

# A Dynamic Theory of Multiple Borrowing\*

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## Abstract

Multiple borrowing—a borrower obtains overlapping loans from multiple lenders—is a common phenomenon in many credit markets. We build a highly tractable, dynamic model of multiple borrowing and show that, because overlapping creditors may impose default externalities to each other, expanding financial access by introducing more lenders may severely backfire. Capital allocation is distorted away from the most productive uses. Entrepreneurs choose inefficient endeavors with low returns-to-scale. These problems are exacerbated when investments become more pledgeable or when borrowers have access to more lenders, explaining why increased access to finance does not always improve outcomes.

Keywords: Commitment, Common Agency, Investment, Misallocation

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# 1 Introduction

Multiple borrowing—that a borrower takes out overlapping loans from multiple lenders—is a common phenomenon in many credit markets. Consumers hold multiple credit cards, student loans, and other debt secured by homes and vehicles, often all financed by different lenders. Firms borrow from multiple banks and can issue multiple public debt securities to a wide range of investors. Having access to many lenders may be a sign of financial development and competition; yet taking out new loans may affect borrower’s ability and incentive to pay off existing loans. Consequently, multiple borrowing may cause creditors to impose default externalities onto each other.

The goal of this paper is to understand how these distortions affect investment decisions and the allocation of capital. To do this, we build a highly tractable, dynamic model of multiple borrowing, where a single borrower takes out overlapping loans from different lenders to finance investment, without the ex-ante ability to commit to an exclusive lending relationship. We show that lack of ex-ante commitment power induces perverse resource misallocation, as more productive projects can receive less investment, and entrepreneurs may deliberately choose inefficient endeavors with low return-to-scale. These problems are exacerbated when borrowers have access to more lenders, explaining why increased access to finance does not always improve outcomes. We further analyze how the inefficiency is affected by pledgeability of investment capital, supply side constraints on the amount of debt that can be issued to a single borrower, and bargaining protocols. We also analyze prudential policies that could alleviate the lack-of-commitment friction.

In the model, an entrepreneur sequentially visits multiple lenders to obtain funds for a new investment opportunity. Crucially, the entrepreneur lacks the ability to commit to exclusive borrowing from a single lender and cannot write loan contracts that are contingent on the terms of those signed with future lenders. The more debt owed the less likely it is to be repaid, giving rise to an externality between lenders—new lenders willingly provide additional investment that existing lenders would not and thereby diluting the value of existing debt. Rational lenders anticipate this additional borrowing and offer loan terms that compensate them for it, making multiple borrowing undesirable ex-ante, but without commitment unavoidable ex-post.

Our model is of a multi-agent dynamic game with multilateral externalities. The potential of future debt dilution implies that, at the time of lending, each lender has to anticipate entrepreneur's future path of debt accumulation, and the future path is itself an equilibrium outcome between the entrepreneur and future lenders. Despite the apparent difficulty of analyzing such a dynamic game, we devise a recursive formulation of this problem and obtain analytic characterizations in closed-form. The analytic solution to the model enables us to sharply characterize the inefficiency due to lack of commitment. First, we show that financial expansion could severely backfire, as high lender availability may lead to over-indebtedness, under-investment, and lower welfare. Second, because more productive entrepreneurs have stronger desire to borrow in the future—and thus face worse commitment problem—they can be penalized in equilibrium and end up raising less investment capital, generating an especially perverse form of investment misallocation. Lastly, given the choice, entrepreneurs may explicitly pick investment opportunities with lower productivity than others available in order to receive better financing terms.

Interestingly, we show that commitment problems are not necessarily alleviated by relaxing financial constraints. Credit market features that are often understood to strengthen credit markets, such as pledgeability, can have negative implications. Pledgeability of investment allows for leveraged borrowing, which on the one hand increases the total investment entrepreneurs can collect, but on the other hand increase the return to borrowing and raise entrepreneur's desire to borrow in the future, increasing the relative severity of commitment problems for more productive projects.

Lower returns to future investment create *endogenous commitment power* not to borrow from additional lenders. Since entrepreneurs cannot commit *ex-ante* to limit future borrowing, investment opportunities with concave returns—those that generate high output for existing investment but have low marginal returns to additional investment—are especially valuable as these opportunities generate low incentives for future debt accumulation. This logic also explains why collateral increases distortions—the static benefit of pledgeability allowing levered investment has negative dynamic consequences because it raises incentives to future borrowing.

We also explore alternative modeling assumptions and potential policy interventions to better understand the source of our results relating lack of commitment in credit markets to capital allocation.

We examine the implications of each lender having limited funds, the observability of borrowing history, and the allocation of bargaining power between borrowers and lenders. Policy interventions such as uniform borrowing limits and interest rate caps can improve outcomes but are challenging to implement in practice. We devise a Pigouvian tax that is easily implementable with only information on borrowing history and can induce the full commitment allocation.

The notion of commitment externalities in credit markets is not new to our paper. The economics of the commitment externality was analyzed in the seminal work of Arnott and Stiglitz (1991, 1993) in the insurance context. There is a large literature that analyzes inefficiencies in credit markets due to lack of commitment, including Bizer and DeMarzo (1992), Kahn and Mookherjee (1998) Parlour and Rajan (2001), Brunnermeier and Oehmke (2013), Donaldson et al. (2017), Admati et al. (2018), and DeMarzo and He (2018).<sup>1</sup> We provide a detailed review of this literature in Section 5. Here we note that the closest paper to ours in this literature is Bizer and DeMarzo (1992). They study the problem of a borrower who can visit multiple lenders to take out loans and smooth consumption. Their environment also features debt dilution and lack of commitment, and they analyze a static equilibrium defined by satiation point of the borrower, so that no future lending happens in equilibrium even if new lenders become available. In contrast, we formulate our model so that the borrower may never satiate—future borrowing is always ex-post desirable—and we analyze the full dynamic path of lending outcomes. Our formulation allows us to study a broader set of questions—most notably how lack of commitment affects resource allocation and how it interacts with the nature of investment opportunities. Our dynamic model also allows a tractable analysis of the relationship between commitment and pledgeability, the availability of lenders, and bargaining between borrowers and lenders.

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<sup>1</sup>Other papers studying lack of commitment in credit markets include Boot and Thakor (1994), Bisin and Rampini (2006), Petersen and Rajan (1995), Bisin and Guaitoli (2004), Attar and Chassagnon (2009), and Attar et al. (2019).

## 2 Model

### 2.1 Setup

An entrepreneur attempts to finance a new investment opportunity with the constraints that it cannot commit to exclusive borrowing from a single lender and that there is limited enforcement of debt repayment, i.e., the borrower only repays if the costs of default are high enough. The model has two stages ( $t = 1, 2$ ) and contains two sets of agents: a single entrepreneur and an infinite sequence of potential lenders. All parties are risk neutral and have no discounting. The entrepreneur is endowed with a *variable-scale* investment opportunity that returns  $R(K) = zK$  deterministically at  $t = 2$  for any  $K \geq 0$  invested at  $t = 1$ . The returns to this investment opportunity are observable but not pledgeable (we relax this assumption in Section 4.1). After the projects returns are realized, the entrepreneur learns of its cost of defaulting on its debt, and only repays if the default cost exceeds the amount of debt owed. Specifically, we assume default costs  $\tilde{c}$  are drawn from a distribution  $p(x) \equiv Pr(\tilde{c} > x)$ . Debt  $D$  is repaid if  $\tilde{c} > D$ , thus  $p(D)$  is also the probability of repayment and is decreasing in  $D$ . Figure 1 summarizes the timing of the model.

We model the entrepreneur raising capital from the lending market at  $t = 1$  as a dynamic game of complete information in which the entrepreneur sequentially and stochastically contacts a potentially infinite number of lenders. All lenders are risk neutral and do not discount between  $t = 1$  and  $t = 2$ . Their opportunity cost of funds is the risk-free rate, which we normalize to one. Upon meeting a lender, the entrepreneur makes a take-it-or-leave-it offer for a simple debt contract  $(k_i, d_i)$  specifying the amount borrowed  $k_i$  and the promised repayment amount  $d_i$ . The lender, who can observe the history of the entrepreneur's borrowing from previous lenders, chooses whether to accept or reject this offer, and if accepted the funds are exchanged. We study alternative bargaining protocols in Section 4.4.

After signing a contract with each lender, the entrepreneur loses access to the lending market with probability  $1 - q$ . Otherwise, with probability  $q$ , the entrepreneur meets a new lender and the process described above is repeated until eventually access to the lending market is lost. After losing access to the lending market, the entrepreneur invests the aggregate financing it raised in the new project, and

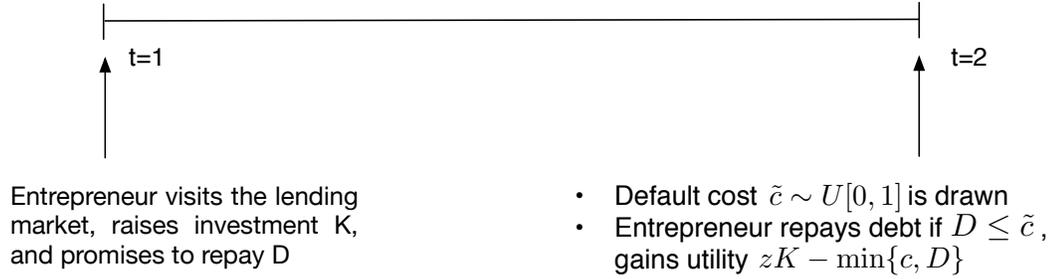


Figure 1: Timing of the Model

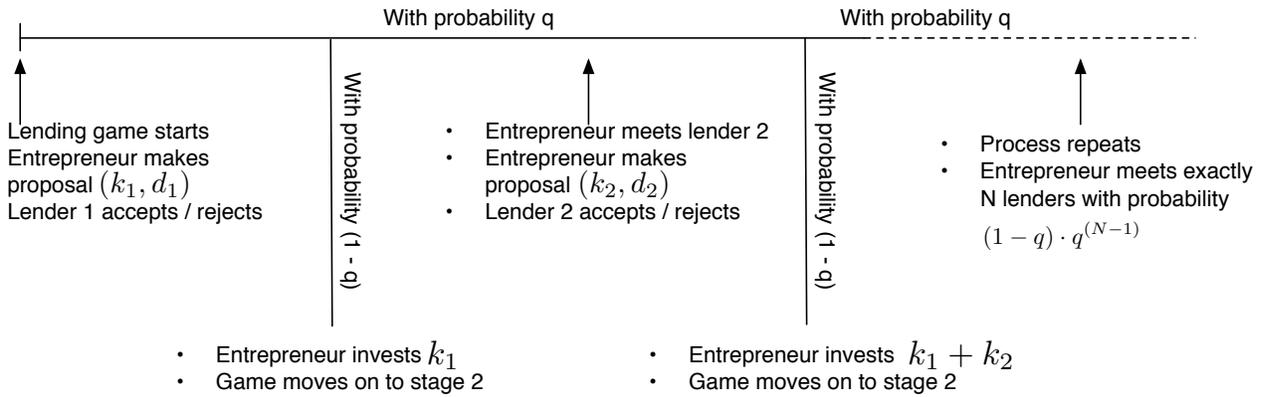


Figure 2: The Lending Market Game at  $t = 1$

the model progresses to stage  $t = 2$ . We assume no lender is visited more than once. The parameter  $q$  measures the availability of lenders relative to the stochastic fundraising deadline of the investment opportunity. The model implies that the probability of meeting at least  $N$  lenders is  $q^{N-1}$ .

Figure 2 provides a graphical exposition of the lending market game of stage  $t = 1$ .

## 2.2 Model Discussion

A crucial feature of our model is that loan contracts cannot be contingent on the terms of loans issued by subsequent lenders. Importantly, by modeling default as a binary decision, we are able to abstract from security design and limit our analysis to the study of debt contracts. Financing arrangements that allow more sensitivity between realized output and repayment could in principal help borrowers internalize the effects of obtaining subsequent financing. While such contingent con-

tracts may be available in some segments of well-developed credit markets, loans without exclusivity clauses prevail in many less-developed credit markets. We study the value of exactly this type of contingent contracting and its role in ensuring the efficient allocation of capital across projects.

The assumption of *sequential* borrowing from a possibly large number of banks is adopted for analytic tractability and need not be taken literally. While the sequential formulation we model implies observability of previous lending, this is not important for our results. What drives our findings is that lack of commitment to an exclusive lending relationship leaves room for the externalities between lenders and perversely affects equilibrium outcomes. While it is possible to instead model borrowing from multiple lenders as simultaneous so that lenders do not observe other borrowing and obtain broadly similar results, such a model is analytically less tractable and subject to the standard critique in the simultaneous contracting literature that the results are sensitive to the specification of agents' off-equilibrium beliefs (Segal and Whinston (2003)). Further, our model of stochastic sequential lending provides a parsimonious way to vary the degree of limited commitment.

We formulate the endogenous choice of default via a random default cost that is realized ex-post. The formulation is similar to a random shock to an outside option, as in Aguiar et al. (2019), and reflects forces that generate ex-ante indeterminacy in the ex-post costs of defaulting on debt. These forces can include weak institution or contractual enforcement. The appendix provides a micro-foundation of the random default through moral hazard.

In Section 4, we introduce a number of extensions to the model, including partial pledgeability of the investment project (Section 4.1), limiting the amount of investment that each lender can provide (Section 4.2), adding concavity to the investment project (Section 4.3), studying the equilibrium under alternative bargaining protocols between the entrepreneur and lenders (Section 4.4), and regulatory policies that could alleviate the lack-of-commitment friction (Section 4.5).

### **2.3 Illustrating the Commitment Problem**

First suppose the entrepreneur meets exactly one lender and makes a take-it-or-leave-it offer  $(D, K)$ —funding  $K$  into the investment project and issuing debt face value  $D$ . The lender accepts

any loan offer that is weakly profitable in expectation. Under the mutual knowledge that the borrower will not take out additional loans from future lenders, the optimal lending contract solves:

$$\max_{D,K} zK - \mathbb{E}[\min(D, \tilde{c})] \text{ s.t } K \leq p(D)D.$$

The maximization problem can be re-written as

$$\max_D z p(D)D - \left[ p(D)D - \int_0^D \tilde{c} p'(\tilde{c}) d\tilde{c} \right].$$

The solution is characterized by the first-order condition:

$$z \times [p(D) + p'(D)D] = p(D), \quad (1)$$

where we have used the property that  $\frac{\partial(\int_0^D \tilde{c} p'(\tilde{c}) d\tilde{c})}{\partial D} = D p'(D)$  to derive equation (1).

Equation (1) characterizes the debt issuance (and  $p(D)D$  the investment level) of the model if entrepreneur were to have full-commitment power not to take out any additional loans from future lenders. At the optimum, the costs and benefits of issuing an additional unit of debt must be equalized. The marginal cost is the increase in expected debt repayment costs, which is simply the probability of actually repaying the marginal unit of debt as shown on the right-hand-side of equation (1).<sup>2</sup> The marginal gain from issuing additional debt is shown on the left-hand-side and is equal to the project's productivity  $z$  times the marginal investment that can be raised from the additional debt issuance. The term  $p(D)$  represents the value of a marginal dollar of promised repayment. Because extra repayment reduces the value of all debt claims, the value of *infra-marginal* debt  $D$  falls; this is reflected in the term  $p'(D)D$ . Hence, after having issued debt  $D$  to the first lender, the additional investment entrepreneur can raise by issuing an additional unit of debt to the first lender is  $p(D) + p'(D)D$ .

To illustrate the source of the commitment problem, now suppose the borrower who has issued debt  $D$  to the first lender gets to meet a second lender. The entrepreneur can raise investment of size

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<sup>2</sup>The right hand side of the first order condition before simplifying is  $p(D) + [p'(D)D - p'(D)D]$ . The term in brackets captures that repayment costs change indirectly through changes in the probability of repayment. But because the borrower is optimizing the default decision ex-post, this indirect effect is zero.

$p(D)$  by issuing one unit of debt to the second lender, whereas if it had tried to promise a marginal dollar of repayment to the first lender, the lender would have only lent  $p(D) + p'(D) < p(D)$ . In other words, the second lender offers better marginal interest rates than the first lender. This is because the second lender does not internalize that its lending dilutes the value of pre-existing debt claims. The same logic holds for all future lenders, and therefore the borrower would always choose to take out additional loans from future lenders if he had the opportunity to do so. Thus in any equilibrium lenders must anticipate all potential future borrowing and charge higher interest rates that compensate them for value they expect to lose. We now formally study the equilibrium of this model.

### 3 Equilibrium Analysis

#### 3.1 Equilibrium Definition and Solution

Our solution concept is Markov Perfect Equilibrium (MPE). The relevant state variable, which both borrower and lenders' strategies are conditioned on, is the cumulative debt that has been issued to all pre-existing lenders. We now proceed to characterize the stationary MPE. We show at the end of this section that the equilibrium we identify captures the limiting outcome of the unique subgame perfect equilibrium (SPE) of finite-lender versions of the model as the number of lenders grows large.

Throughout the paper, let  $(d_i, k_i)$  denote the contract that borrower offers to lender  $i$ , where  $d_i$  is the face value of debt issued and  $k_i$  is the investment that the borrower asks for. Let  $D_i \equiv \sum_{j=1}^i d_j$  and  $K_i \equiv \sum_{j=1}^i k_j$  respectively denote the cumulative debt and investment after contracting with lender  $i$ .

**Lender Best Responses** Lenders have rational expectations of future borrowing and thus only accept contracts that yield non-negative expected returns taking the strategies of the borrower and other lenders as given. Specifically, suppose the borrower has previously obtained total face value of debt  $D$  to all pre-existing lenders and is proposing to issue  $d_i$  to lender  $i$ . Denote  $\tilde{p}_i(D + d_i)$  to be lender  $i$ 's *perceived* repayment probability. Lender  $i$ 's optimal strategy is to accept the loan offer if and only if it is weakly profitable in expectation, i.e. iff  $k_i \leq \tilde{p}_i(D + d_i) \times d_i$ . We focus on a stationary equilibrium in which lenders' best response can be characterized by a function  $\tilde{p}(\cdot)$  such that  $\tilde{p}_i(\cdot) = \tilde{p}(\cdot) \forall i$ . In

equilibrium, rational expectation implies that  $\tilde{p}(\cdot)$  has to be consistent with the borrower's future path of debt accumulation.

**Borrower Best Response** Conditional on meeting a lender, and given lender strategies represented by  $\tilde{p}(\cdot)$ , the borrower makes a loan offer that maximizes its expected continuation utility of potentially being able to meet and borrow from more lenders. Solving for the borrower's best response function thus involves a dynamic optimization problem. Taking lender pricing as given, the entrepreneur forms strategies that maximize its continuation utility at each value of the state variable  $D$ .

The borrower knows that lenders will accept any loan they expect to be profitable, so to maximize its own utility it will only offer loans that lenders expect to make exactly zero profits. If the entrepreneur has cumulative debt  $D$  upon meeting the lender and leaves the lender with cumulative debt  $D' = D + d$ , then the maximum amount of new capital the lender would provide is given by  $\tilde{p}(D')d$ , the probability of repayment times the face value of new debt issuance. Thus, taking loan pricing as given the entrepreneur solves:

$$V(D) = \max_{D'} \{z\tilde{p}(D')(D' - D) - (1 - q)\mathbb{E}[\min(D', \tilde{c})] + qV(D')\}. \quad (2)$$

The first term on the right hand side of Equation 2 is the marginal payoff from the new investment opportunity associated with obtaining additional investment  $k = \tilde{p}(D')d$ . With probability  $1 - q$  the entrepreneur loses access to the lending market, invests total capital raised  $K$  into the investment opportunity, and either repays debt  $D'$  or defaults and pays the default cost  $\tilde{c}$ . Finally, with probability  $q$  the entrepreneur does not lose access to the lending market and will receive continuation utility  $V(D')$  from future borrowing.

Denote a policy function that solves the dynamic programming problem in Equation 2 by  $g(\cdot)$ . Conditional on arriving to a new lender with aggregate face value of debt  $D$ , the borrower will leave with aggregate face value of debt  $D' = g(D)$ . Following this strategy generates a sequence of total aggregate face values of debt that have been accumulated up to a given lender:  $\{g(0), g(g(0)), g^3(0), \dots\}$ . The aggregate face value of debt obtained in the lending market is a random variable, denoted by  $D^{agg}$ , that realizes a particular value of this sequence depending on how many lenders the borrower is able

to visit before the lending market ends.

**Equilibrium Definition** The equilibrium is a fixed point in the pair of functions,  $g(\cdot)$  and  $\tilde{p}(\cdot)$ . Given lender strategies  $\tilde{p}(\cdot)$ , the borrower's optimal strategy of debt accumulation is the policy function  $g(\cdot)$  that solves the dynamic programming problem in Equation 2. Given the probability distribution of total aggregate face value of debt  $D^{agg}$  induced by  $g(\cdot)$ , each lender forms rational expectations over the probability the borrower will repay, denoted by  $\tilde{p}(D') = \mathbb{E}[p(D^{agg}) | D']$ . A set of strategies forms a MPE if lender strategies given by  $\tilde{p}(\cdot)$  are rational given  $g(\cdot)$ , and the borrower strategy  $g(\cdot)$  is optimal given lender strategies embodied in  $\tilde{p}(\cdot)$ .

**Definition 1.** A symmetric Markov Perfect Equilibrium of the lending game is a pair of functions  $\tilde{p}(\cdot)$  and  $g(\cdot)$  that map from cumulative debt level  $D \in [0, 1]$  to the interval  $[0, 1]$  such that:

1.  $g(\cdot)$  solves the dynamic programming problem in Equation 2 taking  $\tilde{p}(\cdot)$  as given.
2. Lenders' perceived expected repayment probabilities  $\tilde{p}(\cdot)$  used to form accept/reject strategies are correct taking  $g(\cdot)$  as given:  $\tilde{p}(D) = \mathbb{E}[p(D^{agg}) | D, g(\cdot)]$ .

**Closed Form Solution** The MPE is defined by a pair of functions that solve an infinite-dimensional fixed point problem, which is in general difficult to characterize. We make the following simplifying assumption on the distribution of default costs, under which the dynamic value function  $V(\cdot)$  becomes quadratic in the state variable, and the model emits a closed form solution.

**Assumption 1.** *Default costs are uniformly distributed between zero and one:  $\tilde{c} \sim U[0, 1]$ .*

Assumption 1 normalizes the entrepreneur's maximum debt capacity to one. Under the assumption, the probability of repayment is simply  $p(D) = 1 - D$ , and the expected cost of debt (either repayment or strategic default) is  $\mathbb{E}[\min(\tilde{c}, D)] = D - D^2/2$ . All of the remaining results in the paper are derived under Assumption 1.

The following proposition characterizes an equilibrium where  $\tilde{p}(D)$  and  $g(D)$  are linear in  $D$ .

**Proposition 1.** For  $z > 1$  and  $1 > q \geq 0$ , the unique stationary MPE in linear strategies can be characterized by two scalars  $b^* \leq 1$  and  $\ell^* \leq 1$ :

$$\begin{aligned} 1 - g(D) &= (1 - D) \cdot b^* \\ \tilde{p}(D) &= (1 - D) \cdot \ell^*. \end{aligned}$$

The two equilibrium scalars are the fixed point of the pair of functions  $L(b)$  and  $B(\ell)$ , i.e.  $\ell^* = L(b^*)$  and  $b^* = B(\ell^*)$ , which satisfy

$$L(b) = \frac{1 - q}{1 - qb}, \quad (3)$$

$$qz\ell(B(\ell))^2 + (1 - q - 2z\ell)B(\ell) + zB(\ell) = 0. \quad (4)$$

The closed-form solution to the two equations is

$$b^* = \frac{z}{(z - \frac{1}{2}) + \sqrt{(z - \frac{1}{2})^2 - qz(z - 1)}}, \quad \ell^* = \frac{(1 - q)(z - 1)}{\sqrt{(z - \frac{1}{2})^2 - qz(z - 1)} - \frac{1}{2}}.$$

Ex-ante borrower welfare, given by  $V(0)$ , is  $V(0) = -\frac{1}{2} + \frac{z^2(1-q)}{2z(1-q) - 1 + 2\sqrt{(z - \frac{1}{2})^2 - qz(z - 1)}}$ .

There is an intuitive interpretation for both  $\ell$  and  $b$ . First, if the borrower arrives to a lender with current debt  $D$ , its remaining debt capacity is  $1 - D$ .<sup>3</sup> When leaving this lender the borrower will have pledged a total of  $g(D)$  and thus the borrower will have remaining debt capacity  $1 - g(D)$ . Therefore  $b^*$  is the fraction of current borrowing capacity that remains after visiting a lender.

Second, recall that with commitment, the repayment probability is exactly  $\tilde{p}(D) = p(D) = 1 - D$ , which corresponds to  $\ell^* = 1$ . Thus  $\ell^* < 1$  corresponds to lower expected repayment probability and higher interest rates. In other words, when  $\ell^*$  is low, the lenders make pricing decisions *as if* they expect the borrower to accumulate substantially more debt from future lenders.

Finally, there is also a static interpretation for the dynamic equilibrium. Taking lender's strategy

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<sup>3</sup>Given that  $\tilde{c} \sim U[0, 1]$ , the borrower would default on all debt with certainty if he were to acquire additional debt of more than  $1 - D$  beyond current outstanding debt  $D$ .

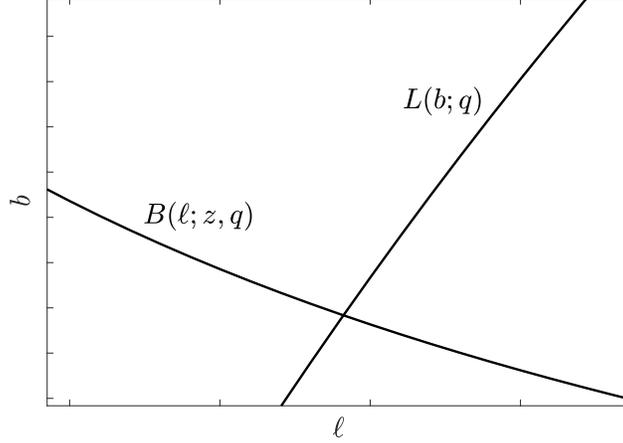


Figure 3: Static Representation of Lending Market Equilibrium.

parameterized by  $\ell$  as given (which determines  $\tilde{p}(D)$  and the interest rates), the entrepreneur chooses how much debt to issue when meeting each lender. This specifies a best response  $b = B(\ell)$  for the entrepreneur in equation (4), which can be thought of as representing a loan demand schedule. Since higher interest rates induce the borrower to take out debt less aggressively, the loan demand schedule is downward-sloping ( $B'(\ell) < 0$ ). On the other hand, given the aggressiveness of entrepreneur's borrowing behavior parameterized by  $b$ , the lenders are able to determine the distribution of the face value of total borrowing. This maps into the distribution of the value of their own debt claims, and lenders set their decision rule such that they only accept loans they expect to be weakly profitable, and thus specify a best response  $\ell = L(b)$  as the loan supply schedule in equation 3. The interest rates lenders have to charge in order to break-even increases as the entrepreneur takes out loans more aggressively, hence the loan supply schedule is upward-sloping ( $L'(b) > 0$ ). The equilibrium  $(b^*, \ell^*)$  is then the intersection of the “loan demand” equation  $B(\ell)$  and the “loan supply” equation  $L(b)$ , which is depicted in Figure 3.

**Describing the Equilibrium Path** Proposition 1 enables us to iterate on the policy function to compute the equilibrium sequence of debt issuance  $\{d_1, d_2, \dots\}$  and to compute the equilibrium sequence of interest rates using the function  $\tilde{p}(\cdot)$ . We find that, in equilibrium, each subsequent lender lends less than previous lenders while charging a higher interest rate.

**Proposition 2.** For  $j > i$ ,  $d_j < d_i$ ,  $k_j < k_i$ , and  $d_j/k_j > d_i/k_i$ .

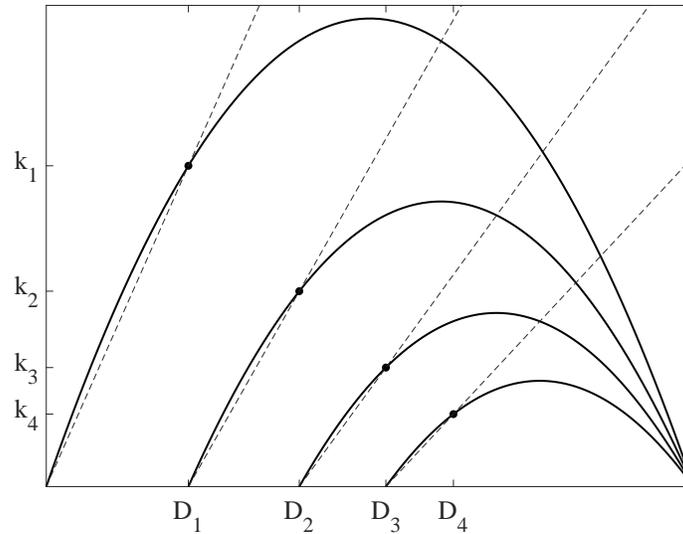


Figure 4: Equilibrium Debt Dynamics

The equilibrium lending path can be visualized in Figure 4. The solid lines denote the sets of loan contracts  $(d, k)$  that each lender  $i$  believes will be zero profit, given the cumulative level of debt  $D_i$  and expectations about future borrowing (i.e. the curves trace out  $\{(d, k) \mid k = d\tilde{p}(D_i + d)\}$  for various  $D_i$ ). The marked point on each curve represents the loan  $(d_i, k_i)$  chosen in equilibrium (recall  $d_i = D_{i+1} - D_i$ ). As the borrower visits more lenders, borrowing amounts fall ( $d_1 > d_2 > \dots$  and  $k_1 > k_2 > \dots$ ) and interest rates  $d_i/k_i$ , represented by the inverse of the slope of dotted lines, rise.

Note that, despite charging higher *average* interest rates, subsequent lenders charge lower *marginal* interest rates—as discussed in Section 2.3—because they do not internalize the effect of their lending on the repayment of previously issued debt. That new lenders provide better marginal interest rates can also be seen from the figure: the slope of the contract curve for the first lender at  $(D_1, k_1)$  is lower than the slope of the curve for the second lender at  $(D_1, 0)$ , indicating that the second lender is willing to provide more capital for a marginal unit of debt.

Why do the sets of zero profit loan contracts  $\{(d, k) \mid k = d\tilde{p}(D_i + d)\}$  have an inverted U shape? Holding repayment probability fixed, higher face value translates into higher present value. However, a higher face value of debt also reduces the probability of repayment. Thus at each lender the borrower faces a “debt Laffer curve”—the value of debt is initially increasing in the amount of debt pledged, but for sufficiently high face values of debt, promising to repay an additional dollar of debt actually

decreases the total amount of financing. Given the contract set  $\{(d, k) \mid k = d\tilde{p}(D_i + d)\}$  for any  $D_i$ , the borrower would always choose contracts to the left of the peak investment, as shown in Figure 4. This is because issuing debt is costly, and, for any  $(d, k)$  to the right of the peak, there exists another contract with the same level of investment  $k$  and a strictly lower level of debt issuance. The borrower always prefers the contract with lower debt.

The debt Laffer curve plays an important role in our comparative statics on how investment changes with entrepreneur productivity, as will be explained in the next section.

**Equilibrium Choice** The linear-stationary MPE we have identified is closely related to the unique subgame-perfect equilibrium of the finite lender version of our game. Hence, our infinite lender model and particular solution concept are merely abstractions for algebraic tractability.

Formally, we define the finite  $N$ -lender game by modifying our infinite lender game as follows. After meeting the  $i$ -th lender, the borrower gets to meet  $(i + 1)$ -th lender with probability  $q$  if and only if  $i + 1 \leq N$ , and with probability zero otherwise. Each finite  $N$ -lender game emits a unique subgame-perfect equilibrium. The following proposition demonstrates that MPE identified in Proposition 1 of the infinite lender game is the limit of the sequence of SPEs of the finite  $N$ -lender games as we take  $N$  to infinity.

**Proposition 3.** *Fix  $i$ . As  $N \rightarrow \infty$ , lender  $i$ 's strategies in the sequence of SPEs of the finite  $N$ -lender game converge uniformly to  $\tilde{p}(\cdot)$ .*

## 3.2 Comparative Statics

We now analyze how equilibrium outcomes depend on availability of lenders  $q$  and productivity  $z$ . Because the outcome of the lending game depends on the stochastic number of lenders the borrower meets, we will focus our attention on *ex-ante, expected outcomes*.

**More Lenders Leads to Worse Allocations** The expected number of lenders a borrower visits is  $\frac{1}{1-q}$  which increases with  $q$ . Higher availability of lenders (higher  $q$ ), leads to a worsening of the commitment problem as we formalize in the following proposition.

**Proposition 4.** *When more lenders are available (higher  $q$ ), in expectation, for any  $z > 1$  and  $q \geq 0$ :*

1. *The aggregate debt issuance increases ( $\partial \mathbb{E}[D^{agg}] / \partial q > 0$ )*
2. *Investment decreases ( $\partial \mathbb{E}[K^{agg}] / \partial q < 0$ ).*
3. *Probability of default increases ( $\partial \mathbb{E}[1 - p(D^{agg})] / \partial q < 0$ ).*
4. *The interest rate increases ( $\partial \left( \frac{\mathbb{E}[D^{agg}]}{\mathbb{E}[K^{agg}]} \right) / \partial q > 0$ ).*
5. *Entrepreneur's welfare decreases in  $q$  and, in the limit as  $q \rightarrow 1$ , converges to zero, the level that would be obtained if the entrepreneur does not have access to the lending market at all.*

Higher  $q$  raises the expected debt issuance as the entrepreneur is more able to smooth out debt issuance over multiple lenders to exploit the fact that future lenders offer better interest rates for marginal debt. Higher debt accumulation thus raises the probability of default, and, in order for lenders to break even, the equilibrium interest rate increases.

Paradoxically, despite higher debt issuance, total investment declines as  $q$  increases, and entrepreneur's ex-ante welfare also declines. In fact, as  $q \rightarrow 1$ , the commitment problem becomes so severe that credit market effectively shuts down: even though there is positive borrowing and investment, all surplus from investment is offset by costly default.<sup>4</sup> These results illustrate the fact that dynamic commitment problems can completely unravel the ability of an agent to capture or generate surplus.

**Better Opportunities, Worse Commitment Problems** Entrepreneurs who have more productive investment technology also have greater incentives to borrow in the future; hence, the commitment problem is more severe for highly productive entrepreneurs. The next result shows that non-exclusivity generates a perverse form of misallocation—high productivity entrepreneurs may not be able to raise as much capital for investment as lower productivity entrepreneurs.

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<sup>4</sup>This result is related to the well-known Coase Conjecture, that a durable good monopolist who lacks commitment not to lower future prices—thereby is forced to compete with itself inter-temporally—is unable to obtain any monopoly rents as consumers become very patient. See Fudenberg and Tirole (Chapter 10, 1991). DeMarzo and He (2018) obtain a similar result in the context of capital structure, finding that firms are unable to capture any tax benefit of debt when they cannot commit ex-ante to a leverage policy.

**Proposition 5.** For any  $q \in (0, 1)$ , in expectation,

1. Debt and interest rates are both increasing in  $z$ :  $\partial \mathbb{E}[D^{agg}] / \partial z > 0$  and  $\partial \left( \frac{\mathbb{E}[D^{agg}]}{\mathbb{E}[K^{agg}]} \right) / \partial z > 0$ .
2. Total investment is non-monotone in  $z$ . In particular, there exists a cutoff  $\bar{z}(q)$  such that

$$\frac{\partial \mathbb{E}[K^{agg}]}{\partial z} \begin{cases} \geq 0 & \text{if } z \leq \bar{z}(q) \\ < 0 & \text{if } z > \bar{z}(q) \end{cases}.$$

3. The more lenders in the lending market, the lower is the cutoff productivity  $\bar{z}(q)$  for investment declines:  $\partial \bar{z}(q) / \partial q < 0$ .
4. More lenders ( $q$ ) lower investment especially for high-productivity ( $z$ ) entrepreneurs, i.e.  $\frac{\partial^2 \mathbb{E}[K^{agg}]}{\partial q \partial z} < 0$ .
5. Borrower welfare is globally increasing in productivity  $z$ :  $\partial V(0) / \partial z > 0$ .

These results can be visualized in Figure 5, which plots equilibrium expected investment as a function of  $z$  for three different values of  $q$ . For all levels of  $q$ , the level of investment that the entrepreneur gets to raise in expectation first increases in  $z$  and then decreases. Entrepreneurs with better opportunities could be facing tighter constraints. The third and fourth part of Proposition 5 shows that such endogenous misallocation of resources is more severe in markets with more lenders. First, when  $q$  is higher, the productivity  $\bar{z}(q)$  that maximizes equilibrium investment is lower. More generally, the negative effect of increased availability of lenders  $q$  on equilibrium investment is more severe the higher is productivity  $z$ . Despite these distortions, borrower welfare is always increasing in the productivity level  $z$ . We show in Section 4.3 that this result is a consequence of imposing linear investment technology—or general non-linear production functions, projects that deliver more output for any level of investment can deliver lower welfare than less productive projects that have more rapidly diminishing returns to scale.

Why can better opportunities be associated with lower levels of investment? When commitment problems are present, it is possible that the equilibrium involves using available debt capacity inefficiently. To better understand the intuition, recall that the present value of repayments to creditors

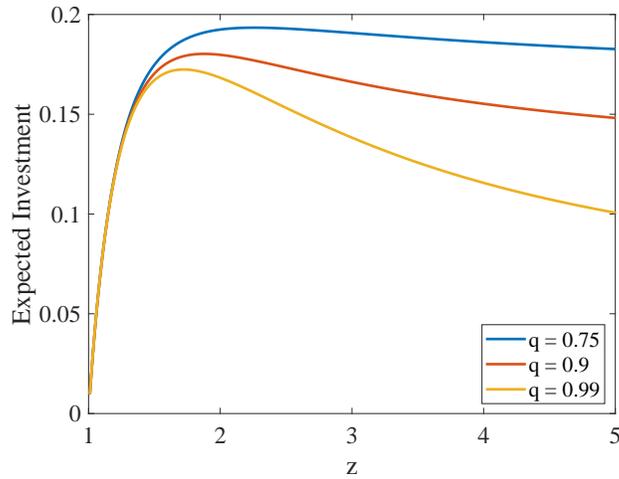


Figure 5: Expected Investment in the Dynamic Lending Game

is not uniformly increasing in the face value of claims issued, because higher levels of debt increase expected dilution. This gives rise to a Laffer curve as discussed above and seen in Figure 4. Of course, facing an *individual* lender, no borrower would propose to borrow so much that it could receive more capital from that lender if it reduced its promised repayment. However, without commitment, the entrepreneur will want to borrow from any future lenders it gets to meet. This means that lenders expect high future borrowing and charge high interest rates, and, from an *ex-ante* perspective, the expected total equilibrium debt and investment could endogenously be on the wrong side of the *aggregate* debt Laffer curve. Proposition 5 shows that this happens precisely when productivity  $z$  is high. If an entrepreneur with a high productivity could have committed to a lower level of debt issuance, it could have received more investment. But because entrepreneurs with the highest returns to investment have the strongest desire to borrow, they are expected to accumulate more debt, and thus end up borrowing at such unfavorable interest rates that they receive lower investment than low-productivity borrowers.

Beyond the fact that investment can decline in productivity, Proposition 5 also shows that higher productivity projects are especially sensitive to lack of commitment. Thus, from an aggregate perspective, the more severe is the lack of entrepreneurs ability to commit to exclusive borrowing, the more investment is distorted into relatively lower productivity projects. This intuition can be formalized by considering aggregate productivity of capital in an economy with a distribution of productive projects

given by  $F(z)$ . We can then define the economy-wide average productivity on invested capital as

$$\tilde{Z} \equiv \frac{\int_1^\infty z \times \mathbb{E}[K^{agg}(z)] dF(z)}{\int_1^\infty \mathbb{E}[K^{agg}(z)] dF(z)}.$$

**Proposition 6.** *For any distribution of productive projects  $F(z)$ , the average productivity on invested capital  $\tilde{Z}$  is decreasing in  $q$ .*

Proposition 6 is consistent with empirical evidence, discussed in detail in Section 6, that increasing access to borrowing does not seem to increase observed returns to investment in environments where commitment to exclusive borrowing may be difficult. 6.

Taken together, these results are surprising and run against common intuition that more productive projects induce higher levels of investment despite the presence of financial constraints. In most classical theories of inefficient investment choice, investment occurs if and only if the net present value of a project exceeds a certain threshold, which can be above or below zero. Such models do not generate the stark patterns of resource misallocation that plague developing countries. The commitment friction we explore in this paper however can generate these distortions. In fact, we show in Section 4.3 that this force is so strong that it can also distort the influence the choice of investment opportunity chosen by entrepreneurs.

## 4 Model Implications

In this section, we discuss implications of our framework through several distinct model extensions. Section 4.1 introduces partial-pledgeability of the investment project to the model. Standard intuition suggests that pledgeability should relax financial constraints, yet we show that without commitment it exacerbates debt accumulation and may actually worsen allocations. Our baseline model abstracts from any benefits of future lenders because the entrepreneur could have obtained as much capital as desired from the initial lender so long as the promised repayment is credible. Section 4.2 instead analyzes a case in which each lender can only provide limited funds to the entrepreneur. We argue in Section 4.3 that the value of exclusivity depends critically on the returns-to-scale of the in-

vestment project. We show that the lack of exclusivity can induce entrepreneurs to choose investment projects with lower *levels* of returns compared to other options if they also feature more rapidly *diminishing* returns. In Section 4.4, we show that shifting bargaining power from the entrepreneur to lenders actually worsens the commitment problem and exacerbates debt accumulation. Lastly, Section 4.5 considers prudential policies that could alleviate the problem of non-exclusivity. We show that debt-limit or interest rate ceiling, as those adopted in several credit markets prone to loan-stacking, could help preventing future lending but are challenging to implement. We also describe a Pigouvian credit tax that restores the full-commitment outcome and can be implemented practically.

## 4.1 Pledgeability

In the baseline model, investment capital raised through borrowing is not pledgeable. If the investment can be (partially) recouped by lenders in case of default, the entrepreneur can increase leverage and raise more investment for any given quantity of risky debt. Therefore, it seems reasonable to expect that pledgeability of investment capital could reduce inefficiency in the lending market. This intuition is incomplete. Leverage induced by partial pledgeability also effectively raises marginal returns of issuing risky debt, and thus there is a channel through which increased pledgeability exacerbates the dynamic incentive to dilute debt with subsequent borrowing.

We explore the implications of this insight through a simple extension of the baseline model. Specifically, suppose a fraction  $\delta$  of the investment is pledgeable. Thus, if the borrower issues risky debt to a lender in exchange for investment capital  $k$ , the borrower can credibly pledge to repay an amount  $\delta \times k$  with certainty. Thus, if the borrower has already promised total risky claims  $D$  to previous lenders and is issuing additional risky debt  $d$ , the current lender's expected repayment is  $\delta k + \tilde{p}(D+d)d$ . The borrower's value function can now be expressed as:

$$V(D) = \max_d \left\{ (z - \delta) \frac{\tilde{p}(D+d)d}{1-\delta} - (1-q) \mathbb{E}[\min(D+d, \tilde{c})] + qV(D+d) \right\}. \quad (5)$$

For any risky debt offering  $d$ , the lender is willing to supply  $k \leq \frac{d\tilde{p}(D+d)}{1-\delta}$ . The multiplier  $\frac{1}{1-\delta}$  can be interpreted as the leverage made possible by pledgeability. Because the borrower has to pay back  $\delta$

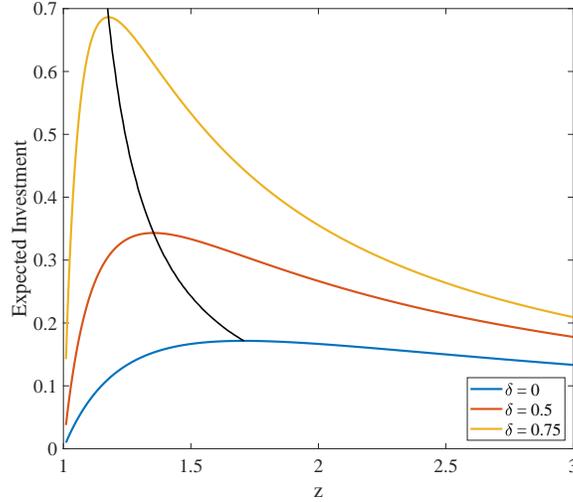


Figure 6: Expected Investment and Pledgeability

fraction of the investment, the return on investment becomes  $(z - \delta)k$ . Defining  $\hat{z} \equiv \frac{z - \delta}{1 - \delta} \geq z$  as the *effective* productivity under pledgeability, Equation 5 becomes isomorphic to Equation 2. This formulation shows that pledgeability exacerbates the incentive to borrow—the effective marginal returns to borrowing, captured by  $\hat{z}$ , are increasing in  $\delta$ . This leads to the following proposition.

**Proposition 7.** Consider the model with partial pledgeability  $\delta \in [0, 1)$ . For any  $q \in (0, 1)$ :

1. Propositions 4, 5, and 6 hold for any  $\delta < 1$ .
2. Investment declines faster with  $q$  under higher  $\delta$ :  $\frac{\partial^2 \mathbb{E}[K^{agg}]}{\partial q \partial \delta} < 0$ .
3. There exists a cutoff  $\bar{z}(q, \delta)$  such that  $\frac{\partial \mathbb{E}[K^{agg}]}{\partial z} \gtrless 0$  if  $z \lesseqgtr \bar{z}(q, \delta)$ , and the cutoff  $\bar{z}(q, \delta)$  decreasing in  $\delta$ :  $\partial \bar{z}(q, \delta) / \partial \delta < 0$ .
4. In expectation, cumulative debt, investment, and borrower welfare are globally increasing in pledgeability  $\delta$ .

The first part of the proposition illustrates that allowing partial pledgeability of investment capital does not qualitatively change the implications of lack of commitment for equilibrium outcomes. More interestingly, *increasing* the extent to which investment capital can be pledgeable has nuanced implications. Welfare is globally increasing in  $\delta$  because pledgeability effectively raises the returns

to all borrowing, current and future. However, under higher  $\delta$ , investment declines faster with  $q$ , and the level of productivity that maximizes equilibrium investment becomes lower. This is because higher marginal returns to future borrowing generate greater incentives for dilutive debt issuance, and pledgeability raises effective marginal returns to borrowing especially for more productive entrepreneurs. The black line in Figure 6 illustrates level of productivity that maximizes investment as  $\delta$  varies.

The borrower is able to raise more investment under higher  $\delta$ . However, because pledgeability raises the desire to borrow disproportionately for high-productivity entrepreneurs, it especially exacerbates their commitment problem, and a perverse effect might kick in: similar to the effect of higher  $q$ , higher pledgeability  $\delta$  might reduce the average productivity of invested capital  $\bar{Z}$ , as described in Proposition 6, because higher  $\delta$  might cause relative resource outflow from high-productivity entrepreneurs and inflow to low-productivity entrepreneurs. A higher  $\delta$  does not always generate this effect; our numerical explorations suggest that this effect happens when the productivity distribution is skewed toward lower productivity projects.

## 4.2 Limited Funds

In our baseline model, higher lender availability  $q$  unambiguously worsens over-indebtedness and under-investment. An important assumption for these results is that lenders have unlimited funds, so that the entrepreneur could have obtained as much capital as desired from the initial lender so long as the promised repayment is credible. Hence, the availability of any subsequent lenders can only deteriorate allocations.

In this section, we relax that assumption and instead assume that each lender can only provide limited investments up to  $F$  to the entrepreneur. Such a constraint could arise, for instance, because of policy regulation or because each lender hopes to limit its exposure to an individual borrower's default risk. The constraint could also arise in settings where lenders simply have limited capital. We then analyze the tradeoff between the two implications of increased lender availability: deteriorating commitment and overcoming per-lender funding limits.

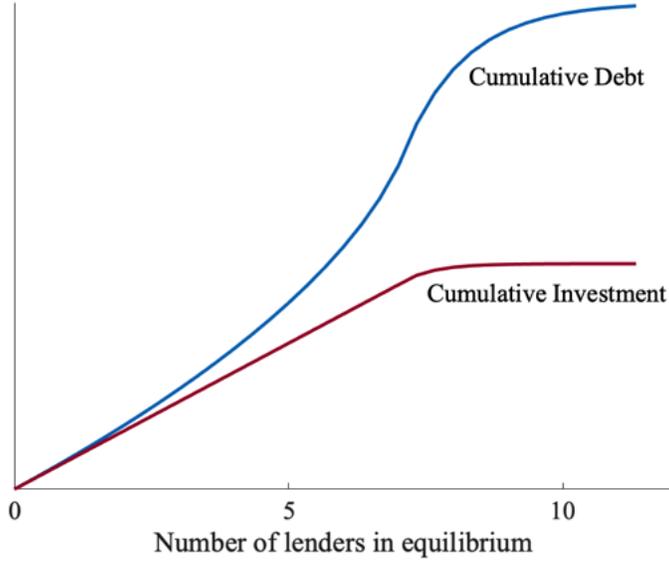


Figure 7: Limited Lender Funds

Specifically, we impose the additional constraint  $k \leq F$  in entrepreneur's optimization problem:

$$V(D) = \max_{d,k} \{(z - \delta)k - (1 - q) \mathbb{E}[\min(D + d, \tilde{c})] + qV(D + d)\} \quad \text{s.t. } k \leq \min\{F, \tilde{p}(D + d)d\}.$$

The additional constraint breaks the stationarity of our model and we are no longer able to provide closed-form solutions. However, we are able to numerically solve the model and analyze the equilibrium path. Debt issuance  $d$  to each new lender is a decreasing function of the cumulative outstanding debt  $D$ ; hence, for sufficiently high levels of cumulative debt  $D$ , the additional debt issuance becomes sufficiently low such that the constraint  $k \leq F$  ceases to bind. On any equilibrium path, the cumulative debt  $D$  increases as the entrepreneur meets more lenders; hence, the limited investment constraint  $k \leq F$  binds for only for a finite number of initial lenders, after which the equilibrium policy and value functions coincides with our baseline model.

Figure 7 demonstrates the equilibrium path of debt issuance given the realized number of lenders the borrower meets. Cumulative investment first increases linearly with number of initial lenders, from whom the entrepreneur raises the maximum investment  $F$ ; after sufficient investment is raised,

the constraint no longer binds for all future lenders.

We numerically investigate the welfare implications of the limited investment constraint  $k \leq F$ . We find that, if the constraint is tight ( $F$  lower than the full-commitment level of investment), then raising lender availability  $q$  could raise entrepreneur welfare, as subsequent lenders are able to provide additional productive investment that the initial lender could not.

Interestingly, however, this does not imply that for a given lender availability  $q$ , lowering the per-lender funding limit  $F$  increases welfare. In fact, lowering  $F$  does not increase entrepreneur welfare for any set of parameters we have investigated. Our intuition for this is that the funding constraint affects welfare through two channels. On the one hand, the constraint onto future lenders lowers the scope of *future* borrowing, thereby alleviating the commitment problem; on the other hand, a binding constraint implies too little investment is available from the initial lender. In other words, lowering  $F$  for future lenders raises welfare, whereas lowering  $F$  for the initial lender lowers welfare. Because the level of investment raised declines at each round of financing, the constraint  $k \leq F$  is always more binding for the initial lender than for subsequent lenders; consequently, a uniform reduction in  $F$  for all lenders always lowers welfare.

### 4.3 Non-exclusivity Distorts Project Choice

The value of exclusivity depends critically on the returns-to-scale of the investment project. For projects with concave returns, obtaining sufficiently large amounts of capital from early lenders would lower the marginal returns to future borrowing, limiting borrowers' ex-post incentives to contract with additional lenders and therefore lowering ex-ante interest rates. Thus, concavity of investment returns can be desirable because it embeds endogenous commitment power. Non-exclusivity may discourage the entrepreneur from starting projects with the highest level of returns, and encourage instead those with lower and more concave returns.

To see this, consider investment projects with linear returns up to a certain size of investment and zero marginal return for any additional investment, with output being  $z \times \min(\bar{K}, K)$ . Clearly, entrepreneur would never borrow further after having raised investment  $\bar{K}$ . This implies that, for  $\bar{K}$

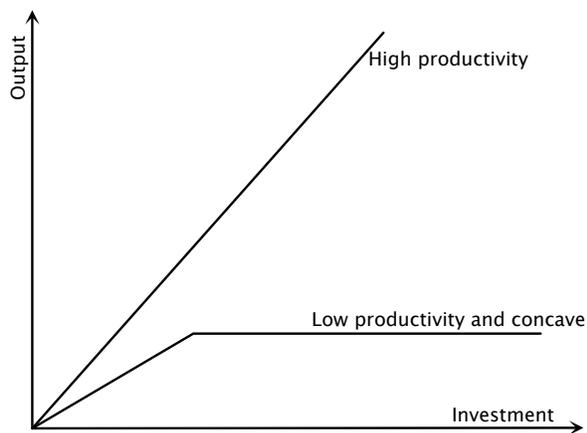


Figure 8: Concave generates endogenous commitment power

The figure demonstrates the payoff functions of two investment opportunities: one with high productivity and linear return, and another with low productivity and concave return. An entrepreneur who lacks commitment power may strictly prefer the investment opportunity with dominated returns because concavity generates endogenous commitment power.

sufficiently low, lack of exclusivity becomes irrelevant, as the entrepreneur does not borrow from any lenders beyond the initial one in equilibrium. Moreover, an appropriate level of  $\bar{K}$  could even restore the second-best level of investment—the level that would have been obtained under full-commitment for a linear-return project with productivity  $z$ —and strictly raise entrepreneur welfare.

Because concavity generates endogenous commitment power, if entrepreneurs can select among a menu of projects to finance, they may prefer an investment opportunity that is strictly dominated by another in terms of having strictly lower and more concave returns for any level of investment (see Figure 8).

**Proposition 8.** *Consider investment opportunities  $R^L(K)$  and  $R^H(K)$  such that  $R^H(K) > R^L(K)$  for all  $K$ . Absent commitment, an entrepreneur may prefer to undertake the dominated project  $R^L(K)$  because it generates endogenous commitment power and delivers higher welfare.*

Proposition 8 generates a strong empirical predictions. If entrepreneurs have scope to choose between projects that can scale significantly and those that cannot, increasing the availability of finance through lender entry in an environment with limited ability to commit could result in lower growth or the failure of increased access to credit to facilitate growth. As discussed in Section 6 this prediction is also consistent with the growing consensus that microfinance has not provided the However, this

prediction is difficult to test empirically, as it would require detailed data on the returns to scale of economic opportunities as well as variation in the degree of lender availability that is uncorrelated with the nature of projects requiring financing.

#### 4.4 Lender's Bargaining Power

In our baseline model the borrower makes take-it-or-leave-it offers to the lender, which effectively gives all bargaining power to borrowers. How does the distribution of surplus and who makes offers affect our results? We now extend the model so that contractual terms are determined by Nash bargaining. Relative to an outside option of zero, each lender is able to capture fraction  $\beta$  of the surplus from the contract. With  $\beta = 1$  we can also interpret the model as lenders making offers instead of borrowers. Borrower surplus is defined with respect to the borrowers outside option of taking no loan from this lender and instead proceed to the next stage of the lending market and possibly meeting another lender.

In developing this analysis it is helpful to reformulate the value function representation of the borrower's problem in terms of its outside option. Define  $W(D) \equiv -(1-q)\mathbb{E}[\min\{D, \tilde{c}\}] + qV(D)$  to be the expected continuation value to the entrepreneur after signing a contract bringing total promised repayment to  $D$ , but *before* the uncertainty about the next lender arrival is realized. Our baseline model's debt accumulation policy can be solved using the recursive value function for  $W(D)$  as

$$W(D) = -(1-q)\mathbb{E}[\min\{D, \tilde{c}\}] + q \times \max_{\{d,k\}} \{zk + W(D+d)\} \quad (6)$$

subject to  $k \leq \tilde{p}(D+d)d$ . Under Nash bargaining, the debt accumulation solves (6) subject to the constraint

$$\frac{1}{\beta} (\tilde{p}(D+d)d - k) = \frac{1}{1-\beta} (zk + W(D+d) - W(D)).$$

The term  $(\tilde{p}(D+d)d - k)$  captures lender surplus from the loan contract  $(d, k)$  relative to an outside option of value zero. The term  $(zk + W(D+d) - W(D))$  captures borrower surplus from the loan contract relative to an outside option of borrowing zero from the current lender and going back to

the lending market, with continuation value  $W(D)$ . The Nash bargaining protocol implies that each lender captures  $\beta$  fraction of the total surplus from the contract  $(d, k)$  and that the borrower captures  $(1 - \beta)$  fraction of the surplus generated by each loan.

**Proposition 9.** *Consider the Nash bargaining extension with  $\beta > 0$ . In equilibrium, lender's assessment of repayment probability  $\tilde{p}(\cdot)$  and borrower's debt accumulation policy  $g(\cdot)$  can be summarized by the endogenous scalars  $b^{Nash}(z, q, \beta)$  and  $\ell^{Nash}(z, q, \beta)$ :*

$$1 - g(D) = (1 - D) b^{Nash}(z, q, \beta)$$

$$\tilde{p}(D) = (1 - D) \ell^{Nash}(z, q, \beta).$$

The two equilibrium scalars are the fixed point of the pair of functions  $L(b)$  and  $B(\ell)$ , i.e.  $\ell^* = L(b^*)$  and  $b^* = B(\ell^*)$ , which satisfy

$$L(b) = \frac{1 - q}{1 - qb}, \quad (7)$$

$$\hat{q}z\ell(B(\ell))^2 + (1 - \hat{q} - 2z\ell)B(\ell) + z\ell = 0, \quad (8)$$

where  $\hat{q} \equiv q \frac{(1-\beta)}{1-\beta+z\beta(1-q)} \leq q$ . Note that  $\hat{q}$  is decreasing in  $\beta$ .

To understand the proposition, note that equation (7) characterizes the probability of repayment as a function of borrower's aggressiveness in future debt accumulation. This function is the same as in the baseline model (3). On the other hand, equation (8) characterizes borrower's policy function as a function of lenders' perceived probability of repayment. Notice that equation (8) is similar to equation (4) of the baseline model except switching  $q$  to  $\hat{q} \leq q$ . In other words, for a given  $\ell$ , debt accumulation policy under Nash bargaining coincides with the policy in a baseline model with lower likelihood of future lender arrival. Consequently, for a given  $\ell$ ,  $b^{Nash}$  is strictly decreasing in lender's bargaining power  $\beta$ , meaning that shifting bargaining power to the lender leads to strictly more aggressive debt accumulation and worsens the commitment problem.

How can this be? To understand this, note that the source of inefficiency lies in default externalities across lenders. When the entrepreneur holds all the bargaining power and lenders exactly

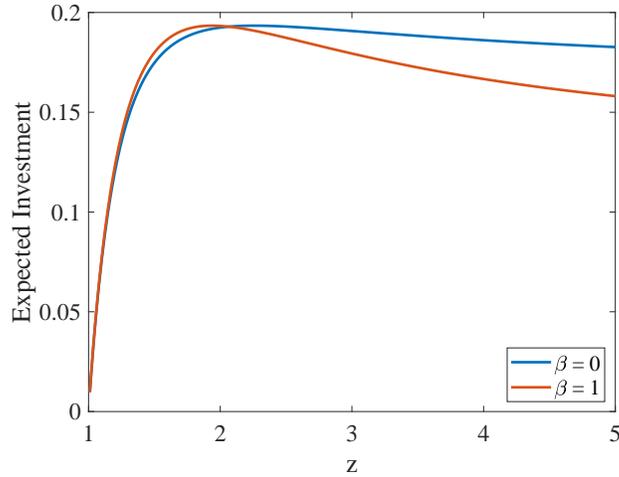


Figure 9: Expected Investment in the Dynamic Lending Game

break even, the entrepreneur takes his own lack-of-commitment into account and makes contractual choices to maximize the total welfare, which is equal to entrepreneur’s own payoff. On the other hand, when lenders hold some bargaining power, a fraction of total surplus will be captured by *future* lenders. Such future surplus is not accounted for when the *current* contracting parties bargain over contractual choice. This makes the contracting parties effective more *myopic*—the parties jointly issue debt contract *as if* the probability of meeting future lenders is lower than the actual probability,  $\hat{q} < q$ —thereby causing more aggressive debt accumulation.

## 4.5 Policy Implications

Regulatory tools can improve outcomes in our model. Intuitively, any prudential policy that discourages borrowing in the future would alleviate the commitment problem. We show such policies may take the form of interest rate caps, borrowing limits, or simply limiting the number of lenders from which a borrower can obtain loans. Because the constrained-optimal investment levels depends on entrepreneur’s productivity  $z$ , the optimal prudential interest rate caps and borrowing limits must also be borrower-specific.

We also solve for the optimal Pigouvian credit tax, levied from new lenders and transferred to

pre-existing lenders. The optimal credit tax can restore the full-commitment outcome and is easily implementable: it can be calculated solely based on equilibrium debt issuance and interest rates, thus the regulator does not need to know any structural parameters of the environment.

#### 4.5.1 Limits on the interest rate, total debt, and number of lenders

Prudential regulations—such as limits on the interest rate, total debt, and number of lenders—may improve welfare because they embed commitment power. Interest rate caps, for instance, ensures that debt beyond a certain level may never be issued, as a binding interest rate cap prevents the issuance of loans that are especially dilutive. Early lenders can then be assured that such future borrowing will not occur, and can provide initial loans at better interest rates. Consequently, interest rate caps may increase investment and improve welfare as long as they are not too severe. Further, for a given investment opportunity the interest rate cap can be set to be the interest rate that would prevail in the full-commitment equilibrium. This resolves the inefficiencies induced by non-exclusivity and induces the full-commitment level of debt, interest rate, and investment. Likewise, debt limits and directly restricting the number of lenders may improve welfare in similar ways.

We denote the equilibrium when  $q = 0$  as the *full-commitment outcome*.

**Proposition 10. (Interest Rate Cap)** *Consider the baseline model with an additional constraint on the interest rate:  $d/k \leq \bar{R}$ . The interest rate cap affects borrower's ex-ante welfare as follows:*

$$\frac{\partial V(0)}{\partial \bar{R}} \begin{cases} \geq 0 \\ \leq 0 \end{cases} \text{ if } \bar{R} \begin{cases} \leq \\ > \end{cases} \frac{2z-1}{z}.$$

Setting  $\bar{R} = \frac{2z-1}{z}$  restores the full-commitment outcome.

**(Debt Limit)** *Consider the baseline model with an additional constraint on the total outstanding debt:  $D^{agg} \leq \bar{D}$ . The debt limit affects borrower's ex-ante welfare as follows:*

$$\frac{\partial V(0)}{\partial \bar{D}} \begin{cases} \geq 0 \\ \leq 0 \end{cases} \text{ if } \bar{D} \begin{cases} \leq \\ > \end{cases} \frac{z-1}{2z-1}.$$

Setting  $\bar{D} = \frac{z-1}{2z-1}$  restores the full-commitment outcome.

Both debt limits and interest rate caps were adopted for microfinance loans in India in 2011 (Reserve Bank of India, 2011), where multiple borrowing was prevalent and posed a threat to the microfinance industry. In general, however, regulators should be cautious to implement market-wide prudential policies to limit debt issuance and interest rates, as caps that are too low can restrict productive investment. Welfare-enhancing prudential policies depend on the distribution of entrepreneur's productivities, information on which might be difficult for the regulators to collect.

#### 4.5.2 Pigouvian Credit Tax

We now show that a simple Pigouvian tax on new loans can restore the full commitment equilibrium. The tax is levied on subsequent loans and rebated to previous lenders so that 1) subsequent lenders internalize the incremental default risk on pre-existing loans and 2) current interest rate does not price-in future debt dilution because current lenders expect to be compensated by future tax rebates. Most importantly, implementing this tax only requires publicly observable information on current and prior loan terms, and—unlike in the case of interest rate caps and debt limits—the planner does not need to know entrepreneur-specific characteristics such as productivity.

Specifically, let  $\tau_n$  be the tax levied on the  $n$ -th lender, who is willing to provide investment  $k_n = p(D_n)d_n - \tau_n$ . At the origination of this loan, each prior lender  $i < n$  is compensated by  $\tau_n^i$ . Government budget balance requires  $\tau_n = \sum_{i=1}^{n-1} \tau_n^i$ .

**Proposition 11. (Optimal Credit Tax)** *The following set of taxes and transfers restore the full-commitment level of investment, debt issuance, and interest rate outcomes:*

$$\tau_n = K_{n-1} \frac{d_n}{D_n} - k_n \frac{D_{n-1}}{D_n} \text{ for } n > 1 \quad \text{and} \quad \tau_n^i = \frac{d_i}{D_{n-1}} \times \tau_n \text{ for } i < n.$$

These taxes and transfers are chosen such that 1) the tax levied on the  $n$ -th lender exactly equals the impact of the loan  $d_n$  on the value of outstanding loans  $D_{n-1}$  ( $\tau_n = [p(D_{n-1}) - p(D_n)]D_{n-1}$ ), and 2) the transfers to all pre-existing lenders  $i < n$  are chosen to exactly offset the effect of debt dilution ( $\tau_n^i = [p(D_{n-1}) - p(D_n)]d_i$ ). In Proposition 11, these taxes and transfers are stated in terms of borrowing history and can be implemented without direct knowledge of how debt accumulation affect

repayment probabilities. This is because lenders charge competitive pricing for loans; consequently, equilibrium interest rates reveal repayment probabilities, and the *difference* in loan terms between sequential lenders reveals changes in repayment probabilities.

The tax instruments we propose is economically similar to the “performance pricing” covenant observed in the broadly syndicated corporate loan market. Performance pricing covenants specify that the interest rate of any existing loan is a pre-specified function of the total debt of the borrower, such that the interest rate on an existing loan can increase if the borrower subsequently issues additional debt. Proposition 11 highlights that even if implementing such a contingent contract is not feasible, the same effect can be achieved through regulatory policy as long as prior borrowing is observable.

## 5 Connection to Theoretical Literature

We now discuss how our paper relates to the broader literature on non-exclusive contracting in credit markets and discussing where differences in modeling assumptions generate different results. First, Parlour and Rajan (2001), Bisin and Guaitoli (2004), and Attar et al. (2019) study models in which lenders, not borrowers, make offers, and find that equilibria are substantially different than in models of multiple borrowing in which borrowers make offers, such as Bizer and DeMarzo (1992) and Kahn and Mookherjee (1998). Specifically, they find that if moral hazard is particularly severe a monopolistic outcome can be attained despite free entry of lenders. Another important feature, as identified by Segal and Whinston (2003) is that allowing the proposal of *menus* of contracts affects equilibrium outcomes. In particular, Attar et al. (2019) highlights that allowing lenders to propose menus of contracts can induce constrained efficient outcomes. While lenders in our model cannot offer menus of contracts, our Nash bargaining model in section 4.4 does nest the case in which lenders make offers, and we find that lenders’ bargaining power strictly worsens allocations.

Our paper also speaks to the role of debt covenants in solving commitment problems between financial claimants of a firm, discussion of which dates back to Smith and Warner (1979). Matvos (2013) and Green (2018) show empirically that corporate debt covenants create value. Covenants that limit indebtedness add value in our model by mechanically serving as an explicit form of commitment,

and these are often observed in credit agreements of large firms in developed countries. Attar et al. (2019) also study the role of covenants in credit markets without commitment and come to a different conclusion.<sup>5</sup> They study a model in which lenders make menus of offers and show that the ability to offer contracts covenants (contingent on total financing raised) can fail to resolve commitment externalities. Their insight is that in an expanded contracting space there is an unraveling of the force that allows non-exclusive competition to induce constrained efficient outcomes. In contrast to this result, we argue in Section 4.5 that in our model, covenants (or an equivalent tax), that allow interest rates to be contingent on subsequent financing do restore the constrained efficient allocation. This happens in our model, but not Attar et al. (2019), because lenders in our model would only write covenants to protect against dilution, and do not have the scope to consider how such covenants could be used strategically to crowd out other lenders and increase profits. Szentes (2015) shows more generally that the ability to contract on contracts in common agency settings generates substantial indeterminacy in equilibria by providing principals scope to collude arbitrarily, including the ability to implement a monopolist outcome.

We also build on the literature studying the concept of debt dilution—that issuing additional debt can reduce the value of a borrower’s existing debt claims.<sup>6</sup> The simplest mechanism is that issuing debt of equal priority to existing debt can reduce existing debt’s value if the new debt derives some of its value from pre-existing cashflows. Thus, seniority arises as a natural resolution of debt dilution (Hart and Moore, 1995). However, as noted in Bizer and DeMarzo (1992) and discussed in the literature on sovereign debt (e.g. Chatterjee and Eyigungor, 2015) debt dilution can also occur with seniority if junior issuances increase the likelihood of costly default on senior claims, for example through moral hazard.<sup>7</sup> The approach in our model blurs distinction between these mechanisms. Dilution in our model arises from moral hazard, though we do not explicitly model new debt issues

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<sup>5</sup>Bisin et al. (2008) study an alternate formulation of endogenous contingency, in which principals are able to monitor the interaction of the agent with other principals, but only at a cost.

<sup>6</sup>The definition of “debt dilution” is varigated in the literature. Some papers consider dilution to mean the loss of value associated with issuing additional pro-rata claims. Other papers, including this one, define debt dilution more broadly as reduction in the value of existing debt arising directly or indirectly from the issuance of any new debt claims.

<sup>7</sup>Also related, Donaldson et al. (2018) study the nuanced relationship between seniority, collateral, and negative pledge covenants when borrowers cannot commit ex-ante to a future debt issuance policy and negative pledge covenants are imperfect in preserving the seniority of unsecured debt.

as subordinate to existing debt. Our model abstracts from seniority, as the cashflows backing the loan are binary. In a version of the model with richer state-contingent payoffs, seniority may dampen the dilution associated with sequential borrowing but cannot eliminate it completely due to moral hazard.<sup>8</sup>

The process of debt accumulation in our model is similar to the temporal dynamics of leverage without commitment in Admati et al. (2018) and DeMarzo and He (2018). With a complete lack of commitment (the limiting case of  $q = 1$  in our model, and assumed throughout in Admati et al. (2018) and DeMarzo and He (2018)), borrowers are unable to obtain *any* value from increasing debt, but do so anyway. The reason, as explained in Section 3.2, is that lenders price in the fact that borrowers cannot commit to avoid taking debt issuance too far in the future, which dilutes the value of current debt.

## 6 Empirical Evidence

How do the implications of our paper, and the broader theoretical literature of multiple borrowing, connect with what we observe in reality? The broadest empirical prediction we generate is that commitment is important for the efficiency of lending outcomes. We thus discuss the relevant empirical evidence in two contexts: in the developing world where weak contracting environments make explicit commitment difficult, and in more sophisticated credit markets where we would not expect commitment externalities to be as large of a problem.

The focus on the commitment is well grounded in the reality of financing in developing countries around the world, which as it expands is becoming increasingly associated with the narrative of *multiple borrowing crises* (e.g. see Faruqee and Khalily (2011)). The quintessential example of multiple borrowing is the spectacular boom and bust of the microfinance industry the Indian state of Andhra Pradesh. A nascent industry in the 1990s, allegedly thousands of lenders entered the market to supply credit to nearly the entire state, much of it in the form of “overlapping” loans from many lenders to the same borrower. Borrowers accumulated large debt balances from many lenders, who individu-

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<sup>8</sup>This is similar to Bisin and Rampini (2006), who study an environment in which a primary lender can use the bankruptcy code to effectively establish itself as senior to other creditors, and find that this cannot completely overcome the distortions that arise from non-exclusive contracting.

ally had no way of observing or controlling a borrower's total indebtedness. This ultimately proved unsustainable and culminated in a default crisis and near-collapse of the industry in 2010. Fear and realization of such multiple borrowing crisis have also arisen around the world where microfinance has grown rapidly.

Several stylized facts have emerged from studies of multiple borrowing in microfinance. First, it appears that episodes of crisis associated with multiple borrowing are common in places where microfinance has experienced rapid growth, with similar episodes and concerns about multiple borrowing described in Peru, Guatemala, Bolivia (de Janvry et al., 2003), Uganda (McIntosh et al., 2005), Bangladesh (Khalily et al., 2016.) At the peak in 2009, surveys estimated that 84% of rural villagers in Andhra Pradesh had loans from multiple lenders, with the median household having four loans from all sources (Johnson and Meka, 2010). The same study also suggests that some borrowers in Andhra Pradesh borrow from multiple lenders because they cannot obtain enough financing from individual lenders. This is true in the equilibrium of our model; no lender will provide a very large amount of financing for fear that their claims will be diluted by other lenders. Of course, it is unclear if in practice limitations on how much lenders will lend to an individual borrower are a consequence of or impetus for multiple borrowing behavior.

Second, consistent with our model, there is some evidence of a *positive* relationship between the intensity of competition between MFI lenders and interest rates. Porteous (2006) explores the relationship between MFI competition and interest rates they charge by examining the cases of Bolivia, Bangladesh, and Uganda, all of which are considered to have very competitive microfinance markets. The study documents that the difference between MFI and formal bank interest rates declined significantly in Bolivia following substantial consolidation, while this spread had remained flat in Bangladesh and Uganda, where such consolidation has not yet occurred.

Third, debt limits and interest rate caps are often discussed as a response to problems arising from multiple borrowing and competition among microfinance lenders. In Bangladesh the microfinance regulator imposed rate caps and the central bank considered broader interest rate limits (Porteous, 2006.) In response to the Andhra Pradesh crisis, in 2011 the Reserve Bank of India imposed new regulations on the banking industry and stated that they were in part meant to address multiple bor-

rowing. A debt limit (USD 790) and interest rate cap (26%) were imposed (Reserve Bank of India, 2011). As shown in Section 4.5, these policies work by explicitly limiting the scope of commitment externalities in lending markets.

It is important to note that while these patterns can be explained by the mechanisms highlighted in our model, they could also be attributed to “growing pains” in new and rapidly expanding industries—the inexperience of new borrowers and lenders, and an intense desire to establish market share at the expense of sound lending practices (Armendáriz and Morduch (2005) and Porteous (2006)). Also, while there is broad evidence that multiple borrowing is viewed as a problem by microfinance lenders and policymakers, empirical evidence is less clear. For example, Khalily et al. (2016)’s study documents correlational evidence that within Bangladesh, households engaging in multiple borrowing are actually better off and do not have excessive levels of debt.

There is, to our knowledge, only indirect evidence on the predictions of our model related to investment productivity and ability to obtain financing.<sup>9</sup> The relationship we characterize between financial constraints and productivity generates a particularly stark form of misallocation: a negative correlation between the *level* of investment in a project and its productivity. This is consistent with a growing body of evidence from micro and experimental studies conducted in developing economies that productive firms may be especially credit constrained. McKenzie and Woodruff (2008) study microenterprise in Mexico and use a randomized experiment to estimate returns to investment capital. In addition to finding very high returns on average, they also find a positive relationship between the returns to investment capital of borrowers and the financial constraints they face, indicating that it is the projects with the best economic fundamentals that are most under-served. Similarly, Banerjee and Duflo (2014), using policy changes in credit access programs in India, find similar evidence of a negative selection effect. Their analysis of the relationship between loan growth and profit growth suggests that it is the least productive firms that are acquiring the most financing.

While this evidence is consistent with our results that higher productivity projects receive less investment, it does not specifically shed light on our prediction that lack of commitment in credit

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<sup>9</sup>Misallocation induced by through frictions in microfinance lending has also been studied theoretically by Liu and Roth (2016), who show monopolistic lenders have incentives to generate debt traps via imposing contractual restrictions when future profits are nonpledgeable.

markets could cause entrepreneurs to purposefully pursue opportunities with lower returns to scale than otherwise available to them. However, this result provides at least a potential explanation for the emerging consensus that microfinance has failed to achieve the widespread poverty reduction once thought possible. Despite the widespread view that expanding credit access would promote entrepreneurial activity and in turn lead to growth, randomized control trials of micro-credit expansions find little impact on borrower income and business size (Banerjee et al., 2015).

Turning to more developed credit markets, multiple and sequential borrowing is common, but there is less systematic concern about its negative implications. In consumer and small and medium enterprise credit markets, lenders are concerned with the possibility of default driven by multiple borrowing, but only in the context of fraud, in which inauthentic credentials are used to open and draw on a large number of credit accounts before information sharing systems can detect it (Phelan, 2016). Why does multiple borrowing not seem to be an explicit concern here outside of cases of pure fraud? It is possible that reputational mechanisms associated with the availability of a credit registry play an important role in aligning borrower incentives and preventing excessive borrowing.<sup>10</sup>

Ivashina and Kovner (2011) find evidence for this in the market for large corporate debt contracts, showing that stronger relationships between a private equity owner of a firm and banks are associated with better pricing and less stringent debt covenants on the firms debt. Prilmeier (2017) similarly finds covenants are stronger earliest in the bank-borrower relationship. Among the most common covenants in corporate loans and bonds are limitations on both subsequent debt issuance and overall leverage and interest coverage ratios of a firm (Billett et al., 2007 and Bradley and Roberts, 2015.) These covenants not only establish the seniority of existing debt, but also prevent the issuance of junior or subordinate debt. This, consistent with the discussion in Section 5, is suggestive that seniority alone cannot ameliorate externalities arising from multiple borrowing. In the syndicated loan market, “performance pricing” covenants tie the interest rate to be paid on debt to the leverage or total debt of a firm.<sup>11</sup> This contractual feature, similar to the Pigouvian tax we analyze in Section 4.5, can force borrowers to internalize the effect that subsequent borrowing has on existing creditors.

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<sup>10</sup>Lieberman (2016) finds that borrowers are willing to pay substantially to eliminate defaults from their credit record.

<sup>11</sup>See Asquith et al. (2005) for a detailed description of performance based pricing covenants.

## 7 Conclusion

This paper argues that commitment problems in lending markets can explain emerging empirical evidence that the rapid expansion of credit access can have perverse effects. When borrowers cannot commit to exclusive contracting, increasing the availability of lenders makes markets appear *less* competitive as interest rates rise and entrepreneur investment and welfare fall. More importantly, commitment problems can result in better projects receiving less investment than worse projects. This force can be so severe that what look like good opportunities are passed over for inferior investment technology. Finally, we show how simple regulatory tools such as interest rate ceilings and debt limits can improve outcomes and ameliorate the misallocative forces we highlight.

The intuition for these result is that the externalities the lenders impose on each other when commitment or contingent contracting is not possible can prevent the borrower from being able to use pledgeable cash flows efficiently. When explicit commitment is impossible, there is value in any implicitly commitment mechanisms that attenuate the demand for further borrowing. Thus, the return profile of an investment opportunities itself is an important driver in the severity of commitment distortions.

Since commitment is less of a problem for projects with lower marginal returns, when given the choice entrepreneurs will endogenously choose investment opportunities that are everywhere less productive than other available opportunities, as long as they are sufficiently more concave. Thus our model provides a new micro-foundation for the idea that commitment problems in lending markets can induce substantial misallocation in capital investment and can explain observations both of low growth and of economic activity below the technological frontier. While we have augmented our study with a sample of the growing anecdotal evidence that multiple borrowing is problematic, there is much more to learn. Formally testing the empirical validity the mechanisms we highlight in explaining the failure of increased access to finance to significantly improve outcomes is an important topic for future research.

# Appendix

## Microfounding Random Default with Moral Hazard

In this section we provide an alternative microfoundation of the random default through limited pledgeability and moral hazard. Instead of assuming the entrepreneur borrows against his default cost  $\tilde{c}$  as we do in the main text of the paper, we endow the entrepreneur with a pledgeable stochastic cashflows from assets-in-place that are realized at  $s = 2$ . The pledgeable cashflows realize one of two values: zero or one. The good payoff occurs with probability  $p$ , which is chosen by the entrepreneur at quadratic effort cost  $p^2$ . Moral hazard arises because effort is chosen after cashflows are (partially) pledged to lenders. Since there are only two realizations of these cashflows and the entrepreneur has no other pledgeable wealth, it is without loss of generality that claims issued to lenders take the form of debt contracts: they are repaid in full when the good realization occurs and the entrepreneur defaults when cashflows are zero. With this in mind, assume the entrepreneur has issued a total face value of debt  $D$  in the first stage. At  $s = 2$  the entrepreneur solves

$$p(D) \equiv \arg \max_p p[1 - D] - p^2$$

The entrepreneur expects the assets in place to pay out with probability  $p$ , and conditional on a positive payout the entrepreneur gets to keep  $1 - D$  of the cashflows as the residual claim. The cost of choosing the probability of positive cashflows to be  $p$  is  $p^2$ . Thus  $p(D)$  denotes the solution to the entrepreneur's choice of effort at stage  $s = 2$  conditional on having a total outstanding face value of debt  $D$ . The solution of entrepreneur's problem yields  $p(D) = \frac{1-D}{2}$ , and entrepreneur's expected payoff from the residual claim is  $\frac{(1-D)^2}{4}$ . The value function in (2) is modified to

$$V(D) = \max_{D'} z\tilde{p}(D')(D' - D) - (1 - q)\frac{(1 - D)^2}{4} + qV(D')$$

and all of our results go through analogously.

## Proof of Proposition 1

Borrower's problem can be formulated recursively as:

$$V(D) = \max_{D'} z \tilde{p}(D') (D' - D) + (1 - q) \left[ \frac{(1 - D')^2}{2} - \frac{1}{2} \right] + qV(D')$$

where

$$\tilde{p}(D') \equiv \mathbb{E}[1 - D^{agg} | D']$$

We guess that borrower's policy function  $g(\cdot)$  and lenders' loan pricing function  $\tilde{p}(\cdot)$  both take a linear form and are each characterized by a single endogenous variable,  $b$  and  $\ell$ , respectively:

$$\tilde{p}(D) = \ell(1 - D)$$

$$1 - g(D) = b(1 - D)$$

To solve for borrower's policy function, we proceed to take first order condition and use the envelope condition for borrower's problem. The first order condition is:

$$-z\ell(g(D) - D) + z\ell(1 - g(D)) - (1 - q)(1 - g(D)) + qV'(g(D)) = 0$$

and the envelope condition is:

$$V'(D) = -z\ell(1 - g(D))$$

Plugging the envelope condition into the first-order condition and after simplifying, we can express the Euler condition as a quadratic function of  $b$ :

$$qz\ell b^2 + (1 - q - 2z\ell)b + z\ell = 0$$

We thus solve for the endogenous parameter  $b$  that governs the borrower's policy function as:<sup>12</sup>

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<sup>12</sup>There are two roots to the quadratic equation, one of which leads to explosive debt accumulation. We choose the other, stable root.

$$b = \frac{(2z\ell - (1-q)) - \sqrt{(1-q - 2z\ell)^2 - 4qz^2\ell^2}}{2qz\ell} \quad (9)$$

To solve for the lender's loan pricing function, not

$$\begin{aligned} \tilde{p}(D) &= \mathbb{E}[1 - D^{agg}|D] \\ &= (1-q) [(1-D) + q(1-g(D)) + q^2(1-g(g(D))) + \dots] \\ &= (1-q) [(1-D) + qb(1-D) + q^2b^2(1-D) + \dots] \\ &= \frac{1-q}{1-qb} (1-D) \end{aligned}$$

Hence

$$\ell = \frac{1-q}{1-qb} \quad (10)$$

Equation (9) characterizes  $b$  as a decreasing function of  $\ell$ . On the other hand, equation (10) characterizes  $\ell$  as an increasing function of  $b$ . The two equations therefore yields a unique solution  $(b^*, \ell^*)$  for each  $q \in [0, 1)$  and  $z \in (1, \infty)$ . In particular, we have

$$\begin{aligned} b^* &= \frac{z}{(z - \frac{1}{2}) + \sqrt{(z - \frac{1}{2})^2 - qz(z-1)}} \\ \ell^* &= \frac{(1-q)(z-1)}{\sqrt{(z - \frac{1}{2})^2 - qz(z-1)} - \frac{1}{2}} \end{aligned}$$

To solve for borrower welfare,  $V(0)$ , we can plug these expressions back into the value function, which we guess and verify has quadratic form  $V(D) = aD^2 + bD + c$ . Observe that  $V(1) = -\frac{1}{2}$  and  $V'(1) = 0$ , which imply the functional form must satisfy  $V(D) = -\frac{1}{2} + a(1-D)^2$  for some constant  $a$ . We can find the constant  $a$  by plugging back into the value function our equilibrium expressions

$$\begin{aligned} V(D) &= z\tilde{p}(D') (D' - D) + (1-q) \left[ \frac{(1-D')^2}{2} - \frac{1}{2} \right] + qV(D') \\ -\frac{1}{2} + a(1-D)^2 &= [z\ell b(1-b) + (1-q)b^2/2 + qab^2] (1-D)^2 - (1-q) \sum_{i=0}^{\infty} q^i \frac{1}{2} \end{aligned}$$

which, solving for  $a$  and simplifying substantially, gives

$$V(D) = -\frac{1}{2} + \left( \frac{z^2(1-q)}{2z(1-q) - 1 + 2\sqrt{(z-\frac{1}{2})^2 - qz(z-1)}} \right) (1-D)^2$$

□

**Lemma 1.** *The best response functions have the following properties:*

$$\begin{aligned} \frac{\partial L(b; \alpha, q)}{\partial b} &\geq 0; \quad \frac{\partial L(b; z, q)}{\partial z} = 0; \\ \frac{\partial B(\ell; z, q)}{\partial z} &\leq 0; \quad \frac{\partial B(\ell; z, q)}{\partial \ell} \leq 0; \end{aligned}$$

where the inequalities are strict for  $q \in (0, 1)$  and  $z > 1$ .

*Proof.* The results with respect to lender's best response immediately follow from equation (10).

We now with with equation (9) to derive the results respect to borrower's best response function.

Let  $x \equiv 2z\ell$ , we have

$$\begin{aligned} b &= \frac{(x - (1-q)) - \sqrt{(1-q-x)^2 - qx^2}}{qx} \\ &= \frac{1}{q} - \frac{1-q + \left( (1-q)(x-1)^2 - q(1-q) \right)^{\frac{1}{2}}}{qx} \end{aligned}$$

Let  $\Delta \equiv \left( (1-q)(x-1)^2 - q(1-q) \right)$  and take derivative with respect to  $x$ , we have

$$\begin{aligned} \frac{\partial b}{\partial x} &= -\frac{qx(1-q)(x-1) - q \left( (1-q)\Delta^{\frac{1}{2}} + (1-q)(x-1)^2 - q(1-q) \right)}{(qx)^2 \Delta^{\frac{1}{2}}} \\ &= -\frac{q(1-q)}{(qx)^2 \Delta^{\frac{1}{2}}} \left( x^2 - x - \Delta^{\frac{1}{2}} - (x-1)^2 + q \right) \\ &= -\frac{q(1-q)}{(qx)^2 \Delta^{\frac{1}{2}}} \left( x + q - 1 - \Delta^{\frac{1}{2}} \right) \end{aligned}$$

Since  $\Delta^{\frac{1}{2}} = \sqrt{(x+q-1)^2 - qx^2} \leq x+q-1$ , we have that

$$\frac{\partial b}{\partial x} \leq 0$$

and the inequality is strict for  $q \in (0,1)$ . Given the definition of  $x$ , we have that  $\frac{\partial b(\ell; z, q)}{\partial z} \leq 0$  and  $\frac{\partial b(\ell; z, q)}{\partial \ell} \leq 0$ . □

## Proof of Proposition 2

The equilibrium sequence of debt issuance is straightforward to compute. Recall that the borrower promises to repay the  $i+1$  lender an amount  $d_{i+1} = g(D_i) - D_i$ , where  $D_i$  is the cumulative repayment already pledged to previous lenders. Also recall that this pledge of  $d_{i+1}$  generates current proceeds of  $k_{i+1} = \tilde{p}(g(D_i))(g(D_i) - D_i)$ . In equilibrium we can write these expressions as simply

$$d_{i+1} = (1 - b^*)(1 - D_i)$$

$$k_{i+1} = \ell^* b^* (1 - b^*) (1 - D_i)^2$$

These can also be expressed recursively as

$$d_{i+1} = b^* \times d_i$$

$$k_{i+1} = (b^*)^2 \times k_i$$

The proposition follows from the fact that  $b^* \leq 1$  with equality only when  $q = 0$  (full commitment).

## Proof of Proposition 3

We begin with an outline of the proof. The proof will first show how to solve the finite-lender game by backwards induction, generating a recursive formulation for borrower and lender strategies. Next, we show that the fixed point of this recursion generates the strategies of the infinite-lender equilibrium defined in the main text. Finally, to demonstrate convergence, we show that this recursive

formulation of strategies is characterized by a contraction mapping. This implies that, considering the strategies at a given lender, as the number of potential subsequent lenders goes to infinity, the equilibrium strategies at this lender converge uniquely to the fixed point and thus to the strategies of the infinite-lender equilibrium.

### Backward Induction in the Finite-Lender Game

Consider a finite version of the game with  $N$  lenders. For this proof, we abuse notation and index periods counting backwards from the end. Thus the last lender is indexed 1, and the first lender is indexed  $N$ . Therefore, after lender  $i$  there are at most  $i - 1$  more lenders for the borrower to visit. Let  $D_i$  denote the amount of cumulative debt the borrower accumulates from meeting lenders  $N$  through  $i + 1$ , thus  $D_0$  denotes the total amount of debt the borrower will accumulate if it gets to meet all  $N$  lenders. The probability of default from the last lender's perspective will be  $\tilde{p}_1(D_0) = 1 - D_0$  if the lender accepts a proposal that brings the borrower's cumulative debt to  $D_0$ . This defines the unique strategy of the final lender to accept only weakly profitable loans. Assume the borrower has any arbitrary face value of debt  $D_1$  upon meeting the final lender. The borrower solves

$$V_1(D_1) \equiv \max_{D_0} z(D_0 - D_1)(1 - D_0) + \left( \frac{D_0^2}{2} - D_0 \right)$$

and the solution is

$$1 - D_0 = (1 - D_1) \underbrace{\frac{z}{2z - 1}}_{\equiv B_1}$$

with borrower's maximized value function being

$$V_1(D_1) = (1 - D_1)^2 \underbrace{\left[ \frac{2z^3 - z^2}{2(2z - 1)^2} \right]}_{\equiv W_1} - \frac{1}{2}$$

We define

$$\begin{aligned} B_1 &\equiv \frac{z}{2z-1} \\ L_1 &\equiv 1 \\ W_1 &\equiv \frac{2z^3 - z^2}{2(2z-1)^2} \end{aligned}$$

Thus the unique subgame perfect equilibrium strategies conditional on arriving to the last lender with some amount of debt  $D$  are:

$$\begin{aligned} 1 - g(D) &= B_1(1 - D) \\ \tilde{p}_1(D) &= L_1(1 - D) \end{aligned}$$

and any borrower considering leaving the second-to-last lender with a total face value of debt  $D$  realizes that continuation utility if it reaches the last lender is given by

$$V_1(D) = W_1(1 - D)^2 - \frac{1}{2}.$$

Thus we know that at lender  $i = 1$  players use strategies linear in  $1 - D$ . Now we show by induction that all lenders use such linear strategies. Assume for some  $n$  that players at all stages  $i < n$  use linear strategies and that the maximized value function at lender  $i$  is proportional to  $(1 - D_{i+1})^2$ . We will show that players at stage  $n$  also use linear strategies and that the maximized value function at lender  $n$  is proportional to  $(1 - D_{n+1})^2$ , and thus by induction prove that these claims do indeed hold for all  $n \in \mathbb{N}$ .

Now consider the subgame where the borrower meets lender  $n$  with cumulative debt  $D_n$  obtained from previous lenders. Since all future lenders and borrowers use linear strategies, we can compute lender  $n$ 's expected probability of repayment:

$$\begin{aligned}
\tilde{p}_n(D_{n-1}) &= (1-q)(1-D_{n-1}) + q\tilde{p}_{n-1}(D_{n-2}) \\
&= (1-q)(1-D_{n-1}) + qL_{n-1}(1-D_{n-2}) \\
&= (1-q)(1-D_{n-1}) + qL_{n-1}B_{n-1}(1-D_{n-1}) \\
&= [(1-q) + qB_{n-1}L_{n-1}](1-D_{n-1})
\end{aligned}$$

where the first to second line follows from the assumption that lender  $n-1$  is using a linear strategy  $\tilde{p}_{n-1}(D) = L_{n-1}(1-D)$ , and moving from the second to the third line relies on the assumption that the borrower at  $n-1$  is using a linear strategy  $1-D_{n-2} = B_{n-1}(1-D_{n-1})$ . Thus we know that lender  $n$  follows the strategy given by

$$L_n = 1 - q + qB_{n-1}L_{n-1}$$

Next, under our inductive hypothesis we can write the borrower's problem visiting lender  $n$  as:

$$\begin{aligned}
V_n(D_n) &= \max_{D_{n-1}} z\tilde{p}_n(D_{n-1})(D_{n-1} - D_n) + (1-q) \left( \frac{D_{n-1}^2}{2} - D_{n-1} \right) + qV_{n-1}(D_{n-1}) \\
&= \max_{D_{n-1}} \left\{ z(D_{n-1} - D_n)(1 - D_{n-1})L_n + (1-q) \left( \frac{D_{n-1}^2}{2} - D_{n-1} \right) \right. \\
&\quad \left. + q \left( W_{n-1}(1 - D_{n-1})^2 - \frac{1}{2} \right) \right\}
\end{aligned}$$

Taking the first order condition and solving for  $D_{n-1}$  verifies that the borrower's strategy does indeed have the hypothesized linear form:

$$1 - D_{n-1} = (1 - D_n) \underbrace{\frac{zL_n}{2zL_n - (1 - q + 2qW_{n-1})}}_{\equiv B_n}$$

and maximized value function

$$V_n(D_n) = (1 - D_n)^2 \underbrace{\left( z(1 - B_n) B_n L_n + (1 - q) \frac{B_n^2}{2} + q B_n^2 W_{n-1} \right)}_{\equiv W_n} - \frac{1}{2}$$

Thus the inductive proof is completed and all strategies satisfy the proposed form.

### Recursive Formulation of Strategies

It is clear from above that the vector  $(B_n, L_n, W_n)$  is generated by a system of 3 difference equations:

$$\begin{aligned} L_n &= (1 - q) + q B_{n-1} L_{n-1} \\ B_n &= \frac{z L_n}{2z L_n - (1 - q + 2q W_{n-1})} \\ W_n &= z(1 - B_n) B_n L_n + (1 - q) \frac{B_n^2}{2} + q B_n^2 W_{n-1} \end{aligned}$$

rearranging so each of  $(L_n, B_n, W_n)$  is a function of only the lagged variables:

$$\begin{aligned} L_n &= 1 - q + q B_{n-1} L_{n-1} \\ B_n &= \frac{z(1 - q + q B_{n-1} L_{n-1})}{2z(1 - q + q B_{n-1} L_{n-1}) - (1 - q + 2q W_{n-1})} \\ W_n &= \frac{z^2 (1 - q + q B_{n-1} L_{n-1})^2}{4z(1 - q + q B_{n-1} L_{n-1}) - 2(1 - q + 2q W_{n-1})} \end{aligned}$$

where the last equation can be simplified to

$$W_n = \frac{z}{2} B_n L_n.$$

## Convergence to Infinite-Lender Strategies

Defining a new variable  $x_n \equiv B_n L_n$ , the set of difference equations above can be rewritten as

$$L_n = (1 - q) + qx_{n-1}$$

$$B_n = \frac{z((1 - q) + qx_{n-1})}{2z((1 - q) + qx_{n-1}) - (1 - q + zqx_{n-1})}$$

$$W_n = \frac{z}{2}x_n$$

Hence the sequence  $\{x_n\}$  is defined by  $x_1 \equiv B_1 L_1 = \frac{z}{2z-1}$  and a continuous function  $f(\cdot)$  such that  $x_n = f(x_{n-1})$ , where

$$f(x) = \frac{z((1 - q) + qx)^2}{(1 - q)(2z - 1) + zqx}$$

First note that  $f(x) > 0$  if  $x > 0$ , and since  $x_1 > 0$ , we have  $x_n > 0$  for all  $n$ . It is also easily verified that  $f(\cdot)$  has a unique fixed point  $x^*$ , which corresponds to the unique fixed point  $(B^*, L^*, W^*)$  of the system of difference equations above. Moreover, it is a matter of algebra to show that the fixed point coincides with our closed form solution of the infinite lender game, implying that  $b^* = B^*$  and  $\ell^* = L^*$ .

We next show that  $f(\cdot)$  defines a contraction mapping which, given the continuity of  $f(\cdot)$ , shows that  $x_n \rightarrow x^*$ . This in turn implies  $(B_n \rightarrow b^*, L_n \rightarrow \ell^*)$  as  $n \rightarrow \infty$ . In words, this means that as the number of potential future lenders in the game after lender  $n$  grows towards infinity, the unique subgame perfect strategies the players at stage  $n$  converge to the stationary strategies employed by all players in the infinite-lender game.

To show  $f(\cdot)$  defines a contraction mapping, we show  $\|f'(x)\| < 1$ . Taking derivatives of  $f$  with

respect to  $x$ , we have (after much simplification)

$$\begin{aligned} f'(x) &= q - \frac{q(z-1)^2(1-q)^2}{(2z+q-2zq+zqx-1)^2} \\ &= q \left( 1 - \left( \frac{(z-1)(1-q)}{(2z-1)(1-q)+zqx} \right)^2 \right) \end{aligned}$$

Since  $x > 0$ ,  $1 > q \geq 0$ ,  $z > 1$ , we have

$$\begin{aligned} 0 &< \frac{(z-1)(1-q)}{(2z-1)(1-q)+zqx} < 1 \\ 0 &< \frac{(z-1)(1-q)}{(z-1)(1-q)+z(1-q)+zqx} < 1 \end{aligned}$$

Hence

$$0 < f'(x) < 1$$

which shows that  $f(\cdot)$  is a contraction mapping. □

## Proof of Proposition 4

Before proving the comparative static propositions, it will be useful to derive some expressions for equilibrium objects of interest in terms of parameters  $z$  and  $q$ . Let  $K^{agg}$  denote the aggregate investment and  $D^{agg}$  denote the aggregate debt that have been attained when the lending market game ends. In equilibrium, ex-ante, these are random variables with respect to the number of lenders the borrower will be able to visit.

**Lemma 2.**  $\mathbb{E}[K^{agg}] = \mathbb{E}[p(D^{agg})D^{agg}]$

*Proof.* Let  $N$  denote the random number of lenders the borrower gets to visit before losing access to the lending market game. The random aggregate face value of debt and aggregate investment can be expressed as:

$$D^{agg} = \sum_{j=1}^{\infty} d_j \mathbf{1}(N \geq j)$$

$$K^{agg} = \sum_{j=1}^{\infty} k_j \mathbf{1}(N \geq j)$$

where  $d_j$  is the amount of debt given by the  $j$ -th lender . Similarly denote  $k_j$  to be the amount of investment capital provided by the  $j$ -th lender. Pick any  $j > 0$ , the zero-profit condition for his loan and investment size is:

$$\mathbb{E} [p(D^{agg}) | N \geq j] d_j \mathbf{1}(N \geq j) = k_j \mathbf{1}(N \geq j)$$

Taking expectation over  $N$  on both sides and applying the law of iterated expectation, we get:

$$\mathbb{E} [p(D^{agg}) d_j \mathbf{1}(N \geq j)] = \mathbb{E} [k_j \mathbf{1}(N \geq j)]$$

We next sum the previous equation over all lenders. By the linearity of the expectations operator, we can bring the sum inside:

$$\mathbb{E} \left[ p(D^{agg}) \sum_{j=1}^{\infty} d_j \mathbf{1}(N \geq j) \right] = \mathbb{E} \left[ \sum_{j=1}^{\infty} k_j \mathbf{1}(N \geq j) \right]$$

Substituting in the defintions of  $D^{agg}$  and  $I^{agg}$ :

$$\mathbb{E} [p(D^{agg}) D^{agg}] = \mathbb{E} [K^{agg}]$$

□

**Lemma 3.** *We can express the expected debt and investment as functions of  $b$ :*

$$\mathbb{E} [D^{agg}] = \frac{1-b}{1-qb}$$

$$\mathbb{E} [K^{agg}] = \frac{b(1-b)(1-q)}{(1-bq)(1-b^2q)}$$

*Proof.* Denote the expected aggregate debt upon leaving a given lender with cumulative debt  $D$  as  $\mathbb{E} [D^{agg} | D]$ . From lender's zero-profit condition, we have

$$\mathbb{E}[D^{agg}|D] = 1 - \tilde{p}(D)$$

The ex-ante expected aggregate debt  $\mathbb{E}[D^{agg}]$  is simply the expected aggregate debt upon leaving a lender with zero outstanding debt, times  $\frac{1}{q}$  (since the borrower meets the first lender with certainty, not probability  $q$ ). Thus we have

$$\begin{aligned}\mathbb{E}[D^{agg}] &= \frac{1}{q}\mathbb{E}[D^{agg}|0] \\ &= \frac{1}{q}\left(1 - \frac{1-q}{1-qb}\right) \\ &= \frac{1-b}{1-qb}\end{aligned}$$

To get the expression for expected investment:

$$\begin{aligned}\mathbb{E}[K^{agg}] &= \tilde{p}(g(0))g(0) + q\tilde{p}(g^2(0))[g^2(0) - g(0)] + q^2\tilde{p}(g^3(0))[g^3(0) - g^2(0)] + \dots \\ &= \ell[(1-g(0))g(0) + q(1-g^2(0))[(1-g(0)) - (1-g^2(0))] + \dots] \\ &= \ell[b(1-b) + qb^2[b-b^2] + q^2b^3[b^2-b^3] + \dots] \\ &= \ell b(1-b)[1 + qb^2 + q^2b^4 + \dots] \\ &= \ell b \frac{1-b}{1-qb^2} \\ &= \frac{b(1-b)(1-q)}{(1-bq)(1-qb^2)}\end{aligned}$$

□

**Lemma 4.** Let  $x \equiv \sqrt{4(1-q)(z^2-z)+1}$ . The analytic solution of expected debt, investment, and welfare can be expressed as the following functions of parameters  $q$  and  $z$ :

$$\mathbb{E}[D^{agg}] = \frac{2z-1-x}{2qz}$$

$$\mathbb{E}[I^{agg}] = \frac{(z-1)(x+1-2z(1-q))}{2zq(2z-1)}$$

$$V(0) = \frac{1-2z(1-q)-2q+x}{4q}$$

*Proof.* These expressions can be obtained by substituting the analytic solution of  $b^*$  from lemma 1 into the expressions in lemma 2. □

Now continuing on, we can express equilibrium  $b^*$  as

$$(b^*) = \frac{2z-1-x}{2q(z-1)}$$

Also note that

$$\frac{\partial x}{\partial z} = x^{-1}(1-q)2(2z-1) > 0$$

$$\frac{\partial x}{\partial q} = -2x^{-1}(z^2-z) < 0$$

We now proceed to prove Proposition 4 claim by claim.

*Claim 1.*  $\mathbb{E}[D^{agg}]$  is decreasing in  $q$ .

*Proof.* We first express  $\mathbb{E}[D^{agg}]$  as a function of  $z$ ,  $q$ , and  $x$ :

$$\mathbb{E}[D^{agg}] = \frac{1-b}{1-qb}$$

$$\begin{aligned}
\mathbb{E}[D^{agg}] &= \frac{1 - \frac{2z-1-x}{2q(z-1)}}{1 - \frac{2z-1-x}{2(z-1)}} \\
&= \frac{2q(z-1) - 2z + 1 + x}{2q(z-1) - 2qz + q + qx} \\
&= \frac{(2qz - 2q - 2z + 1 + x)(x+1)}{q(x-1)(x+1)} \\
&= \frac{(2z-1-x)(1-q)(z-1)}{2zq(1-q)(\alpha-1)} \\
&= \frac{2z-1-x}{2zq}
\end{aligned}$$

Differentiating with respect to  $q$ , we get

$$\frac{d\mathbb{E}[D^{agg}]}{dq} = \frac{-2zq \frac{dx}{dq} - (2z-1-x)2z}{(2qz)^2}$$

which implies

$$\begin{aligned}
\text{sign}\left(\frac{\partial \mathbb{E}[D^{agg}]}{\partial q}\right) &= \text{sign}(2q(z^2 - z) - (2z-1-x)x) \\
&= \text{sign}(2q(z^2 - z) + 4(1-q)(z^2 - z) + 1 - (2z-1)x) \\
&= \text{sign}((z^2 - z)(4 - 2q) + 1 - (2z-1)x)
\end{aligned}$$

Let  $RHS \equiv (z^2 - z)(4 - 2q) + 1 - (2z-1)x$ . The remaining proof consists of three steps: 1) show  $\frac{dRHS}{dz} \geq 0$  for all  $z \geq 1, q \in [0, 1]$ ; 2) show  $\frac{dRHS}{dq} \geq 0$  for all  $z \geq 1, q \in [0, 1]$ , with equality holding only when  $z = 1$  or  $q = 0$ ; 3) RHS evaluated at  $z = 1, q = 0$  is zero, concluding that  $RHS > 0$  for  $z > 1, q > 0$ .

Step 1: show  $\frac{dRHS}{dz} > 0$  for all  $z \geq 1, q \in [0, 1]$ . Differentiating  $RHS$  with respect to  $z$ , we have

$$\begin{aligned}
\frac{dRHS}{dz} &= (2z-1)(4-2q) - 2x - (2z-1) \frac{\partial x}{\partial z} \\
&= (2z-1)(4-2q - x^{-1}(1-q)(4z-2)) - 2x \\
&> (2z-1)(4-2q - x^{-1}(1-q)(4z-2)) \\
&\geq (2z-1) \left( 4 - \max_{q \in [0,1]} (2q) - \max_{q \in [0,1]} x^{-1}(1-q)(4z-2) \right) \\
&= (2z-1)(4-2-2) \\
&= 0
\end{aligned}$$

Step 2: show  $\frac{dRHS}{dq} > 0$  for all  $z \geq 1, q \in [0, 1]$ , with equality holding only when  $z = 1$  or  $q = 0$ .

Differentiating  $RHS$  with respect to  $q$ , we have

$$\begin{aligned}
\frac{dRHS}{dq} &= -2(z^2 - z) - (2z-1) \frac{\partial x}{\partial q} \\
&= (z^2 - z)(2x^{-1}(2z-1) - 2) \\
&= \underbrace{2x^{-1}}_{>0} \underbrace{(z^2 - z)}_{\geq 0} (2z-1-x)
\end{aligned}$$

The last term is non-negative and is zero only when  $q = 0$ . To see this, note

$$\begin{aligned}
x &= \sqrt{4(1-q)(z^2 - z) + 1} \\
(\text{equal only if } q = 0) &\leq \sqrt{4z^2 - 4z + 1} \\
&= 2z - 1
\end{aligned}$$

Hence we have  $\frac{dRHS}{dq} > 0$ .

Step 3: conclude the proof. Note

$$RHS|_{q=0, z=1} = 0$$

Hence we have, for any  $z > 1$  and  $q > 0$ ,  $RHS > 0$ . Thus  $\frac{\partial \mathbb{E}[D^{agg}]}{\partial q} > 0$ .  $\square$

*Claim 2.*  $\mathbb{E}[K^{agg}]$  is decreasing in  $q$ .

*Proof.* Given the result in lemma (4), we first show that investment is decreasing in  $q$  if and only if the ex ante welfare for the borrower is decreasing in  $q$ . To see this, note

$$\begin{aligned}\mathbb{E}[I^{agg}] &= \frac{(z-1)(x+1-2z(1-q))}{2zq(2z-1)} \\ &= \frac{2(z-1)}{z(2z-1)} \cdot \left[ \frac{(x+1-2z(1-q)-2q)}{4q} + \frac{1}{2} \right] \\ &= \frac{2(z-1)}{z(2z-1)} \cdot \left[ V(0) + \frac{1}{2} \right]\end{aligned}$$

To show  $V(0)$  is decreasing in  $q$ , first note

$$\begin{aligned}V(0) &= \frac{1-2z(1-q)-2q+x}{4q} \\ \frac{dV(0)}{dq} &= \frac{4q(2z-2+x_q) - 4(1+2z(q-1)-2q+x)}{16q^2} \\ &= \frac{1}{4q^2} [q(2z-2+x_q) - 1 + 2z(1-q) + 2q - x] \\ &= \frac{1}{4q^2} \left[ -2z \left[ (z-1) \left( \frac{q}{x} \right) - 1 \right] - (1+x) \right]\end{aligned}$$

Define  $\Omega \equiv -2z \left[ (z-1) \left( \frac{q}{x} \right) - 1 \right] - (1+x)$ , we then have  $\text{sign} \left( \frac{dV(0)}{dq} \right) = \text{sign}(\Omega)$ .

To compute the derivative of  $\Omega$  with respect to  $q$ :

$$\begin{aligned}\frac{d\Omega}{dq} &= -2z(z-1) \frac{x-qx_q}{x^2} - x_q \\ &= -\frac{1}{x^3} 4z^2 (z-1)^2 q \leq 0\end{aligned}$$

Therefore  $\Omega$  is declining in  $q$  for all  $z > 1$ . This means to check that  $\frac{dV(0)}{dq} < 0$  it is sufficient to check

that  $\Omega|_{q=0} \leq 0$ .

$$\begin{aligned}\Omega(q=0) &= (2z-1) - x \\ &= 0\end{aligned}$$

Hence welfare is decreasing in  $q$ . □

*Claim 3. Default probability is increasing in  $q$ .*

*Proof.* Note  $\mathbb{E}[Pr(\text{Default})] = \mathbb{E}[D^{agg}]$  and the claim follows directly from claim 1. □

*Claim 3.  $\mathbb{E}[D^{agg}] / \mathbb{E}[K^{agg}]$  is increasing in  $q$ .*

*Proof.* Follows directly from claims 1 and 2. □

*Claim 4.  $\lim_{q \rightarrow 1} V(0) = 0$ .*

*Proof.* We can write the ex-ante welfare as

$$V(0) = \frac{1 + 2z(q-1) - 2q + x}{4q}$$

Using the fact that  $\lim_{q \rightarrow 1} z = 1$  and taking limit, the result is immediate. □

## Proof of Proposition 5

From lemma (3) we have

$$\mathbb{E}[D^{agg}] = \frac{1-b}{1-qb}$$

$$\mathbb{E}[D^{agg}] / \mathbb{E}[K^{agg}] = \frac{1-b^2q}{b(1-q)}$$

both of which are decreasing in  $b$ . It is thus sufficient to show that in equilibrium  $b^*$  is increasing in  $z$ . To see this, consider the equilibrium condition

$$b^* = b(\ell(b^*, z), z)$$

differentiating with respect to  $z$  gives

$$\frac{db^*}{dz} = \frac{\ell_z b_\ell + b_z}{1 - b_\ell \ell_b}.$$

We know from Lemma 1 that  $\ell_z = 0$ ,  $b_z < 0$ ,  $b_\ell < 0$ , and  $\ell_b > 0$ . Plugging these signs in above gives immediately that  $\frac{db^*}{dz} < 0$ , establishing the results to prove part 1 of the proposition.

From Lemma (4) we have

$$\mathbb{E}[K^{agg}] = \frac{(z-1)(x+1-2z(1-q))}{2zq(2z-1)}$$

Differentiating with respect to  $z$ , we get

$$\frac{d\mathbb{E}[K^{agg}]}{dz} = \frac{q(2z - 2z^2x - 10z^2 + 8z^3) - (x - 6z + 4z^2x + 12z^2 - 8z^3 - 4zx + 1)}{2z^2q(2z-1)^2x}$$

From this expression, one can verify that

$$\frac{\partial^2 \mathbb{E}[K^{agg}]}{\partial q \partial z} < 0,$$

and that for any given  $q \in (0, 1)$ , there exists a unique  $\bar{z}(q) \in (1, \infty)$  such that

$$\frac{d\mathbb{E}[K^{agg}]}{dz} \begin{cases} < 0 & \text{for } 1 \leq z < \bar{z}(q) \\ > 0 & \text{for } z > \bar{z}(q) \end{cases}$$

and

$$\frac{d\bar{z}(q)}{dq} < 0.$$

To show welfare is increasing in  $z$  we can start with the expression for welfare and differentiate with respect to  $z$

$$V(0) \propto 1 - 2z(1-q) - 2q + \sqrt{(2z-1)^2(1-q) + q}$$

$$\begin{aligned}\frac{\partial V(0)}{\partial z} &\propto 2(1-q) \left[ \frac{2z-1}{\sqrt{(2z-1)^2(1-q)+q}} - 1 \right] \\ &\propto \left[ \frac{(2z-1)}{\sqrt{(2z-1)^2 - q((2z-1)^2 - 1)}} - 1 \right]\end{aligned}$$

so we have

$$\begin{aligned}\frac{\partial V(0)}{\partial z} > 0 &\iff (2z-1) > \sqrt{(2z-1)^2 - q((2z-1)^2 - 1)} \\ &\iff (2z-1)^2 > (2z-1)^2 - q((2z-1)^2 - 1) \\ &\iff (2z-1)^2 > 1\end{aligned}$$

which is true because  $z > 1$ . Thus welfare is increasing in  $z$ .

□

## Proof of Proposition 6

**Proof:** Define  $i(z; q) \equiv \frac{\mathbb{E}[K^{agg}(z); q]}{\int \mathbb{E}[K^{agg}(z); q] dF(z)}$ , which measures the expected investment of a firm with productivity  $z$  relative to the average investment level in the entire economy. Observe that by construction  $\int i(z) dF(z) = 1$ . We know from Proposition 5 that  $\frac{\partial^2 \mathbb{E}[K^{agg}; q]}{\partial q \partial z} < 0$ ; hence, for any  $q_2 > q_1$ ,  $i(z; q_1)$  and  $i(z; q_2)$  cross exactly once, meaning there exists a cutoff  $z^{cross}$  such that  $i(z; q_2) \gtrless i(z; q_1)$  for  $z \lesseqgtr z^{cross}$ . We can therefore write:

$$\begin{aligned}\tilde{Z}(q_2) - \tilde{Z}(q_1) &= \int_1^{z^{cross}} \underbrace{(i_2(z) - i_1(z))}_{>0} z dF(z) + \int_{z^{cross}}^\infty \underbrace{(i_2(z) - i_1(z))}_{<0} z dF(z) \\ &< \int_1^{z^{cross}} \underbrace{(i_2(z) - i_1(z))}_{>0} z^{cross} dF(z) + \int_{z^{cross}}^\infty \underbrace{(i_2(z) - i_1(z))}_{<0} z^{cross} dF(z) \\ &= \int_1^\infty i_2(z) dF(z) - \int_1^\infty i_1(z) dF(z) \\ &= 0.\end{aligned}$$

Thus we can conclude that  $\tilde{Z}$  is decreasing in  $q$ .

## Proof of Proposition 7

Let  $EK^b(z, q)$  and  $ED^b(z, q)$  denote the expected investment and debt in the baseline model, and let  $EK^p(z, \delta, q)$  and  $ED^p(z, \delta, q)$  denote the expected investment and debt with pledgeability  $\delta$ . Observe that  $EK^p(z, \delta, q) = EK^b\left(\frac{z-\delta}{1-\delta}, q\right) \frac{1}{1-\delta}$  and  $ED^p(z, \delta, q) = ED^b\left(\frac{z-\delta}{1-\delta}, q\right)$ . The proposition follows directly from these observations. □

## Proof of Proposition 9

Let  $S$  be the sum of lender and borrower surplus. We know  $\tilde{p}(D+d)d - k = \beta S$ . We now write  $S$  as a function of  $D$  and  $d$  only, substituting out  $k$ :

$$\begin{aligned} S &= (z-1)k + W(D+d) - W(D) + \tilde{p}(D+d)d \\ &= (z-1)(\tilde{p}(D+d)d - \beta S) + W(D+d) - W(D) + \tilde{p}(D+d)d \\ &= \frac{z\tilde{p}(D+d)d + W(D+d) - W(D)}{1 + (z-1)\beta}. \end{aligned}$$

At the time of contracting, the two parties choose  $d$  to maximize joint surplus and then choose  $k$  according to the Nash bargaining equation to divide surplus. Hence borrower's value function  $V(\cdot)$  can be written recursively as

$$V(D) = \underbrace{W(D)}_{\text{outside option}} + (1-\beta) \underbrace{\max_d \frac{z\tilde{p}(D+d)d + W(D+d) - W(D)}{1 + (z-1)\beta}}_{\text{borrower captures } (1-\beta) \text{ fraction of joint surplus}}$$

Substitute using  $V(D) = \frac{W(D) + (1-q)\mathbb{E}[\min\{D, \tilde{c}\}]}{q}$ , we have

$$\frac{W(D) + (1-q)\mathbb{E}[\min\{D, \tilde{c}\}]}{q} = W(D) + (1-\beta) \max_d \frac{z\tilde{p}(D+d)d + W(D+d) - W(D)}{1 + (z-1)\beta}$$

and, after simplifying and substituting  $\hat{q} \equiv \frac{q(1-\beta)}{1-\beta+z\beta(1-q)}$ ,

$$\begin{aligned} W(D) &= -(1-\hat{q})(D-D^2/2) + \hat{q} \times \max_d \{z\tilde{p}(D')(D'-D) + W(D')\} \\ &= -(1-\hat{q})(D-D^2/2) + \hat{q} \times \max_D \{z\ell(1-g(D))(D'-D) + W(D')\} \end{aligned}$$

*Envelope Condition:*

$$\begin{aligned} W'(D) &= -(1-\hat{q})(1-D) - \hat{q}z\tilde{p}(g(D)) \\ &= -(1-\hat{q})(1-D) - \hat{q}z\ell(1-g(D)) \end{aligned}$$

$$W'(D') = -(1-\hat{q})(1-g(D)) - \hat{q}z\ell(1-g^2(D))$$

FOC

$$0 = -\hat{q}z\ell(g(D)-D) + \hat{q}z\ell(1-g(D)) + \hat{q}W'(g(D))$$

Plugging in envelope into FOC and simplifying gives

$$\hat{q}z\ell b^2 + (1-q-2z\ell)b + z\ell = 0$$

which is the same quadratic as in the proof of Lemma 1. Hence, the best response of  $b$  given  $\ell$ ,  $z$ , and  $\hat{q}$  has the same form as in Lemma 1, but with  $\hat{q}$  replacing  $q$ .

$$b = \frac{(2z\ell - (1-\hat{q})) - \sqrt{(2z\ell - (1-\hat{q}))^2 - 4\hat{q}z^2\ell^2}}{2\hat{q}z\ell}$$

$$\ell = \frac{1-q}{1-qb}$$

Defining  $\gamma \equiv \frac{1-\beta}{1-\beta+z\beta(1-q)}$  and plugging in  $\ell(b)$  into  $b(\ell)$ , rearranging, and squaring and taking the

correct root, we have

$$0 = [b^2\gamma q + 1]z(1-q) + b(1-\gamma q)(1-qb) - 2bz(1-q)$$

$$0 = b^2[(\gamma z - 1) - \gamma q(z-1)] + b[(1-\gamma q) - 2z(1-q)] + [z(1-q)]$$

$$b^* = \frac{2z(1-q) - (1-\gamma q) - \sqrt{((1-\gamma q) - 2z(1-q))^2 - 4q((\gamma z - 1) - \gamma q(z-1))z(1-q)}}{2q((\gamma z - 1) - \gamma q(z-1))}$$

an algebraic rearrangement also gives

$$b^* = \frac{2z(1-q)}{(2z(1-q) - (1-\gamma q)) + \sqrt{((1-\gamma q) - 2z(1-q))^2 - 4q((\gamma z - 1) - \gamma q(z-1))z(1-q)}}$$

which reduces to the expression for  $b^*$  derived above when  $\beta = 0$ .

□

## Proof of Proposition 10

Proof of Claim 1. The first-order condition in equation (1) that characterizes the single-lender equilibrium is

$$z \times [p(D^*) + p'(D^*)D^*] = p(D^*)$$

If an interest rate cap were set to be  $1 + \bar{r}^{SL} \equiv \frac{1}{p(D^*)}$ , the borrower could propose to pledge  $D^*$  and raise  $p(D^*)D^*$  from the very first lender. No future lender would be willing to provide additional investment to the borrower because doing so would require an interest rate higher than  $1 + \bar{r}^{SL}$  to break even, but such a rate is prohibited by the interest rate cap. Hence the full commitment allocation can be achieved under  $\bar{r}^{SL}$ . Substituting for  $p(D) = 1 - D$ , the full-commitment debt issuance is  $D^* = \frac{z-1}{2z-1}$ .

The optimal interest cap is thus

$$\begin{aligned} 1 + \bar{r}^{SL} &= \frac{1}{1 - D^*} \\ &= 1 - \frac{1}{z} \end{aligned}$$

Claim 2 follows directly from the fact that there is an one-to-one relationship between risky debt issuance and the probability of repayment.

Proof of Claim 3. When  $\bar{r} < \bar{r}^{SL}$ , the interest rate cap is inefficiently low and the borrower can pledge less debt facevalue than he would have done under full commitment. The unique equilibrium under the interest rate cap would involve the borrower pledging  $D = 1 - \frac{1}{1+\bar{r}}$  debt and raising  $(1 - D)D$  investment from the very first lender. In the extreme case where  $\bar{r} = 0$ , the borrower would be unable to raise any investment from the lenders, achieving an even lower level of welfare than under the unregulated equilibrium.

Proof of Claim 4. When  $\bar{r} > \bar{r}^{Sl}$ , the full commitment allocation is unattainable as with probability  $q$  the borrower will meet the second lender and pledge a strictly positive amount of debt for any level of outstanding debt below one.

Next we show that for  $\bar{r} < \infty$  the cap unambiguously improves expected investment and welfare while lowering expected debt and interest rate relative to the unregulated equilibrium. Using the techniques in the proof for proposition 3, for any game with finite lenders  $K$  we can find a sequence of aggregate debt  $\{D_1^K, \dots, D_K^K\}$  where  $D_i^K$  corresponds to the aggregate debt level had the borrower reach lender  $i$  in a game with total lender  $K$ , where lender indices start backwards with the last lender being lender 1. Using a simple perturbation argument, we know that for  $K$  such that  $D_1^K < \bar{D} \leq D_1^{K+1}$ , the infinite lender game with debt cap  $\bar{D}$  would have a unique SPE where the borrower reaches the debt cap when borrowing from  $(K + 1)$ -th lender. Furthermore, using the same recursive definition of lender and borrower strategies in equilibrium as we adopted in proposition 3, it is clear that borrower's ex-ante expected investment, aggregate debt level, and welfare with debt cap  $\bar{D}$  is in between the corresponding equilibrium quantities for the finite lender games with  $K$  and  $K + 1$  lenders.

## Proof of Proposition 11

In equilibrium, interest rates are set to exactly compensate lenders for their expected risk of default and to cover their net tax liability. If lenders know they will be compensated for dilution associated with future borrowing, they are willing to price loans as if they expected no future borrowing. Thus

we know that an equilibrium loan satisfies

$$k_n + \tau_n = p(D_n) d_n, \quad (11)$$

and the taxes and transfers are chosen so that

$$\tau_n = [p(D_{n-1}) - p(D_n)] D_{n-1}, \quad (12)$$

$$\tau_n^i = [p(D_{n-1}) - p(D_n)] d_i. \quad (13)$$

We first show  $\tau_n = K_{n-1} \frac{d_n}{D_n} - k_n \frac{D_{n-1}}{D_n}$ . Note that equations (11) and (12) jointly and inductively imply that  $K_n = p(D_n) D_n$ . Hence

$$\begin{aligned} \tau_n &= [p(D_{n-1}) - p(D_n)] D_{n-1} \\ &= K_{n-1} - p(D_n) D_{n-1} \\ &= K_{n-1} - \left( \frac{k_n + \tau_n}{d_n} \right) D_{n-1} \\ &= K_{n-1} \frac{d_n}{D_n} - k_n \frac{D_{n-1}}{D_n}, \end{aligned}$$

where the third equality follows by substituting for  $p(D_n)$  using equation (11).

That  $\tau_n^i = \frac{d_i}{D_{n-1}} \times \tau_n$  follows immediately from equation (13).

□

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