowment. However, with probability \( \lambda \) it will be “redeemed” and start the next period with a good credit rating and renewed access to capital markets. If redeemed, all past debt is forgiven and the economy starts off with zero net assets. We also add a parameter \( \delta \) that governs the additional loss of output in autarky.\(^3\)

In recursive form, we therefore have:

\[
V^B(z_t, \Gamma_t) = u((1-\delta)y_t) + \lambda \beta E_t V^B(0, z_{t+1}, \Gamma_{t+1}) + (1-\lambda) \beta E_t V^B(z_{t+1}, \Gamma_{t+1})
\]

(6)

where \( E_t \) is expectation over next period’s endowment and we have used the fact that \( \lambda \) is independent of realizations of \( y \). If the economy does not default, we have:

\[
V^G(a_t, z_t, \Gamma_t) = \max_{c_t} \{ u(c_t) + \beta E_t V^G(a_{t+1}, z_{t+1}, \Gamma_{t+1}) \}
\]

s.t. \( c_t = y_t + a_t - q_t a_{t+1} \).

(7)

The international capital market consists of risk neutral investors that are willing to borrow or lend at an expected return of \( r^* \), the prevailing world risk free rate. The default function \( D(a_t, z_t, \Gamma_t) = 1 \) if \( V^B(z_t, \Gamma_t) > V^G(a_t, z_t, \Gamma_t) \) and zero otherwise. Then equilibrium in the capital market implies

\[
q(a_{t+1}, z_t, \Gamma_t) = \frac{E_t \{ (1-D_{t+1}) \}}{1+r^*}.
\]

(8)

The higher the expected probability of default the lower the price of the bond.

\(^3\) Rose (2003) finds evidence of a significant and sizeable (8% a year) decline in bilateral trade flows following the initiation of debt renegotiation by a country.
To emphasize the distinction between the role of transitory and permanent shocks we present two extreme cases of the model described above. Model I will correspond to the case when the only shock is the transitory shock $z_t$ and Model II to the case when the only shock has permanent effects, $g_t$. Since few results can be analytically derived we discuss at the outset the calibration and solution method employed.

2.1. Calibration and model solution

Benchmark parameters that are common to all models are reported in Table 2A. Each period refers to a quarter with a quarterly risk free interest rate of 1%. The coefficient of relative risk aversion of 2 is standard. The probability of redemption $\lambda = 0.1$ implies an average stay in autarky of 2.5 years, similar to the estimate by Gelos et al. (2004). The additional loss of output in autarky is set at 2%. We will see in our sensitivity analysis (Section 4.1) that high impatience is necessary for generating reasonable default in equilibrium. Correspondingly, our benchmark calibration sets $\beta = 0.8$. Authors such as Arellano (2004) and Chatterjee et al. (2002) also employ similarly low values of $\beta$ to generate default. The mean quarterly growth rate ($\mu_g$) is calibrated to 0.6% to match the number for Argentina.

The remaining parameters characterize the underlying income process and therefore vary across models (Table 2B). To focus on the nature of the shocks, we ensure that the HP filtered income volatility derived in simulations of both models approximately match the same observed volatility in the data. In Model I, output follows an AR(1) process with stable trend and an autocorrelation coefficient of $q_z = 0.9$, which is similar to the values used in many business cycle models and $r_z = 3.4\%$. We set the mean of log output equal to $-1/2\sigma_z^2$ so that average detrended output in levels is standardized to one. In Model II, $r_z = 0$, $r_g = 3\%$ and $q_g = 0.17$.

To solve the model numerically, we first recast the Bellman equations in detrended form. To detrend, we normalize all variables by $\frac{\log C_t}{C_0}$. This normalization implies that the mean of the detrended endowment is one. A technical appendix available from the authors’ websites derives some key properties of the value functions. In particular, the appendix discusses the equivalence of the original and the detrended problem. We also

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4 One of the reasons we consider the two extremes is to minimize the dimensionality of the problem, which we solve employing discrete state space methods. Using insufficient grids of the state space can generate extremely unreliable results in this set up.

5 Note that we have detrended using the cumulated trend through the previous period. Computationally, the results do not differ with detrending by $\Gamma$. 

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Table 2A
Common benchmark parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>World interest rate</td>
<td>$r^*$</td>
</tr>
<tr>
<td>Loss of output in autarky</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Probability of redemption</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Mean (log) transitory productivity</td>
<td>$\mu_z$</td>
</tr>
<tr>
<td>Mean growth rate</td>
<td>$\mu_g$</td>
</tr>
</tbody>
</table>

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demonstrate the homogeneity properties of the budget sets, value functions, and equilibrium interest rate schedules that allow us to solve the problem in detrended form. We will use the notation $\hat{X}$ to represent the detrended counterpart for any variable or function $X$. Specifically, $\hat{a}_t = \left( a_t / (\log C_t / C_0) \right)$ and $\hat{c}_t = \left( c_t / (\log C_t / C_0) \right)$. The technical appendix proves that the value functions are homogenous of degree 1 in $\hat{a}$ and $C$. Therefore, $(\log C_t / C_0)V(at, zt, C_t) = V(\hat{a}_t, zt, (g_t / \mu_g)) = \hat{V}(\hat{a}_t, zt, \hat{g}_t)$. We define $\hat{V}^B$ and $\hat{V}^G$ in a similar fashion. The interest rate schedule $q$ is homogenous of degree zero in $\hat{a}$ and $\Gamma$. We therefore define $\hat{q}(\hat{a}_{t+1}, zt, g_t) = q(\hat{a}_{t+1}, zt, (g_t / \mu_g)) = q(at_{t+1}, zt, C_t)$. The detrended default indicator function $\hat{D}$ is defined in the same fashion.

To solve the detrended problem, we use the discrete state-space method. We approximate the continuous AR(1) process for income with a discrete Markov chain using 25 equally spaced grids of the original processes steady state distribution. We then integrate the underlying normal density over each interval to compute the values of the Markov transition matrix. The asset space is discretized into 400 possible values. We ensured that the limits of our asset space never bind along the simulated equilibrium paths. The solution algorithm involves the following:

(i) Assume an initial price function $\hat{q}^0(\hat{a}, z, g)$. Our initial guess is the risk free rate at each point in the state space.

(ii) Use this $\hat{q}^0$ and an initial guess for $\hat{V}^B,0$ and $\hat{V}^G,0$ to iterate on the Bellman Eqs. (6) and (7) to solve for the optimal value functions $\hat{V}^B, \hat{V}^G, \hat{V} = \max(\hat{V}^G, \hat{V}^B)$ and the optimal policy functions.

(iii) For the initial guess $\hat{q}^0$, we now have an estimate of the default function $\hat{D}^0(\hat{a}, z, g)$. Next, we update the price function as $\hat{q}^1(\hat{a}_t, (1 - \hat{D}_{t+1})) / (1 + r^*)$ and using this $\hat{q}^1$ repeat steps (ii) and (iii) until $|\hat{q}^{i+1} - \hat{q}^i| < \varepsilon$, where $i$ represents the number of the iteration and $\varepsilon$ is a very small number.

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6 Recall that the default function $D$ is the indicator function that equals one if $V^B > V^G$. This plus the homogeneity of the value functions implies that $D$ is homogenous of degree zero in $\hat{a}$ and $\Gamma$. The interest rate function inherits this property from the definition of $q$ (Eq. (8)). See the technical appendix for a formal proof that this holds in equilibrium.

7 It is important to span the stationary distribution sufficiently so as to include large negative deviations from the average even if these are extremely rare events because default is more likely to occur in these states.
3. Model I: stable trend

Model I assumes a deterministic trend \((\Gamma_t=(\mu_g)')\) and the process for \(z_t\) is given by Eq. (3). In Fig. 1A, we plot the difference between the value function with a good credit rating \(\hat{V}_G\) and that of autarky \(\hat{V}_A\) as a function of \(z\) for our calibration. The agent defaults when output is relatively low. The top panel of Fig. 2A plots the region of default in \((z, \hat{a})\) space. The line that separates the darkly shaded from the lightly shaded region represents combinations of \(z\) and \(\hat{a}\) along which the agent is indifferent between defaulting and not defaulting. The darkly shaded region represents combinations of low productivity and negative foreign assets for which it is optimal to default.

![Graph](image-url)

Fig. 1. (A) Model I. (B) Model II. Note: \(V_G\) represents the (detrended) value function when the agent chooses to repay and is in good credit standing and \(V_B\) is the (detrended) value function when the agent chooses to default. We have plotted here the difference between the two value functions for a given level of assets, across different productivity states. Fig. 2A corresponds to the case when \(z\) varies and Fig. 2B to the case when \(g\) varies. The \((V_G - V_B)\) line is more steeply sloped in the case of \(g\) shocks.
For a given realization of $z$, clearly the agent is more likely to default at lower values of $\hat{a}$. Since $\hat{V}^B$ refers to financial autarky, its value is invariant to $\hat{a}$. Conversely, $\hat{V}^G$ is strictly increasing in assets. This follows straightforwardly from the budget constraint and strict monotonicity of utility. For each $z$, there is a unique point of intersection, say $\hat{a}(z)$, and the agent will default if foreign assets lie below $\hat{a}(z)$.

The default decision as a function of $z$ is less clear-cut. In the case when $\lambda = 0$, it must be the case that $\hat{V}^G$ should be at least as steep as $\hat{V}^B$ at the indifference point, for a given $\hat{a}$. To see this, consider the value of an additional unit of endowment at the indifference point. Clearly, the continuation value for an agent in good credit standing is always (weakly) greater than that of an agent with a bad credit standing (this is true given that $\lambda = 0$). If an agent is indifferent between defaulting or not, current consumption absent default must therefore be (weakly) less than under default, implying an equal or higher marginal utility

Fig. 2. Default region. Note: the darkly shaded region represents combinations of the productivity state and (detrended) assets for which the economy will prefer default. The lightly shaded region accordingly is the nondefault region. The vertical axis represents the realization of the productivity shock. The horizontal axis represents assets normalized by trend income. In both pictures, the agent is more likely to default when holding larger amounts of debt (negative assets) and when in worse productivity states. The line of indifference is less steeply sloped in the case of $g$ shocks.
of consumption. Now consider what happens to utility with an additional unit of endowment at this point. The agent in autarky must consume this additional income. If the agent with a good credit rating consumes the endowment, her utility increases more due to the higher marginal utility of consumption. The agent with a good credit standing also has the option to save the additional endowment. She will only do so if it raises utility by more than consuming it immediately. Therefore, the value function of the agent with good credit rating is more sensitive to an additional unit of endowment at the indifference point. Hence, the slope of $\tilde{V}^G$ relative to $z$ must be (weakly) greater than the slope of $\tilde{V}^B$ relative to $z$, at the indifference point. However, if $\lambda > 0$, then the agent in autarky has an outcome not available to the one in good credit standing, i.e., redemption with debt forgiveness. In this case, the previous argument’s premise does not hold.

The fact that financial autarky is relatively attractive in bad states of the world is not a feature shared by alternative models based on Kehoe and Levine (1993). In that model, optimal “debt” contracts are structured so that the agent never chooses autarky. However, the participation constraint binds strongest in good states of nature, i.e., the states in which the optimal contract calls for payments by the agent. In the bond-only framework, contracted payments are not state contingent and therefore the burden of repayment is greatest in low endowment states.

3.1. Debt and default implications in Model I

A simple calculation à la Lucas (1985) quickly reveals that it is difficult to sustain a quantitatively realistic level of debt in a standard framework without recourse to additional punishment.8 Consider our endowment economy in which the standard deviation of shocks to detrended output are roughly 4%. For this calculation, we stack the deck against autarky by assuming no domestic savings (capital or storage technology), that shocks are iid, and that autarky lasts forever. We stack the deck in favor of financial integration by supposing that integration implies a constant consumption stream (perfect insurance). In order to maintain perfect consumption insurance, we suppose that the agent must make interest payments of $rB$ each period. We now solve for how large $rB$ can be before the agent prefers autarky. We then interpret $B$ as the amount of sustainable debt when interest payments are equal to $r$.

Specifically, let $Y_t = \tilde{Y} e^{\gamma} e^{-\left(\frac{1}{2}\right)\sigma_z^2}$ where $z \sim N(0, \sigma_z^2)$ and iid over time (for this example, we set $\mu_g = 1$). We ensure that $EY_t = \tilde{Y}$ regardless of the volatility of the shocks. Then,

$$V^B = E \left( \frac{\sum_{t=1}^{\infty} \beta^t Y_t^{1-\gamma} c_t^{1-\gamma}}{1 - \gamma} \right)$$

Assuming that financial integration results in perfect consumption insurance,

$$V^G = E \left( \frac{\sum_{t=1}^{\infty} \beta^t c_t^{1-\gamma}}{1 - \gamma} \right)$$

8 See Obstfeld and Rogoff (1996) and Mendoza (1992) for alternative calculations of the welfare gains from financial integration.
The economy will not default as long as \( V^G \geq r^B \), or \((rB)\bar{Y} \leq 1 - \exp\left(-\frac{1}{2}\sigma_z^2\right)\). The volatility of detrended output for Argentina is 4.08\% (i.e. \( \sigma_z^2 = 0.0408^2 = 0.0017 \)). For a coefficient of relative risk aversion of 2, this implies the maximum debt payments as a percentage of GDP is 0.17\%. Or, at a quarterly interest rate of 2\%, debt cannot exceed 8.32\% of output.

Our simulated model will be shown to support higher debt levels because we impose an additional loss of \( \delta \) percent of output during autarky. Introducing such a loss into the above calculation implies a debt cutoff of \((rB)\bar{Y} \leq 1 - (1 - \delta)\exp\left(-\frac{1}{2}\sigma_z^2\right)\). If \( \delta = 0.02 \), we can support debt payments of 20\% of GDP, which implies a potentially large debt to GDP ratio. It is clear that to sustain any reasonable amount of debt in equilibrium in a standard model, we need to incorporate punishments beyond the inability to self-insure, particularly since in reality financial integration does not involve full insurance and autarky does not imply complete exclusion from markets.

A second implication of the model is that default rarely occurs in equilibrium. This arises because of the steepness of the interest rate schedule and the fact that the agent internalizes the effect of his borrowing on the interest rate he must pay. Fig. 3A plots the \( \bar{q} \) schedule as a function of assets for the highest and lowest realizations of \( z \). Over asset regions for which agents never default, the implied interest rate is the risk free rate (\( \bar{q} = (1/ (1 + r^*)) \)). However, the schedule is extremely steep over the range of assets for which default is possible. Consequently, even when the current endowment shock is below average the agent does not borrow much. At the margin, the borrower recognizes that an additional unit of debt raises the average cost of debt by the slope of the interest rate schedule.

The steepness of the interest rate schedule is ultimately tied to the persistence of the endowment process. Let \( \bar{z}(\ddot{a}) \) denote the threshold endowment below which the agent defaults for the given asset level. That is, \( \bar{z} \) is the line separating the shaded region from the unshaded region in Fig. 2. For a given \( \dot{a} \), we can then express the probability of default at time \( t+1 \) as \( \Pr(z_{t+1} < \bar{z}(\ddot{a}_{t+1})|zt) \), and correspondingly \( g(\dot{a}_{t+1}) = \frac{1-\Pr(z_{t+1} < \bar{z}(\ddot{a}_{t+1})|zt)}{(1+r^*)} \).

Suppose the investor is considering saving an additional \( \Delta \ddot{a} \) of assets. The change in the interest rate is given by

\[
\bar{q}(\dot{a}_{t+1} + \Delta \ddot{a}, z_t) - \bar{q}(\dot{a}_{t+1}, z_t) = \frac{-1}{1 + r^*} \sum_{\bar{z}(\ddot{a}_{t+1}) \leq z \leq \bar{z}(\ddot{a} + \Delta \ddot{a})} \pi(z_{t+1}|z_t).
\] (11)

Where \( \pi(z_{t+1}|z_t) \) represents the probability that \( z_{t+1} \) will happen conditional on \( z_t \). From Fig. 2, we see that quantitatively, the distance between \( \bar{z}(\ddot{a}) \) and \( \bar{z}(\ddot{a} + \Delta \ddot{a}) \) for small \( \Delta \ddot{a} \) is extremely large. That is, the steepness of the \( \bar{z}(\ddot{a}) \) translates into the steepness of the \( \bar{q} \) schedule.

We now link the shape of \( \bar{z}(\ddot{a}) \) to the nature of the shock process. Recall that \( \bar{z}(\ddot{a}) \) represents combinations of \( \ddot{a} \) and \( z \) for which the agent is indifferent to default, i.e., \( \ddot{V}^G(\ddot{a}, \bar{z}(\ddot{a})) = \ddot{V}^B(\bar{z}(\ddot{a})) \). An increase in \( \ddot{a} \) directly raises only \( \ddot{V}^G \). Therefore, to maintain equality, we need to lower \( \bar{z} \) (recall that \( V^G \) is more sensitive to \( z \) at the indifference point). How much \( \bar{z} \) needs to fall depends on the difference in sensitivity: \( (\Delta \ddot{V}^G/\Delta z) - (\Delta \ddot{V}^B/\Delta z) \), where \( \Delta \ddot{V}^j, j = G, B \), is the change in \( \ddot{V}^j \) induced by the change in \( z \). In the case of Model I, this difference tends to be a very small number. This can be seen from Fig. 1A that plots the
The difference between \( \hat{V}_G \) and \( \hat{V}_B \) across \( z \) (for a given \( \hat{a} \)). This implies that the slopes of \( \hat{V}_G \) and \( \hat{V}_B \) with respect to \( z \) are not that different.

The similarity in slopes results from the underlying process for \( z \). Suppose that \( z \) is a random walk. In this case, a shock to \( z \) today is expected to persist indefinitely and will have a large impact on expected lifetime utility. However, with a random walk income process there is limited need to save out of additional endowment (the only reason would be precautionary savings and the insurance provided by the option to default). This implies an additional unit of endowment will be consumed, leaving little difference between financial autarky and a good credit history. This issue arises for strictly transitory shock processes as well. Consider the other extreme and suppose that \( z \) is iid over time. Then there is a stronger incentive to borrow and lend. However, the lack of persistence implies

Fig. 3. (A) Model I. (B) Model II. Note: A and B plot the price of the bond (inverse of one plus the interest rate) as a function of (detrended) assets for the highest and lowest values of \( z \) in the case of A and the same for growth shocks in B. The price function is less sensitive to changes in borrowing in the case of \( g \) shocks (B).
the impact of an additional unit of endowment today is limited to its effect on current endowment, resulting in a limited impact on the entire present discounted value of utility. That is, both $\Delta \hat{V}^G$ and $\Delta \hat{V}^B$ are relatively small and therefore so is the difference. Therefore, whether a shock follows a random walk or is iid, a given shock realization has little impact on the difference between $\hat{V}^G$ and $\hat{V}^B$. Consequently, a large movement in $\bar{z}$ is necessary to maintain equality between $\hat{V}^G$ and $\hat{V}^B$ for a given change in assets.

The key point is that in our benchmark Model I, the $q^*$ schedule is extremely steep over the relevant range of borrowing. Moreover, the slope increases dramatically as we move into the range of potential default. Therefore, a large implicit demand for borrowing does not result in additional borrowing and a high probability of default. Instead, it generates minimal borrowing and a large movement in the slope of the interest rate function. The flip side of this is that net exports are extremely stable.

### 3.2. Business cycles implications in Model I

Table 3, column 3A, reports key business cycle moments from Model I.\(^9\) Default is a rare event as it occurs on average only two times in 10,000 periods (i.e., once every 2500 years). The discussion in the previous section explains why this is the case for models with purely transitory shocks. Net export and interest volatility is much lower than in the data. The lack of interest rate volatility is an immediate consequence of the fact that default rarely occurs in equilibrium. The model supports a maximum debt to GDP ratio of 26%.

A typical feature of these models is that the current account and the interest rate tend to be negatively correlated, a counterfactual implication. This follows from the steepness of the interest rate function. That is, if the agent borrows more in good states of the world

---

\(^9\) To analyze the model economy’s stationary distribution we simulate the model for 10,000 periods and extract the last 500 observations to rule out any effect of initial conditions. We then log and HP filter the simulated series in the same way as the empirical data, using an HP filter smoothing parameter of 1600. The reported numbers are averages over 500 such simulations.

### Table 3

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model I (3A)</th>
<th>Model II (3B)</th>
<th>Model II with bailouts (3C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)$</td>
<td>4.08</td>
<td>4.32</td>
<td>4.45</td>
<td>4.43</td>
</tr>
<tr>
<td>$\sigma(c)$</td>
<td>4.85</td>
<td>4.37</td>
<td>4.71</td>
<td>4.68</td>
</tr>
<tr>
<td>$\sigma(TB/Y)$</td>
<td>1.36</td>
<td>0.17</td>
<td>0.95</td>
<td>1.10</td>
</tr>
<tr>
<td>$\sigma(R_s)$</td>
<td>3.17</td>
<td>0.04</td>
<td>0.32</td>
<td>0.12</td>
</tr>
<tr>
<td>$\rho(C, Y)$</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho(TB/Y, Y)$</td>
<td>$-0.89$</td>
<td>$-0.33$</td>
<td>$-0.19$</td>
<td>$-0.12$</td>
</tr>
<tr>
<td>$\rho(R_s, Y)$</td>
<td>$-0.59$</td>
<td>0.51</td>
<td>$-0.03$</td>
<td>$-0.02$</td>
</tr>
<tr>
<td>$\rho(R_s, TB/Y)$</td>
<td>0.68</td>
<td>$-0.21$</td>
<td>0.11</td>
<td>0.38</td>
</tr>
<tr>
<td>Rate of default</td>
<td>75</td>
<td>2</td>
<td>23</td>
<td>92</td>
</tr>
<tr>
<td>Mean debt output ratio (%)</td>
<td>27</td>
<td>19</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Maximum $R_s$ (basis points)</td>
<td>23</td>
<td>151</td>
<td>57</td>
<td></td>
</tr>
</tbody>
</table>

Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). The simulated data is treated in an identical manner to the empirical data. Standard deviations are reported in percentages.
(countercyclical current account) then one effect is for the interest rate to increase as the agent moves up the "loan supply curve". The countering effect is that a persistent good state can imply a lower probability of default and therefore a shift down in the $\hat{q}$ schedule. To generate the empirical fact that countries borrow more in good times at lower interest rates we need the second effect to dominate the first. However, the steepness of the $\hat{q}$ schedule in Model I makes this a less likely outcome. Consequently, in our parameterization of Model I, we obtain a countercyclical current account as the data suggests, however the interest rate process is now procyclical.

We have adopted a relatively persistent process for income that generates a countercyclical current account, but at the cost of procyclical interest rates. That the current account can be countercyclical even in an endowment economy with purely transitory shocks highlights the endogenous response of interest rates. When the income shock is temporarily high, there is the typical incentive to save to smooth consumption. In addition there is the interest rate effect that works through shifts in the interest rate function. That is, all else equal, the expected probability of default is lower when the current income state is high and expected to persist. This was seen in Fig. 3A where the price function shifts in for high $z$. What is less visible, but is also the case, is that the slope of the interest rate function also is reduced. While $\hat{q}$ is countercyclical in equilibrium, we can calculate the total marginal cost of borrowing an additional unit along the equilibrium path and this is found to be countercyclical. This is then consistent with households wanting to borrow more when income is temporarily high. There are alternative parameterizations of Model I that produces a countercyclical interest rate process, as called for by the data. However, this occurs only when the current account is procyclical.\(^{10}\)

Before moving on to an alternative, we summarize how a standard model with transitory shocks has difficulty matching key empirical facts. The rate of default in equilibrium is orders of magnitude too small. This in turn leads to a counterfactually stable interest rate process. Moreover, the cyclicality of the interest rate is the opposite of the cyclicality of net exports, while in the data both are countercyclical and positively correlated in emerging markets.

4. Model II: stochastic trend

In Model II: $y_t = \Gamma_t$, where we let $\Gamma_t$ vary stochastically as in Eqs. (4) and (5). The behavior of this model is captured by the simulation results reported in Table 3B. One important distinction between the model with stochastic trend and Model I is the rate of default in equilibrium. Specifically, the rate of default increases by a factor of 10. The reason for this can be seen by contrasting the behavior of the difference between $\hat{V}^B$ and $\hat{V}^G$ between Fig. 1A (Model I) and Fig. 1B (Model II). High and low states of the growth shock will have substantially different effects on lifetime utility. Moreover, with persistent shocks, the value of financial integration will be high. Consequently, $(\hat{V}^G - \hat{V}^B)$ has a greater slope with respect to the $g$ shocks (Fig. 1B) as compared to $z$ shocks (Fig. 1A). By

\(^{10}\) For example, the countercyclical interest rate found in Arellano (2004) comes with a procyclical current account.
the logic introduced in the context of Model I, this in turn suggests $\bar{z}(\hat{a})$ has a smaller slope. This is seen in Fig. 2. Compared to the figure in the top panel, the region of default is larger and the slope of the line of indifference is smaller in the case of growth shocks. This then translates into a less steep $\hat{q}$ function as seen in Fig. 3B. Moreover, it also suggests that default will occur in equilibrium with more frequency. That is, an implicit increase in the demand for borrowing is not completely offset by a change in slope of the interest rate function, but rather translates into additional borrowing and a higher rate of default.\footnote{It is also the case that a volatile trend generates volatility in consumption and an increased demand for the insurance a defaultable bond provides, implying higher default rates as well. However, quantitatively, this effect is small relative to that induced by the change in the shape of the interest rate function. See Aguiar and Gopinath (2004a) for a full discussion.}

The results of the simulation of the model are reported in Table 3. Some improvements over Model I are immediately apparent. Both the current account and interest rates are countercyclical and positively correlated (though the magnitudes are lower than in the data). A positive shock increases output today, but increases output tomorrow even more (due to the persistence of the growth rate). This induces agents to borrow in good times. In Fig. 3, we plot the $\hat{q}$ schedule as a function of (detrended) $\hat{a}$ for a low and high value of $g$. We see that a positive shock lowers the interest rate (raises $\hat{q}$) for all levels of debt. In the reported parameterization, the shift in the $\hat{q}$ schedule dominates the movement along the schedule induced by additional borrowing. Correspondingly, the volatility of interest rates increases by a factor of 2.5. Net export volatility rises by a factor of 5 to match more closely the volatility in the data. However, interest rate volatility still remains well below that observed empirically for Latin America.

4.1. Sensitivity analysis

In this section we perform sensitivity tests within the framework of Model II. First, we consider a case with endogenous labor supply. The production function now takes the form $y_t = e^{zt} \Gamma_1 L_t^z$. We set $a=0.68$. The utility function takes the GHH form (from Greenwood et al., 1988)

$$u_t = \left( \frac{c_t - \frac{1}{\bar{g}} \mu_0 \Gamma_{t-1} L_t^\omega}{1 - \gamma} \right)^{1-\gamma}.$$  \hspace{1cm} (12)

The $\omega$ parameter is calibrated to 1.455 implying an elasticity of labor supply of 2.2. This is the value employed in previous studies (Mendoza, 1991; Neumeyer and Perri, 2004). The results for the simulation are reported in Table 4. The number of defaults is lower compared to the endowment model, however the business cycle moments line up as in the data.

The importance of a high level of impatience in generating reasonable levels of default is also seen in Table 4 where we report the results for higher levels of $\beta$. As $\beta$ increases the number of defaults drop from 23 to 11 per 2500 years.

Default has an insurance (i.e., state contingent) component absent from a risk-free bond. The agent only repays in the good (nondefault) states of nature. While the agent
cannot explicitly move resources across states of nature in the next period, she can move resources from good states next period forward to today, leaving resources available in bad/default states next period unaffected. Given that default provides insurance, one would expect that the value of this insurance would increase with uncertainty. Table 4 explores this conjecture. For each model, we doubled the innovation variance relative to the case reported in Table 3. We see that the rate of default roughly doubles relative to the benchmark case for both models. It is interesting to note that despite the increased rate of default, the higher volatility economies do not hold more debt in equilibrium, reflecting that these economies face a different interest rate function.

Finally, we explore the sensitivity of our results to our cost of autarky parameter \( \delta \). The last two columns of Table 4 report simulations of the two model economies where we have set \( \delta = 0.005 \) rather than 0.02. As expected, the lower cost of autarky results in very low debt levels observed in equilibrium, where instead of roughly 25% debt to income ratios, we now have ratios closer to 5%. This lack of borrowing is also reflected in the lower volatility of net exports and interest rates. Nevertheless, defaults occur with roughly the same frequency as was the case in the benchmark models.

5. Third party bailouts

Incorporating shocks to trend has improved the model’s performance on a number of dimensions. However, the default rate of once every 125 years remains low, at least compared to the track record of many Latin American countries. In this section, we try to improve on Model II by augmenting the model with a phenomenon observed in many
default episodes—bailouts. For example, Argentina received a $40 billion bailout in 2001 from the IMF, an amount nearly 15% of Argentine GDP in 2001. Such massive bailouts must influence the equilibrium debt market studied in the previous two sections.

We model bailouts as a transfer from a (unmodelled) third party to creditors in the case of default. While in practice bailouts often tend to nominally take the form of loans, we assume that bailouts are grants. To the extent that such loans in practice are extended at below market interest rates, they incorporate a transfer to the defaulting country. We also assume that creditors view bailouts as pure transfers. Again, it may be the case in practice that creditors ultimately underwrite the bailouts through tax payments. A reasonable assumption is that creditors do not internalize this aspect of bailouts.

Specifically, bailouts take the following form. Creditors reimbursed the amount of outstanding debt up to some limit $\hat{a}^*$. Any unpaid debt beyond $\hat{a}^*$ is a loss to the creditor. From the creditors viewpoint, therefore, every dollar lent up to $\hat{a}^*$ is guaranteed. Any lending beyond that has an expected return determined by the probability of default. Specifically, the breakeven price of debt solves

$$\hat{q}_t = \frac{1}{1 + r^*} \left\{ \min \left\{ 1, \frac{\hat{a}^*}{\hat{a}} \right\} + E_t \{ 1 - D_{t+1} \} \max \left\{ 1 - \frac{\hat{a}^*}{\hat{a}}, 0 \right\} \right\}.$$  

The presence of bailouts obviously implies that debt up to $\hat{a}^*$ carries a risk free interest rate. Moreover, the probability of default is used to discount only that fraction of debt that exceeds the limit. The net result is to shift up and flatten the $\hat{q}$ schedule. To consider why the $\hat{q}$ schedule is flatter, consider the case without bailouts, i.e., $\hat{a}^* = 0$. Each additional dollar of debt raises the probability of default. As default implies zero repayment, this lowers the return on all debt. However, with bailouts, an increase in the probability of default affects only the return on debt beyond $\hat{a}^*$. While this may have a large impact on the return of the marginal dollar, the sensitivity of the average return is mitigated by the fact that part of the debt is guaranteed.

From the agent’s perspective, bailouts subsidize default. The increase in the rate of default is not surprising given that bailouts are a pure transfer from a third party. This intuition is confirmed in our simulation results reported in Column 3C of Table 3. We calibrate $\hat{a}^*$ so that the maximum bailout is 18% of (mean detrended) output and we now set the time preference rate at 0.95 so that impatience rates are not as high as in the previous benchmark simulations. We see that the agent now defaults roughly once every 27 years, similar to the Argentine default rate. The increased rate of default however does not raise interest rate volatility. This reflects the fact that bailouts insulate interest rates from changes in the probability of default. In this sense the model with bailouts still leaves unexplained the high interest rate volatility emerging markets face.

6. Conclusion

We present a model of endogenous default that emphasizes the role of switches in growth regimes in matching important business cycle features of Emerging Markets and in generating default levels that are closer to the frequency observed in the data. The reason why a model with growth shocks performs better is that in such an environment a given
probability of default is associated with a smaller borrowing cost at the margin. This in turn rests on the fact that trend shocks have a greater impact on the propensity to default than do standard transitory shocks, making interest rates relatively less sensitive to the amount borrowed and relatively more sensitive to the realization of the shock.

Further, to match the business cycle features of the interest rate and current account, the model should predict that agents borrow more at lower interest rates during booms and the reverse during slumps. Since the interest rate schedule tends to be very steep in these models, the typical prediction is for the interest rate and current account to be negatively correlated. In the model with growth shocks, however, we find that for plausible parameterizations the shift of the interest rate schedule in good states, in anticipation of lower default probabilities, dominates the increase in interest rates that arise because of additional borrowing. While the predictions still remain short of matching quantitatively the magnitudes obtained in the data, the predicted sign of the correlations of income, net exports, and the interest rate are in line with empirical facts.

While we suggest how the presence of a stochastic trend can improve the model’s ability to match important features of the data it remains to be explained what underlies the stochastic trend. Further, the model relies on default penalties to generate sizeable debt to GDP ratios in equilibrium. Accordingly future research should provide us with a better understanding of the costs of default to emerging markets.

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