Online Appendix for “International Friends and Enemies”
(Not for Publication)

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July 2020

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A Introduction

In this section of the online appendix, we report the detailed derivations for the results reported in the paper and further supplementary results. In Section B, we report the proofs of the propositions in the paper. In Section C, we consider the Armington model with a general homothetic utility function in which goods are differentiated by country of origin from Section 2 of the paper. In Section D, we examine the special case of this model that falls within the class of models considered by Arkolakis, Costinot and Rodriguez-Clare (2012, henceforth ACR), which satisfy the four primitive assumptions of (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions;

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and (iv) perfect or monopolistic competition; as well as the three macro restrictions of (i) a constant elasticity import demand system, (ii) a constant share of profits in income, and (iii) balanced trade, as discussed in Section 3 of the paper. Although for convenience of exposition we focus in the paper and Section D of this online appendix on the single-sector Armington model, in Section E we show the same income and welfare exposure measures apply for all models in the ACR class with a constant trade elasticity. In Subsection E.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection E.2, we derive these exposure measures in the Krugman (1980) model.

In Section F, we consider a number of extensions of our baseline friend-enemies results with a constant trade elasticity, as in Section 4 of the paper. In Subsection F.1, we derive the corresponding friend-enemy representation allowing for both productivity and trade cost shocks. In Section F.2, we relax one of the ACR macro restrictions to allow for trade imbalance. In Section F.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system. In Section F.4, we show that our results generalize to a multi-sector version of the constant elasticity Armington model. In Section F.5, we show that they also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section F.6, we further extend the multi-sector Armington model from Section F.4 to introduce input-output linkages following Caliendo and Parro (2015). Finally, in Section F.7, we show that our results also hold for economic geography models with factor mobility, including Helpman (1998), Redding and Sturm (2008), Allen and Arkolakis (2014), Ramondo et al. (2016) and Redding (2016).

In Section G, we provide further details on the empirical specifications reported in the paper, as well as additional supplementary empirical results. Section H contains the data appendix.

B Proofs of Propositions

B.1 Proof of Lemmas 1 and 3

Lemma 1 imposes trade balance, with market clearing conditions \( w_i \ell_i = \sum_n s_{ni} w_n \ell_n \). Define \( q_i = \frac{w_i \ell_i}{\sum_n w_n \ell_n} \); we can rewrite market clearing condition as \( q_i = \sum_n s_{ni} q_n \) and, in matrix form, \( q = q'S \), proving that \( q \) is a left-eigenvector of \( S \) with eigenvalue 1. That \( q' \) is the unique positive left-eigenvector of \( S \) follows from Perron-Frobenius theorem. Under free-trade, every row of \( S \) is identical, and \( \sum_n q_n s_{ni} = s_{1i} \) for all \( i \). The vector \( [s_{11}, s_{12}, \ldots, s_{1N}] \) is therefore also left-eigenvector of \( S \) with eigenvalue 1, and since its entries are all positive, it must be equal to \( q \). Likewise, market-clearing can also be written as \( w_n \ell_n = \sum_i \ell_n w_i \ell_i \), which is equivalent to, in matrix form, \( q' = q'T \). The remaining claims about \( T \) follow analogously.

Lemma 3 introduces trade imbalances to the market clearing conditions, with \( w_i \ell_i = \sum_n [s_{ni} w_n \ell_n + \bar{d}_n] \). Let \( q_i \equiv w_i \ell_i / (\sum_n w_n \ell_n) \) and \( e_i \equiv (w_i \ell_i + \bar{d}_n) / (\sum_n w_n \ell_n) \). Dividing the market clearing condition by \( \sum_j w_j \ell_j \), we have

\[
\frac{w_i \ell_i}{\sum_j w_j \ell_j} = \sum_n \left[ s_{ni} \frac{w_n \ell_n + \bar{d}_n}{\sum_j w_j \ell_j} \right] \quad \iff \quad q_i = \sum_n s_{ni} e_n \quad \iff \quad q' = e'S.
\]

Let \( d_i \equiv q_i/e_i \) and \( D \equiv Diag(d) \). Note \( q' = e'D \) and \( q'D^{-1} = e' \); thus the market clearing condition can be re-written as

\[
e'D = e'S \iff e' = e'SD^{-1} \]
and

\[ q' = e' S \iff q' = q D^{-1} S. \]

\( q \) is therefore the unique positive left-eigenvector of \( D^{-1} S \) with eigenvalue 1, and \( e' \) is the unique positive left-eigenvector of \( SD^{-1} \) with eigenvalue one. The remaining claims about \( T \) follow analogously.

### B.2 Proof of Lemma 2

Note that \( Z = \frac{T + \theta TS}{\theta + 1} \) is a row-stochastic matrix and represents a Markov chain; its eigenvector \( q \) represents the stationary distribution of the Markov chain. Invertibility of \((I - V)\) follows from convergence of the power series \( \sum_{k=0}^{\infty} V^k \equiv \sum_{k=0}^{\infty} (Z - Q)^k \), which we show now. By construction, \( QZ = Q \) and \( ZQ = Q \). Using these two relations, we can show by induction that \( (Z - Q)^k = Z^k - Q \) for any integer \( k > 0 \). That \( \|Z^k - Q\| \leq c \cdot |\mu|^k \), where \( \mu \) is the largest eigenvalue of \( V \) in terms of absolute value (and the second-largest eigenvalue of \( Z \)), follows from standard results on Markov chains (e.g., see Rosenthal (1995)).

### B.3 Proof of Proposition 3

We repeatedly apply the following approximations:

\[ \ln (1 + x) \approx x - \frac{x^2}{2}, \quad x - 1 \approx \ln x + \frac{(\ln x)^2}{2} \]

\[ \ln \left( \sum_i p_i x_i \right) \approx \sum_i p_i (x_i - 1) - \frac{(\sum_i p_i (x_i - 1))^2}{2} \]

\[ \approx \sum_i p_i \left( \ln x_i + \frac{(\ln x_i)^2}{2} \right) - \frac{(\sum_i p_i \ln x_i)^2}{2} \]

\[ = E_p (\ln x_i) + \frac{V_p (\ln x_i)}{2} \]

Let \( \hat{x} \equiv \ln \hat{x} \). The hat-algebra with only TFP shocks can be written as

\[ \hat{w}_i = \sum_n T_{ln} \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}}, \quad \text{where } \hat{c}_i \equiv \frac{w_i}{z_i} \]

Taking logs,

\[ \hat{w}_i = \ln \left( \sum_n T_{ln} \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right) \]

\[ = E_T \left[ \ln \left( \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right) + \frac{V_T \left( \ln \left( \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right) \right)}{2} \right] \]

\[ = E_T \left( \hat{w}_n - \theta \hat{c}_i + E_{S_n} [\theta \hat{c}_k] - \frac{V_{S_n} [\theta \hat{c}_k]}{2} \right) + \frac{V_T (\hat{w}_n + E_{S_n} [\theta \hat{c}_k])}{2} \]

\[ = -\theta \hat{c}_i + E_T [\hat{w}_n] + E_{M_i} [\theta \hat{c}_k] - \frac{E_T V_{S_n} [\theta \hat{c}_k]}{2} + \frac{V_T (\hat{w}_n + E_{S_n} [\theta \hat{c}_k])}{2} \]

To re-write the second-order terms explicitly as a function of the productivity shocks—thereby deriving the Hessian—we express \( \theta \hat{c}_k \) and \( \bar{w}_n + E_{S_n} [\theta \hat{c}_k] \) in terms of productivity shocks to first-order. To do so, note that the first-order approximation is

\[ \bar{w} = T \hat{w} + \theta (TS - I) (\bar{w} - \bar{z}) \]
Lemma 4. We can now re-write the second-order terms as (let $y$

Further, to reduce notational clutter, let $\mathbf{A} \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T)$ and $\mathbf{B} \equiv \frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\}$

We can now re-write the second-order terms as (let $\mathbf{D} (x) \equiv diag (x)$ denote the diagonalization operator)

Hence we have $\epsilon_i \equiv \tilde{z}' \mathbf{H}_{f_i} \tilde{z}$, where

\[ \mathbf{H}_{f_i} = -\frac{1}{2} \left( \mathbf{A}^T (I - S) - \mathbf{S}' \mathbf{D} (T_i) \mathbf{S} \right) \mathbf{A} - \mathbf{B}^T (I - T) \mathbf{B} \]

B.4 Proof of Proposition 4

Lemma 4. $(I - S) \mathbf{A} = -(I - T) \mathbf{B}$, where $\mathbf{A} \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T)$ and $\mathbf{B} \equiv \frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\}$.

Proof. By the definition of $\mathbf{A}$ and $\mathbf{B}$:

\[ (I - S) \mathbf{A} = \frac{\theta}{\theta + 1} (I - S) (I - V)^{-1} (I - T) \]

\[ (I - T) \mathbf{B} = \frac{\theta}{\theta + 1} (I - T) \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\} \]
We have
\[
\frac{\theta + 1}{\theta} ((I - S) A + (I - T) B) = (I - S) (I - V)^{-1} (I - T) \\
+ (I - T) (I - V)^{-1} (TS - I) + (I - T) S (I - V)^{-1} (I - T) \\
= (I - S + (I - T) S) (I - V)^{-1} (I - T) + (I - T) (I - V)^{-1} (TS - I) \\
= (I - TS) (I - V)^{-1} (I - T) + (I - T) (I - V)^{-1} (TS - I) \\
= (I - TS) \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) + I \right) \\
+ \left( \frac{\theta}{\theta + 1} (TS - I) (I - V)^{-1} + I \right) (TS - I) \\
= \left( \frac{\theta}{\theta + 1} (I - TS) (I - V)^{-1} (TS - I) + (I - TS) \right) \\
+ \left( \frac{\theta}{\theta + 1} (TS - I) (I - V)^{-1} (TS - I) + (TS - I) \right) \\
= 0.
\]

Proof of Proposition 4 To show that the second-order terms average to zero across countries, note

\[ q' \epsilon = \tilde{z} \left( \sum q_i H_{f_i} \right) \tilde{z} \]
\[ = -\frac{1}{2} \tilde{z} \left( A' (d - S'dS) A - B' (d - T'dT) B \right) \tilde{z}, \]

where \( d \equiv diag(q) \). We next show \( A' (d - S'dS) A - B' (d - T'dT) B \) is a zero matrix. Using Lemma 1, we have

\[ 2 \left( A' (d - S'dS) A - B' (d - T'dT) B \right) \]
\[ = A' (I - S') d (I + S) A - B' (I + T') d (I - T) B \]
\[ + A' (I + S') d (I - S) A - B' (I - T') d (I + T) B \]

(the next line follows from Lemma 1)
\[ = -B' (I - T') d (I + S) A + B' (I + T') d (I - S) A \]
\[ - A' (I + S') d (I - T) B + A' (I - S') d (I + T) B \]
\[ = -B' - B'T') d (A + SA) + (B' + B'T') d (A - SA) \]
\[ - (A' + A'S') d (B - TB) + (A' - A'S') d (B + BT) \]
\[ = 2 (B' (T'd - dS) A + A' (dT - S'd)) B \]
\[ = 0, \]

where the last equality follows from \( T'd = dS \) and \( dT = S'd \) (recall \( T_{in} q_i = S_{ni} q_n \) and \( d \equiv diag(q) \)).

B.5 Proof of Proposition 5

The fact that \( H_{f_i} \) is real and symmetric, \( \mu_{\max,i} \) is the largest eigenvalue and \( z_{\max,i} \) is the corresponding eigenvector implies that

\[ |\mu_{\max,i}| \equiv \max_z \frac{z^T H_{f_i} z}{z^T z}, \quad z_{\max,i} \equiv \arg \max_z \frac{z^T H_{f_i} z}{z^T z}. \]
B.6 Proof of Proposition 6

The fact that $\mu^A$ is the spectral norm implies that for all $z$,

$$\mu^A \|z\|_2 \geq g(z) = \frac{1}{N} \sum_{i=1}^{N} (z' H_i z)^2 = \frac{1}{N} \sum_{i=1}^{N} \epsilon_i^2 (z)$$

Hence $\sqrt{\frac{1}{N} \sum_{i=1}^{N} \epsilon_i^2 (z)} \leq \sqrt{\mu^A \|z\|_2^2}$, as desired.

B.7 Proof of Proposition 10

To economize on notation we let $x \equiv d \ln w$ and $y \equiv d \ln z$, and we derive the sensitivity of $x$ to $\epsilon$, holding $y$ fixed. Using our assumption of constant returns to scale in production, it is without loss of generality to normalize the weighted mean of productivity shocks $q' y = 0$ and thus $Qy = 0$. Note $(I - V) x = -\theta \frac{1}{\theta + 1} My$, $W = -\frac{1}{\theta + 1} (I - V)^{-1} M$, $x = (W + Q) y$, and $(W + Q)$ is invertible. Differentiating and noting $dV = dM (= \epsilon O)$, we obtain

$$-dV \cdot x + (I - V) \cdot dx = -\frac{\theta}{\theta + 1} dM \cdot y$$

$$\Rightarrow \quad dx = (I - V)^{-1} dV \cdot (x - y)$$

$$\Rightarrow \quad dx = (I - V)^{-1} dV \left( I - (W + Q)^{-1} \right) x$$

By the Cauchy-Schwarz inequality,

$$\|dx\| \leq \|(I - V)^{-1}\| \cdot \|dV\| \cdot \|I - (W + Q)^{-1}\| \cdot \|x\|.$$

We obtain the proposition by substituting $\|dV\| = \frac{\theta}{\theta + 1} \epsilon$ and $\|dx\|/\|x\| = \lim_{\epsilon \to 0} \frac{\|d \ln w - d \ln w\|}{\|d \ln w\|}$.

C General Armington

In this Section of the online appendix, we consider the Armington model with a general homothetic utility function in which goods are differentiated by country of origin from Section 2 of the paper. We consider a world consisting of many countries indexed by $i, n \in \{1, \ldots, N\}$. Each country has an exogenous supply of $L_n$ workers, who are endowed with one unit of labor that is supplied inelastically.

C.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following homothetic indirect utility function:

$$u_n = \frac{w_n}{P(p_n)},$$

where $p_n$ is the vector of prices in country $n$ of the goods produced by each country $i$ with elements $p_{ni} = \tau_{ni} w_i / z_i$; $w_n$ is the income of the representative consumer in country $n$; and $P(p_n)$ is a continuous and twice differentiable function that captures the ideal price index. From Roy’s Identity, country $n$’s demand for the good produced by country $i$ is:

$$c_{ni} = c_{ni}(p_n) = -\frac{\partial (1/P(p_n))}{\partial p_{ni}} w_n P(p_n).$$
C.2 Expenditure Shares

Country \( n \)'s expenditure share on the good produced by country \( i \) is:

\[
\begin{align*}
  s_{ni} &= \frac{p_{ni}c_{ni}}{\sum_{\ell=1}^{N} p_{n\ell}c_{n\ell}} = \frac{x_{ni}(p_n)}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)},
\end{align*}
\]  

(C.3)

Totally differentiating this expenditure share equation, we get:

\[
\begin{align*}
  ds_{ni} &= \frac{1}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)} \sum_{h=1}^{N} \frac{\partial x_{ni}(p_n)}{\partial p_{nh}} dp_{nh} - \frac{x_{ni}(p_n)}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)} \sum_{k=1}^{N} \sum_{h=1}^{N} \frac{\partial x_{nk}(p_n)}{\partial p_{nh}} \frac{dp_{nh}}{p_{nh}}, \\
  ds_{ni} &= \sum_{h=1}^{N} \left[ \frac{\partial x_{ni}(p_n) p_{nh}}{x_{ni}} - \sum_{k=1}^{N} s_{nk} \frac{\partial x_{nk}(p_n) p_{nh}}{x_{nk}} \right] \frac{dp_{nh}}{p_{nh}}, \\
  \frac{ds_{ni}}{s_{ni}} &= \sum_{h=1}^{N} \left[ \theta_{nii} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}}, \\
  d \ln s_{ni} &= \sum_{h=1}^{N} \left[ \theta_{nii} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] d \ln p_{nh},
\end{align*}
\]  

(C.4)

where

\[
\theta_{nii} = \left( \frac{\partial x_{ni}(p_n) p_{nh}}{\partial p_{nh}} \right) \left( \frac{\partial \ln x_{ni}}{\partial \ln p_{nh}} \right).
\]

Totally differentiating prices in equation (3) in the paper, we have:

\[
\begin{align*}
  \frac{dp_{ni}}{p_{ni}} &= \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_{i}}{w_{i}} - \frac{dz_{i}}{z_{i}}, \\
  d \ln p_{ni} &= d \ln \tau_{ni} + d \ln w_{i} - d \ln z_{i}.
\end{align*}
\]  

(C.5)

C.3 Market Clearing

Market clearing implies:

\[
\begin{align*}
  w_{i} \ell_{i} &= \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n}. 
\end{align*}
\]  

(C.6)

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\begin{align*}
  \frac{dw_{i}}{w_{i}} w_{i} \ell_{i} &= \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_{n} \ell_{n} + \sum_{n=1}^{N} s_{ni} \frac{dw_{n}}{w_{n}} w_{n} \ell_{n}, \\
  \frac{dw_{i}}{w_{i}} w_{i} \ell_{i} &= \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right), \\
  \frac{dw_{i}}{w_{i}} &= \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right), \\
  \frac{dw_{i}}{w_{i}} &= \sum_{n=1}^{N} \ell_{ni} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right),
\end{align*}
\]
where we have defined $t_{in}$ as the share of country $i$’s income from market $n$:

$$t_{in} \equiv \frac{s_{ni}w_n}{w_i}.$$ 

Using our result for the derivatives of expenditure shares (C.4), we have:

$$\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \left[ \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nk} \right] \frac{dp_{nh}}{p_{nh}} \right] + \frac{d\tau_{nh}}{\tau_{nh}} + \frac{dw_h}{w_h} - \frac{dz_h}{z_h} \right),$$

Using our results for the derivatives of prices, we have:

$$\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \left[ \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nk} \right] \frac{d\tau_{nh}}{\tau_{nh}} + \frac{dw_h}{w_h} - \frac{d\tau_{nh}}{\tau_{nh}} \right] \right),$$

which can be written as:

$$d\ln w_i = \sum_{n=1}^{N} t_{in} \left( d\ln w_n + \left[ \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nk} \right] \left( d\ln \tau_{nh} + d\ln w_h - d\ln z_h \right) \right] \right), \quad (C.7)$$

which corresponds to equation (8) in the paper.

### C.4 Welfare

Totally differentiating welfare, we have:

$$du_n = dw_n \left( \frac{1}{P(p_n)} \right) + \frac{1}{P(p_n)} \frac{du_n}{P(p_n)} w_n,$$

$$du_n = dw_n \left( \frac{1}{P(p_n)} \right) + \sum_{i=1}^{N} w_n \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} dp_{ni},$$

$$du_n = \frac{dw_n}{w_n} w_n \left( \frac{1}{P(p_n)} \right) + \sum_{i=1}^{N} \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} p_{ni} dp_{ni},$$

$$du_n = \frac{dw_n}{w_n} + \sum_{i=1}^{N} \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} p_{ni} \frac{dp_{ni}}{p_{ni}}.$$

Now recall from equation (C.2) that:

$$c_{ni} = - \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} w_n P(p_n).$$

Using this result in our total derivative for welfare above, we obtain:

$$\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{i=1}^{N} c_{ni} \frac{dp_{ni}}{p_{ni}} \frac{p_{ni}}{p_{ni}}.$$

$$\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{i=1}^{N} s_{ni} \frac{dp_{ni}}{p_{ni}} \frac{p_{ni}}{p_{ni}}.$$

which can be equivalently written as:

$$d\ln u_n = d\ln w_n - \sum_{i=1}^{N} s_{ni} \left[ d\ln \tau_{ni} + d\ln w_i - d\ln z_i \right], \quad (C.8)$$

which corresponds to equation (10) in the paper.
D Constant Elasticity Armington

In this section of the online appendix, we report the derivations for our baseline constant elasticity of substitution Armington model in Section 3 of the paper. This model falls within the class of of quantitative trade models considered by Arkolakis, Costinot and Rodriguez-Clare (2012), which satisfy the three macro restrictions of (i) a constant elasticity import demand system, (ii) profits are a constant share of income, and (iii) trade is balanced. In Section E of this online appendix, we show that our analysis also holds for other models within this class, including those of perfect competition and constant returns to scale with Ricardian technology differences as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution. The world economy consists of many countries indexed by \( i, n \in \{1, \ldots, N\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

D.1 Consumer Preferences

The preferences of the representative consumer in country \( n \) are characterized by the following indirect utility function:

\[
    u_n = \frac{w_n}{p_n}, \quad p_n = \left( \sum_{i=1}^{N} p_{ni}^{1-\sigma} \right)^{1/\sigma}, \quad \sigma > 1,
\]

where \( w_n \) is the wage; \( p_n \) is the consumption goods price index; \( p_{ni} \) is the price in country \( n \) of the good produced by country \( i \); and we focus on the case in which countries’ goods are substitutes (\( \sigma > 1 \)).

D.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that \( \tau_{ni} \geq 1 \) units must be shipped from country \( i \) to country \( n \) in order for one unit to arrive (where \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \)). Therefore, the consumer in country \( n \) of purchasing a good \( \theta \) from country \( i \) is:

\[
    p_{ni} = \frac{\tau_{ni} w_i}{z_i},
\]

where \( z_i \) captures productivity in country \( i \) and iceberg variable trade costs satisfy \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \).

D.3 Expenditure Shares

Using the properties of the CES demand function, country \( n \)’s share of expenditure on goods produced in country \( i \) is:

\[
    s_{ni} = \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}}.
\]

Totally differentiating this expenditure share equation (D.3), we get:

\[
    ds_{ni} = - (\sigma-1) \frac{dp_{ni} p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{mni}^{1-\sigma}} + (\sigma-1) \sum_{h=1}^{N} \frac{d\sum_{m=1}^{N} p_{mni}^{1-\sigma}}{p_{mni}^{1-\sigma} \sum_{m=1}^{N} p_{mni}^{1-\sigma}},
\]

\[
    \frac{ds_{ni}}{s_{ni}} = - (\sigma-1) \frac{dp_{ni}}{p_{ni}} + (\sigma-1) \sum_{h=1}^{N} \frac{d\sum_{m=1}^{N} p_{mni}^{1-\sigma}}{\sum_{m=1}^{N} p_{mni}^{1-\sigma}}.
\]
\[
\frac{ds_{ni}}{s_{ni}} = -(\sigma - 1) \frac{dp_{ni}}{p_{ni}} + (\sigma - 1) \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right),
\]

\[
d ln s_{ni} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d ln p_{nh} - d ln p_{ni} \right),
\]

where, from the definition of \( p_{ni} \) in equation (D.2) above, we have:

\[
\frac{dp_{ni}}{p_{ni}} = \frac{dr_{ni}}{r_{ni}} + \frac{dw_{i}}{w_{i}} - \frac{dz_{i}}{z_{i}},
\]

\[
d ln p_{ni} = d ln r_{ni} + d ln w_{i} - d ln z_{i}.
\]

### D.4 Price Indices

Totally differentiating the consumption goods price index in equation (D.1), we have:

\[
dp_{n} = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}} \left[ \sum_{m=1}^{N} p_{nm}^{1-\sigma} \right]^{\frac{1}{\sigma}},
\]

\[
\frac{dp_{n}}{p_{n}} = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}},
\]

\[
\frac{dp_{n}}{p_{n}} = \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}},
\]

\[
d ln p_{n} = \sum_{m=1}^{N} s_{nm} d ln p_{nm}.
\]

### D.5 Market Clearing

Market clearing requires that income in each country equals expenditure on the goods produced in that country:

\[
w_{i} \ell_{i} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n}.
\]

Totally differentiating this market clearing condition (D.7), holding labor endowments constant, we have:

\[
\frac{dw_{i}}{w_{i}} \frac{w_{i} \ell_{i}}{w_{i} \ell_{i}} = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_{n} \ell_{n} + \sum_{n=1}^{N} s_{ni} \frac{dw_{n}}{w_{n}} w_{n} \ell_{n},
\]

\[
\frac{dw_{i}}{w_{i}} \frac{w_{i} \ell_{i}}{w_{i} \ell_{i}} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right),
\]

\[
\frac{dw_{i}}{w_{i}} \frac{w_{i} \ell_{i}}{w_{i} \ell_{i}} = \sum_{n=1}^{N} \frac{w_{n} \ell_{n}}{w_{i} \ell_{i}} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right).
\]

Using our result for the derivative of expenditure shares in equation (D.4) above, we can rewrite this as:

\[
\frac{dw_{i}}{w_{i}} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right).
\]
\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + (\sigma - 1) \left( \sum_{h \in N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right),
\]
\[
d\ln w_i = \sum_{n=1}^{N} t_{in} \left( d\ln w_n + (\sigma - 1) \left( \sum_{h \in N} s_{nh} d\ln p_{nh} - d\ln p_{ni} \right) \right),
\]
where we have defined \( t_{in} \) as the share of country \( i \)'s income derived from market \( n \):
\[
t_{in} = \frac{s_{ni} w_n}{w_i \ell_i}.
\]

**D.6 Utility Again**

Returning to our expression for indirect utility, we have:
\[
u_n = \frac{w_n}{p_n}
\]
(D.9)

Totally differentiating indirect utility (D.9), we have:
\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}.
\]
Using our total derivative of the sectoral price index in equation (D.6) above, we get:
\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}},
\]
(D.10)
\[
d\ln u_n = d\ln w_n - \sum_{m=1}^{N} s_{nm} d\ln p_{nm}.
\]

**D.7 Wages and Productivity Shocks**

We consider small productivity shocks, holding constant bilateral trade costs:
\[
d\ln \tau_{ni} = 0, \quad \forall \ n, i \in N.
\]
(D.11)

We start with our expression for the log change in wages from equation (D.8) above:
\[
d\ln w_i = \sum_{n=1}^{N} t_{in} \left( d\ln w_n + (\sigma - 1) \left( \sum_{h \in N} s_{nh} d\ln p_{nh} - d\ln p_{ni} \right) \right),
\]
Using the total derivative of prices (D.5) and our assumption of constant bilateral trade costs (D.11), we can write this expression for the log change in wages as:
\[
d\ln w_i = \sum_{n=1}^{N} t_{in} \left( d\ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} [d\ln w_h - d\ln z_h] - [d\ln w_i - d\ln z_i] \right) \right),
\]
\[
d\ln w_i = \sum_{n=1}^{N} t_{in} d\ln w_n + (\sigma - 1) \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} s_{nh} [d\ln w_h - d\ln z_h] - [d\ln w_i - d\ln z_i] \right),
\]
\[
d\ln w_i = \sum_{n=1}^{N} t_{in} d\ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} [d\ln w_h - d\ln z_h] - [d\ln w_i - d\ln z_i] \right),
\]
(D.12)
which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left[ d \ln w_n - d \ln z_n \right] \right), \]

\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]

which has the following matrix representation in the paper:

\[ d \ln w = T d \ln w + \theta M (d \ln w - d \ln z), \quad \text{(D.13)} \]

\[ \theta = \sigma - 1. \]

We solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (D.13) by \( \theta + 1 \), we have:

\[ \frac{1}{\theta + 1} d \ln w = \frac{1}{\theta + 1} T d \ln w + \frac{\theta}{\theta + 1} M (d \ln w - d \ln z), \]

\[ \frac{1}{\theta + 1} (I - T - \theta M) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z. \]

Now using \( M = TS - I \), we have:

\[ \frac{1}{\theta + 1} (I - T - \theta TS + \theta I) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z, \]

\[ \left( I - \frac{T + \theta TS}{\theta + 1} \right) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z. \]

Using our choice of world GDP as numeraire, which implies \( Q d \ln w = 0 \), we have:

\[ \left( I - \frac{T + \theta TS}{\theta + 1} + Q \right) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z, \]

which can be re-written as:

\[ (I - V) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z, \]

\[ V \equiv \frac{T + \theta TS}{\theta + 1} - Q, \]

which yields the following solution of the change in wages in response to a productivity shock:

\[ d \ln w = -\frac{\theta}{\theta + 1} (I - V)^{-1} M d \ln z, \]

which can be re-written as:

\[ d \ln w = W d \ln z, \]

where \( W \) is our friend-enemy income exposure measure:

\[ W \equiv -\frac{\theta}{\theta + 1} (I - V)^{-1} M. \quad \text{(D.14)} \]
D.8 Welfare and Productivity Shocks

From equation (D.10), the log change in utility is given by:

$$
\text{d} \ln u_n = \text{d} \ln w_n - \sum_{m=1}^{N} s_{nm} \text{d} \ln p_{nm},
$$

Using the total derivative of prices (D.5) and our assumption of constant bilateral trade costs (D.11), we can write this log change in utility as:

$$
\text{d} \ln u_n = \text{d} \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ \text{d} \ln w_m - \text{d} \ln z_m \right],
$$

which has the following matrix representation in the paper:

$$
\text{d} \ln u = \text{d} \ln w - S \left( \text{d} \ln w - \text{d} \ln z \right). \tag{D.15}
$$

We can re-write the above relationship as:

$$
\text{d} \ln u = (I - S) \text{d} \ln w + S \text{d} \ln z,
$$

which using our solution for \( \text{d} \ln w \) from above, can be further re-written as:

$$
\text{d} \ln u = (I - S) W \text{d} \ln z + S \text{d} \ln z,
$$

$$
\text{d} \ln u = [(I - S) W + S] \text{d} \ln z,
$$

$$
\text{d} \ln u = U \text{d} \ln z,
$$

where \( U \) is our friend-enemy welfare exposure measure:

$$
U \equiv [(I - S) W + S]. \tag{D.17}
$$

D.9 Market-size and Cross-Substitution Effects

We define the cross-substitution effect as the wage vector that solves equation (D.13) replacing the term \( T \text{d} \ln w \) with \( Q \text{d} \ln w \):

$$
\text{d} \ln w^{\text{Sub}} = Q \text{d} \ln w^{\text{Sub}} + \theta \cdot M \left( \text{d} \ln w^{\text{Sub}} - \text{d} \ln z \right),
$$

where we use the superscript Sub to indicate the cross-substitution effect.

We solve for our friend-enemy measure of income exposure due to the cross-substitution effect using matrix inversion. Dividing both sides of the above equation by \( \theta + 1 \), we have:

$$
\frac{1}{\theta + 1} \text{d} \ln w^{\text{Sub}} = \frac{1}{\theta + 1} Q \text{d} \ln w^{\text{Sub}} + \frac{\theta}{\theta + 1} M \left( \text{d} \ln w^{\text{Sub}} - \text{d} \ln z \right),
$$

$$
\frac{1}{\theta + 1} (I - Q \theta M) \text{d} \ln w^{\text{Sub}} = - \frac{\theta}{\theta + 1} M \text{d} \ln z.
$$

Now using \( M = TS - I \), we have:

$$
\frac{1}{\theta + 1} (I - Q \theta TS + \theta I) \text{d} \ln w^{\text{Sub}} = - \frac{\theta}{\theta + 1} M \text{d} \ln z,
$$

$$
\left( I - \frac{Q + \theta TS}{\theta + 1} \right) \text{d} \ln w^{\text{Sub}} = - \frac{\theta}{\theta + 1} M \text{d} \ln z.
$$
Using our choice of world GDP as numeraire, which implies $\frac{Q}{\theta + 1} + Q$, we have:

\[
\left( I - \frac{Q + \theta TS}{\theta + 1} + Q \right) \, d \ln w_{\text{Sub}} = -\frac{\theta}{\theta + 1} M \, d \ln z,
\]

\[
\left( I - \frac{\theta Q + \theta TS}{\theta + 1} \right) \, d \ln w_{\text{Sub}} = -\frac{\theta}{\theta + 1} M \, d \ln z.
\]

Now using $M = TS - I$, we have:

\[
\left( I - \frac{\theta Q + \theta M}{\theta + 1} - \frac{\theta}{\theta + 1} I \right) \, d \ln w_{\text{Sub}} = -\frac{\theta}{\theta + 1} M \, d \ln z,
\]

\[
\left( I - (\theta Q + \theta M) \right) \, d \ln w_{\text{Sub}} = -\theta M \, d \ln z,
\]

which yields the following solution of the change in wages in response to a productivity shock:

\[
d \ln w_{\text{Sub}} = -\theta (I - (\theta Q + \theta M))^{-1} M \, d \ln z,
\]

which can be re-written as:

\[
d \ln w_{\text{Sub}} = W_{\text{Sub}} \, d \ln z,
\]

where $W_{\text{Sub}}$ is our friend-enemy measure of income exposure due to the cross-substitution effect:

\[
W_{\text{Sub}} \equiv -\theta (I - (\theta Q + \theta M))^{-1} M.
\]

### D.10 Relationship to the ACR Gains from Trade Formula

From equation (D.10), the log change in utility is given by:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \, d \ln p_{nm}.
\]  

(D.18)

Choosing country $n$’s wage as the numeraire and assuming no changes in its productivity or domestic trade costs, we have:

\[
d \ln w_n = 0, \quad d \ln z_n = 0, \quad d \ln \tau_{nn} = 0, \quad d \ln p_{nn} = 0.
\]

The import demand system in equation (D.3) implies:

\[
d \ln s_{nm} - d \ln s_{nn} = -(\sigma - 1) \left( d \ln p_{nm} - d \ln p_{nn} \right).
\]

Using this result in equation (D.18), the log change in welfare can be written as:

\[
d \ln u_n = \sum_{m=1}^{N} s_{nm} \left( \frac{d \ln s_{nm} - d \ln s_{nn}}{\sigma - 1} \right)
\]

Using $\sum_{m=1}^{N} s_{nm} = 1$ and $\sum_{m=1}^{N} ds_{nm} = 0$, we obtain the ACR welfare gains from trade formula for small changes:

\[
d \ln u_n = -\frac{\ln s_{nn}}{\sigma - 1}.
\]  

(D.19)

Integrating both sides of equation (D.19), we get:

\[
\int_{u_n^0}^{u_n^1} d u_n = (\sigma - 1) \int_{s_{nn}^0}^{s_{nn}^1} \frac{d s_{nn}}{s_{nn}}
\]
\[
\ln \left( \frac{u_n^1}{u_n^0} \right) = - (\sigma - 1) \ln \left( \frac{s_{nn}^1}{s_{nn}^0} \right),
\]
\[
\frac{s_n^1}{s_n^0} = \left( \frac{s_{nn}^1}{s_{nn}^0} \right)^{-(\sigma - 1)},
\]
which corresponds to the ACR welfare gains from trade formula for large changes.

E Single-Sector Isomorphisms

While for convenience of exposition we focus on the single-sector Armington model in the paper, we obtain the same income and welfare exposure measures for all models in the ACR class with a constant trade elasticity. In Subsection E.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection E.2, we derive these measures in the Krugman (1980) model.

The Armington model in the paper and the Eaton and Kortum (2002) model assume perfect competition and constant returns to scale, whereas the Krugman (1980) model assumes monopolistic competition and increasing returns to scale. In the Eaton and Kortum (2002) model, the trade elasticity corresponds to the Fréchet shape parameter. In contrast, in the Armington model and the Krugman (1980) model, the trade elasticity corresponds to the elasticity of substitution between varieties. Nevertheless, our income and welfare exposure measures hold across all three models, because they all feature a constant trade elasticity.

In Subsection E.2, we focus on the representative firm Krugman (1980) model of monopolistic competition for simplicity, but our income and welfare exposure measures also hold in the heterogeneous firm model of Melitz (2003) with an untruncated Pareto productivity distribution. In Subsection E.2, we also show that our income and welfare exposure measures with monopolistic competition and increasing returns to scale take a similar form whether we consider shocks to the variable or fixed components of production costs.

E.1 Eaton and Kortum (2002)

We consider a version of Eaton and Kortum (2002) with labor as the sole factor of production. Trade arises because of Ricardian technology differences; production technologies are constant returns to scale; and markets are perfectly competitive. The world economy consists of a set of countries indexed by \( i, n \in \{1, \ldots, N\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

E.1.1 Consumer Preferences

The preferences of the representative consumer in country \( n \) are characterized by the following indirect utility function:

\[
u_n = \frac{w_n}{p_n},
\]

where \( w_n \) is the wage and \( p_n \) is the consumption goods price index, which is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

\[
p_n = \left[ \int_0^1 p_n(\vartheta)^{1-\sigma} \, d\vartheta \right]^\frac{1}{1-\sigma}, \quad \sigma > 1,
\]

where \( p_n(\vartheta) \) denotes the price of good \( \vartheta \) in country \( n \).
E.1.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that \( n_i > 1 \) units must be shipped from country \( i \) to country \( n \) in order for one unit to arrive (where \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \)). Therefore, the price for consumers in country \( n \) of purchasing a good \( \vartheta \) from country \( i \) is:

\[
p_{ni}(\vartheta) = \frac{\tau_{ni}w_i}{z_i a_i(\vartheta)},
\]

where \( z_i \) captures common determinants of productivity across goods within country \( i \) and \( a_i(\vartheta) \) captures idiosyncratic determinants of productivity for each good \( \vartheta \) within that country. Iceberg variable trade costs satisfy \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \). Productivity for each good \( \vartheta \) in each sector \( k \) and each country \( i \) is drawn independently from the following Fréchet distribution:

\[
F_i(a) = \exp\left(-\alpha^{-\theta}\right), \quad \theta > 1,
\]

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \( z_i \).

E.1.3 Expenditure Shares

Using the properties of this Fréchet distribution, country \( n \)'s share of expenditure on goods produced in country \( i \) is:

\[
s_{ni} = \frac{(\tau_{ni}w_i/z_i)^{-\theta}}{\sum_{m=1}^{N} (\tau_{nm}w_m/z_m)^{-\theta}} = \frac{(\rho_{ni})^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm})^{-\theta}},
\]

where we have defined the following price inclusive of trade costs term:

\[
\rho_{ni} = \frac{\tau_{ni}w_i}{z_i}.
\]

Totally differentiating this expenditure share equation (E.5) we get:

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{d\rho_{ni}}{\rho_{ni}} - \theta \frac{\sum_{h=1}^{N} (\rho_{ih})^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm})^{-\theta}} + \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{d\rho_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} \frac{\theta d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{d\rho_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right),
\]

\[
d\ln s_{ni} = \theta \left( \sum_{h=1}^{N} s_{nh} d\ln \rho_{nh} - d\ln \rho_{ni} \right),
\]

where, from the definition of \( \rho_{ni} \) in equation (E.6) above, we have:

\[
\frac{d\rho_{ni}}{\rho_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i},
\]

\[
d\ln \rho_{ni} = d\ln \tau_{ni} + d\ln w_i - d\ln z_i.
\]
E.1.4 Price Indices

Using the properties of the Fréchet distribution (E.4), the consumption goods price index is given by:

\[ p_n = \gamma \left[ \sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\theta}}, \quad \text{(E.9)} \]

where

\[ \gamma \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right) \right]^{-\frac{1}{\theta}}, \]

and \( \Gamma(\cdot) \) is the Gamma function. Totally differentiating this price index (E.9), we have:

\[
\frac{dp_n}{p_n} = \sum_{m=1}^{N} \frac{d\rho_{nm}}{\rho_{nm}} (\rho_{nm})^{-\theta}\left[ \sum_{h=1}^{N} (\rho_{nh})^{-\theta} \right]^{-\frac{1}{\theta}}, \\
\frac{dp_n}{p_n} = \sum_{m=1}^{N} \frac{s_{nm} d\rho_{nm}}{\rho_{nm}}, \\
\frac{dp_n}{p_n} = \sum_{m=1}^{N} s_{nm} d\ln \rho_{nm}, \quad \text{(E.10)}
\]

E.1.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n, \quad \text{(E.11)} \]

Totally differentiating this market clearing condition (E.11), holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n, \\
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( -\frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \\
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( -\frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right).
\]

Using our result for the derivative of expenditure shares in equation (E.7) above, we can rewrite this as:

\[
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right) \right), \\
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right) \right), \\
\frac{d\ln w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{d\ln w_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh} d\ln \rho_{nh} - d\ln \rho_{ni} \right) \right), \quad \text{(E.12)}
\]

where we have defined \( t_{in} \) as country \( n \)'s expenditure on country \( i \) as a share of country \( i \)'s income:

\[ t_{in} = \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \]
E.1.6 Utility Again

Returning to our expression for indirect utility, we have:

\[ u_n = \frac{w_n}{p_n}. \]  
(E.13)

 Totally differentiating indirect utility (E.13), we have:

\[ \frac{du_n}{n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}, \]
\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}. \]

Using our total derivative of the sectoral price index in equation (E.10) above, we get:

\[ \frac{du_n}{n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}}, \]  
(E.14)

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \ln \rho_{nm}. \]

E.1.7 Wages and Common Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[ d \ln \tau_{ni} = 0, \quad \forall n, i \in N. \]  
(E.15)

We start with our expression for the log change in wages from equation (E.12) above:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh} (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right) \right), \]

Using the total derivative of the price term (E.8) and our assumption of constant bilateral trade costs (E.15), we can write this expression for the log change in wages as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \sum_{n=1}^{N} \left( \sum_{h=1}^{N} s_{nh} (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right), \]
\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \left( \sum_{h \in N} \sum_{n=1}^{N} t_{in} s_{nh} (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right). \]  
(E.16)

which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} (d \ln w_n - d \ln z_n) \right), \]

\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]

which has the same matrix representation as in equation (15) in the paper:

\[ d \ln w = T d \ln w + \theta M (d \ln w - d \ln z), \]  
(E.17)

\[ \theta = \sigma - 1. \]
E.1.8 Utility and Common Productivity Shocks

From equation (E.14), the log change in utility is given by:

$$d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}.$$  

Using the total derivative of the price term (E.8) and our assumption of constant bilateral trade costs (E.15), we can write this log change in utility as:

$$d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ d \ln w_m - d \ln z_m \right],$$

which has the same matrix representation as in equation (19) in the paper:

$$d \ln u = d \ln w - S (d \ln w - d \ln z). \quad \text{(E.18)}$$

E.2 Krugman (1980)

We consider a version of Krugman (1980) with labor as the sole factor of production, in which markets are monopolistically competitive, and trade arises from love of variety and increasing returns to scale. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

E.2.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \quad p_n = \left[ \sum_{i=1}^{N} \int_0^{M_i} p_{ni} (j) \frac{1}{\sigma} dj \right] \frac{1}{1-\sigma}, \quad \sigma > 1, \quad \text{(E.19)}$$

where $w_n$ is the wage; $p_n$ is the consumption goods price index; $p_{ni} (j)$ is the price in country $n$ of a variety $j$ produced in country $i$; $M_i$ is the endogenous mass of varieties; and varieties are substitutes ($\sigma > 1$).

E.2.2 Production Technology

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of $F$ units of labor and a constant variable cost in terms of labor that depends on a location’s productivity $z_i$. Therefore the total amount of labor ($l_i (j)$) required to produce $x_i (j)$ units of variety $j$ in location $i$ is:

$$l_i (j) = F_i + \frac{x_i (j)}{z_i}. \quad \text{(E.20)}$$

Varieties can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country $i$ to country $n$ in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Profit maximization and zero profits imply that equilibrium prices are a constant markup over marginal cost:

$$p_{ni} (j) = \left( \frac{\sigma}{\sigma - 1} \right) \rho_{ni}, \quad \rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i}, \quad \text{(E.21)}$$

and equilibrium employment for each variety is equal to a constant:

$$l_i (j) = \bar{l} = (\sigma - 1) F_i. \quad \text{(E.22)}$$
Given this constant equilibrium employment for each variety, labor market clearing implies that the total mass of varieties supplied by each country is proportional to its labor endowment:

\[ M_i = \frac{\ell_i}{\sigma F_i}. \]  

(E.23)

From the definition of \( \rho_{ni} \) in equation (E.21) above, we have:

\[ \frac{d\rho_{ni}}{\rho_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i}, \]  

(E.24)

\[ d \ln \rho_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i. \]

Totally differentiating in the labor market clearing condition (E.23), holding country endowments constant, we have:

\[ \frac{dM_i}{M_i} = - \frac{dF_i}{F_i}, \]  

(E.25)

\[ d \ln M_i = - d \ln F_i. \]

E.2.3 Expenditure Shares

Using the symmetry of equilibrium prices and the properties of the CES demand function, country \( n \)'s share of expenditure on goods produced in country \( i \) is:

\[ s_{ni} = \frac{M_{i} p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_{m} p_{nm}^{1-\sigma}} = \frac{(\ell_i / F_i) \rho_{ni}^{1-\sigma}}{\sum_{m=1}^{N} (\ell_m / F_m) \rho_{nm}^{1-\sigma}}. \]  

(E.26)

Totally differentiating this expenditure share equation (E.26) we get:

\[ \frac{ds_{ni}}{s_{ni}} = - \left[ \frac{dF_i}{F_i} + (\sigma - 1) \frac{d\rho_{ni}}{\rho_{ni}} \right] + \sum_{h=1}^{N} \frac{dF_h}{F_h} + (\sigma - 1) \frac{d\rho_{nh}}{\rho_{nh}} \right), \]  

(E.27)

\[ d \ln s_{ni} = - [d \ln F_i + (\sigma - 1) d \ln \rho_{ni}] + \sum_{h=1}^{N} s_{nh} [d \ln F_h + (\sigma - 1) d \ln \rho_{nh}]. \]

E.2.4 Price Indices

Using the symmetry of equilibrium prices, the price index (E.19) can be re-written as:

\[ p_n = \left[ \sum_{i=1}^{N} M_{i} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \]

Totally differentiating this price index, we have:

\[ \frac{dp_n}{p_n} = \sum_{i=1}^{N} \left[ \frac{1}{1-\sigma} \frac{dM_i}{M_i} + \frac{d\rho_{ni}}{\rho_{ni}} \right] \frac{M_{i} p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} M_{h} p_{nh}^{1-\sigma}} \left[ \sum_{i=1}^{N} M_{i} p_{ni}^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}, \]

\[ \frac{dp_n}{p_n} = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{1-\sigma} \frac{dM_i}{M_i} + \frac{d\rho_{ni}}{\rho_{ni}} \right] \frac{p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}}, \]

which using the equilibrium pricing rule (E.21) and the labor market clearing condition (E.23) can be written as:

\[ \frac{dp_n}{p_n} = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \frac{dF_i}{F_i} + \frac{d\rho_{ni}}{\rho_{ni}} \right], \]  

(E.28)

\[ d \ln p_n = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln \rho_{ni} \right]. \]
E.2.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \]  
\( \text{(E.29)} \)

Totally differentiating this market clearing condition (E.29), holding labor endowments constant, we have:

\[ \frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n, \]
\[ \frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \]
\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right). \]

Using our result for the derivative of expenditure shares in equation (E.27) above, we can rewrite this as:

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{\sum_{h=1}^{N} s_{nh} \frac{dF_h}{F_h} - \frac{dF_i}{F_i}}{\left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right)} \right), \]
\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \frac{\sum_{h=1}^{N} s_{nh} \frac{dF_h}{F_h} - \frac{dF_i}{F_i}}{\left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right)} \right), \]
\[ \frac{d \ln w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{d \ln w_n}{w_n} + \frac{\sum_{h=1}^{N} s_{nh} \frac{d \ln F_h}{F_h} - \frac{d \ln F_i}{F_i}}{\left( \sum_{h=1}^{N} s_{nh} \frac{d \ln \rho_{nh}}{\rho_{nh}} - \frac{d \ln \rho_{ni}}{\rho_{ni}} \right)} \right), \]  
\( \text{(E.30)} \)

where we have defined \( t_{in} \) as the share of country \( i \)'s income derived from market \( n \):

\[ t_{in} = \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \]

E.2.6 Utility Again

Returning to our expression for indirect utility, we have:

\[ u_n = \frac{w_n}{p_n}. \]  
\( \text{(E.31)} \)

Totally differentiating indirect utility (E.31), we have:

\[ du_n = \frac{dw_n}{w_n} \frac{w_n}{p_n} - \frac{dp_n}{p_n} \frac{w_n}{p_n}, \]
\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}. \]

Using our total derivative of the sectoral price index in equation (E.28) above, we get:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \left[ \frac{1}{\sigma - 1} \frac{dF_i}{F_i} + \frac{d\rho_{ni}}{\rho_{ni}} \right], \]  
\( \text{(E.32)} \)
\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln \rho_{ni} \right]. \]
E.2.7 Wages and Productivity Shocks

We consider small shocks to productivity, holding constant bilateral trade costs and fixed costs:

\[ \frac{d \ln \tau_{ni}}{dt} = 0, \quad \frac{d \ln F_i}{dt} = 0, \quad \forall \ n, i \in N. \]  

(E.33)

We start with our expression for the log change in wages from equation (E.30) above:

\[ \frac{d \ln w_i}{dt} = \sum_{n=1}^{N} t_{in} \left( \frac{d \ln w_n}{dt} + \left( \sum_{h=1}^{N} s_{nh} \frac{d \ln F_h}{dt} - \frac{d \ln F_i}{dt} \right) \right), \]

(E.34)

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant fixed costs (E.33), we can write this expression for the log change in wages as:

\[ \frac{d \ln w_i}{dt} = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right) \right), \]

which can be re-written as:

\[ \frac{d \ln w_i}{dt} = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} m_{in} \left[ d \ln w_n - d \ln z_n \right] \right) \right), \]

where

\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]

which has the same matrix representation as in equation (15) in the paper:

\[ \frac{d \ln w}{dt} = T \frac{d \ln w}{dt} + \theta M \left( \frac{d \ln w}{dt} - \frac{d \ln z}{dt} \right), \]

(E.35)

\[ \theta = \sigma - 1. \]

E.2.8 Welfare and Productivity Shocks

From equation (E.32), the log change in utility is given by:

\[ \frac{d \ln u_n}{dt} = \frac{d \ln w_n}{dt} - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \frac{d \ln F_i}{dt} + \frac{d \ln \rho_{ni}}{dt} \right], \]

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant fixed costs (E.33), we can write this log change in utility as:

\[ \frac{d \ln u_n}{dt} = \frac{d \ln w_n}{dt} - \sum_{i=1}^{N} s_{ni} \left[ d \ln w_i - d \ln z_i \right], \]

(E.36)

which has the same matrix representation as in equation (19) in the paper:

\[ \frac{d \ln u}{dt} = d \ln w - S \left( d \ln w - d \ln z \right). \]

(E.37)
E.2.9 Wages and Fixed Cost Shocks

We next consider small shocks to fixed costs, holding constant bilateral trade costs and productivity:

\[ \text{d} \ln \tau_{ni} = 0, \quad \text{d} \ln z_i = 0, \quad \forall \, n, i \in N. \quad (E.38) \]

We start with our expression for the log change in wages from equation (E.30) above:

\[ \text{d} \ln w_i = \sum_{n=1}^{N} t_{in} \left( \text{d} \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \text{d} \ln F_h - \text{d} \ln F_i \right) \right), \]

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant productivity (E.38), we can write this expression for the log change in wages as:

\[ \text{d} \ln w_i = \sum_{n=1}^{N} t_{in} \text{d} \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \text{d} \ln F_h - \text{d} \ln F_i \right) \]

\[ + (\sigma - 1) \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} s_{nh} \text{d} \ln w_h - \text{d} \ln w_i \right) \]

\[ \text{d} \ln w_i = \left[ \sum_{n=1}^{N} t_{in} \text{d} \ln w_n + \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} s_{nh} \text{d} \ln F_h - \text{d} \ln F_i \right) \right], \quad (E.39) \]

which has a similar matrix representation to that for productivity shocks above:

\[ \text{d} \ln w = \mathbf{T} \text{d} \ln w + \theta \mathbf{M} \left( \text{d} \ln w + \frac{1}{\theta} \text{d} \ln \mathbf{F} \right), \quad (E.40) \]

\[ \theta = \sigma - 1. \]

E.2.10 Welfare and Fixed Cost Shocks

From equation (E.32), the log change in utility is given by:

\[ \text{d} \ln u_n = \text{d} \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \text{d} \ln F_i + \text{d} \ln \rho_{ni} \right], \]

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant productivity (E.38), we can write this log change in utility as:

\[ \text{d} \ln u_n = \text{d} \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \text{d} \ln F_i + \text{d} \ln w_i \right], \quad (E.41) \]

which has a similar matrix representation to that for productivity shocks above:

\[ \text{d} \ln u = \mathbf{T} \text{d} \ln w - \mathbf{S} \left( \text{d} \ln w + \frac{1}{\theta} \text{d} \ln \mathbf{F} \right), \quad (E.42) \]

\[ \theta = \sigma - 1. \]
F Extensions

In Subsection F.1, we derive the corresponding friend-enemy matrix representations with both productivity and trade cost shocks, as discussed in Section 4.1 of the paper. In Section F.2, we relax one of the ACR macro restrictions to allow for trade imbalance, as discussed in Section 4.2 of the paper. In Section F.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system, as discussed in Section 4.3 of the paper. In Section F.4, we show that our results generalize to a multi-sector Armington model with a single constant trade elasticity, as discussed in Section 4.4 of the paper. In Section F.5, we show that our results also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section F.6, we further extend the multi-sector specification to introduce input-output linkages following Caliendo and Parro (2015), as discussed in Section 4.5 of the paper. Finally, in Section F.7, we show that our results also hold for economic geography models with factor mobility, including Helpman (1998), Redding and Sturm (2008), Allen and Arkolakis (2014), Ramondo et al. (2016), and Redding (2016), as discussed in Section 4.6 of the paper.

F.1 Trade Cost Reductions

We now show that we obtain similar results incorporating trade cost reductions. We start with our expression for the log change in wages from equation (D.8) above:

\[ \frac{\partial \ln w_i}{\partial \ln p_{ni}} = \sum_{n=1}^{N} \frac{\partial \ln w_n}{\partial \ln p_{nh}} \left( \frac{\partial \ln w_h}{\partial \ln p_{nh}} + \frac{\partial \ln \tau_{nh}}{\partial \ln \tau_{ni}} + \frac{\partial \ln z_h}{\partial \ln z_i} \right) \]

which can be re-written as follows:

\[ \frac{\partial \ln w_i}{\partial \ln p_{ni}} = \sum_{n=1}^{N} t_{in} \left( \frac{\partial \ln w_n}{\partial \ln p_{nh}} + \frac{\partial \ln \tau_{nh}}{\partial \ln \tau_{ni}} + \frac{\partial \ln z_h}{\partial \ln z_i} \right) \]

\[ \frac{\partial \ln w_i}{\partial \ln \tau_{ni}} = \sum_{n=1}^{N} t_{in} \frac{\partial \ln w_n}{\partial \ln \tau_{nh}} \]

\[ \frac{\partial \ln w_i}{\partial \ln z_i} = \sum_{n=1}^{N} t_{in} \frac{\partial \ln w_n}{\partial \ln z_h} \]

We now define inward and outward measures of trade costs as:

\[ \frac{\partial \ln \tau_{in}}{\partial \ln \tau_{ni}} = \sum_{i} s_{ni} \frac{\partial \ln \tau_{ni}}{\partial \ln \tau_{ni}} \]

\[ \frac{\partial \ln \tau_{out}}{\partial \ln \tau_{ni}} = \sum_{n} t_{in} \frac{\partial \ln \tau_{ni}}{\partial \ln \tau_{ni}} \]

Using these definitions of inward and outward trade costs, we can rewrite the above proportional change in wages as follows:

\[ \frac{\partial \ln w_i}{\partial \ln \tau_{ni}} = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} t_{in} \ln w_n + \sum_{n=1}^{N} m_{in} \left[ \frac{\partial \ln w_n}{\partial \ln \tau_{in}} + \frac{\partial \ln \tau_{in}}{\partial \ln \tau_{in}} \right] \right) \]

which can be re-written as:

\[ \frac{\partial \ln w_i}{\partial \ln \tau_{ni}} = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left[ \frac{\partial \ln w_n}{\partial \ln \tau_{in}} + \frac{\partial \ln \tau_{in}}{\partial \ln \tau_{in}} \right] \right) \]
which has the following matrix representation from equation (31) in the paper:

\[
\frac{d \ln w}{d \ln w} = \mathbf{T} \frac{d \ln w}{d \ln z} + \mathbf{M} \left( \frac{d \ln w}{d \ln z} \right) + \mathbf{T} \frac{d \ln \tau_{\text{in}}}{d \ln \tau_{\text{out}}}, \tag{F.4}
\]

\[\theta = \sigma - 1.\]

We next consider our expression for the log change in utility from equation (D.10) above:

\[
\frac{d \ln u}{d \ln w} = \frac{d \ln w}{d \ln z} - \sum_{m=1}^{N} s_{nm} \frac{d \ln p}{d \ln z},
\]

which can be re-written as follows:

\[
\frac{d \ln u}{d \ln w} = \frac{d \ln w}{d \ln z} - \sum_{m=1}^{N} s_{nm} \left( \frac{d \ln w}{d \ln z} + \frac{d \ln \tau_{nm}}{d \ln z_{m}} - \frac{d \ln \tau_{m}}{d \ln z_{m}} \right).
\]

Using our definition of inward trade costs from equation (F.1), we can re-write this proportional change in welfare as:

\[
\frac{d \ln u}{d \ln w} = \frac{d \ln w}{d \ln z} - \sum_{m=1}^{N} s_{nm} \left( \frac{d \ln w}{d \ln z} - \frac{d \ln \tau_{m}}{d \ln z_{m}} - \frac{d \ln \tau_{\text{in}}}{d \ln z_{m}} \right),
\]

which has the following matrix representation from equation (32) in the paper:

\[
\frac{d \ln u}{d \ln w} = \frac{d \ln w}{d \ln z} - \mathbf{S} \left( \frac{d \ln w}{d \ln z} - \frac{d \ln \tau_{\text{in}}}{d \ln z_{m}} \right). \tag{F.5}
\]

\section*{F.2 Trade Imbalance}

In this section of the online appendix, we relax another of the ACR macro restrictions to allow for trade imbalance. In particular, we consider the constant elasticity Armington model from Section 3 of the paper, but allow expenditure to differ from income.

\subsection*{F.2.1 Preferences and Expenditure Shares}

We measure the instantaneous welfare of the representative agent as the real value of expenditure:

\[
u_{n} = \frac{w_{n} \ell_{n} + \bar{d}_{n}}{p_{n}}, \quad p_{n} = \left[ \sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,
\]

where \(\bar{d}_{n}\) is the nominal trade deficit. Expenditure shares take the same form as in equation (12) in Section 3 of the paper.

\subsection*{F.2.2 Market Clearing}

Market clearing requires that income in each location equals expenditure on goods produced in that location:

\[
w_{i} \ell_{i} = \sum_{n=1}^{N} s_{ni} \left[ w_{n} \ell_{n} + \bar{d}_{n} \right]. \tag{F.7}
\]
Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{\partial w_i}{\partial w_i} = \sum_{n=1}^{N} \frac{\partial s_{ni}}{s_{ni}} s_{ni} \left[ w_n \ell_n + \tilde{d}_n \right] + \sum_{n=1}^{N} s_{ni} \frac{\partial w_n}{w_n} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{\partial d_n}{d_n} d_n,
\]

\[
= \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{\partial s_{ni}}{s_{ni}} w_n + \sum_{n=1}^{N} s_{ni} \frac{\partial w_n}{w_n} + \sum_{n=1}^{N} s_{ni} \frac{\partial d_n}{d_n} \right),
\]

or

\[
\frac{\partial w_i}{w_i} = \sum_{n=1}^{N} s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \frac{\partial w_n}{w_n} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \frac{\partial d_n}{d_n} d_n,
\]

\[
\frac{\partial w_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left( \frac{\partial s_{ni}}{s_{ni}} w_n \ell_n + \frac{\partial d_n}{d_n} d_n \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) \left( \frac{\partial s_{ni}}{s_{ni}} w_n \ell_n + \frac{\partial d_n}{d_n} d_n \right),
\]

where we have defined \( t_{in} \) as the share of country \( i \)'s income from market \( n \):

\[
t_{in} = \frac{s_{ni} (w_n \ell_n + \tilde{d}_n)}{w_i \ell_i},
\]

and \( d_n \) as country \( n \)'s ratio of income to expenditure:

\[
d_n = \frac{w_n \ell_n}{w_n \ell_n + \tilde{d}_n}.
\]

Using our result for the derivative of expenditure shares above, we can rewrite this as:

\[
\frac{\partial w_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left( \frac{\partial w_n}{w_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{d p_{nh}}{p_{nh}} - \frac{d p_{ni}}{p_{ni}} \right) \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) \left( \frac{\partial d_n}{d_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{d p_{nh}}{p_{nh}} - \frac{d p_{ni}}{p_{ni}} \right) \right). \tag{F.8}
\]

We can re-write this expression as:

\[
\frac{\partial w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( d_n \frac{\partial w_n}{w_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{d p_{nh}}{p_{nh}} - d \ln p_{ni} \right) \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) \frac{\partial d_n}{d_n} \frac{d \tilde{d}_n}{\tilde{d}_n}.
\]

\section*{F.2.3 Welfare}

Returning to our expression for welfare (F.6), we have:

\[
\omega_n = \frac{w_n \ell_n + \tilde{d}_n}{p_n}.
\]

Using our result for the derivative of welfare, holding labor endowments constant, we have:

\[
\frac{\partial \omega_n}{\partial \omega_n} = \frac{\partial w_n}{w_n} w_n \ell_n + \frac{\partial d_n}{d_n} d_n = \frac{\partial p_n}{p_n} \left[ w_n \ell_n + \tilde{d}_n \right],
\]

\[
\frac{\partial \omega_n}{\omega_n} = \frac{w_n \ell_n}{w_n \ell_n + d_n} \frac{\partial w_n}{w_n} + \frac{\partial d_n}{d_n} \frac{d_n}{d_n} - \frac{\partial p_n}{p_n},
\]

\[
= d_n \frac{\partial w_n}{w_n} + (1 - d_n) \frac{\partial d_n}{d_n} \frac{d p_n}{p_n}.
\]
Using the total derivative of the sectoral price index, we get:

\[
\frac{du_n}{u_n} = d_n \frac{dw_n}{w_n} + (1 - d_n) \frac{d\bar{d}_n}{\bar{d}_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}},
\]

which can be re-written as:

\[
d \ln u_n = d_n \ln w_n + (1 - d_n) \ln \bar{d}_n - \sum_{m=1}^{N} s_{nm} d \ln p_{nm}.
\]

**F.2.4 Wages and Productivity Shocks**

We consider small productivity shocks, holding constant bilateral trade costs:

\[
d \ln \tau_{ni} = 0, \quad \forall \ n, i \in N,
\]

and we assume that trade deficits remain constant in terms of our numeraire of world GDP:

\[
d \ln \bar{d}_n = 0.
\]

Under these assumptions, the change in prices is:

\[
d \ln p_{ni} = d \ln w_i - d \ln z_i.
\]

We start with our expression for the log change in wages in equation (F.8) above. With constant trade deficits \((d \ln \bar{d}_n = 0)\), this expression simplifies to:

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d_n \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} [d \ln w_h - d \ln z_h] - [d \ln w_i - d \ln z_i] \right) \right).
\]

Using the total derivative of prices and our assumption of constant bilateral trade costs (F.10), this further simplifies to:

\[
d \ln w_i = \sum_{n=1}^{N} d_n t_{in} \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} t_{in} s_{nh} [d \ln w_h - d \ln z_h] - [d \ln w_i - d \ln z_i] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} d_n t_{in} \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} [d \ln w_n - d \ln z_n] \right),
\]

\[
m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},
\]

which has the following matrix representation:

\[
d \ln w = TD d \ln w + \theta M \left( d \ln w - d \ln z \right),
\]

\[
\theta = \sigma - 1
\]

where the matrices \(T\) and \(M\) are defined in Section D of this online appendix. The diagonal matrix \(D\) captures trade deficits through the ratio of income to expenditure

\[
D = \begin{bmatrix}
d_1 & 0 & 0 & 0 \\
0 & d_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & d_N
\end{bmatrix}_{N \times N}, \quad d_n = \frac{w_n \ell_n}{w_n \ell_n + d_n}.
\]
F.2.5 Welfare and Productivity Shocks

We start with our expression for the log change in welfare in equation (F.9) above. With constant trade deficits $(d\ln\dd_n = 0)$, this simplifies to:

$$d\ln u_n = d\ln w_n - \sum_{m=1}^{N} s_{nm} d\ln p_{nm}.$$  

Using the total derivative of prices and our assumption of constant bilateral trade costs (F.10), we can write this proportional change in utility as:

$$d\ln u_n = d\ln w_n - \sum_{m=1}^{N} s_{nm} (d\ln w_m - d\ln z_m),$$  

which has the following matrix representation:

$$d\ln u = D d\ln w - S (d\ln w - d\ln z),$$  

where the matrix $S$ is defined in Section D of this online appendix and the diagonal matrix $D$ is defined in the previous subsection of this online appendix.

F.3 Deviations from Constant Elasticity Import Demand

In this section of the online appendix, we consider the generalization of the constant elasticity Armington model in the previous section to incorporate small deviations from a constant elasticity import demand system, as discussed in Section 4.3 of the paper.

F.3.1 Expenditure Shares

We start from the total derivative for expenditure shares in the Armington model with a general homothetic consumption goods price index in equation (C.4) in Section C of this online appendix, as reproduced below:

$$\frac{d s_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}},$$  

where $\theta_{nih}$ is the elasticity of expenditure in country $n$ on the good produced by country $i$ with respect to the price of the good produced by country $h$. In the special case of a constant elasticity import demand, we have:

$$\theta_{nih} = \begin{cases} (s_{nh} - 1) (\sigma - 1) & \text{if } i = h \\ s_{nh} (\sigma - 1) & \text{otherwise} \end{cases}.$$  

Using this result in equation (F.15), we obtain the total derivative for expenditure shares with a constant elasticity import demand system in equation (D.4) in Section D of this online appendix, as reproduced below:

$$\frac{d s_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right).$$  

We now allow for deviations from a constant elasticity import demand system by considering the following generalization of the specification in equation (F.16):

$$\theta_{nih} = \begin{cases} (s_{nh} - 1) (\sigma - 1) + o_{nih} & \text{if } i = h \\ s_{nh} (\sigma - 1) + o_{nih} & \text{otherwise} \end{cases}.$$  

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We assume that this generalized demand specification remains homothetic, which implies:

\[ 0 = \sum_{k} x_{nk} \theta_{nkh}, \]

\[ = \sum_{k=1}^{N} x_{nk} ((\sigma - 1) s_{nh} + o_{nkh}) - (\sigma - 1) x_{nh}, \]

\[ = \sum_{k=1}^{N} x_{nk} o_{nkh}, \]

where \( x_{ni} \) denotes country \( n \)'s expenditure on the goods produced by country \( i \). Therefore, homotheticity implies:

\[ \sum_{k=1}^{N} s_{nk} o_{nkh} = 0. \]  \hspace{1cm} (F.19)

Using our generalized demand specification (F.18) in the total derivative of expenditure shares in equation (F.15), we obtain:

\[
\frac{ds_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nhi} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}},
\]

\[ = (\sigma - 1) \left\{ \sum_{h=1}^{N} \left[ s_{nh} + \frac{o_{nhi}}{\sigma - 1} - 1_{i=h} \right] - \sum_{h=1}^{N} s_{nk} \left( \frac{s_{nh} + o_{nkh}}{\sigma - 1} - s_{nh} \right) \right\} \frac{dp_{nh}}{p_{nh}}, \]

\[ = (\sigma - 1) \left\{ \sum_{h=1}^{N} s_{nh} + \frac{o_{nhi}}{\sigma - 1} - \sum_{k=1}^{N} s_{nk} \frac{o_{nkh}}{\sigma - 1} \right\} \frac{dp_{nh}}{p_{nh}} - (1 + o_{nii}) \frac{dp_{ni}}{p_{ni}} \}.
\]

Using our implication of homotheticity in equation (F.19), this expression simplifies to:

\[
\frac{ds_{ni}}{s_{ni}} = \left\{ \sum_{h=1}^{N} ((\sigma - 1) s_{nh} + o_{nhi}) \frac{dp_{nh}}{p_{nh}} - ((\sigma - 1) + o_{nii}) \frac{dp_{ni}}{p_{ni}} \right\}.
\]  \hspace{1cm} (F.20)

### F.3.2 Market Clearing

Market clearing again requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \]  \hspace{1cm} (F.21)

Totally differentiating this market clearing condition, we obtain:

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right). \]  \hspace{1cm} (F.22)

Using our result for the derivative of expenditure shares (F.20) in the above equation, we obtain:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \left( \sum_{h=1}^{N} ((\sigma - 1) s_{nh} + o_{nhi}) \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right),
\]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \left( \sum_{h=1}^{N} ((\sigma - 1) s_{nh} + o_{nhi}) \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left( \sum_{h=1}^{N} ((\sigma - 1) s_{nh} + o_{nhi}) d \ln p_{nh} - d \ln p_{ni} \right) \right), \]  \hspace{1cm} (F.23)

where we have defined \( t_{in} \) as the share of country \( i \)'s income derived from market \( n \):

\[ t_{in} = \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \]
F.3.3 Welfare

From equation (C.8) in the Armington model with a general homothetic consumption goods price index, we also have the following expression for the total derivative of welfare:

\[
d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ d \ln \tau_{ni} + d \ln w_i - d \ln z_i \right] ,
\]

(F.24)

F.3.4 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[
d \ln \tau_{ni} = 0, \quad \forall n, i \in N.
\]

(F.25)

We start with our expression for the log change in wages in equation (F.23) above. Using the total derivative of prices and our assumption of constant bilateral trade costs (F.25), we can write this expression for the log change in wages as:

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} [(\sigma - 1) s_{nh} + o_{nih}] \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} [(\sigma - 1) s_{nh} + o_{nih}] \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} [(\sigma - 1) s_{nh} + o_{nih}] \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( (\sigma - 1) \sum_{n=1}^{N} \sum_{h=1}^{N} t_{in} s_{nh} \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( (\sigma - 1) \sum_{n=1}^{N} \sum_{h=1}^{N} t_{in} s_{nh} \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( (\sigma - 1) \sum_{n=1}^{N} \sum_{h=1}^{N} t_{in} s_{nh} \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( (\sigma - 1) \sum_{n=1}^{N} \sum_{h=1}^{N} t_{in} s_{nh} \left[ d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right] \right),
\]

which has the following matrix representation in equation (41) in the paper:

\[
d \ln w = T d \ln w + (\theta M + O) (d \ln w - d \ln z),
\]

\[
\theta = (\sigma - 1).
\]

(F.26)

F.3.5 Welfare and Productivity Shocks

Using our assumption of constant bilateral trade costs (F.25) in our expression for the log change in welfare (F.24) above, we obtain:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left( d \ln w_m - d \ln z_m \right),
\]

(F.27)

which the following matrix representation in equation (42) in the paper:

\[
d \ln u = d \ln w - S (d \ln w - d \ln z).
\]

(F.28)
F.4 Multiple Sectors

In this section of the online appendix, we show that our friends-and-enemies exposure measures extend naturally to a multi-sector version of the constant elasticity Armington model, as discussed in Section 4.4 of the paper. The world economy consists of many countries indexed by \( i, n \in \{1, \ldots, N\} \) and a set of sectors indexed by \( k \in \{1, \ldots, K\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

F.4.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[
 u_n = \frac{w_n}{\prod_{k=1}^{K} (p_{nk}^k)^{\alpha_n^k}}, \quad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{F.29}
\]

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

\[
 p_{nk}^k = \left[ \sum_{i=1}^{N} (p_{ni}^k)^{-\theta} \right]^{-\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1. \tag{F.30}
\]

F.4.2 Production Technology

Goods are produced with labor under conditions of perfect competition, such that the cost to a consumer in country \( n \) of purchasing the variety of country \( i \) within sector \( k \) is:

\[
 p_{ni}^k = \tau_{ni}^k w_i z_{ki}, \tag{F.31}
\]

where \( z_{ki} \) captures productivity and iceberg trade costs satisfy \( \tau_{ni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \).

F.4.3 Expenditure Shares

Using the properties of CES demand, country \( n \)'s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[
 s_{ni}^k = \frac{(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}. \tag{F.32}
\]

Totally differentiating the expenditure share equation (F.32), we get:

\[
d s_{ni}^k = -\theta \frac{dp_{nk}^k}{p_{nk}^k} (p_{ni}^k)^{-\theta} + \sum_{h=1}^{N} \frac{\theta dp_{nh}^k}{p_{nh}^k} (p_{nh}^k)^{-\theta},
\]

\[
 \frac{d s_{ni}^k}{s_{ni}^k} = -\theta \frac{dp_{nk}^k}{p_{nk}^k} + \sum_{h=1}^{N} \frac{\theta dp_{nh}^k}{p_{nh}^k} (p_{nh}^k)^{-\theta},
\]

\[
 \frac{d s_{ni}^k}{s_{ni}^k} = -\theta \frac{dp_{nk}^k}{p_{nk}^k} + \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k},
\]

\[
 \frac{d s_{ni}^k}{s_{ni}^k} = \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right), \tag{F.33}
\]

\[
d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k \ln p_{nh}^k - \ln p_{ni}^k \right),
\]

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where, from equilibrium prices in equation (F.31), we have:

\[
\frac{dp^k_{ni}}{p^k_{ni}} = \frac{dr^k_{ni}}{r^k_{ni}} + \frac{dw_i}{w_i} - \frac{dz^k_i}{z^k_i},
\]

\[
d\ln p^k_{ni} = d\ln r^k_{ni} + d\ln w_i - d\ln z^k_i.
\]

F.4.4 Price Indices

Totally differentiating the sectoral price index (F.30), we have:

\[
dp^k_n = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{(p_{nm})^{-\theta}}{\sum_{h=1}^{N} (p_{kh})^{-\theta}} \left[ \sum_{m=1}^{N} (p_{nm})^{-\theta} \right]^{-\frac{1}{\theta}},
\]

\[
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{(p_{nm})^{-\theta}}{\sum_{h=1}^{N} (p_{kh})^{-\theta}},
\]

\[
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} s_{nm}^k \frac{dp_{nm}}{p_{nm}},
\]

\[
d\ln p^k_n = \sum_{m=1}^{N} s_{nm}^k d\ln p^k_{nm}.
\]

F.4.5 Market Clearing

Market clearing requires that income in each location equals expenditure on goods produced in that location:

\[
w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n.
\]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left( \frac{dw_n}{w_n} w_n \ell_n + \frac{ds_{ni}^k}{s_{ni}^k} \right),
\]

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left( \frac{dw_n}{w_n} w_n \ell_n + \frac{ds_{ni}^k}{s_{ni}^k} \right).
\]

Using our result for the derivative of expenditure shares in equation (F.33) above, we can rewrite this as:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}^k}{p_{ni}^k} \right) \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \ell_{in}^k \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}^k}{p_{ni}^k} \right) \right),
\]

\[
d\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \ell_{in}^k \left( d\ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k d\ln p_{nh} - d\ln p_{ni}^k \right) \right),
\]

where we have defined \( \ell_{in}^k \) as the share of country \( i \)'s income derived from market \( n \) and industry \( k \):

\[
\ell_{in}^k = \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.
\]
F.4.6 Utility Again

Returning to our expression for indirect utility in equation (F.29), we have:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_{kn}}}. \]

Totally differentiating indirect utility, we have:

\[ du_n = \frac{dw_n w_n}{w_n} - \sum_{k=1}^{K} \alpha_{kn} \frac{dp_n^k}{p_n^k} w_n. \]

Using our total derivative of the sectoral price index in equation (F.35) above, we get:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_{kn} \sum_{m=1}^{N} s_{nm}^k \frac{dp_{nm}^k}{p_{nm}^k}, \tag{F.38} \]

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_{kn} \sum_{m=1}^{N} s_{nm}^k d \ln p_{nm}^k. \]

F.4.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[ d \ln z_i^k = d \ln z_i, \quad \forall k \in K, \ i \in N, \]
\[ d \ln \tau_{ni}^k = 0, \quad \forall n, i \in N. \tag{F.39} \]

We start with our expression for the log change in wages from equation (F.37) above:

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right) \right). \]

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can write this expression for the log change in wages as:

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right) \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k d \ln w_n + \theta \left( \sum_{h=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{K} t_{kn}^h s_{nh}^k \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \left( \sum_{n=1}^{N} m_{in} \left[ d \ln w_n - d \ln z_n \right] \right), \]

\[ t_{in} = \sum_{k=1}^{K} t_{in}^k, \]

\[ m_{in} = \sum_{h=1}^{N} \sum_{k=1}^{K} t_{kh}^i s_{hn}^k - 1_{n=i}, \]
which has the following matrix representation in the paper:

\[ d \ln w = T d \ln w + \theta M (d \ln w - d \ln z). \quad \text{(F.40)} \]

We again solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (F.40) by \( \theta + 1 \), we have:

\[ \frac{1}{\theta + 1} d \ln w = \frac{1}{\theta + 1} T d \ln w + \frac{\theta}{\theta + 1} M (d \ln w - d \ln z), \]

\[ \frac{1}{\theta + 1} (I - T - \theta M) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z. \]

Now using \( M = TS - I \), we have:

\[ \frac{1}{\theta + 1} (I - T - \theta TS + \theta I) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z, \]

\[ \left( I - \frac{T + \theta TS}{\theta + 1} \right) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z. \]

Using our choice of world GDP as numeraire, which implies \( Q d \ln w = 0 \), we have:

\[ \left( I - \frac{T + \theta TS}{\theta + 1} + Q \right) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z, \]

which can be re-written as:

\[ (I - V) d \ln w = -\frac{\theta}{\theta + 1} M d \ln z, \]

\[ V = \frac{T + \theta TS}{\theta + 1} - Q, \]

which yields the following solution of the change in wages in response to a productivity shock:

\[ d \ln w = -\frac{\theta}{\theta + 1} (I - V)^{-1} M d \ln z, \]

which can be re-written as:

\[ d \ln w = W d \ln z, \quad \text{(F.41)} \]

where \( W \) is our friend-enemy income exposure measure:

\[ W = -\frac{\theta}{\theta + 1} (I - V)^{-1} M. \quad \text{(F.42)} \]

### F.4.8 Welfare and Common Productivity Shocks

We start with our expression for the log change in utility in equation (F.38) above:

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k d \ln p_{nm}^k, \]

or equivalently:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^{k} s_{nm}^k d \ln p_{nm}^k. \]

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can write this change in log utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^{k} s_{nm}^k (d \ln w_m - d \ln z_m), \quad \text{(F.43)} \]
which has the following matrix representation in the paper:
\[
\text{d ln } \mathbf{u} = \text{d ln } \mathbf{w} - \mathbf{S} ( \text{d ln } \mathbf{w} - \text{d ln } \mathbf{z} ).
\] (F.44)

We can re-write the above relationship as:
\[
\text{d ln } \mathbf{u} = (\mathbf{I} - \mathbf{S}) \text{d ln } \mathbf{w} + \mathbf{S} \text{d ln } \mathbf{z},
\]
which, using our solution for \( \text{d ln } \mathbf{w} \) from equation (F.41), can be further re-written as:
\[
\text{d ln } \mathbf{u} = (\mathbf{I} - \mathbf{S}) \mathbf{W} \text{d ln } \mathbf{z} + \mathbf{S} \text{d ln } \mathbf{z},
\]
\[
\text{d ln } \mathbf{u} = [(\mathbf{I} - \mathbf{S}) \mathbf{W} + \mathbf{S}) \text{d ln } \mathbf{z},
\]
\[
\text{d ln } \mathbf{u} = \mathbf{U} \text{d ln } \mathbf{z},
\] (F.45)

where \( \mathbf{U} \) is our friend-enemy welfare exposure measure:
\[
\mathbf{U} = [(\mathbf{I} - \mathbf{S}) \mathbf{W} + \mathbf{S})
\] (F.46)

F.4.9 Industry-Level Sales Exposure

In this multi-sector model, our approach also yields bilateral friend-enemy measures of income exposure to global productivity shocks for each sector. Labor income in each sector and country equals value-added, which in turn equals expenditure on goods produced in that sector and country:
\[
w^k_i = \sum_{n=1}^{N} \alpha_n^k s_n^k w_n^k.
\]

Totally differentiating this industry market clearing condition, we have:
\[
\frac{dy^k_i}{y^k_i} w_i^k \ell_i = \sum_{n=1}^{N} \alpha_n^k s_n^k w_n^k \ell_n^k + \sum_{n=1}^{N} \alpha_n^k s_n^k \frac{dw_n}{w_n} w_n^k \ell_n^k,
\]
\[
\frac{dy^k_i}{y^k_i} w_i^k \ell_i = \sum_{n=1}^{N} \alpha_n^k s_n^k w_n^k \ell_n^k \left( \frac{ds_n^k}{s_n^k} + \frac{dw_n}{w_n} \right),
\]
\[
\frac{dy^k_i}{y^k_i} w_i^k \ell_i = \sum_{n=1}^{K} \sum_{k=1}^{K} \alpha_n^k s_n^k \frac{dw_n}{w_n} + \frac{ds_n^k}{s_n^k} \left( \frac{dw_n}{w_n} \right),
\]

Using the total derivative of expenditure shares in equation (F.33), we can rewrite this as:
\[
\frac{dy^k_i}{y^k_i} = \sum_{n=1}^{N} \alpha_n^k s_n^k \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_n^h \frac{dp_n^h}{p_n^h} - \frac{dp_n^k}{p_n^k} \right),
\]
which can be re-written as:
\[
\text{d ln } y^k_i = \sum_{n=1}^{N} \alpha_n^k s_n^k \left( \text{d ln } w_n + \theta \left( \sum_{h=1}^{N} s_n^h \text{d ln } p_n^h - \text{d ln } p_n^k \right) \right).
\]
Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can re-write this change in sector value-added as:

$$
d_l n_y^k = \sum_{n=1}^{N} \varepsilon_{in}^k \left( d_l n w_n + \theta \left( \sum_{h=1}^{N} \eta_{nh}^k \left( d_l n w_h - d_l n z_h \right) - \left( d_l n w_i - d_l n z_i \right) \right) \right),
$$

which has the following matrix representation:

$$
d_l n Y^k = T^k d_l n w + \theta M^k \left( d_l n w - d_l n z \right). \tag{F.47}
$$

Using our solution for changes in wages as a function of productivity shocks in equation (F.40) above, we have:

$$
d_l n Y^k = T^k W d_l n z + \theta M^k \left( W - I \right) d_l n z,
$$

$$
d_l n Y^k = \left[ T^k W + \theta M^k \left( W - I \right) \right] d_l n z,
$$

which can be re-written as:

$$
d_l n Y^k = W^k d_l n z, \tag{F.48}
$$

$$
W^k = T^k W + \theta M^k \left( W - I \right), \tag{F.49}
$$

which corresponds to our friends-and-enemies measure of sector value-added exposure to productivity shocks.

We now show that our aggregate friends-and-enemies measure of income exposure ($W_i$) in equation (F.42) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure in equation (F.49).

Note that aggregate income equals aggregate value-added:

$$w_i \ell_i = \sum_{k=1}^{K} y_i^k.
$$

Totally differentiating, holding endowments constant, we have:

$$
d w_i \ell_i = \sum_{k=1}^{K} \frac{d y_i^k}{y_i^k} y_i^k.
$$

$$
\frac{d w_i}{w_i} = \sum_{k=1}^{K} \frac{y_i^k}{w_i \ell_i} \frac{d y_i^k}{y_i^k},
$$

$$
\frac{d w_i}{w_i} = \sum_{k=1}^{K} r_i^k \frac{d y_i^k}{y_i^k},
$$

$$
d l n w_i = \sum_{k=1}^{K} r_i^k d l n y_i^k, \tag{F.50}
$$

where $r_i^k = \frac{y_i^k}{w_i \ell_i}$ is the value-added share of industry $k$. Together, equations (F.41), (F.48) and (F.50) imply:

$$W_i d l n z = \sum_{k=1}^{K} r_i^k W_i^k d l n z,$$

where $W_i$ is the income exposure vector for country $i$ with respect to productivity shocks in its trade partners and $W_i^k$ is the sector value-added exposure vector for country $i$ and sector $k$ with respect to productivity shocks in those
trade partners. It follows that our aggregate friends-and-enemies measure of income exposure \((W_i)\) is a weighted average of our industry friends-and-enemies measures of sector income exposure \((W^k_i)\):

\[
W_i = \sum_k r^k_i W^k_i.
\]

Therefore, we can decompose our aggregate income exposure measure into the contributions of the income exposure measures of particular industries, and how much of that income exposure of particular industries is explained by various terms (market-size, cross-substitution etc within the industry).

**F.5 Multi-Sector Isomorphism**

For the convenience of exposition, we focus in Section 3 of the paper and the previous section of this online appendix on a multi-sector version of the constant elasticity Armington model. In this section, we show that the same results hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). The world economy consists of a set of countries indexed by \(i, n \in \{1, \ldots, N\}\) and a set of sectors indexed by \(k \in \{1, \ldots, K\}\). Each country \(n\) has an exogenous supply of labor \(\ell_n\).

**F.5.1 Consumer Preferences**

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[
u_n = \frac{w_n}{\prod_{k=1}^K (p^k_n)^{\alpha^k_n}}, \quad \sum_{k=1}^K \alpha^k_n = 1.
\]  

(F.51)

Each sector contains a fixed continuum of goods that enter the sectoral price index according to the following CES functional form:

\[
p^k_n = \left[ \int_0^1 p^k_n(\vartheta)^{1-\sigma^k_n} d\vartheta \right]^{\frac{1}{1-\sigma^k_n}}, \quad \sigma^k > 1.
\]  

(F.52)

**F.5.2 Production Technology**

Goods are produced with labor and can be traded subject to iceberg variable trade costs, such that the cost to a consumer in country \(n\) of purchasing a good \(\vartheta\) from country \(i\) is:

\[
p^k_{ni}(\vartheta) = \frac{\tau^k_{ni} w_i}{z^k_i a^k_i(\vartheta)},
\]  

(F.53)

where \(z^k_i\) captures determinants of productivity that are common across all goods within a country \(i\) and sector \(k\) and \(a^k_i(\vartheta)\) captures idiosyncratic determinants of productivity for each good within that country and sector. Iceberg trade costs satisfy \(\tau^k_{ni} > 1\) for \(n \neq i\) and \(\tau^k_{nn} = 1\). Productivity for each good \(\vartheta\) in each sector \(k\) and each country \(i\) is drawn independently from the following Fréchet distribution:

\[
F^k_i(a) = \exp (-a^{-\theta} ), \quad \theta > 1,
\]  

(F.54)

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \(z^k_i\).
F.5.3 Expenditure Shares

Using the properties of this Fréchet distribution, country $n$’s share of expenditure on goods produced in country $i$ within sector $k$ is given by:

$$s_{ni}^k = \frac{(z_{ni}^k w_i / z_i^k)^{-\theta}}{\sum_{m=1}^{N} (z_{nm}^k w_m / z_m^k)^{-\theta}} = \frac{(\rho_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta}},$$

where we have defined the following price term:

$$\rho_{ni}^k = \frac{z_{ni}^k w_i}{z_i^k}.$$  \hspace{1cm} (F.56)

 Totally differentiating the expenditure share equation (F.55), we get:

$$ds_{ni}^k = -\frac{\theta \frac{d\rho_{ni}^k}{\rho_{ni}^k} (\rho_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta}} + \sum_{h=1}^{N} \frac{\theta \frac{d\rho_{nh}^k}{\rho_{nh}^k} (\rho_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta}},$$

$$\frac{ds_{ni}^k}{s_{ni}^k} = -\theta \frac{d\rho_{ni}^k}{\rho_{ni}^k} + \sum_{h=1}^{N} \theta \frac{d\rho_{nh}^k}{\rho_{nh}^k},$$

$$\frac{ds_{ni}^k}{s_{ni}^k} = \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{d\rho_{nh}^k}{\rho_{nh}^k} - \frac{d\rho_{ni}^k}{\rho_{ni}^k} \right),$$

$$d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k d \ln \rho_{nh}^k - d \ln \rho_{ni}^k \right),$$

where from the definition of $\rho_{ni}^k$ above we have:

$$\frac{d\rho_{ni}^k}{\rho_{ni}^k} = \frac{d\tau_{ni}^k}{\tau_{ni}^k} + \frac{dw_i}{w_i} - \frac{dz_i^k}{z_i^k},$$

$$d \ln \rho_{ni}^k = d \ln \tau_{ni}^k + d \ln w_i - d \ln z_i^k.$$  \hspace{1cm} (F.58)

F.5.4 Price Indices

Using the properties of the Fréchet distribution (F.54), the sectoral price index is given by:

$$p_n^k = \gamma^k \left[ \sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta} \right]^{-\frac{1}{\theta}},$$

where

$$\gamma^k \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma^k}{\theta} \right) \right]^{-\frac{1}{\theta-\sigma^k}},$$

and $\Gamma(\cdot)$ denotes the Gamma function. Totally differentiating this sectoral price index (F.59), we have:

$$dp_n^k = \sum_{m=1}^{N} \gamma^k \frac{d\rho_{nm}^k}{\rho_{nm}^k} \left[ \sum_{h=1}^{N} (\rho_{nm}^k)^{-\theta} \right]^{-\frac{1}{\theta}} \left[ \sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta} \right]^{-\frac{1}{\theta}} ,$$

$$dp_n^k = \sum_{m=1}^{N} \frac{dp_{nm}^k}{\rho_{nm}^k} \left( \rho_{nm}^k \right)^{-\theta} \sum_{h=1}^{N} (\rho_{nh}^k)^{-\theta} ,$$

$$dp_n^k = \sum_{m=1}^{N} s_{nm}^k \frac{d\rho_{nm}^k}{\rho_{nm}^k} ,$$

$$d \ln p_n^k = \sum_{m=1}^{N} s_{nm}^k d \ln \rho_{nm}^k.$$  \hspace{1cm} (F.60)
F.5.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} K \alpha_n^k s_n^k w_n \ell_n. \] (F.61)

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[ \frac{d w_i}{w_i} \frac{w_i \ell_i}{\ell_i} = \sum_{n=1}^{N} K \alpha_n^k s_n^k w_n \ell_n \left( \frac{d w_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_h^k \frac{d \rho_{nh}^k}{\rho_{nh}^k} - \frac{d \rho_{ni}^k}{\rho_{ni}^k} \right) \right), \]
\[ = \sum_{n=1}^{N} K \alpha_n^k s_n^k w_n \ell_n \left( \frac{d w_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_h^k \frac{d \rho_{nh}^k}{\rho_{nh}^k} - \frac{d \rho_{ni}^k}{\rho_{ni}^k} \right) \right), \] (F.62)

\[ d \ln w_i = \sum_{n=1}^{N} K \alpha_n^k s_n^k w_n \ell_n \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_h^k \ln \rho_{nh}^k - d \ln \rho_{ni}^k \right) \right), \]

where we have defined \( t_{in}^k \) as country \( n \)'s expenditure on country \( i \) in industry \( k \) as a share of country \( i \)'s income:

\[ t_{in}^k = \frac{\alpha_n^k s_n^k w_n \ell_n}{w_i \ell_i}. \]

F.5.6 Utility Again

Returning to our expression for indirect utility in equation (F.51), we have:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}. \]

Totally differentiating indirect utility, we have:

\[ du_n = \frac{d w_n}{w_n} \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}} - \sum_{k=1}^{K} \alpha_n^k \frac{d p_n^k}{p_n^k}, \]
\[ = \frac{d w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{d p_n^k}{p_n^k}. \]

Using our total derivative of the sectoral price index in equation (F.60) above, we get:

\[ \frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{d \rho_{nm}^k}{\rho_{nm}^k}, \] (F.63)
\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k d \ln \rho_{nm}^k. \]
F.5.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[ \frac{d \ln z_i^k}{d \ln z_i} = \frac{d \ln z_i}{d \ln z_i}, \quad \forall \ k \in K, \ i \in N, \]
\[ \frac{d \ln \pi_{ni}^k}{d \ln \pi_{ni}} = 0, \quad \forall \ n, i \in N. \]  

(F.64)

We start with our expression for the log change in wages from equation (F.62) above:

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{d \ln w_h - d \ln z_h}{d \ln w_n} - (d \ln w_i - d \ln z_i) \right) \right), \]

Using the total derivative of prices (F.58) and our assumption of constant bilateral trade costs (F.64), we can write this log change in wages as:

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right), \]

which has the same matrix representation as in equation (46) in the paper:

\[ d \ln w = T d \ln w + \theta M (d \ln w - d \ln z). \]  

(F.65)

F.5.8 Utility and Common Productivity Shocks

We start with our expression for the log change in utility in equation (F.63) above:

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k d \ln \rho_{nm}^k, \]

or equivalently:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k d \ln \rho_{nm}^k. \]

Using the total derivative of prices (F.58) and our assumption of constant bilateral trade costs (F.64), we can write this change in utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k (d \ln w_m - d \ln z_m), \]

which has the same matrix representation as in equation (47) in the paper:

\[ d \ln u = d \ln w - S (d \ln w - d \ln z). \]  

(F.66)
F.6 Multiple Sectors and Input-Output Linkages

In this section of the online appendix, we report the derivations for a version of the model with multiple sectors and input-output linkages following Caliendo and Parro (2015), henceforth CP. In particular, we consider a generalization of the multi-sector Armington model in Section F.4 of this online appendix to incorporate input-output linkages, as discussed in Section 4.5 of the paper. The world economy consists of a set of countries indexed by \( i, n \in \{1, \ldots, N\} \) and a set of sectors referenced by \( k \in \{1, \ldots, K\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

F.6.1 Notations

We use \( i, n, o, r \) to index for countries and \( j, k, l \) for industries. We refer to the varieties in industry \( k \) produced in country \( i \) as "goods \( ik \)". We use subscripts to denote countries and superscripts to denote industries. Let \( I_{(NK)} \) denote the identity matrix with dimension \( NK \times NK \).

F.6.2 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[
    u_n = \frac{w_n}{\prod_{k=1}^{K} (p_{ni}^k)^{\alpha_n^k}}, \quad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{F.68}
\]

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

\[
    p_n^k = \left[ \sum_{i=1}^{N} (p_{ni}^k)^{-\theta} \right]^{-\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1. \tag{F.69}
\]

F.6.3 Production Technology

Goods are produced with labor and can be traded subject to iceberg trade costs, such that the cost to a consumer in country \( n \) of purchasing country \( i \)'s variety within sector \( k \) is:

\[
    p_{ni}^k (\omega) = \tau_{ni}^k c_i^k, \quad c_i^k = \left( \frac{w_i}{z_i} \right)^{\gamma_i} \prod_{j=1}^{K} \left( p_{nj}^i \right)^{\gamma_i^{k,j}}, \quad \sum_{k=1}^{K} \gamma_i^{k,j} = 1 - \gamma_i^k, \tag{F.70}
\]

where \( c_i^k \) denotes the unit cost function within that country and sector; \( \gamma_i^k \) is the share of labor in production costs; \( \gamma_i^{k,j} \) is the share of materials from sector \( j \) used in sector \( k \); \( z_i^k \) captures determinants of productivity that are common across all goods within a country \( i \) and sector \( k \); and it proves convenient to define this common component of productivity in value-added terms (such that it augments labor). Iceberg variable trade costs satisfy \( \tau_{ni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \).

F.6.4 Expenditure Shares

Using the properties of CES demand, country \( n \)'s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[
    s_{ni}^k = \left( \frac{p_{ni}^k}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}} \right)^{-\theta}. \tag{F.71}
\]

Totally differentiating this expenditure share equation, we get:

\[
    ds_{ni}^k = - \frac{\theta \frac{dp_{ni}^k}{p_{ni}^k} (p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}} + \sum_{h=1}^{N} \frac{\theta \frac{dp_{nh}^k}{p_{nh}^k} (p_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}. \tag{F.72}
\]
\[ \frac{d s_{n_i}^k}{s_{n_i}^k} = -\theta \frac{dp_{n_i}^k}{p_{n_i}^k} + \sum_{h=1}^{N} \theta \frac{dp_{n_h}^k}{p_{n_h}^k} \left( p_{n_h}^k \right)^{-\theta}, \]
\[ \frac{d s_{n_i}^j}{s_{n_i}^j} = -\theta \frac{dp_{n_i}^j}{p_{n_i}^j} + \sum_{h=1}^{N} \theta \frac{dp_{n_h}^j}{p_{n_h}^j}, \]
\[ \frac{d s_{n_i}^j}{s_{n_i}^j} = \theta \left( \sum_{h=1}^{N} s_{n_h}^j \frac{dp_{n_h}^k}{p_{n_h}^k} - \frac{dp_{n_i}^k}{p_{n_i}^k} \right), \]
so that
\[ d \ln s_{n_i}^k = \theta \left( \sum_{h=1}^{N} s_{n_h}^k \ln p_{n_h}^k - \ln p_{n_i}^k \right), \] (F.72)
where, from equilibrium prices in equation (F.70), we have:
\[ \frac{dp_{n_i}^k}{p_{n_i}^k} = \frac{dp_{n_i}^j}{p_{n_i}^j} + \gamma_i^k \left( \frac{dw_i}{w_i} - \frac{dz_i^k}{z_i^k} \right) + \sum_{j=1}^{K} \gamma_{i,j}^k \frac{dp_{i}^j}{p_{i}^j}, \] (F.73)
\[ d \ln p_{n_i}^k = d \ln s_{n_i}^k + \gamma_i^k \left( d \ln w_i - d \ln z_i^k \right) + \sum_{j=1}^{K} \gamma_{i,j}^k \ln p_{i}^j. \]

F.6.5 Price Indices

Totally differentiating the sectoral price index (F.69), we have:
\[ dp_{n_i}^k = \sum_{m=1}^{N} \gamma_k^m \frac{dp_{nm}^k}{p_{nm}^k} \left( p_{nm}^k \right)^{-\theta} \left[ \sum_{h=1}^{N} \left( p_{nm}^h \right)^{-\theta} \right]^{-\frac{1}{\theta}}, \]
\[ dp_{n_i}^k = \sum_{m=1}^{N} \frac{dp_{nm}^k}{p_{nm}^k} \left( p_{nm}^k \right)^{-\theta} \sum_{h=1}^{N} \left( p_{nm}^h \right)^{-\theta}, \]
\[ \frac{dp_{n_i}^k}{p_{n_i}^k} = \sum_{m=1}^{N} \frac{dp_{nm}^k}{p_{nm}^k}, \] (F.74)
\[ d \ln p_{n_i}^k = \sum_{m=1}^{N} s_{nm}^k \ln p_{nm}^k. \]

F.6.6 Labor Market Clearing

The labor market clearing condition is:
\[ w_n L_n = \sum_{j=1}^{K} \gamma_n^j y_n^j, \] (F.75)
where \( y_n^j \) is total sales by country \( n \)'s industry \( j \). Totally differentiating this labor market clearing condition, holding endowments constant, we have:
\[ \frac{dw_n}{w_n} w_n L_n = \sum_{j=1}^{K} \gamma_n^j \frac{dy_n^j}{y_n^j} y_n^j, \]
\[ \frac{dw_n}{w_n} = \sum_{j=1}^{K} \left( \gamma_n^j \frac{y_n^j}{w_n L_n} \right) \frac{dy_n^j}{y_n^j}, \]
\[ \frac{dw_n}{w_n} = \sum_{j=1}^{K} \xi_n^j \frac{dy_n^j}{y_n^j}, \] (F.76)
where \( \xi_n^j \) is the share of sector \( j \) in country \( n \)'s total income:
\[ \xi_n^j = \frac{\gamma_n^j y_n^j}{w_n L_n}. \]
F.6.7 Goods Market Clearing

Goods market clearing requires that income in each location and sector equals expenditure on goods produced in that location and sector:

\[ y^k_i = \sum_{n=1}^{N} s^{k}_{ni} k^k, \quad \text{(F.77)} \]

where expenditure in location \( n \) in sector \( k \) is:

\[ x^k_n = \alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma^j_n y^j_n, \quad \text{(F.78)} \]

and recall that \( \gamma^j_n \) is the share of materials from sector \( k \) used in sector \( j \). Combining these two relationships and the labor market clearing \( (\text{F.75}) \), we obtain the following market clearing condition:

\[
y^k_i = \sum_{n=1}^{N} s^{k}_{ni} \left[ \alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma^j_n y^j_n \right],
\]

\[
= \sum_{n=1}^{N} s^{k}_{ni} \left[ \alpha_n^k \sum_{j=1}^{K} \gamma^j_n y^j_n + \sum_{j=1}^{K} \gamma^k_j y^j_n \right],
\]

\[
= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^{k} \left[ \alpha_n^k \gamma^j_n + \gamma^k_j \right] y^j_n.
\]

Totally differentiating this market clearing condition, we have:

\[
\frac{d y^k_i}{y^k_i} = \sum_{n=1}^{N} \sum_{j=1}^{K} \frac{d s^{k}_{ni}}{s^{k}_{ni}} \left[ \frac{\alpha_n^k \gamma^j_n + \gamma^k_j}{y^j_n} \right] y^j_n + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^{k} \left[ \alpha_n^k \gamma^j_n + \gamma^k_j \right] \frac{dy^j_n}{y^j_n}.
\]

Let \( \vartheta^k_{in} \) denote the fraction of \( ik \)'s revenue derived from selling to consumers in country \( n \); \( \Theta \) denote an \( NK \times NK \) matrix with entries \( \Theta_{in}^{kj} \) capturing the fraction of \( ik \)'s revenue derived from selling to producers in country \( n \) industry \( j \); and \( \Delta \) denote the Leontief-inverse of \( \Theta \), such that \( \Delta = (I_{NK} - \Theta)^{-1} \), with the \((ik, nj)\)-th entry, \( \Delta_{in}^{kj} \), capturing the network-adjusted fraction of \( ik \)'s revenue derived from market \( nj \), either directly or indirectly through customers of customers, ad infinitum. The above market clearing can be re-written as

\[
d \ln y^k_i = \sum_{n=1}^{N} \frac{d s^{k}_{ni}}{s^{k}_{ni}} \alpha_n^k \sum_{j=1}^{K} \gamma^j_n \frac{y^j_n}{y^k_i} + \sum_{n=1}^{N} \frac{d s^{k}_{ni}}{s^{k}_{ni}} \gamma^k_j \frac{y^j_n}{y^k_i} y^j_n d \ln y^j_n + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^{k} \left[ \alpha_n^k \gamma^j_n + \gamma^k_j \right] \frac{dy^j_n}{y^j_n}.
\]

Let \( \vartheta^k_{in} \) denote the fraction of \( ik \)'s revenue derived from selling to consumers in country \( n \); \( \Theta \) denote an \( NK \times NK \) matrix with entries \( \Theta_{in}^{kj} \) capturing the fraction of \( ik \)'s revenue derived from selling to producers in country \( n \) industry \( j \); and \( \Delta \) denote the Leontief-inverse of \( \Theta \), such that \( \Delta = (I_{NK} - \Theta)^{-1} \), with the \((ik, nj)\)-th entry, \( \Delta_{in}^{kj} \), capturing the network-adjusted fraction of \( ik \)'s revenue derived from market \( nj \), either directly or indirectly through customers of customers, ad infinitum. The above market clearing can be re-written as

\[
d \ln y^k_i = \sum_{n=1}^{N} \frac{d s^{k}_{ni}}{s^{k}_{ni}} \alpha_n^k \sum_{j=1}^{K} \gamma^j_n \frac{y^j_n}{y^k_i} + \sum_{n=1}^{N} \frac{d s^{k}_{ni}}{s^{k}_{ni}} \gamma^k_j \frac{y^j_n}{y^k_i} y^j_n d \ln y^j_n + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^{k} \left[ \alpha_n^k \gamma^j_n + \gamma^k_j \right] \frac{dy^j_n}{y^j_n}.
\]
where in the second equality we used equations (F.75) and (F.76). Subtracting the latest term on the right hand side from both sides of the equations and taking the Leontief-inverse of $\Theta_{nk}^{ik}$, we obtain:

$$
\begin{align*}
    \frac{d \ln y_i}{y_i} &= \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left[ \sum_{n=1}^{N} \vartheta_{on}^{l} \frac{d \ln w_n}{w_n} + \sum_{n=1}^{N} \left( \vartheta_{on}^{l} + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \frac{d \ln s_{no}^{l}}{s_{no}^{l}} \right]. 
\end{align*}
$$

Combining this result with (F.76), we get

$$
\begin{align*}
    \frac{d \ln w_i}{w_i} &= \sum_{k} \xi_k^{i} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left[ \sum_{n=1}^{N} \vartheta_{on}^{l} \frac{d \ln w_n}{w_n} + \sum_{n=1}^{N} \left( \vartheta_{on}^{l} + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \frac{d \ln s_{no}^{l}}{s_{no}^{l}} \right].
\end{align*}
$$

**F.6.8 Wages and Common Productivity Shocks**

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$
\begin{align*}
    \frac{d \ln z_{ik}}{z_{ik}} &= \frac{d \ln z_i}{z_i}, \quad \forall k \in K, \ i \in N, \\
    \frac{d \ln x_{mi}}{x_{mi}} &= 0, \quad \forall n, i \in N.
\end{align*}
$$

We start with our expression for the change in prices above:

$$
\begin{align*}
    \frac{d \ln p_{ni}^{k}}{p_{ni}^{k}} &= \frac{d \ln x_{mi}^{k}}{x_{mi}^{k}} + \frac{\gamma_{i}^{k}}{\gamma_{i}} \left( \frac{d \ln w_i}{w_i} - \frac{d \ln z_i^{k}}{z_i^{k}} \right) + \sum_{j=1}^{K} \gamma_{i}^{k,j} \frac{d \ln p_{i}^{j}}{p_{i}^{j}}, \\
    &= \frac{\gamma_{i}^{k}}{\gamma_{i}} \left( \frac{d \ln w_i}{w_i} - \frac{d \ln z_i}{z_i} \right) + \sum_{j=1}^{K} \gamma_{i}^{k,j} \frac{d \ln p_{i}^{j}}{p_{i}^{j}}.
\end{align*}
$$

Using our result for the total derivative of price indices (F.74), we can rewrite this expression for the change in prices as:

$$
\begin{align*}
    \frac{d \ln p_{ni}^{k}}{p_{ni}^{k}} &= \frac{\gamma_{i}^{k}}{\gamma_{i}} \left( \frac{d \ln w_i}{w_i} - \frac{d \ln z_i}{z_i} \right) + \sum_{j=1}^{K} \gamma_{i}^{k,j} \sum_{m=1}^{N} s_{im}^{j} \frac{d \ln p_{im}^{j}}{p_{im}^{j}}.
\end{align*}
$$

We use $\Sigma_{nm}^{j,k} = \gamma_{i}^{k,j} s_{im}^{j}$ to denote expenditure in country $i$ and sector $k$ on the goods produced by country $m$ and sector $j$ as a share of revenue in country $i$ and sector $k$. Using this notation, we can rewrite the above expression for the change in prices as:

$$
\begin{align*}
    \frac{d \ln p_{ni}^{k}}{p_{ni}^{k}} &= \frac{\gamma_{i}^{k}}{\gamma_{i}} \left( \frac{d \ln w_i}{w_i} - \frac{d \ln z_i}{z_i} \right) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma_{nm}^{j,k} \frac{d \ln p_{im}^{j}}{p_{im}^{j}}.
\end{align*}
$$

Let $\Sigma$ denote the $NK \times NK$ matrix with entries $\Sigma_{nm}^{j,k}$ capturing the input cost share (relative to revenue) on goods $km$ by producer $ij$. Let us also define the Leontief inverse $\Gamma \equiv (I_{NK} - \Sigma)^{-1}$, with the $(nj, ik)$-th entry, $\Gamma_{nj}^{ik}$, capturing the network-adjusted share of $nj$’s revenue spent on inputs $ik$, either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Finally, let $\lambda_{nj}^{i} \equiv \sum_{k=1}^{K} \lambda_{ii}^{k} \Gamma_{nj}^{ik}$ denote the network-adjusted input cost share of $nj$’s revenue on value-added (labor) in country $i$; note that $\sum_{i=1}^{N} \lambda_{nj}^{i} = 1$ for all $nj$ due to constant returns to scale. Equation (F.81) can be re-written as:

$$
\begin{align*}
    \frac{d \ln p_{ni}^{k}}{p_{ni}^{k}} &= \sum_{i=1}^{N} \lambda_{ii}^{k} \left( \frac{d \ln w_i}{w_i} - \frac{d \ln z_i}{z_i} \right). 
\end{align*}
$$

We can now use (F.82) to re-write the linearized expenditure shares from equation (F.72) as:
\[
\begin{align*}
\text{d ln } s_{ni}^k &= \theta \left( \sum_{h=1}^{N} s_{nh}^k \sum_{r=1}^{N} \Lambda_{hr}^k \left( \text{d ln } w_r - \text{d ln } z_r \right) - \sum_{r} \Lambda_{ir}^k \left( \text{d ln } w_r - \text{d ln } z_r \right) \right), \\
&= \theta \sum_{r=1}^{N} \left( \sum_{h=1}^{N} s_{nh}^k \Lambda_{hr}^k - \Lambda_{ir}^k \right) \left( \text{d ln } w_r - \text{d ln } z_r \right).
\end{align*}
\]

Substitute this into (F.80), we get

\[
\begin{align*}
\text{d ln } w_i &= \sum_{k=1}^{K} \zeta_i^k \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta^{kl}_{io} \sum_{n=1}^{N} \varphi_{on}^l \text{d ln } w_n \\
&\quad + \sum_{k=1}^{K} \zeta_i^k \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta^{kl}_{io} \theta \sum_{n=1}^{N} \left( \varphi_{on}^l + \sum_{j=1}^{K} \Theta_{oj}^l \right) \sum_{r} \left( \sum_{h=1}^{N} s_{nh}^l \Lambda_{hr}^l - \Lambda_{ir}^l \right) \left( \text{d ln } w_r - \text{d ln } z_r \right). \\
&\equiv \text{T}_{in}
\end{align*}
\]

To simplify notation, let us now define \( \Pi_{io}^l \equiv \sum_{k=1}^{K} \zeta_i^k \Delta^{kl}_{io} \) to be the network-adjusted share of income in country \( i \) derived from selling to country \( o \) industry \( l \). Also denote \( \Upsilon_{nor}^l = \sum_{h=1}^{N} s_{nh}^l \Lambda_{hr}^l - \Lambda_{ir}^l \), \( \theta \Upsilon_{nor}^l \) is the elasticity of \( n \)'s expenditure on goods \( lo \) with respect to \( r \)'s factor cost. Then the above expression can be re-written as:

\[
\begin{align*}
\text{d ln } w_i &= \sum_{n=1}^{N} \left( \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^l \varphi_{on}^l \right) \text{d ln } w_n \\
&\quad + \sum_{n=1}^{N} \left( \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^l \left( \varphi_{on}^l + \sum_{j=1}^{K} \Theta_{oj}^l \right) \Upsilon_{ron}^l \right) \left( \text{d ln } w_n - \text{d ln } z_n \right) \\
&\equiv \text{M}_{in}
\end{align*}
\]

Finally, we define \( T \) as an \( N \times N \) matrix with entries \( T_{in} = \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^l \varphi_{on}^l \). Element \( T_{in} \) captures the network-adjusted share of \( i \)'s income derived from selling to consumers in country \( n \); it sums across the network-adjusted income share that country \( i \) derives from selling to country-industry \( ol \), times the revenue share that \( ol \) derives from selling to consumers in country \( n \). We define \( M \) as an \( N \times N \) matrix with entries \( M_{in} \):

\[
M_{in} = \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^l \left( \varphi_{on}^l + \sum_{j=1}^{K} \Theta_{oj}^l \right) \Upsilon_{ron}^l.
\]

To interpret, \( \theta \Upsilon_{ron}^l \) captures how \( r \)'s expenditure on goods \( ol \) responds to factor cost in \( n \). Element \( M_{in} \) sums the cross-substitution effects across \( i \)'s exposure to all markets through network linkages; \( \Pi_{io}^l \) is \( i \)'s network-adjusted income share derived from selling to producers in country \( o \) industry \( l \); goods \( ol \) are then exposed to substitution due to changes in \( n \)'s factor costs through markets that \( ol \) supplies to, including consumers \( (\varphi_{on}^l) \) and producers \( \left( \sum_{j=1}^{K} \Theta_{oj}^l \right) \) in all countries \( r \).

We have thus obtained the same matrix representation as in the paper:

\[
\text{d ln } w = T \text{ d ln } w + \theta M \left( \text{d ln } w - \text{d ln } z \right).
\]

We again solve for our friend-enemy income exposure measure by matrix inversion:

\[
\text{d ln } w = W \text{ d ln } z,
\]
\[ W \equiv -\frac{\theta}{\theta + 1} (I - V)^{-1} M \]  \hspace{1cm} (F.87)

and, using our choice of world GDP as numeraire, which implies \( Q d \ln w = 0, \)

\[ V \equiv \frac{T + \theta TS}{\theta + 1} - Q. \]

### F.6.9 Welfare

Returning to our expression for indirect utility (F.68), we have:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}. \]

Totally differentiating this expression for indirect utility, we have:

\[ d u_n = \frac{d w_n}{w_n} - \sum_{k=1}^{K} \frac{\alpha_n^k p_n^k}{T_n^k} \frac{d p_n^k}{p_n^k}, \]

\[ \frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k s_n^k \frac{d p_n^k}{p_n^k}. \]

Using our total derivative of the sectoral price index above, we get:

\[ \frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{k=1}^{K} \sum_{m=1}^{N} s_n^k s_{nm} \frac{dp_n^k}{p_n^k} \]

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \sum_{m=1}^{N} s_n^k s_{nm} \frac{dp_n^k}{p_n^k}. \]

Plugging (F.82) into the above, we get

\[ d \ln u_n = d \ln w_n - \sum_{i=1}^{N} \left( \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm} \Lambda_{im}^k \right) \left( d \ln w_i - d \ln z_i \right) \]

with \( \Lambda_{im}^k \) defined as:

We define \( S \) as an \( N \times N \) matrix with entries \( S_{ni} \equiv \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm} \Lambda_{im}^k \). Element \( S_{ni} \) captures the network-adjusted expenditure share of consumer \( n \) on value-added by country \( i \); it sums across the expenditure share of consumer \( n \) on goods \( mk \), times the network-adjusted input cost share of \( mk \) on factor \( i \), captured by \( \Lambda_{im}^k \). We have thus obtained the same matrix representation for welfare as in the paper:

\[ d \ln u = d \ln w - S (d \ln w - d \ln z). \]  \hspace{1cm} (F.89)

We can re-write the above relationship as:

\[ d \ln u = (I - S) d \ln w + S d \ln z, \]

which, using our solution for \( d \ln w \) from equation (F.86), can be further re-written as:

\[ d \ln u = (I - S) W d \ln z + S d \ln z, \]

\[ = [(I - S) W + S] d \ln z, \]

\[ = U d \ln z, \]
where \( U \) is our friend-enemy welfare exposure measure:

\[
U \equiv [(I - S) W + S].
\] (F.90)

### F.6.10 Industry-Level Sales Exposure

Similarly to the multi-sector model without input linkages, our approach also yields bilateral friend-enemy measures of sales exposure to global productivity shocks for each sector. From equations (F.79) and (F.83) we obtain the following expression for changes in the sales of industry \( k \) in country \( i \):

\[
d \ln y_i^k = \sum_{n=1}^{N} \left( \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \theta_{io}^l \right) d \ln w_n,
\]

\[
+ \theta \sum_{n=1}^{N} \left( \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left( \theta_{or}^l + \sum_{j=1}^{K} \Theta_{or}^{ij} \right) \gamma_{ron}^l \right) (d \ln w_n - d \ln z_n).
\]

We define \( T^k \) as an \( N \times N \) matrix with entries \( T_{in}^k \). Element \( T_{in}^k \) captures the network-adjusted share of sector \( ik \)'s income derived from selling to consumers in country \( n \); it sums across the network-adjusted income share that sector \( ik \) derives from selling to country-industry \( ol \), times the revenue share that \( ol \) derives from selling to consumers in country \( n \). We define \( M^k \) as an \( N \times N \) matrix with entries \( M_{in}^k \):

\[
M_{in}^k \equiv \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left( \theta_{or}^l + \sum_{j=1}^{K} \Theta_{or}^{ij} \right) \gamma_{ron}^l.
\]

To interpret, \( \theta \gamma_{ron}^l \) captures how \( r \)'s expenditure on goods \( ol \) responds to factor cost in \( n \). Element \( M_{in}^k \) sums the cross-substitution effects across sector \( ik \)'s exposure to all markets through network linkages: \( \Delta_{io}^{kl} \) is \( ik \)'s network-adjusted income share derived from selling to producers in country \( o \) industry \( l \); goods \( ol \) are then exposed to substitution due to changes in \( n \)'s factor costs through markets that \( ol \) supplies to.

We get the following matrix representation:

\[
d \ln Y^k = T^k d \ln W + \theta M^k (d \ln W - d \ln z),
\] (F.91)

\[
= [T^k W + \theta M^k (W - I)] d \ln z,
\]

which can be re-written as:

\[
d \ln Y^k = W^k d \ln z,
\] (F.92)

\[
W^k \equiv T^k W + \theta M^k (W - I),
\] (F.93)

which corresponds to our friends-and-enemies measure of sector sales exposure to productivity shocks. Note that \( W^k \) also captures sector value-added exposure to productivity shocks, as value added is a constant share of revenues in this model. Finally, recall from equation (F.76) that

\[
d \ln w_n = \sum_{j=1}^{K} \xi_{nj}^k d \ln y_n^k.
\]
where $\zeta^k_n$ is the value-added share of industry $k$ in country $n$’s total income. Together, this relationship and equations (F.86) and (F.92) imply that:

$$W_i \, d \ln z = \sum_{k=1}^K \zeta^k_n W^k_i \, d \ln z,$$

where $W_i$ is the income exposure vector for country $i$ with respect to productivity shocks in its trade partners and $W^k_i$ is the sector value-added exposure vector for country $i$ and sector $k$ with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure ($W_i$) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure ($W^k_i$):

$$W_i = \sum_{k=1}^K \zeta^k_n W^k_i.$$

Therefore, we can decompose how much of our aggregate income exposure measure is driven by the value-added exposure of particular industries, and how much of that value-added exposure of particular industries is explained by various terms (market-size, cross-substitution, etc. within the industry).

F.6.11 Isomorphisms

Although for expositional convenience we focus on an extension of the constant elasticity Armington model with multiple sectors and input-output linkages, the same results hold in an extension of the Eaton and Kortum (2002) model to incorporate both multiple sectors following Costinot, Donaldson and Komunjer (2012) and also input-output linkages following Caliendo and Parro (2015).

F.7 Economic Geography

In this section of the online appendix, show that our approach also generalizes to models of economic geography, in which goods and factors are mobile across locations, as in Allen and Arkolakis (2014), Ramondo, Rodriguez-Clare and Saborio-Rodriguez (2016), Redding and Sturm (2008) and Redding (2016), as discussed in Section 4.6 of the paper. The world economy consists of a set of locations indexed by $i, n \in \{1, \ldots, N\}$. The economy as a whole has an exogenous supply of $\bar{\ell}$ workers, who are each endowed with one unit of labor that is supplied inelastically. Workers are perfectly mobile across locations, but have idiosyncratic preferences for each location.

F.7.1 Consumer Preferences

The preferences of worker $\nu$ who chooses to live in location $n$ are characterized by the following indirect utility function:

$$u_n(\nu) = \frac{b_n \epsilon_n(\nu) w_n}{p_n}, \quad (F.94)$$

where $w_n$ is the wage, $p_n$ is the consumption goods price index; $b_n$ captures amenities that are common for all workers (such as climate and scenic views); and $\epsilon_n(\nu)$ is an idiosyncratic amenity draw that is specific to each worker $\nu$ and location $n$. The consumption goods price index is modeled as in our baseline constant elasticity Armington model in Section 3 of the paper and takes the following form:

$$p_n = \left[ \sum_{i=1}^N p_{n1}^{1-\sigma} \right] ^{\frac{1}{\sigma}}, \quad \sigma > 1. \quad (F.95)$$

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Idiosyncratic amenities are drawn independently for each worker and location from the following independent Fréchet distribution:

\[ F_\varepsilon (\varepsilon) = \exp \left( -\varepsilon^{-\kappa} \right), \quad \kappa > 1, \]  

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \( b_n \); and \( \kappa > 1 \) controls the dispersion of idiosyncratic amenities.

### F.7.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between locations subject to iceberg variable costs of trade, such that \( \tau_{ni} \geq 1 \) units must be shipped from location \( i \) to location \( n \) in order for one unit to arrive (where \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \)). Therefore, the cost to the consumer in location \( n \) of purchasing the good produced by location \( i \) is:

\[ p_{ni} = \frac{\tau_{ni} w_i}{z_i}, \]  

where \( z_i \) captures productivity in location \( i \) and iceberg variable trade costs satisfy \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \).

### F.7.3 Expenditure Shares

Using the properties of CES demand, location \( n \)’s share of expenditure on goods produced in location \( i \) is:

\[ s_{ni} = \frac{\left( \tau_{ni} w_i / z_i \right)^{1-\sigma}}{\sum_{m=1}^{N} \left( \tau_{nm} w_m / z_m \right)^{1-\sigma}}. \]  

Totally differentiating this expenditure share equation we get:

\[ \frac{ds_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right), \]  

\[ d \ln s_{ni} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d \ln p_{nh} - d \ln p_{ni} \right), \]

where from the expression for equilibrium prices (F.97) above, we have:

\[ \frac{dp_{ni}}{p_{ni}} = \frac{d \tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i}, \]

\[ d \ln p_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i. \]

### F.7.4 Price Indices

Totally differentiating the consumption goods price index (F.95), we have:

\[ \frac{dp_n}{p_n} = \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}}, \]  

\[ d \ln p_n = \sum_{m=1}^{N} s_{nm} d \ln p_{nm}. \]
F.7.5 Location Choice Probabilities

Using the properties of the Fréchet distribution (F.96), the probability that a worker chooses to live in location \( n \) is:

\[
\xi_n = \frac{\ell_n}{\ell} = \left( \frac{b_n w_n / p_n}{\sum_{h=1}^{N} (b_h w_h / p_h)} \right)^{\kappa},
\]

and expected utility conditional on choosing to live in a location is the same across all locations and given by:

\[
\mathbb{E}[u] = \bar{u} = \Gamma \left( \frac{\kappa - 1}{\kappa} \right) \left[ \sum_{h=1}^{N} (b_h w_h / p_h)^{\kappa} \right]^{\frac{1}{\kappa}}.
\]

Totally differentiating the location choice probabilities, we have:

\[
\frac{d\xi_n}{\xi_n} = \kappa \left( \frac{db_n}{b_n} + \frac{dw_n}{w_n} - \frac{dp_n}{p_n} \right) - \kappa \sum_{h=1}^{N} \xi_h \left( \frac{db_h}{b_h} + \frac{dw_h}{w_h} - \frac{dp_h}{p_h} \right).
\]

Using the total derivative of the consumption goods price index (F.101), we can rewrite this total derivative of the location choice probabilities as:

\[
\frac{d\xi_n}{\xi_n} = \kappa \left( \frac{db_n}{b_n} + \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}} \right) - \kappa \sum_{h=1}^{N} \xi_h \left( \frac{db_h}{b_h} + \frac{dw_h}{w_h} - \sum_{m=1}^{N} s_{hm} \frac{dp_{hm}}{p_{hm}} \right),
\]

which can be further rewritten as:

\[
d \ln \xi_n = \left[ \kappa \left( d \ln b_n + d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln p_{nm} \right) \right] - \kappa \sum_{h=1}^{N} \xi_h \left( d \ln b_h + d \ln w_h - \sum_{m=1}^{N} s_{hm} d \ln p_{hm} \right).
\]

Totally differentiating expected utility, we have:

\[
\frac{d\bar{u}}{\bar{u}} = \sum_{h=1}^{N} \xi_h \left( \frac{db_h}{b_h} + \frac{dw_h}{w_h} - \frac{dp_h}{p_h} \right).
\]

Using the total derivative of the consumption goods price index (F.101), we can rewrite this total derivative of expected utility as:

\[
\frac{d\bar{u}}{\bar{u}} = \sum_{h=1}^{N} \xi_h \left( \frac{db_h}{b_h} + \frac{dw_h}{w_h} - \sum_{m=1}^{N} s_{hm} \frac{dp_{hm}}{p_{hm}} \right),
\]

which equivalently can be written as:

\[
d \ln \bar{u} = \sum_{h=1}^{N} \xi_h \left( d \ln b_h + d \ln w_h - \sum_{m=1}^{N} s_{hm} d \ln p_{hm} \right).
\]

F.7.6 Market Clearing

Market clearing requires that income in each location equals expenditure on goods produced in that location:

\[
w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n,
\]

which can be re-written as:

\[
w_i \xi_i = \sum_{n=1}^{N} s_{ni} w_n \xi_n.
\]
Totally differentiating this market clearing condition, we have:

\[
\frac{dw_i}{w_i} w_i \xi_i + \frac{d\xi_i}{\xi_i} w_i \xi_i = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_i n \xi_n + \sum_{n=1}^{N} \frac{d\xi_n}{\xi_n} s_{ni} w_i n \xi_n + \sum_{n=1}^{N} \frac{d\xi_n}{\xi_n} s_{ni} w_i n \xi_n ,
\]

Using our total derivative of expenditure shares (F.99), this becomes:

\[
\frac{dw_i}{w_i} w_i \xi_i + \frac{d\xi_i}{\xi_i} w_i \xi_i = \sum_{n=1}^{N} s_{ni} w_i n \xi_n \left( \frac{dw_n}{w_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) + \frac{d\xi_n}{\xi_n} \right) ,
\]

\[
\frac{dw_i}{w_i} + \frac{d\xi_i}{\xi_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \frac{d\xi_n}{\xi_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right) ,
\]

where we have defined \( t_{in} \) as the share of location \( i \)'s income from market \( n \):

\[
t_{in} = \frac{s_{ni} w_i n L_n}{w_i L_i} ,
\]

and equivalently we can write this expression as:

\[
d \ln w_i + d \ln \xi_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + d \ln \xi_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d \ln p_{nh} - d \ln p_{ni} \right) \right) . \tag{F.108}
\]

### F.7.7 Productivity Shocks

We consider small productivity shocks, holding constant amenities and bilateral trade costs:

\[
d \ln b_i = 0 , \quad \forall i \in N ,
\]

\[
d \ln \tau_{ni} = 0 , \quad \forall n, i \in N . \tag{F.109}
\]

Using this assumption and the total derivative of prices (F.100) in the total derivative of the choice probabilities (F.105), we obtain:

\[
d \ln \xi_n = \kappa \left( d \ln w_n - \sum_{m=1}^{N} s_{nm} \left( d \ln w_m - d \ln z_m \right) \right) - \kappa \sum_{h=1}^{N} \xi_h \left( d \ln w_h - \sum_{m=1}^{N} s_{hm} \left( d \ln w_m - d \ln z_m \right) \right) ,
\]

which can be rewritten in matrix form as:

\[
d \ln \xi = \kappa (I - \Xi) \left[ d \ln w - S \left( d \ln w - d \ln z \right) \right] , \tag{F.110}
\]

where the term inside the square parentheses is the change in the real wage change in each location and the matrix \( \Xi \) captures the population share of each location:

\[
\Xi = \begin{pmatrix}
\xi_1 & \xi_2 & \cdots & \xi_N \\
\xi_1 & \xi_2 & \cdots & \xi_N \\
\vdots & \vdots & \ddots & \vdots \\
\xi_1 & \xi_2 & \cdots & \xi_N
\end{pmatrix} .
\]

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Using our assumption of no changes in amenities or bilateral trade costs in equation (F.109) and the market clearing condition (F.108), the impact of the productivity shock on income in equation (F.108) can be re-written as:

$$d \ln w_i + d \ln \xi_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + d \ln \xi_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \left( d \ln w_n - d \ln z_h \right) - \left( d \ln w_i - d \ln z_i \right) \right) \right),$$

which can be rewritten in matrix form as:

$$d \ln w + d \ln \xi = T \left( d \ln w + d \ln \xi \right) + (\sigma - 1) \left( TS - I \right) \left( d \ln w - d \ln z \right). \quad (F.111)$$

Substituting for the change in population share ($d \ln \xi$) using equation (F.110), we obtain:

$$\begin{bmatrix}
(1 + \kappa) d \ln w \\
-\kappa S (d \ln w - d \ln z)
\end{bmatrix}
= \begin{bmatrix}
T [(1 + \kappa) d \ln w - \kappa S (d \ln w - d \ln z)] \\
+ (\sigma - 1) (TS - I) (d \ln w - d \ln z)
\end{bmatrix},$$

which can be rewritten as:

$$(1 + \kappa) d \ln w = (1 + \kappa) T d \ln w + \left[ ((\sigma - 1) - \kappa) TS - (\sigma - 1) I + \kappa S \right] (d \ln w - d \ln z),$$

and hence the impact of the productivity shock on wages is:

$$d \ln w = T d \ln w + \left[ \left( \frac{\sigma - 1}{1 + \kappa} \right) TS - \left( \frac{\sigma - 1}{1 + \kappa} \right) I + \frac{\kappa}{1 + \kappa} S \right] (d \ln w - d \ln z).$$

Using our assumed productivity shock (F.109), the total derivative of prices (F.100), and the total derivative of expected utility (F.106), the impact of the productivity shock on welfare is:

$$d \ln \bar{u} = \sum_{h=1}^{N} \xi_h \left[ d \ln w_h - \sum_{m=1}^{N} s_{hm} (d \ln w_m - d \ln z_m) \right],$$

which can be written in matrix form as:

$$d \ln \bar{u} = \xi' \left[ d \ln w - S (d \ln w - d \ln z) \right], \quad (F.112)$$

where the term inside the square parentheses is the change in the real wage in each location.

## G Additional Empirical Results

In this section of the online appendix, we report additional empirical results, as discussed in the paper.

### G.1 Economic Friends and Enemies

We begin by reporting additional empirical results for country income and welfare exposure to productivity and trade cost shocks, as discussed in Section 5 of the paper.

#### G.1.1 Quality of the Approximation

In this section, we report further empirical evidence on the quality of our approximation to the full non-linear solution of the model, as discussed in Section 5.2 of the paper.
Empirical Distribution of Productivity and Trade Cost Shocks  To compare our linearization with exact-hat algebra for empirically-reasonable shocks, we recover the empirical distribution of productivity and trade cost shocks that rationalize the observed trade data within our baseline single-sector constant elasticity Armington model from Section D of this online appendix. Changes in trade costs and productivity are only separately identified up to a normalization or choice of units, because an increase in a country’s productivity is isomorphic to a reduction in its trade costs with all partners (including itself). We use the normalization that there are no changes in own trade costs over time ($\hat{\tau}_{nnt} = 1 \ \forall n, t$), which absorbs common unobserved changes in trade costs across all partners into changes in productivity. But our results are not sensitive to the way in which we recover productivity shocks, as explored in the Monte Carlo simulations discussed in the paper and further below.

Using this normalization, we estimate time-varying bilateral trade cost shocks following Head and Ries (2001). Specifically, let $x_{nit}$ denote the expenditure by country $n$ on goods from country $i$ in year $t$, then

$$\frac{x_{nit}}{x_{nnt}} = \frac{w_{it}^{-\theta} z_{it}^{\theta} (\tau_{nit})^{-\theta}}{w_{nt}^{-\theta} z_{nt}^{\theta} (\tau_{nnt})^{-\theta}},$$

which implies

$$\left( \frac{x_{nit} x_{int}}{x_{nnt} x_{int}} \right)^{\frac{1}{2}} = \left( \frac{\tau_{nit} \tau_{int}}{\tau_{nnt} \tau_{int}} \right)^{-\frac{\theta}{2}}.$$  \hspace{1cm} (G.1)

Denoting by $\hat{x}$ relative changes in variable $x$ across periods and using our normalization that $\hat{\tau}_{nnt} = \hat{\tau}_{int} = 1$,

$$\left( \frac{\hat{x}_{nit} \hat{x}_{int}}{\hat{x}_{nnt} \hat{x}_{int}} \right)^{\frac{1}{2}} = \left( \hat{\tau}_{nit} \hat{\tau}_{int} \right)^{-\frac{\theta}{2}}.$$  \hspace{1cm} (G.2)

Assuming a standard value for the trade elasticity of $\theta = 5$ and that bilateral trade cost shocks are symmetric ($\hat{\tau}_{nit} = \hat{\tau}_{int}$), we can recover all bilateral relative changes in trade costs from the bilateral trade data.

Using the market clearing condition that equates a country’s income with expenditure on its goods, and again assuming a standard value for the trade elasticity of $\theta = 5$, we estimate the changes in productivity in each country ($\hat{z}_{it}$) that exactly rationalize the observed changes in per capita incomes ($\hat{w}_{it}$) and populations ($\hat{\ell}_{it}$), given our estimated changes in bilateral trade costs ($\hat{\tau}_{nnt}$):

$$\hat{w}_{it} \hat{\ell}_{it} w_{it} \ell_{it} = \sum_{n=1}^{N} \sum_{t=1}^{T} s_{nit} \hat{\tau}_{nit}^{-\theta} \hat{w}_{it}^{-\theta} z_{it}^{\theta} \hat{w}_{nt} \hat{\ell}_{nt} w_{nt} \ell_{nt},$$  \hspace{1cm} (G.3)

where any omitted changes in trade costs for an importer $n$ that are common across exporters cancel from the numerator and denominator of the fraction on the right-hand side; any omitted changes in trade costs for an exporter $i$ that are common across importers are implicitly absorbed into the estimated changes in productivities ($\hat{z}_{it}$). Note that the fraction on the right-hand side of equation (G.3) corresponds to an expenditure share and is homogenous of degree zero in these changes in productivities ($\hat{z}_{it}$). Therefore, these changes in productivities ($\hat{z}_{it}$) only can be recovered up to a normalization or choice of units, and we use the normalization that the mean of the log changes in productivities across all countries is equal to zero (a geometric mean of changes in productivities of one).

Having recovered these changes in productivities ($\hat{z}_{it}$) implied by the observed data, we compare the predictions from our (first-order) linearization for the impact of productivity shocks on income to those from the full non-linear solution of the model using the exact-hat algebra approach. In particular, we undertake exact-hat algebra counterfactuals, in which we solve for the counterfactual change in per capita income ($\hat{w}_{it}$) in response to the changes in
productivity alone ($\tilde{z}_{it}$), holding trade costs and populations constant:

$$\hat{w}_{it} = \sum_{n=1}^{N} S_{nit} \frac{\hat{w}_{it} - \tilde{z}_{it}}{\tilde{z}_{it}} = \hat{w}_{nt} \ell_{nt}. \quad (G.2)$$

We compare the results from these exact-hat algebra counterfactuals to the predictions of our linearization, which implies a log change in countries’ per capita incomes in response to these productivity shocks of $\ln \hat{w} = W \ln \tilde{z}$. We also undertake an analogous exercise in which we compare the predictions of our linearization for changes in bilateral trade costs to the counterfactual predictions from exact-hat algebra, as discussed further below.

**Quality of the Approximation for Productivity Shocks** In Section 5.2 of the paper, we report Monte Carlo simulations for our baseline trade elasticity of $\theta = 5$, in which we draw (with replacement) productivity shocks for each country from the empirical distribution of productivity shocks from 2000-2010. Using these simulated productivity shocks, we undertake exact-hat algebra counterfactuals to compute predicted log changes in per capita income, and compare these predictions with those from our linearization. In Figure 3 in the paper, we show that we find slope coefficients from 0.99-1.01 and a correlation coefficient of above 0.999. In Figures G.1 and G.2 below, we explore the robustness of these results to alternative parameter values, by considering trade elasticities of $\theta = 2$ and $\theta = 20$, which spans the range of empirically plausible values for this parameter. Even for trade elasticities as extreme as 2 and 20, we find regressions slope coefficients ranging from 0.85-1.09 and correlation coefficients of above 0.999.

Proposition 5 in Subsection 3.6 of the paper shows that the second-order approximation error for the income exposure of each country is bounded by the product between the largest eigenvalue of the corresponding Hessian matrix and the cross-country variance of productivity shocks (i.e., $|\epsilon_i(\tilde{z})| \leq |\mu_{\max,i}| \cdot \tilde{z}(\tilde{z})$). In Table G.1, we show the eigenvalues of the Hessian matrix. Both for the year 2000 and averaged across our sample period, the largest eigenvalue is small, implying that the second-order approximation error for the income exposure of each country is bounded by 0.26 percent and 0.36 percent of the variance of productivity shocks. Moreover, Table G.1 shows that all other eigenvalues are substantially smaller; the second largest eigenvalue $\mu_{\text{2nd},i}$, for instance, averages to 0.0016 across countries $i$ for the year 2000 and to 0.0020 for the entire sample. We find that for each country $i$, $\tilde{z}_{\max,i} \approx \epsilon_i$, implying that productivity shocks represented by the standard unit vector $\epsilon_i$ come close to achieving the upper bound of the approximation error, and any productivity shock vector that is orthogonal to $\tilde{z}_{\text{max},i}$ induces approximation errors that are bounded above by $\mu_{\text{2nd},i}$. As a result, across our entire sample, we find that the second-order approximation errors in income exposure to own-productivity shocks are, on average, bounded by 0.36 percent of the variance of productivity shocks, and the second-order approximation errors in income exposure to other countries’ productivity shocks are, on average, bounded by 0.20 percent of the variance of productivity shocks.

Proposition 7 in Subsection 3.6 of the paper further shows that the eigenvalues of the Hessian matrix, evaluated at some productivity shock vector $x$, provides the exact error for our first-order approximation, summing across second- and all higher-order errors (i.e., $\tilde{w}_i - [W\tilde{z}]_i = \tilde{z} \cdot H_{f_i}(x) \tilde{z}$ for some $x$). A bound on the eigenvalue of the Hessian evaluated over the entire support $X$ of productivity shocks therefore provides an upper-bound on the exact approximation error. We cannot exhaustively evaluate the eigenvalues of Hessians over the entire support of all possible productivity shocks, because the support of the distribution of productivity shocks is not observed in any finite sample. Nevertheless, the upper envelope of all historical realizations of the Hessian eigenvalues is informative, because year-to-year variations in the Hessian matrices are exactly due to differences in the realizations of productivity
shocks over time. As reported in Table G.1, the largest eigenvalue of the Hessian, averaged across countries and taking the maximum across our sample period, is 0.0062, and subsequent eigenvalues are again substantially smaller. This implies global errors in our first-order approximation of income exposures to own productivity shocks of 0.62 percent of the variance of productivity shocks, and global approximation errors of income exposure to other countries’ productivity shocks of 0.33 percent of the variance of productivity shocks.

Taken together, these results suggest that our friend-enemy income exposure measures for productivity shocks are close to exact for empirically-reasonable changes in productivities and trade elasticities.

Figure G.1: Distributions of Regression Slope Coefficients and Coefficients of Correlation Comparing our Friend-Enemy Approximation to Exact-Hat Algebra Predictions in Monte Carlos using Simulated Productivity Shocks ($\theta = 2$)

![Figure G.1](image)

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 3 of the paper. Monte Carlo simulations using 1,000 replications. Simulated productivity shocks drawn (with replacement) from the empirical distribution of productivity shocks from 2000-10.

Figure G.2: Distributions of Regression Slope Coefficients and Coefficients of Correlation Comparing our Friend-Enemy Approximation to Exact-Hat Algebra Predictions in Monte Carlos using Simulated Productivity Shocks ($\theta = 20$)

![Figure G.2](image)

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 3 of the paper. Monte Carlo simulations using 1,000 replications. Simulated productivity shocks drawn (with replacement) from the empirical distribution of productivity shocks from 2000-10.
Table G.1: Eigenvalues of the Hessian Matrix

<table>
<thead>
<tr>
<th>Eigenvalues of Hessians, ordered by absolute value, averaged across all countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Year 2000</td>
</tr>
<tr>
<td>Average across years 1970–2012</td>
</tr>
<tr>
<td>Max across years 1970–2012</td>
</tr>
</tbody>
</table>

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

**Quality of the Approximation for Trade Cost Shocks** In Figure 1b in the paper, we report a comparison of predicted changes in per capita income using our linearization and exact-hat algebra counterfactuals for productivity shocks. In Figure G.3 below, we report an analogous comparison of our linearization and exact-hat algebra counterfactuals using the empirical distribution of trade cost shocks from 2000-2010. While we again find a strong relationship between the predictions of our linearization and the exact-hat algebra counterfactuals, it is noticeably weaker than for productivity shocks, with a regression slope of less than one. An important reason for this difference is that the productivity shock is common across all trade partners, which means that the direct effect of this productivity shock can be taken outside of the summation across trade partners into a separate first term that is the same in our linearization and the exact hat algebra in equations (26) and (27) in the paper. In contrast, the direct effect of bilateral changes in trade costs cannot be taken outside of this summation sign, because it varies across trade partners. Nevertheless, even though the regression slope now differs from one for trade cost shocks, the correlation coefficient between the predicted changes in per capita income using our linearization and exact-hat algebra counterfactuals remains greater than 0.99. In Monte Carlo simulations in which we draw a shock to bilateral trade costs for an exporter-importer pair from the empirical distribution of trade cost shocks from 2000-2010, and compare our linearization and the exact-hat algebra counterfactuals, we find this same pattern of results with regressions slopes that can differ from one, but correlation coefficients that remain close to one. Furthermore, although our linearization works less well in general for bilateral trade cost shocks, we find many examples in which it provides a close approximation to the full non-linear solution of the model, as shown for a shock to US-China bilateral trade costs in Figure G.4.
Figure G.3: Predicted Impact of Bilateral Trade Cost Shocks on Income: Our Friend-Enemy Approximation Versus Exact-Hat Algebra Predictions for the Empirical Distribution of Trade Cost Shocks from 2000-2010 ($\theta = 5$)

Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

Figure G.4: Predicted Impact of Bilateral Trade Cost Shocks on Income: Our Friend-Enemy Approximation Versus Exact-Hat Algebra Predictions for a 50 Percent Increase in US-China Bilateral Trade Costs ($\theta = 5$)

Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper.
G.1.2 Regional Networks of Welfare Exposure

In this section, we report further empirical evidence on regional networks of welfare exposure, as discussed in Section 5.3.2 of the paper. In Figure G.5, we show bilateral welfare exposure in Central Europe before and after the fall of the Iron Curtain. In 1988 immediately before this event, we observe strong connections between the countries of the former Soviet Union (USR) and Eastern European nations such as the former Czechoslovakia (CSK). By 2012, these connections have substantially weakened, and we observe growing connections between Western European countries such as Italy and Eastern European nations. Although Germany is here the aggregation of the former and East and West Germanies in all years, we also observe a strengthening of its position at the center of the network of welfare exposure. More broadly, we also find an increase in the overall density of connections over time, consistent with trade liberalization increasing countries’ economic interdependence.

Figure G.5: Central European Welfare Exposure, 1988 and 2012

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

G.2 Economic and Political Friends and Enemies

In this section of the online appendix, we report additional information for our analysis of the relationship between bilateral political attitudes and our friends-and-enemies sufficient statistics in Section 6 of the paper. In Subsection G.2.1, we report additional details about the measurement of bilateral political attitudes using voting similarity data. In Subsection G.2.2, we report additional empirical results for the relationship between bilateral strategic rivalries and our friends-and-enemies sufficient statistics.

G.2.1 Measuring Political Attitudes Using Bilateral Voting Similarity

We follow a large literature in political science in measuring countries’ bilateral political attitudes towards one another using the similarity of their votes in the United Nations (UN). The ultimate source for our UN voting data is Voeten (2013), which reports non-unanimous plenary votes in the United Nations General Assembly (UNGA) from 1962-2012, and includes on average around 128 votes each year. Countries are recorded as either voting “no” (coded 1), “abstain”
(coded 2) or “yes” (coded 3). In particular, we use the bilateral measures of voting similarity constructed using these data in the Chance-Corrected Measures of Foreign Policy Similarity (FPSIM) database, as reported in Häge (2017). We denote the outcome of vote \( v \) for country \( i \) by \( O_i(v) \):

\[
O_i(v) \in \{1, 2, 3\} \quad v \in \{1, \ldots, V\}.
\]

Building on a large literature in international relations, we consider a number of different measures of bilateral voting similarity. Our first and simplest measure is the \( S \)-score of Signorino and Ritter (1999), which measures the extent of agreement between the votes of countries \( n \) and \( i \) as one minus the sum of the squared actual deviation between their votes scaled by the sum of the squared maximum possible deviations between their votes:

\[
S_{ni}^S = 1 - \frac{\sum_{v=1}^V (O_n(v) - O_i(v))^2}{\frac{1}{2} \sum_{v=1}^V (d_{\text{max}}(v))^2},
\]

where \((d_{\text{max}}(v))^2 = (\sup\{O_n(v) - O_i(v)\})^2\) represents the maximum possible disagreement for each vote and this measure is bounded between minus one (maximum possible disagreement) and one (maximum possible agreement).

A limitation of this \( S \)-score measure is that it does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which countries agree on a “yes” depends on the frequency with which countries vote “yes.” Similarly, the frequency with which they agree on each of the other voting outcomes (“no” and “abstain”) depends on the frequencies with which countries vote these other voting outcomes. Therefore, we also consider two alternative measures of countries’ bilateral similarity in voting patterns that control in different ways for properties of the empirical distribution of voting outcomes. First, the \( \pi \)-score of Scott (1955) adjusts the observed variability of the countries’ bilateral voting outcomes with the variability of each country’s own voting outcomes around average outcomes across the two countries taken together:

\[
S_{ni}^\pi = 1 - \frac{\sum_{v=1}^V (O_n(v) - O_i(v))^2}{\sum_{n=1}^V (O_n(v) - \bar{O}_n - \bar{O}_i)^2 + \sum_{v=1}^V (O_i(v) - \bar{O}_n + \bar{O}_i)^2}.
\]

where \( \bar{O}_i = (1/V) \sum_{v=1}^V O_i(v) \) is the average outcome for country \( i \).

Second, the \( \kappa \)-score of Cohen (1960) adjusts the observed variability of the countries’ bilateral voting outcomes with the variability of each country’s own voting outcomes around its own average outcome and the difference between the two countries’ average outcomes:

\[
S_{ni}^\kappa = 1 - \frac{\sum_{v=1}^V (O_n(v) - O_i(v))^2}{\sum_{v=1}^V (O_n(v) - \bar{O}_n)^2 + \sum_{v=1}^V (O_i(v) - \bar{O}_i)^2}.
\]

Both the \( \pi \)-score and \( \kappa \)-score have an attractive statistical interpretation, as discussed further in Krippendorf (1970), Fay (2005) and Häge (2011). In the case of binary (0,1) voting outcomes, these indices reduce to the form of \( 1 - (D_o/D_e) \), where \( D_o \) is the observed frequency of agreement and \( D_e \) is the expected frequency of agreement. The key difference between the two indices is in their assumptions about the expected frequency of agreement. The \( \pi \)-score estimates the expected frequency of agreement using the average of the two countries marginal distributions of voting outcomes. In contrast, the \( \kappa \)-score estimates the expected frequency of agreement using each country’s own individual marginal distribution of voting outcomes. Whereas our economic measures of exposure are potentially asymmetric, such that \( n \)’s exposure to \( i \) is not necessarily the same as \( i \)’s exposure to \( n \), each of these measures of foreign policy similarity is necessarily symmetric.
G.2.2 Additional Empirical Results on Political Attitudes

In Section 6.2 of the paper, we examine the relationship between countries’ bilateral political attitudes towards one another and our friends-and-enemies sufficient statistics. We report estimation results using both measures of voting similarity in the United Nations General Assembly (UNGA) and measures of strategic rivalries. In this section of the online appendix, we report additional empirical results for strategic rivalries, using the measures of different types of strategic rivalries (positional, spatial and ideological) from Colaresi et al. (2010). Table G.2 presents the results of estimating the same instrumental variables specification as in Columns (9)-(10) in Table 1 in the paper, but using these three types of strategic rivalries. Columns (1)-(2) use positional rivalry; Columns (3)-(4) use spatial rivalry; and Columns (5)-(6) use ideological rivalry. In each of these pairs of specifications, the first column ((1), (3) and (5)) includes only the exporter-importer and year fixed effects; the second column ((2), (4) and (6)) augments this specification with exporter-year and importer-year fixed effects.

Regardless of which measure of strategic rivalry and econometric specification we consider, we find a negative estimated coefficient that is statistically significant at conventional critical values. Therefore, as countries become greater economic friends in terms of the welfare effects of their productivity growth, they become greater political friends in terms of having a lower propensity to be strategic rivals of each type (positional, spatial and ideological). Beneath the coefficient and standard error for each regression specification, we report the first-stage F-statistic. The first-stage F statistics in each panel are again the same across Columns (1), (3) and (5) and Columns (2), (4) and (6), because the first-stage regression specification (welfare exposure on the instrument) and sample size is the same across each of the different measures of strategic rivalry used in the second-stage regression. Although these first-stage F-statistics naturally fall in the specifications including importer-year and exporter-year fixed effects, they remain well above the conventional threshold of 10.
### Table G.2: Bilateral Political Attitudes and Welfare Exposure (Positional, Spatial and Ideological Strategic Rivalries)

<table>
<thead>
<tr>
<th>Political Outcome</th>
<th>Positional Rivalry</th>
<th>Spatial Rivalry</th>
<th>Ideological Rivalry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(5)</td>
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<td>Panel A: Welfare exposure in single-sector model</td>
<td></td>
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<td></td>
<td>(1.442)</td>
<td>(1.541)</td>
<td>(1.566)</td>
</tr>
<tr>
<td>First-stage F</td>
<td>99.50</td>
<td>78.52</td>
<td>99.50</td>
</tr>
<tr>
<td>Panel B: Welfare exposure in multi-sector model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U^{\text{Multi-sector}}$</td>
<td>-2.904**</td>
<td>-3.133**</td>
<td>-3.100**</td>
</tr>
<tr>
<td></td>
<td>(1.429)</td>
<td>(1.528)</td>
<td>(1.552)</td>
</tr>
<tr>
<td>First-stage F</td>
<td>99.36</td>
<td>78.20</td>
<td>99.36</td>
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<tr>
<td>Panel C: Welfare exposure in input-output model</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$U^{\text{Input-Output}}$</td>
<td>-4.742**</td>
<td>-5.176**</td>
<td>-5.063**</td>
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<tr>
<td></td>
<td>(2.278)</td>
<td>(2.455)</td>
<td>(2.471)</td>
</tr>
<tr>
<td>First-stage F</td>
<td>173.6</td>
<td>119.8</td>
<td>173.6</td>
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Specification: 2SLS

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<th>Year</th>
<th>Exp × Year</th>
<th>Imp × Year</th>
<th>No. of Obs.</th>
<th>No. of Clusters</th>
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<td>No</td>
<td>610954</td>
<td>14761</td>
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<td>No</td>
<td>Yes</td>
<td>No</td>
<td>610954</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>610954</td>
<td>14761</td>
</tr>
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</table>

Standard errors in parentheses are clustered at country-pair level

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
H Data Appendix

Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes. The ultimate source for these data is the United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012.¹ We augment these trade data with information on countries’ gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII.² Note that the NBER World Trade Database lacks direct data on a country’s expenditure on domestic goods ($X_{ntt}$). Therefore, we compute this domestic expenditure as gross output minus exports plus imports. To measure gross output for each country, we multiply its GDP (value added) by 2.2, which is the mean ratio of gross output to GDP in the EU-KLEMS database (which includes the USA and Japan). This method yields a median self-trade-share (expenditure on own goods relative to total expenditure) of 0.92 in 1970 and of 0.83 in 2012, reflecting the increase in global trade over this period. In our multi-sector models, we report results aggregating the products in the NBER World Trade Database to the 20 International Standard Industrial Classification (ISIC) industries listed below. In specifications incorporating input-output linkages, we use a common input-output matrix for all countries from Caliendo and Parro (2015).

Table H.1: International Standard Industrial Classification (ISIC) Industries

<table>
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<tr>
<th>Industry Code</th>
<th>Short Name</th>
<th>Long Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture</td>
<td>Agriculture forestry and Fishing</td>
</tr>
<tr>
<td>2</td>
<td>Mining</td>
<td>Mining and quarrying</td>
</tr>
<tr>
<td>3</td>
<td>Food</td>
<td>Food products, beverages and tobacco</td>
</tr>
<tr>
<td>4</td>
<td>Textile</td>
<td>Textiles, textile products, leather and footwear</td>
</tr>
<tr>
<td>5</td>
<td>Wood</td>
<td>Wood and products of wood and cork</td>
</tr>
<tr>
<td>6</td>
<td>Paper</td>
<td>Pulp, paper, paper products, printing and publishing</td>
</tr>
<tr>
<td>7</td>
<td>Petroleum</td>
<td>Coke, refined petroleum and nuclear fuel</td>
</tr>
<tr>
<td>8</td>
<td>Chemicals</td>
<td>Chemicals</td>
</tr>
<tr>
<td>9</td>
<td>Plastic</td>
<td>Rubber and plastics products</td>
</tr>
<tr>
<td>10</td>
<td>Minerals</td>
<td>Other nonmetallic mineral products</td>
</tr>
<tr>
<td>11</td>
<td>Basic Metals</td>
<td>Basic metals</td>
</tr>
<tr>
<td>12</td>
<td>Metal Products</td>
<td>Fabricated metal products, except machinery and equipment</td>
</tr>
<tr>
<td>13</td>
<td>Machinery nec</td>
<td>Machinery and equipment n.e.c</td>
</tr>
<tr>
<td>14</td>
<td>Office</td>
<td>Office, accounting and computing machinery</td>
</tr>
<tr>
<td>15</td>
<td>Electrical</td>
<td>Electrical machinery and apparatus, n.e.c.</td>
</tr>
<tr>
<td>16</td>
<td>Communication</td>
<td>Radio, television and communication equipment</td>
</tr>
<tr>
<td>17</td>
<td>Medical</td>
<td>Medical, precision and optical instruments, watches and clocks</td>
</tr>
<tr>
<td>18</td>
<td>Auto</td>
<td>Motor vehicles trailers and semi-trailers</td>
</tr>
<tr>
<td>19</td>
<td>Other Transport</td>
<td>Other transport equipment</td>
</tr>
<tr>
<td>20</td>
<td>Other</td>
<td>Manufacturing n.e.c and recycling</td>
</tr>
</tbody>
</table>

¹See https://cid.econ.ucdavis.edu/wix.html.
References


