The Optimal Graduated Minimum Wage

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Abstract

This paper introduces a graduated minimum wage in a model with a government that seeks to maximize social welfare. It shows that the optimal graduated minimum wage always increases the low-productivity workers' consumption and brings it closer to the first-best. The paper also describes how the minimum wage in a social welfare optimum depends on important economy characteristics such as the government's revenue needs and the productivities of the different types of workers.

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1 Introduction

A graduated minimum wage ties the minimum wage a firm must pay to the employment of its eligible workers. Accordingly, it is a nonlinear function of the working hours of a firm's low-productivity workers that forces firms to choose between different combinations of minimum wage and corresponding working hours. The graduated minimum wage is thus analogous to a nonlinear income tax that forces workers to choose between different combinations of consumption and corresponding working hours.

While it has long been recognized that a constant minimum wage cannot improve on the social welfare that can be achieved by an optimal nonlinear income tax in a competitive economy (Allen, 1987; Guesnerie and Roberts, 1987), it has recently been shown that a graduated minimum wage makes superior allocations feasible and can be instrumental in providing a Pareto improvement (Danziger and Danziger, 2013). The reason is that a graduated minimum wage can entice firms to increase the employment of the low-productivity workers above what the firms would have chosen if the minimum wage were constant. More of the firms' revenues are then transferred to the low-productivity workers, thereby increasing their pretax income. This makes it harder for the high-productivity workers to mimic the low-productivity workers' income as they would have to work more. Consequently, the introduction of a graduated minimum wage slackens the high-productivity workers' incentive-compatibility constraint and creates the opportunity for the government to make everyone better off.

This fact, however, leaves many questions unanswered. In particular, if the goal is to maximize social welfare, what effect does the introduction of a graduated minimum wage to complement the optimal income tax have on the low-productivity workers' consumption? And how does the minimum wage in a social optimum depend on central characteristics of the economy such as the government's revenue needs, the masses and productivities of the different types of workers, and the mass of firms?

The purpose of this paper is to address these questions. Our framework is a Stiglitz (1982) type model with low- and high-productivity workers. We follow the optimal tax literature in assuming that taxes can only depend on a worker's income and not be directly conditioned on wages or working hours. We also assume that, as income taxes, a graduated minimum wage can be enforced by a system of self-reporting and whistle blowing leading to inspections and penalties for noncompliance. Indeed, it is easy for workers to know if they are paid too little as the intended minimum wage will be the smallest possible in the graduated minimum wage schedule. Workers also have an incentive to complain if employment is less than intended as the graduated minimum wage schedule would then require them to be paid more.[1] A major finding is that a government that strives to maximize social welfare will always use a graduated minimum wage to increase the low-productivity workers' consumption.[2]

A graduated minimum wage has been implemented in several countries. For example, Colombia, Honduras, and Panama have legislated multi-bracket minimum wage schedules with the minimum wage depending on firm size. Within the U.S., thirteen states have enacted two-bracket state minimum wage systems where the minimum wage depends on the size of the firm. In addition, in Argentina, Greece, Ireland, Puerto Rico and numerous other countries the minimum wage depends on the sector or occupation, both of which might serve as a proxy for the employment of low-productivity workers.

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[1] See also Lee and Saez (2012). Theoretical and empirical aspects of noncompliance with minimum wage laws are considered in Ashenfelter and Smith (1979), Weil (2005), and Danziger (2010).

[2] Previous literature has established that if the environment is not competitive, working hours are fixed, or taxation is not optimal, then a constant minimum wage may be socially beneficial under various circumstances. See Rebitzer and Taylor (1995), De Fraja (1999), Bhashar et al. (2002), Blumkin and Sadka (2005), Flinn (2006), Strobl and Walsh (2007, 2011), Basu et al. (2010), and Lee and Saez (2012). The importance of cultural and institutional factors in the determination of a minimum wage is emphasized in Sobel (1999), Cahuc et al. (2001), Belot et al. (2007), Boeri and Burda (2009), Brown (2009), Checchi and Garcia-Penalosa (2010), Aghion et al. (2011), Azar (2012), and Boeri (2012).

2 The Model

The economy contains a continuum $n_1 > 0$ of low-productivity workers. Their utilities are given by $u(c_1) - h_1$, $u' > 0$ and $u'' < 0$, where c_1 and h_1 denote a low-productivity worker's consumption and working hours. The economy also contains a continuum $n_2 > 0$ of high-productivity workers whose utilities are given by $u(c_2) - h_2$, where c_2 and h_2 denote a high-productivity worker's consumption and working hours.

In addition, there is a continuum $x > 0$ of firms. Their output is given by $f(\ell_1) + q\ell_2$, where $\ell_1 \equiv n_1 h_1 / x$ and $\ell_2 \equiv n_2 h_2 / x$ denote the total hours of low- and high-productivity workers, respectively, and $f(0) = 0$, $f' > 0$, $f'' < 0$, and $f'(0) < q$. The production function therefore exhibits decreasing returns to scale, with the low-productivity workers having a decreasing marginal product which is always less than the high-productivity workers' marginal product.

The government has a utilitarian social welfare function

$$n_1 \left[u(c_1) - h_1 \right] + n_2 \left[u(c_2) - h_2 \right], \tag{1}$$

and the resource constraint of the economy is

$$x[f(\ell_1) + q\ell_2] - n_1 c_1 - n_2 c_2 = R, \tag{2}$$

where $R \geq 0$ is the government's need for revenue to finance public expenditures. The government determines a nonlinear income tax and possibly also a graduated minimum wage, and its objective is to maximize social welfare.

2.1 Social Welfare without a Graduated Minimum Wage

Suppose that wages are competitively determined so that the low-productivity workers' wage is $w_1 = f'(\ell_1)$ and the high-productivity workers' wage is $w_2 = q$. Also suppose that the government enacts a nonlinear income tax but not a graduated minimum wage. The government can differentiate between the two types of workers by designing the income tax so that each type of worker prefers the consumption-work bundle meant for its own type rather than for the other type (and making any other available bundle less attractive). This leads to the following incentive-compatibility constraints of the low- and high-productivity workers

$$
\begin{aligned}
u(\hat{c}_1) - \hat{h}_1 &\geq u(\hat{c}_2) - \frac{\hat{w}_2 \hat{h}_2}{\hat{w}_1}, &(3)\\
u(\hat{c}_2) - \hat{h}_2 &\geq u(\hat{c}_1) - \frac{\hat{w}_1 \hat{h}_1}{\hat{w}_2}, &(4)
\end{aligned}
$$

where a circumflex is used to denote the optimal value of a variable. The last term on the right-hand-side of the constraints are the hours that one type of worker must work in order to earn the same income as the other type of worker.

The government seeks to maximize social welfare (1) by devising an income-tax function that determines the consumption-work bundles for the low- and high-productivity workers subject to the resource constraint (2) and the incentive-compatibility constraints (3) and (4).[3] Assuming an internal solution, it is straightforward to show that in a social welfare optimum with only a nonlinear income tax, constraints (2) and (4) but not (3) are binding. The low-productivity workers' consumption satisfies

$$
\begin{aligned}
u'(\hat{c}_1) &= \frac{n_1 q + n_2 f'(\hat{\ell}_1) + n_2 \hat{\ell}_1 f''(\hat{\ell}_1)}{q[(n_1 + 2n_2) f'(\hat{\ell}_1) - n_2 q + n_2 \hat{\ell}_1 f''(\hat{\ell}_1)]}, &(5)\\
u'(\hat{c}_2) &= \frac{1}{q}.
\end{aligned}
$$

Put in words, for each type of worker the marginal utility of consumption is equal to the marginal social cost of producing that consumption.

[3] Firms' profits, if any, are taxed away. Alternatively, workers own the firms equally with the income tax taking the distributed profits into account.

2.2 Social Welfare with a Graduated Minimum Wage

Suppose that in addition to a nonlinear income tax, the government enacts a graduated minimum wage $m(\ell_1)$ that sets the minimum wage as a function of the total working hours of a firm's low-productivity workers. Thus, in addition to using the income tax to present workers with a choice between different consumption-work bundles, the government now also uses a graduated minimum wage to present firms with a choice between different minimum wage-hours bundles. Accordingly, the wage for low-productivity workers is no longer competitively determined. However, the wage for high-productivity workers is still competitively determined and equal to q.

The incentive-compatibility constraints of the low- and high-productivity workers are modified to

$$\begin{aligned} u(c_1^*) - h_1^* &\geq u(c_2^*) - \frac{w_2^* h_2^*}{m(\ell_1^*)}, \quad (6) \\ u(c_2^*) - h_2^* &\geq u(c_1^*) - \frac{m(\ell_1^*) h_1^*}{w_2^*}, \quad (7) \end{aligned}$$

where an asterisk is used to denote the optimal value of a variable.

Since, by definition, the minimum wage is the lowest wage in the economy, it must be the case that $m(\ell_1) \leq w_2$ for all values of ℓ_1. We can therefore assume that $m(\ell_1^*) < w_2$, while the minimum wage for all other values of ℓ_1 is set to be as prohibitive as possible, i.e., $m(\ell_1) = w_2$ for $\ell_1 \neq \ell_1^*$. A firm could choose to pay its low-productivity workers w_2 and would then be free to set their working hours as it desires. However, since $f'(0) < q \Rightarrow f(\ell_1) - w_2\ell_1 < 0$ for $\ell_1 > 0$, if a firms has to pay w_2 to the low-productivity workers, then its maximum profit will be zero and obtained at $\ell_1 = 0$. For the graduated minimum wage to be relevant, therefore, ℓ_1^* must satisfy the minimum wage constraint

$$f(\ell_1^*) - m(\ell_1^*)\ell_1^* \geq 0. \quad (8)$$

The government's optimization problem is, then, to maximize social welfare (1) by designing an income-tax function that determines the consumption-work bundles for the low- and high-productivity workers, and a graduated minimum wage that determines $m(\ell_1)$. The allocation must meet the resource constraint (2), the modified incentive-compatibility constraints (6) and (7), and the minimum wage constraint (8). Assuming an internal solution, we obtain that in a social welfare optimum with both a nonlinear income tax and a graduated minimum wage, only constraints (2), (7), and (8) are binding and the low- and high-productivity workers' consumptions are such that

$$\begin{aligned} u'(c_1^*) &= \frac{n_1 q + n_2 f'(\ell_1^*)}{q[(n_1 + 2n_2) f'(\ell_1^*) - n_2 q]}, \\ u'(c_2^*) &= \frac{1}{q}. \end{aligned} \tag{9}$$

Therefore, the marginal utilities of low- and high-productivity workers' consumption are equal to what is now the marginal social cost of their production.

3 Optimal Consumption

Since the high-productivity workers' consumption is at the first-best level in a social welfare optimum with only a nonlinear income tax and the social cost of their consumption ($= 1/q$) is not affected by the graduated minimum wage, their consumption remains unchanged at the first-best level.[4] We now show that the low-productivity workers' consumption is positively affected by the graduated minimum wage:

Proposition: $c_1^* > \hat{c}_1$.

Proof: First, if $h_1^* = \hat{h}_1$, then (5) and (9) imply that

$$\begin{aligned} & u'(\hat{c}_1) - u'(c_1^*) \\ = \; & \frac{n_1 q + n_2 f'(\ell_1^*) + n_2 \ell_1^* f''(\ell_1^*)}{q[(n_1 + 2n_2) f'(\ell_1^*) - n_2 q + n_2 \ell_1^* f''(\ell_1^*)]} - \frac{n_1 q + n_2 f'(\ell_1^*)}{q[(n_1 + 2n_2) f'(\ell_1^*) - n_2 q]} \end{aligned}$$

[4] Thus, the optimality of a zero marginal tax rate for the high-productivity workers in the standard optimal tax model (Sadka, 1976; Stiglitz, 1982) carries over to our setting with a graduated minimum wage.

$$
\begin{aligned}
&= \frac{n_2\ell_1^* f''(\ell_1^*)[n_1 f'(\ell_1^*) - (n_1+n_2)q]}{q[(n_1+2n_2)f'(\ell_1^*) - n_2 q + n_2\ell_1^* f''(\ell_1^*)][(n_1+2n_2)f'(\ell_1^*) - n_2 q]} \\
&> 0.
\end{aligned}
$$

Accordingly, if $h_1^* = \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Next, if $h_1^* < \hat{h}_1$, we note that the derivative of the marginal social cost of c_1^* (the right-hand side of (9)) with respect to h_1^* is

$$
-\frac{n_1(n_1+n_2)^2 f''(\ell_1^*)}{x\left[(n_1+2n_2)f'(\ell_1^*) - n_2 q + z\right]^2} > 0.
$$

Hence, the marginal social cost of c_1^* increases with h_1^*. As the marginal utility of c_1^* decreases with c_1^* and since we have just shown that if $h_1^* = \hat{h}_1$, then $c_1^* > \hat{c}_1$, it follows that if $h_1^* < \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Finally, if $h_1^* > \hat{h}_1$, we use that the total differential of the social welfare (1) given the resource constraint (2) is

$$
n_1\left\{\left[u'(c_1) - \frac{1}{q}\right]dc_1 + \left[\frac{f'(\ell_1)}{q} - 1\right]dh_1\right\}.
$$

Since $u'(c_1) - 1/q > 0$ and $f'(\ell)/q < 1$, the graduated minimum wage would decrease social welfare if $c_1^* < \hat{c}_1$ in addition to $h_1^* > \hat{h}_1$. Therefore, it must be the case that if $h_1^* > \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Consequently, it can be concluded that $c_1^* > \hat{c}_1$. □

Since the introduction of a graduated minimum wage loosens the incentive-compatibility constraint of the high-productivity workers, it also reduces the marginal social cost of the low-productivity workers' consumption. As a consequence, a graduated minimum wage raises the consumption of the low-productivity workers. However, due to the modified incentive-compatibility constraint of the high-productivity workers, the graduated minimum wage will not raise the consumption of the low-productivity workers to the extent that it reaches that of the high-productivity workers and becomes first-best. Thus, the graduated minimum wage mitigates, but cannot completely eliminate, the social welfare loss that stems from the inability of the income tax system to condition the tax on wages rather than on incomes.[5]

4 The Optimal Minimum Wage

In Propositions 2-6 which follow, we determine how the characteristics of the economy affect the optimal minimum wage.[6] The fact that the minimum wage constraint is binding in a social optimum is of critical importance. It implies that the optimal minimum wage is equal to the low-productivity workers' average product and, therefore, inversely related to the optimal employment of the low-productivity workers.

Proposition 2: $dm(\ell_1^*)/dR < 0$.

The higher the government's revenue needs, the less output is available for the low-productivity workers' consumption for given working hours. As the marginal utility of low-productivity workers' consumption decreases with their consumption, the social gain from having them work more will be higher. Accordingly, the low-productivity workers' hours increase with the government's revenue needs, thereby lowering the minimum wage.

Proposition 3: $dm(\ell_1^*)/dn_1 < 0$.

The more low-productivity workers there are, the more of them will be employed in each firm. As a result, the total working hours of the low-productivity workers within each firm will be larger which lowers the minimum wage.

[5] The low-productivity workers' working hours need not decrease. On the one hand, the graduated minimum wage increases the low-productivity workers' income for unchanged working hours. This makes it less attractive for the high-productivity workers to mimic their income, which reduces the low-productivity workers' working hours needed to satisfy the high-productivity workers' incentive constraint. On the other hand, the graduated minimum wage boosts the gain of the low-productivity workers' income associated with an increase in their working hours, which tends to increase their hours. As a result of these opposing forces, the graduated minimum wage has an ambiguous effect on the low-productivity workers' hours.

[6] The proofs of the propositions are in the Appendix.

Proposition 4: $dm(\ell_1^*)/dn_2 > 0$.

The more high-productivity workers there are, the more it facilitates the transfer of resources to the low-productivity workers from the high-productivity workers who consume less than they produce. Since the transfer reduces the social gain from the work of the low-productivity workers, they will work less, which raises the minimum wage.

Proposition 5: $dm(\ell_1^*)/dx > 0$.

The more firms there are, the more thinly the low-productivity workers will be spread over firms. The consequent lower total working hours of the low-productivity workers in each firm is accompanied by a higher minimum wage.

Proposition 6: $dm(\ell_1^*)/dq > 0$.

The higher the productivity of the high-productivity workers, the more can be transferred to the low-productivity workers, which reduces the social gain from the work of the latter. Similar to the case of there being more high-productivity workers, a higher productivity of the high-productivity workers leads to the low-productivity workers working less, which causes the minimum wage to be higher.

5 Conclusion

This paper is the first to study the properties of a graduated minimum wage introduced to further social welfare in a competitive environment. This is of great interest in view of the theoretical insight that a constant minimum wage cannot serve as a means to increase social welfare beyond what can be obtained by an optimal income tax alone, and that nonetheless the constant minimum wage is pervasively used in an attempt to achieve exactly that.

We have shown that a welfare-maximizing graduated minimum wage increases the low-productivity workers' consumption above its level with an optimal income tax alone and brings it closer to the first-best. We have also described how the minimum wage in a social welfare optimum depends on important economy characteristics such as the government's revenue needs and the productivities of the different types of workers.

Appendix

We prove Propositions 2-6 by differentiating Condition (9) and the binding constraints (2), (6), and (8). Let

$$\begin{aligned} A &\equiv [(n_1 + 2n_2)f'(\ell_1^*) - n_2 q]^2, \\ B &\equiv -\frac{m(\ell_1^*) - f'(\ell_1^*)}{(n_1+n_2)h_1^* \{n_1(n_1+n_2)\,[n_1 + n_2 q U'(c_1^*)]\, f''(\ell_1^*) + x f'(\ell_1^*) A U''(c_1^*)\}}, \end{aligned}$$

where $m(\ell_1^*) - f'(\ell_1^*) > 0$, $f''(\ell_1^*) < 0$, and $U''(c_1^*) < 0$ imply that $B > 0$.

Differentiating with respect to R yields

$$\frac{dm(\ell_1^*)}{dR} = xABU''(c_1^*).$$

It follows that $dm(\ell_1^*)/dR < 0$, which proves Proposition 2.

Differentiating with respect to n_1 yields

$$\frac{dm(\ell_1^*)}{dn_1} = -\frac{xB}{n_1 q}\left\{n_1 n_2 [q - f'(\ell_1^*)]^2\, [n_1 + n_2 q U'(c_1^*)] - [n_1 c_1^* + n_2 m(\ell_1^*) h_1^*] q A U''(c_1^*)\right\}.$$

It follows that $dm(\ell_1^*)/dn_1 < 0$, which proves Proposition 3.

Differentiating with respect to n_2 yields

$$\frac{dm(\ell_1^*)}{dn_2} = \frac{xB}{q}\left\{n_1 [q - f'(\ell_1^*)]^2\, [n_1 + n_2 q U'(c_1^*)] - q(q h_2^* - c_2^*) A U''(c_1^*)\right\}.$$

Since there is a redistribution away from the high-productivity workers, in a social welfare optimum $qh_2^* > c_2^*$. Hence, $dm(\ell_1^*)/dn_2 > 0$, which proves Proposition 4.

Differentiating with respect to x yields

$$\frac{dm(\ell_1^*)}{dx} = -B[n_2 m(\ell_1^*) h_1^* + x f(\ell_1^*)] A U''(c_1^*).$$

It follows that $dm(\ell_1^*)/dx > 0$, which proves proposition 5.

Differentiating with respect to q yields

$$\frac{dm(\ell_1^*)}{dq} = \frac{n_2 x B}{q^2}\left\{[q - f'(\ell_1^*)]\, [n_1 q + (n_1 + 2n_2) f'(\ell_1^*)]\, [n_1 + n_2 q U'(c_1^*)] - q A U''(c_1^*)[q h_2^* - m(\ell_1^*) h_1^*]\right\}.$$

The term in the braces is positive since $q > f'(\ell_1^*)$ and the high-productivity workers' modified incentive-compatibility constraint implies that $qh_2^* - m(\ell_1^*)h_1^* > 0$. Consequently, $dm(\ell_1^*)/dq > 0$, which proves Proposition 6. $\square$

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