Online Appendix for “International Friends and Enemies”
(Not for Publication)

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A Introduction

In this online appendix, we report the detailed derivations for the results reported in the paper and further supplementary results.

In Section B, we report additional results for the constant elasticity Armington model in Section 2 of the paper, including the proofs of the Propositions. Although in the paper we focus on the single-sector Armington model for expositional convenience, in Section C of this online appendix we establish a number of isomorphisms for the class of models characterized by a constant trade elasticity. In Subsection C.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection C.2, we derive these exposure measures in the Krugman (1980) model.

In Section D, we consider a number of extensions of our baseline friend-enemy exposure measures from Section 2 of the paper. In Subsection D.1, we derive the corresponding friend-enemy exposure measures allowing for both productivity and trade cost shocks. In Section D.2, we allow for trade imbalance. In Section D.3, we show that our results generalize to a multi-sector version of the constant elasticity Armington model. In Section D.4, we further generalize this multi-sector specification to allow for heterogeneous sector trade elasticities. In Section D.5, we show that we obtain analogous results to those in Section D.3 in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section D.6, we further extend the multi-sector Armington model from Section D.3 to introduce input-output linkages following Caliendo and Parro (2015).

In Section E, we generalize our results for a constant elasticity Armington model in Section 2 of the paper to a general neoclassical model of trade with a variable trade elasticity. We use this generalization to derive bounds on the sensitivity of our exposure measures to small deviations from a constant trade elasticity.

In Section F, we report additional empirical results on economic friends and enemies from Section 4.1 of the paper. In Section G, we present further empirical results on economic and political friends and enemies from Section 4.3 of the paper.

In Section H, we report further empirical results for the specification checks in Section 5 of the paper. Section H.1 provides additional evidence on our overidentification check using preferential trade agreements (PTAs). Section H.2 provides an analytical characterization of the quality of the approximation of our linearization to the full non-linear model solution in the single-sector model. We derive bounds for the absolute magnitude of the second and higher-order terms in terms of the observed matrices of trade shares and the trade elasticity. We provide further evidence on the potential role for non-linearities for our quantitative specification with multiple sectors and input-output linkages show that our linearization approximates the full non-
linear model solution even for large shocks, such as cumulative productivity changes over our sample period of more than forty years.

Finally, in Section I, we provide further information on the data sources and definitions.

B Constant Elasticity Armington

In this section of the online appendix, we report the derivations for our baseline constant elasticity of substitution Armington model in Section 2 of the paper. This model falls within the class of quantitative trade models considered by Arkolakis, Costinot and Rodriguez-Clare (2012), which satisfy the three macro restrictions of (i) a constant elasticity import demand system, (ii) profits are a constant share of income, and (iii) trade is balanced. In Section C of this online appendix, we show that our analysis also holds for other models within this class, including those of perfect competition and constant returns to scale with Ricardian technology differences as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution. The world economy consists of many countries indexed by $i, n \in \{1, \ldots, N\}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

B.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \quad p_n = \left[ \sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,$$

(B.1)

where $w_n$ is the wage; $p_n$ is the consumption goods price index; $p_{ni}$ is the price in country $n$ of the good produced by country $i$; and we focus on the case in which countries’ goods are substitutes ($\sigma > 1$).

B.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country $i$ to country $n$ in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the consumer in country $n$ of purchasing a good $i$ from country $i$ is:

$$p_{ni} = \frac{\tau_{ni}w_i}{z_i},$$

(B.2)
where $z_i$ captures productivity in country $i$ and iceberg variable trade costs satisfy $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$.

**B.3 Expenditure Shares**

Using the properties of the CES demand function, country $n$’s share of expenditure on goods produced in country $i$ is:

$$s_{ni} = \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}}.$$  \hfill (B.3)

Totally differentiating this expenditure share equation (B.3), we get:

$$ds_{ni} = -(\sigma - 1) \frac{dp_{ni}}{p_{ni}} + (\sigma - 1) \sum_{h=1}^{N} \frac{dp_{nh}}{p_{nh}} p_{nm}^{1-\sigma},$$

$$\frac{ds_{ni}}{s_{ni}} = - (\sigma - 1) \frac{dp_{ni}}{p_{ni}} + (\sigma - 1) \sum_{h=1}^{N} \frac{dp_{nh}}{p_{nh}},$$

$$\frac{ds_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right),$$

$$d \ln s_{ni} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d \ln p_{nh} - d \ln p_{ni} \right),$$

where, from the definition of $p_{ni}$ in equation (B.2) above, we have:

$$\frac{dp_{ni}}{p_{ni}} = \frac{d \tau_{ni}}{\tau_{ni}} + \frac{d w_i}{w_i} - \frac{d z_i}{z_i},$$  \hfill (B.5)

$$d \ln p_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i.$$

**B.4 Price Indices**

Totally differentiating the consumption goods price index in equation (B.1), we have:

$$dp_n = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}} \left[ \sum_{m=1}^{N} p_{nm}^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

$$\frac{dp_n}{p_n} = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}},$$
\[
\frac{dp_n}{p_n} = \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}}, \tag{B.6}
\]
\[
d\ln p_n = \sum_{m=1}^{N} s_{nm} d\ln p_{nm}.
\]

**B.5 Market Clearing**

Market clearing requires that income in each country equals expenditure on the goods produced in that country:

\[
w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{B.7}
\]

Totally differentiating this market clearing condition (B.7), holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n,
\]

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right).
\]

Using our result for the derivative of expenditure shares in equation (B.4) above, we can rewrite this as:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + (\sigma - 1) \left( \sum_{h\in N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right),
\]

\[
d\ln w_i = \sum_{n=1}^{N} t_{in} \left( d\ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d\ln p_{nh} - d\ln p_{ni} \right) \right), \tag{B.8}
\]

where we have defined \(t_{in}\) as the share of country \(i\)'s income derived from market \(n\):

\[
t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.
\]
B.6 Utility Again

Returning to our expression for indirect utility, we have:

\[ u_n = \frac{w_n}{p_n}. \]  

(B.9)

Totally differentiating indirect utility (B.9), we have:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n} \cdot \frac{w_n}{p_n}, \]

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}. \]

Using our total derivative of the sectoral price index in equation (B.6) above, we get:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}}, \]  

(B.10)

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \ln p_{nm}. \]

B.7 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[ d \ln \tau_{ni} = 0, \quad \forall \ n, i \in N. \]  

(B.11)

We start with our expression for the log change in wages from equation (B.8) above:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \ln p_{nh} - \ln p_{ni} \right) \right), \]

Using the total derivative of prices (B.5) and our assumption of constant bilateral trade costs (B.11), we can write this expression for the log change in wages as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \left[ d \ln w_h - \ln z_h \right] - \left[ d \ln w_i - \ln z_i \right] \right) \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} s_{nh} \left[ d \ln w_h - \ln z_h \right] - \left[ d \ln w_i - \ln z_i \right] \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left[ d \ln w_h - \ln z_h \right] - \left[ d \ln w_i - \ln z_i \right] \right), \]  

(B.12)
which can be re-written as:

\[
d\ln w_i = \sum_{n=1}^{N} t_{in} d\ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left[ d\ln w_n - d\ln z_n \right] \right),
\]

\[
m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},
\]

which has the following matrix representation in the paper:

\[
d\ln w = T d\ln w + \theta M \left( d\ln w - d\ln z \right), \quad \theta = \sigma - 1.
\]

We solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (B.13) by $\theta + 1$, we have:

\[
\frac{1}{\theta + 1} d\ln w = \frac{1}{\theta + 1} T d\ln w + \frac{\theta}{\theta + 1} M \left( d\ln w - d\ln z \right),
\]

\[
\frac{1}{\theta + 1} (I - T - \theta M) d\ln w = -\frac{\theta}{\theta + 1} M d\ln z.
\]

Now using $M = TS - I$, we have:

\[
\frac{1}{\theta + 1} (I - T - \theta TS + \theta I) d\ln w = -\frac{\theta}{\theta + 1} M d\ln z,
\]

\[
\left( I - \frac{T + \theta TS}{\theta + 1} \right) d\ln w = -\frac{\theta}{\theta + 1} M d\ln z.
\]

Using our choice of world GDP as numeraire, which implies $Q d\ln w = 0$, we have:

\[
\left( I - \frac{T + \theta TS}{\theta + 1} + Q \right) d\ln w = -\frac{\theta}{\theta + 1} M d\ln z,
\]

which can be re-written as:

\[
(I - V) d\ln w = -\frac{\theta}{\theta + 1} M d\ln z,
\]

\[
V \equiv \frac{T + \theta TS}{\theta + 1} - Q,
\]

which yields the following solution of the change in wages in response to a productivity shock:

\[
d\ln w = -\frac{\theta}{\theta + 1} (I - V)^{-1} M d\ln z,
\]

which can be re-written as:

\[
d\ln w = W d\ln z,
\]

where $W$ is our friend-enemy income exposure measure:

\[
W \equiv -\frac{\theta}{\theta + 1} (I - V)^{-1} M. \quad (B.14)
\]
B.8 Welfare and Productivity Shocks

From equation (B.10), the log change in utility is given by:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \, d \ln p_{nm}, \]

Using the total derivative of prices (B.5) and our assumption of constant bilateral trade costs (B.11), we can write this log change in utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \, [d \ln w_m - d \ln z_m], \] (B.15)

which has the following matrix representation in the paper:

\[ d \ln u = d \ln w - S (d \ln w - d \ln z). \] (B.16)

We can re-write the above relationship as:

\[ d \ln u = (I - S) \, d \ln w + S \, d \ln z, \]

which using our solution for \( d \ln w \) from above, can be further re-written as:

\[ d \ln u = (I - S) \, W \, d \ln z + S \, d \ln z, \]

\[ d \ln u = [(I - S) \, W + S] \, d \ln z, \]

\[ d \ln u = U \, d \ln z, \]

where \( U \) is our friend-enemy welfare exposure measure:

\[ U \equiv [(I - S) \, W + S]. \] (B.17)

B.9 Relationship to the ACR Gains from Trade Formula

From equation (B.10), the log change in utility is given by:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \, d \ln p_{nm}. \] (B.18)

Choosing country \( n \)'s wage as the numeraire and assuming no changes in its productivity or domestic trade costs, we have:

\[ d \ln w_n = 0, \quad d \ln z_n = 0, \quad d \ln \tau_{nn} = 0, \quad d \ln p_{nn} = 0. \]
The import demand system in equation (B.3) implies:

\[ d \ln s_{nm} - d \ln s_{nn} = - (\sigma - 1) \left( d \ln p_{nm} - d \ln p_{nn} \right). \]

Using this result in equation (B.18), the log change in welfare can be written as:

\[ d \ln u_n = \sum_{m=1}^{N} s_{nm} \left( \frac{d \ln s_{nm} - d \ln s_{nn}}{\sigma - 1} \right) \]

Using \( \sum_{m=1}^{N} s_{nm} = 1 \) and \( \sum_{m=1}^{N} d s_{nm} = 0 \), we obtain the ACR welfare gains from trade formula for small changes:

\[ d \ln u_n = - \frac{d \ln s_{nn}}{\sigma - 1}. \] (B.19)

Integrating both sides of equation (B.19), we get:

\[
\int_{u_n^0}^{u_n^1} \frac{du_n}{u_n} = - (\sigma - 1) \int_{s_{nn}^0}^{s_{nn}^1} \frac{ds_{nn}}{s_{nn}},
\]

\[
\ln \left( \frac{u_n^1}{u_n^0} \right) = - (\sigma - 1) \ln \left( \frac{s_{nn}^1}{s_{nn}^0} \right),
\]

\[
\left( \frac{s_{nn}^1}{s_{nn}^0} \right) = \left( \frac{s_{nn}^1}{s_{nn}^0} \right)^{-(\sigma - 1)}, \quad (B.20)
\]

which corresponds to the ACR welfare gains from trade formula for large changes.

C Single-Sector Isomorphisms

While for convenience of exposition we focus on the single-sector Armington model in the paper, we obtain the same income and welfare exposure measures for all models with a constant trade elasticity in the class considered by Arkolakis, Costinot and Rodriguez-Clare (2012). In Subsection C.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection C.2, we derive these measures in the Krugman (1980) model.

The Armington model in the paper and the Eaton and Kortum (2002) model assume perfect competition and constant returns to scale, whereas the Krugman (1980) model assumes monopolistic competition and increasing returns to scale. In the Eaton and Kortum (2002) model, the trade elasticity corresponds to the Fréchet shape parameter. In contrast, in the Armington model and the Krugman (1980) model, the trade elasticity corresponds to the elasticity of substitution between varieties. Nevertheless, our income and welfare exposure measures hold across all three models, because they all feature a constant trade elasticity.
In Subsection C.2, we focus on the representative firm Krugman (1980) model of monopolistic competition for simplicity, but our income and welfare exposure measures also hold in the heterogeneous firm model of Melitz (2003) with an untruncated Pareto productivity distribution. In Subsection C.2, we also show that our income and welfare exposure measures with monopolistic competition and increasing returns to scale take a similar form whether we consider shocks to the variable or fixed components of production costs.

C.1 Eaton and Kortum (2002)

We consider a version of Eaton and Kortum (2002) with labor as the sole factor of production. Trade arises because of Ricardian technology differences; production technologies are constant returns to scale; and markets are perfectly competitive. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

C.1.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n},$$

(C.1)

where $w_n$ is the wage and $p_n$ is the consumption goods price index, which is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

$$p_n = \left[ \int_0^1 p_n(\vartheta)^{1-\sigma} d\vartheta \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,$$

(C.2)

where $p_n(\vartheta)$ denotes the price of good $\vartheta$ in country $n$.

C.1.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country $i$ to country $n$ in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the price for consumers in country $n$ of purchasing a good $\vartheta$ from country $i$ is:

$$p_{ni}(\vartheta) = \frac{\tau_{ni} w_i}{z_i a_i(\vartheta)},$$

(C.3)

where $z_i$ captures common determinants of productivity across goods within country $i$ and $a_i(\vartheta)$ captures idiosyncratic determinants of productivity for each good $\vartheta$ within that country. Iceberg
variable trade costs satisfy \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \). Productivity for each good \( \vartheta \) in each sector \( k \) and each country \( i \) is drawn independently from the following Fréchet distribution:

\[
F_i(a) = \exp\left(-a^{-\theta}\right), \quad \theta > 1,
\]

(C.4)

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \( z_i \).

### C.1.3 Expenditure Shares

Using the properties of this Fréchet distribution, country \( n \)'s share of expenditure on goods produced in country \( i \) is:

\[
s_{ni} = \frac{\left(\frac{\tau_{ni}w_i}{z_i}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\frac{\tau_{nm}w_m}{z_m}\right)^{-\theta}} = \frac{\left(\rho_{ni}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}},
\]

where we have defined the following price inclusive of trade costs term:

\[
\rho_{ni} \equiv \frac{\tau_{ni}w_i}{z_i}.
\]

(C.6)

Totally differentiating this expenditure share equation (C.5) we get:

\[
ds_{ni} = -\frac{\theta \frac{d\rho_{ni}}{\rho_{ni}} \left(\rho_{ni}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}} + \sum_{h=1}^{N} \frac{\theta \frac{d\rho_{nh}}{\rho_{nh}} \left(\rho_{nh}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{d\rho_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} \frac{\theta \frac{d\rho_{nh}}{\rho_{nh}} \left(\rho_{nh}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{d\rho_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right),
\]

(C.7)

where, from the definition of \( \rho_{ni} \) in equation (C.6) above, we have:

\[
\frac{d\rho_{ni}}{\rho_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i},
\]

(C.8)
C.1.4 Price Indices

Using the properties of the Fréchet distribution (C.4), the consumption goods price index is given by:

\[ p_n = \gamma \left[ \sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\sigma}}, \tag{C.9} \]

where

\[ \gamma \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right) \right]^{\frac{1}{1-\sigma}}, \]

and \( \Gamma(\cdot) \) is the Gamma function. Totally differentiating this price index (C.9), we have:

\[ dp_n = \sum_{m=1}^{N} \gamma \frac{d\rho_{nm}}{\rho_{nm}} \left( \frac{\rho_{nm}}{\sum_{n=1}^{N} (\rho_{nh})^{-\theta}} \right) \left[ \sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\sigma}}, \]

\[ \frac{dp_n}{p_n} = \sum_{m=1}^{N} \frac{d\rho_{nm}}{\rho_{nm}} \frac{\rho_{nm}}{\sum_{h=1}^{N} (\rho_{nh})^{-\theta}}, \]

\[ \frac{dp_n}{p_n} = \sum_{m=1}^{N} s_{nm} \frac{d\rho_{nm}}{\rho_{nm}}, \tag{C.10} \]

\[ d \ln p_n = \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}. \]

C.1.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{C.11} \]

Totally differentiating this market clearing condition (C.11), holding labor endowments constant, we have:

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n, \]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right). \]
Using our result for the derivative of expenditure shares in equation (C.7) above, we can rewrite this as:

\[
\frac{d w_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{d w_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d \rho_{nh}}{\rho_{nh}} - \frac{d \rho_{ni}}{\rho_{ni}} \right) \right),
\]

\[
\frac{d w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{d w_n}{w_n} + \theta \left( \sum_{h \in N} s_{nh} \frac{d \rho_{nh}}{\rho_{nh}} - \frac{d \rho_{ni}}{\rho_{ni}} \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh} d \ln \rho_{nh} - d \ln \rho_{ni} \right) \right),
\]

where we have defined \( t_{in} \) as country \( n \)'s expenditure on country \( i \) as a share of country \( i \)'s income:

\[
t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.
\]

### C.1.6 Utility Again

Returning to our expression for indirect utility, we have:

\[
u_n = \frac{w_n}{p_n}.
\]

Totally differentiating indirect utility (C.13), we have:

\[
d u_n = \frac{d w_n}{w_n} \frac{w_n}{p_n} - \frac{d p_n}{p_n} \frac{w_n}{p_n},
\]

\[
d u_n = \frac{d w_n}{w_n} \frac{w_n}{p_n} - \frac{d p_n}{p_n}.
\]

Using our total derivative of the sectoral price index in equation (C.10) above, we get:

\[
\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{d \rho_{nm}}{\rho_{nm}},
\]

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}.
\]

### C.1.7 Wages and Common Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[
d \ln \tau_{ni} = 0, \quad \forall \ n, i \in N.
\]
We start with our expression for the log change in wages from equation (C.12) above:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh} \left( d \ln \rho_{nh} - d \ln \rho_{ni} \right) \right) \right), \]

Using the total derivative of the price term (C.8) and our assumption of constant bilateral trade costs (C.15), we can write this expression for the log change in wages as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \theta \sum_{h=1}^{N} s_{nh} \left( d \ln w_h - d \ln z_h \right) - (d \ln w_i - d \ln z_i) \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \sum_{n=1}^{N} \sum_{h=1}^{N} t_{in} s_{nh} \left( d \ln w_h - d \ln z_h \right) - (d \ln w_i - d \ln z_i) \right), \]

which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left( d \ln w_n - d \ln z_n \right) \right), \]

\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]

which has the same matrix representation as in equation (11) in the paper:

\[ d \ln w = T d \ln w + \theta M \left( d \ln w - d \ln z \right), \]

\[ \theta = \sigma - 1. \]

C.1.8 Utility and Common Productivity Shocks

From equation (C.14), the log change in utility is given by:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}. \]

Using the total derivative of the price term (C.8) and our assumption of constant bilateral trade costs (C.15), we can write this log change in utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ d \ln w_m - d \ln z_m \right], \]

which has the same matrix representation as in equation (11) in the paper:

\[ d \ln u = d \ln w - S \left( d \ln w - d \ln z \right). \]
C.2 Krugman (1980)

We consider a version of Krugman (1980) with labor as the sole factor of production, in which markets are monopolistically competitive, and trade arises from love of variety and increasing returns to scale. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

C.2.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \quad p_n = \left[ \sum_{i=1}^{N} \int_{0}^{M_i} p_{ni}(j)^{1-\sigma} dj \right]^{-\frac{1}{\sigma}}, \quad \sigma > 1, \quad (C.19)$$

where $w_n$ is the wage; $p_n$ is the consumption goods price index; $p_{ni}(j)$ is the price in country $n$ of a variety $j$ produced in country $i$; $M_i$ is the endogenous mass of varieties; and varieties are substitutes ($\sigma > 1$).

C.2.2 Production Technology

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of $F$ units of labor and a constant variable cost in terms of labor that depends on a country’s productivity $z_i$. Therefore the total amount of labor ($l_i(j)$) required to produce $x_i(j)$ units of variety $j$ in country $i$ is:

$$l_i(j) = F_i + \frac{x_i(j)}{z_i}. \quad (C.20)$$

Varieties can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country $i$ to country $n$ in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Profit maximization and zero profits imply that equilibrium prices are a constant markup over marginal cost:

$$p_{ni}(j) = \left( \frac{\sigma}{\sigma - 1} \right) \rho_{ni}, \quad \rho_{ni} \equiv \frac{\tau_{ni}w_i}{z_i}, \quad (C.21)$$

and equilibrium employment for each variety is equal to a constant:

$$l_i(j) = \bar{l} = (\sigma - 1) F_i. \quad (C.22)$$

Given this constant equilibrium employment for each variety, labor market clearing implies that the total mass of varieties supplied by each country is proportional to its labor endowment:

$$M_i = \frac{\ell_i}{\sigma F_i}. \quad (C.23)$$
From the definition of $\rho_{ni}$ in equation (C.21) above, we have:

$$\frac{d\rho_{ni}}{\rho_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i},$$

(C.24)

$$d \ln \rho_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i.$$  

Totally differentiating in the labor market clearing condition (C.23), holding country endowments constant, we have:

$$\frac{dM_i}{M_i} = -\frac{dF_i}{F_i},$$

(C.25)

$$d \ln M_i = -d \ln F_i.$$  

### C.2.3 Expenditure Shares

Using the symmetry of equilibrium prices and the properties of the CES demand function, country $n$’s share of expenditure on goods produced in country $i$ is:

$$s_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_m p_{nm}^{1-\sigma}} = \frac{\left(\ell_i/F_i\right) p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} \left(\ell_m/F_m\right) p_{nm}^{1-\sigma}}.$$  

(C.26)

Totally differentiating this expenditure share equation (C.26) we get:

$$\frac{ds_{ni}}{s_{ni}} = -\left[ \frac{dF_i}{F_i} + (\sigma - 1) \frac{dp_{ni}}{\rho_{ni}} \right] + \sum_{h=1}^{N} s_{nh} \left[ \frac{dF_h}{F_h} + (\sigma - 1) \frac{dp_{nh}}{\rho_{nh}} \right],$$

(C.27)

$$d \ln s_{ni} = -\left[ d \ln F_i + (\sigma - 1) d \ln \rho_{ni} \right] + \sum_{h=1}^{N} s_{nh} \left[ d \ln F_h + (\sigma - 1) d \ln \rho_{nh} \right].$$  

### C.2.4 Price Indices

Using the symmetry of equilibrium prices, the price index (C.19) can be re-written as:

$$p_n = \left[ \sum_{i=1}^{N} M_i p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$  

Totally differentiating this price index, we have:

$$dp_n = \sum_{i=1}^{N} \left[ \frac{1}{1-\sigma} \frac{dM_i}{M_i} + \frac{dp_{ni}}{p_{ni}} \right] \frac{M_i p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} M_h p_{nh}^{1-\sigma}} \left[ \sum_{i=1}^{N} M_i p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

$$\frac{dp_n}{p_n} = \sum_{i=1}^{N} \left[ \frac{1}{1-\sigma} \frac{dM_i}{M_i} + \frac{dp_{ni}}{p_{ni}} \right] \frac{p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}},$$

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\[
\frac{dp_n}{p_n} = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{1 - \sigma} \frac{dM_i}{M_i} + \frac{dp_{ni}}{p_{ni}} \right],
\]

which using the equilibrium pricing rule (C.21) and the labor market clearing condition (C.23) can be written as:
\[
\frac{dp_n}{p_n} = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \frac{dF_i}{F_i} + \frac{dp_{ni}}{\rho_{ni}} \right],
\]
\[
d \ln p_n = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln \rho_{ni} \right].
\]

C.2.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:
\[
w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n.
\]

Totally differentiating this market clearing condition (C.29), holding labor endowments constant, we have:
\[
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} \frac{d s_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{d w_n}{w_n} w_n \ell_n,
\]
\[
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{d w_n}{w_n} + \frac{d s_{ni}}{s_{ni}} \right),
\]
\[
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{d w_n}{w_n} + \frac{d s_{ni}}{s_{ni}} \right).
\]

Using our result for the derivative of expenditure shares in equation (C.27) above, we can rewrite this as:
\[
\frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{d w_n}{w_n} + \left( \sum_{h=1}^{N} s_{nh} \frac{d F_h}{F_h} - \frac{d F_i}{F_i} \right) \right) + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{d \rho_{nh}}{\rho_{nh}} - \frac{d \rho_{ni}}{\rho_{ni}} \right),
\]
\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{d w_n}{w_n} + \left( \sum_{h=1}^{N} s_{nh} \frac{d F_h}{F_h} - \frac{d F_i}{F_i} \right) \right) + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{d \rho_{nh}}{\rho_{nh}} - \frac{d \rho_{ni}}{\rho_{ni}} \right),
\]
\[
d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left( \sum_{h=1}^{N} s_{nh} d \ln F_h - d \ln F_i \right) \right) + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d \ln \rho_{nh} - d \ln \rho_{ni} \right),
\]

where we have defined \( t_{in} \) as the share of country \( i \)'s income derived from market \( n \):
\[
t_{in} = \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.
\]
C.2.6 Utility Again

Returning to our expression for indirect utility, we have:

\[ u_n = \frac{w_n}{p_n} \]  

(C.31)

Totally differentiating indirect utility (C.31), we have:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}, \]

\[ \frac{du_n}{w_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}. \]

Using our total derivative of the sectoral price index in equation (C.28) above, we get:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \left[ \frac{1}{\sigma - 1} \frac{dF_i}{F_i} + \frac{d\rho_{ni}}{\rho_{ni}} \right], \]  

(C.32)

\[ \frac{du_n}{w_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \left[ \frac{1}{\sigma - 1} \ln F_i + \ln \rho_{ni} \right]. \]

C.2.7 Wages and Productivity Shocks

We consider small shocks to productivity, holding constant bilateral trade costs and fixed costs:

\[ \text{d} \ln \tau_{ni} = 0, \quad \text{d} \ln F_i = 0, \quad \forall n, i \in N. \]  

(C.33)

We start with our expression for the log change in wages from equation (C.30) above:

\[ \text{d} \ln w_i = \sum_{n=1}^{N} t_{in} \left( \text{d} \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \text{d} \ln F_h - \text{d} \ln F_i \right) \right), \]

\[ \text{d} \ln w_i = \sum_{n=1}^{N} t_{in} \left( \text{d} \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \text{d} \ln \rho_{nh} - \text{d} \ln \rho_{ni} \right) \right), \]  

(C.34)
which can be re-written as:

\[
\begin{align*}
\text{dln} w_i &= \sum_{n=1}^{N} t_{in} \text{dln} w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left( \text{dln} w_n - \text{dln} z_n \right) \right), \\
m_{in} &= \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},
\end{align*}
\]

which has the same matrix representation as in equation (8) in the paper:

\[
\begin{align*}
\text{dln} w &= T \text{dln} w + \theta M (\text{dln} w - \text{dln} z), \\
\theta &= \sigma - 1.
\end{align*}
\]

C.2.8 Welfare and Productivity Shocks

From equation (C.32), the log change in utility is given by:

\[
\text{dln} u_n = \text{dln} w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \text{dln} F_i + \text{dln} \rho_{ni} \right],
\]

Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant fixed costs (C.33), we can write this log change in utility as:

\[
\begin{align*}
\text{dln} u_n &= \text{dln} w_n - \sum_{i=1}^{N} s_{ni} \left[ \text{dln} w_i - \text{dln} z_i \right], \\
\end{align*}
\]

which has the same matrix representation as in equation (11) in the paper:

\[
\begin{align*}
\text{dln} u &= \text{dln} w - S (\text{dln} w - \text{dln} z).
\end{align*}
\]

C.2.9 Wages and Fixed Cost Shocks

We next consider small shocks to fixed costs, holding constant bilateral trade costs and productivity:

\[
\begin{align*}
\text{dln} \tau_{ni} &= 0, \quad \text{dln} z_i = 0, \quad \forall n, i \in N.
\end{align*}
\]

We start with our expression for the log change in wages from equation (C.30) above:

\[
\begin{align*}
\text{dln} w_i &= \sum_{n=1}^{N} t_{in} \left( \text{dln} w_n + \left( \sum_{h=1}^{N} s_{nh} \text{dln} F_h - \text{dln} F_i \right) \right) \\
&\quad + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \text{dln} \rho_{nh} - \text{dln} \rho_{ni} \right),
\end{align*}
\]
Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant productivity (C.38), we can write this expression for the log change in wages as:

\[
\frac{d \ln w_i}{d t} = \sum_{n=1}^{N} t_{in} \left( \frac{d \ln w_n}{d t} + \frac{\sum_{h=1}^{N} s_{nh} \frac{d \ln F_h}{d t} - \frac{d \ln F_i}{d t}}{\sigma - 1} \right),
\]

\[
\frac{d \ln w_i}{d t} = \left[ \sum_{n=1}^{N} t_{in} \frac{d \ln w_n}{d t} + \frac{\sum_{n=1}^{N} m_{in} \frac{d \ln F_n}{d t} + (\sigma - 1) \frac{1}{\theta} \frac{d \ln w_n}{d t}}{\sigma - 1} \right],
\]

which can be re-written as:

\[
\frac{d \ln w_i}{d t} = \sum_{n=1}^{N} t_{in} \frac{d \ln w_n}{d t} + \sum_{n=1}^{N} m_{in} \frac{d \ln F_n}{d t} + (\sigma - 1) \frac{1}{\theta} \frac{d \ln w_n}{d t},
\]

where

\[
m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},
\]

which has a similar matrix representation to that for productivity shocks above:

\[
\frac{d \ln w}{d t} = T \frac{d \ln w}{d t} + \theta M \left( \frac{d \ln w}{d t} + \frac{1}{\theta} \frac{d \ln F}{d t} \right),
\]

\[
\theta = \sigma - 1.
\]

### C.2.10 Welfare and Fixed Cost Shocks

From equation (C.32), the log change in utility is given by:

\[
\frac{d \ln u_n}{d t} = \frac{d \ln w_n}{d t} - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \frac{d \ln F_i}{d t} + \frac{d \ln \rho_{ni}}{d t} \right],
\]

Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant productivity (C.38), we can write this log change in utility as:

\[
\frac{d \ln u_n}{d t} = \frac{d \ln w_n}{d t} - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \frac{d \ln F_i}{d t} + \frac{d \ln w_i}{d t} \right],
\]

which has a similar matrix representation to that for productivity shocks above:

\[
\frac{d \ln u}{d t} = \frac{d \ln w}{d t} - S \left( \frac{d \ln w}{d t} + \frac{1}{\theta} \frac{d \ln F}{d t} \right).
\]
D Extensions

In Subsection D.1, we derive the corresponding friend-enemy matrix representations with both productivity and trade cost shocks, as discussed in Section 2 of the paper. In Section D.2, we relax one of the ACR macro restrictions to allow for trade imbalance, as discussed in Section 2 of the paper. In Section D.3, we show that our results generalize to a multi-sector Armington model with a single constant trade elasticity, as discussed in Section 2 of the paper. In Section D.4, we further generalize this specification to allow for heterogeneous sector trade elasticities. In Section D.5, we show that our results also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section D.6, we further extend the multi-sector specification to introduce input-output linkages following Caliendo and Parro (2015), as discussed in Section 2 of the paper.

D.1 Trade Cost Reductions

We now show that we obtain similar results incorporating trade cost reductions. We start with our expression for the log change in wages from equation (B.8) above:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \left( d \ln w_h + d \ln \tau_{nh} - d \ln p_{nh} \right) \right) \right), \]

which can be re-written as follows:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \left[ d \ln w_h + d \ln \tau_{nh} - d \ln \tau_{ni} - d \ln z_h \right] \right) \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + (\sigma - 1) \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} s_{nh} \left[ d \ln w_h + d \ln \tau_{nh} - d \ln \tau_{ni} - d \ln z_h \right] \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} t_{in} s_{nh} \left[ d \ln w_h + d \ln \tau_{nh} - d \ln \tau_{ni} - d \ln z_h \right] \right). \]

We now define inward and outward measures of trade costs as:

\[ d \ln \tau_i^{\text{in}} \equiv \sum_{n} s_{ni} d \ln \tau_{ni}, \quad (D.1) \]

\[ d \ln \tau_i^{\text{out}} \equiv \sum_{n} t_{in} d \ln \tau_{ni}. \quad (D.2) \]

Using these definitions of inward and outward trade costs, we can rewrite the above proportional change in wages as follows:
which can be re-written as:

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right),
\]

which has the following matrix representation:

\[
d \ln \mathbf{w} = \mathbf{T} d \ln \mathbf{w} + \theta \left[ \mathbf{M} \left( d \ln \mathbf{w} - d \ln \mathbf{z} \right) + \mathbf{T} d \ln \tau_{in} - d \ln \tau_{out} \right],
\]

\[
\theta = \sigma - 1.
\]

We next consider our expression for the log change in utility from equation (B.10) above:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln p_{nm},
\]

which can be re-written as follows:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ d \ln w_m + d \ln \tau_{nm} - d \ln z_m \right].
\]

Using our definition of inward trade costs from equation (D.1), we can re-write this proportional change in welfare as:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left( d \ln w_m - d \ln z_m \right) - d \ln \tau_{in}^{nm},
\]

which has the following matrix representation:

\[
d \ln \mathbf{u} = d \ln \mathbf{w} - \mathbf{S} \left( d \ln \mathbf{w} - d \ln \mathbf{z} \right) - d \ln \tau_{in}.
\]

### D.2 Trade Imbalance

In this section of the online appendix, we relax another of the ACR macro restrictions to allow for trade imbalance. In particular, we consider the constant elasticity Armington model from Section 2 of the paper, but allow expenditure to differ from income.
D.2.1 Preferences and Expenditure Shares

We measure the instantaneous welfare of the representative agent as the real value of expenditure:

\[ u_n = \frac{\ell_n}{p_n} + \bar{d}_n, \quad p_n = \left[ \sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{-\frac{1}{1-\sigma}}, \quad \sigma > 1, \]  

(D.6)

where \( \bar{d}_n \) is the nominal trade deficit. Expenditure shares take the same form as in equation (2) in Section 2 of the paper.

D.2.2 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} \left[ w_n \ell_n + \bar{d}_n \right]. \]  

(D.7)

D.2.3 Trade Matrices

We now establish some properties our trade matrices under trade imbalance. We continue to use \( q_i = w_i \ell_i / (\sum_n w_n \ell_n) \) to denote country \( i \)'s share of world income. Let \( e_i = (w_i \ell_i + \bar{d}_n) / (\sum_n w_n \ell_n) \) denote country \( i \)'s share of world expenditures, where we use the fact that the aggregate deficit for the world as a whole is equal to zero. Let \( d_i = q_i / e_i \) denote country \( i \)'s income-to-expenditure ratio, which is equal to one divided by one plus its nominal trade deficit relative to income. Let \( D = Diag(d) \) be the diagonalization of the vector \( d \); note \( q' = e' D \). Under trade balance, \( q_i = e_i \) for all \( i \), and \( D = I \).

We continue to use \( S \) to denote the expenditure share matrix and \( T \) to denote the income share matrix: \( s_{ni} \) captures the expenditure share of importer \( n \) on exporter \( i \) and \( t_{in} \) captures the share of exporter \( i \)'s income derived from selling to importer \( n \). Under trade balance, \( q_i t_{in} = q_n s_{ni} \), but this is no longer the case under trade imbalance. Instead, we have the following results.

**Lemma A.1.** Under trade imbalance, \( q' = e' S \), \( e' = q'T \). Moreover,

1. \( q' \) is the unique left-eigenvector of \( D^{-1}S \) with all positive entries summing to one; the corresponding eigenvalue is one. \( q' \) is also the unique left-eigenvector of \( TD \) and \( TS \) with eigenvalue equal to one.

2. \( e' \) is the unique left-eigenvector of \( SD^{-1} \) with all positive entries summing to one; the corresponding eigenvalue is one. \( e' \) is also the unique left-eigenvector of \( DT \) and \( ST \) with eigenvalue equal to one.
Proof: Dividing the market clearing condition by \( \left( \sum_j w_j \ell_j \right) \), we have

\[
\frac{w_i \ell_i}{\sum_j w_j \ell_j} = \sum_n \left[ s_{ni} w_n \ell_n + \tilde{d}_n \right] \iff q_i = \sum_n s_{ni} \ell_n \iff q' = e' S.
\]

Let \( d_i = q_i/e_i \) and \( D = Diag(\mathbf{d}) \). Note \( q' = e' D \) and \( q'D^{-1} = e' \); thus the market clearing condition can be re-written as

\[
e'D = e'S \iff e' = e'SD^{-1}
\]

and

\[
q' = e'S \iff q' = q'D^{-1}S.
\]

\( q \) is therefore the unique positive left-eigenvector of \( SD^{-1} \) with eigenvalue 1, and \( e' \) is the unique positive left-eigenvector of \( SD^{-1} \) with eigenvalue one. The remaining claims about \( T \) follow analogously.

\[\square\]

D.2.4 Income Comparative Statics

Using these properties of the trade matrices, we now derive countries’ income and welfare exposure to productivity shocks under trade imbalance. As the model does not generate predictions for how trade imbalances respond to shocks, we follow the common approach in the quantitative international trade literature of treating them as exogenous. In particular, we assume that trade imbalances are constant as a share of world GDP, which given our choice of world GDP as the numeraire, corresponds to holding the nominal trade deficits \( \tilde{d}_n \) fixed for all countries \( n \). Totally differentiating the market clearing condition (D.7), holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i \ell_i} = \sum_{n=1}^N s_{ni} \left( \frac{ds_{ni}}{s_{ni}} + \frac{dw_n}{w_n} \right) \left( w_n \ell_n + \tilde{d}_n \right) + \sum_{n=1}^N s_{ni} \ell_n \left( \frac{dd_n}{d_n} \right),
\]

\[
= \sum_{n=1}^N s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \left( \frac{ds_{ni}}{s_{ni}} + \frac{dw_n}{w_n} \right) \left( \frac{w_n \ell_n + \tilde{d}_n}{w_n \ell_n + \tilde{d}_n} \right) + \sum_{n=1}^N s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \left( \frac{dd_n}{d_n} \right) \left( \frac{ds_{ni}}{s_{ni}} + \frac{dd_n}{d_n} \right),
\]

\[
= \sum_{n=1}^N s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \left( \frac{ds_{ni}}{s_{ni}} + \frac{dw_n}{w_n} \right) \left( \frac{w_n \ell_n + \tilde{d}_n}{w_n \ell_n + \tilde{d}_n} \right) + \sum_{n=1}^N s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \left( \frac{dd_n}{d_n} \right) \left( \frac{ds_{ni}}{s_{ni}} + \frac{dd_n}{d_n} \right),
\]

where we have defined \( t_{in} \) as the share of country \( i \)'s income from market \( n \):

\[
t_{in} \equiv \frac{s_{ni} \left( w_n \ell_n + \tilde{d}_n \right)}{w_i \ell_i}
\]
and $d_n$ as country $n$’s ratio of income to expenditure:

$$d_n = \frac{w_n\ell_n}{w_n\ell_n + d_n}.$$  

Using our result for the derivative of expenditure shares above, we can rewrite this as:

$$\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left( \frac{dw_i}{w_i} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) \left( \frac{d\bar{d}_n}{d_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right).$$  

We can re-write this expression as:

$$\frac{d\ln w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( d_n \frac{d\ln w_n}{w_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{d\ln p_{nh}}{p_{nh}} - \frac{d\ln p_{ni}}{p_{ni}} \right) \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) d\ln \bar{d}_n.$$  

### D.2.5 Welfare Comparative Statics

Returning to our expression for welfare (D.6), we have:

$$u_n = \frac{w_n\ell_n + \bar{d}_n}{p_n}.$$  

Totally differentiating welfare, holding labor endowments constant, we have:

$$\frac{du_n}{u_n} = \frac{dw_n}{w_n} \frac{w_n\ell_n}{p_n} + \frac{d\bar{d}_n}{d_n} \frac{d\ell_n}{\ell_n} - \frac{dp_n}{p_n} \left[ \frac{w_n\ell_n + \bar{d}_n}{p_n} \right],$$

$$= \frac{d_n}{w_n} \frac{dw_n}{w_n} + \frac{d\bar{d}_n}{d_n} - \frac{dp_n}{p_n},$$

Using the total derivative of the sectoral price index, we get:

$$\frac{du_n}{u_n} = d_n \frac{dw_n}{w_n} + (1 - d_n) \frac{d\bar{d}_n}{d_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}},$$  

which can be re-written as:

$$\frac{d\ln u_n}{u_n} = d_n \frac{d\ln w_n}{w_n} + (1 - d_n) d\ln \bar{d}_n - \sum_{m=1}^{N} s_{nm} d\ln p_{nm}.$$
D.2.6 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[ \text{d ln } \tau_{ni} = 0, \quad \forall \ n, i \in N, \]  
\[ \text{(D.10)} \]

and we assume that trade deficits remain constant in terms of our numeraire of world GDP:

\[ \text{d ln } \bar{d}_n = 0. \]

Under these assumptions, the change in prices is:

\[ \text{d ln } \bar{p}_n = \text{d ln } \bar{w}_i - \text{d ln } z_i. \]

We start with our expression for the log change in wages in equation (D.8) above. With constant trade deficits (\( \text{d ln } \bar{d}_n = 0 \)), this expression simplifies to:

\[ \text{d ln } \bar{w}_i = \sum_{n=1}^{N} t_{in} \left( d_n \text{d ln } w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \text{d ln } p_{nh} - \text{d ln } p_{ni} \right) \right). \]

Using the total derivative of prices and our assumption of constant bilateral trade costs (D.10), this further simplifies to:

\[ \text{d ln } \bar{w}_i = \sum_{n=1}^{N} t_{in} d_n \text{d ln } w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left( \text{d ln } w_h - \text{d ln } z_h \right) - \left[ \text{d ln } w_i - \text{d ln } z_i \right] \right). \]

\[ \text{(D.11)} \]

\[ \text{d ln } \bar{w}_i = \sum_{n=1}^{N} d_n t_{in} \text{d ln } w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left( \text{d ln } w_n - \text{d ln } z_n \right) \right), \]

\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]

which has the following matrix representation:

\[ \text{d ln } \bar{w} = TD \text{d ln } \bar{w} + \theta M \left( \text{d ln } \bar{w} - \text{d ln } z \right), \]

\[ \text{(D.12)} \]

\[ \theta = \sigma - 1 \]

where the matrices \( T \) and \( M \) are defined in Section B of this online appendix. The diagonal matrix \( D \) captures trade deficits through the ratio of income to expenditure

\[ D = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & d_N \end{pmatrix}, \quad d_n = \frac{w_n \ell_n}{w_n \ell_n + d_n}. \]
D.2.7 Welfare and Productivity Shocks

We start with our expression for the log change in welfare in equation (D.9) above. With constant trade deficits ($d \ln d_n = 0$), this simplifies to:

$$d \ln u_n = d_n \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln p_{nm}.$$  

Using the total derivative of prices and our assumption of constant bilateral trade costs (D.10), we can write this proportional change in utility as:

$$d \ln u_n = d_n \ln w_n - \sum_{m=1}^{N} s_{nm} (d \ln w_m - d \ln z_m),$$  

which has the following matrix representation:

$$d \ln u = D d \ln w - S (d \ln w - d \ln z),$$

where the matrix $S$ is defined in Section B of this online appendix and the diagonal matrix $D$ is defined in the previous subsection of this online appendix.

D.3 Multiple Sectors

In this section of the online appendix, we show that our friends-and-enemies exposure measures extend naturally to a multi-sector version of the constant elasticity Armington model, as discussed in Section 2 of the paper. The world economy consists of many countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors indexed by $k \in \{1, \ldots, K\}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

D.3.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} (p_{n}^k)^{\alpha_n^k}}, \quad \sum_{k=1}^{K} \alpha_n^k = 1.$$  

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

$$p_n^k = \left[ \sum_{i=1}^{N} (p_{ni}^k)^{-\theta} \right]^{-\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1.$$
D.3.2 Production Technology

Goods are produced with labor under conditions of perfect competition, such that the cost to a consumer in country \( n \) of purchasing the variety of country \( i \) within sector \( k \) is:

\[
P_{ni}^k = \frac{\tau_{ni}^k w_i}{z_i^k}, \tag{D.17}
\]

where \( z_i^k \) captures productivity and iceberg trade costs satisfy \( \tau_{ni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \).

D.3.3 Expenditure Shares

Using the properties of CES demand, country \( n' \)’s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[
s_{ni}^k = \frac{(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}, \tag{D.18}
\]

Totally differentiating the expenditure share equation (D.18), we get:

\[
\frac{ds_{ni}^k}{s_{ni}^k} = -\theta \frac{dp_{ni}^k (p_{ni}^k)^{-\theta}}{p_{ni}^k} + \sum_{h=1}^{N} \frac{\theta (p_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}, \tag{D.19}
\]

\[
\frac{d s_{ni}^k}{s_{ni}^k} = -\theta \frac{dp_{ni}^k}{p_{ni}^k} + \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k},
\]

\[
\frac{d s_{ni}^k}{s_{ni}^k} = \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right),
\]

\[
d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k \ d \ln p_{nh}^k - d \ln p_{ni}^k \right),
\]

where, from equilibrium prices in equation (D.17), we have:

\[
\frac{dp_{ni}^k}{p_{ni}^k} = \frac{\tau_{ni}^k w_i}{w_i} - \frac{dz_{ni}^k}{z_i^k}, \tag{D.20}
\]

\[
d \ln p_{ni}^k = d \ln \tau_{ni}^k + d \ln w_i - d \ln z_i^k.
\]
D.3.4 Price Indices

 Totally differentiating the sectoral price index (D.16), we have:

\[
\frac{dp_k^k}{p_n^k} = \frac{\sum_{m=1}^{N} \frac{dp^k_{nm}}{p^k_{nm}} \left(\frac{p^k_{nm}}{p_n^k}\right)^{-\theta} \left[\sum_{m=1}^{N} \left(\frac{p^k_{nm}}{p_n^k}\right)^{-\theta}\right]^{\frac{1}{\theta}}}{\sum_{m=1}^{N} \frac{dp^k_{nm}}{p^k_{nm}} \sum_{h=1}^{N} \left(\frac{p^k_{nh}}{p_n^k}\right)^{-\theta}},
\]

\[
\frac{\partial p_n^k}{p_n^k} = \sum_{m=1}^{N} \frac{dp^k_{nm}}{p^k_{nm}} \left(\frac{p^k_{nm}}{p_n^k}\right)^{-\theta},
\]

\[
\frac{\partial p_n^k}{p_n^k} = \sum_{m=1}^{N} s_{nm}^k \frac{dp^k_{nm}}{p^k_{nm}},
\]

\[
\frac{d\ln p_n^k}{p_n^k} = \sum_{m=1}^{N} s_{nm}^k \frac{dp^k_{nm}}{p^k_{nm}}.
\]

D.3.5 Market Clearing

 Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[
w_i^\ell = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n^\ell_n.
\]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i^\ell = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n^\ell_n + \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k \frac{dw_n}{w_n} w_n^\ell_n,
\]

\[
\frac{dw_i}{w_i} w_i^\ell = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n^\ell_n \left(\frac{dw_n}{w_n} + \frac{ds_{ni}^k}{s_{ni}^k}\right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k \frac{dw_n}{w_n} + \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k}\right),
\]

\[
\frac{d\ln w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} s_{ni}^k \left(\frac{d\ln w_n}{w_n} + \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k}\right)\right),
\]

(D.23)
where we have defined $t_{in}^k$ as the share of country $i$’s income derived from market $n$ and industry $k$:

$$t_{in}^k = \frac{\alpha_n^k s_{ni}^k w_n f_n}{w_i l_i}.$$

### D.3.6 Utility Again

Returning to our expression for indirect utility in equation (D.15), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^K (p_n^k)^{\alpha_n^k}}.$$

Totally differentiating indirect utility, we have:

$$du_n = \frac{dw_n}{w_n} - \sum_{k=1}^K \frac{\alpha_n^k}{\prod_{k=1}^K (p_n^k)^{\alpha_n^k}} dp_n^k.$$

Using the total derivative of the sectoral price index in equation (D.21) above, we get:

$$\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^K \alpha_n^k \sum_{m=1}^N s_{nm}^k \frac{dp_{nm}^k}{p_{nm}^k},$$

(D.24)

### D.3.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$\frac{d \ln z_i^k}{d \ln z_i} = \frac{d \ln z_i^k}{d \ln z_i}, \quad \forall = k \in K, \ i \in N,$$

$$\frac{d \ln \tau_{ni}^k}{d \ln \tau_{ni}} = 0, \quad \forall n, i \in N.$$  

(D.25)

We start with our expression for the log change in wages from equation (D.23) above:

$$d \ln w_i = \sum_{n=1}^N \sum_{k=1}^K t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^N s_{nh}^k d \ln p_{nh}^k - d \ln p_{ni}^k \right) \right).$$

Using the total derivative of prices (D.20) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (D.25), we can write this expression for the log change in wages as:

$$d \ln w_i = \sum_{n=1}^N \sum_{k=1}^K t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^N s_{nh}^k \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right) \right),$$
\[
\begin{align*}
\ln w_i &= \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \ln w_n + \theta \sum_{n=1}^{N} \sum_{k=1}^{K} \left( \sum_{h=1}^{N} s_{nh}^k \left( \ln w_h - \ln z_h \right) - \left[ \ln w_i - \ln z_i \right] \right), \\
\ln w_i &= \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \ln w_n + \theta \left( \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k s_{nh}^k \left( \ln w_h - \ln z_h \right) - \left[ \ln w_i - \ln z_i \right] \right), \\
\ln w_i &= \sum_{n=1}^{N} t_{in} \ln w_n + \theta \left( \sum_{n=1}^{N} m_{in} \left[ \ln w_n - \ln z_n \right] \right), \\
\sum_{k=1}^{K} t_{in}^k &= t_{in}, \\
m_{in} &= \sum_{h=1}^{N} \sum_{k=1}^{K} t_{in}^k s_{nh}^k - 1_{n=i},
\end{align*}
\]
which has the following matrix representation in the paper:
\[
\ln \mathbf{w} = \mathbf{T} \ln \mathbf{w} + \theta \mathbf{M} \left( \ln \mathbf{w} - \ln \mathbf{z} \right). \tag{D.26}
\]

We again solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation \((\text{D.26})\) by \(\theta + 1\), we have:
\[
\frac{1}{\theta + 1} \ln \mathbf{w} = \frac{1}{\theta + 1} \mathbf{T} \ln \mathbf{w} + \frac{\theta}{\theta + 1} \mathbf{M} \left( \ln \mathbf{w} - \ln \mathbf{z} \right),
\]
\[
\frac{1}{\theta + 1} \left( \mathbf{I} - \mathbf{T} - \theta \mathbf{M} \right) \ln \mathbf{w} = - \frac{\theta}{\theta + 1} \mathbf{M} \ln \mathbf{z}.
\]
Now using \(\mathbf{M} = \mathbf{TS} - \mathbf{I}\), we have:
\[
\frac{1}{\theta + 1} \left( \mathbf{I} - \mathbf{T} - \theta \mathbf{TS} + \theta \mathbf{I} \right) \ln \mathbf{w} = - \frac{\theta}{\theta + 1} \mathbf{M} \ln \mathbf{z},
\]
\[
\left( \mathbf{I} - \frac{\mathbf{T} + \theta \mathbf{TS}}{\theta + 1} \right) \ln \mathbf{w} = - \frac{\theta}{\theta + 1} \mathbf{M} \ln \mathbf{z}.
\]
Using our choice of world GDP as numeraire, which implies \(\mathbf{Q} \ln \mathbf{w} = 0\), we have:
\[
\left( \mathbf{I} - \frac{\mathbf{T} + \theta \mathbf{TS}}{\theta + 1} + \mathbf{Q} \right) \ln \mathbf{w} = - \frac{\theta}{\theta + 1} \mathbf{M} \ln \mathbf{z},
\]
which can be re-written as:
\[
\left( \mathbf{I} - \mathbf{V} \right) \ln \mathbf{w} = - \frac{\theta}{\theta + 1} \mathbf{M} \ln \mathbf{z},
\]
\[
\mathbf{V} \equiv \frac{\mathbf{T} + \theta \mathbf{TS}}{\theta + 1} - \mathbf{Q},
\]

31
which yields the following solution of the change in wages in response to a productivity shock:

\[ d \ln w = - \frac{\theta}{\theta + 1} (I - V)^{-1} M d \ln z, \]

which can be re-written as:

\[ d \ln w = W d \ln z, \]  \hspace{1cm} (D.27)

where \( W \) is our friend-enemy income exposure measure:

\[ W \equiv - \frac{\theta}{\theta + 1} (I - V)^{-1} M. \]  \hspace{1cm} (D.28)

### D.3.8 Welfare and Common Productivity Shocks

We start with our expression for the log change in utility in equation (D.24) above:

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_k \sum_{m=1}^{N} s_{nm} d \ln p_{nm}^k, \]

or equivalently:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_{nm} s_{nm}^k d \ln p_{nm}^k. \]

Using the total derivative of prices (D.20) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (D.25), we can write this change in log utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_{nm} s_{nm}^k (d \ln w_m - d \ln z_m), \]  \hspace{1cm} (D.29)

which has the following matrix representation in the paper:

\[ d \ln u = d \ln w - S (d \ln w - d \ln z). \]  \hspace{1cm} (D.30)

We can re-write the above relationship as:

\[ d \ln u = (I - S) d \ln w + S d \ln z, \]

which, using our solution for \( d \ln w \) from equation (D.27), can be further re-written as:

\[ d \ln u = (I - S) W d \ln z + S d \ln z, \]

\[ d \ln u = [(I - S) W + S] d \ln z, \]

\[ d \ln u = U d \ln z, \]  \hspace{1cm} (D.31)

where \( U \) is our friend-enemy welfare exposure measure:

\[ U \equiv [(I - S) W + S]. \]  \hspace{1cm} (D.32)
D.3.9 Industry-Level Sales Exposure

In this multi-sector model, our approach also yields bilateral friend-enemy measures of income exposure to global productivity shocks for each sector. Labor income in each sector and country equals value-added, which in turn equals expenditure on goods produced in that sector and country:

\[
    w_i\ell_i^k = y_i^k = \sum_{n=1}^{N} \alpha_n^k s_{ni}^k w_n\ell_n.
\]

Totally differentiating this industry market clearing condition, we have:

\[
    \frac{dy_i^k}{y_i^k} = \sum_{n=1}^{N} \alpha_n^k \frac{ds_{ni}^k}{s_{ni}^k} w_n\ell_n + \sum_{n=1}^{N} \alpha_n^k s_{ni}^k \frac{dw_n}{w_n}.
\]

Using the total derivative of expenditure shares in equation (D.19), we can rewrite this as:

\[
    \frac{dy_i^k}{y_i^k} = \sum_{n=1}^{N} \alpha_n^k t_{in}^k \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right) \right),
\]

which can be re-written as:

\[
    d\ln y_i^k = \sum_{n=1}^{N} t_{in}^k \left( d\ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k d\ln p_{nh}^k - d\ln p_{ni}^k \right) \right).
\]

Using the total derivative of prices (D.20) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (D.25), we can re-write this change in sector value-added as:

\[
    d\ln y_i^k = \sum_{n=1}^{N} t_{in}^k \left( d\ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k (d\ln w_h - d\ln z_h) - (d\ln w_i - d\ln z_i) \right) \right),
\]

which has the following matrix representation:

\[
    d\ln Y^k = T^k d\ln w + \theta M^k (d\ln w - d\ln z).
\]
Using our solution for changes in wages as a function of productivity shocks in equation (D.26) above, we have:

\[
\begin{align*}
\frac{d \ln Y^k}{dz} &= T^k W \frac{d \ln z}{dz} + \frac{\theta \mathbf{M}^k (\mathbf{W} - I)}{dz} \frac{d \ln z}{dz}, \\
\frac{d \ln Y^k}{dz} &= \left[ T^k W + \frac{\theta \mathbf{M}^k (\mathbf{W} - I)}{dz} \right] \frac{d \ln z}{dz},
\end{align*}
\]

which can be re-written as:

\[
\begin{align*}
\frac{d \ln Y^k}{dz} &= W^k \frac{d \ln z}{dz}, \\
W^k &\equiv T^k W + \frac{\theta \mathbf{M}^k (\mathbf{W} - I)}{dz},
\end{align*}
\]

which corresponds to our friends-and-enemies measure of sector value-added exposure to productivity shocks.

We now show that our aggregate friends-and-enemies measure of income exposure (W) in equation (D.28) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure in equation (D.35). Note that aggregate income equals aggregate value-added:

\[
w_i \ell_i = \sum_{k=1}^{K} y_i^k.
\]

Totally differentiating, holding endowments constant, we have:

\[
\begin{align*}
\frac{dw_i}{w_i} \ell_i &= \sum_{k=1}^{K} \frac{dy_i^k}{y_i^k} y_i^k, \\
\frac{dw_i}{w_i} &= \sum_{k=1}^{K} \frac{y_i^k}{w_i \ell_i} \frac{dy_i^k}{y_i^k}, \\
\frac{dw_i}{w_i} &= \sum_{k=1}^{K} r_i^k \frac{dy_i^k}{y_i^k}, \\
\frac{dl w_i}{w_i} &= \sum_{k=1}^{K} r_i^k \frac{d \ln y_i^k},
\end{align*}
\]

where \( r_i^k \equiv \frac{y_i^k}{w_i \ell_i} \) is the value-added share of industry \( k \). Together, equations (D.27), (D.34) and (D.36) imply:

\[
W_i \frac{d \ln z}{dz} = \sum_{k=1}^{K} r_i^k W_i^k \frac{d \ln z}{dz},
\]

where \( W_i \) is the income exposure vector for country \( i \) with respect to productivity shocks in its trade partners and \( W_i^k \) is the sector value-added exposure vector for country \( i \) and sector \( k \) with
respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure \((W_i)\) is a weighted average of our industry friends-and-enemies measures of sector income exposure \((W^k_i)\):

\[
W_i = \sum_k \alpha_i^k W^k_i.
\]

Therefore, we can decompose our aggregate income exposure measure into the contributions of the income exposure measures of particular industries, and how much of that income exposure of particular industries is explained by various terms (market-size, cross-substitution etc within the industry).

### D.4 Heterogeneous Sector Trade Elasticities

For simplicity, we have so far assumed a common elasticity of substitution \((\sigma)\) and trade elasticity \((\theta)\) across sectors through this section of the online appendix. We now further generalize our analysis to allow for heterogeneous trade elasticities across sectors. The specification remains as in the previous subsection, except that the elasticity of substitution \((\sigma^k)\) and trade elasticity \((\theta^k)\) now vary across sectors \(k\). Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[
w_i \ell_i = \sum_{n=1}^N \sum_{k=1}^K \alpha_n^k s_n^k w_n \ell_n. \tag{D.37}
\]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^N \sum_{k=1}^K \alpha_n^k s_n^k w_n \ell_n \frac{dw_n}{w_n} + \sum_{n=1}^N \sum_{k=1}^K \alpha_n^k s_n^k \left( \frac{ds_n^k}{s_n^k} \right) \frac{dw_n}{w_n} + \sum_{n=1}^N \sum_{k=1}^K \alpha_n^k s_n^k \left( \frac{dp_n^k}{p_n^k} - \frac{dp_n^k}{p_n^k} \right),
\]

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^N \sum_{k=1}^K \alpha_n^k s_n^k w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_n^k}{s_n^k} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^N \sum_{k=1}^K \alpha_n^k s_n^k \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^N s_n^h \frac{dp_n^h}{p_n^h} - \frac{dp_n^h}{p_n^h} \right) \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^N \sum_{k=1}^K \theta \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^N s_n^h \frac{dp_n^h}{p_n^h} - \frac{dp_n^h}{p_n^h} \right) \right). \tag{D.38}
\]
where we have defined \( t_{in}^k \), as the share of country \( i \)'s income derived from market \( n \) and industry \( k \):

\[
t_{in}^k = \frac{\alpha_{n} s_{nh}^k w_{n} \ell_{n}}{w_{i} \ell_{i}}.
\]

Now consider a shock to productivities, holding all else constant:

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \frac{d \ln w_n + \theta^k \left( \sum_{h=1}^{N} s_{nh}^k d \ln \rho_{n}^{h} - d \ln p_{ni}^{h} \right) }{\theta} \right),
\]

where we have multiplied and divided by \( \theta \). We can further rewrite this expression as:

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \frac{d \ln w_n + \theta^k \left( \sum_{h=1}^{N} s_{nh}^k [d \ln w_h - d \ln z_h] - [d \ln w_i - d \ln z_i] \right) }{\theta},
\]

where we have multiplied and divided by \( \theta \). We can further rewrite this expression as:

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \frac{d \ln w_n + \theta^k \left( \sum_{h=1}^{N} s_{nh}^k [d \ln w_h - d \ln z_h] - [d \ln w_i - d \ln z_i] \right) }{\theta},
\]

where the definition of \( m_{in} \) now depends on the ratio of the sectoral trade elasticity (\( \theta^k \)) to the common parameter (\( \theta \)). Equation (D.39) has the following matrix representation as in the paper:

\[
d \ln w = T d \ln w + \theta M \left( d \ln w - d \ln z \right),
\]

(D.40)
where only the definitions and interpretation of $\theta$ and $M$ differ. Totally differentiating the indirect utility function, we again have:

$$
\text{d} \ln u_n = \text{d} \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k \text{d} \ln p_{nm}^k.
$$

Now consider a shock to productivities, holding all else constant:

$$
\text{d} \ln u_n = \text{d} \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k \left( \text{d} \ln w_m - \text{d} \ln z_m \right),
$$

which has the following matrix representation as in the paper:

$$
\text{d} \ln u = \text{d} \ln w - S \left( \text{d} \ln w - \text{d} \ln z \right).
$$

\textbf{D.5 Multi-Sector Isomorphism}

For the convenience of exposition, we focus in Section 2 of the paper and the previous section of this online appendix on a multi-sector version of the constant elasticity Armington model. In this section, we show that the same results hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors indexed by $k \in \{1, \ldots, K\}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

\textbf{D.5.1 Consumer Preferences}

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$
u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}, \quad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{D.43}
$$

Each sector contains a fixed continuum of goods that enter the sectoral price index according to the following CES functional form:

$$
p_n^k = \left[ \int_0^1 \left( \frac{1}{\sigma^k} \right)^{\frac{1}{1-\sigma^k}} \text{d} \vartheta \right]^{\frac{1}{1-\sigma^k}}, \quad \sigma^k > 1. \tag{D.44}
$$

\textbf{D.5.2 Production Technology}

Goods are produced with labor and can be traded subject to iceberg variable trade costs, such that the cost to a consumer in country $n$ of purchasing a good $\vartheta$ from country $i$ is:

$$
p_{ni}^k (\vartheta) = \frac{z_i^k w_i}{z_i^k \alpha_i^k (\vartheta)}, \tag{D.45}
$$
where \( z^k_i \) captures determinants of productivity that are common across all goods within a country \( i \) and sector \( k \) and \( a^k_i (\vartheta) \) captures idiosyncratic determinants of productivity for each good within that country and sector. Iceberg trade costs satisfy \( \tau^k_{ni} > 1 \) for \( n \neq i \) and \( \tau^k_{nn} = 1 \). Productivity for each good \( \vartheta \) in each sector \( k \) and each country \( i \) is drawn independently from the following Fréchet distribution:

\[
F^k_i (a) = \exp \left( -a^{-\theta} \right), \quad \theta > 1, \tag{D.46}
\]

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \( z^k_i \).

### D.5.3 Expenditure Shares

Using the properties of this Fréchet distribution, country \( n' \)’s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[
s^k_{ni} = \frac{\left( \tau^k_{ni} w_i / z^k_i \right)^{-\theta}}{\sum_{m=1}^{N} \left( \tau^k_{nm} w_m / z^k_m \right)^{-\theta}} = \frac{\left( \rho^k_{ni} \right)^{-\theta}}{\sum_{m=1}^{N} \left( \rho^k_{nm} \right)^{-\theta}}, \tag{D.47}
\]

where we have defined the following price term:

\[
\rho^k_{ni} = \frac{\tau^k_{ni} w_i}{z^k_i}. \tag{D.48}
\]

Totally differentiating the expenditure share equation (D.47), we get:

\[
 ds^k_{ni} = -\theta \frac{\rho^k_{ni} d\rho^k_{ni}}{\rho^k_{ni}^2} \left( \frac{\rho^k_{ni}}{\rho^k_{nm}} \right)^{-\theta} + \sum_{h=1}^{N} \frac{\left( \rho^k_{ni} \right)^{-\theta}}{\sum_{m=1}^{N} \left( \rho^k_{nm} \right)^{-\theta}} \theta \frac{d\rho^k_{nh}}{\rho^k_{nh}} \left( \rho^k_{nh} \right)^{-\theta},
\]

\[
 ds^k_{ni} = -\theta \frac{\rho^k_{ni} d\rho^k_{ni}}{\rho^k_{ni}^2} + \sum_{h=1}^{N} \frac{\left( \rho^k_{ni} \right)^{-\theta}}{\sum_{m=1}^{N} \left( \rho^k_{nm} \right)^{-\theta}} \theta \frac{d\rho^k_{nh}}{\rho^k_{nh}} \left( \rho^k_{nh} \right)^{-\theta},
\]

\[
 ds^k_{ni} = -\theta \frac{\rho^k_{ni} d\rho^k_{ni}}{\rho^k_{ni}^2} + \sum_{h=1}^{N} \frac{\left( \rho^k_{ni} \right)^{-\theta}}{\sum_{m=1}^{N} \left( \rho^k_{nm} \right)^{-\theta}} \theta \frac{d\rho^k_{nh}}{\rho^k_{nh}} \left( \rho^k_{nh} \right)^{-\theta},
\]

\[
 ds^k_{ni} = \theta \left( \sum_{h=1}^{N} s^k_{nh} \frac{d\rho^k_{nh}}{\rho^k_{nh}} - \frac{d\rho^k_{ni}}{\rho^k_{ni}} \right), \tag{D.49}
\]

\[
 d \ln s^k_{ni} = \theta \left( \sum_{h=1}^{N} s^k_{nh} d \ln \rho^k_{nh} - d \ln \rho^k_{ni} \right),
\]

where from the definition of \( \rho^k_{ni} \) above we have:

\[
 d \rho^k_{ni} = \frac{d\tau^k_{ni}}{\tau^k_{ni}} + d w_i - d z^k_i, \tag{D.50}
\]

\[
 d \ln \rho^k_{ni} = d \ln \tau^k_{ni} + d w_i - d z^k_i.
\]
D.5.4 Price Indices

Using the properties of the Fréchet distribution (D.46), the sectoral price index is given by:

\[ p_{kn}^k = \gamma^k \left[ \sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{D.51} \]

where

\[ \gamma^k \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma^k}{\theta} \right) \right]^{\frac{1}{1 - \sigma^k}}, \]

and \( \Gamma(\cdot) \) denotes the Gamma function. Totally differentiating this sectoral price index (D.51), we have:

\[
\frac{dp_{kn}^k}{p^k_n} = \sum_{m=1}^{N} \gamma^k \frac{d\rho_{nm}^k}{\rho_{nm}^k} \frac{(\rho_{nm}^k)^{-\theta}}{\sum_{h=1}^{N} (\rho_{nh}^k)^{-\theta}} \left[ \sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta} \right]^{-\frac{1}{\theta}},
\]

\[
\frac{dp_{kn}^k}{p^k_n} = \sum_{m=1}^{N} \frac{d\rho_{nm}^k}{\rho_{nm}^k} \frac{(\rho_{nm}^k)^{-\theta}}{\sum_{h=1}^{N} (\rho_{nh}^k)^{-\theta}},
\]

\[
\frac{dp_{kn}^k}{p^k_n} = \sum_{m=1}^{N} s_{nm}^k \frac{d\rho_{nm}^k}{\rho_{nm}^k}, \tag{D.52}
\]

\[
dl \ln p_{kn}^k = \sum_{m=1}^{N} s_{nm}^k \ln \rho_{nm}^k.
\]

D.5.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n. \tag{D.53} \]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k \frac{ds_{ni}^k}{s_{ni}^k} s_{ni}^k w_n \ell_n + \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k \frac{dw_n}{w_n} w_n \ell_n,
\]

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}^k}{s_{ni}^k} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}^k}{s_{ni}^k} \right),
\]

39
Using our result for the derivative of expenditure shares in equation \((D.49)\) above, we can rewrite this as:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k \frac{\ell_n}{w_i} \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{d \rho_{nh}^k}{p_{nh}^k} - \frac{d \rho_{ni}^k}{p_{ni}^k} \right) \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k \frac{\ell_n}{w_i} \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{d \rho_{nh}^k}{p_{nh}^k} - \frac{d \rho_{ni}^k}{p_{ni}^k} \right) \right),
\]

\[(D.54)\]

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k d \ln \rho_{nh}^k - d \ln \rho_{ni}^k \right) \right),
\]

where we have defined \(t_{in}^k\) as country \(n\)’s expenditure on country \(i\) in industry \(k\) as a share of country \(i\)’s income:

\[
t_{in}^k = \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.
\]

**D.5.6 Utility Again**

Returning to our expression for indirect utility in equation \((D.43)\), we have:

\[
u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.
\]

Totally differentiating indirect utility, we have:

\[
du_n = \frac{dw_n}{w_n} \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}} - \sum_{k=1}^{K} \alpha_n^k \frac{d p_n^k}{p_n^k} \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}},
\]

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{d p_n^k}{p_n^k},
\]

Using our total derivative of the sectoral price index in equation \((D.52)\) above, we get:

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{d \rho_{nm}^k}{p_{nm}^k},
\]

\[(D.55)\]

\[
d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k d \ln \rho_{nm}^k.
\]

**D.5.7 Wages and Common Productivity Shocks**

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[
d \ln z_i^k = d \ln z_i, \quad \forall k \in K, \ i \in N,
\]

\[
d \ln \tau_{nm}^k = 0, \quad \forall n, \ i \in N.
\]

\[(D.56)\]
We start with our expression for the log change in wages from equation (D.54) above:

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k \left( d \ln \rho_{nh}^k - d \ln \rho_{ni}^k \right) \right) \right), \]

Using the total derivative of prices (D.50) and our assumption of constant bilateral trade costs (D.56), we can write this log change in wages as:

\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k d \ln w_n + \theta \sum_{n=1}^{N} \sum_{h=1}^{N} s_{nh}^k \left( d \ln w_n - d \ln z_h - (d \ln w_i - d \ln z_i) \right), \]

which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \left( \sum_{h=1}^{N} m_{in} \left[ d \ln w_n - d \ln z_h \right] \right), \]

\[ t_{in} = \sum_{k=1}^{K} \sum_{n=1}^{N} t_{in}^k, \]

\[ m_{in} = \sum_{h=1}^{N} \sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i}, \]

which has the following matrix representation:

\[ d \ln w = T d \ln w + \theta M \left( d \ln w - d \ln z \right). \] (D.58)

**D.5.8 Utility and Common Productivity Shocks**

We start with our expression for the log change in utility in equation (D.55) above:

\[ d \ln u_n = d \ln w_n - \sum_{h=1}^{N} \alpha_n^h \sum_{m=1}^{N} s_{nm}^k d \ln \rho_{nm}^k, \]

or equivalently:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k d \ln \rho_{nm}^k. \]

Using the total derivative of prices (D.50) and our assumption of constant bilateral trade costs (D.56), we can write this change in utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k \left( d \ln w_m - d \ln z_m \right), \]
which has the following matrix representation:

\[
d \ln u = d \ln w - S \left( d \ln w - d \ln z \right).
\] (D.59)

### D.6 Multiple Sectors and Input-Output Linkages

In this section of the online appendix, we report the derivations for a version of the model with multiple sectors and input-output linkages following Caliendo and Parro (2015), henceforth CP. In particular, we consider a generalization of the multi-sector Armington model in Section D.3 of this online appendix to incorporate input-output linkages, as discussed in Section 2 of the paper.

The world economy consists of a set of countries indexed by \( i, n \in \{1, \ldots, N\} \) and a set of sectors referenced by \( k \in \{1, \ldots, K\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

#### D.6.1 Notations

We use \( i, n, o, r \) to index for countries and \( j, k, l \) for industries. We refer to the varieties in industry \( k \) produced in country \( i \) as “goods \( ik \)”. We use subscripts to denote countries and superscripts to denote industries. Let \( I_{(NK)} \) denote the identity matrix with dimension \( NK \times NK \).

#### D.6.2 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[
u_n = \frac{w_n}{\prod_{k=1}^{K} (p^{k}_{ni})^{\alpha^k_n}}, \quad \sum_{k=1}^{K} \alpha^k_n = 1.
\] (D.60)

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

\[
p^k_{ni} = \left[ \sum_{i=1}^{N} (p^{k}_{ni})^{-\theta} \right]^{\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma^k > 1.
\] (D.61)

#### D.6.3 Production Technology

Goods are produced with labor and can be traded subject to iceberg trade costs, such that the cost to a consumer in country \( n \) of purchasing country \( i \)’s variety within sector \( k \) is:

\[
p^k_{ni}(\omega) = \gamma^k_{ni} c^k_i, \quad c^k_i = \left( \frac{w_i}{z^k_i} \right)^{\gamma^k_i} \prod_{j=1}^{K} (p^j_i)^{\gamma^k_{i,j}}, \quad \sum_{k=1}^{K} \gamma^k_{i,j} = 1 - \gamma^k_i,
\] (D.62)

where \( c^k_i \) denotes the unit cost function within that country and sector; \( \gamma^k_i \) is the share of labor in production costs; \( \gamma^k_{i,j} \) is the share of materials from sector \( j \) used in sector \( k \); \( z^k_i \) captures
determinants of productivity that are common across all goods within a country \( i \) and sector \( k \); and it proves convenient to define this common component of productivity in value-added terms (such that it augments labor). Iceberg variable trade costs satisfy \( \tau_{kni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \).

### D.6.4 Expenditure Shares

Using the properties of CES demand, country \( n' \)'s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[
s_{ni}^k = \frac{(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}.
\]  
(D.63)

Totally differentiating this expenditure share equation, we get:

\[
ds_{ni}^k = -\theta \frac{dp_{ni}^k (p_{ni}^k)^{-\theta}}{s_{ni}^k} + \sum_{h=1}^{N} \frac{\theta dp_{nh}^k (p_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}},
\]

\[
ds_{ni}^k = -\theta \frac{dp_{ni}^k}{s_{ni}^k} + \sum_{h=1}^{N} \frac{\theta dp_{nh}^k}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}},
\]

\[
ds_{ni}^k = -\theta \frac{dp_{ni}^k}{s_{ni}^k} + \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k},
\]

\[
ds_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right).
\]

so that

\[
d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k \ d \ln p_{nh}^k - \ d \ln p_{ni}^k \right),
\]  
(D.64)

where, from equilibrium prices in equation (D.62), we have:

\[
\frac{dp_{ni}^k}{p_{ni}^k} = \frac{d\tau_{ni}^k}{\tau_{ni}^k} + \gamma_i^k \left( \frac{d w_i}{w_i} - \frac{d z_i^k}{z_i^k} \right) + \sum_{j=1}^{K} \gamma_{ij}^k \frac{dp_{ij}^j}{p_{ij}^j},
\]  
(D.65)

\[
d \ln p_{ni}^k = d \ln \tau_{ni}^k + \gamma_i^k \left( d \ln w_i - d \ln z_i^k \right) + \sum_{j=1}^{K} \gamma_{ij}^k d \ln p_{ij}^j.
\]

### D.6.5 Price Indices

Totally differentiating the sectoral price index (D.61), we have:

\[
dp_n^k = \sum_{m=1}^{N} \gamma_k^k \frac{dp_{nm}^k (p_{nm}^k)^{-\theta}}{\sum_{h=1}^{N} (p_{nm}^k)^{-\theta}} \left[ \sum_{m=1}^{N} (p_{nm}^k)^{-\theta} \right]^{-\frac{1}{\theta}},
\]

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\[
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} dp^k_{nm} \frac{(p^k_{nm})^{-\theta}}{\sum_{h=1}^{N} (p^k_{nh})^{-\theta}},
\]
\[
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} s^k_{nm} \frac{dp^k_{nm}}{p^k_{nm}},
\]
\[
d \ln p^k_n = \sum_{m=1}^{N} s^k_{nm} d \ln p^k_{nm}.
\]

(D.66)

D.6.6 Labor Market Clearing

The labor market clearing condition is:
\[
w_n \ell_n = \sum_{j=1}^{K} \gamma^j_n y^j_n,
\]
where \(y^j_n\) is total sales by country \(n\)’s industry \(j\). Totally differentiating this labor market clearing condition, holding endowments constant, we have:
\[
\frac{dw_n}{w_n} w_n \ell_n = \sum_{j=1}^{K} \gamma^j_n \frac{dy^j_n}{y^j_n},
\]
\[
\frac{dw_n}{w_n} = \sum_{j=1}^{K} \left( \frac{\gamma^j_n y^j_n}{w_n L_n} \right) \frac{dy^j_n}{y^j_n},
\]
\[
\frac{dw_n}{w_n} = \sum_{j=1}^{K} \xi^j_n \frac{dy^j_n}{y^j_n},
\]
(D.68)
where \(\xi^j_n\) is the share of sector \(j\) in country \(n\)’s total income:
\[
\xi^j_n = \frac{\gamma^j_n y^j_n}{w_n L_n}.
\]

D.6.7 Goods Market Clearing

Goods market clearing requires that income in each country and sector equals expenditure on goods produced in that country and sector:
\[
y^k_i = \sum_{n=1}^{N} s^k_{ni} x^k_n,
\]
(D.69)
where expenditure in country \(n\) in sector \(k\) is:
\[
x^k_n = \alpha^k_n w_n \ell_n + \sum_{j=1}^{K} \gamma^j_n y^j_n,
\]
(D.70)
and recall that $\gamma_{n}^{j,k}$ is the share of materials from sector $k$ used in sector $j$. Combining these two relationships and the labor market clearing (D.67), we obtain the following market clearing condition:

$$
y_{i}^{k} = \sum_{n=1}^{N} s_{n}^{k} \left[ \alpha_{n}^{k} w_{n} \ell_{n} + \sum_{j=1}^{K} \gamma_{n}^{j,k} y_{n}^{j} \right],$$

$$
= \sum_{n=1}^{N} s_{n}^{k} \left[ \alpha_{n}^{k} \sum_{j=1}^{K} \gamma_{j}^{n} y_{n}^{j} + \sum_{j=1}^{K} \gamma_{n}^{j,k} y_{n}^{j} \right],
$$

$$
= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{n}^{k} \left[ \alpha_{n}^{k} \gamma_{n}^{j} + \gamma_{n}^{j,k} \right] y_{n}^{j}.
$$

Totally differentiating this market clearing condition, we have:

$$
d\frac{y_{i}^{k}}{y_{i}^{k}} y_{i}^{k} = \sum_{n=1}^{N} s_{n}^{k} \left[ \alpha_{n}^{k} w_{n} \ell_{n} + \sum_{j=1}^{K} \gamma_{n}^{j,k} y_{n}^{j} \right],$$

$$
= \sum_{n=1}^{N} s_{n}^{k} \left[ \alpha_{n}^{k} \sum_{j=1}^{K} \gamma_{j}^{n} y_{n}^{j} + \sum_{j=1}^{K} \gamma_{n}^{j,k} y_{n}^{j} \right],
$$

$$
= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{n}^{k} \left[ \alpha_{n}^{k} \gamma_{n}^{j} + \gamma_{n}^{j,k} \right] y_{n}^{j}.
$$

Let $\vartheta_{in}^{k}$ denote the fraction of $ik$’s revenue derived from selling to consumers in country $n$; $\Theta$ denote an $NK \times NK$ matrix with entries $\Theta_{in}^{kj}$ capturing the fraction of $ik$’s revenue derived from selling to producers in country $n$ industry $j$; and $\Delta$ denote the Leontief-inverse of $\Theta$, such that $\Delta \equiv \left( I_{NK} - \Theta \right)^{-1}$, with the $(ik, nj)$-th entry, $\Delta_{nj}^{kj}$, capturing the network-adjusted fraction of $ik$’s revenue derived from market $nj$, either directly or indirectly through customers of customers, ad infinitum. The above market clearing can be re-written as

$$
d \ln y_{i}^{k} = \sum_{n=1}^{N} d \ln s_{n}^{k} \left[ \alpha_{n}^{k} \sum_{j=1}^{K} \gamma_{n}^{j,k} y_{n}^{j} \right] y_{i}^{k} + \sum_{n=1}^{N} \sum_{j=1}^{K} d \ln s_{n}^{k} \frac{s_{n}^{k} \gamma_{n}^{j,k} y_{n}^{j}}{y_{i}^{k}} y_{n}^{j},
$$

$$
+ \sum_{n=1}^{N} s_{n}^{k} \alpha_{n}^{k} w_{n} \ell_{n} \sum_{j=1}^{K} \frac{\gamma_{n}^{j,k} y_{n}^{j}}{w_{n} \ell_{n}} d \ln y_{i}^{k} + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{n}^{k} \frac{s_{n}^{k} \gamma_{n}^{j,k} y_{n}^{j}}{y_{i}^{k}} y_{n}^{j} d \ln y_{n}^{j},
$$

$$
= \sum_{n=1}^{N} \vartheta_{in}^{k} d \ln w_{n} + \sum_{n=1}^{N} \left( \vartheta_{in}^{k} + \sum_{j=1}^{K} \Theta_{in}^{kj} \right) d \ln s_{n}^{k} + \sum_{n=1}^{N} \sum_{j=1}^{K} \Theta_{in}^{kj} d \ln y_{n}^{j}.
$$

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where in the second equality we used equations (D.67) and (D.68). Subtracting the latest term on the right hand side from both sides of the equations and taking the Leontief-inverse of \( \Theta_{nm}^{ik} \), we obtain:

\[
d\ln y_i^k = \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left[ \sum_{n=1}^{N} \varphi_{on}^l d\ln w_n + \sum_{n=1}^{N} \left( \varphi_{on}^l + \sum_{j=1}^{K} \Theta_{on}^{ij} \right) d\ln s_{no}^l \right]. \tag{D.71}
\]

Combining this result with (D.68), we get

\[
d\ln w_i = \sum_k \xi_{i}^{k} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left[ \sum_{n=1}^{N} \varphi_{on}^l d\ln w_n + \sum_{n=1}^{N} \left( \varphi_{on}^l + \sum_{j=1}^{K} \Theta_{on}^{ij} \right) d\ln s_{no}^l \right]. \tag{D.72}
\]

### D.6.8 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[
d\ln z_i^k = d\ln z_i, \quad \forall k \in K, \ i \in N, \quad d\ln \tau_{mi}^k = 0, \quad \forall n, i \in N.
\]

We start with our expression for the change in prices above:

\[
d\ln p_{ni}^k = d\ln \tau_{ni}^k + \gamma_i^k (d\ln w_i - d\ln z_i^k) + \sum_{j=1}^{K} \gamma_{ij}^{kj} d\ln p_{ij}^j,
\]

\[
= \gamma_i^k (d\ln w_i - d\ln z_i) + \sum_{j=1}^{K} \gamma_{ij}^{kj} d\ln p_{ij}^j
\]

Using our result for the total derivative of price indices (D.66), we can rewrite this expression for the change in prices as:

\[
d\ln p_{ni}^k = \gamma_i^k (d\ln w_i - d\ln z_i) + \sum_{j=1}^{K} \gamma_{ij}^{kj} \sum_{m=1}^{N} \xi_{im}^j d\ln p_{im}^j.
\]

We use \( \Sigma_{im}^{kj} = \gamma_{ij}^{kj} \xi_{im}^j \) to denote expenditure in country \( i \) and sector \( k \) on the goods produced by country \( m \) and sector \( j \) as a share of revenue in country \( i \) and sector \( k \). Using this notation, we can rewrite the above expression for the change in prices as:

\[
d\ln p_{ni}^k - d\ln \tau_{ni} = \gamma_i^k (d\ln w_i - d\ln z_i) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma_{im}^{kj} (d\ln p_{im}^j - d\ln \tau_{im}) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma_{im}^{kj} d\ln \tau_{im}. \tag{D.73}
\]
Let $\Sigma$ denote the $NK \times NK$ matrix with entries $\Sigma_{im}^{kj}$ capturing the input cost share (relative to
revenue) on goods $mj$ by producer $ik$. Let us also define the Leontief inverse $\Gamma \equiv (I_{NK} - \Sigma)^{-1}$,
with the $(nj, ik)$-th entry, $\Gamma_{nj}^{ik}$, capturing the network-adjusted share of $nj$’s revenue spent on
inputs $ik$, either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Finally, let $\Lambda_{ni}^{j} \equiv \sum_{k=1}^{K} \gamma_{ik}^{j} \Gamma_{nj}^{ik}$ denote the network-adjusted input cost share of $nj$’s revenue on
value-added (labor) in country $i$; note that $\sum_{i=1}^{N} \Lambda_{ni}^{j} = 1$ for all $nj$ due to constant returns to scale. Equation (D.73) can be re-written as:

$$d \ln p_{ni}^{k} = \sum_{l=1}^{N} \Lambda_{nl}^{k} (d \ln w_{l} - d \ln z_{l}).$$

We can now use (D.74) to re-write the linearized expenditure shares from equation (D.64) as:

$$d \ln s_{ni}^{k} = \theta \left(\sum_{h=1}^{N} s_{nh}^{k} \sum_{r=1}^{N} \Lambda_{hr}^{k} (d \ln w_{r} - d \ln z_{r}) - \sum_{r} \Lambda_{ir}^{k} (d \ln w_{r} - d \ln z_{r})\right),$$

$$= \theta \sum_{r=1}^{N} \left(\sum_{h=1}^{N} s_{nh}^{k} \Lambda_{hr}^{k} - \Lambda_{ir}^{k}\right) (d \ln w_{r} - d \ln z_{r}).$$

Substitute this into (D.72), we get

$$d \ln w_{i} = \sum_{k=1}^{K} \xi_{il} s_{il} \left(\sum_{l=1}^{N} \sum_{n=1}^{N} \Delta_{ni}^{kl} \sum_{n=1}^{N} \theta_{onl} \ln w_{n} \right)$$

$$+ \sum_{k=1}^{K} \xi_{il} s_{il} \left(\sum_{l=1}^{N} \sum_{n=1}^{N} \Delta_{ni}^{kl} \theta \sum_{n=1}^{N} \left(\sum_{j=1}^{K} \Theta_{onj}^{l} \right) \left(\sum_{h=1}^{N} s_{nh}^{l} \Lambda_{hr}^{l} - \Lambda_{ir}^{l}\right) (d \ln w_{r} - d \ln z_{r})\right).$$

To simplify notation, let us now define $\Pi_{io}^{l} \equiv \sum_{k=1}^{K} \xi_{il} \Delta_{io}^{kl}$ to be the network-adjusted share of income in country $i$ derived from selling to country $o$ industry $l$. Also denote $\Upsilon_{nor}^{l} \equiv \sum_{h=1}^{N} s_{nh}^{l} \Lambda_{hr}^{l} - \Lambda_{ir}^{l}; \theta \Upsilon_{nor}^{l}$ is the elasticity of $n$’s expenditure on goods $lo$ with respect to $r$’s factor cost. Then the above expression can be re-written as:

$$d \ln w_{i} = \sum_{n=1}^{N} \sum_{o=1}^{N} \Pi_{io}^{l} \theta_{onl} (d \ln w_{n})$$

$$+ \theta \sum_{n=1}^{N} \left(\sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left(\theta_{onr}^{l} + \sum_{j=1}^{K} \Theta_{onj}^{l} \right) \Upsilon_{ron}^{l} \right) (d \ln w_{r} - d \ln z_{r}).$$

Finally, we define $T$ as an $N \times N$ matrix with entries $T_{in} \equiv \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \theta_{onl}$. Element $T_{in}$ captures the network-adjusted share of $i$’s income derived from selling to consumers in country $n$.
it sums across the network-adjusted income share that country $i$ derives from selling to country-industry $ol$, times the revenue share that $ol$ derives from selling to consumers in country $n$. We define $M$ as an $N \times N$ matrix with entries $M_{in}$:

$$M_{in} = \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left( \vartheta_{or}^{l} + \sum_{j=1}^{K} \Theta_{or}^{lj} \right) Y_{r, on}^{l}.$$ 

To interpret, $\vartheta Y_{r, on}^{l}$ captures how $r$'s expenditure on goods $ol$ responds to factor cost in $n$. Element $M_{in}$ sums the cross-substitution effects across $i$'s exposure to all markets through network linkages: $\Pi_{io}^{l}$ is $i$'s network-adjusted income share derived from selling to producers in country $o$ industry $l$; goods $ol$ are then exposed to substitution due to changes in $n$'s factor costs through markets that $ol$ supplies to, including consumers ($\vartheta_{or}^{l}$) and producers $\left( \sum_{j=1}^{K} \Theta_{or}^{lj} \right)$ in all countries ($r$).

We have thus obtained the same matrix representation as in the paper:

$$d \ln w = T d \ln w + \vartheta M \left( d \ln w - d \ln z \right). \quad (D.77)$$

We again solve for our friend-enemy income exposure measure by matrix inversion:

$$d \ln w = W d \ln z, \quad (D.78)$$

where

$$W \equiv -\frac{\vartheta}{\vartheta + 1} (I - V)^{-1} M \quad (D.79)$$

and, using our choice of world GDP as numeraire, which implies $Q d \ln w = 0$,

$$V \equiv \frac{T + \vartheta TS}{\vartheta + 1} - Q.$$

### D.6.9 Welfare

Returning to our expression for indirect utility ($D.60$), we have:

$$u_{n} = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.$$  

Totally differentiating this expression for indirect utility, we have:

$$du_{n} = \frac{dw_{n}}{w_{n}} w_{n} - \sum_{k=1}^{K} \alpha_n^k \frac{dp_n^k}{p_n^k} w_{n},$$

$$\frac{du_{n}}{u_{n}} = \frac{dw_{n}}{w_{n}} - \sum_{k=1}^{K} \alpha_n^k \frac{dp_n^k}{p_n^k}.$$
Using our total derivative of the sectoral price index above, we get:

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{dp_{nm}^k}{p_{nm}^k},
\]

(D.80)

\[
d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k d \ln p_{nm}^k.
\]

Plugging (D.74) into the above, we get

\[
d \ln u_n = d \ln w_n - \sum_{i=1}^{N} \left( \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm}^k \lambda_{mi}^k \right) (d \ln w_i - d \ln z_i)
\]

We define \( S \) as an \( N \times N \) matrix with entries \( S_{ni} \equiv \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm}^k \lambda_{mi}^k \). Element \( S_{ni} \) captures the network-adjusted expenditure share of consumer \( n \) on value-added by country \( i \); it sums across the expenditure share of consumer \( n \) on goods \( mk \), times the network-adjusted input cost share of \( mk \) on factor \( i \), captured by \( \lambda_{mi}^k \). We have thus obtained the same matrix representation for welfare as in the paper:

\[
d \ln u = d \ln w - S (d \ln w - d \ln z).
\]

(D.81)

We can re-write the above relationship as:

\[
d \ln u = (I - S) d \ln w + S d \ln z,
\]

which, using our solution for \( d \ln w \) from equation (D.78), can be further re-written as:

\[
d \ln u = (I - S) W d \ln z + S d \ln z,
\]

\[
= [(I - S) W + S] d \ln z,
\]

\[
= U d \ln z,
\]

where \( U \) is our friend-enemy welfare exposure measure:

\[
U \equiv [(I - S) W + S].
\]

(D.82)

**D.6.10 Industry-Level Sales Exposure**

Similarly to the multi-sector model without input linkages, our approach also yields bilateral friend-enemy measures of sales exposure to global productivity shocks for each sector. From equations (D.71) and (D.75) we obtain the following expression for changes in the sales of industry \( k \) in country \( i \):

\[
\]
\[
\begin{align*}
d \ln y^k_i &= \sum_{n=1}^N \left( \sum_{o=1}^N \sum_{l=1}^K \Delta_{io}^{kl} \phi_{or}^{lj} \right) d \ln w_n, \\
&= \sum_{n=1}^N \left( \sum_{o=1}^N \sum_{l=1}^K \Delta_{io}^{kl} \left( \phi_{or}^{lj} + \sum_{j=1}^K \Theta_{or}^{lj} \right) \Gamma_{ron}^l \right) (d \ln w_n - d \ln z_n).
\end{align*}
\]

We define \( T^k \) as an \( N \times N \) matrix with entries \( T_{in}^k \equiv \sum_{o=1}^N \sum_{l=1}^K \Delta_{io}^{kl} \phi_{or}^{lj} \). Element \( T_{in}^k \) captures the network-adjusted share of sector \( ik \)'s income derived from selling to consumers in country \( n \); it sums across the network-adjusted income share that sector \( ik \) derives from selling to country-industry \( ol \), times the revenue share that \( ol \) derives from selling to consumers in country \( n \). We define \( M^k \) as an \( N \times N \) matrix with entries \( M_{in}^k \):

\[
M_{in}^k \equiv \sum_{r=1}^N \sum_{o=1}^N \sum_{l=1}^K \Delta_{io}^{kl} \left( \phi_{or}^{lj} + \sum_{j=1}^K \Theta_{or}^{lj} \right) \Gamma_{ron}^l.
\]

To interpret, \( \theta \Gamma_{ron}^l \) captures how \( r \)'s expenditure on goods \( ol \) responds to factor cost in \( n \). Element \( M_{in}^k \) sums the cross-substitution effects across sector \( ik \)'s exposure to all markets through network linkages: \( \Delta_{io}^{kl} \) is \( ik \)'s network-adjusted income share derived from selling to producers in country \( o \) industry \( l \); goods \( ol \) are then exposed to substitution due to changes in \( n \)'s factor costs through markets that \( ol \) supplies to.

We get the following matrix representation:

\[
\begin{align*}
d \ln Y^k &= T^k d \ln w + \theta M^k (d \ln w - d \ln z), \\
&= [T^k W + \theta M^k (W - I)] d \ln z,
\end{align*}
\]

which can be re-written as:

\[
\begin{align*}
d \ln Y^k &= W^k d \ln z, \\
W^k &\equiv T^k W + \theta M^k (W - I),
\end{align*}
\]

which corresponds to our friends-and-enemies measure of sector sales exposure to productivity shocks. Note that \( W^k \) also captures sector value-added exposure to productivity shocks, as value added is a constant share of revenues in this model. Finally, recall from equation (D.68) that

\[
d \ln w_n = \sum_{j=1}^K \xi_n^j d \ln y_n^k.
\]

where \( \xi_n^k \) is the value-added share of industry \( k \) in country \( n \)'s total income. Together, this relationship and equations (D.78) and (D.84) imply that:
\[ W_i \, d \ln z = \sum_{k=1}^{K} \xi_{kn} W_{ik}^k \, d \ln z, \]

where \( W_i \) is the income exposure vector for country \( i \) with respect to productivity shocks in its trade partners and \( W_{ik}^k \) is the sector value-added exposure vector for country \( i \) and sector \( k \) with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure (\( W_i \)) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure (\( W_{ik}^k \)):

\[ W_i = \sum_{k=1}^{K} \xi_{kn} W_{ik}^k. \]

Therefore, we can decompose how much of our aggregate income exposure measure is driven by the value-added exposure of particular industries, and how much of that value-added exposure of particular industries is explained by various terms (market-size, cross-substitution, etc. within the industry).

### D.6.11 Isomorphisms

Although for expositional convenience we focus on an extension of the constant elasticity Armington model with multiple sectors and input-output linkages, the same results hold in an extension of the Eaton and Kortum (2002) model to incorporate both multiple sectors following Costinot, Donaldson and Komunjer (2012) and also input-output linkages following Caliendo and Parro (2015).

### D.6.12 Income and Welfare Exposure to Common Productivity and Trade Cost Shocks

We now derive our income and welfare exposure measures in the multi-sector model with input-output linkages allowing for both productivity and trade cost shocks. We consider small productivity and trade cost shocks for each country that are common across sectors \( k \):

\[
\begin{align*}
    d \ln z_i^k &= d \ln z_i, \quad \forall k \in K, \; i \in N, \\
    d \ln \tau_{ni}^k &= d \ln \tau_{ni}, \quad \forall n, \; i \in N.
\end{align*}
\]

We start with our expression for the change in prices above:

\[
d \ln p_{ni}^k = d \ln \tau_{ni}^k + \gamma_i^k \left( d \ln w_i - d \ln z_i^k \right) + \sum_{j=1}^{K} \gamma_{ij}^{k,j} d \ln p_{im}^j,
\]

Using our result for the total derivative of price indices (D.66), we can rewrite this expression for the change in prices as:

\[
\begin{align*}
    d \ln p_{ni}^k - d \ln \tau_{ni} &= \gamma_i^k \left( d \ln w_i - d \ln z_i \right) + \sum_{j=1}^{K} \gamma_{ij}^{k,j} \sum_{m=1}^{N} \sigma_{im}^j \, d \ln p_{im}^j.
\end{align*}
\]
We use $\Sigma_{im}^{kj} = \gamma_{i}^{k} j^{j} s_{im}^{j}$ to denote expenditure in country $i$ and sector $k$ on the goods produced by country $m$ and sector $j$ as a share of revenue in country $i$ and sector $k$. Using this notation, we can rewrite the above expression for the change in prices as:

$$d \ln p_{ni}^{k} - d \ln \tau_{ni} = \gamma_{i}^{k} (d \ln w_{i} - d \ln z_{i}) + \sum_{j=1}^{K} \sum_{m=1}^{N} \sum_{x=1}^{K} \Sigma_{im}^{jx} d \ln \tau_{im} + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma_{im}^{kj} d \ln \tau_{im}. \quad (D.86)$$

Recall $\Sigma$ is the $NK \times NK$ matrix with entries $\Sigma_{im}^{kj}$ capturing the input cost share (relative to revenue) on goods $mj$ by producer $ik$, and $\Gamma \equiv (I_{(NK)} - \Sigma)^{-1}$ is the Leontief inverse, with the $(n,j,ik)$-th entry, $\Gamma_{ni}^{jk}$, capturing the network-adjusted share of $nj$’s revenue spent on inputs $ik$, either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Equation (D.86) can be re-written as:

$$d \ln p_{ni}^{k} - d \ln \tau_{ni} = \sum_{l=1}^{N} \left( \sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{m=1}^{K} \sum_{x=1}^{K} \Sigma_{im}^{jx} d \ln \tau_{im} + \sum_{j=1}^{K} \Gamma_{il}^{kj} \gamma_{i}^{k} (d \ln w_{l} - d \ln z_{l}) \right) \quad (D.87)$$

Recall $\Lambda_{ni}^{j} \equiv \sum_{k=1}^{K} \gamma_{i}^{k} \Gamma_{ni}^{jk}$ is the network-adjusted input cost share of $nj$’s revenue on value-added (labor) in country $i$. Also let

$$d \ln c_{i}^{k,\tau} \equiv \sum_{l=1}^{N} \sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{m=1}^{K} \sum_{x=1}^{K} \Sigma_{im}^{jx} d \ln \tau_{im},$$

which captures the denote the log changes in the cost of good $ik$ due to changes in trade costs, holding factor costs constant. Equation (D.87) can be re-written as

$$d \ln p_{ni}^{k} = d \ln \tau_{ni} + d \ln c_{i}^{k,\tau} + \sum_{l} \Lambda_{il}^{k} (d \ln w_{l} - d \ln z_{l}) \quad (D.88)$$

We can now use (D.88) to re-write the linearized expenditure shares from equation (D.64) as:

$$d \ln s_{ni}^{k} = \theta \left( \sum_{h=1}^{N} s_{nh}^{k} \left\{ d \ln \tau_{nh} + d \ln c_{h}^{k,\tau} + \sum_{r=1}^{N} \Lambda_{hr}^{k} (d \ln w_{r} - d \ln z_{r}) \right\} \right) - \left\{ d \ln \tau_{ni} + d \ln c_{i}^{k,\tau} + \sum_{r} \Lambda_{ir}^{k} (d \ln w_{r} - d \ln z_{r}) \right\} \quad (D.89)$$

$$= \theta \sum_{r=1}^{N} \left( \sum_{h=1}^{N} s_{nh}^{k} \Lambda_{hr}^{k} - \Lambda_{ir}^{k} \right) (d \ln w_{r} - d \ln z_{r}) + \theta \sum_{h=1}^{N} \left( s_{nh}^{k} - 1_{h=i} \right) \left\{ d \ln \tau_{nh} + d \ln c_{h}^{k,\tau} \right\}. \quad (D.90)$$

Substitute this into (D.72), we get
\[ \begin{aligned}
&\text{d} \ln w_i = \sum_{k=1}^{K} \sum_{a=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \varphi_{in}^{l} \text{d} \ln w_n \\
&+ \theta \sum_{k=1}^{K} \sum_{a=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \left( \varphi_{in}^{l} + \frac{K}{j=1} \Theta_{in}^{jl} \right) \left( \sum_{r} \left( \sum_{h=1}^{N} s_{nh}^{l} \Lambda_{hr}^{l} \right) \right) \left( \text{d} \ln w_r \right) \right) \\
&+ \theta \sum_{k=1}^{K} \sum_{a=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \left( \varphi_{in}^{l} + \frac{K}{j=1} \Theta_{in}^{jl} \right) \left( \sum_{r} \left( s_{nh}^{l} - 1_{h=i} \right) \right) \left( \text{d} \ln \tau_{nh} + \text{d} \ln e_{l,r}^{l} \right) ) .
\end{aligned} \]

Recall we have defined \( \Pi_{io}^{l} \equiv \sum_{k=1}^{K} \alpha_{i}^{l} \Delta_{io}^{kl} \) as the network-adjusted share of income in country \( i \) derived from selling to country \( o \) industry \( l \). Also recall \( \gamma_{nor}^{l} \equiv \sum_{h=1}^{N} s_{nh}^{l} \Lambda_{hr}^{l} \Lambda_{or}^{l} ; \theta \gamma_{nor}^{l} \) is the elasticity of \( n \)'s expenditure on goods \( lo \) with respect to \( r \)'s factor cost. Using these results, we can rewrite income exposure to productivity and trade cost shocks as:

\[ 
\begin{aligned}
&\text{d} \ln w_i = \sum_{n=1}^{N} \left( \sum_{a=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \varphi_{in}^{l} \text{d} \ln w_n \\
&+ \theta \sum_{n=1}^{N} \left( \sum_{r=1}^{N} \sum_{a=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left( \varphi_{in}^{l} + \frac{K}{j=1} \Theta_{in}^{jl} \right) \right) \left( \text{d} \ln w_r \right) \right) \\
&+ \theta \sum_{n=1}^{N} \sum_{r=1}^{N} \sum_{a=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left( \varphi_{in}^{l} + \frac{K}{j=1} \Theta_{in}^{jl} \right) \left( s_{nr}^{l} - 1_{r=i} \right) \left( \text{d} \ln \tau_{nr} + \text{d} \ln e_{l,r}^{l} \right) ) .
\end{aligned} \]

For welfare, note

\[ \text{d} \ln u_n = \text{d} \ln w_n - \sum_{k=1}^{K} \alpha_{n}^{k} \sum_{m=1}^{N} s_{nm}^{k} \text{d} \ln p_{nm}^{k} . \]

Plugging equation (D.88) into the above expression, we can write welfare exposure to productivity and trade cost shocks as:

\[ 
\begin{aligned}
&\text{d} \ln u_n = \text{d} \ln w_n - \sum_{i=1}^{N} \left( \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_{n}^{k} s_{nm}^{k} \Lambda_{mi}^{k} \right) \left( \text{d} \ln w_i \right) \right) \\
&- \sum_{k=1}^{K} \alpha_{n}^{k} \sum_{m=1}^{N} s_{nm}^{k} \left( \text{d} \ln \tau_{nm} + \text{d} \ln e_{m}^{k,r} \right) ) .
\end{aligned} \]

\[ \theta = \sigma - 1 . \]
E  Neoclassical Trade Model

In this Section of the online appendix, we examine income and welfare exposure in a neoclassical trade model with Armington differentiation of goods by origin and a general homothetic utility function. We consider a world consisting of many countries indexed by \(i, n \in \{1, \ldots, N\}\). Each country has an exogenous supply of \(\ell_n\) workers, who are endowed with one unit of labor that is supplied inelastically.

E.1 Consumer Preferences

The preferences of the representative consumer in country \(n\) are characterized by the following homothetic indirect utility function:

\[
u_n = \frac{w_n}{P(p_n)},
\]

(E.1)

where \(p_n\) is the vector of prices in country \(n\) of the goods produced by each country \(i\) with elements \(p_{ni} = \tau_{ni}w_i/z_i\); \(w_n\) is the income of the representative consumer in country \(n\); and \(P(p_n)\) is a continuous and twice differentiable function that captures the ideal price index. From Roy’s Identity, country \(n\)’s demand for the good produced by country \(i\) is:

\[
c_{ni} = c_{ni}(p_n) = -\frac{\partial(1/P(p_n))}{\partial p_{ni}}w_nP(p_n).
\]

(E.2)

E.2 Production

Each country’s good is produced with labor according to a constant returns to scale production technology, with productivity \(z_i\) in country \(i\). Markets are perfectly competitive. Goods can be traded between countries subject to iceberg trade costs, such that \(\tau_{ni} \geq 1\) units of a good must be shipped from country \(i\) in order for one unit to arrive in country \(n\) (where \(\tau_{ni} > 1\) for \(n \neq i\) and \(\tau_{nn} = 1\)). Therefore, the cost in country \(n\) of consuming one unit of the good produced by country \(i\) is:

\[
p_{ni} = \frac{\tau_{ni}w_i}{z_i}.
\]

(E.3)

E.3 Expenditure Shares

Country \(n\)’s expenditure share on the good produced by country \(i\) is:

\[
s_{ni} = \frac{p_{ni}c_{ni}(p_n)}{\sum_{\ell=1}^{N} p_{n\ell}c_{n\ell}(p_n)} = \frac{x_{ni}(p_n)}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)}.
\]

(E.4)

Totally differentiating this expenditure share equation, we get:

\[
ds_{ni} = \frac{dx_{ni}(p_n)}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)} - \frac{x_{ni}(p_n)}{\left[\sum_{\ell=1}^{N} x_{n\ell}(p_n)\right]^2} \left[\sum_{k=1}^{N} dx_{nk}(p_n)\right],
\]

54
\[ ds_{ni} = \frac{1}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)} \sum_{h=1}^{N} \frac{\partial x_{ni}(p_n)}{\partial p_{nh}} dp_{nh} - \frac{x_{ni}(p_n)}{\left(\sum_{\ell=1}^{N} x_{n\ell}(p_n)\right)^2} \sum_{k=1}^{N} \sum_{h=1}^{N} \frac{\partial x_{nk}(p_n)}{\partial p_{nh}} dp_{nh}, \]

\[ ds_{ni} = \sum_{h=1}^{N} \left( \frac{\partial x_{ni}(p_n) p_{nh}}{\partial p_{nh} x_{ni}} \right) - \sum_{k=1}^{N} s_{nk} \left( \frac{\partial x_{nk}(p_n) p_{nh}}{\partial p_{nh} x_{nk}} \right) \right] dp_{nh}, \]

\[ \frac{ds_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}}, \]

\[ d \ln s_{ni} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] d \ln p_{nh}, \]

\[
\text{where}
\theta_{nih} \equiv \left( \frac{\partial x_{ni}(p_n) p_{nh}}{\partial p_{nh} x_{ni}} \right) = \frac{\partial \ln x_{ni}}{\partial \ln p_{nh}}.
\]

Totally differentiating prices in equation (E.3), we have:

\[ \frac{dp_{ni}}{p_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i}, \]

\[ d \ln p_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i. \]

### E.4 Market Clearing

Market clearing requires that income in country \( i \) equals the expenditure on goods produced by that country:

\[ w_i \ell = \sum_{n=1}^{N} s_{ni} w_n \ell_n, \]

where for simplicity we begin by considering the case of balanced trade. Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[ \frac{dw_i}{w_i} = \frac{\sum_{n=1}^{N} ds_{ni} s_{ni} w_n \ell_n + \sum_{n=1}^{N} dw_n w_n}{\sum_{n=1}^{N} s_{ni} w_n \ell_n}, \]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \]
where we have defined $t_{in}$ as the share of country $i$’s income from market $n$:

$$t_{in} \equiv \frac{s_{ni} w_n f_n}{w_i l_i}.$$ 

Using our result for the derivatives of expenditure shares (E.5), we have:

$$\frac{d w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{d w_n}{w_n} + \left[ \frac{\sum_{h=1}^{N} \theta_{nwh} - \sum_{k=1}^{N} s_{nk} \theta_{nkh}}{w_h} \right] \frac{dp_{nh}}{p_{nh}} \right).$$

Using our results for the derivatives of prices, we have:

$$\frac{d w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{d w_n}{w_n} + \left[ \frac{\sum_{h=1}^{N} \theta_{nwh} - \sum_{k=1}^{N} s_{nk} \theta_{nkh}}{w_h} \right] \left[ \frac{d \tau_{nh}}{\tau_{nh}} + \frac{d w_h}{w_h} - \frac{d z_h}{z_h} \right] \right),$$

which can be written as:

$$d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left[ \frac{\sum_{h=1}^{N} \theta_{nwh} - \sum_{k=1}^{N} s_{nk} \theta_{nkh}}{w_h} \right] \left[ d \ln \tau_{nh} + d \ln w_h - d \ln z_h \right] \right), \tag{E.8}$$

## E.5 Welfare

Totally differentiating welfare, we have:

$$d u_n = d w_n \left( \frac{1}{P(p_n)} \right) + \frac{1}{P(p_n)} d \left( \frac{1}{P(p_n)} \right) w_n,$$

$$d u_n = d w_n \left( \frac{1}{P(p_n)} \right) + \sum_{i=1}^{N} w_n \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} d p_{ni},$$

$$d u_n = \frac{d w_n}{w_n} \left( \frac{1}{P(p_n)} \right) + \sum_{i=1}^{N} w_n \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} p_{ni} \frac{d p_{ni}}{p_{ni}},$$

$$\frac{d u_n}{u_n} = \frac{d w_n}{w_n} + \sum_{i=1}^{N} \frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} P(p_n) p_{ni} \frac{d p_{ni}}{p_{ni}}.$$

Now recall from equation (E.2) that:

$$c_{ni} = -\frac{\partial \left( \frac{1}{P(p_n)} \right)}{\partial p_{ni}} w_n P(p_n).$$

Using this result in our total derivative for welfare above, we obtain:

$$\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{i=1}^{N} c_{ni} p_{ni} \frac{d p_{ni}}{p_{ni}},$$

$$\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{i=1}^{N} s_{ni} \frac{d p_{ni}}{p_{ni}}.$$
which can be equivalently written as:

\[ d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ d \ln \tau_{ni} + d \ln w_i - d \ln z_i \right]. \quad (E.9) \]

The market clearing condition for each country (E.8) shapes how exogenous changes in productivities (\(d \ln z_i\)) and trade costs (\(d \ln \tau_{ni}\)) map into endogenous changes in wages (\(d \ln w_i\)). The utility function (E.9) determines how these endogenous changes in wages (\(d \ln w_i\)) and the exogenous changes in productivities (\(d \ln z_i\)) and trade costs (\(d \ln \tau_{ni}\)) translate into endogenous changes in welfare in each country (\(d \ln u_n\)). In general, both the own and cross-price elasticities of expenditure with respect to prices (\(\varnothing_{nih}\)) are variable and depend on the entire price vector (\(p_n\)), complicating the mapping from exogenous to endogenous variables.

E.6 Deviations from Constant Elasticity Import Demand

We now this neoclassical trade model with a variable trade elasticity to derive bounds for the sensitivity of our exposure measures to small deviations from a constant trade elasticity.

Expenditure Shares  We begin by reproducing the total derivative of the expenditure shares in this neoclassical trade model from equation (E.5) above:

\[ \frac{ds_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}}, \quad (E.10) \]

where \(\theta_{nih}\) is the elasticity of expenditure in country \(n\) on the good produced by country \(i\) with respect to the price of the good produced by country \(h\). In the special case of a constant elasticity import demand from Section B of this online appendix, we have:

\[ \theta_{nih} = \begin{cases} (s_{nh} - 1)(\sigma - 1) & \text{if } i = h \\ s_{nh}(\sigma - 1) & \text{otherwise} \end{cases}. \quad (E.11) \]

Using this result in equation (E.10), we obtain the total derivative for expenditure shares with a constant elasticity import demand system in equation (B.4) in Section B of this online appendix, as reproduced below:

\[ \frac{ds_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right). \quad (E.12) \]

We now allow for deviations from a constant elasticity import demand system by considering the following generalization of the specification in equation (E.11):

\[ \theta_{nih} = \begin{cases} (s_{nh} - 1)(\sigma - 1) + \omega_{nih} & \text{if } i = h \\ s_{nh}(\sigma - 1) + \omega_{nih} & \text{otherwise} \end{cases}. \quad (E.13) \]
where \( o_{nih} \) corresponds to the deviation of the variable trade elasticity from the constant trade elasticity. We assume that our generalized demand specification remains homothetic, which implies:

\[
0 = \sum_{k} x_{nk} \theta_{nkh},
\]

\[
= \sum_{k=1}^{N} x_{nk} \left((\sigma - 1) s_{nh} + o_{nkh}\right) - (\sigma - 1) x_{nh},
\]

\[
= \sum_{k=1}^{N} x_{nk} o_{nkh},
\]

where \( x_{ni} \) denotes country \( n \)'s expenditure on the goods produced by country \( i \). Therefore, homotheticity implies:

\[
\sum_{k=1}^{N} s_{nk} o_{nkh} = 0.
\]  (E.14)

Using our generalized demand specification (E.13) in the total derivative of expenditure shares in equation (E.10), we obtain:

\[
\frac{d s_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{d p_{nh}}{p_{nh}},
\]

\[
= (\sigma - 1) \left\{ \sum_{h=1}^{N} \left[ s_{nh} + \frac{o_{nih}}{\sigma - 1} - 1_{i=h} \right] - \sum_{k=1}^{N} s_{nk} \left( s_{nh} + \frac{o_{nkh}}{\sigma - 1} \right) - s_{nh} \right\} \frac{d p_{nh}}{p_{nh}} \},
\]

\[
= (\sigma - 1) \left\{ \sum_{h=1}^{N} s_{nh} + \frac{o_{nih}}{\sigma - 1} - \sum_{k=1}^{N} s_{nk} \frac{o_{nkh}}{\sigma - 1} \right\} \frac{d p_{nh}}{p_{nh}} - (1 + o_{nii}) \frac{d p_{ni}}{p_{ni}} \},
\]

Using our implication of homotheticity in equation (E.14), this expression simplifies to:

\[
\frac{d s_{ni}}{s_{ni}} = \left\{ \sum_{h=1}^{N} [(\sigma - 1) s_{nh} + o_{nih}] \frac{d p_{nh}}{p_{nh}} - [(\sigma - 1) + o_{nii}] \frac{d p_{ni}}{p_{ni}} \right\} .
\]  (E.15)

**Market Clearing**  We now follow the same approach to rewrite our total derivative of the market clearing condition in our neoclassical model in terms of our deviations \( o_{nih} \) from the constant trade elasticity. We can re-write equation (E.8) as follows:

\[
\frac{d w_{i}}{w_{i}} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{d w_{n}}{w_{n}} + \left( \sum_{h=1}^{N} [(\sigma - 1) s_{nh} + o_{nih}] \frac{d p_{nh}}{p_{nh}} - \frac{d p_{ni}}{p_{ni}} \right) \right),
\]

\[
\frac{d w_{i}}{w_{i}} = \sum_{n=1}^{N} t_{in} \left( \frac{d w_{n}}{w_{n}} + \left( \sum_{h=1}^{N} [(\sigma - 1) s_{nh} + o_{nih}] \frac{d p_{nh}}{p_{nh}} - \frac{d p_{ni}}{p_{ni}} \right) \right),
\]
\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] d \ln p_{nh} - d \ln p_{ni} \right) \right), \] (E.16)

where we have defined \( t_{in} \) as the share of country \( i \)'s income derived from market \( n \):

\[ t_{in} \equiv \frac{s_{ni} w_n \ell_{ni}}{w_i \ell_i}. \]

**Welfare**  From equation (E.9) in our neoclassical trade model, the total derivative of welfare takes the same form as in our constant elasticity trade model:

\[ d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ d \ln \tau_{ni} + d \ln w_i - d \ln z_i \right], \] (E.17)

**Wages and Productivity Shocks**  We consider small productivity shocks, holding constant bilateral trade costs:

\[ d \ln \tau_{ni} = 0, \quad \forall \ n, i \in N. \] (E.18)

We start with our expression for the log change in wages in equation (E.16) above. Using the total derivative of prices and our assumption of constant bilateral trade costs (E.18), we can write this expression for the log change in wages as:

\[
\begin{align*}
    d \ln w_i &= \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right) \right), \\
    d \ln w_i &= \sum_{n=1}^{N} t_{in} d \ln w_n + \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right), \\
    d \ln w_i &= \sum_{n=1}^{N} t_{in} d \ln w_n + \left( (\sigma - 1) \sum_{h=1}^{N} \sum_{n=1}^{N} t_{ih} s_{nh} d \ln w_h - d \ln z_h - [d \ln w_i - d \ln z_i] \right), \\
    d \ln w_i &= \sum_{n=1}^{N} t_{in} d \ln w_n + \left( (\sigma - 1) \sum_{h=1}^{N} m_{in} \left( d \ln w_h - d \ln z_h \right) + \sum_{n=1}^{N} o_{in} \left( d \ln w_h - d \ln z_h \right) \right), \\
    m_{in} &= \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \quad o_{in} \equiv \sum_{h=1}^{N} t_{ih} o_{inh},
\end{align*}
\]

which has the following matrix representation:

\[ d \ln w = T d \ln w + (\theta M + O) \left( d \ln w - d \ln z \right), \] (E.19)

\[ \theta = (\sigma - 1). \]
Welfare and Productivity Shocks  Using our assumption of constant bilateral trade costs (E.18) in our expression for the log change in welfare (E.17) above, we obtain:

$$
\ln w_n = \ln w_m - \sum_{m=1}^{N} n_{nm} (\ln w_m - \ln z_m),
$$

(E.20)

which has the following matrix representation:

$$
\ln u = \ln w - S (\ln w - \ln z).
$$

(E.21)

Sensitivity Bounds  We now establish our main result for the sensitivity of our exposure measures to small departures from a constant trade elasticity. Collecting the results for income and welfare in equations (E.19) and (E.21), we have:

$$
\ln w = T \ln w + (\theta M + O) \times (\ln w - \ln z),
$$

(E.22)

$$
\ln U = \ln w - S (\ln w - \ln z),
$$

(E.23)

where $O$ is a matrix with entries $O_{in} \equiv \sum_{h=1}^{N} t_{ih} o_{nh}$ capturing the average across markets $n$ of these deviations weighted by the share of country $i$’s income derived from each market, as shown in Section E.6 of the online appendix. Using homotheticity, we can write $O \equiv \epsilon \cdot \hat{O}$ as the product between a scalar $\epsilon > 0$ and a matrix $\hat{O}$ with an induced 2-norm equal to one ($||\hat{O}|| = 1$). By construction, $||O|| = \epsilon$. Using this representation, we can use results from matrix perturbation to obtain an upper bound on the sensitivity of income exposure to departures from the constant elasticity model, as a function of the observed trade matrices and the trade elasticity.

**Proposition A.1.** Let $\widetilde{\ln w}$ be the solution to the general Armington model in equation (E.8) of this online appendix and let $\ln w$ be the solution to the constant elasticity of substitution (CES) Armington model in equation (8) in the paper. Then

$$
\lim_{\epsilon \to 0} \frac{||\ln w - \ln w||}{\epsilon \cdot ||\ln w||} \leq \frac{\theta}{\theta + 1} ||(I - V)^{-1}|| ||I - (W + Q)^{-1}||.
$$

(E.24)

**Proof.** See Section E.7 below.

Given this upper bound on the sensitivity of income exposure from Proposition A.1, we can use equation (E.23) to compute the corresponding upper bound on the sensitivity of welfare exposure. All terms on the right-hand side of equation (E.24) can be computed using the observed trade matrices and the trade elasticity. Therefore, we can can compute these upper bounds for alternative assumed values of the trade elasticity. An immediate corollary of Proposition A.1 is that as the departures from the constant elasticity model become small ($\epsilon \to 0$), income and welfare exposure under a variable trade elasticity converge towards their values in our constant elasticity specification.
Corollary A.1. As the deviations from a constant elasticity import demand system become small (\(\lim \epsilon \to 0\)), the elasticities of country income and welfare exposure to productivity shocks in the general Armington model converge to those in the constant elasticity of substitution (CES) Armington model in Section 2.3 of the paper.

Proof. This corollary follows immediately from Proposition A.1. \(\square\)

From Corollary A.1, we can interpret the constant elasticity model as a limiting case of the variable elasticity model. In the neighborhood of this limiting case, our analytical expressions for the country income and welfare exposure to productivity shocks approximate those for the variable elasticity model. More generally, from Proposition A.1, we can provide an upper bound for the sensitivity of income and welfare exposure to departures from the constant elasticity model that be computed using the observed trade matrices and assumed values for the trade elasticity. As such, our characterization of income and welfare exposure for a constant trade elasticity provides a useful benchmark for interpreting the results of quantitative trade models outside of this class.

E.7 Proof of Proposition A.1

Proof. To economize on notation we let \(x \equiv \ln w\) and \(y \equiv \ln z\), and we derive the sensitivity of \(x\) to \(\epsilon\), holding \(y\) fixed. Using our assumption of constant returns to scale in production, it is without loss of generality to normalize the weighted mean of productivity shocks \(q'y = 0\) and thus \(Qy = 0\). Note \((I - V)x = -\frac{\theta}{\theta + 1}My\), \(W = -\frac{\theta}{\theta + 1}(I - V)^{-1}M\), \(x = (W + Q)y\), and \((W + Q)\) is invertible. Differentiating and noting \(dV = dM (= \epsilon O)\), we obtain

\[
-dVx + (I - V)dx = -\frac{\theta}{\theta + 1}dM y
\]

\[
\implies dx = (I - V)^{-1}dV(x - y)
\]

\[
\implies dx = (I - V)^{-1}dV(I - (W + Q)^{-1})x
\]

By the Cauchy-Schwarz inequality,

\[
\|dx\| \leq \|(I - V)^{-1}\|\|dV\|\|(I - (W + Q)^{-1})\|\|x\|.
\]

We obtain the proposition by substituting \(\|dV\| = \frac{\theta}{\theta + 1}\epsilon\) and \(\|dx\|/\|x\| = \lim_{\epsilon \to 0} \frac{\|d\ln w - d\ln w\|}{\epsilon\|d\ln w\|} \). \(\square\)
In Section 4.1 of the paper, we provide evidence on our economic exposure measures for our balanced panel of 143 countries over the 43 years from 1970-2012. In this section of the online appendix, we report additional empirical results for these exposure measures.

We begin by showing that our exposure measures are not only theoretically-consistent measures of sensitivity to foreign productivity growth, but are also hard to proxy with simpler measures of trading relationships between countries. In Table F.1, we regress our input-output welfare exposure ($U^{IO}$) and income exposure ($W^{IO}$) measures on a number of simpler measures of trading relationships between countries: (i) log value of bilateral trade; (ii) aggregate import shares (the expenditure share matrix from our single-sector model ($S^{SSM}$)); (iii) the expenditure share matrix from our input-output model ($S^{IO}$); (iv) the income share matrix from our input-output model ($T^{IO}$); and (v) the cross-substitution matrix from our input-output model ($M^{IO}$). In the first panel, we report results for income exposure. In the second panel, we report results for welfare exposure. In the remaining panels, we report results for the three separate components of the market-size effect, cross-substitution effect and cost of living effect. In the interests of brevity, we report results for the year 2000, but find the same pattern for all years.

Our income and welfare exposure measures have statistically significant correlations with all of these proxies. For income exposure in the first panel, we find that the regression R-squared is always less than 0.26. For welfare exposure in the second panel, we find that the expenditure share matrix from the single-sector model ($S^{SSM}$) has substantial explanatory power in Column (2), although around one third of the variation in welfare exposure remains unexplained. We find an even stronger relationship with the expenditure share matrix from the input-output model ($S^{IO}$) in Column (3), with the R-squared rising further to more than 0.80, highlighting the additional information in the input-output structure. We find a similar high R-squared for the income share ($T^{IO}$) and cross-substitution ($M^{IO}$) matrices from the input-output model in Columns (4) and (5), which is consistent with the close relationship between all three matrices in the input-output model.

In the remaining panels of the table, we demonstrate a similar pattern of results for the market-size, cross-substitution and cost of living components of our exposure measures. For both the market-size and cost of living effects, we find regression R-squared of less than 0.30. For the cross-substitution effect, we find higher R-squared for the income share matrix ($T^{IO}$) and the cross-substitution matrix ($M^{IO}$), which is consistent with the central role played by these matrices in determining the cross-substitution effect.
Table F.1: Correlations of Income ($W^{IO}$) and Welfare ($U^{IO}$) Exposure and their Components with Other Measures of Trading Relationships Between Countries

<table>
<thead>
<tr>
<th></th>
<th>(1) log value</th>
<th>(2) $S^{SSM}$</th>
<th>(3) $S^{IO}$</th>
<th>(4) $T^{IO}$</th>
<th>(5) $M^{IO}$</th>
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</thead>
<tbody>
<tr>
<td>$W^{IO}$</td>
<td>-0.000339***</td>
<td>-0.286***</td>
<td>-2.533**</td>
<td>-2.207***</td>
<td>-2.695***</td>
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<tr>
<td></td>
<td>(0.0000104)</td>
<td>(0.0250)</td>
<td>(0.390)</td>
<td>(0.291)</td>
<td>(0.241)</td>
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<tr>
<td>Observations</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
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<td>20592</td>
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<tr>
<td>R-squared</td>
<td>0.0435</td>
<td>0.165</td>
<td>0.152</td>
<td>0.128</td>
<td>0.254</td>
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<tr>
<td>$U^{IO}$</td>
<td>0.00000571***</td>
<td>0.0120***</td>
<td>0.123***</td>
<td>0.113***</td>
<td>0.0976***</td>
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<td></td>
<td>(0.000000231)</td>
<td>(0.000708)</td>
<td>(0.00357)</td>
<td>(0.00621)</td>
<td>(0.00448)</td>
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<tr>
<td>R-squared</td>
<td>0.0290</td>
<td>0.688</td>
<td>0.843</td>
<td>0.781</td>
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<th>(4) $T^{IO}$</th>
<th>(5) $M^{IO}$</th>
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<tr>
<td>Market Size$^{IO}$</td>
<td>-0.000302***</td>
<td>-0.223**</td>
<td>-1.890*</td>
<td>-1.358*</td>
<td>-2.005***</td>
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<td>(0.00000996)</td>
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<td>(0.379)</td>
<td>(0.289)</td>
<td>(0.251)</td>
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<tr>
<td>R-squared</td>
<td>0.0377</td>
<td>0.109</td>
<td>0.0924</td>
<td>0.0529</td>
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<tr>
<td>Cross Substitution$^{IO}$</td>
<td>-0.000370***</td>
<td>-0.0637***</td>
<td>-0.644***</td>
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<td>-0.690***</td>
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<tr>
<td></td>
<td>(0.00000147)</td>
<td>(0.00430)</td>
<td>(0.0391)</td>
<td>(0.00315)</td>
<td>(0.0205)</td>
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<td>20592</td>
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<tr>
<td>R-squared</td>
<td>0.0273</td>
<td>0.431</td>
<td>0.517</td>
<td>0.998</td>
<td>0.878</td>
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<th>(3) $S^{IO}$</th>
<th>(4) $T^{IO}$</th>
<th>(5) $M^{IO}$</th>
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</thead>
<tbody>
<tr>
<td>Price Index$^{IO}$</td>
<td>0.000345***</td>
<td>0.298***</td>
<td>2.657***</td>
<td>2.320***</td>
<td>2.793***</td>
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<tr>
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<td>(0.0000105)</td>
<td>(0.0248)</td>
<td>(0.389)</td>
<td>(0.291)</td>
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<tr>
<td>Observations</td>
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<td>20592</td>
<td>20592</td>
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<td>20592</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0443</td>
<td>0.177</td>
<td>0.165</td>
<td>0.139</td>
<td>0.269</td>
</tr>
</tbody>
</table>

Note: Observations are a cross-section of exporting and importing countries in the year 2000; $W^{IO}$ is our income exposure measure for the input-output model from Section 2.3 of the paper; $U^{IO}$ is our welfare exposure measure for the input-output model from Section 2.3 of the paper; log value is the log of one plus the value of bilateral trade; $S^{SSM}$ is the share of each exporter in aggregate importer expenditure (the expenditure share matrix in the single-sector model); $S^{IO}$ is the expenditure share matrix in the input-output model; $T^{IO}$ is the income share matrix in the input-output model; $M^{IO}$ is the cross-substitution matrix in the input-output model; table reports the regressions of $W^{IO}$ and $U^{IO}$ (rows) on alternative measures of trading relationships between countries (columns); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.
Across all of these specifications, we find that none of these proxies fully captures our theoretically-consistent measures of income and welfare exposure. Furthermore, it would be hard to infer income and welfare exposure without our measures, since the coefficients in these regressions do not have a structural interpretation, and could not be estimated without our measures.

In the remainder of this section, we show that our welfare exposure measure captures the large-scale changes in the network of trade relationships that occurred over our sample period, including regional integration in North America following the North American Free Trade Agreement (NAFTA), the reorientation of European trade relationships following the fall of the Iron Curtain, and the reorganization of trading patterns in East Asia following the emergence of China into the global economy. In Figure F.1 we illustrate global welfare exposure to productivity shocks in 1970, 1985, 2000 and 2012 using a network graph, where the nodes are countries and the edges capture bilateral welfare exposure. For legibility, we display the 50 largest countries in terms of GDP and the 200 edges with the largest absolute values of bilateral welfare exposure.1 The size of each node captures the importance of each country as a source of productivity shocks (as a source of welfare exposure for other countries); the arrow for each edge shows the direction of bilateral welfare exposure (from the source of the productivity shock to the exposed country); and the thickness of each edge shows the absolute magnitude of the bilateral welfare exposure. Countries are grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random).

At the beginning of our sample period in 1970, the global network of welfare exposure is dominated by the U.S., Germany and other Western industrialized countries (top-left panel). Moving forward to 1985, we see the emergence of Japan and a cluster of Newly Industrialized Countries (NICs) in Asia, and we observe Western Europe increasingly emerging as a separate cluster of interdependent nations. By the time we reach 2000, the separate clusters of countries in Asia and Western Europe become even more apparent, with China beginning to displace Japan at the center of the Asian cluster. By the end of our sample period in 2012, China replaces the U.S. at the center of the global network of welfare exposure, with the US more tightly connected to China and other Asian countries than to the cluster of Western European countries. Therefore, we find substantial changes, not only in the mean and dispersion of welfare exposure, but also in the network of bilateral interdependencies between countries.

1 All of the bilateral welfare exposure links shown in the figure are positive.
We next provide further evidence in the change in networks of welfare exposure in North America following the North American Free Trade Agreement (NAFTA), in Central Europe following the fall of the Iron Curtain, and in Asia following the emergence of China into the global economy. We use chord or radial network diagrams, as used for example in comparative genomics in Krzywinski et al. (2009) and for bilateral migration in Sander et al. (2014).

In Figure F.2, we show welfare exposure in 1970 and 2012 for U.S., Canada, Mexico, Japan and China, where each country is labelled by its three-letter International Organization for Standardization (ISO) code. These countries are arranged around a circle, where the size of the inner segment for each country shows its overall outward exposure (the effect of its productivity shocks.

To ensure a consistent treatment of countries over time, we manually assign some three-letter codes, such as the code USR for the members of the former Soviet Union.
on other countries), and the gap between the inner and outer segments shows its overall inward exposure (the effect of foreign productivity shocks upon it). Arrows emerging from the inner segment for each country show the bilateral impact of its productivity shocks on welfare in other countries. Arrows pointing towards the gap between the inner and outer segments show the bilateral impact of other countries’ productivity growth on its welfare.\(^3\) In 1970, the network is dominated by the effect of US productivity shocks on welfare in the other countries, and Japan is substantially more connected to the network than China. By 2012, following Mexican trade liberalization in 1987, the Canada-US Free Trade Agreement (CUSFTA) in 1988 and the North American Free Trade Agreement (NAFTA) in 1994, we observe much deeper integration between the three North American economies. Additionally, we find a reversal of the relative positions of the two Asian economies, with China substantially more integrated into the network than Japan.

Figure F.2: North American Welfare Exposure, 1970 and 2012

(a) 1970

(b) 2012

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 2 of the paper.

In Figure F.3, we display welfare exposure for a broader group of Asian countries. Three features stand out. First, we again find a dramatic change in the relative positions of Japan and China. Whereas in 1970 Japan dominated the network of welfare exposure, in 2012 this position is firmly occupied by China. Second, Vietnam becomes both substantially more exposed to foreign productivity shocks and a much more important source of these productivity shocks for other countries.

\(^3\)We omit own exposure to focus on the impact of foreign productivity shocks on country welfare. Almost all values of our welfare exposure measure in these diagrams are positive. For ease of interpretation, we add a constant to our welfare exposure measure in each year, such that its minimum value is zero, which implies that these diagrams show the impact of the productivity shock on relative levels of welfare.
countries, following its trade liberalization. Third, the overall network of welfare exposure is much denser in 2012 than in 1970, consistent with greater trade integration among these Asian countries increasing their economic interdependence on one another.

Figure F.3: Asian Welfare Exposure, 1970 and 2012

In Figure F.4, we show bilateral welfare exposure in Central Europe before and after the fall of the Iron Curtain. In 1988 immediately before this event, we observe strong connections between the countries of the former Soviet Union (USR) and Eastern European nations such as the former Czechoslovakia (CSK). By 2012, these connections have substantially weakened, and we observe growing connections between Western European countries such as Italy and Eastern European nations. Although Germany is here the aggregation of the former and East and West Germanies in all years, we also observe a strengthening of its position at the center of the network of welfare exposure. More broadly, we also find an increase in the overall density of connections over time, consistent with trade liberalization increasing countries’ economic interdependence.
Figure F.4: Central European Welfare Exposure, 1988 and 2012

(a) 1988

(b) 2012

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 2 of the paper.

G Economic and Political Friends and Enemies

In Section 4.3 in the paper, we provide evidence on the role of changes in economic exposure in inducing changes in political alignment, using variation across all country-partner pairs from the large-scale decline in the cost of air travel over our sample period. The results reported in the paper use the panel data specification from equation (14) in the paper. In this section of the online appendix, we show that we find the same pattern of results in a long-differences specification using five-year differences.

Instrumental Variables Specification  We now introduce our baseline regression specification connecting political and economic interests. In general, a country’s political alignment towards a trade partner could depend on both the sensitivity (elasticity) of its welfare to productivity growth in that partner and the sign and magnitude of the productivity growth. We use our theoretically-consistent measures of the sensitivity of importer welfare to productivity growth in each exporter ($U_{nit}^{IO}$) from Section 2.3 in the paper. We control separately for the sign and magnitude of exporter productivity growth using exporter-year fixed effects, exploiting the property that productivity growth is common across trade partners. We also control separately for importer-year fixed effects, which capture importer expenditure and price indexes, and other macro shocks that are common across trade partners. In particular, we consider the following
second-stage regression specification relating changes in bilateral political alignment ($\Delta A_{nit}$) to changes in bilateral welfare exposure ($\Delta U_{nit}^{IO}$) for importer $n$ and exporter $i$ at time $t$:

$$\Delta A_{nit} = \beta \Delta U_{nit}^{IO} + \eta_{nit}^A + \mu_{it}^A + \epsilon_{nit}^A,$$ (G.1)

where we have differenced out any importer-exporter fixed effect; $\eta_{nit}^A$ and $\mu_{it}^A$ are importer-year and exporter-year fixed effects; and $\epsilon_{nit}^A$ is a stochastic error. We expect countries that benefit more from a partner’s productivity growth (more positive or less negative welfare exposure $U_{nit}^{IO}$) to be more politically aligned with that partner (higher $A_{nit}$). We pool the five-year long differences over time. We report standard errors clustered by country-partner pair to allow for serial correlation in the error term over time.

We consider the following first-stage regression, in which we directly instrument welfare exposure using our interaction between a time trend and log air distance:

$$\Delta U_{nit}^{IO} = \gamma [\ln (\text{airst}_{nit} \times \text{trend}_t)] + \eta_{nit}^U + \mu_{it}^U + \epsilon_{nit}^U,$$

where airdist$_{ni}$ is the population-weighted average of the great circle distance between the largest cities within countries; the interaction between the time trend and log air distance captures secular changes in relative bilateral trade costs at long versus short distances because of improvements in the technology of air travel over our sample period; we have again differenced out any importer-exporter fixed effect; the importer-year ($\eta_{nit}^U$) and exporter-year ($\mu_{it}^U$) fixed effects control for changes over time in country income and price indexes and macro shocks; and $\epsilon_{nit}^U$ is a stochastic error.

### Bilateral Political Alignment Using UNGA Voting Data

In Table G.1, we report the results of estimating this long-differences specification using two-stage least squares and our measures of bilateral political alignment based on UNGA voting data. Columns (1)-(8) of Table G.1 are long-differences specifications that correspond to the panel data specifications in Columns (1)-(8) of Panel B in Table 3 in the paper. Columns (1), (2), (3) and (4) of Table G.1 use the $k$-score, $S$-score, $\pi$-score and ideal distance measures of bilateral political alignment, respectively. Across all four columns, we find positive and statistically significant coefficient on welfare exposure, consistent with the results from the panel data specification in the paper. Therefore, increases in bilateral welfare exposure predicted by our instrument raise bilateral political alignment. In Columns (5)-(8) of Table G.1, we show that these results are robust to controlling for the aggregate share of importer expenditure on each exporter ($S^{SSM}$), again consistent with the panel data results in the paper. Hence, we again find that our measures of welfare exposure are not completely captured by simpler measures of trading relationships between countries. Across all specifications, our instrument has power in the first-stage regression, as shown by the first-stage F-statistic reported at the bottom of each column.
Table G.1: Political and Economic Friends (Long Differences Specification, UN Voting)

<table>
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<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
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</thead>
<tbody>
<tr>
<td>$\Delta U_{nit}$</td>
<td>188.8***</td>
<td>62.79**</td>
<td>226.4***</td>
<td>262.4**</td>
<td>331.3***</td>
<td>110.3**</td>
<td>400.7***</td>
<td>556.8**</td>
</tr>
<tr>
<td></td>
<td>(56.09)</td>
<td>(29.06)</td>
<td>(63.62)</td>
<td>(125.1)</td>
<td>(109.4)</td>
<td>(53.83)</td>
<td>(126.2)</td>
<td>(266.0)</td>
</tr>
<tr>
<td>$\Delta S_{nit}^{SSM}$</td>
<td>-13.74**</td>
<td>-4.579*</td>
<td>-16.80**</td>
<td>-29.31**</td>
<td>(5.907)</td>
<td>(2.591)</td>
<td>(7.025)</td>
<td>(14.42)</td>
</tr>
</tbody>
</table>

Note: Pooled five-year long differences from 1970-2010; all specifications include exporter-year and importer-year fixed effects; $A_{nit}^\kappa$, $A_{nit}^S$, and $A_{nit}^{\pi}$ are the $\kappa$-score, $S$-score and $\pi$-score measures of the bilateral similarity of countries’ votes in the UNGA, respectively; $A_{nit}^{ideal}$ is the bilateral difference in countries’ ideal points from the UNGA voting data; $U_{nit}^{IQ}$ is welfare exposure from our multi-sector model with input-output linkages; all specifications instrument the change in welfare exposure with the change in an interaction between a linear time trend and log air distance; first-stage F-statistic is a test of the statistical significance of the excluded exogenous variables in the first-stage regression; the second-stage R-squared is not reported for these IV specifications, because it does not have a meaningful interpretation; standard errors in parentheses are clustered by exporter-importer pair; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

Alternative Measures of Bilateral Political Alignment  In Table G.2, we re-estimate this long-differences specification using our alternative measures of bilateral political alignment, based on strategic rivalries and formal alliances. Columns (1)-(4) of Table G.2 are long-differences specifications that correspond to the panel data specifications in Columns (5)-(8) in Table 4 in the paper. Columns (5)-(9) of Table G.2 are long-differences specifications that correspond to the panel data specifications in Columns (1)-(5) in Panel B of Table 5 in the paper. For all four strategic rivalries in Columns (1)-(4) of Table G.2, we find negative and statistically significant coefficients on welfare exposure, consistent with the panel data results in the paper. For all five measures of formal alliances, we find positive and statistically significant coefficients on welfare exposure, again confirming the panel data results in the paper. Across all specifications, our instrument has power in the first-stage regression, as shown by the first-stage F statistics reported at the bottom of each column.

Therefore, regardless of whether we consider measures of political alignment based on US voting, strategic rivalries or formal alliances, we find that increases in bilateral welfare exposure predicted by our instrument raise bilateral political alignment. We find that this pattern of results holds whether we consider the panel data specifications in the paper or the long-differences specifications in this section of the online appendix.
Table G.2: Political and Economic Friends (Long Differences Specification, Strategic Rivalries and Formal Alliances)

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<tr>
<td>(U_{nit}^{ITO})</td>
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Note: Pooled five-year long differences from 1970-2010; all specifications include exporter-year and importer-year fixed effects; \(A_{nit}^{Any}\), \(A_{nit}^{Pos}\), \(A_{nit}^{Spa}\), and \(A_{nit}^{Id}\) are indicator variables for any, positional, spatial and ideological strategic rivalries, respectively; \(A_{nit}^{AllAny}\), \(A_{nit}^{AllDef}\), \(A_{nit}^{AllNeu}\), \(A_{nit}^{AllNon}\), and \(A_{nit}^{AllEnt}\) are indicator variables for any, defense, neutrality, non-aggression and entente formal alliances, respectively; \(U_{nit}^{ITO}\) is welfare exposure from our multi-sector model with input-output linkages; all specifications instrument the change in welfare exposure with the change in an interaction between a linear time trend and log air distance; first-stage F-statistic is a test of the statistical significance of the excluded exogenous variables in the first-stage regression; the second-stage R-squared is not reported for these IV specifications, because it does not have a meaningful interpretation; standard errors in parentheses are clustered by exporter-importer pair; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

H Specification Checks

In this section of the online appendix, we provide further evidence on the specification checks in Section 5 of the paper. In Subsection H.1, we report additional results for our overidentification check using preferential trade agreements (PTAs). In Subsection H.2, we present additional evidence on the quality of the approximation of our linearization to the full non-linear model solution. We show that our measures of the sensitivity of country welfare to foreign productivity growth are not only exact for small shocks, but are close to exact for empirically-reasonable productivity shocks and trade elasticities.

H.1 Preferential Trade Agreements (PTAs)

In this section, we use separate data on PTAs to provide an overidentification check on our economic exposure measures. We first examine the determinants of PTAs: If our exposure measures correctly capture gains from economic integration, we would expect them to predict self-selection into PTAs. We next examine the effects of PTAs: If our exposure measures correctly capture economic interdependence between countries, we would expect to observe systematic changes in these measures following the formation of PTAs.

Our empirical analysis builds on a large empirical literature on the determinants and effects of PTAs, including Limao (2007), Romalis (2007), Estevadeordal et al. (2008) and Freund and Ornelas (2010). Since tariff barriers for most countries have been low for several decades, much
of the recent debate about PTAs has focused on non-tariff barriers, regulatory standards and deep integration, as emphasized for example in Grossman et al. (2021). Therefore, we interpret PTAs as a reduction in bilateral trade costs between member countries, abstracting from changes in tariff revenue.

**H.1.1 Determinants of PTAs**

We first examine the ability of our exposure measures at the beginning of our sample in 1970 to predict subsequent selection into PTAs from 1971-2012. We consider the following linear probability model relating the probability that an exporter-importer pair is a member of a PTA in any year from 1971-2012 to welfare exposure in 1970:

\[
\mathbb{I}_{n_i}^{PTA} = \alpha + \beta U_{t,n_i}^{IO} + \gamma \mathbb{I}_{n_i}^{PTA} + \delta X_{n_i} + \eta_n + \mu_i + \epsilon_{n_i},
\]  

(H.1)

where \(\mathbb{I}_{n_i}^{PTA}\) is an indicator variable that equals one if importer \(n\) and exporter \(i\) are members of a PTA in any year \(t \in [1971, 2012]\) and zero otherwise; \(U_{t,n_i}^{IO}\) is our measure of welfare exposure to reductions in bilateral trade costs between importer \(n\) and exporter \(i\) in 1970, which is computed in a similar way to welfare exposure to productivity growth in our input-output specification, as shown in Section D.6.12 of this online appendix; \(\mathbb{I}_{n_i}^{PTA}\) is an indicator variable that equals one if importer \(n\) and exporter \(i\) are members of a PTA in year 1970 and zero otherwise, and controls for past trade agreements for each exporter-importer pair; \(X_{n_i}\) are controls for other characteristics of the importer-exporter pair; \(\eta_n\) and \(\mu_i\) are importer and exporter fixed effects, respectively, which control for unobserved heterogeneity across importers and exporters in their propensity to join PTAs; and \(\epsilon_{n_i}\) is a stochastic error.
Table H.1: Selection into Future Preferential Trade Agreements (PTAs) and Past Welfare Exposure to Bilateral Trade Cost Reductions ($U_{\tau,n0}$)

<table>
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<tr>
<td>$U_{\tau,1970}$</td>
<td>4.874***</td>
<td>3.021**</td>
<td>8.150***</td>
<td>2.754**</td>
<td>2.716**</td>
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<td></td>
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<td>0.554***</td>
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<td>$PTA_{1980}$</td>
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<tr>
<td>R-squared</td>
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<td>0.333</td>
<td>0.319</td>
<td>0.363</td>
<td>0.371</td>
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</table>

Note: Observations are a cross-section of exporter-importer pairs; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $PTA_{1971}$ is a dummy variable that is equal to one if an exporter-importer pair is a member of a preferential trade agreement (PTA) from 1971-2012; $PTA_{1981}$, $PTA_{1991}$, and $PTA_{1980}$ are defined analogously; $U_{\tau,n0}$ is welfare exposure to bilateral trade cost reductions in 1970 in our input-output specification; $PTA_{1970}$ is a dummy variable that is equal to one if an exporter-importer pair is a member of a PTA in 1970 or earlier; log value 1970 is the log of one plus the value of bilateral trade flows in 1970; $SSM_{1970}$ is the share of each exporter in aggregate importer expenditure in 1970 (the expenditure share matrix in the single-sector model); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

In Column (1) of Table H.1, we report the results of estimating equation (H.1) including only initial welfare exposure ($U_{\tau,n0}$), past trade agreements ($PTA_{1971}$) and the exporter and importer fixed effects. We find a positive and statistically significant coefficient on initial welfare exposure ($\beta$), highlighting that our measure of the welfare gains from bilateral trade cost reductions has predictive power for subsequent trade integration in the data. One potential concern about this specification is that initial welfare exposure could be proxying for initial trade flows or other bilateral characteristics. In Columns (2) and (3), we augment this specification with the log of initial bilateral trade in 1970 and the share of importer $n$’s expenditure on exporter $i$ in 1970, respectively. We continue to find a positive and statistically significant coefficient on welfare exposure, again.
highlighting that our exposure measures cannot be completely captured using simpler measures of trading relationships between countries.\textsuperscript{4} In Columns (4) and (5), we consider more stringent specifications, in which we predict subsequent trade agreements from 1981-2012 and 1991-2012, respectively, using initial welfare exposure in 1970. Again we continue to find that our welfare exposure measure has predictive power for subsequent trade integration in the data.

In Table H.2, we report analogous regressions of future membership of a PTA from 1971-2012 on past income exposure to bilateral reductions in trade costs ($W_{tO}$) in 1970 for each importer-exporter pair. We find a similar pattern of results for income exposure as for welfare exposure above. Countries with larger income gains from bilateral trade cost reductions in 1970 are more likely to form future PTAs from 1971-2012. We find that these results for income exposure are marginally weaker than those for welfare exposure, which is consistent with income exposure being only part of welfare exposure.

\textsuperscript{4}We find the same pattern of results if we instead control directly for log bilateral distance, with an estimated coefficient (standard error) on initial welfare exposure of 1.324 (0.646).
Table H.2: Selection into Future Preferential Trade Agreements (PTAs) and Past Income Exposure to Bilateral Trade Cost Reductions ($W^{I/O}_f$)

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<td>0.566*</td>
<td>0.350</td>
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<td>$PTA_{1980}^{1981}$</td>
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<tr>
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<td>17,292</td>
<td>17,292</td>
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</tr>
<tr>
<td>R-squared</td>
<td>0.316</td>
<td>0.332</td>
<td>0.317</td>
<td>0.362</td>
<td>0.370</td>
</tr>
</tbody>
</table>

Note: Observations are a cross-section of importer-exporter pairs; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $PTA_{1971}^{2012}$ is a dummy variable that is equal to one if an exporter-importer pair is a member of a preferential trade agreement (PTA) from 1971-2012; $PTA_{1981}^{2012}$, $PTA_{1991}^{2012}$, $PTA_{1981}^{1971}$ and $PTA_{1991}^{1971}$ are defined analogously; $W^{I/O}_f$ 1970 is welfare exposure to bilateral trade cost reductions in 1970 in the input-output model; PTA 1970 is a dummy variable that is equal to one if an exporter-importer pair is a member of a PTA in 1970 or earlier; log value 1970 is the log of one plus the value of bilateral trade flows in 1970; $S^{SSM}$ 1970 is the share of each exporter in aggregate importer expenditure in 1970 (the expenditure share matrix in the single-sector model); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

H.1.2 Effects of PTAs

We next examine the ability of our exposure measures to detect the effects of PTAs in increasing economic interdependence between countries, as measured by welfare exposure to either productivity growth or bilateral trade cost reductions.

**Econometric Specification** We use an event-study specification to estimate treatment effects for country-partner pairs that form a PTA, both before and after the formation of the agreement. This specification has a “differences-in-differences” interpretation, where the first difference is
between treated and untreated importer-exporter pairs, and the second difference is over time. In empirical settings with variable timing of the treatment and treatment heterogeneity, a recent empirical literature has highlighted that the interpretation of the two-way fixed effects estimator can sometimes be problematic, as discussed in Chaisemartin and D’Haultfouillée (2020), Borusyak et al. (2021) and Goodman-Bacon (2021). We begin by reporting results using the two-way fixed effects estimator, before later showing that we find the same pattern of results using the alternative event-study estimators of Chaisemartin and D’Haultfouillée (2020) and Borusyak et al. (2021). We consider the following regression specification for importer n and exporter i at time t:

$$U_{nit}^{IO} = \sum_{s \in \{s_{-}, s_{+}\}} \beta_s(1_{nit}^{PTA} \times 1_s) + \xi_{ni} + d_{ct} + h_{nit}, \quad (H.2)$$

where $1_{nit}^{PTA}$ is a dummy variable that equals one if an importer-exporter pair ever signs a trade agreement during our sample period; $s$ is a treatment year index, which equals zero in the year an importer-exporter pair joins a PTA, such that negative values of $s$ indicate years before joining a PTA, and zero or positive values represent years after joining a PTA; $1_s$ is a dummy variable that equals one in treatment year $s$ and zero otherwise; we choose treatment year minus one as the excluded category; $\xi_{ni}$ are importer-exporter pair fixed effects, which control for time-invariant factors that affect both bilateral welfare exposure and whether an importer-exporter joins a PTA, such as bilateral distance, contiguity, etc; $d_{ct}$ are continent-year dummies, which control for secular changes over time in welfare exposure and the propensity to join PTAs; and $h_{nit}$ is a stochastic error. We report standard errors clustered by exporter-importer pair to allow for serial correlation in the error term over time.

The key coefficients of interest are $\beta_s$ on the treatment-year interactions, which capture the impact of the PTA on welfare exposure in treatment year $s$, relative to the excluded category of treatment year minus one. The exporter-importer fixed effects control for the non-random selection of exporter-importer pairs into PTAs based on time-invariant factors. Therefore, if exporter-importer pairs with high levels of welfare exposure are more likely to form PTAs in all years, this is controlled for in the exporter-importer fixed effect. The key identifying assumption in equation (H.2) is parallel trends between the treatment and control group within continents. As a check on this identifying assumption, we include the treatment-year interactions for years both before and after joining a PTA, which allows us to provide evidence on whether treated exporter-importer pairs exhibit different trends from untreated pairs even before joining a PTA.

**Welfare Exposure** In Figure H.1a, we display the estimated treatment-year interactions and 95 percent confidence intervals for welfare exposure to productivity growth in our input-output model ($U_{nit}^{IO}$) for our baseline specification using the two-way fixed effects estimator. In Figure
H.1b, we show the corresponding estimates for welfare exposure to reductions in bilateral trade costs in our input-output model ($U_{IO,nit}$). In both cases, we find no evidence of statistically significant differences in trends between the treatment and control group in the years leading up to the formation of a PTA, which is consistent with the idea that the inclusion of the exporter-importer fixed effect largely controls for the non-random selection into PTAs found in the previous subsection. In contrast, we observe a substantial and statistically significant increase in welfare exposure to both productivity shocks ($U_{IO,nit}$) and reductions in bilateral trade costs ($U_{IO,\tau,nit}$) immediately after the formation of a PTA. This pattern of results is consistent with the idea that PTAs are successful in promoting economic interdependence between member countries, which thereby raises member countries’ exposure to productivity growth and bilateral trade cost reductions in other member countries.

Figure H.1: Estimated Treatment Effects of PTAs on Welfare Exposure

(a) Exposure to Productivity Shocks ($U_{IO,nit}$)

(b) Exposure to Bilateral Trade Cost Shocks ($U_{IO,\tau,nit}$)

Note: Estimated treatment-year interactions ($\beta_\tau$) from the event-study specification in equation (H.2) using the two-way fixed effects estimator; Figure H.1b shows results for the welfare exposure of importer $n$ to productivity growth in exporter $i$ at time $t$ ($U_{IO,nit}$) in our input-output specification; Figure H.1b displays results for the welfare exposure of importer $n$ to reductions in bilateral trade costs with exporter $i$ at time $t$ ($U_{IO,\tau,nit}$) in our input-output specification; all regression specifications include exporter-importer fixed effects and continent-year fixed effects; standard errors clustered by exporter-importer pair to allow for serial correlation in the error term over time.

**Income Exposure** In Figure H.2, we show that we find a similar pattern of results for income exposure to productivity growth ($W_{IO,nit}$) and bilateral trade cost reductions ($W_{IO,\tau,nit}$) in our input-output model. We find that these results for income exposure are marginally weaker than those for welfare exposure, which is consistent with the idea that income exposure is only part of welfare exposure, and regional integration reduces the cost of living between member countries.
Figure H.2: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Income Exposure

(a) Importer Income Exposure to Exporter Productivity Growth ($W_{nlt}^{IO}$)

(b) Importer Income Exposure to Exporter Bilateral Trade Cost Reductions ($W_{\tau,nlt}^{IO}$)

Note: Estimated treatment-year interactions ($\beta_\tau$) from the event-study specification in equation (H.2) using the two-way fixed effects estimator; Figure H.2a shows results for an importer’s income exposure to productivity growth in an exporter ($W_{nlt}^{IO}$) in our input-output model; Figure H.2b displays results for an importer’s income exposure to reductions in bilateral trade costs with an exporter ($W_{\tau,nlt}^{IO}$) in our input-output model; all specifications include importer-exporter fixed effects and continent-year fixed effects; standard errors clustered by importer-exporter pair to allow for serial correlation in the error term over time.

Robustness to Controlling for Bilateral Trade  In Figure H.3, we show that we find a similar pattern of results for welfare exposure to productivity growth and trade cost reductions ($U_{nlt}^{IO}$ and $U_{\tau,nlt}^{IO}$) as reported in Figure H.1 above, once we additionally control for the log value of bilateral trade between an importer-exporter in each year. Similarly, we find that our income exposure results in Figure H.2 above are robust to also controlling for the log value of bilateral trade between an importer-exporter in each year (not shown to conserve space). Therefore, we again find that our exposure measures cannot be captured by simpler measures of trading relationships between countries.
Figure H.3: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Welfare Exposure (Controlling for Trade Flows)

(a) Importer Welfare Exposure to Exporter Productivity Growth ($U_{nit}^{IO}$)

(b) Importer Welfare Exposure to Exporter Bilateral Trade Cost Reductions ($U_{\tau,nit}^{IO}$)

Note: Estimated treatment-year interactions ($\beta_3$) from the event-study specification in equation (H.2) using the two-way fixed effects estimator; Figure H.3a shows results for the welfare exposure of importer $n$ to productivity growth in exporter $i$ at time $t$ ($U_{nit}^{IO}$) in our input-output model; Figure H.3b displays results for the welfare exposure of importer $n$ to reductions in bilateral trade costs with exporter $i$ at time $t$ ($U_{\tau,nit}^{IO}$) in our input-output model; both specifications include the log of one plus the value of bilateral trade between an exporter-importer pair in each year, importer-exporter fixed effects and continent-year fixed effects; standard errors clustered by importer-exporter pair to allow for serial correlation in the error term over time.

**Alternative Event-Study Estimators** The event-study estimates above the two-way fixed effects estimator. In Figure H.4, we show that we find the same pattern of results using the alternative event-study estimator of Chaisemartin and D’Haultfleuille (2020), which only exploits variation from transitions from untreated to treated status. In Figure H.5, we show that we also find the same pattern of results using the alternative event-study estimator of Borusyak et al. (2021). Therefore, our findings using our exposure measures of an increase in economic interdependence between countries following regional integration are robust to the use of both of these alternative event-study estimators to control for the variable timing of the treatment and potential treatment heterogeneity.
Figure H.4: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Welfare Exposure (Chaisemartin and D’Haultflocouille (2020) Estimator)

(a) Importer Welfare Exposure to Exporter Productivity Growth ($U_{nit}^{IO}$)

(b) Importer Welfare Exposure to Exporter Bilateral Trade Cost Reductions ($W_{\tau,nit}^{IO}$)

Note: Estimated treatment-year interactions ($\beta_s$) from the event-study specification in equation (H.2) using the Chaisemartin and D’Haultflocouille (2020) Estimator; Figure H.4a shows results for the welfare exposure of importer $n$ to productivity growth in exporter $i$ at time $t$ ($U_{nit}^{IO}$) in our input-output model; Figure H.4b displays results for the welfare exposure of importer $n$ to reductions in bilateral trade costs with exporter $i$ at time $t$ ($U_{\tau,nit}^{IO}$) in our input-output model; both specifications include importer-exporter fixed effects and continent-year fixed effects; standard errors based on 50 bootstrap replications.

Figure H.5: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Welfare Exposure (Borusyak et al. (2021) Estimator)

(a) Importer Welfare Exposure to Exporter Productivity Growth ($U_{nit}^{IO}$)

(b) Importer Welfare Exposure to Exporter Bilateral Trade Cost Reductions ($W_{\tau,nit}^{IO}$)

Note: Estimated treatment-year interactions ($\beta_s$) from the event-study specification in equation (H.2) using the Borusyak et al. (2021) Estimator; Figure H.5a shows results for the welfare exposure of importer $n$ to productivity growth in exporter $i$ at time $t$ ($U_{nit}^{IO}$) in our input-output model; Figure H.5b displays results for the welfare exposure of importer $n$ to reductions in bilateral trade costs with exporter $i$ at time $t$ ($U_{\tau,nit}^{IO}$) in our input-output model; both specifications include importer-exporter fixed effects and continent-year fixed effects. Standard errors are clustered by importer-exporter pair.

Summary  Taken together, the results of this section confirm that our exposure measures have predictive power for separate data on PTAs not used in their estimation, both in terms of pre-
dicting selection into future PTAs, and in terms of detecting the impact of these future PTAs in increasing economic interdependence between countries.

H.2 Non-Linearity

In this subsection of the appendix, we report further empirical evidence on the quality of the approximation of our linearization to the full non-linear model solution for large productivity shocks, as discussed in Section 5.2 of the paper.

In Subsection H.2.1, we compare counterfactual changes in income and welfare in our linearization and the full non-linear model solution in the single-sector model. The difference between these two expressions corresponds to the second and higher-order terms in a Taylor-series expansion around the initial equilibrium. In Subsection H.2.2, we derive analytical bounds on the absolute magnitude of these second and higher-order terms in the single-sector model in terms of the observed trade shares and the trade elasticity ($\theta$). In Subsection H.2.3, we implement these analytical bounds empirically.

In Subsection H.2.4, we discuss our inversion of the single-sector model to recover empirical distributions of productivity and trade cost shocks, as used in Section 5.2 of the paper. In Subsection H.2.5, we compare the predictions of our linearization and the full non-linear model solution in the single-sector model for simulated trade networks.

In Subsection H.2.6, we report additional results for the comparison of the predictions of our linearization and the full non-linear model solution for our quantitative specification with multiple sectors and input-output linkages, as considered in Section 5.2 of the paper.

H.2.1 Single-sector Model

We start with the indirect utility function (equation (1) in the paper) and market clearing condition (equation (4) in the paper) in a counterfactual equilibrium in the full non-linear model. Using exact-hat algebra, we can re-write the counterfactual values of variables in these equations (indicated by a prime) in terms of their observed values in an initial equilibrium (no prime) and the relative changes of these variables between the counterfactual and initial equilibria (denoted by a hat such, that $\hat{x}_i \equiv x'_i/x_i$). Implementing this approach, the counterfactual changes in income per capita ($\hat{w}_i$) and welfare ($\hat{u}_i$) in response to productivity shocks in the non-linear model satisfy the following two equations:

$$\ln \hat{w}_i = \left(\frac{\theta}{\theta + 1}\right) \ln \hat{z}_i + \frac{1}{\theta + 1} \ln \left[ \sum_{n=1}^{N} \frac{\hat{w}_n}{\sum_{\ell=1}^{N} s_{n\ell} \hat{w}_\ell^{-\theta} z_\ell^{\theta}} \right],$$

$$\ln \hat{u}_i = \ln \hat{w}_i + \frac{1}{\theta} \ln \left[ \sum_{n=1}^{N} s_{in} \hat{w}_n^{-\theta} z_n^{\theta} \right].$$

(H.3)
For comparison, we can re-write our friends and enemies exposure measures for income and welfare from equations (8) and (11), respectively, in the following similar but log linear form:

\[
\begin{align*}
\text{d} \ln w_i &= \left(\frac{\theta}{\theta + 1}\right) \text{d} \ln z_i + \frac{1}{\theta + 1} \sum_{n=1}^{N} t_{in} \left[ \text{d} \ln w_n + \theta \sum_{\ell=1}^{N} s_{n\ell} \left(\text{d} \ln w_{\ell} - \text{d} \ln z_{\ell}\right) \right], \\
\text{d} \ln u_i &= \text{d} \ln w_i - \sum_{n=1}^{N} s_{in} \left[ \text{d} \ln w_n - \text{d} \ln z_n \right].
\end{align*}
\] (H.4)

Comparing equations (H.3) and (H.4), it is apparent that the difference between the full non-linear model solution and our linearization corresponds to the difference between the log of a weighted mean and a weighted mean of logs. This difference between the two expressions corresponds to the second and higher-order terms in a Taylor-series expansion around the initial equilibrium. Note that these two expressions take exactly the same value in the two limiting cases of autarky \((t_{nn} \to 1 \text{ and } s_{nn} \to 1 \text{ for all } n)\) and free trade \((t_{in} \to t_i \text{ and } s_{ni} \to s_i \text{ for all } n, i)\). By itself, this property suggests that there could be limited scope for these two exercises to differ from one another for the intermediate case of finite trade costs. In the next subsection, we confirm this intuition by deriving analytical bounds on this approximation error.

### H.2.2 Analytical Bounds on the Approximation Error for Productivity Shocks in the Single-Sector Model

To simplify notation, we define \(\tilde{z}_i\) as \(\ln \hat{z}_i\). We use \(f_i(z)\) to denote the implicit function that defines the log changes in wages \(\tilde{w}_i\) in equation (H.3) as a function of the log productivity shocks \(\{z\}\), and we use \(\epsilon_i(z)\) to denote the second-order term in the Taylor-series expansion of \(f_i(z)\).

Using this notation, we can rewrite equation (H.3) as:

\[
\tilde{w}_i = \underbrace{-\theta (\tilde{w}_i - \tilde{z}_i) + \sum_n t_{in} \tilde{w}_n + \theta \sum_n m_{in} [\tilde{w}_n - \tilde{z}_n]}_{\text{first-order}} + \underbrace{\epsilon_i(z) + O \left( ||z||^3 \right)}_{\text{second-order} \text{ and } \text{higher-order}}.
\]

The properties of the second-order term depend on the Hessian \(H_{f_i}\) of the function \(f_i\) evaluated at \(\tilde{z}_\ell = 0 \forall \ell\):

\[
H_{f_i} = \begin{bmatrix}
\frac{\partial^2 f_i(0)}{\partial z_1^2} & \frac{\partial^2 f_i(0)}{\partial z_1 \partial z_2} & \cdots & \frac{\partial^2 f_i(0)}{\partial z_1 \partial z_N} \\
\frac{\partial^2 f_i(0)}{\partial z_2 \partial z_1} & \frac{\partial^2 f_i(0)}{\partial z_2^2} & \cdots & \frac{\partial^2 f_i(0)}{\partial z_2 \partial z_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2 f_i(0)}{\partial z_N \partial z_1} & \frac{\partial^2 f_i(0)}{\partial z_N \partial z_2} & \cdots & \frac{\partial^2 f_i(0)}{\partial z_N^2}
\end{bmatrix}, \quad (H.5)
\]

where we can write this second-order term as \(\epsilon_i(z) = z^{\dagger} H_{f_i} z\).

We now proceed as follows. First, we derive an expression for this Hessian in terms of matrices of observed trade data (Proposition A.2). Second, we show that a cross-country average of the
second-order terms is exactly zero (Proposition A.3). Third, we show that the absolute magnitude of this second-order term for each country can be bounded by the largest eigenvalue (in absolute) value of this Hessian (Proposition A.4). As this largest eigenvalue can be measured using observed trade data, we can use this result to bound the quality of the approximation for each country given the observed trade matrices. Fourth, we aggregate these results for the second-order terms across countries, and provide an upper bound on their sums of squares (Proposition A.5). Again this bound can be computed using observed trade data and provides a summary measure of the overall performance of our linearization. Finally, Proposition A.6 provides a bound on all higher order terms, including the second-order term and beyond.

In Proposition A.2, we show that the Hessian \((H_{f_i})\) depends solely on the trade elasticity \((\theta)\) and the three observed matrices that capture the market-size effects \((T)\), cross-substitution effects \((M)\), and expenditure shares \((S)\). In particular, the second-order term depends on expectations and variances taken across the elements of these matrices, as summarized in the following proposition.

Proposition A.2. The Hessian matrix can be explicitly written as

\[
H_{f_i} = \frac{1}{2} \left( A^i (\text{diag} ([M + I_i]_i) - S^i \text{diag} (T_i) S) A - B^i (\text{diag} (T_i) - T_i^T T_i) B \right),
\]

where \(A \equiv \frac{\theta}{\theta + 1} (I - V)^{-1}(I - T)\) and \(B \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} M + S A\), and \(T_i, M_i\) are the \(i\)-th rows of \(T\) and \(M\), respectively.

The second-order term \(\epsilon_i (\tilde{z}) \equiv \tilde{z}^T H_{f_i} \tilde{z}\) can be re-written more intuitively as

\[
\epsilon_i (\tilde{z}) = -\frac{\theta^2 \mathbb{E}_{T_i} \mathbb{V}_{S_n} [\ln \tilde{w}_k - \tilde{z}_k]}{2} + \frac{\mathbb{V}_{T_i} (\ln \tilde{w}_i + \theta \mathbb{E}_{S_n} [\ln \tilde{w}_k - \tilde{z}_k])}{2},
\]

where \(\mathbb{E}_{T_i}, \mathbb{E}_{M_i}, \mathbb{E}_{S_n}, \mathbb{V}_{T_i},\) and \(\mathbb{V}_{S_n}\) are expectations and variances taken using \(\{T_{i,n}\}_{n=1}^N, \{M_{i,n}\}_{n=1}^N,\) and \(\{S_{nk}\}_{k=1}^N\) as measures (e.g. \(\mathbb{E}_{T_i} [x_n] \equiv \sum_{n=1}^N T_{i,n} x_n, \mathbb{V}_{T_i} [x_n] \equiv \sum_{n=1}^N T_{i,n} x_n^2 - \left( \sum_{n=1}^N T_{i,n} x_n \right)^2\)).

Proof. See Section H.2.7 of this online appendix.

As a first step towards characterizing the magnitude of the second-order terms in this expression, we next show in Proposition A.3 that the average across countries (weighted by country size in the initial equilibrium before the productivity shock) of these second-order terms is exactly zero: \(q' \epsilon (\tilde{z}) = 0\). Therefore, these second-order terms raise or reduce the predicted change in the wage of individual countries in response to the productivity shock, but when weighted appropriately they average out across countries.

Proposition A.3. Weighted by each country’s income, the second-order terms average to zero for any productivity shock vector: \(q' \epsilon (\tilde{z}) = 0\) for all \(\tilde{z}\).
Proof. See Section H.2.8 of the online appendix.

We now bound the absolute value of the second-order term for the income response of each country, following any vector of productivity shocks. First, note that because the model features constant returns to scale, a uniform shock to the productivity of all countries across the globe does not affect relative income. It is therefore without loss of generality to focus on productivity shocks that average to zero. We now show in Proposition A.4 that the absolute value of the second-order term for the log-change in income of each country \( i \) is bounded, relative to the variance of productivity shocks, by the largest eigenvalue \( \mu_{\text{max},i} \) (by absolute value) of the Hessian matrix \( H_{f_i}(\tilde{\epsilon}_i(\tilde{z})) \leq |\mu_{\text{max},i}| \cdot \tilde{z}^T \tilde{z} \). The corresponding eigenvector \( \tilde{z}_{\text{max},i} \) is the productivity shock vector that achieves the largest second-order term for country \( i \). As these eigenvalues of the Hessian matrix for each country can be evaluated using the observed trade matrices, we thus obtain a bound on the size of the second-order term for each country that can be computed in practice using the observed trade data. In our empirical application below, we show that for each country, even the largest eigenvalue is close to zero, which in turn implies that the second-order term for each country is close to zero.

Proposition A.4. \( |\epsilon_i(\tilde{z})| \leq |\mu_{\text{max},i}| \cdot \tilde{z}^T \tilde{z} \) for all \( \tilde{z} \), where \( \mu_{\text{max},i} \) is the largest eigenvalue of \( H_{f_i} \) by absolute value. Let \( \tilde{z}_{\text{max},i} \) denote the corresponding eigenvector (such that \( H_{f_i} \tilde{z}_{\text{max},i} = \mu_{\text{max},i} \tilde{z}_{\text{max},i} \)). The upper bound for \( |\epsilon_i(\tilde{z})| \) is achieved when productivity shocks are represented by \( \tilde{z}_{\text{max},i} \): \( |\epsilon_i(\tilde{z}_{\text{max},i})| = |\mu_{\text{max},i}| \cdot (\tilde{z}_{\text{max},i})^T \tilde{z}_{\text{max},i} \).

Proof. See Section H.2.9 of this online appendix.

We next aggregate the second-order terms across countries and provide an upper-bound on their sum-of-squares in Proposition A.5, which enables us to assess the overall performance of our linear approximation. As we show in our empirical application later, the standard unit vector \( e^\ell \) comes close to achieving the upper-bound for the \( \ell \)-th equation, i.e. \( e^\ell \approx \tilde{z}_{\text{max},\ell} \) for all \( \ell \). Intuitively, because \( e^i \) is orthogonal to \( e^j \) for all \( i \neq j \), this implies that the productivity shock vectors \( \tilde{z}_{\text{max},i} \) and \( \tilde{z}_{\text{max},j} \) that maximize second-order effects for different countries \( i \neq j \) are almost orthogonal. Hence, given any productivity shock vector \( \tilde{z} \), at most one country \( \ln \tilde{w}_i = f_i(\tilde{z}) \) can have a second-order term close to the upper-bound \( \mu_{\text{max},i} \), which is small, and the second-order terms for all other countries are close to zero. To formalize this intuition, Proposition A.5 constructs a symmetric order-4-tensor \( A \) such that \( \sqrt{\frac{1}{N} \sum_{i=1}^{N} \frac{\epsilon_i^2(\tilde{z})}{\tilde{z}^2}} \) is bounded above by the square-root of the spectral norm of \( A \). Note that \( \frac{1}{N} \sum_{i=1}^{N} \epsilon_i^2(\tilde{z}) \) is exactly the mean-square-residuals from a linear regression of the second-order-approximation on our linearized solution.
Proposition A.5. Let $A : \mathbb{R}^N \to \mathbb{R}_{\geq 0}$ denote the order-4 symmetric tensor defined by the polynomial

$$g(\tilde{z}) = \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{a,b,c,d=1}^{N} [H_{fi}]_{ab} \cdot [H_{fi}]_{cd} \cdot \tilde{z}_a \cdot \tilde{z}_b \cdot \tilde{z}_c \cdot \tilde{z}_d \right),$$

where $[H_{fi}]_{ab}$ is the $ab$-th entry of $H_{fi}$. By construction, $g(\tilde{z}) = \langle A, \tilde{z} \otimes \tilde{z} \otimes \tilde{z} \otimes \tilde{z} \rangle$ represents the inner product and is equal to the cross-equation sum-of-square of the second-order terms $(g(\tilde{z}) = \frac{1}{N} \sum_i c_i^2(\tilde{z}))$ under productivity shock $\tilde{z}$. Let $\mu^A$ be the spectral norm of $A$:

$$\mu^A \equiv \sup_{\tilde{z}} \frac{\langle A, z \otimes z \otimes z \otimes z \rangle}{\|z\|_2^4},$$

where $\| \cdot \|_2$ is the $\ell_2$ norm ($\|z\|_2 \equiv \sqrt{z^Tz}$). Then

$$\sqrt{\frac{1}{N} \sum_i c_i^2(\tilde{z})} \leq \sqrt{\mu^A \|\tilde{z}\|_2^2} = \sqrt{\mu^A \tilde{z}^T \tilde{z}}.$$

Proof. See Section H.2.10 of the online appendix.

The spectral norm of $A$ can be computed using the observed trade data, and the norm being close to zero implies that the second-order terms are close to zero. Furthermore, Lagrange’s remainder theorem implies that if productivity shocks are bounded, we can obtain a bound on all the higher-order terms including second-order and above. Using $H_{fi}(\tilde{z})$ to denote the Hessian of $f_i(\tilde{z})$ evaluated at productivity shock $\tilde{z}$ (not necessarily equal to the zero vector), we have the following result.

Proposition A.6. Suppose productivity shocks are bounded, $\tilde{z} \in \mathcal{X} \equiv \prod_{i=1}^{N} [\tilde{z}, \tilde{z}]$. For any $\tilde{z}$, there exists $x \in \mathcal{X}$ such that

$$\ln \hat{w}_i = -\theta (\ln \hat{w}_i - \tilde{z}_i) + \sum_n t_{in} \ln \hat{w}_n + \theta \sum_n m_{in} \ln \hat{w}_n - \tilde{z}_n + \tilde{z}^T H_{fi}(x) \tilde{z}.$$  

Proof. This is a direct application of Lagrange’s remainder theorem.

Proposition A.6 demonstrates that the Hessian matrix, evaluated at some productivity shock vector $x$, provides the exact error for our first-order approximation. A bound on the eigenvalue of the Hessian evaluated over the entire support $\mathcal{X}$ of productivity shocks therefore provides an upper-bound on the exact approximation error. We exploit this result in our empirical analysis the next subsection and show that approximation errors are close to zero given the observed trade matrices and productivity shocks of the magnitude implied by the observed trade data.
H.2.3 Empirical Bounds on the Approximation Error for Productivity Shocks in the Single-Sector Model

We now use our analytical results from Propositions A.2-A.6 to show that the fact that our linearization provides a close to exact approximation to the non-linear model solution can be explained by the properties of the observed trade matrices. In Table H.3, we report the distribution of the eigenvalues of the Hessian matrix that controls the magnitude of the second-order terms across the years of our sample period. We find that even the largest eigenvalue of the Hessian matrix ($H_{ij}$) is close to zero for each country. Therefore, as we approximate the log-income change for each country $i$ separately, the second-order term $\epsilon_i$, when maximized by a country-specific vector of TFP shocks $\{z_{\text{max},i}\}$, accounts for at most a tiny fraction of the variation in $\ln \hat{w}_i$. For example, for the year 2000 and on average over time, we find that the second-order approximation error for the income exposure of each country is bounded by 0.26 percent and 0.36 percent of the variance of productivity shocks respectively.

Furthermore, for all countries, we find that the second-largest eigenvalues $\mu_{2nd}^{i}$ are substantially closer to zero, which implies that any productivity shock vector that is orthogonal to $z_{\text{max},i}$ generates approximately zero second-order effects. We further find that the standard unit vector $e^\ell$ comes close to achieving the upper bound for the $\ell$-th equation, i.e. $e^\ell \approx z_{\text{max},i}^{\ell}$ for all $\ell$. Hence, the second-order term for evaluating the effect of a productivity shock in country $i \neq \ell$ on income in country $i$ is small (approximately bounded by $|\mu_{2nd}^{i}|$) even relative to the own-effect on country $\ell$ itself, which is already small (approximately bounded by $|\mu_{\text{max},i}|$).

Table H.3: Eigenvalues of the Hessian Matrix

<table>
<thead>
<tr>
<th>Eigenvalues of Hessians, ordered by absolute value, averaged across all countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2000</td>
</tr>
<tr>
<td>----------------------------</td>
</tr>
<tr>
<td>0.0026</td>
</tr>
<tr>
<td>Average across years 1970–2012</td>
</tr>
<tr>
<td>Max across years 1970–2012</td>
</tr>
</tbody>
</table>

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 2 of the paper.

Even when we consider all higher-order terms (second-order and above) in Proposition A.6, using the assumption that the Hessian eigenvalues evaluated over the support of the distribution of productivity shocks are bounded by the Hessian eigenvalues observed during our sample pe-
period, we continue to find that the approximation error remains small. In particular, we find that the global approximation errors for income exposure to own productivity shocks are less than 0.62 percent of the variance of productivity shocks, and that these global approximation errors for welfare exposure to other countries’ productivity shocks are 0.33 percent of the variance of productivity shocks.

### H.2.4 Empirical Distribution of Productivity and Trade Cost Shocks in the Single-Sector Model

To compare our linearization with exact-hat algebra for empirically-reasonable shocks, we recover an empirical distribution of productivity and trade cost shocks from our baseline single-sector constant elasticity Armington model from Section B of this online appendix. Changes in trade costs and productivity are only separately identified up to a normalization or choice of units, because an increase in a country’s productivity is isomorphic to a reduction in its trade costs with all partners (including itself). We use the normalization that there are no changes in own trade costs over time (\( \hat{\tau}_{nnt} = 1 \forall n, t \)), which absorbs common unobserved changes in trade costs across all partners into changes in productivity. But our results are not sensitive to the normalization that we use.

Using this normalization, we estimate time-varying bilateral trade cost shocks following Head and Ries (2001). Specifically, let \( x_{nit} \) denote the expenditure by country \( n \) on goods from country \( i \) in year \( t \), then

\[
\frac{x_{nit}}{x_{nnt}} = \frac{w_{it}^{-\theta} z_{it}^\theta (\tau_{nit})^{-\theta}}{w_{nt}^{-\theta} z_{nt}^\theta (\tau_{nnt})^{-\theta}},
\]

which implies

\[
\left( \frac{x_{nit} x_{int}}{x_{nnt} x_{iit}} \right)^{\frac{1}{2}} = \left( \frac{\tau_{nit} \tau_{int}}{\tau_{nnt} \tau_{iit}} \right)^{-\frac{\theta}{2}}.
\]

Denoting by \( \hat{x} \) relative changes in variable \( x \) across periods and using our normalization that \( \hat{\tau}_{nnt} = \hat{\tau}_{iit} = 1 \),

\[
\left( \frac{\hat{x}_{nit} \hat{x}_{int}}{\hat{x}_{nnt} \hat{x}_{iit}} \right)^{\frac{1}{2}} = \left( \hat{\tau}_{nit} \hat{\tau}_{int} \right)^{-\frac{\theta}{2}}.
\]

Assuming a standard value for the trade elasticity of \( \theta = 5 \) and that bilateral trade cost shocks are symmetric (\( \hat{\tau}_{nit} = \hat{\tau}_{int} \)), we can recover all bilateral relative changes in trade costs from the bilateral trade data.

Using the market clearing condition that equates a country’s income with expenditure on its goods, and again assuming a standard value for the trade elasticity of \( \theta = 5 \), we estimate the changes in productivity in each country (\( \hat{z}_{it} \)) that rationalize the observed changes in per capita
incomes \((\hat{w}_{it})\) and populations \((\hat{\ell}_{it})\), given our estimated changes in bilateral trade costs \((\hat{r}_{nit}^\theta)\):

\[
\hat{w}_{it}\hat{\ell}_{it}w_{it}\ell_{it} = \sum_{n=1}^{N} \sum_{i=1}^{N} s_{nit} \hat{r}_{nit}^\theta \hat{w}_{it}^\theta \hat{\ell}_{it}^\theta \hat{z}_{it}^\theta s_{nti} \hat{r}_{nti}^\theta \hat{w}_{ti}^\theta \hat{\ell}_{ti}^\theta \hat{z}_{ti}^\theta, \tag{H.6}
\]

where any omitted changes in trade costs for an importer \(n\) that are common across exporters cancel from the numerator and denominator of the fraction on the right-hand side; any omitted changes in trade costs for an exporter \(i\) that are common across importers are implicitly absorbed into the estimated changes in productivities \((\hat{z}_{it})\). Note that the fraction on the right-hand side of equation (H.6) corresponds to an expenditure share and is homogenous of degree zero in these changes in productivities \((\hat{z}_{it})\). Therefore, these changes in productivities \((\hat{z}_{it})\) only can be recovered up to a normalization or choice of units, and we use the normalization that the mean of the log changes in productivities across all countries is equal to zero (a geometric mean of changes in productivities of one).

### H.2.5 Quality of the Approximation for Productivity Shocks in Simulated Trade Networks in the Single-Sector Model

In Section 5.2 of the paper, we show that our linearization provides an almost-exact approximation to the full non-linear model solution for empirically-reasonable productivity shocks and trade elasticities in actual trade networks. We now report the results of an extensive search over simulated trade networks to try to find counterexamples where our approximation performs less well for empirically-reasonable productivity shocks.

We begin by comparing the predictions of our linearization and the full non-linear model solution for the empirical distribution of productivity shocks from 2000-2010. We start from the observed equilibrium in the data in 2000 and shock productivity in each country by the empirical distribution of productivity shocks from 2000-2010. We compare the two sets of predictions for both our baseline value of trade elasticity of \(\theta = 5\) and the empirically-relevant range of values for the trade elasticity from \(\theta = 2\) to \(\theta = 20\). Across all of these parameter values, we find that our linearization provides an almost-exact approximation to the full non-linear model solution, consistent with the results reported in Section 5.2 of the paper.

As discussed in Section 5.2 of the paper, our linearization and the full non-linear model solution are identical in the two limiting cases of autarky and free trade. Furthermore, we find that our linearization also performs well for random trade matrices. We now show that observed trade matrices are well approximated by a weighted average of autarky, free trade, and random noise, which provides an intuition why our approximation works so well for actual trade networks. Under autarky, the expenditure share matrix is an identity \((S = I)\); under free trade, \(S = Q\) where \(Q_{ni} = q_i\), the size of country \(i\), for all \(n\). Figure H.6 shows the histogram of the entries...
of the matrix \((S - (1 - \alpha)Q - \alpha I)\) for the year 2000, where \(\alpha \equiv \frac{1}{N} \sum_{i=1}^{N} S_{ii}\) is the average expenditure share on domestic goods. The mass close to zero shows that the observed trade matrix \(S\) in 2000 is well approximated by a weighted average of autarky, free trade, and random noise. This is a robust pattern that holds across all years in our sample.

Figure H.6: Histogram of the entries in the off-diagonal elements of the matrix \((S - Q)\)

Note: Figure shows the histogram of the entries of the matrix \((S - (1 - \alpha)Q - \alpha I)\) for the year 2000, where \(\alpha \equiv \frac{1}{N} \sum_{i=1}^{N} S_{ii}\) is the average expenditure share on domestic goods.

We now assess quality of the linearized solution in simulated trade networks. We construct several types of artificial trade networks to validate our intuition above. Specifically, we assess the quality of approximation for TFP shocks in four types of simulated networks, each with varying number of simulated countries \(N \in \{10, 20, 50, 100, 150\}\):

1. Networks that are well-approximated by weighted averages of autarky, free-trade, and noise: \(S^{(1)} \equiv \alpha I + (1 - \alpha)Q + \bar{\epsilon};\)

2. Networks that are purely noise: \(S^{(2)} \equiv \bar{\epsilon};\)

3. Networks that represent circular graphs, with every country \(i\) consuming only goods \(i + 1\) and \(i + 2\), i.e., \(S_{ij}^{(3)} = \begin{cases} 1/2 & \text{if } (j - i) \mod N \in \{1, 2\}\; \\
0 & \text{otherwise}; \end{cases}\)

4. Networks that represent circular graphs with noise: \(S^{(4)} \equiv S^{(3)} + \bar{\epsilon}.\)

We use a bar above a variable to denote random draws. We draw the random error \(\bar{\epsilon}\) (with replacement) from the residuals shown in Figure H.6 above, the simulated free-trade \(\bar{Q}\) is obtained \footnote{When constructing a circular graph, each country needs to consume at least two goods in order for country-level income to respond to TFP shocks; world income distribution is invariant to TFP shocks in a trade network with \(S_{ij} = \begin{cases} 1 & \text{if } (j - 1) \mod N = 1 \\
0 & \text{otherwise}. \end{cases}\).}

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by drawing (with replacement) a random row vector $\bar{q}'$ of country level income shares from the empirical distribution $q'$ in the year 2000, normalizing such that the total world income sums to one, and then stacking the rows $N$ times. In every simulation, expenditure shares are rescaled if necessary to ensure that the row sums of $\bar{S}$ always equal one.

Simulated networks of type 1 are constructed to imitate the observed actual world trade matrix. Simulated networks of type 3 are constructed as polar opposites of type 1, where the expenditure shares deviate substantially from autarky, free trade, or random noise. Simulated networks of types 2 and 4 are introduced to assess the role of noise that is residualized from the autarky and free trade matrices.

We simulate each network type for each given number of countries 1,000 times. For each simulation we draw (with replacement) country-level TFP shocks from empirical distribution of productivity shocks from 2000-2010, and we compare the counterfactual predictions of our linearization and the non-linear model solution for changes in income ($\bar{w}_{it}$) and welfare ($\bar{u}_{it}$) in response to productivity shocks ($\bar{z}_{it}$).

In Figures H.7 through H.10, we display histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations for each type of network. The top row shows the histograms for the regression slope coefficient for log changes in income. The second row shows the histograms for the regression slope coefficient for log changes in welfare. The third row shows the histograms for the correlation coefficient for log changes in income. The fourth row shows the histograms for the correlation coefficient for log changes in welfare. The columns of each figure correspond to different assumed numbers of countries. As the regression slope coefficients and correlation coefficients become more concentrated towards one, our linearization provides a closer approximation to the full non-linear solution of the model.

We now summarize the main take-aways from these simulations. First, the approximation quality increases in the number of countries. With 150 countries, approximation quality is almost-exact in simulated networks of types 1, 2, and 4. Second, even with a small number of countries, the approximation quality is extremely high in simulated networks of type 1, which is constructed to imitate the actual trade network in the observed data. The only networks in which our linearization performs substantially less well are those with both a small number of countries and an extreme, circular trade pattern that is quite different from that observed in the data. Even with this extreme trade pattern, we find that as we increase the number of countries, the quality of the approximation improves. This pattern of results using simulated trade networks provides further intuition for why our linearization performs so well using the actual trade network in the observed data: this actual trade network is well-approximated by a weighted average of autarky, free-trade, and noise, and it features a large number of 140 countries.
Figure H.7: Approximation quality for simulated network type 1 (expenditure share matrix is a weighted average of free trade, autarky, and noise)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
Figure H.8: Approximation quality for simulated network type 2 (expenditure share matrix is pure noise)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
Figure H.9: Approximation quality for simulated network type 3 (expenditure share matrix represents a circular graph)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
Figure H.10: Approximation quality for simulated network type 4 (expenditure share matrix represents a circular graph with noise)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
**H.2.6 Quality of the Approximation for the Multi-Sector Input-Output Model**

We now report additional results comparing the linear and non-linear predictions in our quantitative model with multiple sectors and input-output linkages.

In Figure 8 in Section 5.2 of the paper, we show that we find a correlation between the counterfactual predictions of our linearization and the full non-linear model solution that is close to one regardless of the magnitude of the productivity shocks that we consider, at least up to the (large) empirical value of the shocks from Figure 7 in the paper. For trade cost shocks up to around half as large as the empirical shocks, we also find a correlation close to one, as shown in Figure 8 in Section 5.2 of the paper. As we increase the magnitude of the trade cost shocks towards the (large) empirical value of the shocks, we find that the correlation begins to fall, but it remains above 0.5 for most of the interval, and is always positive and statistically significant at conventional levels.\(^6\)

In Figure H.11, we provide further evidence on the approximation error for the full empirical value of shocks over our entire sample period (at the right-most point on the horizontal axis in Figure 8 with a weight of one). We define the approximation error as the absolute value of the difference in the log changes in welfare in response to shocks between the non-linear model and our linearization. We show the empirical distribution of this approximation error (in log points) for productivity shocks (left panel) and trade cost shocks (right panel). For productivity shocks, we find that the approximation error is always concentrated close to zero, which is consistent with the high correlation between the non-linear and linear predictions for the full magnitude of productivity shocks, as shown in Figure 8 in the paper. For trade cost shocks, we find that the approximation error is close to zero for most countries, but is large for a few countries, which results in the lower correlation between the non-linear and linear predictions for the full empirical magnitude of trade cost shocks, as shown in Figure 8 in the paper. Therefore, even for the large values of trade cost shocks for which the correlation falls, we find that for most countries the linear and non-linear predictions are close to one another.

\(^6\)Whereas productivity shocks are common across all trade partners, trade shocks are bilateral, which results in a three-tensor rather than a matrix representation (see Section D.1 of this online appendix). Our friend-enemy representation reduces this three-tensor down to a matrix using the observed trade shares, which contributes to the lower correlation between the non-linear and linear predictions for large changes in trade costs.
Figure H.11: Histograms of Approximation Errors for Log Changes in Welfare for the Full Empirical Magnitude of Productivity and Trade Cost Shocks from 1970-2012

Note: Histograms of approximation errors for the full empirical magnitude of shocks from 1970-2012 from Figure 7 in the paper; approximation error corresponds to the absolute value of the difference in the log counterfactual changes in welfare in response to shocks between the non-linear model and our linearization; left panel shows results for productivity shocks; right panel shows results for trade cost shocks.

H.2.7 Proof of Proposition A.2

Proof. We repeatedly apply the following approximations:

\[
\ln (1 + x) \approx x - \frac{x^2}{2}, \quad x - 1 \approx \ln x + \frac{(\ln x)^2}{2}
\]

\[
\ln \left( \sum_i p_i x_i \right) \approx \sum_i p_i (x_i - 1) - \frac{(\sum_i p_i (x_i - 1))^2}{2}
\]

\[
\approx \sum_i p_i \left( \ln x_i + \frac{(\ln x_i)^2}{2} \right) - \frac{(\sum_i p_i \ln x_i)^2}{2}
\]

\[
= \mathbb{E}_p [\ln x_i] + \frac{\nabla_p (\ln x_i)}{2}
\]

Let \( \hat{x} = \ln \hat{x} \). The hat-algebra with only TFP shocks can be written as

\[
\hat{w}_i = \sum_n T_{in} \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}}, \quad \text{where} \quad \hat{c}_i \equiv w_i / \hat{z}_i
\]
Taking logs,

\[
\tilde{w}_i = \ln \left( \sum_n T_{in} \tilde{w}_n \frac{\tilde{c}_i^{\theta}}{\sum_k s_{nk} c_k^{\theta}} \right)
\]

\[
= \mathbb{E}_{T_i} \left[ \ln \left( \tilde{w}_n \frac{\tilde{c}_i^{\theta}}{\sum_k s_{nk} c_k^{\theta}} \right) \right] + \frac{\mathcal{V}_{T_i} \left( \ln \left( \tilde{w}_n \frac{\tilde{c}_i^{\theta}}{\sum_k s_{nk} c_k^{\theta}} \right) \right)}{2} 
\]

\[
= \mathbb{E}_{T_i} \left[ \tilde{w}_n - \theta \tilde{c}_i + \mathbb{E}_{S_n} [\theta \tilde{c}_k] - \frac{\mathcal{V}_{S_n} [\theta \tilde{c}_k]}{2} + \frac{\mathcal{V}_{T_i} (\tilde{w}_n + \mathbb{E}_{S_n} [\theta \tilde{c}_k])}{2} \right] 
\]

\[
= -\theta \tilde{c}_i + \mathbb{E}_{T_i} [\tilde{w}_n] + \mathbb{E}_{M_i} [\theta \tilde{c}_k] - \frac{\mathcal{V}_{T_i} \mathbb{E}_{S_n} [\theta \tilde{c}_k]}{2} + \frac{\mathcal{V}_{T_i} (\tilde{w}_n + \mathbb{E}_{S_n} [\theta \tilde{c}_k])}{2} 
\]

To re-write the second-order terms explicitly as a function of the productivity shocks—thereby deriving the Hessian—we express \(\theta \tilde{c}_k\) and \(\tilde{w}_n + \mathbb{E}_{S_n} [\theta \tilde{c}_k]\) in terms of productivity shocks to first-order. To do so, note that the first-order approximation is

\[
\tilde{w} = T\tilde{w} + \theta (TS - I)(\tilde{w} - \tilde{z}) 
\]

\[
\implies \tilde{w} = -\frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) \tilde{z} 
\]

where \(V \equiv \frac{T + \theta TS}{1 + \theta} - Q\). We can therefore rewrite

\[
\theta (\tilde{w} - \tilde{z}) = -\theta \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) + I \right) \tilde{z} 
\]

\[
= -\theta \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) + (I - V)^{-1} (I - V) \right) \tilde{z} 
\]

\[
= -\frac{\theta}{\theta + 1} (I - V)^{-1} (I - T) \tilde{z} 
\]

Where we have used the fact \(q'\tilde{z} = 0\), which follows from constant returns to scale and our normalization that world GDP is constant, to drop \(Q\tilde{z}\) from the right-hand side.

We further have

\[
\tilde{w} + \theta S (\tilde{w} - \tilde{z}) = -\frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) \tilde{z} - S \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T + Q) \right) \tilde{z} 
\]

\[
= -\frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T + Q) \right\} \tilde{z} 
\]

Further, to reduce notational clutter, let \(A \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T)\) and

\[B \equiv \frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\},\]

\(y \equiv \theta(\tilde{w} - \tilde{z}), x \equiv \tilde{w} - \theta S (\tilde{w} - \tilde{z})\),

thus

\[y = -A\tilde{z}, \quad x = +B\tilde{z}\]
We can now re-write the second-order terms as (let $D(x) \equiv \text{diag}(x)$ denote the diagonalization operator)

$$
\begin{align*}
\mathbb{E}_T V_s [\theta \tilde{c}_k] - \nabla^2_{T_i} (\tilde{w}_n + \mathbb{E}_s [\theta \tilde{c}_k]) \\
= \mathbb{E}_T V_s [y_k] - \nabla^2_{T_i} (x_n) \\
= \sum_n T_in \left[ \sum S_{nky_k}^2 - (\sum S_{nk}y_k)^2 \right] - \left( \sum_n T_inx_n^2 - (\sum_n T_inx_n)^2 \right) \\
= y'D([M + I]x) - y'S'D(T_i)Sy - (x'D(T_i)x - x'T'Tx) \\
= \tilde{z}'A'D([M + I]x)A\tilde{z} - z'A'S'D(T_i)SA\tilde{z} - (\tilde{z}'B'D(T_i)B\tilde{z} - \tilde{z}'B'T_iT_iB\tilde{z}) \\
= \tilde{z}'(A'([M + I]x) - S'D(T_i)x - B'(D(T_i) - T_i'T_iB)\tilde{z}.
\end{align*}
$$

Hence we have $\epsilon_i \equiv \tilde{z}H_{f_i}\tilde{z}$, where

$$
H_{f_i} = -\frac{1}{2} (A^T (\text{diag}([M + I]x) - S'\text{diag}(T_i)x) A - B^T (\text{diag}(T_i) - T_i'T_i)B).
$$

### H.2.8 Proof of Proposition A.3

We begin by establishing the following intermediate result.

**Lemma A.2.** $(I - S) A = -(I - T)B$, where $A \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T)$ and $B \equiv \frac{\theta}{\theta + 1} \{ (I - V)^{-1} (TS - I) + S(I - V)^{-1} (I - T) \}$.

**Proof.** By the definition of $A$ and $B$:

$$
(I - S) A = \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T) \\
(I - T) B = \frac{\theta}{\theta + 1} (I - T) \{ (I - V)^{-1} (TS - I) + S(I - V)^{-1} (I - T) \}
$$

We have

$$
\frac{\theta + 1}{\theta} ((I - S)A + (I - T)B) = (I - S) (I - V)^{-1} (I - T) \\
+ (I - T) (I - V)^{-1} (TS - I) + (I - T) S(I - V)^{-1} (I - T) \\
= (I - S + (I - T)S)(I - V)^{-1} (I - T) + (I - T)(I - V)^{-1} (TS - I) \\
= (I - TS)(I - V)^{-1} (I - T) + (I - T)(I - V)^{-1} (TS - I) \\
= (I - TS) \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) + I \right) \\
+ \left( \frac{\theta}{\theta + 1} (TS - I)(I - V)^{-1} I \right) (TS - I) \\
= \left( \frac{\theta}{\theta + 1} (I - TS)(I - V)^{-1} (TS - I) + (I - TS) \right) \\
+ \left( \frac{\theta}{\theta + 1} (TS - I)(I - V)^{-1} (TS - I) + (TS - I) \right) \\
= 0.
$$

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We now use this intermediate result in Lemma A.2 prove Proposition A.3:

**Proof.** To show that the second-order terms average to zero across countries, note

\[ q' \varepsilon = \tilde{z}' \left( \sum q_i H_{f_i} \right) \tilde{z} = -\frac{1}{2} \tilde{z}' \left( A' (d - S'dS) A - B' (d - T'dT) B \right) \tilde{z}, \]

where \( d \equiv \text{diag}(q) \). We next show \( (A' (d - S'dS) A - B' (d - T'dT) B) \) is a zero matrix. Using Lemma A.2, we have

\[ 2 \left( A' (d - S'dS) A - B' (d - T'dT) B \right) \]
\[ = A' (I - S') d (I + S) A - B' (I + T') d (I - T) B \]
\[ + A' (I + S') d (I - S) A - B' (I - T') d (I + T) B \]

(the next line follows from Lemma A.2)
\[ = -B' (I - T') d (I + S) A + B' (I + T') d (I - S) A \]
\[ - A' (I + S') d (I - T) B + A' (I - S') d (I + T) B \]
\[ = - (B' - B'T') d (A + SA) + (B' + B'T') d (A - SA) \]
\[ - (A' + A'S') d (B - TB) + (A' - A'S') d (B + BT) \]
\[ = 2 (B'(T'd - dS) A + A'(dT - S'd) B) \]
\[ = 0, \]

where the last equality follows from \( T'd = dS \) and \( dT = S'd \) (recall \( T_{in} q_i = S_{ni} q_n \) and \( d \equiv \text{diag}(q) \)). \( \square \)

**H.2.9 Proof of Proposition A.4**

**Proof.** The fact that \( H_{f_i} \) is real and symmetric, \( \mu^{\text{max},i} \) is the largest eigenvalue and \( \tilde{z}^{\text{max},i} \) is the corresponding eigenvector implies that

\[ |\mu^{\text{max},i}| \equiv \max_z \frac{z^T H_{f_i} z}{z^T z}, \quad \tilde{z}^{\text{max},i} \equiv \arg \max_z \frac{z^T H_{f_i} z}{z^T z}. \]

\( \square \)

**H.2.10 Proof of Proposition A.5**

**Proof.** The fact that \( \mu^A \) is the spectral norm implies that for all \( z \),

\[ \mu^A \|z\|_2^4 \geq g(z) = \frac{1}{N} \sum_{i=1}^{N} (z'H_{f_i} z)^2 = \frac{1}{N} \sum_{i=1}^{N} \varepsilon^2_i (z) \]

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Hence $\sqrt{\frac{1}{N} \sum_{i=1}^{N} \epsilon_i^2(z)} \leq \sqrt{\mu^A \|z\|^2}$, as desired.

I Data Appendix

In this section of the online appendix, we provide additional information on our data sources and definitions, as summarized in Section 3 of the paper. In Section I.1, we report further information on our economic data. In Section I.2, we give additional details for our political data and measures of bilateral political alignment.

I.1 Economic Data

NBER World Trade Database Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes. The ultimate source for these data is the United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012.

We augment these trade data with information on countries’ gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII.

Note that the NBER World Trade Database lacks direct data on a country’s expenditure on domestic goods ($X_{nn}$). Therefore, we compute this domestic expenditure as gross output minus exports plus imports. To measure gross output for each country, we multiply its GDP (value added) by 2.2, which is the mean ratio of gross output to GDP in the EU-KLEMS database (which includes the USA and Japan).

In our multi-sector models, we report results aggregating the products in the NBER World Trade Database to the 20 International Standard Industrial Classification (ISIC) industries listed below. In specifications incorporating input-output linkages, we use a common input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015). We allow some variation in self-expenditure shares across sectors using the following procedure. First, we compute global sector-level expenditure shares that take into account both the final-demand coefficients and the input-output matrix. We then multiply these shares by the imputed gross output of each country to get sectoral expenditure levels for each country. Finally, we allocate these sectoral expenditure levels to various origin countries using the imports data. The residual is treated as sectoral self-expenditure. We show below that this procedure for measuring sectoral self-expenditure generates income and welfare exposure measures that are strongly correlated with those from the EORA database that reports sectoral self-expenditure shares.

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Table I.1: International Standard Industrial Classification (ISIC) Tradeable Sectors

<table>
<thead>
<tr>
<th>Industry Code</th>
<th>Short Name</th>
<th>Long Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture</td>
<td>Agriculture, forestry and Fishing</td>
</tr>
<tr>
<td>2</td>
<td>Mining</td>
<td>Mining and quarrying</td>
</tr>
<tr>
<td>3</td>
<td>Food</td>
<td>Food products, beverages and tobacco</td>
</tr>
<tr>
<td>4</td>
<td>Textile</td>
<td>Textiles, textile products, leather and footwear</td>
</tr>
<tr>
<td>5</td>
<td>Wood</td>
<td>Wood and products of wood and cork</td>
</tr>
<tr>
<td>6</td>
<td>Paper</td>
<td>Pulp, paper, paper products, printing and publishing</td>
</tr>
<tr>
<td>7</td>
<td>Petroleum</td>
<td>Coke, refined petroleum and nuclear fuel</td>
</tr>
<tr>
<td>8</td>
<td>Chemicals</td>
<td>Chemicals</td>
</tr>
<tr>
<td>9</td>
<td>Plastic</td>
<td>Rubber and plastics products</td>
</tr>
<tr>
<td>10</td>
<td>Minerals</td>
<td>Other nonmetallic mineral products</td>
</tr>
<tr>
<td>11</td>
<td>Basic Metals</td>
<td>Basic metals</td>
</tr>
<tr>
<td>12</td>
<td>Metal Products</td>
<td>Fabricated metal products, except machinery and equipmen</td>
</tr>
<tr>
<td>13</td>
<td>Machinery nec</td>
<td>Machinery and equipment n.e.c.</td>
</tr>
<tr>
<td>14</td>
<td>Office</td>
<td>Office, accounting and computing machinery</td>
</tr>
<tr>
<td>15</td>
<td>Electrical</td>
<td>Electrical machinery and apparatus, n.e.c.</td>
</tr>
<tr>
<td>16</td>
<td>Communication</td>
<td>Radio, television and communication equipment</td>
</tr>
<tr>
<td>17</td>
<td>Medical</td>
<td>Medical, precision and optical instruments, watches and clocks</td>
</tr>
<tr>
<td>18</td>
<td>Auto</td>
<td>Motor vehicles trailers and semi-trailers</td>
</tr>
<tr>
<td>19</td>
<td>Other Transport</td>
<td>Other transport equipment</td>
</tr>
<tr>
<td>20</td>
<td>Other</td>
<td>Manufacturing n.e.c and recycling</td>
</tr>
</tbody>
</table>

**EORA Database**  Our baseline specification using the NBER World Trade Database has the advantage of including a large number of more than 140 countries over a long time period of more than 40 years and with a relatively disaggregated industry classification of 40 sectors (out of which 20 are tradable sectors). A limitation is that we need to measure self-expenditure shares using the procedure discussed in the previous paragraph, and assume the same input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015).

To assess the sensitivity of our results to these assumptions, we replicated our analysis using the EORA Global Supply Chain Database ([https://www.worldmrio.com/](https://www.worldmrio.com/)), which contains country-specific input-output tables and can be used to directly measure self-expenditure shares in each sector. Limitations of this dataset are that it considers a much shorter time period from 1990-2015, a more aggregated industry classification of 26 sectors (out of which 11 are tradable sectors), and is constructed with substantial imputation.

We replicate our input-output analysis from Section 4.1 of the paper using the EORA data for this shorter time period and more aggregated industry classification. For comparability with our baseline specification, we use a country-specific but time invariant input-output table for each.
country in the EORA data. We compare results using the two datasets for the 126 countries in common to both datasets over the time period from 1990-2015 for which both data are available. Recall that welfare exposure ($U^{IO}$) is invariant to our choice of numeraire. To ensure that our results for income exposure ($W^{IO}$) are not sensitive to our choice of numeraire, we normalize income exposure by the income-weighted average for the OECD (excluding the country experiencing the productivity shock).

As shown in Figure I.1, we find a strong, positive and statistically significant correlation between our input-output exposure measures computed using each dataset, which is equal to 0.78 for relative income exposure ($W^{IO}$) and 0.87 for welfare exposure ($U^{IO}$) in the year 2000. As shown in Figure I.2, we also find a strong, positive and statistically significant correlation between the underlying trade share matrices, which is equal to 0.92 for expenditure shares ($S^{IO}$) and 0.86 for income shares ($T^{IO}$) in the year 2000. Although both figures show results for the year 2000, we find the same high correlation for all years from 1990-2015 for which both data are available. Taken together, this strong correlation between results using the COMTRADE and EORA databases provides evidence that our findings are robust to different assumptions about self-expenditure shares and input-output matrices.

Figure I.1: Comparison of Our Baseline Exposure Measures using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)

Note: Baseline specification uses the NBER World Trade Database; EORA specification uses the EORA Database; Welfare exposure to productivity growth ($U^{IO}$) is measured using our multi-sector input-output model and is invariant to our choice of numeraire; Relative income exposure equals income exposure to productivity growth ($W^{IO}$) measured using our multi-sector input-output model and normalized by its income-weighted mean across OECD countries (excluding the shocked country), to ensure that results for income exposure are not sensitive to our choice of numeraire.
I.2 Political Data

We next provide further details on the data sources and definitions for our political alignment measures. We begin by discussing our three main sources of data on bilateral political alignment. First, we use data on observed voting behavior in the United Nations General Assembly (UNGA) to reveal the bilateral similarity of countries’ foreign policies. Second we use measures of strategic rivalries, as classified by political scientists, based on contemporary perceptions by political decision makers of whether countries regard one another as competitors, a source of actual or latent threats, or enemies. Third, we use information on formal alliances, including mutual defense pacts, neutrality and non-aggression treaties and ententes. A key advantage of each of these measures relative to data on military conflict is that much international political influence does not involve open hostilities, including international treaties, other supra-national agreements, international institutions, and back-room diplomacy.

**United Nations Voting Behavior**  We follow a large literature in political science in measuring countries’ bilateral political attitudes towards one another using the similarity of their votes in the United Nations (UN). The ultimate source for our UN voting data is Voeten (2013), which reports non-unanimous plenary votes in the United Nations General Assembly (UNGA) from 1962-2012, and includes on average around 128 votes each year. Countries are recorded as either voting “no” (coded 1), “abstain” (coded 2) or “yes” (coded 3). In particular, we use the bilateral measures of voting similarity constructed using these data in the Chance-Corrected Measures of
Foreign Policy Similarity (FPSIM) database, as reported in Häge (2017). We denote the outcome of vote $v$ for country $i$ by $O_i (v)$:

$$O_i (v) \in \{1, 2, 3\} \quad v \in \{1, \ldots, V\}. \quad (I.1)$$

Building on a large literature in international relations, we consider a number of different measures of bilateral voting similarity. Our first and simplest measure is the $S$-score of Signorino and Ritter (1999), which measures the extent of agreement between the votes of countries $n$ and $i$ as one minus the sum of the squared actual deviation between their votes scaled by the sum of the squared maximum possible deviations between their votes:

$$S_{ni} = 1 - \frac{\sum_{v=1}^{V} (O_n (v) - O_i (v))^2}{\frac{1}{2} \sum_{v=1}^{V} (d_{\text{max}} (v))^2} \quad (I.2)$$

where $(d_{\text{max}} (v))^2 = (\sup \{O_n (v) - O_i (v)\})^2$ represents the maximum possible disagreement for each vote and this measure is bounded between minus one (maximum possible disagreement) and one (maximum possible agreement).

A limitation of this $S$-score measure is that it does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which countries agree on a “yes” depends on the frequency with which countries vote “yes.” Similarly, the frequency with which they agree on each of the other voting outcomes (“no” and “abstain”) depends on the frequencies with they choose these other voting outcomes. Therefore, we also consider two alternative measures of countries’ bilateral similarity in voting patterns that control in different ways for properties of the empirical distribution of voting outcomes. First, the $\pi$-score of Scott (1955) adjusts the observed variability of the countries’ bilateral voting outcomes with the variability of each country’s own voting outcomes around average outcomes across the two countries taken together:

$$S_{ni}^\pi = 1 - \frac{\sum_{v=1}^{V} (O_n (v) - O_i (v))^2}{\sum_{n=1}^{V} (O_n (v) - \frac{O_n + O_i}{2})^2 + \sum_{v=1}^{V} (O_i (v) - \frac{O_n + O_i}{2})^2}, \quad (I.3)$$

where $\bar{O}_i = (1/V) \sum_{v=1}^{V} O_i (v)$ is the average outcome for country $i$.

Second, the $\kappa$-score of Cohen (1960) adjusts the observed variability of the countries’ bilateral voting outcomes with the variability of each country’s own voting outcomes around its own average outcome and the difference between the two countries’ average outcomes:

$$S_{ni}^\kappa = 1 - \frac{\sum_{v=1}^{V} (O_n (v) - O_i (v))^2}{\sum_{v=1}^{V} (O_n (v) - \bar{O}_n)^2 + \sum_{v=1}^{V} (O_i (v) - \bar{O}_i)^2 + \sum_{v=1}^{V} (\bar{O}_n - \bar{O}_i)^2}. \quad (I.4)$$

---

9 See https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/ALVXLM
Both the $\pi$-score and $\kappa$-score have an attractive statistical interpretation, as discussed further in Krippendorf (1970), Fay (2005) and Häge (2011). In the case of binary (0,1) voting outcomes, these indices reduce to the form of $1 - (D_o/D_e)$, where $D_o$ is the observed frequency of agreement and $D_e$ is the expected frequency of agreement. The key difference between the two indices is in their assumptions about the expected frequency of agreement. The $\pi$-score estimates the expected frequency of agreement using the average of the two countries marginal distributions of voting outcomes. In contrast, the $\kappa$-score estimates the expected frequency of agreement using each country’s own individual marginal distribution of voting outcomes. Whereas our economic measures of exposure are potentially asymmetric, such that $n$’s exposure to $i$ is not necessarily the same as $i$’s exposure to $n$, each of these measures of foreign policy similarity is necessarily symmetric.

Finally, as Bailey et al. (2017) point out, measures based on dyadic similarity of vote choices—such as the $S$, $\pi$, and $\kappa$ scores—do not account for the heterogeneity in resolutions being voted on. As a result, these measures could incorrectly attribute changes in agenda as changes in state preferences. To resolve this issue, Bailey et al. (2017) apply spatial voting models from the roll call literature to estimate each country’s political preferences embedded in its UN votes. The outcome of this statistical procedure is a time-varying, one-dimensional measure called “ideal points”, which reflects each country’s preference. Bailey et al. (2017) show that ideal points consistently capture the position of states vis-à-vis a US-led liberal order. We derive a measure of bilateral distance by taking the absolute difference between the ideal points of countries $i$ and $j$ in year $t$.

**Strategic Rivalries** Our second set of measures of countries’ bilateral alignment are indicator variables that pick up whether country $i$ is a strategic rival of $j$ in year $t$, as classified by Thompson (2001) and Colaresi et al. (2010). These rivalry measures capture the risk of conflict with a country of significant relative size and military strength, based on contemporary perceptions by political decision makers, gathered from historical sources on foreign policy and diplomacy. Specifically, rivalries are identified by whether two countries regard each other as competitors, a source of actual or latent threats that pose some possibility of becoming militarized, or enemies.

Colaresi et al. (2010) further refine the data to distinguish between three types of rivalries: spatial, where rivals contest the exclusive control of a territory; positional, where rivals contest relative shares of influence over activities and prestige within a system or subsystem; and ideological, where rivals contest the relative virtues of different belief systems relating to political.

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10Specifically, Bailey et al. (2017)’s methodology identifies preference change over time by exploiting duplicate resolutions that are voted repeatedly in consecutive sessions. This methodology also weights resolutions based on how much they reflect the main policy preference dimension in order to ensure that ideal points are not heavily influenced by resolutions that reflect idiosyncratic factors.
economic or religious activities.

Strategic rivalry is much more prevalent than military conflict, as shown in Aghion et al. (2018). In our sample from 1970-2012, we find that a total of 42 countries have had at least one strategic rival; 74 country-pairs have been strategic rivals at some point; and the total number of country-pair-years that exhibit strategic rivalry is 2,452. For example, China is classified as a strategic rival of the U.S. (1970–1972 and 1996–present), India (the entire sample period), Japan (1996–present), the former Soviet Union (1970–1989), and Vietnam (1973–1991). By comparison, the United States is coded as a strategic rival of China (1970-72 and 1996-2012), Cuba (1970-2012), and the former Soviet Union (1970-89 and 2007-2012).

**Formal Alliances** Our third set of political alignment measures are indicator variables for whether country \( i \) is in a formal alliance with country \( j \) in year \( t \) from the Correlates of War Formal Alliances v4.1 (Gibler 2008). This dataset records all formal alliances among states between 1816 and 2012, including mutual defense pacts, neutrality and non-aggression treaties, and ententes. A defense pact is the highest level of military commitment, requiring alliance members to come to each other’s aid militarily if attacked by a third party. Neutrality and non-aggression pacts pledge signatories to either remain neutral in case of conflict or not use force against the other alliance members. Ententes obligate members to consult in times of crisis or armed attack. Over our entire sample period from 1970-2012, 1,946 country-pairs are in a formal alliance, and 117 countries have at least one formal ally. In the year 2010, China had four allies: Iran, North Korea, Russia, and Pakistan. In contrast, the United States was in alliance with 49 nations in the same year, a significantly greater number than the median country, which has 10 allies.
References


