Online Appendix for “Dynamic Spatial General Equilibrium” (Not for Publication)

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October 2021

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A Introduction

In this online appendix, we report the detailed derivations for the results reported in the paper and further supplementary results. In Section B, we report the derivations for our baseline model with a single traded sector from Section 2 of the paper. We characterize the existence and uniqueness of the general equilibrium of the model and present the proofs of the other propositions in the paper. In Section C, we establish a number of isomorphisms, in which we show that our results hold throughout the class of trade models with a constant trade elasticity.

In Section D, we introduce a number of extensions of our baseline specification, as discussed in Section 4 of the paper. Subsection D.1 shows that our framework naturally accommodates shocks to trade and migration costs. Subsection D.2 allows for agglomeration forces in production and residence and provides a characterization of the existence and uniqueness of the equilibrium in the presence of these agglomeration forces.

Subsection D.3 introduces multiple final goods sectors with region-specific capital. Section D.4 incorporates multiple final goods sectors with region-sector-specific capital. Section D.5 further generalizes the analysis to allow for multiple final goods sectors with region-sector-specific capital and input-output linkages. Subsection D.6 incorporates trade deficits following the conventional approach of the quantitative international trade literature in treating these deficits as exogenous. Section D.7 allows capital to be used residually (for housing) as well as commercially (in production).

In Section E, we present the derivations for the extension of our baseline model with a single traded sector and single non-traded sector used for our baseline quantitative analysis in Section 5 of the paper.

Section F reports additional empirical results that are discussed in the paper. Subsection F.1 shows that individual U.S. states differ substantially in terms of the dynamics of their capital-labor ratios, highlighting the empirical relevance of capital accumulation for income convergence. Subsection F.2 provides evidence of substantial net migration between U.S. states, highlighting the empirical salience of migration for the population dynamics of U.S. states. Subsection F.3 shows that the model’s gravity equation predictions provide a good approximation to the observed data on trade and migration flows.

Subsection F.4 reports additional evidence on the predictive power of convergence to the initial steady-state for the observed population growth of U.S. states. Subsection F.5 provides further information about the implied fundamentals from inverting the non-linear model. Subsection F.6 presents additional details about the solution algorithm used to solve for the economy’s transition path in the non-linear model.

Section G reports further details about the data sources and definitions.
B Baseline Dynamic Spatial Model

In this section of the online appendix, we introduce our baseline dynamic spatial model, which features a model of trade between locations with a constant trade elasticity, a dynamic discrete choice model of migration with a constant migration elasticity, and an optimal consumption-investment decision for the accumulation of capital. We derive our main sufficient statistics results for the comparative statics of the spatial distribution of economic activity in steady-state and along the full transition path, using our four observed matrices of expenditure shares, income shares, outmigration shares and immigration shares.

To simplify the exposition, we model trade between locations as in Armington (1969), in which goods are differentiated by origin. In Section C of this online appendix, we establish a number of isomorphisms, in which we show that our results hold throughout the class of models with a constant trade elasticity in Arkolakis, Costinot and Rodriguez-Clare (2012). For expositional clarity, we also focus in this section on shocks to productivities and amenities, but we show in Section D of this online appendix that our approach also holds for shocks to trade and migration costs. As part of that section, we also show that our approach admits a large number of other extensions and generalizations, including agglomeration economies, multiple factors, multiple sectors, and input-output linkages, among others.

We consider an economy that consists of many locations indexed by \( i \in \{1, \ldots, N\} \). Time is discrete and is indexed by \( t \). There are two types of infinitely-lived agents: workers and landlords. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile subject to migration costs. Workers do not have access to an investment technology and hence live "hand to mouth," as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed and depreciates gradually at a constant rate \( \delta \).

In Subsections B.1-B.6, we introduce our specifications of worker migration and landlord investment decisions. In Subsection B.7, we provide a characterization of the existence and uniqueness of the deterministic steady-state equilibrium of the model. In Subsection B.10, we derive our sufficient statistics for the first-order general equilibrium effect of shocks to productivities and amenities on the entire spatial distribution of economic activity in steady-state and along the transition path. In Subsection B.11, we characterize the distributional consequences of shocks to productivity and amenities.

In Subsection B.12, we report the derivations for the expression for expected utility in the paper. In Subsection B.13, we provide the derivations for the expression for the migration choice probabilities in the paper.
B.1 Worker Migration Decisions

At the beginning of each period \( t \), the economy inherits a mass of workers in each location \( i \) (\( \ell_{it} \)), where the total labor endowment of the economy is given by \( \ell = \sum_{i=1}^{N} \ell_{it} \). Workers produce and consume in their current location during period \( t \), before observing mobility shocks \( \{\varepsilon_{gt}\} \) and subsequent location fundamentals \( \{z_{gt+1}, b_{gt+1}\} \) for all possible locations \( g \in \{1, \ldots, N\} \), and deciding where to move for the next period \( t+1 \), given bilateral migration costs \( \{\gamma_{igt}\} \). Therefore, the value function for a worker in location \( i \) in period \( t \) (\( V_{w}^{it} \)) is equal to the current flow of utility in that location plus the expected continuation value next period from the optimal choice of location:

\[
V_{w}^{it} = \ln u_{it}^{w} + \max_{\{g\}_{i=1}^{N}} \left\{ \beta \mathbb{E}_{t} \left[ V_{w}^{gt+1} \right] - \kappa_{git} + \rho \xi_{git} \right\}, \tag{B.1}
\]

where we use the superscript \( w \) to denote workers; we assume logarithmic utility (\( \ln u_{it}^{w} \)); \( \beta \) is the discount rate; \( \mathbb{E}_{t} [\cdot] \) denotes an expectation taken over future location characteristics; the distribution for idiosyncratic mobility shocks; \( \rho \) controls the dispersion of idiosyncratic mobility shocks; and we assume \( \kappa_{it} = 1 \) and \( \kappa_{nit} > 1 \) for \( n \neq i \).

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

\[
F(\varepsilon) = e^{-e^{-\varepsilon}}, \tag{B.2}
\]

where \( \gamma \) is the Euler-Mascheroni constant.

Under this assumption, the value for a worker of living in location \( i \) at time \( t \) after taking expectation with respect to the idiosyncratic mobility shocks \( \{\varepsilon_{gt}\} \) (\( v_{w}^{it} \equiv \mathbb{E}_{t} [V_{w}^{it}] \)) can be re-written in the following form:

\[
v_{w}^{it} = \ln u_{it}^{w} + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_{t} v_{w}^{gt+1} \right) / \kappa_{git} \right)^{1/\rho}, \tag{B.3}
\]

as shown in Subsection B.12 below, where the expectation in \( \mathbb{E}_{t} v_{w}^{gt+1} = \mathbb{E}_{t} \mathbb{E}_{t} [V_{w}^{it}] \) is taken over future fundamentals, \( \{z_{is}, b_{its}\}_{s=t+1}^{\infty} \). The corresponding probability of migrating from location \( i \) to location \( g \) satisfies the following gravity equation:

\[
D_{igt} = \frac{(\exp (\beta \mathbb{E}_{t} v_{w}^{gt+1}) / \kappa_{git})^{1/\rho}}{\sum_{m=1}^{N} (\exp (\beta \mathbb{E}_{t} v_{w}^{mt+1}) / \kappa_{mit})^{1/\rho}}, \tag{B.4}
\]

as shown in Subsection B.13 below.

B.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each
period to maximize their flow utility in the location in which they have chosen to live. Worker
flow utility depends on local amenities \((b_{nt})\) and goods consumption \((c_{nt}^{w})\) and is assumed to take
the logarithmic form:

\[
\ln u_{nt}^{w} = \ln b_{nt} + \ln c_{nt}^{w},
\]

where \(c_{nt}^{w}\) is a consumption index for workers in location \(n\) defined over the consumption of the
variety supplied by each location \(i\) \((c_{ni}^{w})\):

\[
c_{nt}^{w} = \left[ \sum_{i=1}^{N} \left( c_{ni}^{w} \right)^{\theta \mu \sigma} \right]^{-\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1,
\]

where \(\sigma > 1\) is the constant elasticity of substitution (CES) between varieties and \(\theta = \sigma - 1\) is the
trade elasticity. Amenities \((b_{nt})\) capture characteristics of a location that make it a more attractive
place to live regardless of goods consumption (e.g. climate and scenic views). In this section, we
assume that amenities are exogenous, but in Section D of this online appendix, we allow them to
be endogenous to the surrounding concentration of economic activity through agglomeration or
congestion forces.

The corresponding worker indirect utility each period depends on amenities \((b_{nt})\), the wage
\((w_{nt})\) and the consumption goods price index \((p_{nt})\):

\[
\ln u_{nt}^{w} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt},
\]

where the consumption goods price index \((p_{nt})\) in location \(n\) depends of the price of the variety
sourced from each location \(i\) \((p_{nit})\):

\[
p_{nt} = \left[ \sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-1/\theta}.
\]

Using the properties of constant elasticity of substitution (CES) preferences, the share of ex-
penditure of importer \(n\) on the goods supplied by exporter \(i\) is:

\[
S_{nit} = \frac{(p_{nit})^{-\theta}}{\sum_{m=1}^{N} (p_{ntm})^{-\theta}}.
\]

### B.3 Production

Firms in each location use labor \((\ell_{it})\) and capital \((k_{it})\) to produce output \((y_{it})\) of the variety supplied
by that location. Production is assumed to occur under conditions of perfect competition and
subject to the following constant returns to scale technology:

\[
y_{it} = z_{it} \left( \frac{\ell_{it}}{\mu} \right)^{\mu} \left( \frac{k_{it}}{1-\mu} \right)^{1-\mu}, \quad 0 < \mu < 1,
\]
where $z_{it}$ denotes productivity in location $i$ at time $t$. As for amenities above, we assume in this section that productivity is exogenous, but in Section 4 below we allow it to be endogenous to the surrounding concentration of economic activity through agglomeration forces.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit} \geq 1$ units of a good must be shipped from location $i$ in order for one unit to arrive in location $n$, where $\tau_{nit} > 1$ for $n \neq i$ and $\tau_{iit} = 1$. From profit maximization, the cost to a consumer in location $n$ of sourcing the good produced by location $i$ depends solely on iceberg trade costs and constant marginal costs:

$$p_{nit} = \tau_{nit} p_{iit} = \frac{\tau_{nit} w_{iit}^{\mu} (1-\mu)}{z_{it}},$$

(B.11)

where $p_{iit}$ is the “free on board” price of the good supplied by location $i$ before trade costs.

From profit maximization and zero profits, total payments to each factor of production are a constant share of total revenue:

$$w_{it} \ell_{it} = \mu p_{iit} y_{it}.$$  

(B.12)

$$r_{it} k_{it} = (1 - \mu) p_{iit} y_{it}.$$  

(B.13)

### B.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords’ intertemporal utility equals the present discounted value of their flow utility:

$$v_{it}^k = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left( c_{it+s}^k \right)^{1-1/\psi} \frac{1}{1 - 1/\psi},$$

(B.14)

where $c_{it}^k$ is a consumption index defined over the consumption of the good supplied by each location ($c_{imt}^k$), as in equation (B.6) for workers above; $\psi$ is the elasticity of intertemporal substitution.

We assume that the investment technology in each location uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location can produce one unit of capital in that location using one unit of the consumption index in that location.\(^1\) We assume that capital is geographically immobile once installed and depreciates at a constant rate $\delta$. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($r_{it} k_{it}$) equals the total value of their consumption ($p_{iit} c_{it}^k$) plus the total value of net investment ($p_{iit} (k_{it+1} - (1-\delta) k_{it})$):

$$r_{it} k_{it} = p_{iit} \left( c_{it}^k + k_{it+1} - (1-\delta) k_{it} \right).$$

(B.15)

\(^1\)Although this assumption that consumption and investment use goods in the same proportions is the standard specification, we obtain similar results in an alternative specification in which investment in each location uses only the good produced by that location.
Combining landlords’ intertemporal utility (B.14) and budget constraint (B.15), the landlord’s intertemporal optimization problem is:

\[
\max \{ c_k^t, s_t+1 \} \sum_{s=0}^{\infty} \beta^{s+1} \left( \frac{c_{k+s}^t}{1 - \beta} \right),
\]

subject to \( p_i t c_{k+t}^k + p_i t (k_{i+t+1} - (1 - \delta) k_{i+t}) = r_{i+t} k_{i+t} \).

Lemma. (Lemma 1 in the paper) We denote \( R_{i+t} \equiv 1 - \delta + r_{i+t}/p_{i+t} \) as the gross return on capital. The optimal consumption of location \( i \)’s landlords satisfies \( c_{i+t} = \varsigma_{i+t} R_{i+t} k_{i+t} \), where \( \varsigma_{i+t} \) is defined recursively as

\[
\varsigma_{i+t}^{-1} = 1 + \beta^t \left( \mathbb{E}_t \left[ \frac{R_{i+t+1} \varsigma_{i+t+1}}{R_{i+t+1} \varsigma_{i+t+1}} \right] \right)^{1/\psi}.
\]

Landlord’s optimal saving and investment satisfies \( k_{i+t+1} = (1 - \varsigma_{i+t}) R_{i+t} k_{i+t} \).

Proof. For notational simplicity we drop the locational subscript. Consider a landlord facing linear returns \( R_t \) on wealth \( k_t \) for all \( t \). Let \( v(k_t; t) \) denote the value function at time \( t \); we can rewrite the landlord’s consumption-saving problem recursively as:

\[
v(k_t; t) = \max_{\{c_t, k_{t+1}\}} \frac{c_{t}^{1-\phi}}{1 - \psi} + \beta \mathbb{E}_t v(k_{t+1}; t+1) \quad \text{s.t.} \quad c_t + k_{t+1} = R_t k_t,
\]

where, with a slight abuse of notation, we denote landlord consumption as \( c \) instead of \( c^k \) for the purpose of this proof. We guess-and-verify that there exists \( a_t, \varsigma_t \) such that \( v(k_t; t) = \frac{\langle a_t R_t k_t \rangle^{1-\phi}}{1 - \psi} \), and that optimal \( c_t = \varsigma_t R_t k_t \).

Under the conjecture, \( v_k(k_t; t) = a_t^{1-\phi} R_t^{1-\phi} k_t^{-\phi} \), we setup the Lagrangian as:

\[
\mathcal{L}_t = \frac{c_t^{1-\phi}}{1 - \psi} + \beta \mathbb{E}_t v(k_{t+1}; t+1) + \xi_t [R_t k_t - c_t - k_{t+1}].
\]

The first-order conditions imply:

\[
\{c_t\} \quad c_t^{-\phi} = \xi_t,
\]

\[
\{k_t\} \quad \xi_{t+1} = \beta \mathbb{E}_t \left[ a_{t+1}^{1-\phi} R_{t+1}^{1-\phi} \right].
\]

Hence:

\[
c_t = \beta^{-\phi} k_{t+1} \mathbb{E}_t \left[ a_{t+1}^{1-\phi} R_{t+1}^{1-\phi} \right]^{-\phi}.
\]

(B.17)

The Envelope condition \( v_k(k_t; t) = \xi_t R_t \) implies

\[
a_t^{1-\phi} R_t^{1-\phi} k_t^{-\phi} = c_t^{-\phi} R_t.
\]

(B.18)
Substituting our guess that \( c_t \equiv \zeta_t R_t k_t \) into the Envelope condition (B.18), we obtain:

\[
a_t^{1-\psi} = \zeta_t.
\]

The budget constraint implies \( k_{t+1} = (1 - \zeta_t) R_t k_t \), and substituting this result into (B.17), we get:

\[
\begin{align*}
\zeta_t &= \beta^{-\psi} E_t \left[ a_{t+1}^{1-1/\psi} R_{t+1}^{1-1/\psi} \right]^{-\psi} (1 - \zeta_t) \\
\iff \zeta_t^{-1} &= 1 + \beta^\psi E_t \left[ R_{t+1}^{\psi-1} \zeta_{t+1}^{-1/\psi} \right]^\psi, 
\end{align*}
\]

(B.19)
as desired. \( \Box \)

Note that, in the special case of logarithmic flow utility (\( \psi = 1 \)), landlord’s optimal consumption and saving rate is independent of future returns to capital, and \( \zeta_t = (1 - \beta) \) for all \( t \), as in Moll (2014).

**B.5 Market Clearing**

Goods market clearing implies that income in each location, which equals the sum of the income of workers and landlords, is equal to expenditure on the goods produced by that location:

\[
(w_{it} \ell_{it} + r_{it} k_{it}) = \sum_{n=1}^{N} S_{nit} (w_{nt} \ell_{nt} + r_{nt} k_{nt}).
\]

(B.20)

Using the property that payments to capital and labor are constant shares of total revenue in equations (B.12) and (B.13), we can rewrite this goods market clearing condition as follows:

\[
\begin{align*}
&\frac{1}{\mu} w_{it} \ell_{it} + \frac{1 - \mu}{\mu} w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} \left( w_{nt} \ell_{nt} + \frac{1 - \mu}{\mu} w_{nt} h_{nt} h_{nt} \right), \\
&\frac{1}{\mu} w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} \frac{1}{\mu} w_{nt} \ell_{nt}, \\
&w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}.
\end{align*}
\]

(B.21)

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords’ income from the ownership of capital equals payments for its use. Using the property that payments to capital and labor are constant shares of total revenue in equations (B.12) and (B.13), we can write this capital market clearing condition as:

\[
r_{it} = \frac{1 - \mu}{\mu} \frac{w_{it} \ell_{it}}{k_{it}}.
\]

(B.22)
B.6 General Equilibrium

Given the state variables \( \{ \ell_{it}, k_{it} \} \), the general equilibrium of the economy is the stochastic process of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets, with the appropriate measurability constraint with respect to the realization of location fundamentals. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \( \{ \ell_{it}, k_{it}, w_{it}, R_{it}, v_{it} \} \). All other endogenous variables of the model can be recovered as a function of these variables.

Capital Returns and Accumulation: Using capital market clearing (B.22), the gross return on capital in each location \( i \) must satisfy:

\[
R_{it} = \left( 1 - \delta + \frac{1 - \mu}{\mu} \frac{w_{it} \ell_{it}}{p_{it} k_{it}} \right),
\]

where the price index (B.8) follows

\[
p_{nt} = \left[ \sum_{i=1}^{N} \left( w_{it} \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \ell_{it}/k_{it} \right)^{1-\mu} \frac{\tau_{ni}}{z_{i}} \right) \right]^{-\theta}.
\]  

(B.23)

The law of motion for capital is

\[
k_{it+1} = (1 - \varsigma_{it}) \left( 1 - \delta + \frac{1 - \mu}{\mu} \frac{w_{it} \ell_{it}}{p_{it} k_{it}} \right) k_{it},
\]

where \( (1 - \varsigma_{it}) \) is the saving rate defined recursively as in Lemma 1 in the paper:

\[
\varsigma_{it}^{-1} = 1 + \beta^{\psi} \left( E_{t} \left[ R_{it+1} \varsigma_{it+1}^{\psi} \right] \right)^{\psi}.
\]

Goods Market Clearing: Using the equilibrium pricing rule (B.11), the expenditure share (B.9) and capital market clearing (B.22), the goods market clearing condition (B.20) can be written as:

\[
w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt},
\]

\[
S_{nit} = \frac{\left( w_{it} \left( \ell_{it}/k_{it} \right)^{1-\mu} \frac{\tau_{ni}}{z_{i}} \right)^{-\theta}}{\sum_{m=1}^{N} \left( w_{mt} \left( \ell_{mt}/k_{mt} \right)^{1-\mu} \frac{\tau_{nm}}{z_{m}} \right)^{-\theta}};
\]

\[
T_{int} = \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},
\]

where \( S_{nit} \) is the expenditure share of importer \( n \) on exporter \( i \) at time \( t \); we have defined \( T_{int} \) as the corresponding income share of exporter \( i \) from importer \( n \) at time \( t \); and note that the order of subscripts switches between the expenditure share \( (S_{nit}) \) and the income share \( (T_{int}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.
Population Flow: Using the out-migration probabilities \((B.4)\), the population flow condition for the evolution of the population distribution over time is given by:

\[
\ell_{g,t+1} = \sum_{i=1}^{N} D_{igt} \ell_{it},
\]

\[
D_{igt} = \frac{\left(\exp \left(\frac{\beta \mathbb{E}_t v_{n,t}^w}{\kappa_{igt}}\right)\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp \left(\frac{\beta \mathbb{E}_t v_{m,t+1}^w}{\kappa_{mit}}\right)\right)^{1/\rho}},
\]

where \(D_{igt}\) is the outmigration probability from location \(i\) to location \(g\) between time \(t\) and \(t+1\); we have defined \(E_{git}\) as the corresponding immigration probability to location \(g\) from location \(i\) between time \(t\) and \(t+1\); and again note that the order of subscripts switches between the outmigration probability \((D_{igt})\) and the immigration probability \((E_{git})\), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function \((B.7)\) in the value function \((B.3)\), the expected value from living in location \(n\) at time \(t\) can be written as:

\[
v_{n,t}^w = \ln b_{nt} + \ln \left(\frac{w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp \left(\frac{\beta \mathbb{E}_t v_{g,t+1}^w}{\kappa_{gnt}}\right)\right)^{1/\rho}.
\]

(B.27)

B.7 Existence and Uniqueness (Proof of Proposition 1 in the Paper)

We now use the system of equations for general equilibrium \((B.24)-(B.27)\) to prove the existence and uniqueness of a deterministic steady-state equilibrium with time-invariant fundamentals \(\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}\) and endogenous variables \(\{v_i^*, w_i^*, R_i^*, \ell_i^*, k_i^*\}\). Given these time-invariant fundamentals, we can drop the expectation over future fundamentals, such that \(\mathbb{E}_t v_{n,t+1}^w = v_{n,t+1}^w\).

B.7.1 Capital Labor Ratio

In steady-state, \(k_{it+1} = k_{it} = k_i^*\), \(c_{it+1} = c_i^* = c_i^{k*}\), and \(s_{it+1} = s_{it} = s_i^*\), which implies:

\[
1 - s_i^* = \beta.
\]

Using these results and the capital accumulation condition \((B.24)\), we can solve for the steady-state capital-labor ratio:

\[
k_i^* = \beta \left(1 - \delta + \frac{1 - \mu}{\mu} \frac{w_i^* \ell_i^*}{p_i^* k_i^*}\right) k_i^*,
\]

\[
k_i^* = \beta \frac{1 - \mu}{\mu} \frac{w_i^* \ell_i^*}{p_i^* k_i^*} + \beta (1 - \delta) k_i^*.
\]
\[
\begin{align*}
k^*_i &= \frac{\beta}{1 - \beta (1 - \delta)} \frac{1 - \mu w^*_i \ell^*_i}{p^*_i}, \\
k^*_m \ell^*_i &= \frac{\beta}{1 - \beta (1 - \delta)} \frac{1 - \mu w^*_i}{p^*_i}.
\end{align*}
\] (B.28)

### B.7.2 Price Index

Using this result for the steady-state capital-labor ratio, we can re-write the price index in equation (B.23) as follows:

\[
(p^*_n)^{-\theta} = \sum_{i=1}^{N} \left( w^*_i \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \ell^*_i / k^*_i \right)^{1-\mu} \frac{\tau_{ni}}{z_i} \right)^{-\theta},
\]

\[
(p^*_n)^{-\theta} = \sum_{i=1}^{N} \left( w^*_i \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \ell^*_i / k^*_i \right)^{1-\mu} \frac{\tau_{ni}}{z_i} \right)^{-\theta},
\]

\[
(p^*_n)^{-\theta} = \sum_{i=1}^{N} \left( w^*_i \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{1-\mu} \left( p^*_i / w^*_i \right)^{1-\mu} \frac{\tau_{ni}}{z_i} \right)^{-\theta},
\]

\[
(p^*_n)^{-\theta} = \sum_{i=1}^{N} \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)} \left( w^*_i \right)^{-\theta \mu} \left( p^*_i \right)^{-\theta(1-\mu)} \left( \tau_{ni} / z_i \right)^{-\theta},
\]

\[
(p^*_n)^{-\theta} = \sum_{i=1}^{N} \tilde{\tau}_{ni} \left( w^*_i \right)^{-\theta \mu} \left( p^*_i \right)^{-\theta(1-\mu)},
\] (B.29)

\[
\psi \equiv \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)}, \quad \tilde{\tau}_{ni} \equiv \left( \tau_{ni} / z_i \right)^{-\theta}.
\]

### B.7.3 Goods Market Clearing Condition

Using this result for the steady-state capital-labor ratio, we can also re-write the goods market clearing condition (B.25) as follows:

\[
\begin{align*}
\ell^*_i &= \sum_{n=1}^{N} \left( w^*_i \left( \ell^*_i / k^*_i \right)^{1-\mu} \frac{\tau_{ni}}{z_i} \right)^{-\theta} w^*_n \ell^*_n, \\
\ell^*_i &= \sum_{n=1}^{N} \left( w^*_i \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \ell^*_i / k^*_i \right)^{1-\mu} \frac{\tau_{ni}}{z_i} \right)^{-\theta} w^*_n \ell^*_n, \\
\ell^*_i &= \sum_{n=1}^{N} \left( w^*_i \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{1-\mu} \left( p^*_i / w^*_i \right)^{1-\mu} \frac{\tau_{ni}}{z_i} \right)^{-\theta} w^*_n \ell^*_n.
\end{align*}
\]
Using this solution in the definition of $B.7.4$ Value Function

\[ w_i^* \ell_i^* = \sum_{n=1}^{N} \frac{(w_i^*)^{-\theta \mu} \left( \frac{0 \left(1-\delta \right) - \theta (1-\mu) (p_i^*)^{-\theta (1-\mu)} (\tau_{ni}/z_i)^{-\theta}}{(p_i^*)^{-\theta}} \right)}{w_i^* \ell_{n_i}^*} \]

\[ \ell_i^*(w_i^*)^{1+\theta \mu} (p_i^*)^\theta (1-\mu) = \sum_{n=1}^{N} \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta (1-\mu)} \left( p_n^* \right)^\theta (\tau_{ni}/z_i)^{-\theta} w_i^* \ell_{n_i}^*. \]

\[ \ell_i^*(w_i^*)^{1+\theta \mu} (p_i^*)^\theta (1-\mu) = \sum_{n=1}^{N} \psi_{ni} p_n^*(\theta) \cdot w_i^* \ell_{n_i}^*. \] (B.30)

**B.7.4 Value Function**

We now show that the value function (B.27) can be re-written as follows:

\[ v_n^{w*} = b_n + \ln \left( \frac{w_n^*}{p_n^*} \right) + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \beta v_g^{w*} \right) / \kappa_{gn} \right)^{1/\rho}, \]

\[ \exp (v_n^{w*}) = b_n \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \left[ \sum_{g=1}^{N} \left( \exp \left( \beta v_g^{w*} \right) / \kappa_{gn} \right)^{1/\rho} \right]^\beta, \]

\[ \exp \left( \frac{\beta v_n^{w*}}{\rho} \right) = b_n^{\beta/\rho} \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \left[ \sum_{g=1}^{N} \kappa_{gn}^{-1/\rho} \exp \left( \frac{\beta v_g^{w*}}{\rho} \right) \right]^\beta, \]

\[ \exp \left( \frac{\beta v_n^{w*}}{\rho} \right) = \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \left[ \sum_{g=1}^{N} \bar{\kappa}_{gn} \exp \left( \frac{\beta v_g^{w*}}{\rho} \right) \right]^\beta, \]

\[ \exp \left( \frac{\beta v_n^{w*}}{\rho} \right) = \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \left[ \sum_{g=1}^{N} \bar{\kappa}_{gn} \exp \left( \frac{\beta v_g^{w*}}{\rho} \right) \right]^\beta, \]

\[ \exp \left( \frac{\beta v_n^{w*}}{\rho} \right) = \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \phi_n, \quad \phi_n \equiv \sum_{g=1}^{N} \bar{\kappa}_{gn} \exp \left( \frac{\beta v_g^{w*}}{\rho} \right). \] (B.31)

Using this solution in the definition of $\phi_n$ immediately above, we have:

\[ \phi_n = \sum_{g=1}^{N} \bar{\kappa}_{gn} \left( p_g^* \right)^{-\beta/\rho} (w_g^*)^{\beta/\rho} \phi_g^{\beta}. \] (B.32)
B.7.5 Population Flow Condition

We now show that the population flow condition (B.26) can be re-written as follows:

\[
\ell_i^* = \sum_{i=1}^{N} \left( \exp \left( \beta v_{i}^{w} \right) / \kappa_{gi} \right)^{1/\rho} \ell_i^*,
\]

\[
\ell_g^* = \sum_{i=1}^{N} \kappa_{gi}^{1-1/\rho} \exp \left( \frac{\beta v_{i}^{w}}{\rho} \right) \ell_i^*,
\]

\[
\ell_g^* = \sum_{i=1}^{N} \kappa_{gi}^{1-1/\rho} \exp \left( \frac{\beta v_{i}^{w}}{\rho} \right) \left[ \sum_{m=1}^{N} \kappa_{mi}^{1-1/\rho} \exp \left( \frac{\beta v_{m}^{w}}{\rho} \right) \right]^{-1} \ell_i^*,
\]

\[
\ell_g^* = \sum_{i=1}^{N} \kappa_{gi}^{1-1/\rho} \exp \left( \frac{\beta v_{i}^{w}}{\rho} \right) \left[ \sum_{m=1}^{N} \kappa_{mi}^{1-1/\rho} \exp \left( \frac{\beta v_{m}^{w}}{\rho} \right) \right]^{-1} \ell_i^*,
\]

\[
\ell_g^* = \sum_{i=1}^{N} \kappa_{gi} \exp \left( \frac{\beta v_{i}^{w}}{\rho} \right) \left[ \sum_{m=1}^{N} \kappa_{mi} \exp \left( \frac{\beta v_{m}^{w}}{\rho} \right) \right]^{-1} \ell_i^*, \quad \kappa_{gi} \equiv \left( \kappa_{gi} / b_i^{\beta} \right)^{1-1/\rho},
\]

\[
\ell_g^* = \sum_{i=1}^{N} \kappa_{gi} \exp \left( \frac{\beta v_{i}^{w}}{\rho} \right) \phi_i^{-1} \ell_i^*, \quad \phi_i \equiv \sum_{m=1}^{N} \kappa_{mi} \exp \left( \frac{\beta v_{m}^{w}}{\rho} \right).
\]

Now using the value function result (B.31) above, we have:

\[
\ell_g^* = \sum_{i=1}^{N} \kappa_{gi} \left( \frac{w_g^*}{p_g} \right)^{\beta/\rho} \phi_i^{\beta/\rho} \phi_i^{-1} \ell_i^*,
\]

\[
(p_g^*)^{\beta/\rho} (w_g^*)^{-\beta/\rho} \ell_g^* \phi_g^{-\beta} = \sum_{i=1}^{N} \kappa_{gi} \ell_i^* \phi_i^{-1}. \tag{B.33}
\]

B.7.6 System of Equations

Collecting together these results, the steady-state equilibrium of the model \((p_i^*, w_i^*, \ell_i^*, \phi_i^*)\) can be expressed as the solution to the following system of equations:

\[
(p_i^*)^{-\theta} = \sum_{n=1}^{N} \psi_{\tau_n} \left( p_n^* \right)^{-\theta \left( 1 - \mu \right)} \left( w_n^* \right)^{-\theta \mu}, \tag{B.34}
\]

\[
(p_i^*)^{\theta \left( 1 - \mu \right)} \left( w_i^* \right)^{1 + \theta \mu} \ell_i^* = \sum_{n=1}^{N} \psi_{\tau_n} \left( p_n^* \right)^{\theta} w_n^* \ell_n^*, \tag{B.35}
\]

\[
(p_i^*)^{\beta/\rho} \left( w_i^* \right)^{-\beta/\rho} \ell_i^* \left( \phi_i^* \right)^{-\beta} = \sum_{n=1}^{N} \kappa_{in} \ell_n^* \left( \phi_n^* \right)^{-1}. \tag{B.36}
\]
\[ \phi^*_i = \sum_{n=1}^{N} \kappa_n \phi^*_n \beta/\rho \left( \frac{u^*_n}{\beta} \right)^{\beta/\rho} \left( \phi^*_n \right)^{\beta}, \]  

(B.37)

where we have the following definitions:

\[ \psi \equiv \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)}, \quad \tau_{ni} \equiv \left( \frac{\tau_{ni}}{z_i} \right)^{-\theta}, \]

\[ \phi^*_i \equiv \sum_{n=1}^{N} \kappa_n \exp \left( \frac{\beta w^*_n}{\rho} \right), \quad \kappa_{in} \equiv \left( \frac{\kappa_{in}}{b^*_n} \right)^{-1/\rho}. \]

The exponents on the variables on the left-hand side of the system of equations (B.34)-(B.37) can be represented as the following matrix:

\[
\Lambda = \begin{bmatrix}
-\theta & 0 & 0 & 0 \\
\theta (1 - \mu) & (1 + \theta \mu) & 1 & 0 \\
\beta/\rho & -\beta/\rho & 1 & -\beta \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

The exponents on the variables on the right-hand side of the system of equations (B.34)-(B.37) can be represented as the following matrix:

\[
\Gamma = \begin{bmatrix}
-\theta (1 - \mu) & -\theta \mu & 0 & 0 \\
\theta & 1 & 1 & 0 \\
0 & 0 & 1 & -1 \\
-\beta/\rho & \beta/\rho & 0 & \beta
\end{bmatrix}.
\]

Let \( A \equiv \Gamma \Lambda^{-1} \) and denote the spectral radius (eigenvalue with the largest absolute value) of this matrix by \( \rho(A) \). Building on the arguments in Allen, Arkolakis and Li (2020), we begin by showing that the equilibrium exists and is unique up to scale (up to a choice of numeraire for prices and a choice of units for population (population shares)) when \( \rho(A) = 1 \).

**Proposition.** Existence and Uniqueness (Proposition 1 in the paper). There exists a unique steady-state spatial distribution of economic activity \( \{ \ell^*_i, k^*_i, w^*_i, R^*_i, v^*_i \} \) (up to a choice of units) given time-invariant locational fundamentals \( \{ z^*_i, b^*_i, \tau^*_{ni}, \kappa^*_ni \} \) that is independent of initial conditions \( \{ \ell_{i0}, k_{i0} \} \).

**Proof.** Existence of solutions to the system of equations (B.34)-(B.37) follows from Brouwer’s fixed point theorem. We now establish uniqueness (up to a choice of units).

Consider a system of \( N \times H \) equations with endogenous variables \( \{ y_{ih} \} \) and exogenous coefficients \( \{ A_{hh'} \} \) and \( \{ \alpha_{ijh} \} \):

\[ \exp \left( y_{ih} \right) = \sum_{j=1}^{N} \alpha_{ijh} \prod_{h'=1}^{H} \exp \left( A_{hh'} y_{jh'} \right). \]
Suppose (i) The $H \times H$ matrix $A$ is invertible; (ii) The spectral radius (the largest eigenvalue) of $A = [A_{h'j}]$ is equal to one; (iii) $\alpha_{ijh} > 0$ for all $i, j, h$. We now show that the solution $\{y_{ih}\}$ is unique up to a scaling constant: if $\{y_{ih}\}$ and $\{y'_{ih}\}$ are both solutions to the system, then there $y_{ih} - y'_{ih}$ is independent of $i$. Furthermore, the vector $\{y_{ih} - y'_{ih}\}_h$ (enumerating over $h$) must be an eigenvector of the matrix $A$ with associated eigenvalue of one. To establish this result, define $f_{ijh} \left( \{y_{j'k'}\}_{h' = 1}^H \right) \equiv \alpha_{ijh} \prod_{h' = 1}^H \exp (A_{h'j} y_{j'k'}).$ Let $g_{ih} \equiv \ln \sum_{j=1}^N f_{ijh} \left( \{y_{j'k'}\}_{h' = 1}^H \right).$ Consider $y$ and $y'$ both solutions to the equation system, with $y \neq y'$.

$$y_{ih} = g_{ih} (y), \quad y'_{ih} = g_{ih} (y'). \quad \text{(B.38)}$$

We show it must be the case that $y_{ih} - y'_{ih}$ is independent of $i$. By the mean value theorem, for every $ih$, there exists $t_{(ih)} \in [0, 1]$ such that for $\hat{y} = (1 - t_{(ih)}) y + t_{(ih)} y'$. Using this result, we have:

$$g_{ih} (y) - g_{ih} (y') = \sum_{j,h'} \frac{\partial g_{ih} (\hat{y})}{\partial y_{j'h'}} (y_{j'h'} - y'_{j'h'}) ,$$

$$= \sum_{h'} A_{hh'} \sum_j \frac{f_{ijh} (\hat{y})}{\sum_j f_{ijh} (\hat{y})} (y_{j'h'} - y'_{j'h'}) ,$$

$$= \sum_j \frac{f_{ijh} (\hat{y})}{\sum_j f_{ijh} (\hat{y})} \sum_{h'} A_{hh'} (y_{j'h'} - y'_{j'h'}). \quad \text{(B.39)}$$

Note that the spectral radius of a matrix is equal to its operator norm induced by the infinity-norm, $\rho (A) = \|A\|_\infty \equiv \sup \left\{ \frac{\|Ax\|_\infty}{\|x\|_\infty} \right\}$, where the infinity norm over vectors is defined as $\|x\|_\infty \equiv \max_i \{|x_1|, \ldots, |x_n|\}$.

Note that (B.39) implies that for all $i$,

$$\max_h |g_{ih} (y) - g_{ih} (y')| = \max_h \left| \sum_j \frac{f_{ijh} (\hat{y})}{\sum_j f_{ijh} (\hat{y})} \sum_{h'} A_{hh'} (y_{j'h'} - y'_{j'h'}) \right| ,$$

$$\leq \max_j \max_h \left| \sum_{h'} A_{hh'} (y_{j'h'} - y'_{j'h'}) \right| ,$$

$$\leq \rho (A) \max_j \max_h |y_{j'h'} - y'_{j'h'}| . \quad \text{(B.40)}$$

Note $\alpha_{ijh} > 0$ implies $\sum_{j'k'} f_{ijh} (\hat{y}) > 0$ for all $i, j, h$, hence the first inequality achieves equality only if $\sum_{h'} A_{hh'} (y_{j'h'} - y'_{j'h'})$ is independent of $j$. When $A$ is invertible, this implies that $y_{j'h'} - y'_{j'h'}$ must be independent of $j$. Note also that the second inequality achieves equality only if $\{y_{j'h'} - y'_{j'h'}\}_{h'}$ is an eigenvector of $A$ with associated eigenvalue of one. Further note that (B.38) implies

$$\max_j \max_h |y - y'| = \max_i \max_h |g_{ih} (y) - g_{ih} (y')| ,$$

$$\leq \rho (A) \max_j \max_h |y_{j'h} - y'_{j'h}| .$$
where the inequality follows from the fact that (B.40) holds for any \(i\) and thus must hold as we take the maximum over \(i\)'s. Hence, when \(\rho(A) < 1\), the solution to the system must be unique: \(\max_j \max_h |y - y_h' - y_j' - y_j'| = 0\). When \(\rho(A) = 1\) and \(A\) invertible, the solution must be unique up-to-scale—\((y_h - y_{ih}')\) is independent of \(i\)—and \(\{y_{ih} - y_{ih}'\}_h\) must be an eigenvector of \(A\) with associated eigenvalue of one. The definition of \(\Gamma\) and \(\Lambda\) implies that both matrices are invertible for parameters throughout the domain \(\beta \in (0, 1), \mu \in [0, 1], \rho > 0, \) and \(\theta \geq 0\), thereby implying \(A \equiv \Gamma\Lambda^{-1}\) is invertible as well. Evaluating the eigenvalues of \(A\), we have:

\[
\begin{pmatrix}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4
\end{pmatrix}
= \begin{pmatrix}
1 \\
\frac{1}{a+c} \\
\frac{1}{a-c} \\
\frac{a+c}{a-c}
\end{pmatrix},
\]

where the functions \(a, b, c\) are defined in terms of the parameters \(\beta, \mu, \rho, \theta\) as:

\[
a = \rho + \beta \rho - \mu \rho - (1 - \beta) \mu \rho \theta, \\
b = [\rho (\mu \theta + \mu - 1) - \beta (2 - \rho - \mu \theta)]^2 + 4 \beta (1 + \beta) \rho \mu (1 + 2 \theta), \\
c = 2 (\beta + \rho + \mu \rho \theta).
\]

We now show that for all parameters satisfying \(\theta \geq 0, \mu \in [0, 1], \beta \in (0, 1), \rho > 0\), it must be that \(\left|\frac{a \pm \sqrt{b}}{c}\right| < 1\). Note that these parameter inequalities imply that \(b, c > 0\). Note also that:

\[
\left|\frac{a \pm \sqrt{b}}{c}\right| < 1 \iff \left|a \pm \sqrt{b}\right| < c.
\]

We first establish a few results based on algebraic simplifications.

\[
a + c = (\rho (3 + \mu (\theta - 1)) + \beta (2 + \rho + \mu \rho \theta)) > 0 \quad \text{(B.41)}
\]

\[
c - a = (\rho (1 + \mu + 3 \mu \theta) - \beta (-2 + \rho + \mu \rho \theta)) \quad \text{(B.42)}
\]

\[
(c + a)^2 - b = 4 (1 + \beta)(2 - \mu) \rho (\beta + \rho + \mu \rho \theta) > 0 \quad \text{(B.43)}
\]

\[
(c - a)^2 - b = 4 (1 - \beta) \rho \mu (1 + 2 \theta)(\beta + \rho + \mu \rho \theta) > 0 \quad \text{(B.44)}
\]

We know \(b > 0\). First we need to show \(\left|a + \sqrt{b}\right| < c\). If \(a + \sqrt{b} > 0\), then it’s equivalent to show \((c - a)^2 > b\), which is implied by (B.43). Otherwise, we need to show \(-\sqrt{b} < c + a\), which is implied by \(a + c > 0\) as in (B.41).

Next we show \(\left|a - \sqrt{b}\right| < c\). If \(a - \sqrt{b} < 0\) then it is equivalent to show \(\sqrt{b} - a < c \iff (c + a)^2 > b\) as implied by (B.43). Otherwise, \(a + c > 0\) implies the inequality holds if \(a - \sqrt{b} < 0\), as desired.
As the expenditure shares ($S$) and income shares ($T$) are homogeneous of degree zero in factor prices, we require a choice of units or numeraire in order to solve for changes in wages. We choose the total income of all locations as our numeraire: $\sum_{i=1}^{N} q^*_i \ln q^*_i = \sum_{i=1}^{N} q^*_i \frac{dq^*_i}{q^*_i} = \sum_{i=1}^{N} dq^*_i = 0$. Similarly, the outmigration shares ($D$) and immigration shares ($E$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell^*_i = 1$, which implies $\sum_{i=1}^{N} \ell^*_i \ln \ell^*_i = \sum_{i=1}^{N} \ell^*_i \frac{d\ell^*_i}{\ell^*_i} = \sum_{i=1}^{N} d\ell^*_i = 0$.

### B.8 Dynamic Exact-hat Algebra (Proof of Proposition 2 in the Paper)

Given an initial allocation of the economy $\left\{ (l_{0i})_{i=1}^{N}, (k_{0i})_{i=1}^{N}, (k_{ii})_{i=1}^{N}, (S_{n0i})_{i=1}^{N}, (D_{ni,-1})_{n=1}^{N} \right\}$, and an anticipated sequence of changes in fundamentals, $\left\{ (\dot{z}_{it})_{i=1}^{N}, (\dot{b}_{it})_{i=1}^{N}, (\dot{\tau}_{ijt})_{i,j=1}^{N}, (\dot{k}_{ijt})_{i,j=1}^{N} \right\}_{t=1}^{\infty}$, the solution to the sequential equilibrium in time differences solves the following system of non-linear equations:

\[
\dot{D}_{igt+1} = \frac{D_{igt} (\dot{u}_{igt+2}/\dot{k}_{igt+1})^{1/\rho}}{\sum_{m=1}^{N} D_{imt} (\dot{u}_{imt+2}/\dot{k}_{imt+1})^{1/\rho}},
\]

\[
\dot{u}_{it+1} = \left( \frac{\dot{b}_{it+1}}{\dot{p}_{it+1}} \right)^\beta \left( \sum_{m=1}^{N} (\dot{u}_{mmt+2}/\dot{k}_{mmt+1})^{1/\rho} \right)^\beta^\rho,
\]

\[
\dot{p}_{it+1} = \left( \sum_{m=1}^{N} S_{imt} \left( \dot{\tau}_{imt+1} \dot{w}_{imt+1} \left( \dot{i}_{mmt+1}/\dot{k}_{mmt+1} \right)^{1-\mu}/\dot{z}_{mmt+1} \right) \right)^{-1/\theta},
\]

\[
\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it},
\]

\[
\dot{w}_{it+1} \dot{\ell}_{it+1} = \sum_{n=1}^{N} S_{nnt+1} w_{nt} \ell_{nt} \dot{w}_{nt+1} \dot{\ell}_{nt+1},
\]

\[
\dot{S}_{nkt+1} = \frac{\left( \dot{\tau}_{nkt+1} \dot{w}_{kkt+1} \left( \dot{i}_{kkt+1}/\dot{k}_{kkt+1} \right)^{1-\mu}/\dot{z}_{kkt+1} \right)^{-\theta}}{\sum_{k=1}^{N} S_{kkt+1} \left( \dot{\tau}_{kkt+1} \dot{w}_{kkt+1} \left( \dot{i}_{kkt+1}/\dot{k}_{kkt+1} \right)^{1-\mu}/\dot{z}_{kkt+1} \right)^{-\theta}}.
\]

\[
\zeta_{it+1} = \beta R_{it+1}^{\psi-1} \zeta_{it} / (1 - \zeta_{it}),
\]

\[
k_{it+1} = (1 - \zeta_{it}) R_{it} k_{it},
\]

\[
(R_{it} - (1 - \delta)) = \frac{\dot{w}_{it+1} \dot{k}_{it+1}}{\dot{w}_{it+1} \dot{\ell}_{it+1}} (R_{it+1} - (1 - \delta)),
\]
where we use a dot above a variable to denote a time difference: \( \dot{x}_{it+1} = x_{it+1}/x_{it} \). Note that the solution to this system of equations does not require information on the level of fundamentals, \( \{z_{it}\}_{i=1}^{N}, \{b_{it}\}_{i=1}^{N}, \{\tau_{ijt}\}_{i,j=1}^{N}, \{\kappa_{ijt}\}_{i,j=1}^{N} \}_{t=0}^{\infty} \).

**B.9 Model Inversion**

In this section of the online appendix, we show how our generalization of dynamic exact-hat algebra to incorporate forward-looking capital investments in Proposition 2 of the paper can be used to invert the model and recover the unobserved changes in fundamentals \((z_{it}, b_{it}, \tau_{nit}, k_{git})\) implied by the observed data. We solve for the unobserved changes in these fundamentals from the general equilibrium conditions of the model and the observed data on bilateral trade and migration flows, population, capital stock and labor income per capita under the assumption of perfect foresight. We recover these unobserved fundamentals, without making assumptions about where the economy lies on the transition path to steady-state or the specific trajectory of fundamentals, because the observed changes in migration flows and the capital stock capture agents’ expectations about this sequence of future fundamentals. We show that this model inversion has a sequential structure, such that we can recover unobserved fundamentals in a sequence of steps, where we make the minimal set of assumptions in each step, before adding further assumptions in the next step to recover additional fundamentals.

We use our baseline values for the model’s parameters from Section 5.1 of the paper based on central values from the existing empirical literature. In a first step, we recover bilateral trade frictions \((\tau_{nit})\) from observed bilateral trade shares \((S_{nit})\). Assuming that bilateral trade frictions are symmetric \((\tau_{nit} = \tau_{int})\), and normalizing own trade frictions to one \((\tau_{nnt} = \tau_{iit} = 1)\), the model’s gravity equation predictions for goods trade in equation (13) in the paper imply:

\[
\frac{S_{nit}S_{int}}{S_{nmt}S_{sit}} = \left( \frac{\tau_{nit} \tau_{int}}{\tau_{nnt} \tau_{iit}} \right)^{-\theta} = (\tau_{nit})^{-2\theta}. \tag{B.45}
\]

In second step, we solve for productivity \((z_{it})\) using observed population \((\ell_{it})\), labor income per capita \((w_{it})\), and the capital stock \((k_{it})\) and our solutions for bilateral trade frictions \((\tau_{nit})\) from the previous step. From the model’s goods market clearing condition in equation (12) in the paper, we have:

\[
w_{it}\ell_{it} = \sum_{n=1}^{N} \frac{w_{it} (\ell_{it}/k_{it})^{1-\mu} \tau_{nit}/z_{it}^{-\theta}}{\sum_{m=1}^{N} w_{mt} (\ell_{mt}/k_{mt})^{1-\mu} \tau_{nmt}/z_{mt}^{-\theta}} w_{nt}\ell_{nt}, \tag{B.46}
\]

which uniquely determines productivity \((z_{it})\) up to normalization (or a choice of units). Since we normalize own trade frictions to one \((\tau_{nnt} = \tau_{iit} = 1)\), a change in trade costs with all trade partners (including oneself) is captured in productivity \((z_{it})\). As these solutions for bilateral trade frictions \((\tau_{nit})\) and productivity \((z_{it})\) only use the predictions of the static Armington trade
model and condition on the observed capital stock and population, they hold regardless of what assumptions are made about capital accumulation and migration.

In a third step, we recover bilateral migration frictions ($\kappa_{git}$) from observed bilateral migration flows ($D_{igt}$). Assuming that bilateral migration frictions are symmetric ($\kappa_{git} = \kappa_{igt}$), and normalizing own migration frictions to one ($\kappa_{ggt} = \kappa_{iit} = 1$), the model’s gravity equation predictions for migration in equation (16) in the paper imply:

$$\frac{D_{igt}}{D_{ggt}} = \left(\frac{\kappa_{git} \kappa_{igt}}{\kappa_{ggt} \kappa_{iit}}\right)^{-1/\rho} = (\kappa_{git})^{-2/\rho}. \quad (B.47)$$

In a fourth step, we solve for the expected value of living in each location ($v_{gt+1}^w$) from observed population ($\ell_{it}$) and our solutions for bilateral migration frictions ($\kappa_{git}$) from the previous step. From the model’s population flow condition in equation (15) in the paper and the assumption of perfect foresight, we have:

$$\ell_{gt+1} = \sum_{i=1}^{N} \frac{\left(\exp\left(\beta v_{gt+1}^w / \kappa_{git}\right)\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta v_{mt+1}^w / \kappa_{mit}\right)\right)^{1/\rho}} \ell_{it}, \quad (B.48)$$

which uniquely determines the expected values ($v_{gt+1}^w$) up to a normalization (or choice of units). Since we normalize own migration frictions to one ($\kappa_{ggt} = \kappa_{iit} = 1$), a change in migration frictions with all locations (including oneself) is captured in the expected value ($v_{gt+1}^w$) and hence in amenities ($b_{gt}$) in the next step. As these solutions for bilateral migration frictions ($\kappa_{git}$) and expected values ($v_{gt+1}^w$) use only the predictions of the migration model, they hold regardless of what assumptions are made about patterns of trade in goods.

In a fifth and final step, we recover amenities in each location ($b_{it}$) from observed goods trade ($S_{nit}$), observed migration flows ($D_{git}$), and our solutions for productivity ($z_{it}$) and expected values ($v_{gt+1}^w$) from the previous steps. Using the model’s value function (14) for a pair of time periods and the assumption of perfect foresight, we have:

$$\ln b_{it} = \left(v_{it}^w - v_{it+1}^w\right) + (1 - \beta) v_{it+1}^w - \ln \frac{S_{it}^{-\frac{\beta}{\rho}}}{D_{it}^{\frac{1}{\rho}}} - \ln z_{it}, \quad (B.49)$$

which uniquely determines amenities ($b_{it}$) up to our choices of units for productivity and expected values. In this final step, we use the predictions of both the migration and trade blocs of the model. Note that this final step for amenities in equation (B.49) requires expected values for both periods $t$ and $t+1$, and hence requires migration flows for both periods from equations (B.47) and (B.48).

We thus obtain values for the unobserved fundamentals ($z_{it}$, $b_{it}$, $\tau_{nit}$, $\kappa_{nit}$) implied by the observed values of the endogenous variables under the assumption of perfect foresight, without making assumptions about where on the transition path to steady-state the economy lies or about
the particular expected future trajectory of fundamentals. Note that these fundamentals are derived under the assumption of symmetric trade and migration costs, which need not necessarily be satisfied in the data. Therefore, these fundamentals do not exactly rationalize the observed expenditure shares \( (S_{nit}) \) and outmigration probabilities \( (D_{igt}) \), although we find that the model’s predictions under this symmetry assumption are strongly correlated with the observed data.

As general equilibrium allocations in the model are homogenous of degree zero in productivity and amenities, multiplying these fundamentals by scalars leaves allocations unchanged. Therefore, without loss of generality, we focus on shocks to relative productivity and amenities, which are invariant to the units in which these variables are measured.

### B.10 Linearization

We now derive our main sufficient statistics results for the response of the spatial distribution of economic activity with respect to shocks to the economic environment. In Subsection B.10.1, we totally differentiate the general equilibrium conditions of the model to obtain comparative statics. In Subsections B.10.2-B.10.3, we derive our sufficient statistics for changes in steady-state. In Subsection B.10.4, we derive our sufficient statistics for the entire transition path of the spatial distribution of economic activity. Throughout the following, we use bold math font to denote a vector (lowercase letters) or matrix (uppercase letters).

#### B.10.1 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

**Expenditure Shares**  Totally differentiating this expenditure share equation (B.9), we get:

\[
\frac{dS_{nit}}{S_{nit}} = \theta \left( \sum_{h=1}^{N} S_{nht} \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nit}}{p_{nit}} \right),
\]

\[d \ln S_{nit} = \theta \left( \sum_{h=1}^{N} S_{nht} d \ln p_{nht} - d \ln p_{nit} \right).
\]

**Prices**  Using the relationship between capital and labor payments (B.22), the pricing rule (B.11) can be re-written as follows:

\[p_{nit} = \frac{\tau_{nit} w_{it} \left( \frac{1-\mu}{\mu} \right)^{1-\mu} \left( \frac{1}{\lambda_{it}} \right)^{1-\mu}}{z_{it}},\]
where $\chi_{it}$ is the capital-labor ratio:

$$\chi_{it} = \frac{k_{it}}{l_{it}}.$$  

Totally differentiating this pricing rule, we have:

$$\frac{dp_{nit}}{p_{nit}} = \frac{d\tau_{nit}}{\tau_{nit}} + \frac{dw_{it}}{w_{it}} - (1 - \mu) \frac{d\chi_{it}}{\chi_{it}} - \frac{dz_{it}}{z_{it}},$$

$$d \ln p_{nit} = d \ln \tau_{nit} + d \ln w_{it} - (1 - \mu) d \ln \chi_{it} - d \ln z_{it}. \quad (B.51)$$

**Price Indices**  Totally differentiating the consumption goods price index in equation (B.8), we have:

$$\frac{dp_{nt}}{p_{nt}} = \sum_{m=1}^{N} S_{nmt} \frac{dp_{nmt}}{p_{nmt}}, \quad (B.52)$$

$$d \ln p_{nt} = \sum_{m=1}^{N} S_{nmt} d \ln p_{nmt}.$$  

**Real Income.**  Totally differentiating real income we have:

$$d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - d \ln p_{it},$$

$$d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - \sum_{m=1}^{N} S_{nmt} d \ln p_{nmt},$$

$$d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - \sum_{m=1}^{N} S_{nmt} [d \ln \tau_{nmt} + d \ln w_{mt} - (1 - \mu) d \ln \chi_{mt} - d \ln z_{mt}], \quad (B.53)$$

**Migration Shares**  Totally differentiating the outmigration share in equation (B.4), we get:

$$\frac{dD_{igt}}{D_{igt}} = \frac{1}{\rho} \left[ \beta \mathbb{E}_t d\nu_{igt+1} - \frac{d\kappa_{git}}{\kappa_{git}} \right] - \sum_{h=1}^{N} D_{ih} \left( \beta \mathbb{E}_t d\nu_{ih+1} - \frac{d\kappa_{hit}}{\kappa_{hit}} \right), \quad (B.54)$$

$$d \ln D_{igt} = \frac{1}{\rho} \left[ \beta \mathbb{E}_t d\nu_{igt+1} - d \ln \kappa_{git} \right] - \sum_{h=1}^{N} D_{ih} \left( \beta \mathbb{E}_t d\nu_{ih+1} - d \ln \kappa_{hit} \right).$$

**Goods Market Clearing**  Totally differentiating the goods market clearing condition (B.20), we have:

$$\frac{dw_{it}}{w_{it}} + \frac{d\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt} \left( \frac{dw_{nt}}{w_{nt}} + \frac{d\ell_{nt}}{\ell_{nt}} + \frac{dS_{nit}}{S_{nit}} \right).$$
Using our results for the derivatives of expenditure shares (B.50) and prices (B.51), we can rewrite this as:

\[
\frac{dw_{it}}{w_{it}} + \frac{dl_{it}}{\ell_{it}} = \sum_{n=1}^{N} T_{nt} \left( \frac{dw_{nt}}{w_{nt}} + \frac{dl_{nt}}{\ell_{nt}} + \theta \left( \sum_{h \in N} S_{nh} \left( \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nit}}{p_{nit}} \right) \right) \right),
\]

\[
T_{nt} = \frac{S_{nt} w_{nt} \ell_{nt}}{w_{it} \ell_{it}}.
\]

\[
\begin{bmatrix}
\frac{d \ln w_{it}}{d \ln \ell_{it}}
\end{bmatrix} = \left[ +\theta \sum_{n=1}^{N} \sum_{m=1}^{N} T_{nt} S_{nt} \left( d \ln w_{nt} + d \ln \ell_{nt} \right) -\theta \sum_{n=1}^{N} T_{nt} \left( d \ln \tau_{nt} + d \ln w_{it} - (1 - \mu) d \ln \chi_{nt} - d \ln z_{it} \right) \right].
\]

(B.55)

**Population Flow.** Totally differentiating the population flow condition (B.26) we have:

\[
d \ln \ell_{gt+1} = \sum_{i=1}^{N} E_{git} \left[ d \ln \ell_{it} + d \ln D_{igt} \right],
\]

\[
d \ln \ell_{gt+1} = \sum_{i=1}^{N} E_{git} \left[ d \ln \ell_{it} + \frac{1}{\rho} \left( \beta \mathbb{E}_t d v_{gt+1} - d \ln \kappa_{gi} - \sum_{n=1}^{N} D_{nt} (\beta \mathbb{E}_t d v_{nt+1} - d \ln \kappa_{nit}) \right) \right].
\]

(B.56)

**Value Function.** Note that the value function can be re-written using the following results:

\[
v_{it} = \ln \left( \frac{w_{it}}{\sum_{m=1}^{N} \frac{\theta}{p_{imt}} \left( \frac{p_{nit}}{\kappa_{iit}} \right)^{1/\theta}} \right) + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_t v_{gt+1} / \kappa_{gvt} \right) \right)^{1/\rho},
\]

\[
\left( \sum_{m=1}^{N} \frac{\theta}{p_{imt}} \right)^{-1/\theta} = \left( \frac{p_{nit}}{\kappa_{iit}} \right)^{-1/\theta}, \quad \tau_{iit} = 1,
\]

\[
\sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_t v_{gt+1} / \kappa_{gvt} \right) \right)^{1/\rho} = \frac{\left( \exp \left( \beta \mathbb{E}_t v_{it+1} / \kappa_{iit} \right) \right)^{1/\rho}}{D_{iit}}, \quad \kappa_{iit} = 1,
\]

\[
v_{it} = -\frac{1}{\theta} \ln S_{iit} + \ln w_{it} - \ln p_{iit} + \ln b_{it} + \beta \mathbb{E}_t d v_{it+1} - \rho \ln D_{iit}.
\]

(B.57)

Totally differentiating this expression for the value function, we have:

\[
d v_{it} = -\frac{1}{\theta} d \ln S_{iit} + d \ln w_{it} - d \ln p_{iit} + d \ln b_{it} + \beta \mathbb{E}_t d v_{it+1} - \rho d \ln D_{iit},
\]

where

\[
d \ln S_{iit} = -\theta d \ln p_{iit} + \theta \left[ \sum_{m=1}^{N} S_{imt} d \ln p_{imt} \right],
\]
\[
\frac{d\ln D_{it}}{\rho} \left[ \beta E_t dv_{it+1} - d\ln \kappa_{it} - \sum_{m=1}^{N} D_{imt} (\beta E_t v_{mt+1} - d\ln \kappa_{mit}) \right].
\]

Using these results for \(d\ln S_{it}\) and \(d\ln D_{it}\) in the expression for \(dv_{it}\) above, we have:
\[
dv_{it} = \left[ d\ln w_{it} - \sum_{m=1}^{N} S_{imt} d\ln p_{imt} + \sum_{m=1}^{N} D_{imt} (\beta E_t dv_{mt+1} - d\ln \kappa_{mit}) \right],
\]
where we have used \(d\ln \kappa_{it} = 0\). Using the total derivative of the pricing rule (B.51), we can re-write this derivative of the value function as follows:
\[
dv_{it} = \left[ d\ln w_{it} - \sum_{m=1}^{N} S_{imt} \left( d\ln \tau_{nt} + d\ln w_{mt} - (1 - \mu) d\ln \chi_{mt} - d\ln z_{mt} \right) + \sum_{m=1}^{N} D_{imt} (\beta E_t dv_{mt+1} - d\ln \kappa_{mit}) \right].
\]

(B.58)

**B.10.2 Steady-State Sufficient Statistics**

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: \(k_{it+1} = k_{it} = k^*_i\), \(\ell_{it+1} = \ell_{it} = \ell^*_i\), \(w_{it+1} = w_{it} = w^*_i\) and \(v_{it+1} = v_{it} = v^*_i\), where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity (\(d\ln z\)) and amenities (\(d\ln b\)) in each location, holding constant the economy’s aggregate labor endowment (\(d\ln \ell = 0\)), trade costs (\(d\ln \tau = 0\)) and commuting costs (\(d\ln \kappa = 0\)).

**Capital Accumulation.** From the capital accumulation equation (B.24), the steady-state stock of capital solves:
\[
(1 - \beta (1 - \delta)) \chi^*_i = (1 - \beta (1 - \delta)) \frac{k^*_i}{\ell^*_i} = \beta \frac{1 - \mu}{\mu} \frac{w^*_i}{p^*_i}.
\]
Totally differentiating, we have:
\[
d\ln \chi^*_i = d\ln \left( \frac{w^*_i}{p^*_i} \right).
\]
Using the total derivative of real income (B.53) above, this becomes:
\[
d\ln \chi^*_i = d\ln w^*_i - \sum_{m=1}^{N} S^*_im \left[ d\ln w^*_m - (1 - \mu) d\ln \chi^*_m - d\ln z_m \right],
\]
where we have used and \(d\ln \tau_{nm} = 0\). This relationship has the matrix representation:
\[
d\ln \chi^* = d\ln w^* - S d\ln w^* + (1 - \mu) S d\ln \chi^* + S d\ln z,
\]
\[
(I - (1 - \mu) S) d\ln \chi^* = (I - S) d\ln w^* + S d\ln z.
\]
(B.59)
Goods Market Clearing. The total derivative of the goods market clearing condition \((B.55)\) has the following matrix representation:

\[
d\ln w_t + d\ln \ell_t = T (d\ln w_t + d\ln \ell_t) + \theta (TS - I) (d\ln w_t - (1 - \mu) d\ln \chi_t - d\ln z),
\]

where we have used \(d\ln \tau = 0\). We can re-write this relationship as:

\[
[I - T + \theta (I - TS)] d\ln w_t = -(I - T) d\ln \ell_t + \theta (I - TS) (d\ln z + (1 - \mu) d\ln \chi_t).
\]

In steady-state we have:

\[
[I - T + \theta (I - TS)] d\ln w^* = [- (I - T) d\ln \ell^* + \theta (I - TS) (d\ln z + (1 - \mu) d\ln \chi^*)]. \tag{B.60}
\]

Population Flow. The total derivative of the population flow condition \((B.56)\) has the following matrix representation:

\[
d\ln \ell_{t+1} = E d\ln \ell_t + \frac{\beta}{\rho} (I - ED) dv_{t+1}.
\]

In steady-state, we have:

\[
d\ln \ell^* = E d\ln \ell^* + \frac{\beta}{\rho} (I - ED) dv^*.
\] \tag{B.61}

Value function. The total derivative of the value function \((B.58)\) has the following matrix representation:

\[
dv_t = (I - S) d\ln w_t + S (d\ln z + (1 - \mu) d\ln \chi_t) + d\ln b + \beta D dv_{t+1},
\]

where we have used \(d\ln \tau = d\ln \kappa = 0\). In steady-state, we have:

\[
dv^* = (I - S) d\ln w^* + S (d\ln z + (1 - \mu) d\ln \chi^*) + d\ln b + \beta D dv^*. \tag{B.62}
\]

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

\[
d\ln \chi^* = (I - (1 - \mu) S)^{-1} ((I - S) d\ln w^* + S d\ln z). \tag{B.63}
\]

\[
d\ln w^* = (I - T + \theta (I - TS))^{-1} (- (I - T) d\ln \ell^* + (I - TS) \theta (d\ln z + (1 - \mu) d\ln \chi^*)). \tag{B.64}
\]

\[
d\ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) dv^*. \tag{B.65}
\]

\[
dv^* = (I - \beta D)^{-1} \{ d \ln w^* - S (d\ln w^* - d\ln z - (1 - \mu) d\ln \chi^*) + d\ln b \}. \tag{B.66}
\]
As the expenditure shares ($S$) and income shares ($T$) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire ($\sum_{i=1}^{N} w_i^* T_i^* = \sum_{i=1}^{N} q_i^* = \bar{q} = 1$), which implies that the log changes in incomes satisfy $Q^* d \ln q^* = \sum_{i=1}^{N} q_i^* d \ln q_i^* = \sum_{i=1}^{N} q_i^* \frac{d q_i^*}{q_i^*} = \sum_{i=1}^{N} d q_i^* = 0$, where $Q$ is a row vector of the income of each location. Similarly, the outmigration shares ($D$) and immigration shares ($E$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell_{it} = \bar{\ell} = 1$, which implies $L^* d \ln \ell^* = \sum_{i=1}^{N} \ell_i^* d \ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{d \ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} d \ell_i^* = 0$, where $L$ is a row vector of the population of each location.

### B.10.3 Steady-State Elasticities

We now use equation (B.63) to substitute for $d \ln \chi^*$ in the value function equation (B.66) to obtain:

$$
\begin{align*}
\text{d} v^* &= (I - \beta D)^{-1} \{ d \ln w^* - S (d \ln w^* - d \ln z - (1 - \mu) d \ln \chi^*) + d \ln b \}, \quad (B.67) \\
(I - T + \theta (I - TS)) d \ln w^* &= - (I - T) d \ln \ell^* + (I - TS) \theta (d \ln z^* + (1 - \mu) d \ln \chi^*), \\
(I - T + \theta (I - TS)) d \ln w^* &= \left[ - (I - T) d \ln \ell^* + (I - TS) \theta (1 - \lambda) (I - (1 - \mu) S)^{-1} ((I - S) d \ln w^* + S z) \right], \\
(I - T + \theta (I - TS)) d \ln w^* &= \left[ - (I - T) d \ln \ell^* + (I - TS) \theta \left( I + (1 - (1 - \mu) S)^{-1} (1 - \mu) S \right) d \ln z \right], \\
(I - T + \theta (I - TS)) d \ln w^* &= - (I - T) d \ln \ell^* + (I - TS) \theta (d \ln z + (1 - \mu) d \chi^*) \\
&= - (I - T) d \ln \ell^* + (I - TS) \theta d \ln z \\
&\quad + (I - TS) \theta (1 - \mu) (I - (1 - \mu) S)^{-1} ((I - S) d \ln w^* + S d \ln z) \\
&= - (I - T) d \ln \ell^* + (I - TS) \theta \left( I + (1 - (1 - \mu) S)^{-1} (1 - \mu) S \right) d \ln z \\
&\quad + (I - TS) \theta (1 - \mu) (I - (1 - \mu) S)^{-1} (I - S) d \ln w^* \\
\left( I - T + \theta (I - TS) \left( I - (1 - \mu) (I - (1 - \mu) S)^{-1} (I - S) \right) \right) d \ln w^* \\
&= - (I - T) d \ln \ell^* + \theta (I - TS) (I - (1 - \mu) S)^{-1} d \ln z,
\end{align*}
$$

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Using the value function (B.67), we now show that we can further simplify this system of equations. We begin by defining the following composite matrices:

\[
\begin{pmatrix}
I - T + \theta (I - TS) \\
(I - (1 - \mu) S)^{-1} - (1 - \mu) (I - (1 - \mu) S)^{-1}
\end{pmatrix}
\] d ln w^* = - (I - T) d ln \ell^* + \theta (I - TS) (I - (1 - \mu) S)^{-1} d ln z,

\[
\begin{pmatrix}
I - T + \theta (I - TS) \mu (I - (1 - \mu) S)^{-1}
\end{pmatrix}
\] d ln w^* = - (I - T) d ln \ell^* + \theta (I - TS) (I - (1 - \mu) S)^{-1} d ln z.

(B.68)

Collecting together the capital accumulation equation (B.63), the population equation (B.65), the value function (B.67) and the wage equation (B.68), we have:

\[
d\nu^* = (I - \beta D)^{-1} (I - (1 - \mu) S)^{-1} [(I - S) d\ln w^* + S d\ln z + d\ln b],
\]

(B.69)

\[
d\ln w^* = \left[ I - T + \theta (I - TS) \mu (I - (1 - \mu) S)^{-1} \right]^{-1} \left[ - (I - T) d\ln \ell^* + \theta (I - TS) (I - (1 - \mu) S)^{-1} d\ln z \right],
\]

(B.70)

\[
d\ln \chi^* = (I - (1 - \mu) S)^{-1} [(I - S) d\ln w^* + S d\ln z],
\]

(B.71)

\[
d\ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) d\nu^*.
\]

(B.72)

We now show that we can further simplify this system of equations. We begin by defining the following composite matrices:

\[
G \equiv (I - E)^{-1} (I - ED) (I - \beta D)^{-1},
\]

(B.73)

\[
O \equiv (1 - (1 - \mu) S)^{-1},
\]

\[
M \equiv (TS - I).
\]

which implies the following relationships:

\[
I + (1 - \mu) SO = O,
\]

\[
I - (1 - \mu) O (I - S) = I + (1 - \mu) OS - (1 - \mu) O = \mu O.
\]

Using these definitions and relationships, we can re-write the wage equation (B.70) as:

\[
(I - T - \theta M) d\ln w^* = - (I - T) d\ln \ell^* - \theta M [d\ln z + (1 - \mu) O (I - S) d\ln w^* + (1 - \mu) OS d\ln z],
\]

\[
[I - T - \theta M (I - (1 - \mu) O (I - S))] d\ln w^* = - (I - T) d\ln \ell^* - \theta MO d\ln z,
\]

\[
d\ln w^* = \left[ I - T + \theta (I - TS) \mu (I - (1 - \lambda) S)^{-1} \right]^{-1} \left[ - (I - T) d\ln \ell^* + \theta (I - TS) (I - (1 - \mu) S)^{-1} d\ln z \right].
\]

(B.74)

Using the value function (B.69), we can re-write the employment equation (B.72) as:

\[
d\ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) (I - \beta D)^{-1} (I - (1 - \mu) S)^{-1} [(I - S) d\ln w^* + S d\ln z + d\ln b].
\]

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Using the capital accumulation equation (B.71) and our definitions (B.73), we can further re-write this employment equation as:
\[
\begin{align*}
\frac{d\ln \ell^*}{b} &= \frac{\beta}{\rho} G \left[ \frac{d\ln x^*}{b} + (I - (1 - \mu)S)^{-1} \frac{d\ln b}{b} \right].
\end{align*}
\]
Using the definitions (B.73), we can re-write the capital accumulation equation (B.71) as follows:
\[
\begin{align*}
\frac{d\ln x^*}{b} &= O \left[ (I - S)(I - T - \theta M_\mu O)^{-1} \left( - (I - T) \frac{\beta}{\rho} G \frac{d\ln x^*}{b} \right) - \theta MO \frac{d\ln z}{b} \right] + S \frac{d\ln z}{b},
\end{align*}
\]
\[
\begin{align*}
\frac{d\ln x^*}{b} &= \left[ (I + O(I - S)(I - T - \theta M_\mu O)^{-1}(I - T) \frac{\beta}{\rho} G) \left( OS - \theta O(I - S)(I - T - \theta M_\mu O)^{-1} MO \right) \right] \frac{d\ln z}{b}.
\end{align*}
\]
We thus obtain the following representation of the steady-state elasticity of the endogenous variables in each location with respect to a shock in any location (omitted from the paper for brevity).

**Proposition A.1.** The general equilibrium response of the steady-state distribution of economic activity \( \{w^*_i, v^*_i, \ell^*_i, k^*_i\} \) to small productivity \( \frac{d\ln z}{b} \) and amenity shocks \( \frac{d\ln b}{b} \) is uniquely determined by the matrices \( \{L^{z*}, K^{z*}, W^{z*}, V^{z*}, L^{bs}, K^{bs}, W^{bs}, V^{bs}\} \), which depend solely on the structural parameters \( \{\theta, \beta, \rho, \mu, \delta\} \) and the observed matrices of expenditure shares \( \{S\} \), income shares \( \{T\} \), outmigration shares \( \{D\} \) and inmigration shares \( \{E\} \):
\[
\begin{align*}
\begin{bmatrix}
\frac{d\ln \ell^*}{b} \\
\frac{d\ln k^*}{b} \\
\frac{d\ln w^*}{b} \\
\frac{d\ln v^*}{b}
\end{bmatrix} &= \begin{bmatrix}
L^{z*} & K^{z*} \\
W^{z*} & V^{z*}
\end{bmatrix} \frac{d\ln z}{b} + \begin{bmatrix}
L^{bs} & K^{bs} \\
W^{bs} & V^{bs}
\end{bmatrix} \frac{d\ln b}{b},
\end{align*}
\]

**Proof.** The proposition follows from the value function (B.69), wage equation (B.74), population equation (B.75), and capital-labor equation (B.76). In particular, from the population equation (B.75) and the capital-labor equation (B.76), we have:
\[
L^{z*} \equiv \frac{\beta}{\rho} G \left[ I + O(I - S)(I - T - \theta M_\mu O)^{-1}(I - T) \frac{\beta}{\rho} G \right]^{-1} \left( OS - \theta O(I - S)(I - T - \theta M_\mu O)^{-1} MO \right),
\]
\[
L^{bs} \equiv \frac{\beta}{\rho} G (I - (1 - \mu)S)^{-1}.
\]
From the capital-labor equation (B.76) and population equation (B.75), we have:
\[
K^{z*} \equiv \left[ I + \frac{\beta}{\rho} G \right] \left[ I + O(I - S)(I - T - \theta M_\mu O)^{-1}(I - T) \frac{\beta}{\rho} G \right]^{-1} \left( OS - \theta O(I - S)(I - T - \theta M_\mu O)^{-1} MO \right),
\]
\[
K^{bs} \equiv \left[ I + \frac{\beta}{\rho} G \right] (I - (1 - \mu)S)^{-1}.
\]

From the wage equation (B.74) and population equation (B.75), we have:

\[ W^{bs} \equiv [I - T - \theta M \mu O]^{-1} \left[ -(I - T) L^{bs} - \theta MO \right], \]

\[ W^{bs} \equiv [I - T - \theta M \mu O]^{-1} \left[ -(I - T) L^{bs} \right]. \]

From the value function (B.69) and the wage equation (B.74), we have:

\[ V^{bs} \equiv (I - \beta D)^{-1} (I - (1 - \mu) S)^{-1} [(I - S) W^{zs} + S], \]

\[ V^{bs} \equiv (I - \beta D)^{-1} (I - (1 - \mu) S)^{-1}. \]

Note that the matrices of steady-state elasticities \( \{L^{zs}, K^{zs}, W^{zs}, V^{zs}, L^{bs}, K^{bs}, W^{bs}, V^{bs}\} \) are linear combinations of the structural parameters \( \{\theta, \beta, \rho, \mu, \delta\} \) and the observed matrices of expenditure shares \( (S) \), income shares \( (T) \), outmigration shares \( (D) \) and immigration shares \( (E) \). Therefore, the steady-state changes in the endogenous variables \( \{w_i^*, v_i^*, \ell_i^*, k_i^*\} \) in response to productivity and amenity shocks are unique (up to a choice of numeraire for wages). We choose the total income of all locations as our numeraire \( \left( \sum_{i=1}^N w_i^* \ell_i^* = \sum_{i=1}^N q_i^* = \bar{q} = 1 \right) \), which implies that the log changes in incomes satisfy \( Q^{*} d \ln q^{*} = \sum_{t=1}^N q_t^* d \ln q_t^* = \sum_{t=1}^N q_t^* \frac{dq_t^*}{q_t^*} = \sum_{t=1}^N dq_t^* = 0 \), where \( Q \) is a row vector of the income of each location. Similarly, the outmigration shares \( (D) \) and immigration shares \( (E) \) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: \( \sum_{i=1}^N \ell_{it} = \bar{\ell} = 1 \), which implies \( L^{*} d \ln \ell^{*} = \sum_{i=1}^N \ell_{t}^* d \ln \ell_{t}^* = \sum_{i=1}^N \ell_{t}^* \frac{d\ell_{t}^*}{\ell_{t}^*} = \sum_{i=1}^N d\ell_{t}^* = 0 \), where \( L \) is a row vector of the population of each location.

\[ \square \]

**B.10.4 Derivations for the Linearized Equilibrium Conditions in Section 3.1 of the Paper**

In this section of the online appendix, we derive the linearized equilibrium conditions (equations 20 through 17) in Section 3.1 of the paper. We suppose that we observe the initial values of the state variables \( (\ell_0, k_0) \) and the trade and migration share matrices \( (S, T, D, E) \) at time \( t = 0 \), which need not correspond to a steady-state of the model. Throughout the following, we use a tilde above a variable to denote a log deviation from the steady-state implied by the initial fundamentals (the “initial steady-state”), such that \( \tilde{\chi}_{it+1} = \ln \chi_{it+1} - \ln \chi_{i}^* \), for all variables except for the worker value function \( v_{it} \); with a slight abuse of notation we use \( \tilde{v}_{it} \equiv v_{it} - v_{i}^* \) to denote the deviation in levels for the worker value function. We consider stochastic shocks to productivity \( (d \ln z_t) \) and amenities \( (d \ln b_t) \) in each location, holding constant the economy’s aggregate labor endowment \( (d \ln \bar{\ell} = 0) \), trade costs \( (d \ln \tau_t = 0) \) and commuting costs \( (d \ln \kappa_t = 0) \).
Population Flow (equation (20) in the Paper). The total derivative of the population flow condition (B.56) relative to the initial steady-state has the following matrix representation:

$$\tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \tilde{v}_{t+1}. \tag{B.78}$$

Capital Accumulation (equation (18) in the Paper). Note that in a deterministic steady-state, $\beta R^* = 1$, and $\zeta^{*-1} = 1 + \beta^\psi (R^*)^{\psi-1} \zeta^{*-1}$, thereby implying $\zeta^* = 1 - \beta$. We now linearize (B.19) relative to the deterministic steady-state (let $\bar{x}_t \equiv \ln x_t - \ln x^*$),

$$\zeta_t \approx -\mathbb{E}_t \ln \frac{1 + \beta^\psi (R^*)^{\psi-1} (R_{t+1}/R^*)^{\psi-1} \zeta^{*-1}_{t+1}}{1 + \beta \zeta^{*-1}_{t+1}}$$

$$= -\mathbb{E}_t \ln \frac{1 + \beta^\psi (R^*)^{\psi-1} (\zeta^{*-1}_{t+1}/\zeta^*)^{-1}}{1 + \beta/(1 - \beta)}$$

$$\approx -\mathbb{E}_t \ln \left(1 + \beta \left((R_{t+1}/R^*)^{\psi-1} - 1\right) + \beta \left((\zeta^{*-1}_{t+1}/\zeta^*)^{-1} - 1\right)\right)$$

$$= \beta \mathbb{E}_t \zeta_{t+1} - (\psi - 1) \beta \mathbb{E}_t \tilde{R}_{t+1}$$

$$\tilde{c}_t = \tilde{k}_t + \tilde{R}_t + \zeta_t$$

$$= \tilde{k}_t + \tilde{R}_t - (\psi - 1) \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \tilde{R}_{t+s}$$

$$\tilde{k}_{t+1} = \tilde{k}_t + \tilde{R}_t + (1 - \zeta_t)$$

$$= \tilde{k}_t + \tilde{R}_t - \frac{1 - \beta}{\beta} \zeta_t$$

$$= \tilde{k}_t + \tilde{R}_t + \frac{1 - \beta}{\beta} (\psi - 1) \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \tilde{R}_{t+s} \tag{B.79}$$

We now derive $\tilde{R}_{t+s}$. Note $R_{it} = 1 - \delta + r_{it}/p_{it}$, and we know in steady-state $\beta (1 - \delta + r^*/p^*) = 1$ and $r^*/p^* = \beta^{-1} + \delta - 1$. Thus

$$\tilde{R}_{it} = \ln \left(\frac{1 - \delta + r_{it}/p_{it}}{1 - \delta + r^*/p^*}\right)$$

$$= \ln \left(\beta \left((1 - \delta + r^*/p^*)^* - 1 + 1\right) (p^*/p_{it} - 1 + 1)\right)$$

$$= \ln \left(1 + \beta r^*/p^* \left((r_{it}/r^* - 1) + (p^*/p_{it} - 1)\right)\right)$$

$$= \beta r^*/p^* (\tilde{r}_{it} - \tilde{p}_{it})$$

$$= (1 - \beta (1 - \delta)) (\tilde{w}_{it} - \tilde{p}_{it} - \tilde{\chi}_{it})$$

$$= (1 - \beta (1 - \delta)) \left(\tilde{w}_{it} - \tilde{p}_{it} - \tilde{\chi}_{it}\right) \tag{B.80}$$
where recall $\chi_{it} \equiv k_{it}/\ell_{it}$ and the last equality follows from $r_{it} = \frac{1-\lambda}{\lambda} w_{it} / k_{it}$. Note (B.79) and (B.80) imply:

$$\tilde{k}_{t+1} = \tilde{k}_t + (1 - \beta (1 - \delta)) \left[ (\tilde{w}_t - \tilde{p}_t - \tilde{x}_t) + \frac{1 - \beta}{\beta} (\psi - 1) \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s (\tilde{w}_{t+s} - \tilde{p}_{t+s} - \tilde{x}_{t+s}) \right], \quad (B.81)$$

as desired.

**Value Function (equation (21) in the Paper).** The total derivative of the value function (B.58) relative to the initial steady-state has the following matrix representation:

$$\tilde{v}_t = \tilde{w}_t - \tilde{p}_t + \tilde{b}_t + \beta \mathbb{E}_t \tilde{v}_{t+1}. \quad (B.82)$$

**Goods Market Clearing (equation (19) in the Paper).** The total derivative of the goods market clearing condition (B.55) relative to the initial steady-state has the following matrix representation:

$$\tilde{w}_t + \tilde{\ell}_t = T (\tilde{w}_t + \tilde{\ell}_t) + \theta (TS - I) (\tilde{w}_t - (1 - \mu) \tilde{x}_t - \tilde{z}_t),$$

where we have used $\ln \tau = 0$. We can re-write this relationship as:

$$[I - T + \theta (I - TS)] \tilde{w}_t = \left[ -(I - T) \tilde{\ell}_t + \theta (I - TS) (\tilde{z}_t + (1 - \mu) \tilde{x}_t) \right]. \quad (B.83)$$

**Price Index (equation (17) in the Paper).** We obtain the equation (17) by substituting (B.51) into (B.52) and stack into a matrix to obtain:

$$\tilde{p}_t = S \left( \tilde{w}_t - \tilde{z}_t - (1 - \mu) \left( \tilde{k}_t - \tilde{\ell}_t \right) \right) \quad (B.84)$$

**System of Equations for Transition Dynamics Relative to the Initial Steady-State.** Collecting together the capital accumulation equation (B.81), the goods market clearing condition (B.83), the population flow condition (B.78), the value function (B.82), and the price index equation (B.84), the system of equations for the transition dynamics relative to the initial steady-state is:

$$\tilde{k}_{t+1} = \tilde{k}_t + (1 - \beta (1 - \delta)) \left[ (\tilde{w}_t - \tilde{p}_t - \tilde{k}_t + \tilde{\ell}_t) + (1 - \beta (1 - \delta)) \frac{1 - \beta}{\beta} (\psi - 1) \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s (\tilde{w}_{t+s} - \tilde{p}_{t+s} - \tilde{k}_{t+s} + \tilde{\ell}_{t+s}) \right] \quad (B.85)$$

$$\tilde{w}_t = [I - T + \theta (I - TS)]^{-1} \left[ -(I - T) \tilde{\ell}_t + \theta (I - TS) (\tilde{z}_t + (1 - \mu) \tilde{x}_t) \right]. \quad (B.86)$$
\[ \tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1}. \]  
\[ \tilde{v}_t = (I - S) \tilde{w}_t + S \tilde{z}_t + (1 - \mu) S \tilde{\chi}_t + \tilde{b}_t + \beta D E_t \tilde{v}_{t+1}. \]  
\[ \tilde{p}_t = S \left( \tilde{w}_t - \tilde{z}_t - (1 - \mu) \left( \tilde{k}_t - \tilde{\ell}_t \right) \right) \] (B.89)

### B.10.5 Equilibrium Conditions in terms of the State Variables

We now re-express the equilibrium conditions (B.85) through (B.89) and solve for the law of motion of the endogenous state variables (\( \ell_t \) and \( k_t \)). For notational convenience, we re-express the state variables as labor and capital labor ratio (\( \ell_t \) and \( \chi_t \)), but note that a law of motion for capital can always recovered since \( k_{it} = \ell_{it} \chi_{it} \).

We begin by using the wage equation (B.86) to substitute for \( d \ln \tilde{w}_t \) in the value function (B.88):

\[
\tilde{v}_t = \left[ (I - S) [I - T + \theta (I - TS)]^{-1} \left\{ - (I - T) \tilde{\ell}_t + S \tilde{z}_t + (1 - \mu) \tilde{\chi}_t \right\} + \theta (I - TS) (\tilde{z}_t + (1 - \mu) \tilde{\chi}_t) \right],
\]

\[
\tilde{v}_t = \left[ - (I - S) [I - T + \theta (I - TS)]^{-1} (I - T) \tilde{\ell}_t + (1 - \mu) \left\{ S + \theta (I - S) [I - T + \theta (I - TS)]^{-1} (I - TS) \right\} \tilde{\chi}_t + \theta S [I - T + \theta (I - TS)]^{-1} (I - TS) \tilde{z}_t \right] + \tilde{b}_t + \beta D E_t \tilde{v}_{t+1}
\]

which can be re-written more compactly as:

\[ \tilde{v}_t = A \tilde{\ell}_t + B \tilde{\chi}_t + C \tilde{z}_t + \tilde{b}_t + \beta D E_t \tilde{v}_{t+1}, \] (B.91)

where

\[ A \equiv - (I - S) [I - T + \theta (I - TS)]^{-1} (I - T), \]
\[ B \equiv (1 - \mu) \left\{ S + \theta (I - S) [I - T + \theta (I - TS)]^{-1} (I - TS) \right\}, \]
\[ C \equiv S + \theta (I - S) [I - T + \theta (I - TS)]^{-1} (I - TS). \]

Iterating equation (B.91) forward in time, we have:

\[ \tilde{v}_t = E_t \sum_{s=0}^{\infty} (\beta D)^s \left( A \tilde{\ell}_{t+s} + B \tilde{\chi}_{t+s} + C \tilde{z}_{t+s} + \tilde{b}_{t+s} \right). \] (B.92)

Using equation (B.92) to substitute for \( \tilde{v}_{t+1} \) in equation (B.87), we obtain the following autoregressive representation of the deviation in the log change in population from its steady-state value (\( \tilde{\ell}_t \)):

\[ \tilde{\ell}_{t+1} - E \tilde{\ell}_t = \left[ \frac{\beta}{\rho} (I - ED) E_t \sum_{s=0}^{\infty} (\beta D)^s \left( A \tilde{\ell}_{t+s+1} + B \tilde{\chi}_{t+s+1} + C \tilde{z}_{t+s+1} + \tilde{b}_{t+s+1} \right) \right]. \] (B.93)
Likewise, the capital law of motion (B.85) can be re-written as (noting $\tilde{w}_t - \tilde{p}_t = A\tilde{e}_t + B\tilde{\chi}_t + C\tilde{z}_t$):

$$\tilde{x}_{t+1} + \tilde{\ell}_{t+1} = \tilde{x}_t + \tilde{\ell}_t + \left(1 - \beta (1 - \delta)\right) \left( A\tilde{e}_t + (B - I)\tilde{\chi}_t + C\tilde{z}_t \right) + (1 - \beta (1 - \delta)) \frac{1 - \beta}{\beta} \left( \psi - 1 \right) \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \left( A\tilde{e}_{t+s} + (B - I)\tilde{\chi}_{t+s} + C\tilde{z}_{t+s} \right)$$  \hspace{1cm} (B.94)

B.10.6 Proof of Proposition 3 in the Paper

We now solve for the economy’s transition path in response to a one-time shock. We suppose that agents learn at time $t = 0$ about a one-time, unexpected, and permanent change in productivity and amenities from time $t = 1$ onwards. Under this assumption, we can write the sequence of future fundamentals (productivities and amenities) relative to the initial level as $(\tilde{z}_t, \tilde{b}_t) = (\tilde{z}, \tilde{b})$ for $t \geq 1$.

**Proposition. Transition Path (Proposition 3 in the paper).** There exists a $2N \times 2N$ transition matrix $(P)$ and a $2N \times 2N$ impact matrix $(R)$ such that the second-order difference equation system in (22) has a closed-form solution of the form:

$$\tilde{x}_{t+1} = P\tilde{x}_t + RF_t \quad \text{for } t \geq 0,$$  \hspace{1cm} (B.95)

where $\tilde{x}_t = \begin{bmatrix} \tilde{\ell}_t \\ \tilde{\chi}_t \end{bmatrix}$ is a $2N \times 2N$ vector of the state variables; $\tilde{f}_t = \begin{bmatrix} \tilde{z}_t \\ \tilde{b}_t \end{bmatrix}$ is a $2N \times 2N$ vector of the shocks to fundamentals; and $(P, R)$ are $2N \times 2N$ matrices that depend only on the structural parameters $[\theta, \beta, \rho, \mu, \delta]$ and the observed trade and migration matrices $(S, T, D, E)$.

**Proof.** We prove the proposition using the equivalent representation of $\tilde{\ell}_t$ and $\tilde{\chi}_t \equiv \tilde{\ell}_t - \tilde{\ell}_t$ as the state variables, where $\tilde{\chi}_t$ is the vector of capital-labor ratios in each location. Since agents expect fundamentals to be constant for all $t \geq 1$, we can drop the expectation signs in equations (B.93) and (B.94) and write $(\tilde{z}_t, \tilde{b}_t) = (\tilde{z}, \tilde{b})$:

$$(I - ED)^{-1} \left( \tilde{\ell}_{t+1} - E\tilde{\ell}_t \right) = \frac{\beta}{\rho} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta D)^s \left( A\tilde{e}_{t+s+1} + B\tilde{\chi}_{t+s+1} + C\tilde{z} + \tilde{b} \right)$$  \hspace{1cm} (B.96)

$$\tilde{x}_{t+1} + \tilde{\ell}_{t+1} = \tilde{x}_t + \tilde{\ell}_t + \left(1 - \beta (1 - \delta)\right) \left( A\tilde{e}_t + (B - I)\tilde{\chi}_t + C\tilde{z} \right) + (1 - \beta (1 - \delta)) \frac{1 - \beta}{\beta} \left( \psi - 1 \right) \sum_{s=1}^{\infty} \beta^s \left( A\tilde{e}_{t+s} + (B - I)\tilde{\chi}_{t+s} + C\tilde{z} \right)$$  \hspace{1cm} (B.97)

Analogously,

$$(I - ED)^{-1} \left( \tilde{\ell}_{t+2} - E\tilde{\ell}_{t+1} \right) = \frac{\beta}{\rho} \sum_{s=0}^{\infty} (\beta D)^s \left( A\tilde{e}_{t+s+2} + B\tilde{\chi}_{t+s+2} + C\tilde{z} + \tilde{b} \right)$$  \hspace{1cm} (B.98)
\[ \tilde{x}_{t+2} + \tilde{e}_{t+2} = \tilde{x}_{t+1} + \tilde{e}_{t+1} + (1 - \beta (1 - \delta)) \left( A \tilde{x}_{t+1} + (B - I) \tilde{x}_{t+1} + C \tilde{z} \right) + (1 - \beta (1 - \delta)) \sum_{s=1}^{\infty} \beta^s \left( A \tilde{x}_{t+s+1} + (B - I) \tilde{x}_{t+s+1} + C \tilde{z} \right) \] (B.99)

Multiply (B.98) by \( \beta D \), subtract from (B.96), and re-arrange to obtain:

\[ \beta D \left( I - ED \right)^{-1} \tilde{e}_{t+2} = \left[ \frac{\beta D (I - ED)^{-1} E + (I - ED)^{-1} - \frac{\beta}{\rho} A}{\beta (I - ED)^{-1} E \tilde{e}_{t} - \frac{\beta}{\rho} B \tilde{x}_{t+1} - \frac{\beta}{\rho} C \tilde{z} - \frac{\beta}{\rho} b} \right]. \]

Likewise, multiply (B.99) by \( \beta \), subtract from (B.97) to obtain:

\[ \beta \left( \tilde{x}_{t+2} + \tilde{e}_{t+2} \right) = (\beta - (1 - \beta (1 - \delta)) \left( -I - (1 - \beta (1 - \delta)) A \right) \tilde{e}_{t} + (\beta - (1 - \beta (1 - \delta)) \left( I - (1 - \beta (1 - \delta)) (B - I) \right) \tilde{x}_{t} + ((1 + \beta) I - (1 - \beta (1 - \delta)) \left( \psi - 1 - \beta \psi \right) (B - I) \tilde{x}_{t+1} + ((1 + \beta) I - (1 - \beta (1 - \delta)) \left( \psi - 1 - \beta \psi \right) A \tilde{e}_{t+1} - (1 - \beta (1 - \delta)) \psi (1 - \beta) C \tilde{z} \]

Stacking these two, second-order difference equations, we obtain:

\[ \begin{bmatrix} \beta D (I - ED)^{-1} E + (I - ED)^{-1} - \frac{\beta}{\rho} A \\ \frac{\beta}{\rho} B \tilde{x}_{t+1} - \frac{\beta}{\rho} C \tilde{z} - \frac{\beta}{\rho} b \end{bmatrix} = \begin{bmatrix} \tilde{y}_{11} & \tilde{y}_{12} \\ \tilde{y}_{21} & \tilde{y}_{22} \end{bmatrix} \begin{bmatrix} \tilde{e}_{t+1} \\ \tilde{x}_{t+1} \end{bmatrix} + \begin{bmatrix} \Theta_{11} \\ \Theta_{21} \end{bmatrix} \begin{bmatrix} \tilde{e}_{t} \\ \tilde{x}_{t} \end{bmatrix} + \begin{bmatrix} \frac{\beta}{\rho} C \\ -H \end{bmatrix} \begin{bmatrix} \tilde{z} \\ b \end{bmatrix}. \] (B.100)

where

\[ \tilde{y}_{11} \equiv \beta D (I - ED)^{-1} E + (I - ED)^{-1} - \frac{\beta}{\rho} A, \]

\[ \tilde{y}_{12} \equiv -\frac{\beta}{\rho} B, \]

\[ \tilde{y}_{21} \equiv I + \beta \left[ I - (1 - \beta (1 - \delta)) \left( \psi - 1 - \beta \psi \right) (I - S) [I - T + \theta (I - TS)]^{-1} (I - T) \right], \]

\[ \tilde{y}_{22} \equiv I + \beta \left[ \left( 1 - \mu \right) (1 - \beta (1 - \delta)) \left( \psi - 1 - \beta \psi \right) \times \left( S + \theta (I - S) (I - T + \theta (I - TS))^{-1} (I - TS) \right) \right], \]

\[ \Theta_{11} \equiv - (I - ED)^{-1} E \]

\[ \Theta_{21} \equiv - I + (1 - \beta (1 - \delta)) (I - S) (I - T + \theta (I - TS))^{-1} (I - T) \]

\[ \Theta_{22} \equiv - I - (1 - \beta (1 - \delta)) \left( 1 - \mu \right) \left( S + \theta (I - S) (I - T + \theta (I - TS))^{-1} (I - TS) \right) - I \]

\[ H \equiv \psi (1 - \beta) (1 - \beta (1 - \delta)) \left( \theta (I - S) (I - T + \theta (I - TS))^{-1} (I - TS) + S \right). \]

We first conjecture the linear closed-form solution (B.95) and substitute it into the second-order difference equation (B.100) to obtain a matrix system of quadratic equations. We next solve this matrix system of quadratic equations and confirm that our conjecture of a linear closed-form solution is indeed satisfied. Using our conjecture (B.95) in the system of second-order difference equations (B.100), we obtain:

\[ (\Psi P^2 - \Gamma P - \Theta) \begin{bmatrix} \tilde{e}_{t} \\ \tilde{x}_{t} \end{bmatrix} + \left[ (\Psi P^2 - \Gamma P - \Theta) R - \Pi \right] \begin{bmatrix} \tilde{z} \\ b \end{bmatrix} = 0, \] (B.101)
\[ \Psi \equiv \left[ \begin{array}{cc} (\beta D) (I - ED)^{-1} & 0 \\ \beta I & \beta I \end{array} \right], \]

\[ \Gamma \equiv \left[ \begin{array}{cc} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{array} \right], \]

\[ \Theta \equiv \left[ \begin{array}{cc} \theta_{11} & 0 \\ \theta_{21} & \theta_{22} \end{array} \right], \]

\[ \Pi \equiv \left[ \begin{array}{cc} -\frac{a}{\rho} C & -\frac{a}{\rho} I \\ -H & 0 \end{array} \right]. \]

For the system of matrix quadratic equations (B.101) to have a solution for \( \left[ \begin{array}{c} \tilde{\ell} \\ \tilde{\chi} \end{array} \right] \neq 0 \) and \( \left[ \begin{array}{c} \tilde{z} \\ \tilde{b} \end{array} \right] \neq 0 \), we require:

\[ \Psi P^2 - \Gamma P - \Theta = 0, \quad (B.102) \]

\[ R = (\Psi P + \Psi - \Gamma)^{-1} \Pi. \quad (B.103) \]

Following Uhlig (1999), we can write this first condition (B.102) as the following generalized eigenvector-eigenvalue problem, where \( e \) is a generalized eigenvector and \( \xi \) is a generalized eigenvalue of \( \Xi \) with respect to \( \Delta \):

\[ \xi \Delta e = \Xi e, \]

where:

\[ \Xi \equiv \left[ \begin{array}{cc} \Gamma & \Theta \\ I & 0 \end{array} \right], \quad \Delta \equiv \left[ \begin{array}{cc} \Psi & 0 \\ 0 & I \end{array} \right]. \]

If \( e_h \) is a generalized eigenvector and \( \xi_h \) is a generalized eigenvalue of \( \Xi \) with respect to \( \Delta \), then \( e_h \) can be written for some \( h \in \mathbb{R}^N \) as:

\[ e_h = \left[ \begin{array}{c} \xi_h e_h \\ e_h \end{array} \right]. \]

Assuming that that the transition matrix has distinct eigenvalues, which we verify empirically, there are \( 2N \) linearly independent generalized eigenvectors \( (e_1, \ldots, e_N) \) and corresponding eigenvalues \( (\xi_1, \ldots, \xi_N) \), and the transition matrix \( (P) \) is given by:

\[ P = \Omega \Lambda \Omega^{-1}, \]

where \( \Lambda \) is the diagonal matrix of the \( 2N \) eigenvalues and \( \Omega \) is the matrix stacking the corresponding \( 2N \) eigenvectors. The impact matrix \( (R) \) in the second condition (B.103) can be recovered using:

\[ R = (\Psi P + (\Psi - \Gamma)^{-1} \Pi, \]

and our conjecture (B.95) is satisfied.
B.10.7 Properties of the Transition Path.

We now use the eigenvalue-eigenvector representation in Proposition 3 in the paper to establish some properties of the transition path towards the new steady-state. In Subsection B.10.8, we report some additional results for convergence dynamics versus fundamental shocks from Subsection 3.2.1 of the paper. In Subsection B.10.9, we provide additional results for our spectral analysis of the transition matrix \( P \) from Subsection 3.2.2 of the paper.

B.10.8 Convergence Dynamics Versus Fundamental Shocks

In particular, we now consider the case in which agents at time \( t = 0 \) learn of a permanent change in fundamentals \((\zeta, \tilde{b})\) at time \( t = 1 \). From Proposition 3 in the paper and equation (B.95) above, the initial impact of the productivity \((\zeta)\) and amenity \((\tilde{b})\) shocks in the first period is:

\[
\tilde{x}_1 = R \tilde{f}.
\]

More generally, the impact of these productivity and amenity shocks in period \( t \geq 1 \) is:

\[
\tilde{x}_{t+1} = P \tilde{x}_t + R \tilde{f},
\]

\[
= \left( \sum_{s=0}^{t} P^s \right) R \tilde{f}.
\]

If the spectral radius of \( P \) is less than one, a condition that we verify empirically, the summation \( \lim_{t \to \infty} \sum_{s=0}^{t} P^s \) converges, and we can re-write the impact of the productivity and amenity shocks in period \( t \geq 1 \) as:

\[
\tilde{x}_{t+1} = \left( \sum_{s=0}^{\infty} P^s - \sum_{s=t+1}^{\infty} P^s \right) R \tilde{f},
\]

\[
= (I - P^{t+1}) (I - P)^{-1} R \tilde{f}.
\]

From this relationship, the new steady-state must satisfy:

\[
\lim_{t \to \infty} \tilde{x}_t = x_{n}^* - \tilde{x}_{\text{initial}} = (I - P)^{-1} R \tilde{f},
\]

where \((I - P)^{-1} R\) coincides with the explicit solution for the changes-in-steady-states in Proposition A.1 in Section B.10.3 of this online appendix:

\[
(I - P)^{-1} R = \begin{bmatrix}
L^z & L^b \\
K^z & K^b
\end{bmatrix}.
\]

Using Proposition 3 in the paper, we can also decompose the evolution of the spatial distribution of economic activity across locations into the contributions of convergence towards
steady-state and shocks to fundamentals. In particular, from Proposition 3, we have:

\[ \bar{x}_t = P\bar{x}_{t-1} + \tilde{R}f, \]
\[ \bar{x}_{t-1} = P\bar{x}_{t-2} + \tilde{R}f, \]
\[ \vdots \]
\[ \bar{x}_1 = P\bar{x}_0 + \tilde{R}f, \]
\[ \bar{x}_0 = P\bar{x}_{-1}, \]

where the last equation at \( t = 0 \) is different from the other periods, because agents become aware at time \( t = 0 \) of the shock to fundamentals a time \( t = 1 \), after they have migrated between time \( t = -1 \) and time \( t = 0 \). Taking the difference between the equations for time \( t \) and \( t - 1 \), we have:

\[ \ln x_t - \ln x_ {t-1} = P (\ln x_{t-1} - \ln x_{t-2}) \]
\[ \vdots \]
\[ = P^{t-1} (\ln x_1 - \ln x_0) \]
\[ = P^t (\ln x_0 - \ln x_{-1}) + P^{t-1} R\tilde{f}. \]

Therefore, we have:

\[ \ln x_t - \ln x_{-1} = [\ln x_t - \ln x_{t-1}] + [\ln x_{t-1} - \ln x_{t-2}] + \cdots + [\ln x_1 - \ln x_0] + [\ln x_0 - \ln x_{-1}] \]
\[ = P^t (\ln x_0 - \ln x_{-1}) + P^{t-1} R\tilde{f} \]
\[ \vdots \]
\[ + \cdots + P (\ln x_0 - \ln x_{-1}) + R\tilde{f} + [\ln x_0 - \ln x_{-1}] \]
\[ = \sum_{s=0}^{t} P^s (\ln x_0 - \ln x_{-1}) + \sum_{s=0}^{t-1} P^s R\tilde{f}, \]

which corresponds to equation (24) in the paper.

B.10.9 Spectral Analysis of the Transition Matrix \( P \)

Our characterization so far in Proposition 3 in the paper requires knowledge of the entire transition matrix \( (P) \). We now show that we can further characterize the economy’s transition path in terms of lower-dimensional components, namely the eigenvectors and eigenvalues of the transition matrix \( (P) \). Such a lower-dimensional representation is not possible within the full nonlinear model, for which only numerical solutions for the transition path exist. We use this lower-dimensional representation to characterize the economy’s speed of convergence to steady-state and the heterogeneous impact of local shocks.
In equation (24) in the paper, we have already shown in that we can decompose the dynamic path of the economy into one component capturing shocks to fundamentals and another component capturing convergence to the initial steady-state. Therefore, for the remainder of this subsection, we focus for expositional simplicity on an economy that is initially in steady-state.

**Eigendecomposition of the Transition Matrix** We begin by undertaking an eigendecomposition of the transition matrix, \( \mathbf{P} \equiv \mathbf{U} \Lambda \mathbf{V} \), where \( \Lambda \) is a diagonal matrix of eigenvalues arranged in decreasing order by absolute values, and \( \mathbf{V} = \mathbf{U}^{-1} \). For each eigenvalue \( \lambda_h \), the \( h \)-th column of \( \mathbf{U} \) \( (u_h) \) and the \( h \)-th row of \( \mathbf{V} \) \( (v'_h) \) are the corresponding right- and left-eigenvectors of \( \mathbf{P} \), respectively, such that

\[
\lambda_h u_h = Pu_h, \quad \lambda_h v'_h = v'_h \mathbf{P}.
\]

That is, \( u_h \) \( (v'_h) \) is the vector that, when left-multiplied (right-multiplied) by \( \mathbf{P} \), is proportional to itself but scaled by the corresponding eigenvalue \( \lambda_h \).\(^2\) We refer to \( u_h \) simply as eigenvectors. Both \( \{u_h\} \) and \( \{v'_h\} \) are bases that span the \( 2N \)-dimensional vector space.

We next introduce a particular type of shock to productivity and amenities that proves useful for characterizing the model’s transition dynamics. We define an *eigen-shock* as a shock to productivity and amenities \( (\bar{\mathbf{f}}(h)) \) for which the initial impact of these shocks on the state variables \( (\mathbf{R} \mathbf{f}(h)) \) coincides with a real eigenvector of the transition matrix \( (u_h) \). Assuming that the impact matrix is invertible, which we verify empirically, we can recover these eigen-shocks from the impact matrix \( (\mathbf{R}) \) and the eigenvectors of the transition matrix \( (u_h) \), using \( \bar{\mathbf{f}}(h) = \mathbf{R}^{-1}u_h \).

Recall that both the impact matrix \( (\mathbf{R}) \) and the eigenvectors of the transition matrix \( (u_h) \) can be computed using only our observed trade and migration share matrices \( (\mathbf{S}, \mathbf{T}, \mathbf{D}, \mathbf{E}) \) and the structural parameters of the model \( \{\psi, \theta, \beta, \rho, \mu, \delta\} \). Therefore, we can solve for the eigen-shocks from these observed data and the structural parameters of the model.

Using our eigendecomposition and definition of an eigen-shock, we can undertake a spectral analysis of the economy’s dynamic response to shocks. In particular, we can express the transition path of the state variables in response to any empirical productivity and amenity shocks \( (\mathbf{f}) \) in terms of a linear combination of the eigenvectors and eigenvalues of the transition matrix. Additionally, the weights or loadings in this linear combination can be recovered from a linear projection (regression) of the observed shocks \( (\bar{\mathbf{f}}) \) on the eigen-shocks \( (\bar{\mathbf{f}}(h)) \).

**Proposition. Spectral Analysis (Proposition 4 in the paper).** Consider an economy that is initially in steady-state at time \( t = 0 \) when agents learn about one-time, permanent shocks to productivity and amenities \( (\bar{\mathbf{f}} = \begin{bmatrix} \bar{z} \\ \bar{b} \end{bmatrix}) \) from time \( t = 1 \) onwards. The transition path of the state variables in response to these shocks is given by

\[
\mathbf{R}^{-1}u_h \text{ for all } h.
\]

\(^2\)Note that \( \mathbf{P} \) need not be symmetric. This eigendecomposition can be undertaken as long the transition matrix has distinct eigenvalues, a condition that we verify is satisfied empirically. We construct the right-eigenvectors such that the 2-norm of \( u_h \) is equal to 1 for all \( h \), where note that \( v'_i u_h = 1 \) if \( i = h \) and is equal to zero otherwise.
variables can be written as a linear combination the eigenvalues \((\lambda_h)\) and eigenvectors \((u_h)\) of the transition matrix:

\[
\tilde{x}_t = \sum_{s=0}^{t-1} P^s R \tilde{f} = \sum_{h=1}^{2N} \frac{1 - \lambda_h^s}{1 - \lambda_h} u_h v_h' R \tilde{f} = \sum_{h=1}^{2N} \frac{1 - \lambda_h^s}{1 - \lambda_h} u_h a_h,
\]

where the weights in this linear combination \((a_h)\) can be recovered as the coefficients from a linear projection (regression) of the observed shocks \((\tilde{f})\) on the eigen-shocks \((\tilde{f}_{(h)})\).

**Proof.** The proposition follows from the eigendecomposition of the transition matrix: \(P \equiv U \Lambda V\), which implies \(P^s = \sum_{h=1}^{2N} \lambda_h^s u_h v_h'\) and hence:

\[
\tilde{x}_t = \sum_{s=0}^{t-1} P^s R \tilde{f},
\]

\[
= \sum_{s=0}^{t-1} \left( \sum_{h=1}^{2N} \lambda_h^s u_h v_h' \right) R \tilde{f},
\]

\[
= \sum_{h=1}^{2N} \left( \sum_{s=0}^{t-1} \lambda_h^s \right) u_h v_h' R \tilde{f},
\]

\[
= \sum_{h=1}^{2N} \frac{1 - \lambda_h^t}{1 - \lambda_h} u_h v_h' R \tilde{f}.
\]

Now note that \(v_h' R \tilde{f} = v_h' \sum_{i=1}^{2N} a_i \tilde{f}_{(i)} = \sum_{i=1}^{2N} a_i v_h' u_i = a_h\). Finally, note that

\[
a = VR \tilde{f},
\]

\[
= U^{-1} R \tilde{f},
\]

\[
= (R^{-1} U)^{-1} \tilde{f},
\]

\[
= (R^{-1} U)^{-1} \left((R^{-1} U)^T \right)^{-1} \left((R^{-1} U)^T \right) \tilde{f},
\]

\[
= \left((R^{-1} U)^T \right)^{-1} \left(R^{-1} U\right)^T \tilde{f}.
\]

\(\square\)

We now show how this proposition can be used to characterize both the speed of convergence to steady-state and the heterogeneous impact of shocks across locations.

**Speed of Convergence** We measure the speed of convergence to steady-state using the conventional measure of the half-life. In particular, we define the half-life of a shock \(\tilde{f}\) for the \(i\)-th state variable as the time it takes for that state variable to converge half of the way to steady-state:

\[
\arg \max_t \frac{|\tilde{x}_{it} - \tilde{x}_{i\infty}|}{\max_s |\tilde{x}_{is} - \tilde{x}_{i\infty}|} \geq \frac{1}{2},
\]

(B.107)
We begin by considering the speed of convergence for eigen-shocks, for which the initial impact on the state variables corresponds to a real eigenvector of the transition matrix ($\tilde{\mathbf{f}}(h) = \mathbf{R}^{-1}\mathbf{u}_h$). For these eigen-shocks, the state variables converge exponentially towards steady-state, and the speed of convergence depends solely on the corresponding eigenvalue ($\lambda_h$).

**Proposition. Speed of Convergence (Proposition 5 in the paper).** Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\mathbf{e}_f(\mathbf{h})$) from time $t = 1$ onwards. Suppose that these shocks are an eigen-shock ($\tilde{\mathbf{f}}(h)$), for which the initial impact on the state variables at time $t = 1$ coincides with a real eigenvector ($\mathbf{u}_h$) of the transition matrix ($\mathbf{P}$): $\mathbf{R} \tilde{\mathbf{f}}(h) = \mathbf{u}_h$. The transition path of the state variables ($\mathbf{x}_t$) in response to such an eigen-shock ($\mathbf{e}_f(\mathbf{h})$) is:

$$
\tilde{x}_t = \sum_{j=1}^{2N} \frac{1 - \lambda^j}{1 - \lambda} \mathbf{u}_j v'_j \mathbf{u}_h = \frac{1 - \lambda^i}{1 - \lambda} \mathbf{u}_h \quad \Rightarrow \quad \ln x_{t+1} - \ln x_t = \lambda^i u_h,
$$

and the half-life is given by:

$$
\tau^{(1/2)} = -\left\lceil \frac{\ln 2}{\ln \lambda} \right\rceil
$$

for all state variables $i = 1, \cdots, 2N$, where $\lceil \cdot \rceil$ is the ceiling function.

**Proof.** If the initial impact impact of the shock to productivity and amenities on the state variables ($\mathbf{R} \tilde{\mathbf{f}}$) coincides with a real eigenvector ($\mathbf{R} \tilde{\mathbf{f}}(h) = \mathbf{u}_h$), we can re-write equation (28) in Proposition 4 in the paper as follows:

$$
\tilde{x}_t = \sum_{h=1}^{2N} \left( \frac{\lambda^i_h}{1 - \lambda} \right) \mathbf{u}_h v'_h \mathbf{R} \tilde{\mathbf{f}} = \sum_{j=1}^{2N} \frac{1 - \lambda^j}{1 - \lambda} \mathbf{u}_j v'_j \mathbf{u}_h = \frac{1 - \lambda^i}{1 - \lambda} \mathbf{u}_h,
$$

where we have used $v'_i \mathbf{u}_h = 0$ for $i \neq h$ and $v'_i \mathbf{u}_h = 1$ for $i = h$. Taking differences between periods $t + 1$ and $t$, we have:

$$
\tilde{x}_{t+1} - \tilde{x}_t = \frac{1 - \lambda^i_{t+1}}{1 - \lambda} \mathbf{u}_h - \frac{1 - \lambda^i_t}{1 - \lambda} \mathbf{u}_h,
$$

which simplifies to:

$$
(1 - \lambda_h) (\tilde{x}_{t+1} - \tilde{x}_t) = (1 - \lambda_h) \lambda^i_h \mathbf{u}_h,
$$

and hence:

$$
(\tilde{x}_{t+1} - \tilde{x}_t) = \lambda^i_h \mathbf{u}_h.
$$

Noting that $\tilde{x}_t = \ln x_t - \ln x_{\text{initial}}^*$, we have:

$$
\ln x_{t+1} - \ln x_t = \lambda^i_h \mathbf{u}_h.
$$

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which implies exponential convergence to steady-state, such that for each location $i$:
\[
\frac{x_{it+1}}{x_{it}} = \exp \left( \lambda^i_h u_{ih} \right).
\]
Using the half-life definition (B.107), we can solve for the half-life as:
\[
\frac{1 - \lambda^i_h u_{ih}}{1 - \lambda_h u_{ih}} = \frac{1}{2},
\]
which simplifies to:
\[
\lambda^i_h = \frac{1}{2},
\]
and hence:
\[
\ln \frac{1}{2} = t \ln \lambda_h,
\]
\[
t = -\frac{\ln 2}{\ln \lambda_h}.
\]
Imposing the requirement that $t$ is an integer, we obtain:
\[
t = \left\lceil \frac{\ln 2}{\ln \lambda_h} \right\rceil,
\]
for all state variables $i = 1, \ldots, 2N$, where $\lceil \cdot \rceil$ is the ceiling function.

**B.10.10 Convergent Sequence of Shocks Under Perfect Foresight**

We now generalize our analysis of the model’s transition dynamics to any convergent sequence of future shocks to productivities and amenities under perfect foresight. In particular, we consider an economy that is somewhere on a convergence path towards an initial steady-state with constant fundamentals at time $t = 0$, when agents learn about a convergent sequence of future shocks to productivity and amenities $\{\tilde{e}_s\}_{s \geq 1}$ from time $t = 1$ onwards, where $\tilde{f}_s$ is a vector of log differences in fundamentals between times $s$ and $0$ for each location.

**Proposition. Sequence of Shocks Under Perfect Foresight (Proposition 6 in the paper).** Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about a convergent sequence of future shocks to productivity and amenities $\{\tilde{e}_s\}_{s \geq 1}$ from time $t = 1$ onwards. There exists a $2N \times 2N$ transition matrix ($P$) and a $2N \times 2N$ impact matrix ($R$) such that the dynamic path of state variables relative to the initial steady-state follows:
\[
\bar{x}_t = \sum_{s=t+1}^{\infty} (\Psi^{-1} \Gamma - P)^{-(s-t)} R \left( \tilde{f}_s - \tilde{f}_{s-1} \right) + R\tilde{f}_t + P\bar{x}_{t-1} \quad \text{for all } t \geq 1, \quad (B.108)
\]
with initial condition $\bar{x}_0 = 0$ and where $\Psi, \Gamma$ are matrices from the second-order difference equation (22) in the paper.
Proof. We start by proving the case with a single fundamental shock \( \tilde{f}_s \) at future time \( s \), with \( \tilde{f}_t = 0 \) for all \( t \neq s \). We then exploit the linear structure and consider a sequence of shocks. Given that the shock takes place at time \( s \), we know from the Proposition 3 that:

\[ \tilde{x}_s = R \tilde{f}_s + P \tilde{x}_{s-1}. \]  

(B.109)

Following our derivations for (B.101), we know the state variables follow a system of second-order difference equations:

\[
\Psi \tilde{x}_{t+2} = \begin{cases} 
\Gamma \tilde{x}_{t+1} + \Theta \tilde{x}_t + \Pi \tilde{f}_t & t \geq s-1, \\
\Gamma \tilde{x}_{t+1} + \Theta \tilde{x}_t & 0 \leq t < s-1.
\end{cases}
\]  

(B.110)

We now solve the second-order difference equation (B.110) backwards for \( 0 \leq t < s-1 \). Starting from \( t = s-2 \), we have:

\[
\Gamma \tilde{x}_{s-1} = \Psi \tilde{x}_s - \Theta \tilde{x}_{s-2}.
\]

Substitute using (B.109) and (B.102), we obtain:

\[
\tilde{x}_{s-1} = (\Psi^{-1} \Gamma - P)^{-1} \Psi \tilde{f} + Ps.
\]

We can show by induction that, for all \( t \geq 1 \):

\[
\tilde{x}_t = (\Psi^{-1} \Gamma - P)^{-(s-t)} \Psi \tilde{f} + P \tilde{x}_{t-1}.
\]

Hence, with a single shock \( \tilde{f}_s \) at time \( s > 0 \), the law of motion of the state variables follows:

\[
\tilde{x}_t = \begin{cases} 
R \tilde{f}_s + P \tilde{x}_{t-1} & t \geq s, \\
(\Psi^{-1} \Gamma - P)^{-(s-t)} R \tilde{f}_s + P \tilde{x}_{t-1} & 1 \leq t < s.
\end{cases}
\]

Given linearity, the law of motion with a sequence of convergent fundamentals follows:

\[
\tilde{x}_t = \sum_{s=t+1}^{\infty} (\Psi^{-1} \Gamma - P)^{-(s-t)} R (\tilde{f}_s - \tilde{f}_{s-1}) + R \tilde{f}_t + P \tilde{x}_{t-1} \quad \text{for all } t \geq 1,
\]

where \( \tilde{f}_s - \tilde{f}_{s-1} \) is the change in fundamental in period \( s \) and \( \tilde{f}_t \) is the cumulative change in fundamental at time \( t \) relative to time 0. That the sequence of fundamentals converges \( \lim_{s \to \infty} (\tilde{f}_s - \tilde{f}_{s-1}) \to 0 \) ensures the summation is well defined. \( \square \)

**B.10.11 Stochastic Location Characteristics and Rational Expectations**

Finally, we prove the equilibrium stochastic process under stochastic fundamentals and rational expectations. Note that we let \( \Delta \ln x_{t+1} \equiv \ln x_{t+1} - \ln x_t \) for any variable \( x \).
Proposition. Stochastic Fundamentals and Rational Expectations (Proposition 7 in the paper). Suppose that productivity and amenities evolve stochastically according to the AR(1) process in equation (30) in the paper and agents have rational expectations. There exists a $2N \times 2N$ transition matrix ($P$) and a $2N \times 2N$ impact matrix ($R$) such that the evolution of the economy’s state variables ($x_t$) has the following closed-form solution:

$$\Delta \ln x_{t+1} = P \Delta \ln x_t + R \Delta \ln f_t + \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R N^{s+1} \left( \Delta \ln f_t - \Delta \ln f_{t-1} \right). \tag{B.111}$$

Proof. The state variables at time $t + 1$ are chosen by agents as functions of past state variables and fundamental shocks realized up to time $t$. Under rational expectation, agents at each time $t$ expect a sequence of future fundamental shocks according to (31). Thus, from Proposition 6, we know

$$\ln x_{t+1} - \ln x_t = P \ln x_t - \ln x_t^* + \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R \Delta \ln f_{t+s+1} + \Delta \ln x_t^*,$$

where $x_t^*$ is the steady-state implied by fundamentals at time $t$. Taking the difference between the two equations, we get

$$\ln x_{t+1} - \ln x_t = P \ln x_t - \ln x_t^* - (\ln x_t - \ln x_t^*) + P \ln x_{t-1} - \ln x_{t-1}^* + \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R \Delta \ln f_{t+s},$$

The left-hand side (LHS) of the above equation can be written as:

$$LHS = \Delta \ln x_{t+1} - P \Delta \ln x_t - (I - P) \Delta \ln x_t^*.$$

We know $E_t \hat{f}_{t+s} = N^s \hat{f}_t$, and hence the right-hand side (RHS) of the above equation can be written as:

$$RHS = \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R E_t \Delta \ln f_{t+s+1} - \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R E_{t-1} \Delta \ln f_{t+s},$$

$$= \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R N^{s+1} \Delta \ln f_t - \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R N^{s+1} \Delta \ln f_{t-1},$$

$$= \sum_{s=0}^{\infty} (\Psi^{-1} \Gamma - P)^{-s} R N^{s+1} \left( \Delta \ln f_t - \Delta \ln f_{t-1} \right).$$

We also know:

$$\Delta \ln x_t^* = (I - P)^{-1} R \Delta \ln f_t.$$

We obtain the Proposition by setting LHS to be equal to RHS. \qed


B.11 Distributional Consequences

The presence of gradual adjustment in the model from migration frictions and capital accumulation has two important implications for the welfare effects of shocks to productivity and amenities. First, these welfare effects depend not only on the change in steady-state, but also on the transition dynamics. Second, there is a distribution of these welfare effects, both across landlords because they are geographically immobile, and across workers because of migration frictions, which imply that a worker’s initial location matters for the welfare impact of these shocks.

As our approach provides sufficient statistics for the economy’s transition path in response to shocks to fundamentals, it also provides sufficient statistics for the welfare effects of these shocks. In the remainder of this subsection, we illustrate these sufficient statistics for welfare, using changes in migration flows to reveal information about continuation values. In particular, we suppose that the economy starts from steady-state at time $t = 0$, at which point agents become aware of a permanent change in fundamentals ($f$) at time $t = 1$. Since fundamentals change from time $t = 1$ onwards, the change in workers’ welfare at time $t = 0$ is completely determined by the change in the continuation value from their optimal location choice:

$$\tilde{v}_0 = \beta D \tilde{v}_1,$$

where this change in continuation value ($\beta D \tilde{v}_1$) depends on workers’ initial location at time $t = 0$, because of migration frictions, as captured by the outmigration matrix ($D$).

We now show that the expression for population dynamics in equation (20) can be used to infer relative changes in continuation values in response to shocks to fundamentals from these population movements:

$$\tilde{\ell}_1 = E\tilde{\ell}_0 + \frac{\beta}{\rho} (I - ED) (\tilde{v}_1 + \varsigma),$$

where the first term ($E\tilde{\ell}_0$) is equal to zero, because of our assumption that the economy starts from an initial steady state at time $t = 0$ ($\tilde{\ell}_0 = \ln \ell_0 - \ln \ell^* = 0$); the presence of the constant $\varsigma$ reflects the fact that migration decisions depend on relative expected values across locations, and hence are invariant to a common change in expected values across all locations.

To compute the impact on the overall level of welfare, we set this constant equal to the average change in expected values across all locations weighted by population shares ($\ell^r \cdot \tilde{v}_1$), where we stack the $\ell^r$ vector $N$ times into an $N \times N$ matrix $L \equiv [\ell^r, \ldots, \ell^r]$, such that $\varsigma = -L\tilde{v}_1$. This convenient choice has two simplifying properties: (i) $L^2 = L$; (ii) $LD = L$, because $\ell^r$ is the Perron-eigenvector of $D$.\footnote{Since $\ell^r$ is the Perron-eigenvector of $D$ and $E$, we have $LD = DL = LE = EL = L$. Since population share sum to one, $L \times \tilde{\ell}_1 = 0$.} Using these properties, we can re-write the above population
dynamics equation as follows:\(^4\)

\[(I - L) \tilde{v}_1 = \frac{\rho}{\beta} (I - ED + L)^{-1} \tilde{\ell}_1.\]

Combining this result with equation (B.112), we obtain the following key implication that population movements at time \(t = 1\) in response to these shocks to fundamentals are sufficient statistics for their impact on relative expected values for workers in different locations at time \(t = 0\):\(^5\)

\[(I - L) \tilde{v}_0 = \rho D (I - ED + L)^{-1} \tilde{\ell}_1,\]

where \(L \tilde{v}_0\) is again a constant vector that represents the average change in expected values across all locations weighted by initial population shares, and the right-hand side captures relative changes in expected values across locations, as revealed by the first-period population movements.

Finally, we can connect these first-period population movements (\(\tilde{\ell}_1\)) to the productivity (\(\tilde{z}\)) and amenity (\(\tilde{b}\)) shocks using our closed-form solution for the economy’s transition path (26), which yields our sufficient statistic for workers’ welfare exposure to these shocks.

**Proposition A.2.** Consider an economy that is initially in steady-state at time \(t = 0\) when agents learn about one-time, permanent shocks to productivity and amenities (\(\tilde{f} = \begin{bmatrix} \tilde{z} \\ \tilde{b} \end{bmatrix}\)) from time \(t = 1\) onwards.

(i) The relative welfare impact for agents initially in each location at time 0 is

\[\tilde{v}_0 - L \tilde{v}_0 = \rho D (I - ED + L)^{-1} R^f \tilde{f},\]

where \(R^f\) is the matrix representing the first \(N\) rows of \(R\).

(ii) The average welfare impact on all agents, weighted by initial population shares, is

\[L \tilde{v}_0 = \frac{\beta}{1 - \beta} L \left( \begin{bmatrix} C & I \end{bmatrix} \tilde{f} + \begin{bmatrix} A & B \end{bmatrix} (I - (1 - \beta) P (I - \beta P)^{-1}) (I - P)^{-1} R^f \tilde{f} \right),\]

where \(A, B, C\) are matrices from equation (B.91).

**Proof.** We start from the migration equation (20) in the paper:

\[
\tilde{\ell}_1 = \frac{\beta}{\rho} (I - ED) (\tilde{v}_1 - L \tilde{v}_1)
\]

\[
= \frac{\beta}{\rho} (I - ED + L) (\tilde{v}_1 - L \tilde{v}_1)
\]

\[\text{In particular, we use } (I - ED) (\tilde{v}_1 - L \tilde{v}_1) = (I - ED + L) (\tilde{v}_1 - L \tilde{v}_1), \text{ because } L^2 = L.\]

\[\text{We pre-multiply both sides of equation (B.112) by } (I - L) \text{ and use } (I - L) D \tilde{v}_1 = D (I - L) \tilde{v}_1.\]
where the second equality follows from $L=L^2$. Hence
\[ (I - L) \tilde{v}_1 = \frac{\rho}{\beta} (I - ED + L)^{-1} \tilde{t}_1 \]
\[ = \frac{\rho}{\beta} (I - ED + L)^{-1} R^t \tilde{f} \]
and the changes in welfare at $t=0$ follow
\[ \tilde{v}_0 = \beta D \tilde{v}_1 \]
\[ = \beta L \tilde{v}_1 + \beta D (I - L) \tilde{v}_1 \]
\[ = L \tilde{v}_0 + \rho D (I - ED + L)^{-1} R^t \tilde{f}, \]
where the third equality follows from $LD = L$; this complete the proof of the first part of the Proposition.

To prove the second part, note
\[ L \hat{v}_0 = L \beta D \hat{v}_1 \]
\[ = \frac{L}{1 - \beta} \left( C \tilde{z} + \bar{b} + A \hat{\ell}_s + B \hat{\chi}_s \right) \sum_{s=1}^\infty \beta^s (I - P^s) (I - P)^{-1} R^t \tilde{f} \]
\[ = \frac{\beta}{1 - \beta} L \left( C \tilde{f} + A \right) (I - (1 - \beta) P (I - \beta P)^{-1}) (I - P)^{-1} R^t \tilde{f}, \]
where the third equality follows from Proposition 4 and the fact $LD = L$. \qed

B.12 Derivation of Expected Utility

In this subsection, we derive the expected the expected value for a worker of living in location $i$ at time $t$ ($v^w_w$) in equation (B.3) above. Recall that idiosyncratic mobility shocks are drawn from an extreme value distribution with the following cumulative distribution function:
\[ F(\epsilon) = e^{-e^{-\epsilon-\gamma}}, \]
and corresponding probability density function:
\[ f(\epsilon) = e^{(-\epsilon-\gamma)} e^{-e^{-\epsilon-\gamma}}. \]
Using this extreme value distribution, note that:
\[ \text{Prob} \left[ \beta \tilde{v}^w_{gt+1} - \kappa_{gi} + \rho \epsilon_{gt} \geq \beta \tilde{v}^w_{mt+1} - \kappa_{mi} + \rho \epsilon_{mt}, \quad \forall m \neq g \right], \]
Using these definitions:

\[
\text{Prob} \left[ \beta \left( \mathbb{E}_t v_{gt+1}^w - \mathbb{E}_t v_{mt+1}^w \right) - (\kappa_{gi} - \kappa_{mi}) + \rho \epsilon_{it} \geq \rho \epsilon_{mt} \right],
\]

\[
\text{Prob} \left[ \rho \epsilon_{mt} \leq \beta \left( \mathbb{E}_t v_{gt+1}^w - \mathbb{E}_t v_{mt+1}^w \right) - (\kappa_{gi} - \kappa_{mi}) + \rho \epsilon_{gt} \right],
\]

\[
\text{Prob} \left[ \rho \epsilon_{mt} \leq \rho \epsilon_{igt} + \rho \epsilon_{gt} \right],
\]

\[
\bar{\epsilon}_{igt} = \frac{\beta \left( \mathbb{E}_t v_{gt+1}^w - \mathbb{E}_t v_{mt+1}^w \right) - (\kappa_{gi} - \kappa_{mi})}{\rho},
\]

\[
\text{Prob} \left[ \epsilon_{kt} \leq \epsilon_{igt} + \epsilon_{gt} \right].
\]

Now define the expected continuation value for an agent in location \( i \) at time \( t \):

\[
\Phi_{it} = \max_{(g)} \left\{ \beta \mathbb{E}_t \mathbb{E}_t \left[ V_{gt+1}^w \right] - \kappa_{gi} + \rho \epsilon_{gt} \right\}
\]

\[
\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left( \beta \mathbb{E}_t v_{gt+1}^w - \kappa_{gi} + \rho \epsilon_{gt} \right) e^{(-\epsilon_{gt} - \gamma)} e^{-e^{(-\epsilon_{gt} - \gamma)}} e^{-\sum_{m \neq g} e^{(-\epsilon_{igt} - \epsilon_{gt} - \gamma)}} d\epsilon_{gt},
\]

\[
\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left( \beta \mathbb{E}_t v_{gt+1}^w - \kappa_{gi} + \rho \epsilon_{gt} \right) e^{(-\epsilon_{gt} - \gamma)} e^{-e^{(-\epsilon_{gt} - \gamma)}} e^{-\sum_{m=1}^{N} e^{(-\epsilon_{igt} - \epsilon_{gt} - \gamma)}} d\epsilon_{gt},
\]

since \( \bar{\epsilon}_{mmt} = 0 \).

\[
\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left( \beta \mathbb{E}_t v_{gt+1}^w - \kappa_{gi} + \rho \epsilon_{gt} \right) e^{(-\epsilon_{gt} - \gamma)} e^{-e^{(-\epsilon_{gt} - \gamma)}} e^{-\sum_{m=1}^{N} e^{(-\epsilon_{igt})}} d\epsilon_{gt},
\]

Define:

\[
\lambda_{igt} \equiv \log \sum_{m=1}^{N} e^{-\epsilon_{igt}},
\]

\[
e^{\lambda_{igt}} = \sum_{m=1}^{N} e^{-\epsilon_{igt}},
\]

\[
\zeta_{gt} \equiv \epsilon_{gt} + \bar{\gamma},
\]

Using these definitions:

\[
\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left( \beta \mathbb{E}_t v_{gt+1}^w - \kappa_{gi} + \rho \left( \zeta_{gt} - \bar{\gamma} \right) \right) e^{(-\zeta_{gt})} e^{-e^{(-\zeta_{gt})}} e^{-\sum_{m=1}^{N} e^{(-\epsilon_{igt})}} d\zeta_{gt},
\]

\[
\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left( \beta \mathbb{E}_t v_{gt+1}^w - \kappa_{gi} + \rho \left( \zeta_{gt} - \bar{\gamma} \right) \right) e^{(-\zeta_{gt})} e^{-e^{(-\zeta_{gt})}} e^{\lambda_{igt}} d\zeta_{gt},
\]
\[ \Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} (\beta E_{gt}v_{gt+1}^{w} - \kappa_{gi} + \rho (\zeta_{gt} - \bar{\gamma})) e^{-(\zeta_{gt} - \lambda_{igt})} e^{-\zeta_{gt}} e^{-\zeta_{gt}} d\zeta_{gt}. \]

Now define another change of variables:

\[ \tilde{\gamma}_{igt} \equiv \zeta_{gt} - \lambda_{igt}. \]

Using this definition:

\[ \Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} (\beta E_{gt}v_{gt+1}^{w} - \kappa_{gi} + \rho (\tilde{\gamma}_{igt} + \lambda_{igt} - \bar{\gamma})) e^{-(\tilde{\gamma}_{igt} + \lambda_{igt}) - e^{-(\tilde{\gamma}_{igt})}} d\tilde{\gamma}_{igt}. \]

Now note that:

\[ \frac{d}{d\tilde{\gamma}} \left[ e^{-e^{-\tilde{\gamma}} - y} \right] = e^{-y} e^{-y}, \]

\[ \int_{-\infty}^{\infty} e^{-(\tilde{\gamma}_{igt} - e^{-(\tilde{\gamma}_{igt})})} d\tilde{\gamma}_{igt} = \left[ e^{-e^{-\tilde{\gamma}_{igt}}} \right]_{-\infty}^{\infty} = [1 - 0], \]

which implies:

\[ \Phi_{it} = \sum_{g=1}^{N} e^{-\lambda_{igt}} \left( \beta E_{gt}v_{gt+1}^{w} - \kappa_{gi} + \rho (\lambda_{igt} - \bar{\gamma}) \right) e^{-(\tilde{\gamma}_{igt} + \lambda_{igt}) - e^{-(\tilde{\gamma}_{igt})}} d\tilde{\gamma}_{igt}. \]

Now note also that:

\[ \rho \bar{\gamma} = \rho \int_{-\infty}^{\infty} \tilde{\gamma}_{igt} e^{-(\tilde{\gamma}_{igt} + \lambda_{igt}) - e^{-(\tilde{\gamma}_{igt})}} d\tilde{\gamma}_{igt}. \]

Therefore:

\[ \Phi_{it} = \sum_{g=1}^{N} e^{-\lambda_{igt}} \left( \beta E_{gt}v_{gt+1}^{w} - \kappa_{gi} + \rho \lambda_{igt} \right). \]

Using the definition of \( \lambda_{igt} \), we have:

\[ \Phi_{it} = \sum_{g=1}^{N} e^{-\log \sum_{m=1}^{N} e^{-\tilde{\gamma}_{igt}}} \left( \beta E_{gt}v_{gt+1}^{w} - \kappa_{gi} + \rho \log \sum_{m=1}^{N} e^{-\tilde{\gamma}_{igt}} \right). \]
Recall that
\[ \bar{\epsilon}_{igt} \equiv \frac{\beta (E_t v_{igt+1}^w - E_t v_{mt+1}^w) - (\kappa_{gi} - \kappa_{mi})}{\rho}. \]

Therefore
\[
\left( \beta E_t v_{igt+1}^w - \kappa_{gi} + \rho \log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igt}} \right) = \left( \beta E_t v_{igt+1}^w - \kappa_{gi} + \rho \log \sum_{k=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \right)
\]
\[= \rho \log \left( \sum_{m=1}^{N} e^{\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \right), \]
\[= \rho \log \left( \sum_{m=1}^{N} e^{\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \right), \]
and
\[\sum_{g=1}^{N} e^{-\bar{\epsilon}_{igt}} \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igt}} = \sum_{g=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \sum_{m=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \]
\[= \sum_{g=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \sum_{m=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \]
\[= \sum_{g=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \sum_{m=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \]
\[= 1. \]

Therefore, we have:
\[\Phi_{it} = \max_{(g)} \left\{ \beta E_t E_e [v_{igt+1}^w] - \kappa_{gi} + \rho \epsilon_{gt} \right\} = \rho \log \left( \sum_{g=1}^{N} e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \right). \]

Using this result in equation (B.1), we obtain the expression for expected utility in equation (B.3) above:
\[v_{it}^w = \ln \left( \frac{W_{it}}{p_{it}} \right) + \ln b_{it} + \rho \log \sum_{g=1}^{N} \left( e^{-\frac{\beta E_t v_{igt+1}^w - \kappa_{gi} - \kappa_{mi}}{\rho}} \right)^{1/\rho}. \]

**B.13 Derivation of Outmigration Probabilities**

In this subsection, we derive the outmigration probabilities \(D_{igt}\) in equation (B.4) above. The probability that a worker migrates from location \(i\) to location \(g\) at the end of period \(t\) is given by:
\[D_{igt} = \text{Prob} \left[ \frac{\beta E_t v_{igt+1}^w - \kappa_{gi}}{\rho} + \epsilon_{gt} \geq \max_{m \neq g} \left\{ \frac{\beta E_t v_{igt+1}^w - \kappa_{mi}}{\rho} + \epsilon_{mt} \right\} \right], \]
\[ D_{igt} = \text{Prob} \left[ \beta \left( \mathbb{E}t_{igt}^{w} - \mathbb{E}t_{igt}^{w} \right) - (\kappa_{gi} - \kappa_{mi}) \over \rho \right] + \epsilon_{gt} \geq \max_{m \neq g} \{ \epsilon_{mt} \} . \]

Therefore this outmigration probability can be written as:

\[ D_{igt} = \int_{-\infty}^{\infty} f(\epsilon_{gt}) \prod_{m \neq g} F \left( \beta \left( \mathbb{E}t_{igt}^{w} - \mathbb{E}t_{igt}^{w} \right) - (\kappa_{gi} - \kappa_{mi}) \over \rho \right) + \epsilon_{gt} \right) \, d\epsilon_{gt}. \]

Using our extreme value distributional assumption and the definition of \( \bar{\epsilon}_{igm} \) in the previous subsection, we can write this as:

\[ D_{igt} = \int_{-\infty}^{\infty} e^{(-\epsilon_{gt} - \bar{\gamma})} e^{(-\epsilon_{igt} - \bar{\gamma})} \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igm}} \, d\epsilon_{gt}. \]

Recall from the previous subsection the following definitions:

\[ \lambda_{igt} \equiv \log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igm}}, \]

\[ e^{\lambda_{igt}} = \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igm}}, \]

\[ \zeta_{gt} \equiv \epsilon_{gt} + \bar{\gamma}. \]

Using these definitions, our outmigration probability can be written as follows:

\[ D_{igt} = \int_{-\infty}^{\infty} e^{-\zeta_{gt}} e^{-\zeta_{gt} - \lambda_{igt}} e^{\lambda_{igt}} \, d\zeta_{gt}, \]

Now recall the following additional definition from the previous subsection:

\[ \tilde{y}_{igt} \equiv \zeta_{gt} - \lambda_{igt}. \]

\[ D_{igt} = \int_{-\infty}^{\infty} e^{-(\tilde{y}_{igt} + \lambda_{igt})} e^{-\tilde{y}_{igt} - \lambda_{igt}} e^{\lambda_{igt}} \, d\tilde{y}_{igt}, \]

\[ D_{igt} = e^{-\lambda_{igt}} \int_{-\infty}^{\infty} e^{-\tilde{y}_{igt}} e^{-\tilde{y}_{igt}} \, d\tilde{y}_{igt}, \]

\[ D_{igt} = e^{-\lambda_{igt}} \int_{-\infty}^{\infty} e^{-\tilde{y}_{igt} - \tilde{y}_{igt}} \, d\tilde{y}_{igt}, \]

Recall that:

\[ \int_{-\infty}^{\infty} e^{(-\tilde{y}_{igt} - \tilde{y}_{igt})} \, d\tilde{y}_{igt} = \left[ e^{-\tilde{y}_{igt}} \right]_{-\infty}^{\infty} = [1 - 0]. \]

Therefore we have

\[ D_{igt} = e^{-\lambda_{igt}}. \]
Recall
\[ \lambda_{igt} \equiv \log \sum_{m=1}^{N} e^{-\xi_{igm}}. \]

Therefore
\[ D_{igt} = e^{-\log \sum_{m=1}^{N} e^{-\xi_{igm}}}, \]

Recall
\[ \bar{\xi}_{igm} \equiv \beta \left( \mathbb{E}_t v_{gt+1} - \mathbb{E}_t v_{mt+1} \right) - \left( \kappa_{gi} - \kappa_{mi} \right). \]

Therefore
\[ D_{igt} = e^{-\log \left[ \sum_{m=1}^{N} e^{\beta \left( \mathbb{E}_t v_{gt+1} - \mathbb{E}_t v_{mt+1} \right) - \left( \kappa_{gi} - \kappa_{mi} \right) / \rho} \right]}, \]
\[ D_{igt} = e^{-\log \left[ \sum_{m=1}^{N} e^{-\left( \mathbb{E}_t v_{gt+1} - \kappa_{gi} \right) / \rho} \sum_{m=1}^{N} e^{-\left( \mathbb{E}_t v_{mt+1} - \kappa_{mi} \right) / \rho} \right]}, \]
\[ D_{igt} = e^{-\log \left[ \sum_{m=1}^{N} e^{-\left( \mathbb{E}_t v_{gt+1} - \kappa_{gi} \right) / \rho} \sum_{m=1}^{N} e^{-\left( \mathbb{E}_t v_{mt+1} - \kappa_{mi} \right) / \rho} \right]}, \]
\[ D_{igt} = e^{\log \left[ \sum_{m=1}^{N} e^{-\left( \mathbb{E}_t v_{gt+1} - \kappa_{gi} \right) / \rho} \sum_{m=1}^{N} e^{-\left( \mathbb{E}_t v_{mt+1} - \kappa_{mi} \right) / \rho} \right]}, \]
\[ D_{igt} = \frac{\exp \left( \beta \mathbb{E}_t v_{gt+1} / \kappa_{gi} \right) / \rho}{\sum_{m=1}^{N} \exp \left( \beta \mathbb{E}_t v_{mt+1} / \kappa_{mi} \right) / \rho}. \]

which yields equation (B.4) above:
\[ D_{igt} = \frac{\exp \left( \beta \mathbb{E}_t v_{gt+1} / \kappa_{gi} \right) / \rho}{\sum_{m=1}^{N} \exp \left( \beta \mathbb{E}_t v_{mt+1} / \kappa_{mi} \right) / \rho}. \]

C Isomorphisms

In Section 2 of the paper, we derive our baseline results using the Armington (1969) model of trade, in which goods are differentiated by location. In this section of the online appendix, we show that our results also hold in the class of trade models with a constant trade elasticity considered by Arkolakis et al. (2012), henceforth ACR.

In Section C.1, we consider the Ricardian model of trade based on technology differences of Eaton and Kortum (2002), in which markets are perfectly competitive and production technologies are constant returns to scale. In Section C.2 we consider the new trade theory model of Krugman (1980), in which markets are monopolistically competitive and production technologies are increasing return to scale. Although for simplicity we assume a representative firm in Section C.2, analogous results also hold in the heterogeneous firm model of Melitz (2003) with an untruncated Pareto productivity distribution.
In Section C.1, the goods market clearing condition in the Eaton and Kortum (2002) model takes exactly the same form as in our Armington (1969) model in Section 2 of the paper. Combining this goods market clearing condition with our specifications of migration decisions and capital accumulation, we obtain the same system of equations for general equilibrium as in Section 2 of the paper. The only difference is that the trade elasticity (θ) in the Eaton Kortum (2002) depends on the shape parameter of the Fréchet productivity distribution rather than the elasticity of substitution between varieties.

In Section C.2, the presence of love of variety, increasing returns and transport costs in the Krugman (1980) model gives rise to agglomeration forces. As a result, the goods market clearing condition takes a similar form as in the extension of our baseline Armington (1969) model to incorporate agglomeration forces. Combining this goods market clearing condition with our specifications of migration decisions and capital accumulation, we obtain a similar system of equations for general equilibrium as in the extension of our baseline Armington (1969) model to incorporate agglomeration forces.

C.1  Ricardian Technology Differences

We consider a version of Eaton and Kortum (2002) with labor and capital as the two factors of production. Migration and capital accumulation are modelled in the same way as in Section 2 of the paper. The only difference from our baseline Armington model in that section of the paper is the specification of preferences and production.

C.1.1  Preferences

Workers’ indirect utility function in location n at time t is assumed to take the following form:

\[ \ln u_{nt} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}, \]

where \( b_{nt} \) are amenities; \( w_{nt} \) is the wage; and \( p_{nt} \) is the consumption goods price index. Landowners’ indirect utility function takes the same form, but their income depends on the rental rate for capital (\( r_{nt} \)) rather than the wage (\( w_{nt} \)). The consumption goods price index (\( p_{nt} \)) is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

\[ p_{nt} = \left[ \int_0^1 p_{nt}(\vartheta)^{1-\sigma} \, d\vartheta \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1, \]

where \( p_{nt}(\vartheta) \) denotes the price of good \( \vartheta \) in location n.
C.1.2 Production

Goods are produced with labor and capital according to a constant returns to scale production technology. These goods can be traded between locations subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from location $i$ to location $n$ in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the price for consumers in location $n$ of purchasing a good $\vartheta$ from location $i$ is:

$$p_{nit}(\vartheta) = \frac{\tau_{nit} w_{it}^\mu r_{it}^{1-\mu}}{z_{it} a_i(\vartheta)}, \quad 0 < \mu < 1,$$

where $z_{it}$ captures common determinants of productivity across goods in location $i$ and $a_i(\vartheta)$ captures idiosyncratic determinants of productivity for each good $\vartheta$ within that location. Productivity for each good $\vartheta$ in each location $i$ is drawn independently from the following Fréchet distribution:

$$F_i(a) = \exp \left( -a^{-\theta} \right), \quad \theta > 1,$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to $z_{it}$. Using the properties of this Fréchet distribution, location $n$’s share of expenditure on goods produced in location $i$ is:

$$s_{nit} = \frac{\left( \tau_{nit} w_{it}^\mu r_{it}^{1-\mu} / z_{it} \right)^{-\theta}}{\sum_{m=1}^N \left( \tau_{nmt} w_{mt}^\mu r_{mt}^{1-\mu} / z_{mt} \right)^{-\theta}},$$

and location $n$’s price index can be expressed as:

$$p_{nt} = \left[ \sum_{m=1}^N \left( \tau_{nmt} w_{mt}^\mu r_{mt}^{1-\mu} / z_{mt} \right)^{-\theta} \right]^{-\frac{1}{\theta}}.$$

C.1.3 Market Clearing

Goods market clearing implies that income in each location, which equals the sum of the income of workers and landlords, is equal to expenditure on the goods produced by that location:

$$(w_{it} \ell_{it} + r_{it} k_{it}) = \sum_{n=1}^N S_{nit} (w_{nt} \ell_{nt} + r_{nt} k_{nt}).$$

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords’ income from the ownership of capital equals payments for its use. Using profit maximization and zero profits, this capital market clearing condition can be expressed as follows:

$$r_{it} k_{it} = \frac{1 - \mu}{\mu} w_{it} \ell_{it}.$$
C.1.4 General Equilibrium

Given the state variables \( \{i_0, k_0\} \), the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \( \{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty} \). All other endogenous variables of the model can be recovered as a function of these variables. We now show that the system of equations for general equilibrium in this version of the Eaton and Kortum (2002) model takes exactly the same form as in our baseline Armington model.

**Capital Accumulation:** Using capital market clearing (C.7), the price index (C.5) and the analogous derivations for landlords’ consumption–investment decision as in our baseline Armington model, the capital accumulation equation can be expressed as:

\[
k_{it+1} = \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1 - \delta) k_{it},
\]

where for simplicity we assume logarithmic intertemporal utility.

**Goods Market Clearing:** Using the expenditure share (C.4) and capital market clearing (C.7) in the goods market clearing condition (C.6), we obtain:

\[
S_{nit} = \frac{(w_{it} (\ell_{it}/k_{it})^{1-\mu} \tau_{ni} / z_{i})^{-\theta}}{\sum_{m=1}^{N} (w_{mt} (\ell_{mt}/k_{mt})^{1-\mu} \tau_{nm} / z_{m})^{-\theta}},
\]

\[
T_{int} = \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},
\]

where \( S_{nit} \) is the expenditure share of importer \( n \) on exporter \( i \) at time \( t \); we have defined \( T_{int} \) as the corresponding income share of exporter \( i \) from importer \( n \) at time \( t \); and note that the order of subscripts switches between the expenditure share (\( S_{nit} \)) and the income share (\( T_{int} \)), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.
**Population Flow:** Using the analogous derivations for migration decisions as in our baseline Armington model, the population flow condition for the evolution of the population distribution over time is given by:

\[ \ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \]  

(C.12)

\[ D_{igt} = \frac{\exp \left( \frac{\beta E_{it} v_{gt+1}^w}{\kappa_{igt}} \right)^{1/\rho}}{\sum_{m=1}^{N} \exp \left( \frac{\beta E_{im} v_{mt+1}^w}{\kappa_{mit}} \right)^{1/\rho}}, \quad E_{git} = \frac{\ell_{it} D_{igt}}{\ell_{gt+1}}, \]  

(C.13)

where \( D_{igt} \) is the outmigration probability from location \( i \) to location \( g \) between time \( t \) and \( t+1 \); we have defined \( E_{git} \) as the corresponding immigration probability to location \( g \) from location \( i \) between time \( t \) and \( t+1 \); and again note that the order of subscripts switches between the outmigration probability (\( D_{igt} \)) and the immigration probability (\( E_{git} \)), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Worker Value Function:** Using the analogous derivations for migration decisions as in our baseline Armington model, the expected value from living in location \( n \) at time \( t \) can be written as:

\[ v_{nt}^w = \ln b_{nt} + \ln \left( \frac{w_{nt}}{p_{nt}} \right) + \rho \ln \sum_{g=1}^{N} \left( \frac{\exp \left( \beta E_{it} v_{gt+1}^w \right)}{\kappa_{gnt}} \right)^{1/\rho}. \]  

(C.14)

**C.2 New Trade Model**

We consider a version of Krugman (1980) with labor and capital as the two factors of production. Migration and capital accumulation are modelled in the same way as in Section 2 of the paper. The only difference from our baseline Armington model in that section of the paper is the specification of preferences and production. We now show that the system of equations for general equilibrium in this version of the Krugman (1980) model takes a similar form as in the extension of our baseline Armington model with agglomeration economies.

**C.2.1 Preferences**

Workers’ indirect period utility function in location \( n \) at time \( t \) is assumed to take the following form:

\[ \ln u_{nt} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}, \]  

(C.15)

where \( b_{nt} \) are amenities; \( w_{nt} \) is the wage; and \( p_{nt} \) is the consumption goods price index. Landowners’ indirect utility function takes the same form, but their income depends on the rental rate for
capital \((r_{nt})\) rather than the wage \((w_{nt})\). The consumption goods price index \((p_{nt})\) is defined over the consumption of a mass of varieties \((M_{it})\) from each location \(i\) according to the constant elasticity of substitution (CES) functional form:

\[
p_{nt} = \left[ \sum_{i=1}^N \int_0^{M_{it}} p_{nit} (j)^{1-\sigma} dj \right]^{1/\sigma}, \quad \sigma > 1, \tag{C.16}
\]

where \(p_{nit} (j)\) is the price in country \(n\) of a variety \(j\) produced in country \(i\) at time \(t\); the mass of varieties \((M_{it})\) is endogenously determined by free entry; and varieties are substitutes \((\sigma > 1)\).

### C.2.2 Production

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost \((F)\) and a constant marginal cost that depends on a location’s productivity \((z_{it})\). The production technology is assumed to be homothetic, such that the fixed and marginal cost use labor and capital with the same intensity. In particular, the total cost of producing \(x_{it} (j)\) units of variety \(j\) in location \(i\) is given by:

\[
\varpi_{i} (j) = F + \frac{x_{it} (j)}{z_{it}} \mu w_{it} \ell_{it}^{1-\mu}, \quad 0 < \mu < 1. \tag{C.17}
\]

Varieties can be traded between countries subject to iceberg variable costs of trade, such that \(\tau_{ni} \geq 1\) units must be shipped from country \(i\) to country \(n\) in order for one unit to arrive (where \(\tau_{ni} > 1\) for \(n \neq i\) and \(\tau_{nn} = 1\)). The cost to the consumer in location \(n\) of sourcing a variety from location \(i\) is thus:

\[
p_{nit} (j) = \tau_{nit} p_{nit} (j), \tag{C.18}
\]

where \(p_{nit} (j)\) is the “free on board” price before transport costs. Profit maximization and zero profits imply that this free on board price is a constant markup over marginal cost:

\[
p_{it} (j) = \bar{p}_{it} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{w_{it} \mu \ell_{it}^{1-\mu}}{z_{it}}, \tag{C.19}
\]

and equilibrium variety output is equal to a constant that depends on location productivity:

\[
x_{it} (j) = \bar{x}_{it} = z_{it} (\sigma - 1) F. \tag{C.20}
\]

Multiplying equilibrium prices and output, variety revenue is given by:

\[
\bar{y}_{it} = \bar{p}_{it} \bar{x}_{it} = \sigma F w_{it} \mu \ell_{it}^{1-\mu}.
\]

Additionally, cost minimization implies that capital payments are a constant multiple of labor payments:

\[
r_{it} k_{it} = \frac{1 - \mu}{\mu} w_{it} \ell_{it}. \tag{C.21}
\]
Using this implication of cost minimization, variety revenue can be re-written as:

\[ y_{it} = \sigma F w_{it} \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \frac{1}{x_{it}} \right)^{1-\mu}. \]  

(C.22)

The mass of varieties in each location equals aggregate revenue divided by variety variety:

\[ M_{it} = \frac{r_{it} k_{it} + w_{it} \ell_{it}}{y_{it}}. \]

Using the constant relationship between capital payments and labor payments (C.21) and the expression for variety revenue (C.22), the mass of varieties can be expressed as:

\[ M_{it} = \frac{\ell_{it}}{\sigma F \lambda \left( \frac{1 - \mu}{\mu} \right)^{1-\mu}}. \]  

(C.23)

Using the properties of CES demand, country \( n \)'s share of expenditure on goods produced in country \( i \) is:

\[ s_{ni} = \frac{M_{it} P_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_{mt} P_{nm}^{1-\sigma}}. \]

Using equilibrium prices in equations (C.18) and (C.19) and the mass of varieties (C.23), we can re-write this expenditure share as:

\[ s_{ni} = \frac{\ell_{it} (\chi_{it})^{1-\mu} \left( \tau_{nit} w_{it}^\mu r_{it}^1 / z_{it} \right)^{1-\sigma}}{\sum_{m=1}^{N} \ell_{mt} (\chi_{mt})^{1-\mu} \left( \tau_{nmt} w_{mt}^\mu r_{mt}^1 / z_{mt} \right)^{1-\sigma}}, \]  

(C.24)

and the price index (C.16) as:

\[ p_{nt} = \left[ \sum_{m=1}^{N} \frac{\ell_{mt} (\chi_{mt})^{1-\mu} \left( \frac{\sigma}{\sigma - 1} \tau_{nmt} w_{mt}^\mu r_{mt}^1 / z_{mt} \right)^{1-\sigma}}{\sigma F \lambda \left( \frac{1 - \mu}{\mu} \right)^{1-\mu}} \right]^{\frac{1}{1-\sigma}}. \]  

(C.25)

Using the relationship between capital and labor payments (C.21), we can further re-write the expenditure share (C.24) as:

\[ s_{ni} = \frac{\ell_{it} (\chi_{it})^{\sigma(1-\mu)} \left( \tau_{nit} w_{it}^\mu / z_{it} \right)^{1-\sigma}}{\sum_{m=1}^{N} \ell_{mt} (\chi_{mt})^{\sigma(1-\mu)} \left( \tau_{nmt} w_{mt}^\mu / z_{mt} \right)^{1-\sigma}}, \]  

(C.26)

and the price index (C.25) as:

\[ p_{nt} = \left[ \sum_{m=1}^{N} \frac{(\sigma - 1) \ell_{mt} (\chi_{mt})^{\sigma(1-\mu)} \left( \tau_{nmt} w_{mt}^\mu \left( \frac{1 - \mu}{\mu} \right)^{1-\sigma} / z_{mt} \right)^{1-\sigma}}{\sigma F \lambda \left( \frac{1 - \mu}{\mu} \right)^{1-\mu}} \right]^{\frac{1}{1-\sigma}}. \]  

(C.27)
C.2.3 Market Clearing

Goods market clearing implies that income in each location, which equals the sum of the income of workers and landlords, is equal to expenditure on the goods produced by that location:

\[
(w_{it} \ell_{it} + r_{it} k_{it}) = \sum_{n=1}^{N} S_{nit} (w_{nt} \ell_{nt} + r_{nt} k_{nt}).
\]  
(C.28)

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords’ income from the ownership of capital equals payments for its use. Using profit maximization and zero profits, this capital market clearing condition can be expressed as in equation (C.21) above.

C.2.4 General Equilibrium

Given the state variables \{\ell_{i0}, k_{i0}\}, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}. All other endogenous variables of the model can be recovered as a function of these variables. We now show that the system of equations for general equilibrium in this version of the Eaton and Kortum (2002) model takes a similar form as in the extension of our baseline Armington model with agglomeration economies.

Capital Accumulation: Using capital market clearing (C.21), the price index (C.27) and the analogous derivations for landlords’ consumption-investment decision as in our baseline Armington model, the capital accumulation equation can be expressed as:

\[
k_{it+1} = \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1 - \delta) k_{it},
\]
(C.29)

\[
\left[ \sum_{m=1}^{N} \frac{(\sigma - 1)}{\sigma - 1} \frac{1 - \sigma}{1 - \mu} \frac{\ell_{mt} (\lambda_{mt})^{\sigma(1 - \mu)}}{\sigma F \lambda_{mt} (1 - \mu)} \left( \tau_{nmt} w_{mt} \left( \frac{1 - \mu}{\mu} \right)^{1 - \mu} / z_{mt} \right)^{1 - \sigma} \right]^{1/\sigma},
\]
(C.30)

where for simplicity we assume logarithmic intertemporal utility.

Goods Market Clearing: Using the expenditure share (C.26) and capital market clearing (C.21) in the goods market clearing condition (C.28), we obtain:
where $S_{nit}$ is the expenditure share of importer $n$ on exporter $i$ at time $t$; we have defined $T_{int}$ as the corresponding income share of exporter $i$ from importer $n$ at time $t$; and note that the order of subscripts switches between the expenditure share ($S_{nit}$) and the income share ($T_{int}$), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Population Flow:** Using the analogous derivations for migration decisions as in our baseline Armington model, the population flow condition for the evolution of the population distribution over time is given by:

$$
\ell_{g_{t+1}} = \sum_{i=1}^{N} D_{igt} \ell_{it},
$$

$$
D_{igt} = \frac{\left(\exp \left(\beta T_{it} v_{nt}^{w} / \kappa_{nit}\right) / \kappa_{nit}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp \left(\beta T_{it} v_{mt}^{w} / \kappa_{nit}\right) / \kappa_{mit}\right)^{1/\rho}},
$$

where $D_{igt}$ is the outmigration probability from location $i$ to location $g$ between time $t$ and $t+1$; we have defined $E_{git}$ as the corresponding immigration probability to location $g$ from location $i$ between time $t$ and $t+1$; and again note that the order of subscripts switches between the outmigration probability ($D_{igt}$) and the immigration probability ($E_{git}$), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Worker Value Function:** Using the analogous derivations for migration decisions as in our baseline Armington model, the expected value from living in location $n$ at time $t$ can be written as:

$$
v_{nt}^{w} = \ln b_{nt} + \ln \left(\frac{w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp \left(\beta T_{it} v_{gt}^{w} / \kappa_{git}\right) / \kappa_{git}\right)^{1/\rho}.
$$

## D Extensions

In this section of the online appendix, we consider a number of extensions of our baseline specification from Section 2 of the paper and Section B of this online appendix. In Subsection D.1,
we show that our results naturally generalize to accommodate shocks to trade and migration frictions, in addition to shocks to productivity and amenities.

In Subsection D.2, we allow for agglomeration and dispersion forces, such that both productivity and amenities are endogenous to the surrounding concentration of economic activity. In Subsection D.3 we introduce multiple final goods sectors with region-specific capital. In Section D.4, we incorporate multiple final goods sectors with region-sector-specific capital. In Section D.5, we further generalizes the analysis to allow for multiple final goods sectors with region-sector-specific capital and input-output linkages.

In Subsection D.6, we generalize our baseline specification to allow for trade deficits, following the standard approach in the quantitative international trade literature of treating these trade deficits as exogenous. Finally, in Subsection D.7, we allow capital to be used residentially (housing) as well as commercially.

D.1 Shocks to Trade and Migration Costs

In this Subsection, we derive sufficient statistics for changes in steady-states and the transition path, allowing for shocks to trade and migration costs, as well as to productivity and amenities, as discussed in Section 4.1 of the paper. In the interests of brevity, we focus on the case in which the economy starts from a steady-state, for which we observe the trade and migration share matrices (S, T, D, E). For simplicity, we also assume logarithmic intertemporal utility. We derive sufficient statistics for changes in steady-states and the transition path in response to small changes in productivities (d ln z), amenities (d ln b), trade costs (d ln τ) and migration costs (d ln κ), using the observed trade and migration matrices from the initial steady-state.

D.1.1 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1} = k_{it} = k_i^*, \ell_{it+1} = \ell_{it} = \ell_i^*, w_{it+1} = w_{it} = w_i^*$ and $v_{it+1} = v_{it} = v_i^*$, where we use an asterisk to denote a steady-state value.

**Capital Accumulation.** From the capital accumulation equation (B.24), the steady-state stock of capital solves:

$$\left(1 - \beta (1 - \delta)\right) \chi_i^* = \left(1 - \beta (1 - \delta)\right) \frac{k_i^*}{\ell_i^*} = \beta \frac{1 - \mu}{\mu} \frac{w_i^*}{p_i^*}.$$

Totally differentiating, we have:

$$d \ln \chi_i^* = \ln \left( \frac{w_i^*}{p_i^*} \right).$$
Using the total derivative of real income (B.53) above, this becomes:

\[
d\ln \chi_i^* = d\ln w_i^* - \sum_{m=1}^N S_{im}^* \left[ d\ln \tau_{im} + d\ln w_m^* - (1 - \mu) \ d\ln \chi_m^* - d\ln z_m \right],
\]

which can be re-written as:

\[
d\ln \chi_i^* = d\ln w_i^* - \sum_{m=1}^N S_{im}^* \left[ d\ln w_m^* - (1 - \mu) \ d\ln \chi_m^* - d\ln z_m \right] - d\ln \tau_{im},
\]

where \(d\ln \tau_{im}\) is a measure of weighted-average incoming trade costs defined as:

\[
d\ln \tau_{im}^\text{in} \equiv \sum_{m=1}^N S_{im}^* \ d\ln \tau_{im}.
\]

This relationship has the matrix representation:

\[
(I - (1 - \mu) S) \ d\ln \chi^* = (I - S) \ d\ln w^* + S \ d\ln z - d\ln \tau^\text{in},
\]

(D.1)

**Goods Market Clearing.** The total derivative of the goods market clearing condition (B.55) can be re-written as:

\[
\begin{bmatrix}
d\ln w_{it} \\
+ d\ln \ell_{it}
\end{bmatrix} = \begin{bmatrix}
+ \theta \sum_{n=1}^N \sum_{m=1}^N T_{itn} (d\ln w_{nt} + d\ln \ell_{nt}) \\
- \theta \sum_{n=1}^N T_{int} (d\ln w_{it} - (1 - \mu) \ d\ln \chi_{it} - d\ln z_{it}) \\
+ \theta \sum_{n=1}^N T_{int} \ d\ln \tau_{it}^\text{in} - \theta \ d\ln \tau_{it}^\text{out}
\end{bmatrix},
\]

where \(d\ln \tau_{it}^\text{in}\) is defined above and \(d\ln \tau_{it}^\text{out}\) is defined as:

\[
d\ln \tau_{it}^\text{out} \equiv \sum_{n=1}^N T_{int} \ d\ln \tau_{nit}.
\]

This relationship above has the following matrix representation:

\[
\begin{bmatrix}
d\ln w_t + d\ln \ell_t = T (d\ln w_t + d\ln \ell_t) + \theta \left[ (TS - I) (d\ln w_t - (1 - \mu) \ d\ln \chi_t - d\ln z_t) + T d\ln \tau_t^\text{in} - d\ln \tau_t^\text{out} \right].
\end{bmatrix}
\]

We can re-write this relationship as:

\[
[I - T + \theta (I - TS)] \ d\ln w_t = - (I - T) d\ln \ell_t + \theta (I - TS) (d\ln z_t + (1 - \mu) \ d\ln \chi_t) + \theta [T d\ln \tau_t^\text{in} - d\ln \tau_t^\text{out}].
\]

In steady-state we have:

\[
[I - T + \theta (I - TS)] \ d\ln w^* = - (I - T) d\ln \ell^* + \theta (I - TS) (d\ln z + (1 - \mu) \ d\ln \chi^*) + \theta [T d\ln \tau^\text{in} - d\ln \tau^\text{out}].
\]

(D.2)
Population Flow. The total derivative of the population flow condition (B.56) can be re-written as:

\[ \frac{d}{dt} \ln \ell_{t+1} = \sum_{i=1}^{N} E_{git} \ln \ell_{it} + \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \beta \mathbb{E}_t dv_{gt+1} - \frac{1}{\rho} \sum_{i=1}^{N} E_{git} d \ln \kappa_{git} \]

\[ - \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \sum_{m=1}^{N} D_{imt} \beta \mathbb{E}_t dv_{mt+1} + \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \sum_{m=1}^{N} D_{imt} d \ln \kappa_{mit}, \]

and hence:

\[ \frac{d}{dt} \ln \ell_{t+1} = \sum_{i=1}^{N} E_{git} \ln \ell_{it} + \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \beta \mathbb{E}_t dv_{gt+1} - \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \ln \kappa_{in} \]

\[ - \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \sum_{m=1}^{N} D_{imt} \beta \mathbb{E}_t dv_{mt+1} + \frac{1}{\rho} \sum_{i=1}^{N} E_{git} \ln \kappa_{out}, \]

where we have defined:

\[ d \ln \kappa_{in} = \sum_{i=1}^{N} E_{git} d \ln \kappa_{git}, \]

\[ d \ln \kappa_{out} = \sum_{m=1}^{N} D_{imt} d \ln \kappa_{mit}. \]

This total derivative has the following matrix representation:

\[ \frac{d}{dt} \ln \ell_{t+1} = E \frac{d}{dt} \ln \ell_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t dv_{t+1} - \frac{1}{\rho} \left( d \ln \kappa_{in} - E d \ln \kappa_{out} \right). \]

In steady-state, we have:

\[ \frac{d}{dt} \ln \ell^* = E \frac{d}{dt} \ln \ell^* + \frac{\beta}{\rho} (I - ED) \frac{dv^*}{dt} - \frac{1}{\rho} \left( d \ln \kappa_{in} - E d \ln \kappa_{out} \right). \] (D.3)

Value function. The total derivative of the value function (B.58) can be re-written as:

\[ dv_{it} = \left[ \frac{d}{dt} w_{it} - \sum_{m=1}^{N} S_{imt} \left( \frac{d}{dt} w_{mt} - (1 - \mu) \frac{d}{dt} \chi_{mt} - \frac{d}{dt} \tau_{in} - d \ln \tau_{it} \right) + \frac{d}{dt} b_{it} + \sum_{m=1}^{N} D_{imt} \left( \beta \mathbb{E}_t dv_{mt+1} - d \ln \kappa_{out} \right) \right]. \]

where \( d \ln \tau_{in} \) and \( d \ln \kappa_{out} \) are defined above. The above relationship has the following matrix representation:

\[ dv_t = \left[ (I - S) \frac{d}{dt} w_t + S \left( \frac{d}{dt} z_t + (1 - \mu) \frac{d}{dt} \chi_t - \frac{d}{dt} \tau_{in} \right) + \frac{d}{dt} b_t - d \ln \kappa_{out} + \beta \mathbb{E}_t dv_{t+1} \right]. \]

In steady-state, we have:

\[ dv^* = \left[ (I - S) \frac{d}{dt} w^* + S \left( \frac{d}{dt} z + (1 - \mu) \frac{d}{dt} \chi^* - \frac{d}{dt} \tau_{in} \right) + \frac{d}{dt} b - d \ln \kappa_{out} + \beta \mathbb{E} dv^* \right]. \] (D.4)
System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

\[
\begin{align*}
\frac{d \ln \chi^*}{dt} &= [I - (1 - \mu) S]^{-1} \left[ (I - S) \frac{d \ln w^*}{dt} + S \frac{d \ln z}{dt} - \frac{d \ln \tau^z}{dt} \right], \quad (D.5) \\
\frac{d \ln w^*}{dt} &= [I - T + \theta (I - TS)]^{-1} \left[ \left( - (I - T) \frac{d \ln \ell^*}{dt} + (I - TS) \theta \left( \frac{d \ln z + (1 - \mu) \frac{d \ln \chi^*}{dt}}{\lambda \frac{d \ln \tau^z}{dt}} \right) \right) \right]. \quad (D.6) \\
\frac{d \ln \ell^*}{dt} &= (I - E)^{-1} \left[ \frac{\beta}{\rho} (I - ED) \frac{d \nu^*}{dt} - \frac{1}{\rho} \left( \frac{d \ln \kappa^z - E \frac{d \ln \kappa^{out}}{dt}}{\lambda \frac{d \ln \tau^z}{dt}} \right) \right]. \quad (D.7) \\
\frac{d \nu^*}{dt} &= [I - \beta D]^{-1} \left[ (I - S) \frac{d \ln w^*}{dt} + S \left( \frac{d \ln z + (1 - \mu) \frac{d \ln \chi^*}{dt}}{\lambda \frac{d \ln \tau^z}{dt}} \right) - \frac{d \ln \tau^z}{dt} \right]. \quad (D.8)
\end{align*}
\]

As the expenditure shares (\(S\)) and income shares (\(T\)) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire (\(\sum_{i=1}^{N} q_i^* = \bar{q} = 1\)), which implies that the log changes in incomes satisfy \(Q^* \frac{d \ln q^*}{dt} = \sum_{i=1}^{N} q_i^* \frac{d \ln q_i^*}{dt} = \sum_{i=1}^{N} q_i^* \frac{dq_i^*}{q_i^*} = \sum_{i=1}^{N} dq_i^* = 0\), where \(Q\) is a row vector of the income of each location. Similarly, the outmigration shares (\(D\)) and immigration shares (\(E\)) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: \(\sum_{i=1}^{N} \ell_i^* = \bar{\ell} = 1\), which implies \(L^* \frac{d \ln \ell^*}{dt} = \sum_{i=1}^{N} \ell_i^* \frac{d \ln \ell_i^*}{dt} = \sum_{i=1}^{N} \ell_i^* \frac{d \ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} d \ell_i^* = 0\), where \(L\) is a row vector of the population of each location.

D.1.2 Sufficient Statistics for Transition Dynamics Starting from Steady-State

We suppose that the economy starts from an initial steady-state distribution of economic activity \(\{k_i^*, \ell_i^*, w_i^*, v_i^*\}\). We consider small shocks to productivity (\(d \ln z\)), amenities (\(d \ln b\)), trade costs (\(d \ln \tau\)) and commuting costs (\(d \ln \kappa\)), holding constant the economy’s aggregate labor endowment (\(d \ln \bar{\ell} = 0\)). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that \(\tilde{\chi}_{it+1} = \ln \chi_{it+1} - \ln \chi_i^*\), for all variables except for the worker value function \(v_{it}\); with a slight abuse of notation we use \(\tilde{v}_{it} \equiv v_{it} - v_i^*\) to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (B.24), we have:

\[
\begin{align*}
\frac{k_{it+1}}{k_{it}} &= \beta (1 - \delta) \frac{k_{it}}{k_{it-1}} + \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}} \frac{\ell_{it}}{\ell_{it-1}}, \\
\frac{k_{it+1}}{\ell_{it+1}} &= \beta (1 - \delta) \frac{k_{it}}{\ell_{it}} + \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}}, \\
\frac{\chi_{it+1}}{\ell_{it+1}} &= \beta (1 - \delta) \frac{\chi_{it}}{\ell_{it}} + \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}}. \quad (D.9)
\end{align*}
\]
while in steady-state we have:

\[
\frac{k^*}{\ell^*} = \beta (1 - \delta) \frac{k^*_i}{\ell^*_i} + \beta \frac{1 - \mu w_i^*}{\mu p_i^*},
\]

\[
\chi^*_i = \beta (1 - \delta) \chi^*_i + \beta \frac{1 - \mu w_i^*}{\mu p_i^*},
\]

\[
\chi^*_i = \frac{\beta}{(1 - \beta (1 - \delta))} \frac{1 - \mu w_i^*}{\mu p_i^*}.
\]  

(D.10)

Dividing both sides of equation (D.9) by \( \chi^*_i \), we have:

\[
\frac{\chi_{it+1} \ell_{it+1}}{\chi_i^* \ell_{it}} = \beta (1 - \delta) \frac{\chi_{it}}{\chi_i^*} + \frac{1 - \mu w_{it}}{p_{it}} - 1,
\]

which using (D.10) can be re-written as:

\[
\frac{\chi_{it+1} \ell_{it+1}}{\chi_i^* \ell_{it}} - 1 = \beta (1 - \delta) \left( \frac{\chi_{it}}{\chi_i^*} - 1 \right) + \frac{1 - \mu w_{it}}{p_{it}} - 1.
\]

Noting that:

\[
\frac{x_{it}}{x_i^*} - 1 \simeq \ln \left( \frac{x_{it}}{x_i^*} \right),
\]

\[
\frac{\chi_{it+1} \ell_{it+1}}{\chi_i^* \ell_{it}} - 1 \simeq \ln \left( \frac{\chi_{it+1} \ell_{it+1}^*}{\chi_i^* \ell_{it}^*} \right),
\]

we have:

\[
\ln \left( \frac{\chi_{it+1}}{\chi_i^*} \right) + \ln \left( \frac{\ell_{it+1}}{\ell_{it}^*} \right) = \beta (1 - \delta) \ln \left( \frac{\chi_{it}}{\chi_i^*} \right) + (1 - \beta (1 - \delta)) \ln \left( \frac{w_{it}/w_i^*}{p_{it}/p_i^*} \right),
\]

\[
\ln \left( \frac{\chi_{it+1}}{\chi_i^*} \right) + \ln \left( \frac{\ell_{it+1} \ell_{it}^*}{\ell_{it} \ell_{it}^*} \right) = \beta (1 - \delta) \ln \left( \frac{\chi_{it}}{\chi_i^*} \right) + (1 - \beta (1 - \delta)) \ln \left( \frac{w_{it}/w_i^*}{p_{it}/p_i^*} \right),
\]

which can be re-written as follows:

\[
\tilde{\chi}_{it+1} = \beta (1 - \delta) \tilde{\chi}_{it} + (1 - \beta (1 - \delta)) (\tilde{w}_{it} - \tilde{p}_{it}) - \tilde{\ell}_{it+1} + \tilde{\ell}_{it},
\]

We can re-write the above relationship for the log deviation of the capital-labor ratio from the initial steady-state as:

\[
\tilde{\chi}_{t+1} = \beta (1 - \delta) \tilde{\chi}_t + (1 - \beta (1 - \delta)) (\tilde{w}_t - \tilde{p}_t) - \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\]  

(D.11)
Taking the total derivative of real income relative to the initial steady-state, we have:

\[
\tilde{w}_{it} - \tilde{p}_{it} = \tilde{w}_{it} - \sum_{m=1}^{N} S_{int} \left[ \tilde{r}_{int} + \tilde{w}_{mt} - (1 - \mu) \tilde{\chi}_{mt} - \tilde{z}_m \right],
\]

\[
\tilde{w}_{it} - \tilde{p}_{it} = \tilde{w}_{it} - \sum_{m=1}^{N} S_{int} \left[ \tilde{w}_{mt} - (1 - \mu) \tilde{\chi}_{mt} - \tilde{z}_m \right] - \tilde{r}_{int},
\]

where

\[
\tilde{r}_{int} \equiv \sum_{m=1}^{N} S_{int} \tilde{r}_{int}.
\]

We can re-write this relationship in matrix form as:

\[
\tilde{w}_t - \tilde{p}_t = (I - S) \tilde{w}_t + (1 - \mu) S \tilde{\chi}_t + S \tilde{z} - \tilde{r}^{in}.
\]

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

\[
\tilde{\chi}_{t+1} = \left[ \beta (1 - \delta) I + (1 - \beta (1 - \delta)) (1 - \mu) S \right] \tilde{\chi}_t
+ \left( 1 - \beta (1 - \delta) \right) (I - S) \tilde{w}_t + \left( 1 - \beta (1 - \delta) \right) S \tilde{z} \\
- (1 - \beta (1 - \delta)) \tilde{r}^{in} - \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\]

(Goods Market Clearing.) The total derivative of the goods market clearing condition \((B.55)\) relative to the initial steady-state has the following matrix representation:

\[
\tilde{w}_t + \tilde{\ell}_t = \left[ T \left( \tilde{w}_t + \tilde{\ell}_t \right) + \theta (TS - I) (\tilde{w}_t - (1 - \mu) \tilde{\chi}_t - \tilde{z}) \right],
\]

where

\[
\tilde{r}_{it}^{out} \equiv \sum_{n=1}^{N} T_{int} \tilde{r}_{nit}.
\]

We can re-write this relationship as:

\[
\tilde{w}_t = \left[ I - T + \theta (I - TS) \right]^{-1} \left[ (I - T) \tilde{\ell}_t + \theta (I - TS) (\tilde{z} + (1 - \mu) \tilde{\chi}_t) \right],
\]

(Population Flow.) The total derivative of the population flow condition \((B.56)\) relative to the initial steady-state has the following matrix representation:

\[
\tilde{\ell}_{t+1} = E \tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) \tilde{v}_{t+1} - \frac{1}{\rho} (\tilde{r}^{in} - E \tilde{r}^{out}),
\]

where

\[
\tilde{r}^{in}_{git} \equiv \sum_{i=1}^{N} E_{git} \tilde{r}_{git},
\]

\[
\tilde{r}^{out}_{git} \equiv \sum_{m=1}^{N} D_{mit} \tilde{r}_{mit}.
\]
Value Function. The total derivative of the value function (B.58) relative to the initial steady-state has the following matrix representation:

\[
\tilde{v}_t = \begin{bmatrix}
(I - S) \tilde{w}_t + S \tilde{z} + (1 - \mu) S \tilde{x}_t - \tilde{\tau}^{in} \\
+ b - \tilde{\kappa}^{out} + \beta D E_t \tilde{v}_{t+1}
\end{bmatrix}.
\] (D.15)

System of Equations for Transition Dynamics Relative to the Initial Steady-State. Collecting together the capital accumulation equation (D.12), the goods market clearing condition (D.13), the population flow condition (D.14), and the value function (D.15), the system of equations for the transition dynamics relative to the initial steady-state takes the following form:

\[
\tilde{x}_{t+1} = \begin{bmatrix}
[\beta (1 - \delta) I + (1 - \beta (1 - \delta)) (1 - \mu) S] \tilde{x}_t \\
+ (1 - \beta (1 - \delta)) (I - S) \tilde{w}_t + (1 - \beta (1 - \delta)) S \tilde{z} \\
-(1 - \beta (1 - \delta)) \tilde{\tau}^{in} - \tilde{\ell}_{t+1} + \tilde{\ell}_t
\end{bmatrix}.
\] (D.16)

\[
\tilde{w}_t = [I - T + \theta (I - TS)]^{-1} \begin{bmatrix}
- (I - T) \tilde{\ell}_t + \theta (I - TS) (\tilde{z} + (1 - \mu) \tilde{x}_t)
+ \theta [T \tilde{\tau}^{in} - \tilde{\tau}^{out}]
\end{bmatrix}.
\] (D.17)

\[
\tilde{\ell}_{t+1} = E \tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1} - \frac{1}{\rho} (\tilde{\kappa}^{in} - E \tilde{\kappa}^{out}).
\] (D.18)

\[
\tilde{v}_t = \begin{bmatrix}
(I - S) \tilde{w}_t + S \tilde{z} + (1 - \mu) S \tilde{x}_t - \tilde{\tau}^{in} \\
+ b - \tilde{\kappa}^{out} + \beta D E_t \tilde{v}_{t+1}
\end{bmatrix}.
\] (D.19)

D.2 Agglomeration Forces

In this section of the online appendix, we generalize our baseline specification from Section 2 of the paper to introduce agglomeration forces. We allow productivity and amenities to both have an exogenous component, which captures locational fundamentals such as climate and access to natural water, and an endogenous component, which captures agglomeration forces, and depends on the surrounding concentration of economic activity.

D.2.1 Productivity and Amenities

We follow the standard approach in the spatial economics literature of modelling these agglomeration forces as a power function of a location’s own population: \( z_{it} = \tilde{z}_{it} \eta^z_{it} \) and \( b_{it} = \tilde{b}_{it} \eta^b_{it} \), where \( \eta^z > 0 \) and \( \eta^b > 0 \) parameterize the strength of agglomeration forces in productivity and amenities respectively.\(^6\) In this extension, the general equilibrium conditions of the model remain as in Section 2.6 above, except that the price index (10) and the expenditure share (13) are

\(^6\)While for simplicity we assume that agglomeration and dispersion forces only depend on a location’s own population, it is straightforward to also introduce spillovers across locations, as in Ahlfeldt, Redding, Sturm and Wolf (2015) and Allen, Arkolakis and Li (2020). Dispersion forces in productivity and amenities can be introduced through \( \eta^z < 0 \) and \( \eta^b < 0 \) respectively.
modified to incorporate agglomeration forces in production \( z_{it} = z_{it}^0 \), and the value function (14) is adjusted to include agglomeration forces in amenities \( b_{it} = b_{it}^0 \).

D.2.2 General Equilibrium

Given the state variables \( \{ i_0, k_{i0} \} \), the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords capitalists make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \( \{ i_t, k_{it}, w_{it}, v_{it} \} \). All other endogenous variables of the model can be recovered as a function of these variables.

**Capital Accumulation:** Using capital market clearing in equation (9) in the paper, the price index in equation (4) in the paper and the equilibrium pricing rule in equation (2) in the paper, the law of motion for capital is:

\[
k_{it+1} = \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1 - \delta) k_{it}, \tag{D.20}
\]

\[
p_{nt} = \left[ \sum_{i=1}^{N} \left( w_{it} \left( \frac{1 - \mu}{\mu} \right) \frac{\ell_{it}}{k_{it}} \right)^{1-\mu} \frac{\tau_{ni}}{\left( z_{it}^0 \right)^{1-\mu}} \right]^{-1/\theta}, \tag{D.21}
\]

where for simplicity we assume logarithmic intertemporal utility. The presence of agglomeration forces implies that the term in \( z_{it}^0 \) appears in the expression for the price index \( p_{nt} \) in equation (D.21).

**Goods Market Clearing:** Using the equilibrium pricing rule in equation (2) in the paper, the CES expenditure share, and capital market clearing in equation (9) in the paper, together with the goods market clearing condition in equation (8), we obtain:

\[
w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}, \tag{D.22}
\]

\[
S_{nit} = \frac{w_{it} \left( \frac{\ell_{it}}{k_{it}} \right)^{1-\mu} \frac{\tau_{ni}}{\left( z_{it}^0 \right)^{1-\mu}}}{\sum_{m=1}^{N} \left( w_{mt} \left( \frac{\ell_{mt}}{k_{mt}} \right)^{1-\mu} \frac{\tau_{nm}}{\left( z_{mt}^0 \right)^{1-\mu}} \right)^{-\theta}}, \quad T_{int} = \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}}, \tag{D.23}
\]

where \( S_{nit} \) is the expenditure share of importer \( n \) on exporter \( i \) at time \( t \); we have defined \( T_{int} \) as the corresponding income share of exporter \( i \) from importer \( n \) at time \( t \); and the only difference from our baseline specification in the paper is the terms in \( z_{it}^0 \) in the expenditure share \( (S_{nit}) \) in
equation (D.23). Note that the order of subscripts switches between the expenditure share \((S_{nit})\) and the income share \((T_{int})\), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Population Flow:** Using the outmigration probabilities, the population flow condition for the evolution of the population distribution over time is given by:

\[
\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it},
\]

\[
D_{igt} = \frac{\exp \left( \beta E_t v^w_{gt+1} \right) / \kappa_{git}}{\sum_{m=1}^{N} \exp \left( \beta E_t v^w_{mt+1} \right) / \kappa_{mit}}^{1/\rho},
\]

\[
E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}},
\]

where \(D_{igt}\) is the outmigration probability from location \(i\) to location \(g\) between time \(t\) and \(t + 1\), and we have defined \(E_{git}\) as the corresponding immigration probability to location \(g\) from location \(i\) between time \(t\) and \(t + 1\). Note that the order of subscripts switches between the outmigration probability \((D_{igt})\) and the immigration probability \((E_{git})\), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Worker value function:** Using the worker indirect utility function in equation (3) in the paper in the value function, the expected value from living in location \(n\) at time \(t\) can be written as:

\[
v^w_{nt} = \ln \left( \frac{E_{nt} \ell^b_{nt} w_{nt}}{p_{nt}} \right) + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \beta E_t v^w_{gt+1} \right) / \kappa_{gmt} \right)^{1/\rho},
\]

where the only difference from our baseline specification in the paper is the term in \(\ell^b_{nt}\) in the value function in equation (D.26).

**D.2.3 Existence and Uniqueness (Proof of Proposition 8 in the Paper)**

We now use the system of equations for general equilibrium (D.20)-(B.27) to establish the existence and uniqueness of a deterministic steady-state equilibrium with time-invariant fundamentals \(\{\pi_t, \bar{h}_t, \tau_{ni}, \kappa_{nt}\}\) and endogenous variables \(\{v^*_i, w^*_i, \ell^*_i, k^*_i\}\). Given these time-invariant fundamentals, we can drop the expectation over future fundamentals, such that \(E_t v^w_{gt+1} = v^w_{gt+1}\).

**Capital-Labor Ratio** In steady-state, \(k_{it+1} = k_{it} = k^*_i\), and we can use the capital accumulation condition (B.24) to solve for the steady-state capital-labor ratio:

\[
k^*_i = \beta \frac{1 - \mu}{\mu} \frac{w^*_i \ell^*_i}{p^*_i} + \beta (1 - \delta) k^*_i,
\]
\[
\begin{align*}
    k_i^* &= \frac{\beta}{1 - \beta(1 - \delta)} \frac{1 - \mu w_i^*}{p_i^*} \ell_i^*, \\
    \ell_i^* &= \frac{\beta}{1 - \beta(1 - \delta)} \frac{1 - \mu w_i^*}{p_i^*}.
\end{align*}
\] (D.27)

**Price Index** Using this result for the steady-state capital-labor ratio, we can re-write the price index in equation (D.21) as follows:

\[
\begin{align*}
    (p_n^*)^{-\theta} &= \sum_{i=1}^{N} \left( w_i^* \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \ell_i^*/k_i^* \right)^{1-\mu} \tau_{ni} / \left( \tilde{w}_i (\ell_i^*)^{\nu} \right) \right)^{-\theta}, \\
    (p_n^*)^{-\theta} &= \sum_{i=1}^{N} \left( w_i^* \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{1-\mu} \left( \ell_i^*/k_i^* \right)^{1-\mu} \tau_{ni} / \left( \tilde{w}_i (\ell_i^*)^{\nu} \right) \right)^{-\theta}, \\
    (p_n^*)^{-\theta} &= \sum_{i=1}^{N} \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)} \left( w_i^* \right)^{-\theta\mu} \left( p_n^* \right)^{-\theta(1-\mu)} \left( \ell_i^* \right)^{\eta^2 \theta} \left( \tau_{ni} / \tilde{z}_i \right)^{-\theta}, \\
    (p_n^*)^{-\theta} &= \sum_{i=1}^{N} \psi \left( w_i^* \right)^{-\theta\mu} \left( p_n^* \right)^{-\theta(1-\mu)} \left( \ell_i^* \right)^{\eta^2 \theta}, \\
    \psi &\equiv \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)}, \quad \tilde{\tau}_{ni} \equiv \left( \tau_{ni} / \tilde{z}_i \right)^{-\theta}. \tag{D.28}
\end{align*}
\]

**Goods Market Clearing Condition** Using this result for the steady-state capital-labor ratio, we can also re-write the goods market clearing condition (D.22) as follows:

\[
\begin{align*}
    w_i^* \ell_i^* &= \sum_{n=1}^{N} \left( w_i^* \left( \ell_i^*/k_i^* \right)^{1-\mu} \tau_{ni} / \left( \tilde{w}_i (\ell_i^*)^{\nu} \right) \right)^{-\theta} w_n^* \ell_n^*, \\
    w_i^* \ell_i^* &= \sum_{n=1}^{N} \left( w_i^* \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \left( \ell_i^*/k_i^* \right)^{1-\mu} \tau_{ni} / \left( \tilde{w}_i (\ell_i^*)^{\nu} \right) \right)^{-\theta} w_n^* \ell_n^*, \\
    w_i^* \ell_i^* &= \sum_{n=1}^{N} \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)} \left( p_n^* \right)^{-\theta(1-\mu)} \left( \ell_i^* \right)^{\eta^2 \theta} \left( \tau_{ni} / \tilde{z}_i \right)^{-\theta} w_n^* \ell_n^*, \\
    w_i^* \ell_i^* &= \sum_{n=1}^{N} \left( w_i^* \right)^{-\theta\mu} \left( \frac{1 - \beta (1 - \delta)}{\beta} \right)^{-\theta(1-\mu)} \left( p_n^* \right)^{-\theta(1-\mu)} \left( \ell_i^* \right)^{\eta^2 \theta} \left( \tau_{ni} / \tilde{z}_i \right)^{-\theta} w_n^* \ell_n^*, \tag{D.29}
\end{align*}
\] 68
\[(\ell^*_i)^{1-\eta^*\theta} (w^*_i)^{1+\theta\mu} (p^*_i)^{\theta(1-\mu)} = \sum_{n=1}^{N} \left( 1 - \beta (1 - \delta) \right)^{-\theta(1-\mu)} (p^*_n)^\theta (\tau_{ni}/\bar{z}_i)^{-\theta} w^*_n \ell^*_n, \]

\[(\ell^*_i)^{1-\eta^*\theta} (w^*_i)^{1+\theta\mu} (p^*_i)^{\theta(1-\mu)} = \sum_{n=1}^{N} \psi_\tau_{ni} (p^*_n)^\theta w^*_n \ell^*_n. \quad (D.29)\]

**Value Function** We now show how the value function \((D.26)\) can be re-written using our change of variables:

\[v_{w_n}^* = \ln \left( \frac{\tilde{b}_n (\ell_n^*)^{\phi} w_n^*}{p_n^*} \right) + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \frac{\beta v_{w_g}^*}{\kappa_{gn}} \right) \right)^{1/\rho}, \]

\[\exp \left( v_{w_n}^* \right) = \left( \frac{\tilde{b}_n (\ell_n^*)^{\phi} w_n^*}{p_n^*} \right) \left[ \sum_{g=1}^{N} \left( \exp \left( \frac{\beta v_{w_g}^*}{\kappa_{gn}} \right) \right)^{1/\rho} \right]^{\rho}, \]

\[\exp \left( \frac{\beta}{\rho} v_{w_n}^* \right) = \tilde{b}_n (\ell_n^*)^{\phi/\rho} \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \left[ \sum_{g=1}^{N} \left( \frac{\kappa_{gn}}{\tilde{b}_n} \right)^{-1/\rho} \exp \left( \frac{\beta v_{w_g}^*}{\rho} \right) \right]^{\rho}, \]

\[\exp \left( \frac{\beta}{\rho} v_{w_n}^* \right) = (\ell_n^*)^{\phi/\rho} \left( \frac{w_n^*}{p_n^*} \right)^{\beta/\rho} \left[ \sum_{g=1}^{N} \kappa_{g_n} \exp \left( \frac{\beta v_{w_g}^*}{\rho} \right) \right]^{\beta}, \quad \phi_n \equiv \sum_{g=1}^{N} \kappa_{g_n} \exp \left( \frac{\beta v_{w_g}^*}{\rho} \right), \quad (D.30)\]

Using this solution in the definition of \(\phi_n\) immediately above, we have:

\[\phi_n = \sum_{g=1}^{N} \tilde{\kappa}_{gn} (\ell_g^*)^{\beta \phi/\rho} (p_g^*)^{-\beta/\rho} (w_g^*)^{-\beta/\rho} \phi_g^\beta. \quad (D.31)\]

**Population Flow Condition** We now show how the population flow condition \((D.24)\) can be re-written using our change of variables:

\[\ell^*_g = \frac{\sum_{i=1}^{N} \left( \exp \left( \frac{\beta v_{w_i}^*}{\kappa_{mi}} \right) \right)^{1/\rho}}{\sum_{m=1}^{N} \left( \exp \left( \frac{\beta v_{w_m}^*}{\kappa_{mi}} \right) \right)^{1/\rho}}, \]

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\[
\ell_g^* = \sum_{i=1}^N \frac{\kappa_{gi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_g^* \right)}{\sum_{m=1}^N \kappa_{mi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_m^* \right)} \ell_i^* ,
\]

\[
\ell_g^* = \sum_{i=1}^N \kappa_{gi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_g^* \right) \left[ \sum_{m=1}^N \kappa_{mi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_m^* \right) \right]^{-1} \ell_i^* ,
\]

\[
\ell_g^* = \sum_{i=1}^N \kappa_{gi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_g^* \right) \left[ \sum_{m=1}^N \kappa_{mi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_m^* \right) \right]^{-1} \ell_i^* ,
\]

\[
\ell_g^* = \sum_{i=1}^N \kappa_{gi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_g^* \right) \left[ \sum_{m=1}^N \kappa_{mi}^{-1/\rho} \exp \left( \frac{\beta}{\rho} v_m^* \right) \right]^{-1} \ell_i^* ,
\]

Now using the value function result (D.30) above, we have:

\[
\ell_g^* = \sum_{i=1}^N \kappa_{gi} \left( \ell_g^* \right)^{\beta/\rho} \left( \frac{w_g^*}{p_g} \right)^{\beta/\rho} \phi_g \phi_i^{-1} \ell_i^* ,
\]

\[
\left( p_g^* \right)^{\beta/\rho} \left( w_g^* \right)^{-\beta/\rho} \left( \ell_g^* \right)^{1-\beta/\rho} \phi_g^{-\beta} = \sum_{i=1}^N \kappa_{gi} \ell_i^* \phi_i^{-1} .
\]

**D.2.4 System of Equations**

Collecting together these results, the steady-state equilibrium of the model \( \{ p_i^*, w_i^*, \ell_i^*, \phi_i^* \} \) can be expressed as the solution to the following system of equations:

\[
\left( p_i^* \right)^{-\theta} = \sum_{n=1}^N \psi_{ini} \left( p_n^* \right)^{-\theta (1-\mu)} \left( w_n^* \right)^{-\theta \mu} \left( \ell_n^* \right)^{\eta^\theta} ,
\]

(D.33)

\[
\left( p_i^* \right)^{\theta (1-\mu)} \left( w_i^* \right)^{1+\theta \mu} \left( \ell_i^* \right)^{1-\eta^\theta} = \sum_{n=1}^N \psi_{ini} \left( p_n^* \right)^{\theta} \left( w_n^* \right) \ell_n^* ,
\]

(D.34)

\[
\left( p_i^* \right)^{\beta/\rho} \left( w_i^* \right)^{-\beta/\rho} \left( \ell_i^* \right)^{1-\beta \eta^\rho} \phi_i^{-\beta} = \sum_{n=1}^N \kappa_{mi} \left( p_n^* \right)^{\beta/\rho} \left( w_n^* \right)^{\beta/\rho} \left( \ell_n^* \right)^{\beta \eta^\rho} \phi_n^\beta ,
\]

(D.35)

\[
\phi_i^* = \sum_{n=1}^N \kappa_{mi} \left( p_n^* \right)^{-\beta/\rho} \left( w_n^* \right)^{\beta/\rho} \left( \ell_n^* \right)^{\beta \eta^\rho} \phi_n^\beta ,
\]

(D.36)
where we have the following definitions:

\[
\psi \equiv \left(1 - \beta \left(1 - \delta \right) \right)^{-\theta (1-\mu)} , \quad \tilde{\tau}_{ni} \equiv \left(\tau_{ni}/z_i \right)^{-\theta},
\]

\[
\phi_i^* \equiv \sum_{n=1}^{N} \tilde{\kappa}_{ni} \exp \left(\frac{\beta \nu_{ni}^{w*}}{\rho} \right) , \quad \tilde{\kappa}_{in} \equiv \left(\kappa_{in}/\bar{b}_n^\beta \right)^{-1/\rho}.
\]

The exponents on the variables on the left-hand side of the system of equations (D.33)-(D.36) can be represented as the following matrix:

\[
\Lambda = \begin{bmatrix}
-\theta & 0 & 0 & 0 \\
\theta (1-\mu) & (1+\theta \mu) & (1-\eta^2 \theta) & 0 \\
\beta/\rho & -\beta/\rho & (1-\beta \eta /\rho) & -\beta \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

The exponents on the variables on the right-hand side of the system of equations (D.33)-(D.36) can be represented as the following matrix:

\[
\Gamma = \begin{bmatrix}
-\theta (1-\mu) & -\theta \mu & \eta^2 \theta & 0 \\
\theta & 1 & 1 & 0 \\
0 & 0 & 1 & -1 \\
-\beta/\rho & \beta/\rho & \beta \eta /\rho & \beta
\end{bmatrix}.
\]

Let \( A \equiv \Gamma \Lambda^{-1} \) and denote the spectral radius (eigenvalue with the largest absolute value) of this matrix by \( \rho (A) \). Building on the arguments in Allen, Arkolakis and Li (2020), we begin by showing that the equilibrium exists and is unique up to scale (up to a choice of numeraire for prices and a choice of units for population (population shares)) when \( \rho (A) = 1 \).

**Proposition.** *(Proposition 8 in the paper)* *(A)* There exists a unique steady-state spatial distribution of economic activity \( \{\ell_i^*, k_i^*\} \) (up to a choice of units) given the exogenous fundamentals \( \{z_i, \bar{b}_i, \tau_{ni}, \kappa_{ni}\} \) if the following parameter inequalities hold:

\[
1 \geq \eta^b + \eta^2 \theta + \eta^b \mu \theta ,
\]

\[
\rho (\beta + \rho + \rho \mu \theta) (1 - \beta) (1 + 2 \theta) \geq (1 + 2 \theta) \rho \mu \beta (3 + \beta - \mu \theta \beta) \eta^b + \eta^2 \theta \beta (2 \beta - \rho (2 \theta \mu \beta - 2 - \mu (1 - \beta) - 4 \theta \mu)) .
\]

*(B)* A sufficient condition for the parameter inequalities (D.37) to hold is that agglomeration forces are sufficiently small \((\eta^b, \eta^2 \rightarrow 0)\).

**Proof.** *(A)* Existence of solutions to the system of equations (D.33)-(D.36) follows from Brouwer’s fixed point theorem. Following the same line of argument as in the proof of Proposition 1 in Section B.7.6 of this online appendix, the equilibrium is unique when \( \rho (A) < 1 \) and is unique
up to a choice of units when \( A \) invertible, which is true for parameters throughout the domain \( \beta \in (0, 1), \mu \in [0, 1], \rho > 0, \) and \( \theta, \eta^b, \eta^r \geq 0, \) and \( \rho (A) = 1. \) Evaluating the eigenvalues of \( A \), we have:

\[
\begin{pmatrix}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4
\end{pmatrix} = \begin{pmatrix}
1 \\
1 \\
\frac{a + \sqrt{b}}{c} \\
\frac{a - \sqrt{b}}{c}
\end{pmatrix},
\]

where the functions \( a, b, c \) are defined in terms of the parameters \( \beta, \mu, \rho, \eta^b, \eta^r \) as:

\[
a = \beta \eta^b \mu + \rho + \beta \rho - \mu \rho + \beta \eta^r \theta + 2 \beta \eta^b \mu \theta - \mu \rho \theta + \beta \mu \rho \theta,
\]

\[
b = \rho (1 - \mu - \mu \theta) + \beta \rho (1 + \mu \theta) + \beta \eta^r \theta + \eta^b \beta \mu (1 + 2 \beta),
\]

\[
\begin{align*}
\xi_4 &= 2 \beta (1 - \eta^b - \eta^r \theta - \eta^b \mu \theta) + 2 (\rho + \mu \rho \theta).
\end{align*}
\]

We now provide sufficient conditions under which:

\[
\left| \frac{a \pm \sqrt{b}}{c} \right| < 1.
\]

Suppose \( 1 - \eta^b - \eta^r \theta - \eta^b \mu \theta > 0 \), then we know \( c > 0 \), and we just need to show \( | a \pm \sqrt{b} | < c. \)

Furthermore, we can re-write \( b \) as

\[
b = (\rho (1 - \mu - \mu \theta) + \beta \left[ 2 + \eta^b \mu - 2 \eta^b - \eta^r \theta - \rho (1 + \mu \theta) \right])^2 + 4 \rho \beta (1 + \beta) (\mu (1 + 2 \theta) + \eta^r \theta (1 - \mu - \mu \theta)),
\]

\[
b > 0,
\]

Also note

\[
a + c = \rho (2 - \mu) + (\beta + 1) \rho (1 + \mu \theta) + \beta \eta^r \theta + \eta^b \beta \mu (1 + 2 \beta)
\]

\[
+ 2 \beta (1 - \eta^b - \eta^r \theta - \eta^b \mu \theta),
\]

\[
> 0.
\]

Following the same logic as in the proof for Proposition 1 in Section B.7.6 of this online appendix, \( b, (a + c) > 0 \) implies that it suffices to show \( (c + a)^2 - b > 0 \) and \( (c - a)^2 - b > 0. \) Note:

\[
c - a = 2 \beta (1 - \eta^b - \eta^r \theta - \eta^b \mu \theta) + (2 - \beta) \rho (1 + \mu \theta) + \rho \mu (1 + \theta)
\]

\[
- \beta \eta^r \theta - \eta^b \beta \mu (1 + 2 \beta),
\]

\[
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\]
Thus a sufficient condition for \( \frac{\alpha \pm \sqrt{\delta}}{c} \) < 1 and a unique steady-state equilibrium (up to a choice of units) is that the following two inequalities hold:

\[
1 \geq \eta^b + \eta^\theta + \eta^b \mu \theta, \\
\rho \mu (\beta + \rho + \rho \mu \theta) (1 - \beta) (1 + 2\theta) \geq (1 + 2\theta) \rho \mu \beta (3 + \beta - \mu \theta \beta) \eta^b + \eta^\theta \beta (2\theta \mu \beta - 2 - \mu (1 - \beta) - 4\theta \mu) - 2\beta),
\]

First, expanding, and then simplifying, we have:

\[
\frac{(c - a)^2 - b}{4} = 2\eta^\theta \beta^2 x + [\beta^2 x (2\eta^b + \rho) \mu + \rho \mu^2 (1 + \mu \theta)] (1 + 2\theta) \\

= \frac{\mu \rho^2 (1 + 2\theta) (1 + \mu \theta) + \beta^2 \eta^b + \eta^\theta + \eta^b \mu \theta \beta (2\eta^\theta + 2\eta^b \mu (1 + 2\theta) + \mu \rho (1 + 2\theta))}{4} - \frac{-\beta \rho (2\eta^\theta + \mu^2 (3\eta^b + \rho) \theta (1 + 2\theta) + \mu (-1 + \rho - 2\theta + \eta^\theta + 2\rho \theta + 4\eta^\theta \theta^2 + \eta^b (3 + 6\theta))}{4} \\
\mu \rho^2 (1 + 2\theta) (1 + \mu \theta) + \beta^2 (-1 + \eta^b + \eta^\theta + \eta^b \mu \theta \beta (2\eta^\theta + (2\eta^b + \rho) \mu (1 + 2\theta)) \\

- \beta \rho (-\mu (1 + 2\theta) + (3\eta^b + \rho) \mu (1 + \mu \theta) (1 + 2\theta) + 2\eta^\theta + \eta^\theta \mu + 4\eta^\theta \theta^2 \mu).
\]

Let \( x \equiv -1 + \eta^b + \eta^\theta \theta + \eta^b \mu \theta \), \( y \equiv \eta^b (1 + \mu \theta) + \eta^\theta \theta \) then:

\[
\frac{(c - a)^2 - b}{4} = 2\eta^\theta \beta^2 x + [\beta^2 x (2\eta^b + \rho) \mu + \rho \mu^2 (1 + \mu \theta)] (1 + 2\theta) \\

= \frac{\mu \rho^2 (1 + 2\theta) (1 + \mu \theta) + \beta^2 \eta^b + \eta^\theta + \eta^b \mu \theta \beta (2\eta^\theta + 2\eta^b \mu (1 + 2\theta) + \mu \rho (1 + 2\theta))}{4} - \frac{-\beta \rho (2\eta^\theta + \mu^2 (3\eta^b + \rho) \theta (1 + 2\theta) + \mu (-1 + \rho - 2\theta + \eta^\theta + 2\rho \theta + 4\eta^\theta \theta^2 + \eta^b (3 + 6\theta))}{4} \\
\mu \rho^2 (1 + 2\theta) (1 + \mu \theta) + \beta^2 (-1 + \eta^b + \eta^\theta + \eta^b \mu \theta \beta (2\eta^\theta + (2\eta^b + \rho) \mu (1 + 2\theta)) \\

- \beta \rho (-\mu (1 + 2\theta) + (3\eta^b + \rho) \mu (1 + \mu \theta) (1 + 2\theta) + 2\eta^\theta + \eta^\theta \mu + 4\eta^\theta \theta^2 \mu).
\]
For sufficiently small agglomeration forces \((\eta^b, \eta^z \to 0)\) both of these inequalities above are necessarily satisfied.

As the expenditure shares \((S)\) and income shares \((T)\) are homogeneous of degree zero in factor prices, we require a choice of units or numeraire in order to solve for wages. We choose the total income of all locations as our numeraire \((\sum_{i=1}^{N} w_{it} \ell_{it} = \sum_{i=1}^{N} q_{it} = 1)\).

Similarly, the outmigration shares \((D)\) and immigration shares \((E)\) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels.

We solve for population shares, imposing the requirement that the population shares sum to one:

\[
\sum_{i=1}^{N} \ell_{it} = 1, \quad \text{which implies} \quad \sum_{i=1}^{N} \ell_{i}^{*} d \ln \ell_{i}^{*} = \sum_{i=1}^{N} \ell_{i}^{*} \frac{d \ell_{i}^{*}}{\ell_{i}^{*}} = \sum_{i=1}^{N} d \ell_{i}^{*} = 0.
\]

D.2.5 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path. In the interests of brevity, we focus on differences from our baseline specification without agglomeration economies.

**Goods Market Clearing**

Totally differentiating the goods market clearing condition, we have:

\[
\left[ \frac{d \ln w_{it}}{d \ln \ell_{it}} + \frac{d \ln w_{it}}{d \ln \ell_{it}} \right] = \left[ \theta \sum_{m=1}^{N} \sum_{n=1}^{N} T_{int} S_{nt} \left( d \ln w_{nt} + d \ln \ell_{nt} \right) - \theta \sum_{m=1}^{N} T_{int} \left( d \ln \tau_{nt} + (1 - \mu) d \ln \chi_{nt} \right) - \eta^z d \ln \ell_{mt} - d \ln z_{it} \right].
\]

(D.38)

**Value Function.**

Totally differentiating the value function, we have:

\[
dv_{it} = \left[ d \ln w_{it} - \sum_{m=1}^{N} S_{int} \left( d \ln \tau_{nt} + d \ln w_{nt} - (1 - \mu) d \ln \chi_{nt} - \eta^z d \ln \ell_{mt} - d \ln z_{it} \right) \right].
\]

(D.39)

D.2.6 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: \(k_{it+1} = k_{it} = k_{i}^{*}, \ell_{it+1} = \ell_{it} = \ell_{i}^{*}, w_{it+1} = w_{it} = w_{i}^{*}\), and \(v_{it+1}^{*} = v_{it}^{*} = v_{i}^{*}\), where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity \((d \ln z)\) and amenities \((d \ln b)\) in each location, holding constant the economy’s aggregate labor endowment \((d \ln \ell = 0)\), trade costs \((d \ln \tau = 0)\) and commuting costs \((d \ln \kappa = 0)\).
**Capital Accumulation.** From the capital accumulation equation (B.24), the steady-state stock of capital solves:

\[
(1 - \beta (1 - \delta)) \chi_t^* = (1 - \beta (1 - \delta)) \frac{k_t^*}{\ell_t^*} = \beta \frac{1 - \mu}{\mu} \frac{w_t^*}{p_t^*}.
\]

Totally differentiating, we have:

\[
d \ln \chi_t^* = d \ln \left( \frac{w_t^*}{p_t^*} \right).
\]

Totally differentiating real income, we have:

\[
d \ln \chi_t^* = d \ln w_t^* - \sum_{m=1}^{N} S_{im}^* \left[ d \ln w_m^* - (1 - \mu) d \ln \chi_m^* - \gamma d \ln \ell_m^* - d \ln z_m \right],
\]

where we have used and \(d \ln \tau_{nm} = 0\). This relationship has the matrix representation:

\[
(\mathbf{I} - (1 - \mu) \mathbf{S}) d \ln \chi_t^* = (\mathbf{I} - \mathbf{S}) d \ln w_t^* + \gamma \mathbf{S} d \ln \ell_t^* + \mathbf{S} d \ln z.
\](D.40)

**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.38) has the following matrix representation:

\[
d \ln \mathbf{w}_t + d \ln \ell_t = \begin{bmatrix}
\mathbf{T} (d \ln \mathbf{w}_t + d \ln \ell_t) \\
+ \theta (\mathbf{T} \mathbf{S} - \mathbf{I}) \left( d \ln \mathbf{w}_t - (1 - \mu) d \ln \chi_t - \gamma d \ln \ell_t - d \ln z \right)
\end{bmatrix},
\]

where we have used \(d \ln \tau = 0\). We can re-write this relationship as:

\[
[I - \mathbf{T} + \theta (\mathbf{I} - \mathbf{T} \mathbf{S})] d \ln \mathbf{w}_t = \begin{bmatrix}
- (\mathbf{I} - \mathbf{T} - \theta \gamma^2 (\mathbf{I} - \mathbf{T} \mathbf{S})) d \ln \ell_t \\
+ \theta (\mathbf{I} - \mathbf{T} \mathbf{S}) (d \ln z + (1 - \mu) d \ln \chi_t)
\end{bmatrix}.
\]

In steady-state we have:

\[
[I - \mathbf{T} + \theta (\mathbf{I} - \mathbf{T} \mathbf{S})] d \ln \mathbf{w}^* = \begin{bmatrix}
- (\mathbf{I} - \mathbf{T} - \theta \gamma^2 (\mathbf{I} - \mathbf{T} \mathbf{S})) d \ln \ell^* \\
+ \theta (\mathbf{I} - \mathbf{T} \mathbf{S}) (d \ln z + (1 - \mu) d \ln \chi^*)
\end{bmatrix}.
\](D.41)

**Population Flow.** The total derivative of the population flow condition has the same matrix representation as in our baseline model:

\[
d \ln \ell_{t+1} = \mathbf{E} d \ln \ell_t + \frac{\beta}{\rho} (\mathbf{I} - \mathbf{E} \mathbf{D}) \mathbb{E}_t d \mathbf{v}_{t+1}.
\]

In steady-state, we have:

\[
d \ln \ell^* = \mathbf{E} d \ln \ell^* + \frac{\beta}{\rho} (\mathbf{I} - \mathbf{E} \mathbf{D}) d \mathbf{v}^*.
\](D.42)
Value function. The total derivative of the value function (D.39) has the following matrix representation:

\[
\text{d}v_t = \begin{bmatrix}
(I - S) \ d \ln \ w_t + S \ (\ d \ln z + (1 - \mu) \ d \ln \chi_t) \\
\quad + d \ln b + (\eta^b S + \eta^b) \ d \ln \ell + \beta D E_t \ d v_{t+1}
\end{bmatrix},
\]

where we have used \(d \ln \tau = d \ln \kappa = 0\) and \(\eta^b\) is a \(N \times N\) diagonal matrix with the parameter \(\eta^b\) along its diagonal. In steady-state, we have:

\[
\text{d}v^* = \begin{bmatrix}
(I - S) \ d \ln w^* + S \ (\ d \ln z + (1 - \mu) \ d \ln \chi^*) \\
\quad + d \ln b + (\eta^b S + \eta^b) \ d \ln \ell^* + \beta D d v^*
\end{bmatrix}.
\]

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

\[
d \ln \chi^* = (I - (1 - \mu) S)^{-1} ((I - S) d \ln w^* + \eta^b S d \ln \ell^* + S d \ln z).
\]

\[
d \ln w^* = (I - T + \theta (I - TS))^{-1} \left[-(I - T - \theta \eta^b (I - TS)) d \ln \ell^* + (I - TS) \theta (\ d \ln z + (1 - \mu) \ d \ln \chi^*) \right].
\]

\[
d \ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) d v^*.
\]

\[
d v^* = (I - \beta D)^{-1} \left\{ d \ln w^* - S (d \ln w^* - d \ln z - (1 - \mu) d \ln \chi^*) + d \ln b + \eta^b S + \eta^b \right\} d \ln \ell^*.
\]

As the expenditure shares \((S)\) and income shares \((T)\) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire \((\sum_{i=1}^{N} \ w_i \ell_i = \sum_{i=1}^{N} \ q_i^* = \bar{q} = 1)\), which implies that the log changes in incomes satisfy \(Q^* d \ln q^* = \sum_{i=1}^{N} \ q_i^* d \ln q_i^* = \sum_{i=1}^{N} \ q_i^* \frac{d q_i^*}{q_i^*} = \sum_{i=1}^{N} \ d q_i^* = 0\), where \(Q^*\) is a row vector of the income of each location. Similarly, the outmigration shares \((D)\) and immigration shares \((E)\) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: \(\sum_{i=1}^{N} \ell_{it} = \bar{\ell} = 1\), which implies \(L^* d \ln \ell^* = \sum_{i=1}^{N} \ell_i^* d \ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{d \ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} \ d \ell_i^* = 0\), where \(L\) is a row vector of the population of each location.

D.2.7 Sufficient Statistics for Transition Dynamics Starting from Steady-State

We suppose that the economy starts from an initial steady-state distribution of economic activity \(\{k_i^*, \ell_i^*, \ w_i^*, \ v_i^*\}\). We consider small shocks to productivity \((d \ln z)\) and amenities \((d \ln b)\) in each location, holding constant the economy’s aggregate labor endowment \((d \ln \bar{\ell})\), trade costs
where we have used relative to the initial steady-state has the following matrix representation:

\[
\text{Goods Market Clearing.}
\]

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

\[
\text{where we have used}
\]

Taking the total derivative of real income relative to the initial steady-state, we have:

\[
\text{We can re-write the above relationship for the log deviation of the capital-labor ratio from the initial steady-state as:}
\]

\[
\text{We can re-write the above relationship for the log deviation of the capital-labor ratio from the initial steady-state as:}
\]

Taking the total derivative of real income relative to the initial steady-state, we have:

\[
\text{where we have used } \frac{d \ln \tau}{\ln \kappa} = 0. \text{ We can re-write this relationship in matrix form as:}
\]

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

\[
\text{Goods Market Clearing.} \text{ The total derivative of the goods market clearing condition (D.38) relative to the initial steady-state has the following matrix representation:}
\]

\[
\text{where we have used } \frac{d \ln \tau}{\ln \kappa} = 0. \text{ We can re-write this relationship as:}
\]

\[
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\]
Population Flow. The total derivative of the population flow condition relative to the initial steady-state has the same matrix representation as in the baseline model without agglomeration economies:

\[ \tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \tilde{v}_{t+1}. \] (D.51)

Value Function. The total derivative of the value function (D.39) relative to the initial steady-state has the following matrix representation:

\[ \tilde{v}_t = \begin{bmatrix} (I - S) \tilde{w}_t + S\tilde{z} + (1 - \mu) S\tilde{x}_t \\ + (\eta^z S + \eta^h) d\ln \ell^* + b + \beta E_t \tilde{v}_{t+1} \end{bmatrix}, \] (D.52)

where we have used \( d\ln \tau = d\ln \kappa = 0 \).

System of Equations for Transition Dynamics Relative to the Initial Steady-State. Collecting together the capital accumulation equation (D.49), the goods market clearing condition (D.50), the population flow condition (D.51), and the value function (D.52), the system of equations for the transition dynamics relative to the initial steady-state takes the following form:

\[ \tilde{x}_{t+1} = \left[ \begin{array}{c} \beta (1 - \delta) I + (1 - \beta (1 - \delta)) (1 - \mu) S\tilde{x}_t \\ + (1 - \beta (1 - \delta)) (I - S) \tilde{w}_t + (1 - \beta (1 - \delta)) S\tilde{z} \\ - \tilde{\ell}_{t+1} + [1 + \eta^z (1 - \beta (1 - \delta)) S] \tilde{\ell}_t \end{array} \right]. \] (D.53)

\[ \tilde{w}_t = [I - T + \theta (I - TS)]^{-1} \left[ \begin{array}{c} - (I - T - \theta \eta^z (I - TS)) \tilde{\ell}_t \\ + \theta (I - TS) (\tilde{z} + (1 - \mu) \tilde{x}_t) \end{array} \right]. \] (D.54)

\[ \tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \tilde{v}_{t+1}. \] (D.55)

\[ \tilde{v}_t = \begin{bmatrix} (I - S) \tilde{w}_t + S\tilde{z} + (1 - \mu) S\tilde{x}_t \\ + (\eta^z S + \eta^h) d\ln \ell^* + b + \beta E_t \tilde{v}_{t+1} \end{bmatrix}. \] (D.56)

D.3 Multiple Sectors (Region-Specific Capital)

We consider an economy that consists of many locations indexed by \( i \in \{1, \ldots, N\} \) and many sectors indexed by \( j \in \{1, \ldots, J\} \). Time is discrete and is indexed by \( t \). The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelastically and are geographically mobile across locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate \( \delta \).
D.3.1 Worker Migration Decisions

At the beginning of period $t$, the economy inherits a mass of workers in each location $i$ and sector $j$ ($\ell_{it}^j$), with the total labor endowment of the economy given by $\bar{\ell} = \sum_{i=1}^N \sum_{j=1}^J \ell_{it}^j$. Workers first produce and consume in their location and sector in period $t$, before observing mobility shocks $\{\ell_{git}^h\}$ for all possible locations $g \in \{1, \ldots, N\}$ and sectors $h \in \{1, \ldots, J\}$ and deciding where to move for period $t + 1$. Workers face bilateral migration costs $\{\delta_{git}^h\}$, which vary by both location and sector. The value function for a worker in location $i$ and sector $j$ at time $t$ ($V_{j,w}^{it}$) is equal to the current flow of utility in that location and sector plus the expected continuation value next period from the optimal choice of location and sector:

$$V_{j,w}^{it} = w_{it}^j + \max_{\{g,h\}^t} \left\{ \beta \mathbb{E}_t \left[ \left( V_{h,w}^{gt+1} - \delta_{git}^h + \rho \ell_{git}^h \right) \right] - \bar{\ell}_{git} \right\},$$

(D.57)

where we use the superscript $w$ to denote workers; $\beta$ is the discount rate; $\mathbb{E}[\cdot]$ denotes an expectation taken over the distribution for idiosyncratic mobility shocks; $\rho$ captures the dispersion of idiosyncratic mobility shocks; and we assume $\kappa_{jit}^{ij} = 1$ and $\kappa_{git}^{gh} > 1$ for $g \neq i$ and $h \neq j$.

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

$$F(\epsilon) = e^{-e^{-\epsilon}},$$

(D.58)

where $\gamma$ is the Euler-Mascheroni constant.

Under this assumption, the expected value for a worker of living in location $i$ at time $t$ ($V_{j,w}^{it}$) can be re-written in the following form:

$$V_{j,w}^{it} = w_{it}^j + \rho \log \sum_{g=1}^N \sum_{h=1}^J \left( \exp \left( \beta \mathbb{E}_t \left[ \frac{V_{h,w}^{gt+1}}{\kappa_{git}^{gh}} \right] \right) \right)^{1/\rho}.$$

(D.59)

The corresponding probability of migrating from location-sector $ij$ to location-sector $gh$ satisfies a gravity equation:

$$D_{ij}^{gh} = \frac{\left( \exp \left( \beta \mathbb{E}_t V_{h,w}^{gt+1} \right) / \kappa_{git}^{ij} \right)^{1/\rho}}{\sum_{m=1}^N \sum_{o=1}^J \left( \exp \left( \beta \mathbb{E}_t V_{o,w}^{mt+1} \right) / \kappa_{nit}^{oj} \right)^{1/\rho}}.$$

(D.60)

D.3.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in their location and sector that period. Worker flow indirect utility
in location $n$ and sector $j$ depends on local amenities ($b_{nt}^j$), the wage ($w_{nt}^j$), and the consumption goods price index ($p_{nt}$):

$$\ln u_{nt}^{j,w} = \ln b_{nt}^j + \ln w_{nt}^j - \ln p_{nt}, \quad (D.61)$$

where amenities ($b_{nt}^j$) capture characteristics of a location and sector that make it a more attractive place to live and work regardless of the wage and cost of consumption goods (e.g. climate and rewarding work). In this section of the online appendix, we assume that amenities are exogenous.

The consumption goods price index ($p_{nt}$) in location $n$ depends on the consumption goods price index for each sector $h$ in that location ($p_{nt}^h$):

$$p_{nt} = \prod_{h=1}^{J} (p_{nt}^h)^{\psi_h}, \quad 0 < \psi_h < 1, \quad \sum_{h=1}^{J} \psi_h, \quad (D.62)$$

where the consumption goods price index for each sector $h$ in location $n$ depends on the price of the variety sourced from each location $i$ within that sector $h$ ($p_{nit}^h$):

$$p_{nit}^h = \left[ \sum_{i=1}^{N} (p_{nit}^h)^{-\theta} \right]^{-1/\theta}, \quad \theta = \sigma - 1, \quad \sigma > 1, \quad (D.63)$$

where $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties; $\theta = \sigma - 1$ is the trade elasticity; and for simplicity, we assume a common elasticity of substitution and trade elasticity across all sectors.

Utility maximization implies that goods consumption expenditure on each sector ($p_{nt}^h c_{nt}^h$) is a constant share of overall goods consumption expenditure ($p_{nt} c_{nt}$) in each location:

$$p_{nt}^h c_{nt}^h = \psi_h p_{nt} c_{nt} = \psi_h \sum_{j=1}^{J} w_{nt}^j \epsilon_{nt}^j. \quad (D.64)$$

Using constant elasticity of substitution (CES) demand for individual varieties of goods, the share of location $n$’s expenditure within sector $h$ on the goods produced by location $i$ is:

$$\epsilon_{nit}^h = \frac{(p_{nit}^h)^{-\theta}}{\sum_{m=1}^{N} (p_{ntm}^h)^{-\theta}}. \quad (D.65)$$

### D.3.3 Production

Producers in each location $i$ and sector $j$ use labor ($\ell_{it}^j$) and capital ($k_{it}^j$) to produce output ($y_{it}^j$) of the variety supplied by that location in that sector. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

$$y_{it}^j = z_{it}^j \left( \frac{\ell_{it}^j}{\mu^j} \right)^{\mu^j} \left( \frac{k_{it}^j}{1 - \mu^j} \right)^{1 - \mu^j}, \quad 0 < \mu^j < 1, \quad (D.66)$$
where $z_{ijt}$ denotes productivity in location $i$ in sector $j$ at time $t$. As for amenities above, we assume in this section of the online appendix that productivity is exogenous.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit} \geq 1$ units of a good must be shipped from location $i$ in order for one unit to arrive in location $n$, where $\tau_{nit} > 1$ for $n \neq i$ and $\tau_{iit} = 1$. From profit maximization, the cost to a consumer in location $n$ of sourcing the good produced by location $i$ in sector $j$ depends on iceberg trade costs and constant marginal costs:

$$p_{nit} = \frac{\tau_{nit} \left( w_{nit} \right)^{\mu_j} \left( r_{it} \right)^{1-\mu_j}}{z_{it}^j}, \quad (D.67)$$

where $p_{iit}$ is the “free on board” price of the good supplied by location $i$ before transport costs.

From profit maximization and zero profits, total payments to each factor of production are a constant share of total revenue:

$$w_{iit} = \mu_j p_{iit} y_{iit}, \quad (D.68)$$

$$r_{iit} = \left( 1 - \mu_j \right) p_{iit} y_{iit}, \quad (D.69)$$

where capital mobility within regions ensures the same return to capital across sectors within regions ($r_{iit} = r_{it}$).

### D.3.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords’ intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v_{it}^k = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^k, \quad (D.70)$$

where we use the superscript $k$ to denote landlords; $c_{it}^k$ is the consumption goods index for landlords; and $\beta$ is the discount rate. Since landlords are geographically immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

The consumption goods index for landlords ($c_{it}^k$) takes exactly the same form as for workers and is a Cobb-Douglas aggregate of consumption indexes for each sector, where these consumption indexes for each sector are constant elasticity of substitution (CES) functions of the consumption of varieties from each location. Therefore, the consumption goods price index ($p_{nit}$) takes the same form as in equation (D.62), and the consumption goods price index for each sector
\((p^j_{it})\) takes the same form as in equation (D.63). Under these assumptions, the landlords’ utility maximization problem is weakly separable. First, we solve for the optimal consumption-savings decision across time periods for overall goods consumption. Second, we solve for the optimal allocation of consumption across sectors within each time period. Third, we solve for the optimal allocation of consumption across location varieties within each sector.

Beginning with landlords’ optimal consumption-saving decision, we assume that the investment technology for capital in each location uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location can produce one unit of capital in that location using one unit of the consumption index in that location. We assume that capital is geographically immobile once installed and depreciates at a constant rate \(\delta\). The intertemporal budget constraints for landlords in each location requires that total income from the existing stock of capital \((\sum_{j=1}^J r_{it}k^j_{it})\) equals the total value of goods consumption \((p_{it}c^k_{it})\) and net investment \((p_{it}(k_{it+1} - (1 - \delta) k_{it}))\):

\[
r_{it}k_{it} = \sum_{j=1}^J r_{it}k^j_{it} = p_{it}c^k_{it} + r_{it}k^k_{it} + p_{it}(k_{it+1} - (1 - \delta) k_{it}).
\]  

(D.71)

Combining landlords’ intertemporal utility (D.70) and budget constraint (D.71), the landlords’ intertemporal optimization problem is:

\[
\max \{c_{it}, k_{it+1}\} \sum_{t=0}^{\infty} \beta^t \ln c^k_{it},
\]

subject to 
\[p_{it}c^k_{it} + p_{it}(k_{it+1} - (1 - \delta) k_{it}) = r_{it}(k_{it} - k^k_{it}).\]

We can write this problem as the following Lagrangian:

\[
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c^k_{it} - \xi_t \left[p_{it}c^k_{it} + p_{it}(k_{it+1} - (1 - \delta) k_{it}) - r_{it}(k_{it} - k^k_{it})\right].
\]  

(D.73)

The first-order conditions are:

\[
\{c_{it}\} \quad \frac{\beta^t}{c_{it}} - p_{it}\xi_t = 0,
\]

\[
\{k_{it+1}\} \quad (r_{it+1} + p_{it+1}(1 - \delta))\xi_{t+1} - p_{it}\xi_t = 0.
\]

Together these first-order conditions imply:

\[
\frac{c_{it+1}}{c_{it}} = \beta \frac{p_{it}\mu_t}{p_{it+1}\mu_{t+1}} = \beta \left(r_{it+1}/p_{it+1} + (1 - \delta)\right),
\]  

(D.74)

where the transversality condition implies:

\[
\lim_{t \to \infty} \beta^t \frac{k_{it+1}}{c_{it}} = 0.
\]
Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords’ optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

\[ p_{it} c_{kt}^i = (1 - \beta) \left( r_{it} + p_{it} (1 - \delta) \right) k_{it}, \]  
\[ k_{it+1} = \beta \left( r_{it}/p_{it} + (1 - \delta) \right) k_{it}. \]  

(D.75)  
(D.76)

Substituting the consumption policy function (D.75) into the Euler equation (D.74), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

\[ \frac{c_{k_{it+1}}}{c_{k_{it}}} = \frac{(r_{it+1}/p_{it+1} + (1 - \delta)) k_{it+1}}{(r_{it}/p_{it} + (1 - \delta)) k_{it}}, \]  
\[ = \beta \left( r_{it+1}/p_{it+1} + (1 - \delta) \right). \]  

Given this optimal consumption-saving decision in equations (D.75)-(D.76), our assumption of Cobb-Douglas preferences across sectors implies that landlords allocate constant shares of consumption expenditure across sectors within time periods, as for workers in equation (D.64). Similarly, our assumption of constant elasticity of substitution (CES) preferences across locations within sectors implies that landlords in location \( n \) allocate the same share of expenditure on location \( i \) within sector \( j \), as for workers in equation (D.65).

### D.3.5 Market Clearing

Goods market clearing implies that revenue in each location in each sector equals expenditure on the goods produced by that location and sector:

\[ p_{it} y_{it}^j = \psi^j \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^j \left( w_{nt}^h \ell_{nt}^h + r_{nt} k_{nt}^h \right), \]

\[ w_{it}^j \ell_{it}^j + r_{nt} k_{nt}^j = \psi^j \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^j \left( w_{nt}^h \ell_{nt}^h + r_{nt} k_{nt}^h \right), \]

\[ w_{it}^j \ell_{it}^j + \frac{1 - \mu^j}{\mu^j} w_{it}^j \ell_{it}^j = \psi^j \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^j \left( w_{nt}^h \ell_{nt}^h + \frac{1 - \mu^h}{\mu^h} w_{nt}^h \ell_{nt}^h \right), \]

\[ \frac{1}{\mu^j} w_{it}^j \ell_{it}^j = \psi^j \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^j \frac{1}{\mu^h} w_{nt}^h \ell_{nt}^h. \]  

(D.77)

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords’ income from the ownership of capital equals payments for its use. Using the
property that payments to capital and labor are constant shares of total revenue in equations (D.68) and (D.69), we can write payments for capital in each sector as:

\[ r_{it}k_{it}^j = \frac{1 - \mu_j^j}{\mu_j^j} w_{it}^j e_{it}^j. \]  

(D.78)

Therefore capital market clearing implies:

\[ k_{it}^j = \frac{\frac{1 - \mu_j^j}{\mu_j^j} w_{it}^j e_{it}^j}{\sum_{o=1}^{J} \frac{1 - \mu_o^o}{\mu_o^o} w_{it}^o e_{it}^o} k_{it}^o. \]  

(D.79)

Re-arranging the relationship between sector-level payments to capital and labor in equation (D.78), the equilibrium rental rate for capital is given by:

\[ r_{it} = \frac{1 - \mu_j^j}{\mu_j^j} \frac{w_{it}^j e_{it}^j}{k_{it}^j}. \]  

Using this result in capital market clearing (D.79), we can re-write this capital market clearing condition as:

\[ r_{it} = \left( \sum_{o=1}^{J} \frac{1 - \mu_o^o}{\mu_o^o} w_{it}^o e_{it}^o \right) / k_{it}^j. \]  

(D.80)

D.3.6 General Equilibrium

Given the state variables \{\ell_{it}^j, k_{it}^j\} for each sector \(j\) and location \(i\), the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \{\ell_{it}^j, k_{it}^j, w_{it}^j, v_{it}^j\}_t^\infty. All other endogenous variables of the model can be recovered as a function of these variables.

**Capital Accumulation:** Using capital market clearing (D.80), the price index (D.62) and the equilibrium pricing rule (D.67), the capital accumulation equation (D.76) becomes:

\[ k_{it+1} = \beta (1 - \delta) k_{it} + \beta \sum_{o=1}^{J} \varphi_{it}^o e_{it}^o, \]  

(D.81)

\[ \varphi_{it}^o = \frac{1 - \mu_o^o}{\mu_o^o} \frac{w_{it}^o e_{it}^o}{p_{it}}, \]

\[ k_{it}^j = \frac{\varphi_{it}^j e_{it}^j}{\sum_{o=1}^{J} \varphi_{it}^o e_{it}^o} k_{it}^o. \]

\[ p_{it} = \prod_{j=1}^{J} \left[ \sum_{i=1}^{N} \left( w_{it}^j \left( \frac{1 - \mu_j^j}{\mu_j^j} \right)^{1 - \mu_j^j} \left( \frac{\ell_{it}^j}{k_{it}^j} \right)^{1 - \mu_j^j} \frac{\tau_{it}^j}{z_{it}^j} \right)^{-\theta_j} \right]^{-\psi_j/j}. \]  

(D.82)
Goods Market Clearing: From the goods market clearing condition (D.77), we have:

\[ \frac{1}{\mu_j} w_{it} \phi_j = \psi_j \sum_{n=1}^N \sum_{h=1}^J S_{nit}^j \frac{1}{\mu_h} w_{nt}^h \phi_h, \quad (D.83) \]

\[ S_{nit}^j = \frac{\left( \tau_{nit}^j w_{it}^j (r_{it}^j)^{1-\mu_j} / z_{it}^j \right)^{-\theta}}{\sum_{m=1}^N \left( \tau_{mnt}^j w_{mt}^j (r_{mt}^j)^{1-\mu_j} / z_{mt}^j \right)^{-\theta}}, \]

\[ T_{int}^{jh} \equiv \psi_j S_{nit}^j \frac{1}{\mu_h} w_{nt}^h \phi_h, \]

where \( S_{nit}^j \) is the expenditure share of importer \( n \) on exporter \( i \) in sector \( j \) at time \( t \), and we have defined \( T_{int}^{jh} \) as the corresponding income share of exporter \( i \) from importer \( n \) at time \( t \). Note that the order of subscripts switches between the expenditure share \( (S_{nit}^j) \) and the income share \( (T_{int}^{jh}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the out-migration probabilities (D.60), the population flow condition for the evolution of the population distribution over time is given by:

\[ P_{gt+1}^h = \sum_{i=1}^N \sum_{j=1}^J D_{igt}^{jh}, \quad (D.84) \]

\[ D_{igt}^{jh} = \frac{\exp \left( \beta E_{t}^{h,w} v_{gt+1}^{h,w} / \kappa_{git}^{h,j} \right)^{1/\rho}}{\sum_{m=1}^N \sum_{o=1}^J \left( \exp \left( \beta E_{t}^{o,w} v_{mt+1}^{o,w} / \kappa_{mit}^{o,j} \right)^{1/\rho} \right)}, \]

\[ E_{gt}^{ij} = \frac{\theta_{ij} D_{igt}^{jh}}{P_{gt+1}^{h}}, \]

where \( D_{igt}^{jh} \) is the outmigration probability from sector \( j \) in location \( i \) to sector \( h \) in location \( g \) at time \( t \), and we have defined \( E_{gt}^{ij} \) as the corresponding immigration probability to sector \( h \) in location \( g \) from sector \( j \) in location \( i \) at time \( t \). Note that the order of subscripts switches between the outmigration probability \( (D_{igt}^{jh}) \) and the immigration probability \( (E_{gt}^{ij}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function (D.61) in the value function (D.57), the expected utility from working in sector \( j \) in location \( n \) at time \( t \) can be written as:

\[ v_{nt}^{j,w} = \ln b_{it}^j + \ln \left( \frac{w_{nt}^j}{p_{nt}} \right) + \rho \log \sum_{g=1}^N \sum_{h=1}^J \left( \exp \left( \beta E_{t}^{h,w} v_{gt+1}^{h,w} / \kappa_{git}^{h,j} \right)^{1/\rho} \right). \quad (D.85) \]

D.3.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.
Prices. Using the relationship between capital and labor payments (D.79), the pricing rule (D.67) can be re-written as follows:

\[ p_{nit}^j = \frac{\tau_{nit}^j (w_{it}^j)^{\left(1-\mu^j\right)\chi_{it}^j}}{z_{it}^j} \]  

(D.86)

where \( \chi_{it}^j \) is the capital-labor ratio in sector \( j \):

\[ \chi_{it}^j \equiv \frac{k_{it}^j}{l_{it}^j} \]

Totally differentiating this pricing rule, we have:

\[ \frac{dp_{nit}^j}{p_{nit}^j} = \left[ \frac{d\tau_{nit}^j}{\tau_{nit}^j} + \frac{dw_{it}^j}{w_{it}^j} - (1 - \mu^j) \frac{d\chi_{it}^j}{\chi_{it}^j} - \frac{dz_{it}^j}{z_{it}^j} \right], \]

\[ d\ln p_{nit}^j = \left[ d\ln \tau_{nit}^j + \mu^j d\ln w_{it}^j - (1 - \mu^j) d\ln \chi_{it}^j - d\ln z_{it}^j \right]. \]  

(D.87)

Expenditure Shares. Totally differentiating this expenditure share equation (D.65), we get:

\[ \frac{dS_{nit}}{S_{nit}} = \theta \left( \sum_{h=1}^{N} S_{nht} \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nit}}{p_{nit}} \right), \]  

(D.88)

\[ d\ln S_{nit} = \theta \left( \sum_{h=1}^{N} S_{nht} \ln p_{nht} - \ln p_{nit} \right). \]

Price Indices. Totally differentiating the consumption goods price index in equation (D.63), we have:

\[ \frac{dp_{nt}}{p_{nt}} = \sum_{m=1}^{N} S_{nmt} \frac{dp_{nmt}}{p_{nmt}}, \]  

(D.89)

\[ d\ln p_{nt} = \sum_{m=1}^{N} S_{nmt} d\ln p_{nmt}. \]

Migration Shares. Totally differentiating the outmigration share in equation (D.60), we get:

\[ \frac{dD_{ijt}^h}{D_{ijt}^h} = \frac{1}{\rho} \left[ \left( \beta \mathbb{E}_t d_{ijt}^h - \frac{d_{ijt}^h}{\kappa_{ijt}} \right) - \sum_{m=1}^{N} \sum_{o=1}^{J} D_{imt}^{jo} \left( \beta \mathbb{E}_t d_{imt+1}^{o,w} - \frac{d_{imt}^{o,w}}{\kappa_{imt+1}} \right) \right], \]  

(D.90)

\[ d\ln D_{ijt}^h = \frac{1}{\rho} \left[ \left( \beta \mathbb{E}_t d_{ijt}^h - d\ln \kappa_{ijt}^h \right) - \sum_{m=1}^{N} \sum_{o=1}^{J} D_{imt}^{jo} \left( \beta \mathbb{E}_t d_{imt+1}^{o,w} - d\ln \kappa_{imt+1}^{o,j} \right) \right]. \]
Real Income. Totally differentiating real income we have:

\[
\begin{align*}
\frac{\partial \ln \varphi^j_{st}}{\partial \ln \varphi^j_{st}} &= \frac{\partial \ln \left( w^j_{it} / p_{it} \right)}{\partial \ln \varphi^j_{st}} = \frac{\partial \ln w^j_{it}}{\partial \ln \varphi^j_{st}} - \sum_{h=1}^{J} \psi^h \frac{\partial \ln p^h_{it}}{\partial \ln \varphi^j_{st}}, \\
\frac{\partial \ln \varphi^j_{st}}{\partial \ln \varphi^j_{st}} &= \frac{\partial \ln \left( w^j_{it} / p_{it} \right)}{\partial \ln \varphi^j_{st}} = \frac{\partial \ln w^j_{it}}{\partial \ln \varphi^j_{st}} - \sum_{h=1}^{J} \psi^h \sum_{m=1}^{N} S_{nmt} \frac{\partial \ln p^h_{nt}}{\partial \ln \varphi^j_{st}}, \\
\frac{\partial \ln \varphi^j_{st}}{\partial \ln \varphi^j_{st}} &= \frac{\partial \ln \left( w^j_{it} / p_{it} \right)}{\partial \ln \varphi^j_{st}} = \frac{\partial \ln w^j_{it}}{\partial \ln \varphi^j_{st}} - \sum_{h=1}^{J} \psi^h \left[ - (1 - \mu^h) \frac{\partial \ln \chi^h_{mt}}{\partial \ln \varphi^j_{st}} - \frac{\partial \ln \chi^h_{nt}}{\partial \ln \varphi^j_{st}} \right],
\end{align*}
\]

Goods Market Clearing. Totally differentiating the goods market clearing condition (D.77), we have:

\[
\frac{\partial w^j_{it}}{\partial \ell^j_{it}} + \frac{\partial \ell^j_{it}}{\partial \ln \ell^j_{it}} = \sum_{n=1}^{N} \sum_{h=1}^{J} \psi^h \frac{S_{nmt} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it}}{w^h_{nt} \ell^j_{it}} \left( \frac{\partial w^h_{nt}}{\partial \ln \varphi^j_{st}} - \frac{\partial \ln \varphi^j_{st}}{\partial \mu^j_{nmt}} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it} + \frac{\partial \ln \varphi^j_{st}}{\partial \mu^j_{nmt}} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it} \right),
\]

Using our result for the derivative of expenditure shares in equation (D.88) above, we can rewrite this as:

\[
\frac{\partial w^j_{it}}{\partial \ell^j_{it}} + \frac{\partial \ell^j_{it}}{\partial \ln \ell^j_{it}} = \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{T^{jh}_{nt}}{\mu^h} \frac{1}{w^h_{nt} \ell^j_{it}} \left( \frac{\partial w^h_{nt}}{\partial \ln \varphi^j_{st}} + \frac{\partial \ln \varphi^j_{st}}{\partial \mu^j_{nmt}} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it} - \frac{\partial \ln \varphi^j_{st}}{\partial \mu^j_{nmt}} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it} \right),
\]

\[
\left[ \frac{\partial \ln w^j_{nt}}{\partial \ln \ell^j_{it}} + \frac{\partial \ln \ell^j_{it}}{\partial \ln \ell^j_{it}} \right] = \left[ + \theta \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{T^{jh}_{nt}}{\partial \mu^h} \frac{1}{w^h_{nt} \ell^j_{it}} \left( \frac{\partial w^h_{nt}}{\partial \ln \varphi^j_{st}} + \frac{\partial \ln \varphi^j_{st}}{\partial \mu^j_{nmt}} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it} - \frac{\partial \ln \varphi^j_{st}}{\partial \mu^j_{nmt}} \frac{1}{\mu^h} w^h_{nt} \ell^j_{it} \right) \right].
\]

Population Flow. Totally differentiating the population flow condition (D.84) we have:

\[
\frac{\partial e^j_{gt+1}}{\partial \ell^j_{gt+1}} = \sum_{i=1}^{N} \sum_{j=1}^{J} E^{hj}_{git} \left( \frac{\partial e^j_{it}}{\partial \ell^j_{it}} + \frac{\partial \ell^j_{it}}{\partial \ln \ell^j_{it}} \right) + \frac{\partial D^{j}_{igt}}{\partial D^{j}_{igt}},
\]

\[
\frac{\partial \ln \ell^j_{gt+1}}{\partial \ln \ell^j_{gt+1}} = \sum_{i=1}^{N} \sum_{j=1}^{J} E^{hj}_{git} \left[ \frac{\partial \ln \ell^j_{it}}{\partial \ln \ell^j_{it}} + \frac{1}{\rho} \left( \beta E_{it} \ln \chi_{gt+1}^{hj} - \frac{\partial \ln \chi_{gt+1}^{hj}}{\partial \ln \ell^j_{it}} \right) - \sum_{m=1}^{N} \sum_{a=1}^{J} D^{j}_{imat} \left( \beta E_{it} \ln \chi_{mat+1}^{hj} - \frac{\partial \ln \chi_{mat+1}^{hj}}{\partial \ln \ell^j_{it}} \right) \right].
\]

Value Function. Note that the value function is:

\[
\psi^j_{it} = \frac{w^j_{it}}{\prod_{h=1}^{J} \left( \sum_{m=1}^{N} \left( p^h_{nmt} \right)^{-\theta} \right)^{-\psi^j_{it}}} + b^j_{it} + \rho \ln \sum_{g=1}^{G} \sum_{h=1}^{J} \left( \exp \left( \beta E_{it} \psi^{hj}_{git} \right) \cdot \kappa^{hj}_{git} \right)^{1/\rho},
\]
\[
\left[ \sum_{m=1}^{N} (p_{imt}^h)^{-\theta} \right]^{-1/\theta} = \left( \frac{(p_{ist}^h)^{-\theta}}{S_{ist}^h} \right)^{-1/\theta}, \quad \tau_{ist}^h = 1,
\]

\[
\prod_{h=1}^{J} \left[ \sum_{m=1}^{N} (p_{imt}^h)^{-\theta} \right]^{-\psi^h/\theta} = \prod_{h=1}^{J} \left( \frac{(p_{ist}^h)^{-\theta}}{S_{ist}^h} \right)^{-\psi^h/\theta},
\]

\[
\sum_{g=1}^{J} \sum_{h=1}^{J} \exp \left( \beta E_t q_{gjt}^{jw} / \gamma_{gjt}^j \right)^{1/\rho} = \frac{\exp \left( \beta E_t q_{gjt+1}^{jw} / \gamma_{gjt}^j \right)^{1/\rho}}{D_{ij}^{jt}}, \quad \kappa_{ij}^{jt} = 1,
\]

\[
v_{it}^{jw} = \ln w_{it}^{j} - \sum_{h=1}^{J} \psi^h \left( \frac{1}{\theta} \ln S_{ist}^h + \ln p_{ist}^h \right) + \ln b_{it}^{j} + \beta E_t v_{it+1}^{jw} - \rho \ln D_{it}^{jt}.
\]

Note that the value function can be equivalently written as:

\[
v_{it}^{jw} = \ln \xi^j + \ln \psi_{it}^j + \ln b_{it}^j + \beta E_t \psi_{it+1}^{jw} - \rho \ln D_{it}^{jt},
\]

where \( \xi^j = \frac{\lambda^j}{1 - \lambda^j} \). Totally differentiating the value function (D.94) we have:

\[
v_{it}^{jw} = \ln w_{it}^{j} - \sum_{h=1}^{J} \psi^h \left( \frac{1}{\theta} \ln S_{ist}^h + \ln p_{ist}^h \right) + \ln b_{it}^{j} + \beta E_t v_{it+1}^{jw} - \rho \ln D_{it}^{jt}.
\]

\[
\frac{dv_{it}^{jw}}{dt} = \frac{d \ln w_{it}^{j}}{dt} - \sum_{h=1}^{J} \psi^h \left( \frac{1}{\theta} \ln S_{ist}^h + \ln p_{ist}^h \right) + \frac{d \ln b_{it}^{j}}{dt} + \beta E_t \psi_{it+1}^{jw} - \rho \frac{d \ln D_{it}^{jt}}{dt},
\]

\[
\frac{d \ln S_{ist}^h}{dt} = -\theta \frac{d \ln p_{ist}^h}{dt} + \theta \left[ \sum_{m=1}^{N} S_{imt}^h \frac{d \ln p_{imt}^h}{dt} \right],
\]

\[
\frac{d \ln D_{it}^{jt}}{dt} = \frac{1}{\rho} \left[ \beta E_t \psi_{it+1}^{jw} - \frac{d \ln \kappa_{ij}^{jt}}{dt} - \sum_{m=1}^{N} \sum_{o=1}^{J} D_{im}^{jo} \left( \beta E_t \psi_{mt+1}^{o,w} - \frac{d \ln \kappa_{mot}}{dt} \right) \right].
\]

Using these results in the derivative of the value function, we have:

\[
\frac{dv_{it}^{j}}{dt} = \left[ \frac{d \ln w_{it}^{j}}{dt} - \sum_{h=1}^{J} \psi^h \left( \frac{1}{\theta} \ln S_{ist}^h + \ln p_{ist}^h \right) + \frac{d \ln b_{it}^{j}}{dt} + \sum_{m=1}^{N} \sum_{o=1}^{J} D_{im}^{jo} \left( \beta E_t \psi_{mt+1}^{o,w} - \frac{d \ln \kappa_{mot}}{dt} \right) \right],
\]

where we have used \( \frac{d \ln \kappa_{ij}^{jt}}{dt} = 0 \). Using the total derivative of the pricing rule (D.87), we can re-write this derivative of the value function as follows:

\[
\frac{dv_{it}^{j}}{dt} = \left[ \frac{d \ln w_{it}^{j}}{dt} - \sum_{h=1}^{J} \psi^h \sum_{m=1}^{N} S_{imt}^h \left( \frac{d \ln E_{m}^{jw}}{dt} + \frac{d \ln w_{mt}^{h}}{dt} - \frac{(1 - \mu^h)}{\rho} \frac{d \ln \chi_{mt}^{h}}{dt} - \frac{d \ln z_{mt}^{h}}{dt} \right) + \frac{d \ln b_{it}^{j}}{dt} + \sum_{m=1}^{N} \sum_{o=1}^{J} D_{im}^{jo} \right],
\]

which can be equivalently written as:

\[
\frac{dv_{it}^{j}}{dt} = \left[ \frac{d \ln \psi_{it}^j}{dt} + \frac{d \ln b_{it}^{j}}{dt} + \sum_{m=1}^{N} \sum_{o=1}^{J} D_{im}^{jo} \right].
\]
D.3.8 Steady-state Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables:

\[ k_{it+1}^j = k_{it}^j = k_i^j, \quad \ell_{it+1}^j = \ell_{it}^j = \ell_i^j, \quad w_{it+1}^j = w_{it}^j = w_i^j \quad \text{and} \quad v_{it+1}^j = v_{it}^j = v_i^j, \]

where we use an asterisk to denote a steady-state value. We consider a small common shock to productivity across all sectors \((\ln z)\) and amenities across all sectors \((\ln b)\) in each location, holding constant the economy’s aggregate labor endowment \((\ln \bar{l} = 0)\), trade costs \((\ln \bar{\tau} = 0)\) and commuting costs \((\ln \kappa = 0)\).

**Capital Accumulation.** From the capital accumulation equation (D.81), the steady-state stock of capital solves:

\[
k_i^* = \beta (1 - \delta) k_i^* + \beta \sum_{o=1}^{J} \varphi_i^{os} \ell_i^{os},
\]

\[
(1 - \beta (1 - \delta)) k_i^* = \beta \sum_{o=1}^{J} \varphi_i^{os} \ell_i^{os}.
\]

But from capital market clearing, we also have:

\[
k_i^j = \frac{\varphi_i^{j} \ell_i^{j}}{\sum_{o=1}^{J} \varphi_i^{os} \ell_i^{os}} k_i^*.
\]

and hence:

\[
\sum_{o=1}^{J} \varphi_i^{os} \ell_i^{os} = \varphi_i^{j} \ell_i^{j} \frac{k_i^*}{k_i^j}.
\]

Using this result in the capital accumulation equation, we have:

\[
(1 - \beta (1 - \delta)) k_i^j = \beta \varphi_i^{j} \ell_i^{j},
\]

and hence:

\[
\varphi_i^{j} = (1 - \beta (1 - \delta)) \frac{k_i^j}{\ell_i^{j}} = \frac{1 - \beta (1 - \delta)}{\beta} \chi_i^{j}.
\]

Totally differentiating, we have:

\[
d \ln \chi_i^* = d \ln \varphi_i^{j},
\]

\[
d \ln \varphi_i^{j} = d \ln w_i^* - d \ln p_i^*.
\]

From the total derivative of real income (D.91) above, this becomes:

\[
d \ln \varphi_i^{j} = d \ln w_i - \sum_{m=1}^{N} \sum_{h=1}^{J} \psi^h S_{imt}^h \left[ d \ln \tau_{imt}^h + d \ln w_{mt}^h - (1 - \mu^h) d \ln \varphi_i^{j} - d \ln z_{mt}^h \right].
\]
where we have used $d \ln \tau_{int}^h = 0$. This relationship has the following matrix representation:

$$d \ln \vartheta^* = d \ln w^* - S \left( d \ln w - (I - \mu) \ d \ln \vartheta^* - d \ln z \right),$$

(D.97)

where $d \ln \vartheta^*$ and $d \ln w^*$ are $N \times 1$ vectors; $S$ is a $NJ \times NJ$ matrix with elements:

$$S_{nit} = S^j_{nit} = \sum_{h=1}^J \psi^h S_{nit},$$

and $\mu$ is $NJ \times NJ$ diagonal matrix whose $(ij)$-th element on the diagonal is $\mu^j$. Note that the evolution of the regional capital stock is given by:

$$k_{it} = \sum_{j=1}^J k^j_{it},$$

$$d k_{it} = \sum_{j=1}^J d k^j_{it},$$

$$\frac{d k_{it}}{k_{it}} = \sum_{j=1}^J \frac{k^j_{it}}{k_{it}} \frac{d k^j_{it}}{k^j_{it}},$$

$$d \ln k_{it} = \sum_{j=1}^J \frac{k^j_{it}}{k_{it}} \left( d \ln \vartheta^j_{it} + d \ln \ell^j_{it} \right),$$

which can be written as:

$$d \ln k_{reg}^{it} = \mathcal{K} \left( d \ln \vartheta_t + d \ln \ell_t \right),$$

where $k_{reg}^{it}$ is a $N \times 1$ vector of regional capital stocks; $\mathcal{K}$ is the $N \times NJ$ matrix whose $(i, jo)$ element is the steady-state share of capital in location $i$ employed in location $j$ sector $o$; $\vartheta_t$ and $\ell_t$ are $NJ \times 1$ vectors.

**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.92) has the following matrix representation:

$$d \ln w_t + d \ln \ell_t = T \left( d \ln w_t + d \ln \ell_t \right) + \theta (TS - I) \left( d \ln w_t - (I - \mu) \ d \ln x_t - d \ln z \right),$$

where these matrices have $NJ \times NJ$ elements and we have used $d \ln \tau = 0$. In steady-state we have:

$$d \ln w^* + d \ln \ell^* = T \left( d \ln w^* + d \ln \ell^* \right) + \theta (TS - I) \left( d \ln w^* - (I - \mu) \ d \ln \vartheta^* - d \ln z \right).$$

(D.98)
Population Flow. The total derivative of the population flow condition (D.93) has the following matrix representation:

$$
\frac{d \ln \ell_{t+1}}{dt} = E \frac{d \ln \ell_t}{dt} + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \frac{d v}{dt},
$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$
\frac{d \ln \ell^*}{dt} = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) \frac{d v^*}{dt}.
$$

(D.99)

Value function. The total derivative of the value function has the following matrix representation:

$$
\frac{dv_t}{dt} = \frac{d \ln \theta_t}{dt} + \frac{d \ln b}{dt} + \beta D E \frac{dv}{dt+1},
$$

where these matrices again have $NJ \times NJ$ elements and we have used $d \ln \kappa = 0$. In steady-state, we have:

$$
\frac{dv^*}{dt} = (I - \beta D)^{-1} \left[ \frac{d \ln \theta^*}{dt} + \frac{d \ln b}{dt} \right].
$$

(D.100)

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$
\frac{d ln \theta^*}{dt} = \frac{d ln w^*}{dt} - S \left( \frac{d ln w}{dt} - (I - \mu) \frac{d ln \theta^*}{dt} - \frac{d ln z}{dt} \right),
$$

(D.101)

$$
\frac{d ln w^*}{dt} + \frac{d ln \ell^*}{dt} = T \left( \frac{d ln w^*}{dt} + \frac{d ln \ell^*}{dt} \right) + \theta (TS - I) \left( \frac{d ln w^*}{dt} - (I - \mu) \frac{d ln \theta^*}{dt} - \frac{d ln z}{dt} \right),
$$

(D.102)

$$
\frac{d ln \ell^*}{dt} = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) \frac{d v^*}{dt}.
$$

(D.103)

$$
\frac{dv^*}{dt} = (I - \beta D)^{-1} \left[ \frac{d ln \theta^*}{dt} + \frac{d ln b}{dt} \right].
$$

(D.104)

$$
\frac{d ln k^{reg}}{dt} = K \left( \frac{d ln \theta^*}{dt} + \frac{d ln \ell^*}{dt} \right).
$$

(D.105)

D.3.9 Sufficient Statistics for Transition Dynamics

Suppose that the economy starts from an initial steady-state. Consider a small shock to productivity ($d \ln z$) and amenities ($d \ln b$) in each sector and location, holding constant the economy’s aggregate labor endowment ($d \ln \ell = 0$), trade costs ($d \ln \tau = 0$) and commuting costs ($d \ln \kappa = 0$). We use a tilde above a variable to denote a log-deviation from the initial steady-state, such that $\tilde{\ell}_{it+1} = \ln \ell_{it+1} - \ln \ell^*_i$, for all variables except for the worker value function $v_{it}$; with a slight abuse of notation we use $\tilde{\ell}_{it} \equiv v_{it} - v^*_i$ to denote the deviation in levels for the worker value function.

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Capital Accumulation. From the capital accumulation equation (D.81), we have:

\[ k_{it+1} = \beta (1 - \delta) k_{it} + \beta \sum_{o=1}^{J} \vartheta_{ot} \ell_{ot}. \]

Dividing by the steady-state capital stock we have:

\[ \frac{k_{it+1}}{k^*_i} = \beta (1 - \delta) \frac{k_{it}}{k^*_i} + \beta \sum_{o=1}^{J} \frac{\vartheta_{it} \ell_{it}^o}{k^*_i}. \]

We know from above that the steady-state capital stock is given by:

\[ k^*_i = \frac{\beta}{1 - \beta (1 - \delta)} \sum_{o=1}^{J} \vartheta_{i}^{ox} \ell_{i}. \]

Therefore we can re-write the capital accumulation equation as:

\[ \frac{k_{it+1}}{k^*_i} = \beta (1 - \delta) \frac{k_{it}}{k^*_i} + (1 - \beta (1 - \delta)) \sum_{o=1}^{J} \frac{\vartheta_{it} \ell_{it}^o}{k^*_i}, \]

which can be further re-written as:

\[ \frac{k_{it+1}}{k^*_i} - 1 = \beta (1 - \delta) \left( \frac{k_{it}}{k^*_i} - 1 \right) + (1 - \beta (1 - \delta)) \sum_{o=1}^{J} \frac{\vartheta_{i}^{ox} \ell_{i}^{ox}}{\vartheta_{i}^{ox} \ell_{i}^{ox}} \left( \frac{\vartheta_{it} \ell_{it}^o}{\vartheta_{i}^{ox} \ell_{i}^{ox}} - 1 \right), \]

Noting that

\[ \frac{k_{it}}{k^*_i} - 1 \simeq \ln \left( \frac{k_{it}}{k^*_i} \right), \]

we have:

\[ \ln \left( \frac{k_{it+1}}{k^*_i} \right) = \beta (1 - \delta) \ln \left( \frac{k_{it}}{k^*_i} \right) + (1 - \beta (1 - \delta)) \sum_{o=1}^{J} \frac{\vartheta_{i}^{ox} \ell_{i}^{ox}}{\vartheta_{i}^{ox} \ell_{i}^{ox}} \ln \left( \frac{\vartheta_{it} \ell_{it}^o}{\vartheta_{i}^{ox} \ell_{i}^{ox}} \right), \]

which can be re-written in matrix form as:

\[ \bar{k}_{t+1}^{reg} = \beta (1 - \delta) \bar{k}_{t}^{reg} + (1 - \beta (1 - \delta)) \mathcal{K} \left( \hat{\vartheta} + \hat{\ell} \right), \]

(D.106)

where \( \bar{k}_{t+1}^{reg} \) and \( \bar{k}_{t}^{reg} \) are \( N \times 1 \) vectors; \( \mathcal{K} \) is the \( N \times NK \) matrix whose \((i, jo)\) element is the steady-state share of capital in location \( i \) employed in location \( j \) sector \( o \); \( \hat{\vartheta} \) and \( \hat{\ell} \) are \( NJ \times 1 \) vectors.
To derive the cross-industry allocation, note:

\[
\frac{k_{it}^j}{k_{it}^j} = \sum_{o=1}^J \frac{\varphi_{it}^o}{\varphi_{it}^o} \frac{k_{it}^o}{k_{it}^o} \text{k}_{it}^o,
\]

\[
\frac{k_{it}^j}{k_{it}^j} = \frac{\varphi_{it}^j}{\varphi_{it}^j} \frac{k_{it}^j}{k_{it}^j} \text{k}_{it}^j,
\]

\[
\frac{k_{it}^j}{k_{it}^j} = \frac{\varphi_{it}^j}{\varphi_{it}^j} \frac{\varphi_{it}^j}{\varphi_{it}^j} \text{k}_{it}^j.
\]

\[
\ln \left( \frac{k_{it}^j}{k_{it}^j} \right) = \ln \left( \frac{\varphi_{it}^j}{\varphi_{it}^j} \frac{\varphi_{it}^j}{\varphi_{it}^j} \text{k}_{it}^j \right) - \ln \left( \sum_{o=1}^J \frac{\varphi_{it}^o}{\varphi_{it}^o} \frac{\varphi_{it}^o}{\varphi_{it}^o} \text{k}_{it}^o \right) + \ln \left( \frac{k_{it}^j}{k_{it}^j} \right).
\]

Note that:

\[
\ln \left( \sum_{o=1}^J \frac{\varphi_{it}^o}{\varphi_{it}^o} \frac{\varphi_{it}^o}{\varphi_{it}^o} \text{k}_{it}^o \right) \approx \sum_{o=1}^J \frac{\varphi_{it}^o}{\varphi_{it}^o} \frac{\varphi_{it}^o}{\varphi_{it}^o} \ln \left( \frac{\varphi_{it}^o}{\varphi_{it}^o} \right), \quad \frac{\varphi_{it}^o}{\varphi_{it}^o} \approx 1.
\]

Using this result above, we have:

\[
\ln \left( \frac{k_{it}^j}{k_{it}^j} \right) = \ln \left( \frac{\varphi_{it}^j}{\varphi_{it}^j} \frac{\varphi_{it}^j}{\varphi_{it}^j} \text{k}_{it}^j \right) - \sum_{o=1}^J \frac{\varphi_{it}^o}{\varphi_{it}^o} \frac{\varphi_{it}^o}{\varphi_{it}^o} \ln \left( \frac{\varphi_{it}^o}{\varphi_{it}^o} \right) + \ln \left( \frac{k_{it}^j}{k_{it}^j} \right).
\]

In matrix representation, we have:

\[
\tilde{k}_{\ell t} - \tilde{\ell}_{\ell t} = \tilde{\ell}_{\ell t} = \frac{1}{N \times 1} \otimes \left( K \left( \tilde{\vartheta}_{\ell t} + \tilde{\ell}_{\ell t} \right) + k_{it}^{reg} \right). \tag{D.107}
\]

**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.92) relative to the initial steady-state has the following matrix representation:

\[
\tilde{w}_t + \tilde{\ell}_t = \left[ T \left( \tilde{w}_t + \tilde{\ell}_t \right) + \theta (TS - I) \left( w_t - \left( I - \mu \right) \left( k_{it}^j - \ell_t \right) - \tilde{z} \right) \right],
\]

where these matrices have \( N \times J \) elements; we have used \( \vartheta \ln \tau = 0 \); and we again use the superscript \( j \) for capital \( (k_{it}^j) \) to distinguish sector-location capital from aggregate location capital \( (k_{it}^{reg}) \). We can re-write this matrix representation as:

\[
\begin{align*}
\tilde{w}_t &= [I - T + \theta (I - TS)]^{-1} \left[ - (I - T) \tilde{\ell}_t + \theta (I - TS) \left( (I - \mu) \left( k_{it}^j - \tilde{\ell}_t \right) + \tilde{z} \right) \right], \\
\tilde{w}_t &= [I - T + \theta (I - TS)]^{-1} \left[ - (I - T) \tilde{\ell}_t + \theta (I - TS) \left( (I - \mu) \left( k_{it}^j - \tilde{\ell}_t \right) + \tilde{z} \right) \right]. \tag{D.108}
\end{align*}
\]
**Population Flow.** The total derivative of the population flow condition (D.93) relative to the initial steady-state has the following matrix representation:

\[ \tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1}, \]  

(D.109)

where again these matrices have \( NJ \times NJ \) elements.

**Value Function.** The total derivative of the value function relative to the initial steady-state has the following matrix representation:

\[ \tilde{v}_t = (I - S) \tilde{w}_t + S \left[ (I - \mu) \left( \tilde{k}_t^j - \tilde{\ell}_t \right) + \tilde{z} \right] + \tilde{b} + \beta DE_t \tilde{v}_{t+1}, \]  

(D.110)

where again these matrices have \( NJ \times NJ \) elements; we have used \( d \ln \tau = d \ln \kappa = 0 \); and we use the superscript \( j \) for capital \( (k_t^j) \) to distinguish sector-location capital from aggregate location capital \( (k_t^{reg}) \).

**Real Income.** The total derivative of real income relative to the initial steady-state has the following matrix representation:

\[ \tilde{\theta}_t = (I - S) \tilde{w}_t + S \left[ (I - \mu) \left( \tilde{k}_t^j - \tilde{\ell}_t \right) + \tilde{z} \right], \]  

(D.111)

where again these matrices have \( NJ \times NJ \) elements and we have used \( d \ln \tau = 0 \).

**System of Equations for Transition Dynamics.** Collecting together the system of equations for the transition dynamics, we have:

\[ \tilde{\theta}_t = (I - S) \tilde{w}_t + S \left[ (I - \mu) \left( \tilde{k}_t^j - \tilde{\ell}_t \right) + \tilde{z} \right], \]  

(D.112)

\[ \tilde{w}_t = [I - T + \theta (I - TS)]^{-1} \left[ \left( I - T \right) \tilde{\ell}_t + \theta (I - TS) \left[ (I - \mu) \left( \tilde{k}_t^j - \tilde{\ell}_t \right) + \tilde{z} \right] \right], \]  

(D.113)

\[ \tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1}, \]  

(D.114)

\[ \tilde{v}_t = (I - S) \tilde{w}_t + S \left[ (I - \mu) \left( \tilde{k}_t^j - \tilde{\ell}_t \right) + \tilde{z} \right] + \tilde{b} + \beta DE_t \tilde{v}_{t+1}. \]  

(D.115)

\[ \tilde{k}_t^{reg} = \beta (1 - \delta) \tilde{k}_t^{reg} + (1 - \beta (1 - \delta)) K \left( \tilde{\theta}_t + \tilde{\ell}_t \right), \]  

(D.116)

\[ \tilde{k}_t^j - \tilde{\ell}_t^j = \tilde{\theta}_t^j - \sum_{N \times 1} \left( K \left( \tilde{\theta}_t + \tilde{\ell}_t \right) + \tilde{k}_t^{reg} \right). \]  

(D.117)
D.4 Multiple Sector-Regions (Sector-Location Specific Capital)

We consider an economy that consists of many locations indexed by \( i \in \{1, \ldots, N\} \) and many sectors indexed by \( j \in \{1, \ldots, J\} \). Time is discrete and is indexed by \( t \). The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile across sectors and locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate \( \delta \).

D.4.1 Worker Migration Decisions

At the beginning of each period \( t \), the economy inherits a mass of workers in each sector \( j \) and location \( i \) (\( \ell_{it}^j \)), with the total labor endowment of the economy given by \( \bar{\ell} = \sum_{i=1}^{N} \sum_{j=1}^{J} \ell_{it} \). Workers first produce and consume in their sector and location in period \( t \), before observing mobility shocks \( \{\xi_{git}\} \) for all possible sectors \( h \in \{1, \ldots, J\} \) and locations \( g \in \{1, \ldots, N\} \) and deciding where to move for period \( t + 1 \). Workers face bilateral migration costs that vary by sector and location, where \( \kappa_{git}^{hj} \) denotes the cost of moving from sector \( j \) in location \( i \) to sector \( h \) in location \( g \). The value function for a worker in sector \( j \) and location \( i \) at time \( t \) \( (V_{j,w}^{it}) \) is equal to the current flow of utility in that sector and location plus the expected continuation value next period from the optimal choice of sector and location:

\[
V_{j,w}^{it} = \ln u_{it}^{j,w} + \max_{\{g\} \in \{h\}^{1,J}} \left\{ \beta \mathbb{E}_t [V_{g,w}^{h,t+1}] - \kappa_{git}^{hj} + \rho \xi_{git}^{hj} \right\},
\]  

(D.118)

where we use the superscript \( w \) to denote workers; we assume logarithmic flow utility \( (\ln u_{it}^{j,w}) \); \( \beta \) denotes the discount rate; \( \mathbb{E} [\cdot] \) denotes an expectation taken over the distribution for idiosyncratic mobility shocks; \( \rho \) captures the dispersion of idiosyncratic mobility shocks; and we assume \( \kappa_{iit}^{JJ} = 1 \) and \( \kappa_{git}^{hj} > 1 \) for \( g \neq i \) and \( h \neq j \).

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

\[
F(\epsilon) = e^{-e^{-(\epsilon - \bar{\xi})}},
\]  

(D.119)

where \( \bar{\xi} \) is the Euler-Mascheroni constant.
Under this assumption, the expected value for a worker of living in location $i$ at time $t$ ($v_{it}^{j,w}$) can be re-written in the following form:

$$v_{it}^{j,w} = \ln u_{it}^{j,w} + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{K} \left( \exp \left( \beta \mathbb{E}_t v_{gt+1}^{h,w} / \kappa_{git}^{h,j} \right) \right)^{1/\rho},$$  \hspace{1cm} (D.120)

The corresponding probability of migrating from location-sector $ij$ to location-sector $gh$ satisfies a gravity equation:

$$D_{igt}^{jh} = \frac{\left( \exp \left( \beta \mathbb{E}_t v_{gt+1}^{h,w} / \kappa_{git}^{h,j} \right) \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left( \exp \left( \beta \mathbb{E}_t v_{mt+1}^{o,w} / \kappa_{kit}^{o,j} \right) \right)^{1/\rho}}.$$  \hspace{1cm} (D.121)

### D.4.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their utility in the location and sector that period. Worker flow indirect utility in location $n$ and sector $j$ depends on local amenities ($b_{nt}^{j}$), the wage ($u_{nt}^{j}$), and the consumption goods price index ($p_{nt}$):

$$\ln u_{nt}^{j,w} = \ln b_{nt}^{j} + \ln u_{nt}^{j} - \ln p_{nt},$$  \hspace{1cm} (D.122)

where amenities ($b_{nt}$) capture characteristics of a location that make it a more attractive place to live regardless of the wage and cost of consumption goods (e.g. climate and scenic views). In this section of the online appendix, we assume that amenities are exogenous.

The consumption goods price index ($p_{nt}$) in location $n$ depends on the consumption goods price index for each sector $h$ in that location ($p_{nt}^{h}$):

$$p_{nt} = \prod_{h=1}^{J} \left( p_{nt}^{h} \right)^{\psi^{h}}, \hspace{1cm} 0 < \psi^{h} < 1, \hspace{1cm} \sum_{h=1}^{J} \psi^{h},$$  \hspace{1cm} (D.123)

where the consumption goods price index for each sector $h$ in location $n$ depends on the price of the variety sourced from each location $i$ within that sector $h$ ($p_{nit}^{h}$):

$$p_{nt}^{h} = \left[ \sum_{i=1}^{N} \left( p_{nit}^{h} \right)^{-\theta} \right]^{-1/\theta}, \hspace{1cm} \theta = \sigma - 1, \hspace{1cm} \sigma > 1,$$  \hspace{1cm} (D.124)

where $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties; $\theta = \sigma - 1$ is the trade elasticity; and for simplicity, we assume a common elasticity of substitution and trade elasticity across all sectors.

Utility maximization implies that goods consumption expenditure on each sector ($p_{nt}^{h}c_{nt}^{h}$) is a constant share of overall goods consumption expenditure ($p_{nt}c_{nt}$) in each location:

$$p_{nt}^{h}c_{nt}^{h} = \psi^{h}p_{nt}c_{nt}.$$  \hspace{1cm} (D.125)
Using constant elasticity of substitution (CES) demand for individuals varieties of goods, the share location $n$’s expenditure within sector $h$ on the goods produced by location $i$ is:

$$ S_{nit}^h = \frac{\left(p_{nit}^h\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nit}^m\right)^{-\theta}}. \tag{D.126} $$

### D.4.3 Production

Producers in each location $i$ and sector $j$ use labor ($l_{it}^j$) and capital ($k_{it}^j$) to produce output ($y_{it}^j$) of the variety supplied by that location in that sector. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

$$ y_{it}^j = z_{it}^j \left(\frac{l_{it}^j}{\mu^j}\right)^{\mu^j} \left(\frac{k_{it}^j}{1-\mu^j}\right)^{1-\mu^j}, \quad 0 < \mu^j < 1, \tag{D.127} $$

where $z_{it}^j$ denotes productivity in location $i$ in sector $j$ at time $t$. As for amenities above, we assume in this section of the online appendix that productivity is exogenous.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit}^j \geq 1$ units of a good must be shipped from location $i$ in order for one unit to arrive in location $n$, where $\tau_{nit}^j > 1$ for $n \neq i$ and $\tau_{ii}^j = 1$. From profit maximization, the cost to a consumer in location $n$ of sourcing the good produced by location $i$ within sector $j$ is:

$$ p_{nit}^j = \tau_{nit}^j \frac{w_{it}^j \left(\frac{l_{it}^j}{\mu^j}\right)^{\mu^j} \left(\frac{k_{it}^j}{1-\mu^j}\right)^{1-\mu^j}}{z_{it}^j}, \tag{D.128} $$

where $p_{nit}^j$ is the “free on board” price of the good supplied by location $i$ before transport costs; $r_{it}^j$ is the rate of return to capital, which now varies across both sectors $j$ and locations $i$, because capital is specific to both a sector and location.

From profit maximization problem and zero profits, payments for labor and building capital are constant shares of revenue:

$$ w_{it}^j l_{it}^j = \mu^j p_{nit}^j y_{it}^j, \tag{D.129} $$

$$ r_{it}^j k_{it}^j = \left(1-\mu^j\right) p_{nit}^j y_{it}^j, \tag{D.130} $$

where the immobility of capital across sectors once installed implies that the rate of return on capital need not be equalized across sectors and locations out of steady-state ($r_{it}^j \neq r_{nt}^h$).

### D.4.4 Landlord Consumption

Landlords in each location and sector choose their consumption and investment in capital to maximize their intertemporal utility subject to the intertemporal budget constraint. Landlords’
intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

\[ v_{it}^{j,k} = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k}, \quad (D.131) \]

where we use the superscript \( k \) to denote landlords; \( c_{it}^{j,k} \) is the consumption index for landlords in location \( i \) and sector \( j \); and \( \beta \) denotes the discount rate. Since landlords are immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

The consumption goods index for landlords \( (c_{it}^{j,k}) \) takes exactly the same form as for workers and is a Cobb-Douglas aggregate of consumption indexes for each sector, where these consumption indexes for each sector are constant elasticity of substitution (CES) functions of the consumption of varieties from each location. Therefore, the consumption goods price index \( (p_{it}) \) takes the same form as in equation \( (D.123) \), and the consumption goods price index for each sector \( (p_{it}^j) \) takes the same form as in equation \( (D.124) \). Under these assumptions, the landlords’ utility maximization problem is weakly separable. First, we solve for the optimal consumption-savings decision across time periods for overall goods consumption. Second, we solve for the optimal allocation of consumption across sectors within each time period. Third, we solve for the optimal allocation of consumption across location varieties within each sector.

Beginning with landlords’ optimal consumption-saving, we assume that the investment technology for capital in each location and sector uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location and sector can produce one unit of capital for that sector and location using one unit of the consumption index for that sector and location. We assume that capital is geographically immobile once installed and depreciates at a constant rate \( \delta \). The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital \( (r_{it}^j k_{it}^j) \) equals the total value of goods consumption \( (p_{it} c_{it}^{j,k}) \) and net investment \( (p_{it} k_{it+1}^j - (1 - \delta^j) k_{it}^j) \):

\[ r_{it}^j k_{it}^j = p_{it} c_{it}^{j,k} + p_{it} \left( k_{it+1}^j - (1 - \delta^j) k_{it}^j \right). \quad (D.132) \]

Combining landlords’ intertemporal utility \( (D.131) \) and budget constraint \( (D.132) \), their intertemporal optimization problem is:

\[ \max_{\{c_{it}^{j,k}, k_{it+1}^j\}} \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k}, \quad (D.133) \]

subject to

\[ p_{it} c_{it}^{j,k} + p_{it} \left( k_{it+1}^j - (1 - \delta^j) k_{it}^j \right) = r_{it}^j k_{it}^j. \]
We can write this problem as the following Lagrangian:

\[ \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k} - \xi_t^j \left[ p_{it} c_{it}^{j,k} + p_{it} \left( k_{it+1}^j - (1 - \delta^j) k_{it}^j \right) - r_{it}^j k_{it}^j \right]. \]  

(D.134)

The first-order conditions are:

\[ \left\{ c_{it}^{j,k} \right\} \quad \frac{\beta^t}{c_{it}^{j,k}} - p_{it} \xi_t^j = 0; \]

\[ \left\{ k_{it+1}^j \right\} \quad (r_{it+1}^j + p_{it+1} (1 - \delta^j)) \xi_{t+1}^j - p_{it} \xi_t^j = 0, \]

Together these first-order conditions imply:

\[ \frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} = \beta \frac{p_{it} \mu_t^j}{p_{it+1} \mu_{t+1}^j} = \beta \left( \frac{r_{it+1}^j}{p_{it+1} + (1 - \delta^j)} \right), \]  

(D.135)

where the transversality condition implies:

\[ \lim_{t \to \infty} \beta^t \frac{k_{it+1}^j}{c_{it}^{j,k}} = 0. \]

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords’ optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

\[ p_{it} c_{it}^{j,k} = (1 - \beta) \left( r_{it}^j + p_{it} \left( 1 - \delta^j \right) \right) k_{it}^j, \]  

(D.136)

\[ k_{it+1}^j = \beta \left( r_{it}^j / p_{it} + (1 - \delta^j) \right) k_{it}^j. \]  

(D.137)

Substituting the consumption policy function (D.136) into the Euler equation (D.135), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

\[ \frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} = \left( \frac{r_{it+1}^j / p_{it+1} + (1 - \delta^j)}{r_{it}^j / p_{it} + (1 - \delta^j)} \right) \frac{k_{it+1}^j}{k_{it}^j}, \]

\[ = \beta \left( \frac{r_{it+1}^j / p_{it+1} + (1 - \delta^j)}{r_{it+1}^j / p_{it+1} + (1 - \delta^j)} \right). \]

Given this optimal consumption-saving decision in equations (D.136)-(D.137), our assumption of Cobb-Douglas preferences across sectors implies that landlords allocate constant shares of consumption expenditure across sectors within time periods, as for workers in equation (D.125). Similarly, our assumption of constant elasticity of substitution (CES) preferences across locations within sectors implies that landlords in location \( n \) allocate the same share of expenditure in location \( i \) within sector \( j \) as for workers in equation (D.126).
D.4.5 Market Clearing

Goods market clearing implies that revenue in each region-sector equals expenditure on the goods produced by that region-sector:

\[ p_j^t y_j^t = \sum_{n=1}^{N} \psi_j^t S_n^j \sum_{h=1}^{J} (w_{nt}^h \ell_{nt}^h + r_{nt}^h k_{nt}^h), \]

\[ w_{jt}^h \ell_{jt}^h + r_{nt}^j k_{nt}^j = \sum_{n=1}^{N} \psi_j^t S_n^j \sum_{h=1}^{J} (w_{nt}^h \ell_{nt}^h + r_{nt}^j k_{nt}^j), \]

\[ w_{jt}^h \ell_{jt}^h + \frac{1 - \mu_j^h}{\mu_j^h} w_{jt}^h \ell_{jt}^h = \sum_{n=1}^{N} \psi_j^t S_n^j \sum_{h=1}^{J} \left( w_{nt}^h \ell_{nt}^h + \frac{1 - \mu_h^j}{\mu_h^j} w_{nt}^h \ell_{nt}^h \right), \]

\[ \left( \frac{1}{\mu_j^h} \right) w_{jt}^h \ell_{jt}^h = \sum_{n=1}^{N} \sum_{h=1}^{J} \psi_j^t S_n^j \left( \frac{1}{\mu_h^j} \right) w_{nt}^h \ell_{nt}^h. \] (D.138)

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords’ income from the ownership of capital equals payments for its use. Using the property that payments to capital and labor are constant shares of total revenue in equations (D.129) and (D.130), we can write payments for capital in each sector as:

\[ r_{jt}^j k_{jt}^j = \frac{1 - \mu_j^j}{\mu_j^j} w_{jt}^j \ell_{jt}^j. \] (D.139)

D.4.6 General Equilibrium

Given the state variables \( \{ \ell_{jt}, k_{jt} \} \) for each sector \( j \) and location \( i \), the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \( \{ \ell_{jt}, k_{jt}, w_{jt}, v_{jt} \}_{t=0}^{\infty} \). All other endogenous variables of the model can be recovered as a function of these variables.

**Capital Accumulation:** Using capital market clearing (D.139), the price index (D.123) and the equilibrium pricing rule (D.128), the capital accumulation equation (D.137) can be re-written as:

\[ k_{jt+1}^j = \beta \frac{1 - \mu_j^j}{\mu_j^j} \frac{w_{jt}^j \ell_{jt}^j}{\ell_{jt}^j} + \beta (1 - \delta^j) k_{jt}^j, \] (D.140)

\[ p_{nt} = \prod_{h=1}^{H} \left[ \sum_{i=1}^{N} \left( w_{jt}^j \left( \frac{1 - \mu_j^j}{\mu_j^j} \right) - \mu_j^j \left( \ell_{jt}^j / k_{jt}^j \right) \right)^{-\theta} \right]. \] (D.141)
**Goods Market Clearing:** Using the equilibrium pricing rule (D.128), the expenditure share (D.126) and capital market clearing (D.139) in the goods market clearing condition (D.138), we obtain:

\[
\left( \frac{1}{\mu^j} \right) w^{j}_{it} \ell^{j}_{it} = \sum_{n=1}^{N} \sum_{h=1}^{J} q^{j}_{n} S^{j}_{nit} \left( \frac{1}{\mu^h} \right) w^{h}_{nt} \ell^{h}_{nt}, \quad \text{(D.142)}
\]

\[
S^{h}_{nit} = \frac{\left( w^{j}_{it} \left( \ell^{j}_{it} / k_{it}^{j} \right)^{1-\mu^j} \tau^{j}_{nit} / z^{j}_{it} \right)^{-\theta}}{\sum_{m=1}^{N} \left( w^{j}_{mt} \left( \ell^{j}_{mt} / k_{mt}^{j} \right)^{1-\mu^j} \tau^{j}_{nmt} / z^{j}_{mt} \right)^{-\theta}}, \quad \text{and} \quad T^{jh}_{int} = \frac{\left( \frac{1}{\mu^h} \right) w^{h}_{nt} \ell^{h}_{nt}}{\left( \frac{1}{\mu^j} \right) w^{j}_{it} \ell^{j}_{it}}, \quad \text{(D.143)}
\]

where \( S^{h}_{nit} \) is the expenditure share of importer \( n \) on each exporter \( i \) at time \( t \), and we have defined \( T^{jh}_{int} \) as the corresponding income share of exporter \( i \) from each importer \( n \) at time \( t \). Note that the order of subscripts switches between the expenditure share \( (S^{h}_{nit}) \) and the income share \( (T^{jh}_{int}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Population Flow:** Using the outmigration probabilities (D.121), the population flow condition for the evolution of the employment distribution over time is given by:

\[
\ell^{h}_{gt+1} = \sum_{i=1}^{N} \sum_{j=1}^{J} D^{j}_{igt} \ell^{j}_{it}, \quad \text{(D.144)}
\]

\[
D^{j}_{igt} = \frac{\left( \exp \left( \beta \mathbb{E}_{t}^{j,w} v^{j}_{igt+1} \right) / k^{j}_{igt+1} \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left( \exp \left( \beta \mathbb{E}_{t}^{o,w} v^{o}_{mgt+1} \right) / k^{o}_{mgt+1} \right)^{1/\rho}}, \quad E^{j}_{igt} = \frac{\ell^{j}_{it} D^{j}_{igt}}{\ell^{h}_{gt+1}}, \quad \text{(D.145)}
\]

where \( D^{j}_{igt} \) is the outmigration probability from sector \( j \) in location \( i \) to sector \( h \) in location \( g \) between time \( t \) and \( t+1 \), and we have defined \( E^{j}_{igt} \) as the corresponding immigration probability to sector \( h \) in location \( g \) from sector \( j \) in location \( i \) between time \( t \) and \( t+1 \). Note that the order of subscripts switches between the outmigration probability \( (D^{j}_{igt}) \) and the immigration probability \( (E^{j}_{igt}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Worker Value Function:** Using the worker indirect utility function (D.122) in the value function (D.120), the expected value from living in location \( n \) at time \( t \) can be written as:

\[
v^{j,w}_{it} = \ln \left[ \frac{b^{j}_{nt} u^{j}_{nt}}{p_{nt}} \right] + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{K} \left( \exp \left( \beta \mathbb{E}_{t}^{h,w} v^{h}_{glt+1} \right) / k^{h}_{glt} \right)^{1/\rho}. \quad \text{(D.146)}
\]
D.4.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

Prices. Using the relationship between capital and labor payments (D.139), the pricing rule (D.128) can be re-written as follows:

\[
p^j_{nit} = \frac{\tau^j_{nit} w^j_{it} \left(1 - \mu^j\right)}{z^j_{it}} \left(\frac{1}{\chi^j_{it}}\right)^{1-\mu^j},
\]

(D.147)

where \(\chi^j_{it}\) is the capital-labor ratio in sector \(j\) in region \(i\):

\[
\chi^j_{it} = \frac{k^j_{it}}{l^j_{it}}.
\]

Totally differentiating this pricing rule, we have:

\[
\frac{dp^j_{nit}}{p^j_{nit}} = \frac{d\tau^j_{nit}}{\tau^j_{nit}} + \frac{d\tau^j_{nit}}{\tau^j_{nit}} - (1 - \mu^j) \frac{d\chi^j_{it}}{\chi^j_{it}} - \frac{dz^j_{it}}{z^j_{it}},
\]

\[
d \ln p^j_{nit} = d \ln \tau^j_{nit} + d \ln w^j_{it} - (1 - \mu^j) d \ln \chi^j_{it} - d \ln z^j_{it}.
\]

(D.148)

Expenditure Shares. Totally differentiating the expenditure share equation (D.126), we get:

\[
\frac{dS^j_{nit}}{S^j_{nit}} = \theta \left( \sum_{h=1}^{N} S^j_{nht} \frac{dp^j_{nht}}{p^j_{nht}} - \frac{dp^j_{nit}}{p^j_{nit}} \right),
\]

(D.149)

\[
d \ln S^j_{nit} = \theta \left( \sum_{h=1}^{N} S^j_{nht} d \ln p^j_{nht} - d \ln p^j_{nit} \right).
\]

Price Indices. Totally differentiating the industry consumption goods price index in equation (D.123), we have:

\[
\frac{dp^j_{nit}}{p^j_{nit}} = \sum_{m=1}^{N} S^j_{nmt} \frac{dp^j_{nmt}}{p^j_{nmt}},
\]

(D.150)

\[
d \ln p^j_{nit} = \sum_{m=1}^{N} S^j_{nmt} d \ln p^j_{nmt}.
\]
Migration Shares. Totally differentiating the outmigration share equation (D.121), we get:

\[
\frac{dD^{j_{ht}}}{D^{j_{ht}}} = \frac{1}{\rho} \left[ \left( \beta \mathbb{E}_t \delta v^{h,w}_{gt+1} - d \kappa^{h_{jt}}_{git} \right) - \sum_{m=1}^{N} \sum_{o=1}^{J} D^{j_{om}} \left( \beta \mathbb{E}_t \delta v^{o,w}_{mt+1} - d \kappa^{o_j}_{mit} \right) \right],
\]

\[
d \ln D^{j_{ht}} = \frac{1}{\rho} \left[ \left( \beta \mathbb{E}_t \delta v^{h,w}_{gt+1} - d \ln \kappa^{h_{jt}}_{git} \right) - \sum_{m=1}^{N} \sum_{o=1}^{J} D^{j_{om}} \left( \beta \mathbb{E}_t \delta v^{o,w}_{mt+1} - d \ln \kappa^{o_j}_{mit} \right) \right].
\]

Real Income. Totally differentiating real income we have:

\[
d \ln \left( \frac{w^{j}_{it}}{p_{it}} \right) = d \ln w^{j}_{it} - d \ln p_{it},
\]

\[
d \ln \left( \frac{w^{j}_{it}}{p_{it}} \right) = d \ln w^{j}_{it} - \sum_{h=1}^{J} \psi^{h} \sum_{m=1}^{N} S^{h}_{m_{it}} d \ln p^{h}_{m_{it}},
\]

\[
d \ln \left( \frac{w^{j}_{it}}{p_{it}} \right) = d \ln w^{j}_{it} - \sum_{h=1}^{J} \sum_{m=1}^{N} S^{h}_{m_{it}} \left[ d \ln \tau^{h}_{m_{it}} + d \ln w^{h}_{m_{it}} - (1 - \mu^{h}) d \ln \chi^{h}_{m_{it}} - d \ln z^{h}_{m_{it}} \right],
\]

Goods Market Clearing. Totally differentiating the goods market clearing condition (D.138), we have:

\[
\frac{dw^{j}_{it}}{w^{j}_{it}} + \frac{d\ell^{j}_{it}}{\ell^{j}_{it}} = \sum_{n=1}^{N} \sum_{h=1}^{J} \mathbb{E}^{j_{nt}}_{m_{it}} \left( \frac{1}{\mu^{h}} \right) \left( \frac{dw^{h}_{nt}}{w^{h}_{nt}} \ell^{h}_{it} - \frac{d\ell^{h}_{nt}}{\ell^{h}_{nt}} + \frac{dS^{j}_{nt}}{S^{j}_{nt}} \right).
\]

Using our result for the derivative of expenditure shares in equation (D.149) above, we can rewrite this as:

\[
\frac{dw^{j}_{it}}{w^{j}_{it}} + \frac{d\ell^{j}_{it}}{\ell^{j}_{it}} = \sum_{n=1}^{N} \sum_{h=1}^{J} T^{j}_{nt} \left( \frac{dw^{h}_{nt}}{w^{h}_{nt}} + \frac{d\ell^{h}_{nt}}{\ell^{h}_{nt}} + \theta \left( \frac{dS^{j}_{nt}}{S^{j}_{nt}} \right) \right),
\]

\[
T^{j}_{nt} = \frac{\psi^{j} S^{j}_{nt} \left( \frac{1}{\mu^{h}} \right) w^{h}_{nt} \ell^{h}_{it}}{\left( \frac{1}{\mu^{h}} \right) w^{h}_{nt} \ell^{h}_{it}}.
\]

\[
\left[ d \ln w^{j}_{it} + d \ln \ell^{j}_{it} \right] = \left[ + \theta \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{h=1}^{J} T^{j}_{nt} \left( d \ln w^{h}_{nt} + d \ln \ell^{h}_{nt} \right) \right.
\]

\[
\left. - \theta \left( d \ln \tau^{h}_{m_{it}} + d \ln w^{h}_{m_{it}} - (1 - \mu^{h}) d \ln \chi^{h}_{m_{it}} - d \ln z^{h}_{m_{it}} \right) \right].
\]

Population Flow. Totally differentiating the population flow condition (D.144) we have:

\[
\frac{d\ell^{h}_{g_{t+1}}}{\ell^{h}_{g_{t+1}}} = \sum_{i=1}^{N} \sum_{j=1}^{J} E^{hij}_{g_{it}} \left[ \frac{d\ell^{j}_{it}}{\ell^{j}_{it}} + \frac{dD^{j_{ht}}}{D^{j_{ht}}} \right],
\]

\[
d \ln \ell^{h}_{g_{t+1}} = \sum_{i=1}^{N} \sum_{j=1}^{J} E^{hij}_{g_{it}} \left[ \frac{d\ell^{j}_{it}}{\ell^{j}_{it}} + \frac{1}{\rho} \left( \beta \mathbb{E}_t \delta v^{h}_{g_{t+1}} - d \ln \kappa^{h_{jt}}_{git} - \sum_{m=1}^{N} \sum_{o=1}^{J} D^{j_{om}} \left( \beta \mathbb{E}_t \delta v^{o,w}_{mt+1} - d \ln \kappa^{o_j}_{mit} \right) \right) \right].
\]
**Value Function.** Note that the value function can be re-written using the following results:

\[
 v_{it}^{j,w} = \ln \left( \frac{w_{it}^j}{\Pi_{o=1}^J \left( \sum_{m=1}^N p_{imt}^o \right)^{-\psi_o/\theta}} \right) - \frac{1}{\theta} \ln S_{sit}^o - \ln p_{it}^o + \rho \ln \sum_{g=1}^N \sum_{h=1}^J \left( \exp \left( \frac{\beta E_t v_{gt+1}^{h,w}}{\kappa_{git}^{h}} \right) \right)^{1/\rho},
\]

\[
 D.148
 \]

\[
 v_{it}^{j,w} = \ln w_{it}^j + \sum_{o=1}^J \psi^o \left[ -\frac{1}{\theta} \ln S_{sit}^o - \ln p_{it}^o \right] + \ln b_{it}^j + \beta E_t v_{it+1}^{j,w} - \rho \ln D_{it}^{j,j},
\]

(D.155)

Totally differentiating the value function (D.155) we have:

\[
 dv_{it}^{j,w} = d \ln w_{it}^j + \sum_{o=1}^J \psi^o \left[ -\frac{1}{\theta} \ln S_{sit}^o - d \ln p_{it}^o \right] + d \ln b_{it}^j + \beta E_t d v_{it+1}^{j,w} - \rho \ln D_{it}^{j,j},
\]

\[
 d \ln S_{it}^o = \rho \ln \left( \sum_{m=1}^N \ln p_{imt}^o \right),
\]

\[
 d \ln D_{it}^{j,j} = \frac{1}{\rho} \left[ \beta E_t d v_{it+1}^{j,w} - d \ln \kappa_{it}^{j,j} \right] - \sum_{m=1}^N \sum_{h=1}^J D_{imt}^{j,h} \left( \beta E_t v_{mt+1}^{h,w} - d \ln \kappa_{mt}^{h} \right) \cdot
\]

Using these results in the derivative of the value function, we have:

\[
 dv_{it}^{j,w} = \left[ d \ln w_{it}^j - \sum_{o=1}^J \psi^o S_{imt}^o \ln p_{imt}^o + \ln b_{it}^j + \sum_{m=1}^N \sum_{h=1}^J D_{imt}^{j,h} \left( \beta E_t d v_{mt+1}^{h,w} - d \ln \kappa_{mt}^{h} \right) \right].
\]

where we have used \( d \ln \kappa_{it}^{j,j} = 0 \). Using the total derivative of the pricing rule (D.148), we can re-write this derivative of the value function as follows:

\[
 dv_{it}^{j,w} = \left[ d \ln w_{it}^j - \sum_{o=1}^J \psi^o \sum_{m=1}^N S_{imt}^o \ln p_{imt}^o + \ln w_{it}^j - (1 - \mu^o) \ln \chi_{mt}^o - \ln z_{mt}^o \right] + \ln b_{it}^j + \sum_{m=1}^N \sum_{h=1}^J D_{imt}^{j,h} \left( \beta E_t d v_{mt+1}^{h,w} - d \ln \kappa_{mt}^{h} \right).
\]

(D.156)

**D.4.8 Steady-state**

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: \( k_{it+1}^j = k_{it}^j, \ell_{it+1}^j = \ell_{it}^j, w_{it+1}^{j,s} = w_{it}^{j,s} \) and \( v_{it+1}^{j,s} = v_{it}^{j,s} = v_{it}^{s} \), where we use an asterisk to denote a steady-state value. We consider small common shocks to productivities across all sectors (\( d \ln z \)) and to amenities across all sectors (\( d \ln b \)) in each location, holding constant the economy’s aggregate labor endowment (\( d \ln \ell = 0 \)), trade costs (\( d \ln \tau = 0 \)) and commuting costs (\( d \ln \kappa = 0 \)).
Capital Accumulation. From the capital accumulation equation (D.140), the steady-state stock of building capital solves:

\[
\begin{align*}
    k_j^* &= \beta \left[ \frac{r_j}{p_i} + (1 - \delta^j) \right] k_i^* \\
    (1 - \beta (1 - \delta^j)) k_i^* &= \beta \frac{r_j}{p_i} k_j^*.
\end{align*}
\]

From the relationship between labor and capital payments, we have:

\[
\frac{r_j^i k_j^i}{p_i^i} = \frac{1 - \mu^j w_j^i}{p_i^i}.
\]

Using this result in the expression for the steady-state capital stock above, we have:

\[
(1 - \beta (1 - \delta^j)) k_i^* = \beta \frac{1 - \mu^j w_j^i}{\mu^j} \frac{k_j^*}{p_i^*},
\]

Totally differentiating, we have:

\[
d \ln \chi_j^* = d \ln \left( \frac{w_j^*}{p_i^*} \right).
\]

From the total derivative of real income (D.152) above, this becomes:

\[
d \ln \chi_j^* = d \ln w_j^* - \sum_{m=1}^{N} \sum_{h=1}^{J} \psi^h S^h_{imt} \left[ d \ln w^h_{mt} - (1 - \mu^h) d \ln \chi^h_{mt} - d \ln z_{mt} \right].
\]

which has the matrix representation:

\[
d \ln \chi^* = d \ln w^* - S \left( d \ln w + (I - \mu) d \ln \chi^* + d \ln z \right),
\]

where \( d \ln \chi^* \) and \( d \ln w^* \) are \( NJ \times 1 \) vectors; \( S \) is a \( NJ \times NJ \) matrix with elements:

\[
S_{imt} = S^j_{imt} = \sum_{h=1}^{J} \psi^h S^h_{imt}.
\]

and \( \mu \) is \( NJ \times NJ \) diagonal matrix whose \((ij)\)-th element on the diagonal is \( \mu^j \).

Goods Market Clearing. The total derivative of the goods market clearing condition (D.153) has the following matrix representation:

\[
d \ln w_t + d \ln \ell_t = T (d \ln w_t + d \ln \ell_t) + \theta (TS - I) \left( d \ln w - (I - \mu) d \ln \chi_t - d \ln z \right),
\]

where these matrices have \( NJ \times NJ \) elements. In steady-state we have:

\[
d \ln w^* + d \ln \ell^* = T (d \ln w^* + d \ln \ell^*) + \theta (TS - I) (d \ln w^* - (I - \mu) d \ln \chi^* - d \ln z).
\]
Population Flow. The total derivative of the population flow condition (D.154) has the following matrix representation:

$$\text{d} \ln \ell_{t+1} = \mathbf{E} \text{d} \ln \ell_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \text{d}v_{t+1},$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$\frac{\beta}{\rho} (I - E)^{-1} (I - ED) \text{d}v^*.$$  \hspace{1cm} (D.159)

Value function. The total derivative of the value function has the following matrix representation:

$$\text{d} v_t = (I - S) \text{d} \ln w_t + S (\text{d} \ln z + (1 - \mu) \text{d} \ln \chi_t) + \text{d} \ln b + \beta D \mathbb{E}_t \text{d}v_{t+1},$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$\text{d} v^* = (I - \beta D)^{-1} [(I - S) \text{d} \ln w^* + S (\text{d} \ln z + (I - \mu) \text{d} \ln \chi^*) + \text{d} \ln b].$$  \hspace{1cm} (D.160)

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\text{d} \ln \chi^* = (I - S) \text{d} \ln w^* + S (I - \mu) \text{d} \ln \chi^* + S \text{d} \ln z.$$  \hspace{1cm} (D.161)

$$\text{d} \ln w^* + \text{d} \ln \ell^* = T (\text{d} \ln w^* + \text{d} \ln \ell^*) + \theta (TS - I) (\text{d} \ln w^* - (I - \mu) \text{d} \ln \chi^* - \text{d} \ln z).$$  \hspace{1cm} (D.162)

$$\text{d} \ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) \text{d}v^*.$$  \hspace{1cm} (D.163)

$$\text{d} v^* = (I - \beta D)^{-1} [(I - S) \text{d} \ln w^* + S (\text{d} \ln z + (I - \mu) \text{d} \ln \chi^*) + \text{d} \ln b].$$  \hspace{1cm} (D.164)

D.4.9 Transition Dynamics

Suppose that the economy starts from an initial steady-state. Consider a small shock to productivity ($\text{d} \ln z$) and amenities ($\text{d} \ln b$) in each sector and location, holding constant the economy’s aggregate labor endowment ($\text{d} \ln \widetilde{\ell} = 0$), trade costs ($\text{d} \ln \tau = 0$) and commuting costs ($\text{d} \ln \kappa = 0$). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that $\tilde{\ell}_{it+1} = \ell_{it+1} - \ell^*_t$, for all variables except for the worker value function $v_{it}$, where with a slight abuse of notation we use $\widetilde{v}_{it} = v_{it} - v^*_i$ to denote the deviation in levels for the worker value function.
**Capital Accumulation.** From the capital accumulation equation (D.140), we have:

\[ k_{jt+1}^j = \beta \frac{r_j^j}{p_{it}} k_{jt}^j + \beta \left( 1 - \delta^j \right) k_{jt}^j. \]

From the relationship between labor and capital payments, we have:

\[ \frac{r_j^j}{p_{it}} k_{jt}^j = 1 - \mu_j^j w_{jt}^j \hat{\ell}_{jt}^j. \]

Using this result in the capital accumulation equation above, we have:

\[ k_{jt+1}^j = \beta \left( 1 - \delta^j \right) k_{jt}^j + \beta \frac{1 - \mu_j^j w_{jt}^j \hat{\ell}_{jt}^j}{p_{it}}. \]

while in steady-state we have:

\[ k_{jt+1}^{j*} = \beta \left( 1 - \delta^j \right) k_{jt}^{j*} + \beta \frac{1 - \mu_j^j w_{jt}^{j*} \hat{\ell}_{jt}^{j*}}{p_{it}^{j*}}. \]

\[ \chi_{jt+1}^{j*} = \beta \left( 1 - \delta^j \right) \chi_{jt}^{j*} + \beta \frac{1 - \mu_j^j w_{jt}^{j*}}{p_{it}^{j*}}. \]

\[ \chi_{jt+1}^{j*} = \beta \left( 1 - \delta^j \right) \chi_{jt}^{j*} + \beta \frac{1 - \mu_j^j w_{jt}^{j*}}{p_{it}^{j*}}. \]

while in steady-state we have:

\[ \frac{k_{jt+1}^{j*}}{\ell_{jt}^{j*}} = \beta \left( 1 - \delta^j \right) \frac{k_{jt}^{j*}}{\ell_{jt}^{j*}} + \beta \frac{1 - \mu_j^j w_{jt}^{j*}}{p_{it}^{j*}}, \]

\[ \chi_{jt+1}^{j*} = \beta \left( 1 - \delta^j \right) \chi_{jt}^{j*} + \beta \frac{1 - \mu_j^j w_{jt}^{j*}}{p_{it}^{j*}}. \]

\[ \chi_{jt+1}^{j*} = \beta \left( 1 - \delta^j \right) \chi_{jt}^{j*} + \beta \frac{1 - \mu_j^j w_{jt}^{j*}}{p_{it}^{j*}}. \]

Dividing both sides of equation (D.165) by \( \chi_{jt}^{j*} \), we have:

\[ \frac{\chi_{jt+1}^{j*} \hat{\ell}_{jt+1}^{j*}}{\chi_{jt}^{j*} \hat{\ell}_{jt}^{j*}} = \beta \left( 1 - \delta^j \right) \frac{\chi_{jt}^{j*}}{\chi_{jt}^{j*}} + \beta \frac{1 - \mu_j^j w_{jt}^{j*}}{p_{it}^{j*}}. \]

which using (D.166) can be re-written as:

\[ \frac{\chi_{jt+1}^{j*} \hat{\ell}_{jt+1}^{j*}}{\chi_{jt}^{j*} \hat{\ell}_{jt}^{j*}} = \beta \left( 1 - \delta^j \right) \frac{\chi_{jt}^{j*}}{\chi_{jt}^{j*}} + \beta \left( 1 - \delta^j \right) \frac{w_{jt}^{j*}}{p_{it}^{j*}}. \]

which can be further re-written as:

\[ \frac{\chi_{jt+1}^{j*} \hat{\ell}_{jt+1}^{j*}}{\chi_{jt}^{j*} \hat{\ell}_{jt}^{j*}} - 1 = \beta \left( 1 - \delta^j \right) \frac{\chi_{jt}^{j*}}{\chi_{jt}^{j*}} + \beta \left( 1 - \delta^j \right) \frac{w_{jt}^{j*}}{p_{it}^{j*}} - 1, \]

\[ \frac{\chi_{jt+1}^{j*} \hat{\ell}_{jt+1}^{j*}}{\chi_{jt}^{j*} \hat{\ell}_{jt}^{j*}} - 1 = \beta \left( 1 - \delta^j \right) \left( \frac{\chi_{jt}^{j*}}{\chi_{jt}^{j*}} - 1 \right) + \beta \left( 1 - \delta^j \right) \left( \frac{w_{jt}^{j*}}{p_{it}^{j*}} - 1 \right). \]

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Noting that:

\[
\frac{x_{it}}{x_i^*} - 1 \simeq \ln \left( \frac{x_{it}}{x_i^*} \right),
\]

\[
\frac{\chi^j_{it+1}}{\chi^j_i} \frac{\ell^j_{it+1}}{\ell^j_i} - 1 \simeq \ln \left( \frac{\chi^j_{it+1}}{\chi^j_i} \frac{\ell^j_{it+1}}{\ell^j_i} \right),
\]

we have:

\[
\ln \left( \frac{\chi^j_{it+1}}{\chi^j_i} \right) + \ln \left( \frac{\ell^j_{it+1}}{\ell^j_i} \right) = \beta \left( 1 - \delta^j \right) \ln \left( \frac{\chi^j_i}{\chi^j_i} \right) + \left( 1 - \beta \left( 1 - \delta^j \right) \right) \ln \left( \frac{w^j_{it}}{w^j_i} \frac{\ell^j_{it}}{\ell^j_i} \right),
\]

\[
\ln \left( \frac{\chi^j_{it+1}}{\chi^j_i} \right) + \ln \left( \frac{\ell^j_{it+1}}{\ell^j_i} \right) = \beta \left( 1 - \delta^j \right) \ln \left( \frac{\chi^j_i}{\chi^j_i} \right) + \left( 1 - \beta \left( 1 - \delta^j \right) \right) \ln \left( \frac{w^j_{it}}{w^j_i} \frac{\ell^j_{it}}{\ell^j_i} \right),
\]

which can be re-written as follows:

\[
\tilde{\chi}_{it+1} = \beta \left( 1 - \delta^j \right) \tilde{\chi}_{it} + \left( 1 - \beta \left( 1 - \delta^j \right) \right) \left( \tilde{w}_{it} - \tilde{p}_{it} \right) - \tilde{\ell}_{it+1} + \tilde{\ell}_{it},
\]

We can rewrite this relationship in matrix form as:

\[
\tilde{\chi}_{t+1} = \beta \left( I - \delta \right) \tilde{\chi}_{t} + \left( I - \beta \left( I - \delta \right) \right) \left( \tilde{w}_{t} - \tilde{p}_{t} \right) - \tilde{\ell}_{t+1} + \tilde{\ell}_{t},
\]

where these matrices have \(NJ \times NJ\) elements. Now, from the total derivative of real income (D.152), we have:

\[
\tilde{w}_{it} - \tilde{p}_{it} = (I - S) \tilde{w}_{t} + S (I - \mu) \tilde{\chi}_{t} + S \tilde{z}.
\]

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

\[
\tilde{\chi}_{t+1} = \left[ \beta \left( I - \delta \right) + \left( I - \beta \left( I - \delta \right) \right) S \left[ (I - \mu) \tilde{\chi}_{t} + S \tilde{z} \right] \right] + \left( I - \beta \left( I - \delta \right) \right) \left( I - S \right) \tilde{w}_{t} - \tilde{\ell}_{t+1} + \tilde{\ell}_{t},
\]

\[
\text{(D.167)}
\]

**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.153) relative to the initial steady-state has the following matrix representation:

\[
\tilde{w}_{t} + \tilde{\ell}_{t} = T \left( \tilde{w}_{t} + \tilde{\ell}_{t} \right) + \theta \left( TS - I \right) \left( \tilde{w}_{t} - \left( I - \mu \right) \tilde{\chi}_{t} - \tilde{z} \right),
\]

where these matrices have \(NJ \times NJ\) elements and we have used \(d \ln \tau = 0\). This expression can be re-written as:

\[
\tilde{w}_{t} = [I - T + \theta \left( I - TS \right)]^{-1} \left[ - \left( I - T \right) \tilde{\ell}_{t} + \theta \left( I - TS \right) \left[ (I - \mu) \tilde{\chi}_{t} + \tilde{z} \right] \right].\]

\[
\text{(D.168)}
\]
Population Flow. The total derivative of the population flow condition \( (D.154) \) relative to the initial steady-state has the following matrix representation:

\[
\tilde{\ell}_{t+1} = \tilde{E} \tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \tilde{v}_{t+1}, \tag{D.169}
\]

where again these matrices have \( NJ \times NJ \) elements.

Value function. The total derivative of the value function relative to the initial steady-state has the following matrix representation:

\[
\tilde{v}_t = (I - S) \tilde{w}_t + S [(I - \mu) \tilde{\chi}_t + \tilde{z}] + \tilde{b} + \beta D \mathbb{E}_t \tilde{v}_{t+1}, \tag{D.170}
\]

where again these matrices have \( NJ \times NJ \) elements and we have used \( d \ln \tau = 0 \) and \( d \ln \kappa = 0 \).

System of Equations for Transition Dynamics. Collecting together the system of equations for the transition dynamics, we have:

\[
\tilde{\chi}_{t+1} = \begin{bmatrix}
\beta \left( \frac{1}{2} \right) & \left( I - \delta \right) I + \left( \frac{1}{2} \delta \right) \left( I - \delta \right) S \left( I - \mu \right) \tilde{\chi}_t + \tilde{z} \\
& + \left( I - \beta \left( I - \delta \right) \right) \left( I - S \right) \tilde{w}_t - \tilde{\ell}_{t+1} + \tilde{\ell}_t 
\end{bmatrix}, \tag{D.171}
\]

\[
\tilde{w}_t = \left[ I - T + \theta (I - TS) \right]^{-1} \left[ - (I - T) \tilde{\ell}_t + \theta (I - TS) [I - \mu] \tilde{\chi}_t + \tilde{z} \right]. \tag{D.172}
\]

\[
\tilde{\ell}_{t+1} = \tilde{E} \tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t \tilde{v}_{t+1}. \tag{D.173}
\]

\[
\tilde{v}_t = (I - S) \tilde{w}_t + S \left( I - \mu \right) \tilde{\chi}_t + \tilde{z} + \tilde{b} + \beta D \mathbb{E}_t \tilde{v}_{t+1}. \tag{D.174}
\]

D.5 Input-Output Linkages (Sector-Location Specific Capital)

We consider an economy that consists of many locations indexed by \( i \in \{1, \ldots, N\} \) and many sectors indexed by \( j \in \{1, \ldots, J\} \). Time is discrete and is indexed by \( t \). The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile across sectors and locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate \( \delta \).
D.5.1 Worker Migration Decisions

At the beginning of each period $t$, the economy inherits a mass of workers in each sector $j$ and location $i$ ($\ell^j_{it}$), with the total labor endowment of the economy given by $\bar{\ell} = \sum_{i=1}^{N} \sum_{j=1}^{J} \ell^j_{it}$.

Workers first produce and consume in their sector and location in period $t$, before observing mobility shocks $\{\epsilon^h_{git}\}$ for all possible sectors $h \in \{1, \ldots, J\}$ and locations $g \in \{1, \ldots, N\}$ and deciding where to move for period $t + 1$. Workers face bilateral migration costs that vary by sector and location, where $\gamma^j_{git}$ denotes the cost of moving from sector $j$ in location $i$ to sector $h$ in location $g$. The value function for a worker in sector $j$ and location $i$ at time $t$ ($V^{j,w}_{it}$) is equal to the current flow of utility in that sector and location plus the expected continuation value next period from the optimal choice of sector and location:

$$V^{j,w}_{it} = \ln u^{j,w}_{it} + \max_{\{g\}^N \{h\}^J} \left\{ \beta \mathbb{E}_t [V^{h,w}_{gt+1}] - \gamma^j_{git} + \rho \epsilon^h_{git} \right\},$$

where we use the superscript $w$ to denote workers; we assume logarithmic flow utility ($\ln u^{j,w}_{it}$); $\beta$ denotes the discount rate; $\mathbb{E} [\cdot]$ denotes an expectation taken over the distribution for idiosyncratic mobility shocks; $\rho$ captures the dispersion of idiosyncratic mobility shocks; and we assume $\kappa^{j,j}_{iit} = 1$ and $\kappa^{h,j}_{git}$ > 1 for $g \neq i$ and $h \neq j$.

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

$$F(\epsilon) = e^{-e^{-(\epsilon - \bar{\gamma})}},$$

where $\bar{\gamma}$ is the Euler-Mascheroni constant.

Under this assumption, the expected value for a worker of living in location $i$ at time $t$ ($V^{j,w}_{it}$) can be re-written in the following form:

$$V^{j,w}_{it} = \ln u^{j,w}_{it} + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{J} \left( \exp \left( \beta \mathbb{E}_t V^{h,w}_{gt+1} \right) / \kappa^{h,j}_{git} \right)^{1/\rho},$$

The corresponding probability of migrating from location-sector $ij$ to location-sector $gh$ satisfies a gravity equation:

$$D^{j,h}_{igt} = \frac{\left( \exp \left( \beta \mathbb{E}_t V^{h,w}_{gt+1} / \kappa^{h,j}_{git} \right) \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left( \exp \left( \beta \mathbb{E}_t V^{o,w}_{mt+1} / \kappa^{o,j}_{kit} \right) \right)^{1/\rho}}.$$

The population flow condition implies:

$$\ell^h_{gt+1} = \sum_{i=1}^{N} \sum_{j=1}^{J} D^{j,h}_{igt} \ell^j_{it},$$

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We also define a corresponding immigration probability \( E_{git}^{hj} \), which captures the share of workers in destination \( g \) and sector \( h \) at time \( t + 1 \) that immigrated from origin \( i \) and sector \( j \) at time \( t \):

\[
E_{git}^{hj} = \frac{\tilde{e}^{i} D_{igt}^{jh}}{\tilde{p}_{git}^{hj + 1}}.
\]  

(D.180)

Note that the order of subscripts switches between the outmigration probability and the immigration probability, because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

### D.5.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in their location and sector that period. Worker flow indirect utility in location \( n \) and sector \( j \) depends on local amenities \( (b_{nt}^{j}) \), the wage \( (w_{nt}^{j}) \), and the consumption goods price index \( (p_{nt}) \):

\[
\ln u_{nt}^{j,w} = \ln b_{nt}^{j} + \ln w_{nt}^{j} - \ln p_{nt},
\]

where amenities \( (b_{nt}) \) capture characteristics of a location that make it a more attractive place to live regardless of the wage and cost of consumption goods (e.g. climate and scenic views). In this section of the online appendix, we assume that amenities are exogenous.

The consumption goods price index \( (p_{nt}) \) in location \( n \) depends on the consumption goods price index for each sector \( h \) in that location \( (p_{nt}^{h}) \):

\[
p_{nt} = \prod_{h=1}^{J} (p_{nt}^{h})^{\psi_{h}}, \quad 0 < \psi_{h} < 1, \quad \sum_{h=1}^{J} \psi_{h},
\]

where the consumption goods price index for each sector \( h \) in location \( n \) depends on the price of the variety sourced from each location \( i \) within that sector \( h \) \( (p_{nit}^{h}) \):

\[
p_{nt}^{h} = \left[ \sum_{i=1}^{N} (p_{nit}^{h})^{-\theta} \right]^{-1/\theta}, \quad \theta = \sigma - 1, \quad \sigma > 1
\]

(D.182)

where \( \sigma > 1 \) is the constant elasticity of substitution (CES) between varieties; \( \theta = \sigma - 1 \) is the trade elasticity; and for simplicity, we assume a common elasticity of substitution and trade elasticity across all sectors.

Utility maximization implies that goods consumption expenditure on each sector \( (p_{nt}^{h}c_{nt}^{h}) \) is a constant share of overall goods consumption expenditure \( (p_{nt}c_{nt}) \) in each location:

\[
p_{nt}^{h}c_{nt}^{h} = \psi_{h} p_{nt}c_{nt}.
\]

(D.184)
Using constant elasticity of substitution (CES) demand for individual varieties of goods, the share of location $n$’s expenditure within sector $h$ on the goods produced by location $i$ is:

$$S_{nit}^h = \frac{\left(\frac{p_{nit}^h}{P_{nit}}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\frac{p_{nit}^m}{P_{nit}}\right)^{-\theta}}.$$  \hspace{1cm} (D.185)

### D.5.3 Production

Producers in each location $i$ and sector $j$ use labor, capital and intermediate inputs to produce the variety supplied by that location in that sector. Production is assumed to occur under conditions of perfect competition and subject to the following unit cost function:

$$C_{jit} = \left[ \left( \frac{w_{jti}}{z_{jti}} \right)^{\mu_j} (r_{jt})^{1-\mu_j} \right]^{\gamma_j} \prod_{h=1}^{J} \left( \frac{p_{jht}}{P_{jht}} \right)^{\gamma_j^{j,h}}, \quad \sum_{h=1}^{J} \gamma_j^{j,h} = 1 - \gamma_j,$$  \hspace{1cm} (D.186)

where $(1 - \gamma_j)$ is the share of intermediates in production costs; $\gamma_j^{j,h}$ is the share of materials from sector $h$ used in sector $j$; $z_{jti}$ denotes labor-augmenting productivity in location $i$ in sector $j$ at time $t$. As for amenities above, we assume in this section of the online appendix that productivity is exogenous.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit}^j \geq 1$ units of a good must be shipped from location $i$ in order for one unit to arrive in location $n$, where $\tau_{nit}^j > 1$ for $n \neq i$ and $\tau_{iit}^j = 1$. From profit maximization, the cost to a consumer in location $n$ of sourcing the good produced by location $i$ within sector $j$ is:

$$p_{nit}^j = \tau_{nit}^j P_{jit}^i = \tau_{nit}^j \left[ \left( \frac{w_{jti}}{z_{jti}} \right)^{\mu_j} (r_{jt})^{1-\mu_j} \right]^{\gamma_j} \prod_{h=1}^{J} \left( \frac{p_{jht}}{P_{jht}} \right)^{\gamma_j^{j,h}},$$  \hspace{1cm} (D.187)

where $P_{jit}^i$ is the “free on board” price of the good supplied by location $i$ before transport costs.

From profit maximization problem and zero profits, payments for labor and capital in each sector are constant shares of revenue in that sector:

$$w_{jti} \ell_{jti} = \gamma_j^{j} \mu_j y_{jti}^j,$$  \hspace{1cm} (D.188)

$$r_{jti}^{j} k_{jti}^{j} = \gamma_j^{j} (1 - \mu_j) y_{jti}^j,$$  \hspace{1cm} (D.189)

where $\ell_{jti}$ is labor input; $k_{jti}^{j}$ is capital input; and $y_{jti}^j$ denotes revenue; the immobility of capital across sectors and locations once installed implies that the rate of return on capital need not be equalized across sectors and locations out of steady-state ($r_{jti}^{j} \neq r_{hti}$).
### D.5.4 Landlord Consumption

Landlords in each location and sector choose their consumption and investment in capital to maximize their intertemporal utility subject to the intertemporal budget constraint. Landlords’ intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

\[
v_{it}^{j,k} = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k},
\]

where we use the superscript \( k \) to denote landlords; \( c_{it}^{j,k} \) is the consumption index for landlords in location \( i \) and sector \( j \); and \( \beta \) denotes the discount rate. Since landlords are immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

The consumption goods index for landlords \( (c_{it}^{j,k}) \) takes exactly the same form as for workers and is a Cobb-Douglas aggregate of consumption indexes for each sector, where these consumption indexes for each sector are constant elasticity of substitution (CES) functions of the consumption of varieties from each location. Therefore, the consumption goods price index \( (p_{nt}) \) takes the same form as in equation (D.182), and the consumption goods price index for each sector \( (p_{jnt}) \) takes the same form as in equation (D.183). Under these assumptions, landlords’ utility maximization problem is weakly separable. First, we solve for the optimal consumption-savings decision across time periods for overall goods consumption. Second, we solve for the optimal allocation of consumption across sectors within each time period. Third, we solve for the optimal allocation of consumption across location varieties within each sector.

Beginning with landlords’ optimal consumption-saving decision, we assume that the investment technology for capital in each location and sector uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location and sector can produce one unit of capital in that location and sector using one unit of the consumption index for that location and sector. We assume that capital is geographically immobile once installed and depreciates at a constant rate \( \delta^j \). The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital \( (r_{it}^{j,k} k_{it}^{j,k}) \) equals the total value of goods consumption \( (p_{it} c_{it}^{j,k}) \) and net investment \( (p_{it} (k_{it+1}^{j,k} - (1 - \delta^j) k_{it}^{j,k})) \):

\[
r_{it}^{j,k} k_{it}^{j,k} = p_{it} c_{it}^{j,k} + p_{it} (k_{it+1}^{j,k} - (1 - \delta^j) k_{it}^{j,k}).
\]

Combining landlords’ intertemporal utility (D.190) and budget constraint (D.191), their intertemporal optimization problem is:

\[
\max_{\{c_{it}^{j,k}, k_{it+1}^{j,k}\}} \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k},
\]
subject to \[ p_{it}c_{it}^{j,k} + p_{it} \left( k_{it+1}^{j} - (1 - \delta^j) k_{it}^{j} \right) = r_{it}^{j} k_{it}^{j}. \]

We can write this problem as the following Lagrangian:

\[
L = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k} - \xi_t^{j,k} \left[ p_{it}c_{it}^{j,k} + p_{it} \left( k_{it+1}^{j} - (1 - \delta^j) k_{it}^{j} \right) - r_{it}^{j} k_{it}^{j} \right].
\]  \(\text{(D.193)}\)

The first-order conditions are:

\[
\left\{ \frac{c_{it}^{j,k}}{c_{it}^{j,k}} - p_{it} \xi_t^{j} = 0, \right. \\
\left. \{ k_{it+1}^{j} \right\} \left( r_{it+1}^{j} + p_{it+1} \left( 1 - \delta^j \right) \right) \xi_{t+1}^{j} - p_{it+1} \xi_{t}^{j} = 0,
\]

Together these first-order conditions imply:

\[
\frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} = \beta \frac{p_{it}h_{it}^{j}}{p_{it+1}h_{it+1}^{j}} = \beta \left( r_{it+1}^{j}/p_{it+1} + (1 - \delta^j) \right),
\]  \(\text{(D.194)}\)

where the transversality condition implies:

\[
\lim_{t \to \infty} \beta^t \frac{k_{it+1}^{j}}{c_{it}^{j,k}} = 0.
\]

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords’ optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

\[
p_{it}c_{it}^{j,k} = (1 - \beta) \left( r_{it}^{j} + p_{it} \left( 1 - \delta^j \right) \right) k_{it}^{j},
\]  \(\text{(D.195)}\)

\[
k_{it+1}^{j} = \beta \left( r_{it+1}/p_{it} + (1 - \delta^j) \right) k_{it}^{j}.
\]  \(\text{(D.196)}\)

Substituting the consumption policy function (D.195) into the Euler equation (D.194), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

\[
\frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} = \beta \left( r_{it+1}/p_{it+1} \right) \frac{k_{it+1}^{j}}{k_{it}^{j}},
\]

where the optimal consumption-saving decision in equations (D.195)-(D.196), our assumption of Cobb-Douglas preferences across sectors implies that landlords allocate constant shares of consumption expenditure across sectors within time periods, as for workers in equation (D.184). Similarly, our assumption of constant elasticity of substitution (CES) preferences across locations within sectors implies that landlords in location \(n\) allocate the same share of expenditure on location \(i\) within sector \(j\), as for workers in equation (D.185).
D.5.5 Goods Market Clearing

Goods market clearing implies that income in each location and sector equals expenditure on the goods produced in that location and sector:

\[ y_{jt} = \sum_{n=1}^{N} S_{nit}^j x_{nt}^j, \]

where \( y_{jt} \) is total income in sector \( j \) in location \( i \) and \( x_{nt}^j \) is total expenditure on industry \( j \) in region \( n \) at time \( t \). Total expenditure is the sum of final consumption and intermediate goods expenditure and is given by:

\[ x_{nt}^j = \psi^j \sum_{h=1}^{J} \left( w_{nt}^h \ell_{nt}^h + r_{nt}^h k_{nt}^h \right) + \sum_{h=1}^{J} \gamma_{h,j} y_{nt}^h. \]

Combining these two relationships, we have:

\[ y_{jt} = \sum_{n=1}^{N} S_{nit}^j \left[ \psi^j \sum_{h=1}^{J} \gamma_{h}^j y_{nt}^h + \sum_{h=1}^{J} \gamma_{h,j} y_{nt}^h \right], \]

which can be re-written as:

\[ y_{jt} = \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^j \left[ \psi^j \gamma_{h}^j + \gamma_{h,j} \right] y_{nt}^h. \]

D.5.6 Capital Market Clearing

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords’ income from the ownership of capital equals payments for its use. Using the property that payments to capital and labor are constant shares of total revenue in equations (D.188) and (D.189), we can write payments for capital in each sector as:

\[ r_{it}^j k_{it}^j = \frac{1 - \mu^j}{\mu^j} w_{it}^j \ell_{it}^j. \]

D.5.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.
Industry Price Indices. Totally differentiating the industry consumption goods price index in equation (D.182), we have:

\[
\frac{dp^j_{nt} \mid p^j_{nt}}{p^j_{nt}} = \sum_{m=1}^{N} S^j_{nmt} \frac{dp^j_{nmt} \mid p^j_{nmt}}{p^j_{nmt}},
\]

\[
d \ln p^j_{nt} = \sum_{m=1}^{N} S^j_{nmt} \ d \ln p^j_{nmt}.
\] (D.200)

Prices. Using the relationship between capital and labor payments (D.199), the pricing rule (D.187) can be re-written as follows:

\[
p^j_{nit} = \tau^j_{nit} \left( \frac{w^j_{it}}{z^j_{it}} \right)^{\gamma^j} \left( 1 - \mu^j \right)^{\left( 1 - \mu^j \right) \gamma^j} \left( \frac{1}{\lambda^j_{it}} \right)^{\left( 1 - \mu^j \right) \gamma^j} \prod_{h=1}^{J} (p^h_{it})^{\gamma^j \gamma^h},
\] (D.201)

where \( \chi^j_{it} \) is the capital-labor ratio in sector \( j \) in region \( i \):

\[
\chi^j_{it} \equiv \frac{k^j_{it}}{L^j_{it}}.
\]

Totally differentiating this pricing rule, we have:

\[
\frac{dp^j_{nit} \mid p^j_{nit}}{p^j_{nit}} = \frac{d\tau^j_{nit} \mid \tau^j_{nit}}{\tau^j_{nit}} + \gamma^j \frac{dw^j_{it} \mid w^j_{it}}{w^j_{it}} - \gamma^j (1 - \mu^j) \frac{d\chi^j_{it} \mid \chi^j_{it}}{\chi^j_{it}} - \gamma^j \frac{dz^j_{it} \mid z^j_{it}}{z^j_{it}} + \sum_{h=1}^{J} \gamma^j \gamma^h \frac{dp^h_{it} \mid p^h_{it}}{p^h_{it}},
\]

\[
d \ln p^j_{nit} = \left[ d \ln \tau^j_{nit} + \gamma^j d \ln w^j_{it} - (1 - \mu^j) \gamma^j d \ln \chi^j_{it} \right]
- \gamma^j d \ln z^j_{it} + \sum_{h=1}^{J} \gamma^j \gamma^h \ \sum_{j,h=1}^{J} \frac{dp^j_{nit} \mid p^j_{nit}}{p^j_{nit}}.
\] (D.202)

Combining the total derivatives of the price index (D.200) and prices (D.204), we have:

\[
d \ln p^j_{nit} = \left[ d \ln \tau^j_{nit} + \gamma^j d \ln w^j_{it} - (1 - \mu^j) \gamma^j d \ln \chi^j_{it} \right]
- \gamma^j d \ln z^j_{it} + \sum_{h=1}^{J} \gamma^j \gamma^h \ \sum_{j,h=1}^{J} \frac{dp^j_{nit} \mid p^j_{nit}}{p^j_{nit}}
+ \sum_{h=1}^{J} \sum_{m=1}^{N} \gamma^j \gamma^h S^j_{imt} \ d \ln p^h_{imt},
\]

which can be re-written as:

\[
d \ln p^j_{nit} = \left[ d \ln \tau^j_{nit} + \gamma^j d \ln w^j_{it} - (1 - \mu^j) \gamma^j d \ln \chi^j_{it} \right]
- \gamma^j d \ln z^j_{it} + \sum_{h=1}^{J} \sum_{m=1}^{N} \gamma^j \gamma^h S^j_{imt} \ d \ln \chi^j_{it},
\]

where \[ \sum_{i,m=1}^{N} \gamma^j \gamma^h S^j_{imt}. \]

Define \( \Gamma \equiv [I - \Sigma]^{-1} \) as the Leontief inverse of the shares \( \Sigma^j_{imt} \), we can write this relationship as:

\[
d \ln p^j_{nit} = \sum_{m=1}^{N} \sum_{o=1}^{J} \Gamma^j_{im} \left[ d \ln \tau^o_{nmt} + \gamma^o d \ln w^o_{mt} - (1 - \mu^o) \gamma^o d \ln \chi^o_{mt} - \gamma^o d \ln z^o_{mt} \right].
\] (D.203)
Expenditure Shares. Totally differentiating the expenditure share equation (D.185), we get:

\[
\frac{dS_{nit}^j}{S_{nit}^j} = \theta \left( \sum_{h=1}^{N} S_{nih}^j \frac{dp_{nht}^j}{p_{nht}^j} - \frac{dp_{nht}^j}{P_{nht}^j} \right),
\]

\[
d \ln S_{nit}^j = \theta \left( \sum_{h=1}^{N} S_{nih}^j d \ln p_{nht}^j - d \ln p_{nht}^j \right). \tag{D.204}
\]

Using the total derivatives of prices above (D.203), this total derivative of the expenditure shares can be written as:

\[
d \ln S_{nit}^j = \theta \left[ \sum_{h=1}^{N} S_{nih}^j \sum_{m=1}^{N} \sum_{o=1}^{J} \Gamma_{hnm}^o - \sum_{m=1}^{N} \sum_{o=1}^{J} \Gamma_{im}^o \right] \left[ -d \ln \tau_{nit} + \gamma^o d \ln w_{nit}^o - (1 - \mu^o) \gamma^o d \ln \chi_{nit}^o - \gamma^o d \ln z_{nit}^o \right]. \tag{D.205}
\]

Migration Shares. Totally differentiating this expenditure share equation (D.178), we get:

\[
\frac{dD_{igt}^j}{D_{igt}^j} = \frac{1}{\rho} \left[ \left( \beta E_t \tau_{igt} - \frac{d\kappa_{igt}^j}{\kappa_{igt}^j} \right) - \sum_{m=1}^{N} \sum_{o=1}^{J} D_{int}^o \left( \beta E_t \tau_{int} - \frac{d\kappa_{int}^o}{\kappa_{int}^o} \right) \right], \tag{D.206}
\]

\[
d \ln D_{igt}^j = \frac{1}{\rho} \left[ \left( \beta E_t \tau_{igt} - d \ln \kappa_{igt}^j \right) - \sum_{m=1}^{N} \sum_{o=1}^{J} D_{int}^o \left( \beta E_t \tau_{int} - d \ln \kappa_{int}^o \right) \right].
\]

Labor Payments. Totally differentiating labor payments (D.188), we have:

\[
w_{it}^j \ell_{it}^j = \gamma^j \mu^j y_{it}^j,
\]

\[
\frac{dw_{it}^j}{w_{it}^j} \ell_{it}^j + \frac{d\ell_{it}^j}{\ell_{it}^j} w_{it}^j = \gamma^j \mu^j y_{it}^j \frac{dy_{it}^j}{y_{it}^j},
\]

\[
\frac{dw_{it}^j}{w_{it}^j} + \frac{d\ell_{it}^j}{\ell_{it}^j} = \gamma^j \mu^j y_{it}^j \frac{dy_{it}^j}{y_{it}^j},
\]

\[
\frac{dw_{it}^j}{w_{it}^j} + \frac{d\ell_{it}^j}{\ell_{it}^j} = \frac{\xi_i^j}{y_{it}^j},
\]

\[
d ln w_{it}^j + d \ln \ell_{it}^j = \xi_i^j d \ln y_{it}^j, \tag{D.207}
\]

\[
\xi_i^j = \frac{\gamma^j \mu^j y_{it}^j}{w_{it}^j \ell_{it}^j}.
\]
Goods Market Clearing. Totally differentiating the goods market clearing condition (D.198), we have:

\[ y_j^t = \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^{j} \left[ \gamma_j^h + \gamma_j^{h,j} \right] y_{nt}^h, \]

\[ \frac{d y_j^t}{y_j^t} = \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^{j} \left[ \gamma_j^h + \gamma_j^{h,j} \right] y_{nt}^h \frac{d S_{nit}^{j}}{y_j^t} = \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^{j} \left[ \gamma_j^h + \gamma_j^{h,j} \right] y_{nt}^h \frac{d y_{nt}^h}{y_j^t}, \]

which can be re-written as:

\[ d \ln y_j^t = \left[ \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right) + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^{h,j}}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln \ell_{nt}^h}{y_j^t} \right) \right], \]

where

\[ \vartheta_j^{h,j} = \frac{S_{nit}^{j} \gamma_j^{h,j}}{y_j^t} \]

Using equation (D.207), we can re-write this relationship as:

\[ d \ln y_j^t = \left[ + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right) + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^{h,j}}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln \ell_{nt}^h}{y_j^t} \right) \right]. \]

Using the definition of \( x_j^{i} \), we have:

\[ d \ln y_j^t = \left[ + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right) + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^{h,j}}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln \ell_{nt}^h}{y_j^t} \right) \right]. \]

Using \( w_{it}^j \ell_{it}^j = \gamma_j^h \mu_j^j y_j^t \), we can re-write this further as:

\[ d \ln y_j^t = \left[ + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right) \right] \]

where

\[ \vartheta_j^{h,j} = \frac{S_{nit}^{j} \gamma_j^{h,j}}{y_j^t} \]

\[ d \ln y_j^t \left[ 1 - \sum_{n=1}^{N} \sum_{h=1}^{J} \vartheta_j^{h,j} \frac{d \ln y_j^t}{y_j^t} \right] = \left[ + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln S_{nit}^{j}}{y_j^t} \left( \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right) \right]. \]

Taking the Leontief inverse of \( \vartheta_j^{h,j} \), we have:

\[ d \ln y_j^t = \sum_{m=1}^{N} \sum_{o=1}^{J} \Delta_j^{om} \left[ + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right]. \]

Using equation (D.207), this becomes:

\[ d \ln w_{it}^j + d \ln \ell_{it}^j = \sum_{m=1}^{N} \sum_{o=1}^{J} \Delta_j^{om} \left[ + \sum_{n=1}^{N} \sum_{h=1}^{J} \frac{\gamma_j^h}{ \gamma_j^{h,j} + \gamma_j^h} \frac{d \ln w_{nt}^h + d \ln \ell_{nt}^h}{y_j^t} \right]. \]
Population Flow. Totally differentiating the population flow condition (D.179) we have:

\[
\begin{align*}
  d \ln \ell_{gt}^h & = \sum_{i=1}^{N} \sum_{j=1}^{J} E_{git}^{hj} \left[ d \ln \ell_{it}^i + d \ln D_{igt}^h \right], \\
d \ln \ell_{gt}^h & = \sum_{i=1}^{N} \sum_{j=1}^{J} E_{git}^{hj} \left[ d \ln \ell_{it}^i + \frac{1}{\rho} \left( \beta E_t d v_{it+1}^h - d \ln \kappa_{git}^h - \sum_{m=1}^{N} \sum_{h=1}^{J} D_{imt}^{hj} \left( \beta E_t v_{imt+1}^h - d \ln \kappa_{mit}^j \right) \right) \right].
\end{align*}
\]

Value Function. Note that the value function can be re-written using the following results:

\[
v_{it}^{j,w} = \ln \frac{w_{it}^{j'}}{\prod_{o=1}^{J} \left( \sum_{m=1}^{N} \phi_{o}^{\theta} \right)^{-\psi'/\theta}} + \ln b_{it}^{j'} + \rho \ln \left( \sum_{g=1}^{N} \sum_{h=1}^{J} \left( \exp \left( \beta E_t v_{gt+1}^{h,w} / \kappa_{git}^{j} \right) \right)^{1/\rho} \right),
\]

\[
\begin{align*}
  \prod_{o=1}^{J} \left( \sum_{m=1}^{N} \phi_{o}^{\theta} \right)^{-\psi'/\theta} & = \prod_{o=1}^{J} \left( \frac{\phi_{o}^{\theta}}{S_{o}^{\theta}} \right)^{-\psi'/\theta}, \\
  \tau_{it}^{o} & = 1, \\
  \kappa_{it}^{j}_{j} & = 1,
\end{align*}
\]

\[
\begin{align*}
  v_{it}^{j,w} & = \ln w_{it}^{j'} + \sum_{o=1}^{J} \psi_{o}^{j'} \left[ -\frac{1}{\theta} \ln S_{o}^{\phi} - \ln p_{o}^{\phi} \right] + \ln b_{it}^{j'} + \beta E_t v_{it+1}^{j,w} - \rho \ln D_{it}^{j},
\end{align*}
\]

Totally differentiating the value function (D.210) we have:

\[
\begin{align*}
  dv_{it}^{j,w} & = d \ln w_{it}^{j'} + \sum_{o=1}^{J} \psi_{o}^{j'} d \ln S_{o}^{\phi} - d \ln p_{o}^{\phi} + d \ln b_{it}^{j'} + \beta E_t dv_{it+1}^{j,w} - \rho d \ln D_{it}^{j}, \\
  d \ln S_{o}^{\phi} & = -\theta d \ln p_{o}^{\phi} + \theta \sum_{m=1}^{N} S_{o}^{\phi} d \ln p_{o}^{\phi}, \\
  d \ln D_{it}^{j} & = \frac{1}{\rho} \left[ \beta E_t dv_{it+1}^{j} - d \ln \kappa_{it}^{j} - \sum_{m=1}^{N} \sum_{h=1}^{J} D_{imt}^{j} \left( \beta E_t v_{imt+1}^{h} - d \ln \kappa_{mit}^{j} \right) \right].
\end{align*}
\]

Using these results in the derivative of the value function, we have:

\[
\begin{align*}
  dv_{it}^{j,w} & = d \ln w_{it}^{j'} - \sum_{o=1}^{J} \psi_{o}^{j'} \sum_{m=1}^{N} S_{o}^{\phi} d \ln p_{o}^{\phi} + d \ln b_{it}^{j'} + \beta E_t dv_{it+1}^{j,w} - d \ln \kappa_{it}^{j},
\end{align*}
\]

where we have used \(d \ln \kappa_{it}^{j} = 0\). From the total derivative of prices in equation (D.203), we have:

\[
\begin{align*}
  d \ln p_{o}^{\phi} & = \sum_{h=1}^{J} \sum_{n=1}^{N} R_{o}^{nh} \left[ d \ln \tau_{nt}^{h} + \gamma^{h} d \ln w_{nt}^{h} - (1 - \mu_{o}^{h}) \gamma^{h} d \ln \chi_{nt}^{h} - \gamma^{h} d \ln Z_{nt}^{h} \right].
\end{align*}
\]

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Using this result in the value function above, we obtain:

\[
\begin{align*}
\text{d} v_j^{i,t} & = \left[ \frac{d \ln w_i^j - \sum_{m=1}^J \sum_{n=1}^N \psi^m \Sigma_{i,m}^{h} + \sum_{n=1}^N \sum_{h=1}^H \Gamma_{i,n}^{h*} }{p_i^j} \right] \\
& \quad + \frac{d \ln b_{i,t}^j + \sum_{n=1}^N \sum_{h=1}^H D_{i,n}^{h*} }{p_i^j} \left( \beta E_{i,t}^{h*} \text{d} \ln \rho_{i,n}^{h*} - d \ln \zeta_{i,n}^{h*} \right). 
\end{align*}
\]  

(D.211)

### D.5.8 Steady-state

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables:

\[
\begin{align*}
k_j^{i,t+1} &= k_i^j, \\
\ell_j^{i,t+1} &= \ell_i^j, \\
w_j^{i,t+1} &= w_i^j, \\
v_j^{i,t+1} &= v_i^j, \\
\end{align*}
\]

where we use an asterisk to denote a steady-state value. We consider small common shocks to productivities across all sectors (d ln z) and to amenities across all sectors (d ln b) in each location, holding constant the economy’s aggregate labor endowment (d ln \( \bar{\ell} = 0 \)), trade costs (d ln \( \tau = 0 \)) and commuting costs (d ln \( \kappa = 0 \)).

#### Capital Accumulation.

From the capital accumulation equation (D.196), the steady-state stock of building capital solves:

\[
(1 - \beta (1 - \delta)) k_i^j = \beta \frac{r_i^j}{p_i} k_i^j.
\]

Using this result in the expression for the steady-state capital stock above, we have:

\[
(1 - \beta (1 - \delta)) k_i^j = \beta \frac{1 - \mu}{\mu} \frac{w_i^j}{p_i^j}.
\]

We now derive an expression for the total derivative of real income:

\[
\text{d} \ln \left( \frac{w_i^j}{p_i^j} \right) = \text{d} \ln w_i^j - \text{d} \ln p_i^j.
\]

The total derivative of the aggregate price index is given by:

\[
\text{d} \ln p_i^j = \sum_{m=1}^N \sum_{h=1}^H \psi^h \Sigma_{i,m}^{h*} \text{d} \ln p_i^h.
\]

Using our expression for the total derivative of prices above (D.203), we can re-write this total derivative of the aggregate price index as:

\[
\text{d} \ln p_i^j = \sum_{m=1}^N \sum_{h=1}^H \psi^h \Sigma_{i,m}^{h*} \sum_{n=1}^N \sum_{a=1}^H \Gamma_{i,mn}^{h*} \gamma^a \text{d} \ln w_{i,n}^{a*} - (1 - \mu^a) \gamma^a \text{d} \ln \chi_{i,n}^{a*} - \gamma^a \text{d} \ln \zeta_{i,n}^{a*}.
\]
where have used $d \ln \tau_{nit}^j = 0$. Using these results in equation (D.212), the change the steady-state capital labor ratio is given by:

\[
d \ln \chi_{i}^{j*} = d \ln w_{it}^{j*} - \sum_{m=1}^{N} \sum_{h=1}^{J} \psi_{hm}^{j} S_{lnm}^{j} \sum_{n=1}^{N} \sum_{o=1}^{J} \Gamma_{mn}^{ho} \left[ \gamma^{o} d \ln w_{nt}^{o} - (1 - \mu^{o}) \gamma^{o} d \ln \chi_{nt}^{o*} - \gamma^{o} d \ln z_{nt}^{o} \right],
\]

which has the matrix representation:

\[
d \ln \chi^{*} = d \ln w^{*} - S \left( d \ln w + (I - \mu) d \ln \chi^{*} + d \ln z \right),
\]

where $d \ln \chi^{*}$ and $d \ln w^{*}$ are $NJ \times NJ$ matrices; $\lambda$ is a $NJ \times NJ$ diagonal matrix whose $(ij)$-th element on the diagonal is $\lambda^{ij}$; and $S$ is a $NJ \times NJ$ matrix with elements:

\[
S_{nt}^{j} = \sum_{n=1}^{N} \sum_{o=1}^{J} \sum_{m=1}^{J} \sum_{h=1}^{J} \Gamma_{mn}^{ho} \gamma^{o}.
\]

**Goods Market Clearing.** Recall that the total derivative of the expenditure share in equation (D.205) is:

\[
d \ln S_{nmt}^{j} = \theta \left[ \sum_{h=1}^{N} S_{nht}^{j} \sum_{g=1}^{J} \sum_{o=1}^{J} \Gamma_{bhg}^{ho} - \sum_{g=1}^{J} \sum_{o=1}^{J} \Gamma_{bg}^{o} \right] \left[ \gamma^{o} d \ln w_{gt}^{o} - (1 - \mu^{o}) \gamma^{o} d \ln \chi_{gt}^{o} - \gamma^{o} d \ln z_{gt}^{o} \right],
\]

where we have used $d \ln \tau_{nit}^j = 0$. We can re-write this total derivative of the expenditure share as:

\[
d \ln S_{nmt}^{j} = \theta \left[ \sum_{h=1}^{N} S_{nht}^{j} \sum_{g=1}^{J} \sum_{o=1}^{J} A_{bhg}^{ho} - \sum_{g=1}^{J} \sum_{o=1}^{J} A_{bg}^{o} \right] \left[ \gamma^{o} d \ln w_{gt}^{o} - (1 - \mu^{o}) \gamma^{o} d \ln \chi_{gt}^{o} - \gamma^{o} d \ln z_{gt}^{o} \right],
\]

where $A_{bhg}^{ho} \equiv \gamma^{o} \Gamma_{bg}^{ho}$.

Recall that the total derivative of the goods market clearing condition is:

\[
d \ln w_{it}^{j} + d \ln \ell_{it}^{j} = \xi_{i}^{j} \sum_{n=1}^{N} \sum_{m=1}^{J} \Delta_{im}^{j} \left[ \sum_{n=1}^{N} \sum_{h=1}^{J} \psi_{mh}^{j} \left( d \ln w_{nt}^{h} + d \ln \ell_{nt}^{h} \right) \right] + \sum_{g=1}^{N} \sum_{o=1}^{J} \left( \gamma^{o} d \ln w_{gt}^{o} - (1 - \mu^{o}) \gamma^{o} d \ln \chi_{gt}^{o} - \gamma^{o} d \ln z_{gt}^{o} \right) d \ln S_{nmt}^{o}.
\]

Using this expression for the total derivative of the expenditure share in the total derivative of the goods market clearing condition in equation (D.208), we obtain:

\[
d \ln w_{it}^{j} + d \ln \ell_{it}^{j} = \left\{ \xi_{i}^{j} \sum_{m=1}^{N} \sum_{o=1}^{J} \Delta_{im}^{j} \sum_{n=1}^{N} \sum_{h=1}^{J} \psi_{mh}^{j} \left( d \ln w_{nt}^{h} + d \ln \ell_{nt}^{h} \right) \right\} + \left\{ \sum_{g=1}^{N} \sum_{o=1}^{J} S_{nht}^{j} \sum_{m=1}^{J} \sum_{h=1}^{J} A_{mhg}^{ho} - \sum_{o=1}^{J} \sum_{h=1}^{J} \Delta_{mht}^{j} \right\} \left\{ \gamma^{o} d \ln w_{gt}^{o} - (1 - \mu^{o}) \gamma^{o} d \ln \chi_{gt}^{o} - \gamma^{o} d \ln z_{gt}^{o} \right\}.
\]

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To simplify notation, we define \( \Pi_{im}^{jo} \equiv \xi_i^j \Delta_{im}^{jo} \) as the network-adjusted share of income in sector \( j \) in location \( i \) derived from selling to sector \( o \) in location \( m \). We also define \( \Upsilon_{nmg} \equiv \sum_{h=1}^{N} S_{nh}^j \sum_{o=1}^{J} \Lambda_{ho}^{jo} - \sum_{o=1}^{J} \Lambda_{mg}^{jo} \) as the elasticity of location \( n \)'s expenditure in sector \( j \) on goods from location \( i \) with respect to the price of goods in that sector from location \( m \). Using this notation, we can re-write the above goods market clearing condition as:

\[
\begin{align*}
&\frac{\partial \ln w^j_h}{\partial \ln P} + \frac{\partial \ln \ell^j_{it}}{\partial \ln P} = \\
&= \left\{ \sum_{n=1}^{N} \left( \sum_{m=1}^{M} \sum_{o=1}^{O} \Pi_{im}^{jo} \frac{\partial h_{mn}}{\partial m} \left( \frac{\partial \ln w_{nt}^{h}}{\partial \ln P} + \frac{\partial \ln \ell_{nt}^{h}}{\partial \ln P} \right) \right) \right. \\
&\left. + \theta \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{o=1}^{O} \Pi_{im}^{jo} \frac{\partial \ell_{nt}^{o}}{\partial m} \left( \frac{\partial \ln w_{nt}^{o}}{\partial \ln P} + \frac{\partial \ln \ell_{nt}^{o}}{\partial \ln P} \right) \right. \\
&\left. \times \Upsilon_{nmg} \left[ \frac{\partial \ln w_{gt}^{o}}{\partial h} - (1 - \mu^{o}) \frac{\partial \ln \chi_{gt}^{o}}{\partial h} - \frac{\partial \ln z_{gt}^{o}}{\partial h} \right] \right\}.
\end{align*}
\]

We can write this goods market clearing condition in matrix form as:

\[
\frac{\partial \ln w_t}{\partial \ln P} + \frac{\partial \ln \ell_t}{\partial \ln P} = T \left( \frac{\partial \ln w^*}{\partial \ln P} + \frac{\partial \ln \ell^*}{\partial \ln P} \right) + \theta M \left( \frac{\partial \ln w^* - (I - \mu^*)}{\partial \ln P} \frac{\partial \ln \chi^* - \frac{\partial \ln z}{\partial \ln P} \right),
\]

where these matrices have \( NJ \times NJ \) elements. In particular, \( T \) is a \( NJ \times NJ \) matrix with elements:

\[
T_{in} = \sum_{m=1}^{N} \sum_{o=1}^{O} \Pi_{im}^{jo} \frac{\partial h_{mn}}{\partial m},
\]

and \( M \) is a \( NJ \times NJ \) matrix with elements:

\[
M_{in} = \sum_{m=1}^{N} \sum_{o=1}^{O} \sum_{g=1}^{G} \Pi_{im}^{jo} \frac{\partial \ell_{nt}^{o}}{\partial m} + \sum_{h=1}^{H} \Theta_{mn}^{oh} \Upsilon_{nmg}.
\]

In steady-state we have:

\[
\frac{\partial \ln w^*}{\partial \ln P} + \frac{\partial \ln \ell^*}{\partial \ln P} = T \left( \frac{\partial \ln w^*}{\partial \ln P} + \frac{\partial \ln \ell^*}{\partial \ln P} \right) + \theta M \left( \frac{\partial \ln w^* - (I - \mu^*)}{\partial \ln P} \frac{\partial \ln \chi^* - \frac{\partial \ln z}{\partial \ln P} \right),
\]

\[
\frac{\partial \ln w^*}{\partial \ln P} + \frac{\partial \ln \ell^*}{\partial \ln P} = T \left( \frac{\partial \ln w^*}{\partial \ln P} + \frac{\partial \ln \ell^*}{\partial \ln P} \right) + \theta M \left( \frac{\partial \ln w^* - (I - \mu^*)}{\partial \ln P} \frac{\partial \ln \chi^* - \frac{\partial \ln z}{\partial \ln P} \right).
\]

**Population Flow.** The total derivative of the population flow condition (D.209) has the following matrix representation:

\[
\frac{\partial \ln \ell_{t+1}}{\partial \ln P} = E \frac{\partial \ln \ell_t}{\partial \ln P} + \frac{\beta}{\rho} (I - ED) E_t \frac{\partial \ln \ell^*}{\partial \ln P},
\]

where these matrices again have \( NJ \times NJ \) elements. In steady-state, we have:

\[
\frac{\partial \ln \ell^*}{\partial \ln P} = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) \frac{\partial \ln \ell^*}{\partial \ln P}.
\]

**Value function.** Recall from equation (D.211) that the total derivative of the value function is given by:

\[
\frac{\partial v^j_{it}}{\partial \ln \ell^j_{it}} = \left\{ \frac{\partial \ln w^j_h}{\partial \ln \ell^j_{it}} - \sum_{n=1}^{N} \sum_{h=1}^{H} \sum_{o=1}^{O} \sum_{g=1}^{G} \left( \frac{\partial \ln w_{nt}^{h}}{\partial \ln \ell^j_{it}} - \gamma^h \frac{\partial \ln \chi_{nt}^{h}}{\partial \ln \ell^j_{it}} \right) \right. \\
\left. + \gamma^h \frac{\partial \ln \chi_{nt}^{h}}{\partial \ln \ell^j_{it}} \right\}.
\]

\[
\left. + \frac{\partial \ln b^j_{it}}{\partial \ln \ell^j_{it}} + \sum_{n=1}^{N} \sum_{h=1}^{H} \sum_{o=1}^{O} \sum_{g=1}^{G} \frac{\partial \ln b^j_{nt}^{h}}{\partial \ln \ell^j_{it}} \right\}.
\]
where we have used \( \text{d} \ln \tau_{nit}^j = 0 \) and \( \text{d} \ln \kappa_{nit}^{kj} = 0 \). This total derivative of the value function has the following matrix representation:

\[
\text{d} v_t = \text{d} \ln w_t - S [ \text{d} \ln w_t - (I - \mu) \text{d} \ln \chi_t - \text{d} \ln z] + \text{d} \ln b + \beta D E_t \text{d} v_{t+1},
\]

where these matrices again have \( NJ \times NJ \) elements. Recall that the matrix \( S \) has elements \( S_{nit}^j \) given by:

\[
S_{nit}^j = \sum_{h=1}^J \sum_{m=1}^N \sum_{o=1}^J J \sum_{m=1}^N \sum_{o=1}^J \psi^o S_{nit}^o \Gamma_{mh}^o \).
\]

The matrix \( D \) has elements \( D_{nit}^j \) given by:

\[
D_{nit}^j = \sum_{h=1}^J D_{nit}^h.
\]

In steady-state, this total derivative of the value function becomes:

\[
d v^* = (I - \beta D)^{-1} [(I - S) \text{d} \ln w^* + S (\text{d} \ln z + (I - \mu) \text{d} \ln \chi^*) + \text{d} \ln b]. \quad \text{(D.216)}
\]

**System of Steady-State Equations.** Collecting together the system of steady-state equations, we have:

\[
\begin{align*}
\text{d} \ln \chi^* &= (I - S) \text{d} \ln w^* + S (I - \mu) \text{d} \ln \chi^* + S \text{d} \ln z. \quad \text{(D.217)} \\
\text{d} \ln w^* + \text{d} \ln \ell^* &= T (\text{d} \ln w^* + \text{d} \ln \ell^*) + \theta M (\text{d} \ln w^* - (I - \mu) \text{d} \ln \chi^* - \text{d} \ln z). \quad \text{(D.218)} \\
\text{d} \ln \ell^* &= \frac{\beta}{\rho} (I - E)^{-1} (I - ED) \text{d} v^*. \quad \text{(D.219)} \\
d v^* &= (I - \beta D)^{-1} [(I - S) \text{d} \ln w^* + S (\text{d} \ln z + (I - \mu) \text{d} \ln \chi^*) + \text{d} \ln b]. \quad \text{(D.220)}
\end{align*}
\]

**D.5.9 Transition Dynamics**

Suppose that the economy starts from an initial steady-state. Consider a small common shock to productivity across sectors (\( \text{d} \ln z \)) and amenities across sectors (\( \text{d} \ln b \)) in each location, holding constant the economy’s aggregate labor endowment (\( \text{d} \ln \ell = 0 \)), trade costs (\( \text{d} \ln \tau = 0 \)) and commuting costs (\( \text{d} \ln \kappa = 0 \)). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that \( \tilde{\ell}_{it+1} = \ell_{it+1} - \ell_{it}^* \), for all variables except for the worker value function \( v_{it} \), where with a slight abuse of notation we use \( \tilde{v}_{it} = v_{it} - v_{it}^* \) to denote the deviation in levels for the worker value function.
**Capital Accumulation.** From the capital accumulation equation (D.196), we have:

\[ k_{it+1}^j = \beta \frac{r^j}{p_t} k_{it}^j + \beta \left( 1 - \delta^j \right) k_{it}^j. \]

From the relationship between labor and capital payments, we have:

\[ \frac{r^j}{p_t} k_{it}^j = \frac{1 - \mu^j w_{it}^j \ell_{it}^j}{\mu^j}. \]

Using this result in the capital accumulation equation above, we have:

\[ k_{it+1}^j = \beta \left( 1 - \delta^j \right) k_{it}^j + \beta \frac{1 - \mu^j w_{it}^j \ell_{it}^j}{\mu^j}, \]

\[ \frac{k_{it+1}^j \ell_{it+1}^j}{\ell_{it+1}^j} = \beta \left( 1 - \delta^j \right) \frac{k_{it}^j \ell_{it}^j}{\ell_{it}^j} + \beta \frac{1 - \mu^j w_{it}^j \ell_{it}^j}{\mu^j}, \]

\[ \chi_{it+1}^j \ell_{it+1}^j = \beta \left( 1 - \delta^j \right) \chi_{it}^j + \beta \frac{1 - \mu^j w_{it}^j \ell_{it}^j}{\mu^j}. \] (D.221)

while in steady-state we have:

\[ \frac{k_{it}^{j*}}{\ell_{it}^{j*}} = \beta \left( 1 - \delta^j \right) \frac{k_{it}^{j*}}{\ell_{it}^{j*}} + \beta \frac{1 - \mu^j w_{it}^{j*}}{\mu^j}, \]

\[ \chi_{it}^{j*} = \beta \left( 1 - \delta^j \right) \chi_{it}^{j*} + \beta \frac{1 - \mu^j w_{it}^{j*}}{\mu^j}. \]

**Using** (D.222) **can be re-written as:**

\[ \chi_{it+1}^j \ell_{it+1}^j = \beta \left( 1 - \delta^j \right) \chi_{it}^j \frac{w_{it}^j}{w_{it}^{j*}} + (1 - \beta \left( 1 - \delta^j \right)) \frac{w_{it}^j}{w_{it}^{j*}}, \]

which can be further re-written as:

\[ \chi_{it+1}^j \ell_{it+1}^j = \beta \left( 1 - \delta^j \right) \chi_{it}^j \frac{w_{it}^j}{w_{it}^{j*}} + (1 - \beta \left( 1 - \delta^j \right)) \frac{w_{it}^j}{w_{it}^{j*}} - 1, \]

\[ \frac{\chi_{it+1}^j \ell_{it+1}^j}{\chi_{it}^j \ell_{it}^j} - 1 = \beta \left( 1 - \delta^j \right) \left( \frac{\chi_{it}^j}{\chi_{it}^{j*}} - 1 \right) + \left( 1 - \beta \left( 1 - \delta^j \right) \right) \left( \frac{w_{it}^j}{w_{it}^{j*}} - 1 \right). \]

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Noting that:
\[
\frac{x_{it}}{x_i^*} - 1 \simeq \ln \left( \frac{x_{it}}{x_i^*} \right),
\]
\[
\frac{\chi_i^{j+1}}{\chi_i^{j+1}} = \frac{\ell_i^{j+1}}{\ell_i^j} - 1 \simeq \ln \left( \frac{\chi_i^{j+1}}{\chi_i^{j+1}} \right) ,
\]
we have:

\[
\ln \left( \frac{\chi_i^{j+1}}{\chi_i^{j+1}} \right) + \ln \left( \frac{\ell_i^{j+1}}{\ell_i^j} \right) = \beta \left( 1 - \delta^i \right) \ln \left( \frac{\chi_i^{j+1}}{\chi_i^{j+1}} \right) + \ln \left( \frac{\ell_i^{j+1}}{\ell_i^j} \right) ,
\]

\[
\ln \left( \frac{\chi_i^{j+1}}{\chi_i^{j+1}} \right) + \ln \left( \frac{\ell_i^{j+1}}{\ell_i^j} \right) = \beta \left( 1 - \delta^i \right) \ln \left( \frac{\chi_i^{j+1}}{\chi_i^{j+1}} \right) + \ln \left( \frac{\ell_i^{j+1}}{\ell_i^j} \right) ,
\]

which can be re-written as follows:
\[
\tilde{\chi}_{i+1} = \beta \left( 1 - \delta^i \right) \tilde{\chi}_i + \left( 1 - \beta \left( 1 - \delta^i \right) \right) \left( \tilde{w}_i^j - \tilde{p}_i^j \right) - \tilde{\ell}_i^j + \tilde{\ell}_i.
\]

We can rewrite this relationship in matrix form as:
\[
\tilde{\chi}_{i+1} = \beta \left( I - \delta^i \right) \tilde{\chi}_i + \left( I - \beta \left( I - \delta^i \right) \right) \left( \tilde{w}_i^j - \tilde{p}_i^j \right) - \tilde{\ell}_{i+1} + \tilde{\ell}_i,
\]

where these matrices have \(NJ \times NJ\) elements.

Following an analogous analysis as for steady-state above, the total derivative of real income relative to the initial steady-state can be written in matrix form as:
\[
\tilde{w}_i - \tilde{p}_i = (I - S) \tilde{w}_i + S (I - \mu) \tilde{\chi}_i + \tilde{S} \tilde{z},
\]
where we have used \(d \ln \tilde{\tau} = 0\). Using this result in our expression for the dynamics of the capital-labor ratio above, we have:
\[
\tilde{X}_{i+1} = \left[ \beta \left( I - \delta^i \right) + \left( I - \beta \left( I - \delta^i \right) \right) S \left( I - \mu \right) \tilde{\chi}_i + \tilde{S} \tilde{z} \right].
\]
\[
+ \left( I - \beta \left( I - \delta^i \right) \right) \left( I - S \right) \tilde{w}_i - \tilde{\ell}_{i+1} + \tilde{\ell}_i.
\]

(D.223)

**Goods Market Clearing.** Following an analogous analysis as for steady-state above, the total derivative of the goods market clearing condition can be written in matrix form as:
\[
\tilde{w}_i + \tilde{\ell}_i = T \left( \tilde{w}_i + \tilde{\ell}_i \right) + \theta M \left( \tilde{w}_i - (I - \mu) \tilde{\chi}_i - \tilde{z} \right),
\]
where these matrices have \(NJ \times NJ\) elements and we have used \(d \ln \tilde{\tau} = 0\). This expression can be re-written as:
\[
\tilde{w}_i = [I - T - \theta M]^{-1} \left[ - (I - T) \tilde{\ell}_i - \theta M \left[ (I - \mu) \tilde{\chi}_i + \tilde{z} \right] \right].
\]
\[
(D.224)
\]
**Population Flow.** The total derivative of the population flow condition (D.209) relative to the initial steady-state has the following matrix representation:

\[
\tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1},
\]

(D.225)

where again these matrices have $NJ \times NJ$ elements.

**Value function.** Following an analogous analysis as for steady-state above, the total derivative of the value function relative to the initial steady-state can be written in matrix form as:

\[
\tilde{v}_t = (I - S) \tilde{w}_t + S [(I - \mu) \tilde{\chi}_t + \tilde{z}] + \tilde{b} + \beta D E_t \tilde{v}_{t+1},
\]

where again these matrices have $NJ \times NJ$ elements and we have used $d \ln \tau = 0$ and $d \ln \kappa = 0$.

**System of Equations for Transition Dynamics.** Collecting together the system of equations for the transition dynamics, we have:

\[
\tilde{\chi}_{t+1} = \left[ \frac{\beta (I - \delta) I + (I - \beta (I - \delta)) S (I - \mu) \tilde{\chi}_t + \tilde{z}}{(I - \beta (I - \delta)) (I - S) \tilde{w}_t - \ell_{t+1} + \ell_t} \right].
\]

(D.227)

\[
\tilde{w}_t = [I - T - \theta M]^{-1} \left[ - (I - T) \tilde{\ell}_t - \theta M \tilde{\chi}_t + \tilde{z} \right].
\]

(D.228)

\[
\tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1}.
\]

(D.229)

\[
\tilde{v}_t = (I - S) \tilde{w}_t + S [(I - \mu) \tilde{\chi}_t + \tilde{z}] + \tilde{b} + \beta D E_t \tilde{v}_{t+1}.
\]

(D.230)

**D.6 Trade Deficits.**

In this section of the appendix, we consider an extension of our baseline model in Section 2 of the paper and Section B of this online appendix to allow for trade deficits. As the model does not generate predictions for how trade imbalances respond to shocks, we follow the standard approach in the quantitative international trade literature of treating these imbalances as exogenous. We apportion these trade deficits fully to worker income, assuming that expenditure equals income for landlords. In particular, we allow the ratio of per capita expenditure to per capita income ($d_{nt}$) for workers to differ exogenously across locations and over time. When workers choose whether to move to a location, they take into account not only the labor income in that region but also this exogenous ratio of expenditure to income, which corresponds to a transfer to workers by location.
D.6.1 General Equilibrium

Given the state variables \{i_0, k_{i0}\} and a path for the ratio of expenditure to income \{d_{it}\}, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}. All other endogenous variables of the model can be recovered as a function of these variables. The conditions for general equilibrium take a similar form as in our baseline model in Section 2 of the paper and Section B of this online appendix.

**Capital Accumulation:** Using capital market clearing (B.22), the price index (B.8) and the equilibrium pricing rule (B.11), and assuming logarithmic intertemporal utility for simplicity, the capital accumulation equation becomes:

\[
k_{it+1} = \beta \frac{1 - \mu}{\mu} w_{it} \ell_{it} + \beta (1 - \delta) k_{it}.
\]

(D.231)

\[
p_{nt} = \left[ \sum_{i=1}^{N} \left( w_{it} \left( \frac{1 - \mu}{\mu} \left( \ell_{it}/k_{it} \right)^{1-\mu} \tau_{ni}/z_i \right)^{-\theta} \right) \right]^{-1/\theta}.
\]

(D.232)

**Goods Market Clearing:** Using the equilibrium pricing rule (B.11), the expenditure share (B.9) and capital market clearing (B.22), the goods market clearing condition (B.20) with trade deficits can be written as:

\[
w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} d_{nt} w_{nt} \ell_{nt},
\]

(D.233)

\[
S_{nit} = \frac{\left( w_{it} \left( \ell_{it}/k_{it} \right)^{1-\mu} \tau_{ni}/z_i \right)^{-\theta}}{\sum_{m=1}^{N} \left( w_{mt} \left( \ell_{mt}/k_{mt} \right)^{1-\mu} \tau_{nm}/z_m \right)^{-\theta}}, \quad T_{int} = \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},
\]

where \(S_{nit}\) is the expenditure share of importer \(n\) on exporter \(i\) at time \(t\), and we have defined \(T_{int}\) as the corresponding income share of exporter \(i\) from importer \(n\) at time \(t\). Note that the order of subscripts switches between the expenditure share (\(S_{nit}\)) and the income share (\(T_{int}\)), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.
Population Flow: Using the out-migration probabilities (B.4), the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it},$$

$$D_{igt} = \exp \left( \frac{\beta E_{t} v_{wt}^{w}}{\kappa_{gt}} \right)^{1/\rho} / \sum_{m=1}^{N} \exp \left( \frac{\beta E_{t} v_{mt}^{w}}{\kappa_{mt}} \right)^{1/\rho},$$

$$E_{git} = \ell_{it} D_{igt} / \ell_{gt+1},$$

where $D_{igt}$ is the out-migration probability from location $i$ to location $g$ between time $t$ and $t + 1$, and we have defined $E_{git}$ as the corresponding in-migration probability to location $g$ from location $i$ between time $t$ and $t + 1$. Note that the order of subscripts switches between the out-migration probability ($D_{igt}$) and the in-migration probability ($E_{git}$), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function (B.7) in the value function (B.3), the expected value from living in location $n$ at time $t$ can be written as:

$$v_{nt}^{w} = \ln b_{nt} + \ln \left( \frac{d_{nt} w_{nt}}{p_{nt}} \right) + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \frac{\beta E_{t} v_{gt+1}^{w}}{\kappa_{gt}} \right) / \kappa_{gt} \right)^{1/\rho}.$$

(D.235)

D.6.2 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics. In the interests of brevity, we focus on relationships that differ from the baseline model in Section 2 of the paper and Section B of this online appendix.

Real Expenditure. Totally differentiating real expenditure we have:

$$d \ln \left( \frac{d_{it} w_{it}}{p_{it}} \right) = d \ln d_{it} + d \ln w_{it} - d \ln p_{it},$$

$$d \ln \left( \frac{d_{it} w_{it}}{p_{it}} \right) = - \sum_{m=1}^{N} S_{nt} \left[ d \ln \tau_{nt} + d \ln \chi_{nt} - d \ln \chi_{mt} - d \ln \zeta_{nt} \right].$$

(D.236)

Goods Market Clearing. Totally differentiating the goods market clearing condition (B.20), we have:

$$\frac{dw_{it}}{w_{it}} + \frac{d\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} S_{nt} d_{nt} w_{nt} \ell_{nt} \left( \frac{dd_{nt}}{d_{nt}} + \frac{dw_{nt}}{w_{nt}} + \frac{d\ell_{nt}}{\ell_{nt}} + \theta \left( \sum_{h=1}^{N} s_{nht} \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nt}}{p_{nt}} \right) \right).$$
\[
\frac{dw_{it}}{w_{it}} + \frac{dl_{it}}{l_{it}} = \sum_{n=1}^{N} T_{int} \left( \frac{dd_{nt}}{d_{nt}} + \frac{dw_{nt}}{w_{nt}} + \frac{dl_{nt}}{l_{nt}} + \theta \left( \sum_{h \in N} S_{nht} \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nit}}{p_{nht}} \right) \right),
\]

\[
T_{int} = \frac{S_{nit} w_{nt} l_{nt}}{w_{it} l_{it}},
\]

\[
\left[ \frac{d \ln w_{it}}{d \ln l_{it}} \right] = \left[ \frac{d \ln w_{it}}{d \ln l_{it}} + \frac{d \ln w_{nt}}{d \ln l_{nt}} \right] = \left[ \theta \sum_{n=1}^{N} \sum_{m=1}^{N} T_{int} \frac{d \ln d_{nt} + d \ln w_{nt} + d \ln l_{nt}}{d \ln \tau_{nmt} + d \ln w_{nt} - (1 - \mu) d \ln \chi_{nt} - d \ln z_{nt}} \right].
\]

**Value Function.** Note that the value function can be re-written using the following results:

\[
v_{it} = \ln \left[ \frac{d_{it} w_{it}}{\left( \sum_{m=1}^{N} p_{itm} \right)^{-1/\theta}} \right] + b_{it} + \rho \ln \left( \frac{\exp \left( \beta \mathcal{E}_t v_{gt+1} \right)}{\kappa_{it}} \right)^{1/\rho},
\]

\[
\left( \sum_{m=1}^{N} p_{itm} \right)^{-1/\theta} = \left( \frac{p_{itm}^{\theta}}{S_{nit}} \right)^{-1/\theta} = \tau_{nmt} = 1,
\]

\[
\sum_{g=1}^{N} \left( \frac{\exp \left( \beta \mathcal{E}_t v_{gt+1} \right)}{\kappa_{it}} \right)^{1/\rho} = \frac{\exp \left( \beta \mathcal{E}_t v_{it+1} \right)}{D_{it}}, \quad \kappa_{it} = 1
\]

\[
v_{it} = -\frac{1}{\theta} \ln S_{nit} + d \ln d_{it} + d \ln w_{it} - d \ln p_{it} + d \ln b_{it} + \beta \mathcal{E}_t v_{it+1} - \rho d \ln D_{it}.
\]  

Totally differentiating this expression for the value function, we have:

\[
dv_{it} = -\frac{1}{\theta} d \ln S_{nit} + d \ln d_{it} + d \ln w_{it} - d \ln p_{it} + d \ln b_{it} + \beta \mathcal{E}_t d v_{it+1} - \rho d \ln D_{it},
\]

where

\[
d \ln S_{nit} = -\theta d \ln p_{it} + \theta \left[ \sum_{m=1}^{N} S_{nitm} d \ln p_{itm} \right],
\]

\[
d \ln D_{it} = \frac{1}{\rho} \left[ \beta \mathcal{E}_t d v_{it+1} - d \ln \kappa_{it} - \sum_{m=1}^{N} D_{itm} (\beta \mathcal{E}_t d v_{mt+1} - d \ln \kappa_{mit}) \right].
\]

Using these results for \( d \ln S_{nit} \) and \( d \ln D_{it} \) in the expression for \( dv_{it} \) above, we have:

\[
dv_{it} = \left[ d \ln d_{it} + d \ln w_{it} - \sum_{m=1}^{N} S_{nitm} d \ln p_{itm} + d \ln b_{it} + \sum_{m=1}^{N} D_{itm} (\beta \mathcal{E}_t d v_{mt+1} - d \ln \kappa_{mit}) \right],
\]

where we have used \( d \ln \kappa_{it} = 0 \). Using the pricing rule, we can re-write this derivative of the value function as follows:

\[
dv_{it} = \left[ d \ln d_{it} + d \ln w_{it} - \sum_{m=1}^{N} S_{nitm} (d \ln \tau_{nmt} + d \ln w_{nt} - (1 - \mu) d \ln \chi_{nt} - d \ln z_{nt}) + d \ln b_{it} + \sum_{m=1}^{N} D_{itm} (\beta \mathcal{E}_t d v_{mt+1} - d \ln \kappa_{mit}) \right].
\]
D.6.3 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant fundamentals \(\{z, b, d, \tau_n, \kappa_n\}\) and constant values of the endogenous variables: \(\kappa_{it+1} = \kappa_{it} = \kappa_{it}^*, \ell_{it+1} = \ell_{it} = \ell_{it}^*, w_{it+1} = w_{it} = w_{it}^*\) and \(v_{it+1} = v_{it} = v_{it}^*\), where we use an asterisk to denote a steady-state value. We consider a small common shock to productivity across sectors \((\ln z)\), amenities across sectors \((\ln b)\) and trade deficits across sectors \((\ln d)\) in each location, holding constant the economy’s aggregate labor endowment \((\ln \tilde{\ell})\), trade costs \((\ln \tau)\) and commuting costs \((\ln \kappa = 0)\).

**Capital Accumulation.** From the capital accumulation equation (B.24), the steady-state stock of capital solves:

\[
(1 - \beta (1 - \delta)) \chi_i^* = (1 - \beta (1 - \delta)) \frac{k_i^*}{\ell_i^*} = \beta \frac{1 - \mu w_i^*}{\mu} p_i^*.
\]

Totally differentiating, we have:

\[
\ln \chi_i^* = \ln \left( \frac{w_i^*}{p_i^*} \right).
\]

Using the total derivative of real income, this becomes:

\[
\frac{d \ln \chi_i}{d \ln w_i} = S d \ln w_i - \sum_{m=1}^{N} S_{im}^* \frac{d \ln w_m}{d \ln z_m},
\]

where we have used and \(d \ln \tau_{nm} = 0\). This relationship has the matrix representation:

\[
(I - (1 - \mu) S) d \ln \chi^* = (I - S) d \ln w^* + S d \ln z.
\]

**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.237) has the following matrix representation:

\[
\ln w_t + d \ln \ell_t = \begin{bmatrix}
T \left( d \ln d + d \ln w_t + d \ln \ell_t \right) \\
+ \theta (TS - I) \left( d \ln w_t - (1 - \mu) d \ln \chi_t - d \ln z \right)
\end{bmatrix},
\]

where we have used \(d \ln \tau = 0\). We can re-write this relationship as:

\[
[I - T + \theta (I - TS)] d \ln w_t = \begin{bmatrix}
T d \ln d - (I - T) d \ln \ell_t \\
+ \theta (I - TS) \left( d \ln z + (1 - \mu) d \ln \chi_t \right)
\end{bmatrix}.
\]

In steady-state we have:

\[
[I - T + \theta (I - TS)] d \ln w^* = \begin{bmatrix}
T d \ln d - (I - T) d \ln \ell^* \\
+ \theta (I - TS) \left( d \ln z + (1 - \mu) d \ln \chi^* \right)
\end{bmatrix}.
\]

(D.241)
**Population Flow.** The total derivative of the population flow condition has the same matrix representation as in our baseline model:

\[
\frac{d \ln \ell_{t+1}}{dt} = E \frac{d \ln \ell_t}{dt} + \frac{\beta}{\rho} (I - ED) d v_{t+1}.
\]

In steady-state, we have:

\[
\frac{d \ln \ell^*}{dt} = E \frac{d \ln \ell^*}{dt} + \frac{\beta}{\rho} (I - ED) d v^*.
\] (D.242)

**Value function.** The total derivative of the value function (D.239) has the following matrix representation:

\[
\frac{dv_t}{dt} = \left[ d \frac{d \ln d}{dt} + (I - S) \frac{d \ln w_t}{dt} \right.
\]
\[
+ S (d \ln z + (1 - \mu) d \ln x_t) + d \ln b + \beta D d v_{t+1}
\].

where we have used \(d \ln \tau = d \ln \kappa = 0\). In steady-state, we have:

\[
\frac{dv^*}{dt} = \left[ d \frac{d \ln d}{dt} + (I - S) \frac{d \ln w^*}{dt} \right.
\]
\[
+ S (d \ln z + (1 - \mu) d \ln x^*) + d \ln b + \beta D d v^*
\].

(D.243)

**System of Steady-State Equations.** Collecting together the system of steady-state equations, we have:

\[
\frac{d \ln x^*}{dt} = [I - (1 - \mu) S]^{-1} [(I - S) \frac{d \ln w^*}{dt} + S \frac{d \ln z}{dt}].
\] (D.244)

\[
\frac{d \ln w^*}{dt} = [I - T + \theta (I - TS)]^{-1} \left[ T \frac{d \ln d}{dt} - (I - T) \frac{d \ln \ell^*}{dt} \right.
\]
\[
+ (I - TS) \theta (d \ln z + (1 - \mu) d \ln x^*)
\]. (D.245)

\[
\frac{d \ln \ell^*}{dt} = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) d v^*.
\] (D.246)

\[
\frac{dv^*}{dt} = [I - \beta D]^{-1} \left[ -S \left( \frac{d \ln w^*}{dt} - d \ln z - (1 - \mu) \frac{d \ln x^*}{dt} \right) + d \ln b \right].
\] (D.247)

As the expenditure shares \((S)\) and income shares \((T)\) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire \((\sum_{i=1}^{N} w^*_i \ell^*_i = \sum_{i=1}^{N} q^*_i = \overline{q} = 1)\), which implies that the log changes in incomes satisfy \(Q^* \frac{d \ln q^*}{dt} = \sum_{i=1}^{N} q^*_i \frac{d \ln q^*_i}{dt} = \sum_{i=1}^{N} \frac{d q^*_i}{d q^*_i} = \sum_{i=1}^{N} d q^*_i = 0\), where \(Q^*\) is a row vector of the income of each location. Similarly, the outmigration shares \((D)\) and immigration shares \((E)\) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: \(\sum_{i=1}^{N} \ell^*_i = \overline{\ell} = 1\), which implies \(L^* \frac{d \ln \ell^*}{dt} = \sum_{i=1}^{N} \ell^*_i \frac{d \ln \ell^*_i}{dt} = \sum_{i=1}^{N} \ell^*_i \frac{d \ell^*_i}{d \ell^*_i} = \sum_{i=1}^{N} d \ell^*_i = 0\), where \(L^*\) is a row vector of the population of each location.
In the interest of brevity, we focus above on deriving sufficient statistics for changes in steady-states in the presence of trade deficits. Nevertheless, analogous sufficient statistics results for transition paths can be derived in the presence of trade deficits, as in our baseline model in Section 2 of the paper and Section B of this online appendix.

D.7 Residential Capital (Housing)

In this section of the appendix, we consider an extension of our baseline model in Section 2 of the paper and Section B of this web appendix to allow capital to be used residentially as well as commercially. We consider an economy that consists of many locations indexed by \( i \in \{1, \ldots, N\} \). Time is discrete and is indexed by \( t \). The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelastically and are geographically mobile across locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed and depreciates gradually at a constant rate \( \delta \).

D.7.1 Worker Migration Decisions

Worker migration decisions are modeled as in our baseline model in Section 2 of the paper and Section B of this online appendix. The expected value for a worker of living in location \( i \) at time \( t \) \( (v_{it}^w) \) is:

\[
v_{it}^w = \ln u_{it}^w + \rho \log \sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_t v_{gt+1}^w \right) / \kappa_{git} \right)^{1/\rho}.
\]  
(D.248)

The probability of migrating from location \( i \) to location \( g \) is:

\[
D_{igt} = \frac{\left( \exp \left( \beta \mathbb{E}_t v_{gt+1}^w \right) / \kappa_{git} \right)^{1/\rho}}{\sum_{m=1}^{N} \left( \exp \left( \beta \mathbb{E}_t v_{mt+1}^w \right) / \kappa_{kit} \right)^{1/\rho}}.
\]  
(D.249)

D.7.2 Worker Consumption

As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in the location in which they have chosen to live in that period. Worker static utility depends on local amenities \( (b_{nt}) \), goods consumption \( (c_{nt}^w) \) and residential use of capital \( (k_{nt}^w) \):
\[ \ln u_{nt} = \ln b_{nt} + \alpha \ln c_{nt}^{w} + (1 - \alpha) \ln k_{nt}^{w}, \quad 0 < \alpha < 1, \]  
(D.250)

where \( c_{nt}^{w} \) is a consumption index for workers in location \( n \) defined over the consumption of the variety supplied by each location \( i \) (\( c_{ni}^{w} \)):

\[
c_{nt}^{w} = \left[ \sum_{i=1}^{N} \left( c_{ni}^{w} \right) \frac{\theta}{\sigma + 1} \right] ^{-\frac{\sigma + 1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1, \]  
(D.251)

where \( \sigma > 1 \) is the constant elasticity of substitution (CES) between varieties and \( \theta = \sigma - 1 \) is the trade elasticity. Amenities (\( b_{nt} \)) capture exogenous characteristics of a location that make it a more attractive place to live regardless of the wage and cost of consumption goods (e.g. climate and scenic views).

The corresponding worker indirect utility function depends on amenities (\( b_{nt} \)), the wage (\( w_{nt} \)), the rental rate for capital (\( r_{nt} \)) and the consumption goods price index (\( p_{nt} \)):

\[ \ln u_{nt} = \ln b_{nt} + \ln w_{nt} - \alpha \ln p_{nt} - (1 - \alpha) \ln r_{nt}, \]  
(D.252)

where the consumption goods price index (\( p_{nt} \)) in location \( n \) depends of the price of the variety sourced from each location \( i \) (\( p_{nit} \)):

\[ p_{nt} = \left[ \sum_{i=1}^{N} p_{nit}^{-\theta} \right] ^{-1/\theta}. \]  
(D.253)

From the first-order conditions for worker utility maximization, total worker payments for goods consumption and residential capital use are constant multiples of total worker income:

\[ p_{nt} c_{nt} = \alpha w_{nt} \ell_{nt}, \]  
(D.254)

\[ r_{nt} k_{nt}^{w} = (1 - \alpha) w_{nt} \ell_{nt}. \]  
(D.255)

Using constant elasticity of substitution (CES) demand for individuals varieties of goods, the share location \( n \)'s expenditure on the goods produced by location \( i \) is:

\[ S_{nit} = \frac{p_{nit}^{-\theta}}{\sum_{m=1}^{N} p_{nit}^{-\theta}}. \]  
(D.256)

### D.7.3 Production

Producers in each location use labor (\( \ell_{it} \)) and commercial capital (\( k_{it}^{y} \)) to produce output (\( y_{it} \)) of the variety supplied by that location. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

\[ y_{it} = z_{it} \left( \frac{\ell_{it}}{\mu} \right)^{\mu} \left( \frac{k_{it}^{y}}{1 - \mu} \right)^{1-\mu}, \quad 0 < \mu < 1, \]  
(D.257)
where $z_{it}$ denotes exogenous productivity in location $i$ at time $t$.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit} \geq 1$ units of a good must be shipped from location $i$ in order for one unit to arrive in location $n$, where $\tau_{nit} > 1$ for $n \neq i$ and $\tau_{iit} = 1$. From profit maximization, the cost to a consumer in location $n$ of sourcing the good produced by location $i$ is:

$$p_{nit} = \tau_{nit} p_{iit} = \frac{\tau_{nit} w_{it}^{1-\mu}}{z_{it}},$$

(D.258)

where $p_{iit}$ is the “free on board” price of the good supplied by location $i$ before trade costs.

From profit maximization problem and zero profits, total payments to each factor of production are a constant share of total revenue:

$$w_{it} = \mu p_{iit} y_{it},$$

(D.259)

$$r_{it} k_{it}^y = (1 - \mu) p_{iit} y_{it},$$

(D.260)

### D.7.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords’ intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v_{it}^k = \sum_{t=0}^\infty \beta^t \left[ \alpha \ln c_{it}^k + (1 - \alpha) \ln k_{it}^k \right],$$

(D.261)

where $c_{it}^k$ is a consumption index defined over the consumption of the good supplied by each location ($c_{it}^{k_{nit}}$) as in equation (D.251); $k_{nit}$ denotes is landlords’ residential use of capital; and $\beta$ denotes the discount rate.

We assume that the investment technology for capital in each location uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location can produce one unit of capital in that location using one unit of the consumption index in that location. We assume that capital is geographically immobile once installed and depreciates at a constant rate $\delta$. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($r_{it} k_{it}$) equals the total value of goods consumption ($p_{it} c_{it}^k$), residential capital use ($r_{it} k_{it}^y$), and net investment ($p_{it} (k_{it+1} - (1 - \delta) k_{it})$):

$$r_{it} k_{it} = p_{it} c_{it}^k + r_{it} k_{it}^y + p_{it} (k_{it+1} - (1 - \delta) k_{it}),$$

(D.262)
Combining the landlords intertemporal utility (D.261) and budget constraint (D.262), the landlord’s intertemporal optimization problem is:

$$\max_{\{c_t, k_{it+1}\}} \sum_{t=0}^{\infty} \beta^t \left[ \alpha \ln c^k_{it} + (1 - \alpha) \ln k^k_{it} \right],$$

subject to $$p_{it} c^k_{it} + p_{it} (k_{it+1} - (1 - \delta) k_{it}) = r_{it} (k_{it} - k^k_{it}).$$

We can write this problem as the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ \alpha \ln c^k_{it} + (1 - \alpha) \ln k^k_{it} \right] - \xi_t \left[ p_{it} c^k_{it} + p_{it} (k_{it+1} - (1 - \delta) k_{it}) - r_{it} (k_{it} - k^k_{it}) \right].$$

The first-order conditions are:

$$\{c_t\} \quad \alpha \frac{\beta^t}{c_t} - p_{it} \xi_t = 0,$$

$$\{k_{it+1}\} \quad (r_{it+1} + p_{it+1} (1 - \delta)) \xi_{t+1} - p_{it} \xi_t = 0,$$

$$\{k^k_{it}\} \quad (1 - \alpha) \frac{\beta^t}{k^k_{it}} - r_{it} \xi_t = 0.$$

Together these first-order conditions imply:

$$\frac{c_{it+1}}{c_{it}} = \beta \frac{p_{it+1}}{p_{it+1} \mu_{it+1}} = \beta \left( \frac{r_{it+1}}{p_{it+1} \mu_{it+1}} + (1 - \delta) \right),$$

$$\frac{r_{it} k^k_{it}}{p_{it} c^k_{it}} = \frac{1 - \alpha}{\alpha},$$

where the transversality condition implies:

$$\lim_{t \to \infty} \beta^t \frac{k_{it+1}}{c_{it}} = 0.$$

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

$$p_{it} c^k_{it} = \alpha (1 - \beta) \left( r_{it} + p_{it} (1 - \delta) \right) k_{it},$$

$$r_{it} k^k_{it} = (1 - \alpha) (1 - \beta) \left( r_{it} + p_{it} (1 - \delta) \right) k_{it},$$

$$k_{it+1} = \beta \left( \frac{r_{it}}{p_{it}} + (1 - \delta) \right) k_{it}.$$

Substituting the consumption policy function (D.267) into the Euler equation (D.265), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

$$\frac{c^k_{it+1}}{c^k_{it}} = \frac{\left( r_{it+1} / p_{it+1} + (1 - \delta) \right) k_{it+1}}{\left( r_{it} / p_{it} + (1 - \delta) \right) k_{it}},$$

$$= \beta \left( \frac{r_{it+1}}{p_{it+1}} + (1 - \delta) \right).$$
D.7.5 Market Clearing

Goods market clearing implies that revenue in each location equals expenditure on the goods produced by that location:

\[ p_{it}y_{it} = \alpha \sum_{n=1}^{N} S_{nit} (w_{nt}\ell_{nt} + r_{nt}k_{nt}) , \]

\[ w_{it}\ell_{it} + r_{nt}k_{nt} = \alpha \sum_{n=1}^{N} S_{nit} (w_{nt}\ell_{nt} + r_{nt}k_{nt}) , \]

\[ w_{it}\ell_{it} + \frac{1 - \mu}{\mu} w_{it}\ell_{it} = \alpha \sum_{n=1}^{N} S_{nit} (w_{nt}\ell_{nt} + r_{nt}k_{nt}) , \]

\[ w_{it}\ell_{it} + \frac{1 - \mu}{\mu} w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} \left( \frac{1}{\alpha} w_{nt}\ell_{nt} + \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) w_{nt}\ell_{nt} \right) , \]

\[ \frac{1}{\mu} w_{it}\ell_{it} = \frac{\mu}{\mu} S_{nit} w_{nt}\ell_{nt} , \]

\[ \frac{1}{\mu} w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt}\ell_{nt} , \]

\[ w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt}\ell_{nt} . \]  

(D.270)

The capital market clearing condition equates the income received by landlords from ownership of capital to payments for the residential and commercial use of capital. Using workers’ expenditure on residential capital (D.255), payments for labor (D.259) and capital (D.260) in production, and landlords’ expenditure on residential capital (D.266), this capital market clearing condition can be expressed as:

\[ r_{it}k_{it} = r_{it}k_{it}^k + r_{it}k_{it}^w + r_{it}k_{it}^y , \]

\[ r_{it}k_{it} = (1 - \alpha) r_{it}k_{it} + (1 - \alpha) w_{it}\ell_{it} + \frac{1 - \mu}{\mu} w_{it}\ell_{it} \]

\[ r_{it}k_{it} = \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) w_{it}\ell_{it} . \]  

(D.271)
D.7.6 General Equilibrium

Given the state variables \( \{i_0, k_{i0}\} \), the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \( \{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty} \). All other endogenous variables of the model can be recovered as a function of these variables.

Capital Accumulation: Using capital market clearing (D.271), the price index (D.253) and the equilibrium pricing rule (D.258), the capital accumulation equation (D.269) becomes:

\[
k_{it+1} = \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w_{it} \ell_{it}}{p_{it}} + \beta (1 - \delta) k_{it}.
\]

\[p_{nt} = \left[ \sum_{i=1}^{N} \left( w_{it} \left( \frac{1 - \mu}{\mu} \right) \frac{(1 - \mu)}{(1 + \mu)} \frac{k_{it}}{\ell_{it}} \frac{(1 - \mu)}{(1 + \mu)} \frac{\tau_{nit}}{z_{it}} \right) \right]^{-\theta/\alpha}.
\]

Goods Market Clearing: Using the equilibrium pricing rule (D.258), the expenditure share (D.256) and capital market clearing (D.271), the goods market clearing condition (D.270) can be written as:

\[
w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}.
\]

\[
S_{nit} \equiv \frac{p_{nit}^{-\theta}}{\sum_{m=1}^{N} p_{nmt}^{-\theta}}, \quad T_{int} \equiv \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},
\]

where \( S_{nit} \) is the expenditure share of importer \( n \) on exporter \( i \) at time \( t \), and we have defined \( T_{int} \) as the corresponding income share of exporter \( i \) from importer \( n \) at time \( t \). Note that the order of subscripts switches between the expenditure share \( (S_{nit}) \) and the income share \( (T_{int}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the outmigration probabilities (D.249), the population flow condition for the evolution of the population distribution over time is given by:

\[
\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}.
\]
\[ D_{igt} = \frac{\left( \exp \left( \frac{\beta E_{igt} v_{gt_{t+1}}^w}{\kappa_{git}} \right) \right)^{1/\rho}}{\sum_{m=1}^{N} \left( \exp \left( \frac{\beta E_{igt} v_{mt_{t+1}}^w}{\kappa_{mit}} \right) \right)^{1/\rho}}, \]

\[ E_{git} = \frac{\ell_{it} D_{igt}}{E_{git}^{1/\rho}}, \]

where \( D_{igt} \) is the outmigration probability from location \( i \) to location \( g \) between time \( t \) and time \( t + 1 \), and we have defined \( E_{git} \) as the corresponding immigration probability to location \( g \) from location \( i \) between time \( t \) and \( t + 1 \). Note that the order of subscripts switches between the outmigration probability \( D_{igt} \) and the immigration probability \( E_{git} \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

**Worker Value Function:** Using the worker indirect utility function \( D.250 \) in the value function \( D.248 \), the expected value from living in location \( n \) at time \( t \) can be written as:

\[ v_{it}^w = \ln b_{nt} + \ln \left( \frac{u_{nt}^\alpha}{p_{nt}^\alpha \left( \frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha \mu} \right) k_{nt}^{1-\alpha}} \right) + \rho \log \sum_{g=1}^{N} \exp \left( \frac{\beta E_{git} v_{gt_{t+1}}^w}{\kappa_{git}} \right)^{1/\rho}. \]

**D.7.7 Comparative Statics**

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

**Prices** Using the relationship between capital and labor payments \( D.271 \), the pricing rule \( D.258 \) can be re-written as follows:

\[ p_{nit} = \tau_{nit} w_{it} \left( \frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha \mu} \right)^{1-\mu} \left( \frac{1}{\chi_{it}} \right)^{1-\mu}, \quad (D.275) \]

where \( \chi_{it} \) is the capital-labor ratio:

\[ \chi_{it} = \frac{k_{it}}{\ell_{it}}. \]

Totally differentiating this pricing rule, we have:

\[ d \ln p_{nit} = d \ln \tau_{nit} + d \ln w_{it} - (1-\mu) \ln \chi_{it} - d \ln z_{it}. \quad (D.276) \]

**Expenditure Shares** Totally differentiating this expenditure share equation \( D.256 \), we get:

\[ \frac{dS_{nit}}{S_{nit}} = \theta \left( \sum_{h=1}^{N} S_{nht} \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nit}}{p_{nit}} \right), \quad (D.277) \]

\[ d \ln S_{nit} = \theta \left( \sum_{h=1}^{N} S_{nht} d \ln p_{nht} - d \ln p_{nit} \right). \]
Price Indices

Totally differentiating the consumption goods price index in equation (D.253), we have:

\[
\frac{dp_{nt}}{p_{nt}} = \sum_{m=1}^{N} S_{ntm} \frac{dp_{nmt}}{p_{nmt}},
\]
\[\text{(D.278)}\]

\[
d \ln p_{nt} = \sum_{m=1}^{N} S_{ntm} d \ln p_{nmt}.
\]

Migration Shares

Totally differentiating the outmigration share in equation (D.249), we get:

\[
\frac{dD_{igt}}{D_{igt}} = \frac{1}{\rho} \left[ \left( \beta \mathbb{E}_t d\nu_{gt+1} - \frac{d\kappa_{git}}{\kappa_{git}} \right) - \sum_{h=1}^{N} D_{iht} \left( \beta \mathbb{E}_t d\nu_{ht+1} - \frac{d\kappa_{hit}}{\kappa_{hit}} \right) \right],
\]
\[\text{(D.279)}\]

\[
d \ln D_{igt} = \frac{1}{\rho} \left[ \left( \beta \mathbb{E}_t d\nu_{gt+1} - d \ln \kappa_{git} \right) - \sum_{h=1}^{N} D_{iht} \left( \beta \mathbb{E}_t d\nu_{ht+1} - d \ln \kappa_{hit} \right) \right].
\]

Real Income

Totally differentiating real income we have:

\[
d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - d \ln p_{it},
\]

\[
d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - \sum_{m=1}^{N} S_{ntm} d \ln p_{nmt},
\]

\[
d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - \sum_{m=1}^{N} S_{ntm} \left[ d \ln \tau_{nmt} + d \ln w_{mt} - (1 - \mu) d \ln \lambda_{nt} - d \ln z_{mt} \right].
\]
\[\text{(D.280)}\]

Goods Market Clearing

Totally differentiating the goods market clearing condition (D.270), we have:

\[
\frac{dw_{it}}{w_{it}} + \frac{d\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt} \left( \frac{dw_{nt}}{w_{nt}} + \frac{d\ell_{nt}}{\ell_{nt}} + \frac{dS_{nit}}{S_{nit}} \right).
\]

Using our result for the derivative of expenditure shares in equation (D.277) above, we can rewrite this as:

\[
\frac{dw_{it}}{w_{it}} + \frac{d\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} T_{int} \left( \frac{dw_{nt}}{w_{nt}} + \frac{d\ell_{nt}}{\ell_{nt}} + \theta \left( \sum_{h \in N} \frac{S_{nht}}{p_{nht}} \frac{dp_{nht}}{p_{nht}} - \frac{dp_{nt}}{p_{nt}} \right) \right),
\]

\[
T_{int} = \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}}.
\]

\[
d \ln w_{it} + d \ln \ell_{it} = \left[ +\theta \sum_{n=1}^{N} \sum_{m=1}^{N} T_{int} S_{nmt} \left( d \ln w_{nt} + d \ln \ell_{nt} \right) + \sum_{m=1}^{N} T_{int} S_{nmt} \left( d \ln \tau_{nmt} + d \ln w_{mt} - (1 - \mu) d \ln \lambda_{nt} - d \ln z_{mt} \right) \right].
\]
\[\text{(D.281)}\]
Population Flow. Totally differentiating the population flow condition (D.274) we have:

\[ \ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \]

\[ d \ln \ell_{gt+1} = \sum_{i=1}^{N} E_{git} \left[ d \ln \ell_{it} + \frac{1}{\rho} \left( \beta E_t d v_{gt+1} - d \ln \kappa_{gi} - \sum_{m=1}^{N} D_{imt} (\beta E_t d v_{mt+1} - d \ln \kappa_{mit}) \right) \right]. \quad (D.282) \]

Value Function. Note that the value function can be re-written using the following results:

\[ v_{it} = \ln \frac{w_{it}}{\left( \sum_{m=1}^{N} p_{imt}^{-\theta} \right)^{-1/\theta}} + \ln b_{it} + \rho \ln \sum_{g=1}^{N} (\exp (\beta E_t v_{gt+1} / \kappa_{gi}))^{1/\rho}, \]

\[ \left( \sum_{m=1}^{N} p_{imt}^{-\theta} \right)^{-1/\theta} = \frac{p_{it}^{-\theta}}{S_{it}}, \quad \tau_{ii} = 1, \]

\[ \sum_{g=1}^{N} (\exp (\beta E_t v_{gt+1} / \kappa_{gi}))^{1/\rho} = \frac{(\exp (\beta E_t v_{it+1} / \kappa_{iit}))^{1/\rho}}{D_{iit}}, \quad \kappa_{iit} = 1, \]

\[ v_{it} = -\frac{1}{\theta} \ln S_{it} + \ln w_{it} - \ln p_{iit} + \ln b_{it} + \beta E_t v_{it+1} - \rho \ln D_{iit}. \quad (D.283) \]

Totally differentiating this value function (D.283) we have:

\[ dv_{it} = -\frac{1}{\theta} d \ln S_{it} + d \ln w_{it} - d \ln p_{iit} + d \ln b_{it} + \beta E_t d v_{it+1} - \rho d \ln D_{iit}, \]

\[ d \ln S_{it} = -\theta d \ln p_{iit} + \theta \left[ \sum_{m=1}^{N} S_{imt} d \ln p_{imt} \right], \]

\[ d \ln D_{iit} = \frac{1}{\rho} \left[ \beta E_t d v_{it+1} - d \ln \kappa_{iit} - \sum_{m=1}^{N} D_{imt} (\beta E_t d v_{mt+1} - d \ln \kappa_{mit}) \right]. \]

Using these results in the derivative of the value function, we have:

\[ dv_{it} = \left[ d \ln w_{it} - \sum_{m=1}^{N} S_{imt} d \ln p_{imt} + d \ln b_{it} + \sum_{m=1}^{N} D_{imt} (\beta E_t d v_{mt+1} - d \ln \kappa_{mit}) \right], \]

where we have used \( d \ln \kappa_{iit} = 0 \). Using the total derivative of the pricing rule (D.276), we can re-write this derivative of the value function as follows:

\[ dv_{it} = \left[ d \ln w_{it} - \sum_{m=1}^{N} S_{imt} \left( d \ln \tau_{imt} + d \ln w_{mt} - (1 - \mu) d \ln \chi_{mt} - d \ln z_{mt} \right) \right]. \quad (D.284) \]
D.7.8 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: \( k_{it+1} = k_{it} = k^*_i \), \( \ell_{it+1} = \ell_{it} = \ell^*_i \), \( w_{it+1} = w_{it} = w^*_i \) and \( v_{it+1} = v_{it} = v^*_i \), where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity (\( d \ln z \)) and amenities (\( d \ln b \)) in each location, holding constant the economy’s aggregate labor endowment (\( d \ln \bar{\ell} = 0 \)), trade costs (\( d \ln \tau \)) and commuting costs (\( d \ln \kappa \)).

**Capital Accumulation.** From the capital accumulation equation (D.272), the steady-state stock of capital solves:

\[
k^*_i = \beta \left[ \frac{r_i}{p_i} + (1 - \delta) \right] k^*_i.
\]

\[
(1 - \beta (1 - \delta)) k^*_i = \beta \frac{r_i}{p_i} k^*_i.
\]

From the relationship between labor and capital payments, we have:

\[
\frac{r_{it}}{p_{it}} k_{it} = \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w_{it} \ell_{it}}{p_{it}},
\]

Using this result in the expression for the steady-state capital stock above, we have:

\[
(1 - \beta (1 - \delta)) k^*_i = \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w^*_i \ell^*_i}{p^*_i}.
\]

Totally differentiating, we have:

\[
d \ln \chi^*_i = d \ln \left( \frac{w^*_i}{p^*_i} \right).
\]

From the total derivative of real income (D.280) above, this becomes:

\[
d \ln \chi^*_i = d \ln w^*_i - \sum_{m=1}^{N} S_{im} [d \ln w^*_m - (1 - \mu) d \ln \chi^*_m - d \ln z_m],
\]

where we have used \( d \ln \tau_{im} = 0 \). This relationship has the following matrix representation:

\[
\begin{align*}
    d \ln \chi^* &= d \ln w^* - S d \ln w^* + (1 - \mu) S d \ln \chi^* + S d \ln z, \\
    (I - (1 - \mu) S) d \ln \chi^* &= (I - S) d \ln w^* + S d \ln z,
\end{align*}
\]

which can be written as:

\[
d \ln \chi^* = (I - (1 - \mu) S)^{-1} [(I - S) d \ln w^* + S d \ln z]. \tag{D.285}
\]
**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.281) has the following matrix representation:

\[
d \ln w_t + d \ln \ell_t = T (d \ln w_t + d \ln \ell_t) + \theta (TS - I) (d \ln w_t - (1 - \mu) d \ln \chi_t - d \ln z),
\]

where we have used \( d \ln \tau = 0 \). We can re-write this relationship in steady-state as:

\[
d \ln w^* = [I - T + \theta (I - TS)]^{-1} \left[ - (I - T) d \ln \ell^* + \theta (I - TS) (d \ln z + (1 - \mu) d \ln \chi^*) \right].
\]  

**Population Flow.** The total derivative of the population flow condition (D.282) has the following matrix representation:

\[
d \ln \ell_{t+1} = E d \ln \ell_t + \frac{\beta}{\rho} (I - ED) E_t d v_{t+1},
\]

which can be written in steady-state as:

\[
d \ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) d v^*.
\]  

**Value function.** The total derivative of the value function (D.284) has the following matrix representation:

\[
d v_t = (I - S) d \ln w_t + S (d \ln z + (1 - \mu) d \ln \chi_t) + d \ln b + \beta \Delta E_t d v_{t+1},
\]

where we have used \( d \ln \tau = d \ln \kappa = 0 \). We can re-write this relationship in steady-state as:

\[
d v^* = (I - \beta D)^{-1} [(I - S) d \ln w^* + S (d \ln z + (1 - \mu) d \ln \chi^*) + d \ln b].
\]  

**System of Steady-State Equations.** Collecting together the system of steady-state equations, we have:

\[
d \ln \chi^* = (I - (1 - \mu) S)^{-1} [(I - S) d \ln w^* + S d \ln z].
\]  

\[
d \ln w^* = [I - T + \theta (I - TS)]^{-1} \left[ - (I - T) d \ln \ell^* + \theta (I - TS) (d \ln z + (1 - \mu) d \ln \chi^*) \right].
\]  

\[
d \ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) d v^*.
\]  

\[
d v^* = (I - \beta D)^{-1} [(I - S) d \ln w^* + S (d \ln z + (1 - \mu) d \ln \chi^*) + d \ln b].
\]  

**D.7.9 Sufficient Statistics for Transition Dynamics**

Suppose that the economy starts from an initial steady-state. Consider a small shock to productivity \( d \ln z \) and amenities \( d \ln b \) in each location, holding constant the economy’s aggregate labor endowment \( d \ln \ell = 0 \), trade costs \( d \ln \tau = 0 \) and commuting costs \( d \ln \kappa = 0 \). We use a tilde above a variable to denote a log-deviation from the initial steady-state, such that
\[ \tilde{\ell}_{it+1} = \ln \ell_{it+1} - \ln \ell_t^*, \text{ for all variables except for the worker value function } v_{it}; \text{ with a slight abuse of notation we use } \tilde{v}_{it} \equiv v_{it} - v_t^* \text{ to denote the deviation in levels for the worker value function.}\]

**Capital Accumulation.** From the capital accumulation equation (D.272), we have:

\[ k_{it+1} = \beta \frac{r_{it}}{p_{it}} k_{it} + \beta (1 - \delta) k_{it}. \]

From the relationship between labor and capital payments, we have:

\[ \frac{r_{it}}{p_{it}} k_{it} = \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) w_{it} \ell_{it}, \]

\[ k_{it+1} = \beta (1 - \delta) k_{it} + \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) w_{it} \ell_{it}, \]

\[ \frac{k_{it+1} \ell_{it+1}}{\ell_{it+1} \ell_{it}} = \beta (1 - \delta) k_{it} + \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) w_{it} \ell_{it}, \]

\[ \frac{\chi_{it+1} \ell_{it+1}}{\ell_{it} \ell_{it}} = \beta (1 - \delta) \chi_{it} + \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) w_{it} \ell_{it}, \]

while in steady-state we have:

\[ \frac{\tilde{k}^*_{it}}{\tilde{\ell}^*_{it}} = \beta (1 - \delta) \frac{\tilde{k}^*_{it}}{\tilde{\ell}^*_{it}} + \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w^*_{it}}{p^*_{it}}, \]

\[ \chi^*_{it} = \beta (1 - \delta) \chi^*_{it} + \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w^*_{it}}{p^*_{it}}, \]

\[ \chi^*_{it} = \beta \left( \frac{1}{1 - \beta (1 - \delta)} \right) \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w^*_{it}}{p^*_{it}} . \]

(D.294)

Dividing both sides of equation (D.293) by \( \chi^*_{it} \), we have:

\[ \frac{\chi_{it+1} \ell_{it+1}}{\chi^*_{it} \ell_{it}} = \beta (1 - \delta) \frac{\chi_{it}}{\chi^*_{it}} + \beta \left( \frac{1 - \alpha}{\alpha} + \frac{1 - \mu}{\alpha \mu} \right) \frac{w_{it}}{p_{it}}, \]

which using (D.294) can be re-written as:

\[ \frac{\chi_{it+1} \ell_{it+1}}{\chi^*_{it} \ell_{it}} = \beta (1 - \delta) \frac{\chi_{it}}{\chi^*_{it}} + (1 - \beta (1 - \delta)) \frac{w_{it}}{w^*_{it}} \frac{p_{it}}{p^*_{it}}, \]

which can be further re-written as:

\[ \frac{\chi_{it+1} \ell_{it+1}}{\chi^*_{it} \ell_{it}} - 1 = \beta (1 - \delta) \frac{\chi_{it}}{\chi^*_{it}} + (1 - \beta (1 - \delta)) \frac{w_{it}}{w^*_{it}} \frac{p_{it}}{p^*_{it}} - 1, \]

\[ \frac{\chi_{it+1} \ell_{it+1}}{\chi^*_{it} \ell_{it}} - 1 = \beta (1 - \delta) \left( \frac{\chi_{it}}{\chi^*_{it}} - 1 \right) + (1 - \beta (1 - \delta)) \left( \frac{w_{it}}{w^*_{it}} \frac{p_{it}}{p^*_{it}} - 1 \right). \]
Noting that:
\[
\frac{x_{it}}{x_i^*} - 1 \simeq \ln \left( \frac{x_{it}}{x_i^*} \right),
\]
\[
\frac{\chi_{it+1}}{\chi_i^*} \frac{\ell_{it+1}}{\ell_{it}} - 1 \simeq \ln \left( \frac{\chi_{it+1} \ell_{it+1}}{\chi_i^* \ell_{it}} \right),
\]
we have:
\[
\ln \left( \frac{\chi_{it+1}}{\chi_i^*} \right) + \ln \left( \frac{\ell_{it+1}}{\ell_{it}} \right) = \beta (1 - \delta) \ln \left( \frac{\chi_{it}}{\chi_i^*} \right) + (1 - \beta (1 - \delta)) \ln \left( \frac{w_{it}/w_i^*}{p_{it}/p_i^*} \right),
\]
\[
\ln \left( \frac{\chi_{it+1}}{\chi_i^*} \right) + \ln \left( \frac{\ell_{it+1}/\ell_i^*}{\ell_{it}/\ell_i^*} \right) = \beta (1 - \delta) \ln \left( \frac{\chi_{it}}{\chi_i^*} \right) + (1 - \beta (1 - \delta)) \ln \left( \frac{w_{it}/w_i^*}{p_{it}/p_i^*} \right),
\]
which can be re-written as follows:
\[
\tilde{\chi}_{it+1} = \beta (1 - \delta) \tilde{\chi}_{it} + (1 - \beta (1 - \delta)) (\tilde{w}_{it} - \tilde{p}_{it}) - \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\]
We can rewrite this relationship in matrix form as:
\[
\tilde{\chi}_{t+1} = \beta (1 - \delta) \tilde{\chi}_t + \beta (1 - \delta) \left( I - S \right) (\tilde{w}_t - \tilde{p}_t) - \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\]
Now note that:
\[
\tilde{w}_{it} - \tilde{p}_{it} = \tilde{w}_{it} - \sum_{m=1}^N S_{mnt} [\tilde{w}_{mt} - (1 - \mu) \tilde{\chi}_{mt} - \tilde{z}_m],
\]
which can be written in matrix form as:
\[
\tilde{w}_t - \tilde{p}_t = (I - S) \tilde{w}_t + S \left[ (1 - \mu) \tilde{\chi}_t + \tilde{z} \right].
\]
Using this result in our expression for the dynamics of the capital-labor ratio above, we have:
\[
\tilde{\chi}_{t+1} = \beta (1 - \delta) \tilde{\chi}_t + (1 - \beta (1 - \delta)) \left( I - S \right) (\tilde{w}_t - \tilde{p}_t) - \tilde{\ell}_{t+1} + \tilde{\ell}_t,
\]
\[
\tilde{\chi}_{t+1} = \left[ \beta (1 - \delta) I + (1 - \beta (1 - \delta)) (1 - \mu) S \right] \tilde{\chi}_t + (1 - \beta (1 - \delta)) S \tilde{z} + (1 - \beta (1 - \delta)) (I - S) \tilde{w}_t - \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\]
\[
\text{(D.295)}
\]

**Goods Market Clearing.** The total derivative of the goods market clearing condition (D.281) relative to the initial steady-state has the following matrix representation:
\[
\tilde{w}_t + \tilde{\ell}_t = T \left( \tilde{w}_t + \tilde{\ell}_t \right) + \theta (TS - I) (\tilde{w}_t - (1 - \mu) \tilde{\chi}_t - \tilde{z}),
\]
which can be re-written as:
\[
[I - T + \theta (I - TS)] \tilde{w}_t = - (I - T) \tilde{\ell}_t + \theta (I - TS) [(1 - \mu) \tilde{\chi}_t + \tilde{z}],
\]
\[
\tilde{w}_t = [I - T + \theta (I - TS)]^{-1} \left[ - (I - T) \tilde{\ell}_t + \theta (I - TS) [(1 - \mu) \tilde{\chi}_t + \tilde{z}] \right].
\]
\[
\text{(D.296)}
\]
Population Flow. The total derivative of the population flow condition \( \text{(D.282)} \) relative to the initial steady-state has the following matrix representation:

\[
\tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{w}_{t+1}.
\] (D.297)

Value function. The total derivative of the value function \( \text{(D.284)} \) relative to the initial steady-state has the following matrix representation:

\[
\tilde{v}_t = (I - S) \tilde{w}_t + S \left[ (1 - \mu) \tilde{\chi}_t + \tilde{z} \right] + \tilde{b} + \beta D E_t \tilde{v}_{t+1}.
\] (D.298)

System of Equations for Transition Dynamics. Collecting together the system of equations for the transition dynamics, we have:

\[
\tilde{\chi}_{t+1} = \left[ (1 - \delta) I + (1 - \beta (1 - \delta)) (1 - \mu) S \right] \tilde{\chi}_t + (1 - \beta (1 - \delta) S \tilde{z}) \tilde{\ell}_t + \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\] (D.299)

\[
\tilde{w}_t = [I - T + \theta (I - TS)]^{-1} \left[ -(I - T) \tilde{\ell}_t + \theta (I - TS) \left[ (1 - \mu) \tilde{\chi}_t + \tilde{z} \right] \right].
\] (D.300)

\[
\tilde{\ell}_{t+1} = E\tilde{\ell}_t + \frac{\beta}{\rho} (I - ED) E_t \tilde{v}_{t+1}.
\] (D.301)

\[
\tilde{v}_t = (I - S) \tilde{w}_t + S \left[ (1 - \mu) \tilde{\chi}_t + \tilde{z} \right] + \tilde{b} + \beta D E_t \tilde{v}_{t+1}.
\] (D.302)

D.8 Landlord Investment in Other Locations

In this section of the appendix, we consider an extension of our baseline model in Section 2 of the paper and Section B of this web appendix to allow landlords to invest in other locations in addition to their own location. We consider an economy that consists of many locations indexed by \( i \in \{1, \ldots, N\} \). Time is discrete and is indexed by \( t \). The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelastically and are geographically mobile across locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords in each location are geographically immobile and own a stock of capital that can be allocated to any location. Landlords make a forward-looking decision over consumption and investment in this stock of capital, which depreciates gradually at a constant rate \( \delta \).
D.8.1 Capital Allocation

At the beginning of period $t$, the landlords in location $n$ inherit an existing stock of capital $k_{nt}$, and decide where to allocate this existing capital and how much to invest in accumulating additional capital. Once these decisions have been made, production and consumption occur. At the end of period $t$, new capital is created from the investment decisions made at the beginning of the period, and the depreciation of existing capital occurs. In the remainder of this subsection, we characterize landlords’ decisions at the beginning of period $t$ of where to allocate the existing stock of capital. In the next subsection, we characterize landlords’ optimal consumption-investment decision.

The stock of existing capital owned by landlords in source location $n$ can be employed in each host location $i$. The productivity of each unit of capital owned in location $n$ is subject to an idiosyncratic productivity shock for each of the possible locations $i$ to which it can be allocated $\alpha_{nit}$. This productivity shock determines the number of effective units of capital, and has an interpretation as a Keynesian marginal efficiency of capital draw, which captures all the idiosyncratic factors that affect the productivity of capital invested in a location. Landlords face financial frictions or management costs in allocating capital to other locations, which are assumed to take the iceberg form, such that $\phi_{nit} \geq 1$ units of capital from location $n$ must be allocated to location $i$ in order for one unit to be available for production, where $\phi_{nnt} = 1$ and $\phi_{nit} > 1$ for $n \neq i$. The realized rate of return to a landlord in location $n$ from allocating one unit of capital to location $i$ is:

$$R_{nit} = \frac{\alpha_{nit}r_{it}}{\phi_{nit}},$$

where the idiosyncratic productivity shock ($\alpha_{nit}$) corresponds to the number of effective units of capital before financial frictions or management costs ($\phi_{nit}$) are incurred; and $r_{it}$ corresponds to the rate of return per effective unit of capital.

We assume that these idiosyncratic shocks to the productivity of capital are drawn independently across source and host locations and units of capital from the following Fréchet distribution:

$$F_{nit}(\alpha) = e^{-(\alpha/a_{it})^{-\epsilon}}, \quad a_{it} > 0, \quad \epsilon > 1,$$

where the Fréchet scale parameter ($a_{it}$) controls the average productivity of capital allocated to host location $i$. The Fréchet shape parameter $\epsilon$ controls the dispersion of idiosyncratic shocks to the productivity of capital, and regulates the sensitivity of the capital allocation to economic variables such as the rate of return per effective unit of capital.

Using the properties of this Fréchet distribution, the share of capital from location $n$ that is
employed in location $i$ satisfies a gravity equation:

$$\zeta_{nit} = \frac{k_{nit}}{k_{nt}} = \frac{(a_{nit}/\phi_{nit})^\epsilon}{\sum_{h=1}^N (a_{ht}r_{ht}/\phi_{nht})^\epsilon},$$

which provides a natural explanation for findings of home bias in capital investments, because financial frictions or management costs abroad are greater than those at home ($\phi_{nit} > \phi_{nht}$ for $n \neq i$).

Using the properties of this Fréchet distribution, the realized rate of return on capital owned by location $n$ at time $t$ is the same across all host locations $i$ and given by:

$$\mathcal{R}_{nit} = \mathcal{R}_{nt} = \Gamma \left( \frac{\epsilon - 1}{\epsilon} \right) \left[ \sum_{h=1}^N (a_{ht}r_{ht}/\phi_{nht})^\epsilon \right]^{\frac{1}{\epsilon}},$$

where $\Gamma(\cdot)$ is the Gamma function.

Using the properties of the Fréchet distributions for productivity ($a_{nit}$) and the realized rate of return ($\mathcal{R}_{nt}$), the average productivity of capital from source country $n$ in host country $i$ conditional on capital being allocated to that host country is given by:

$$\bar{a}_{nit} = \Gamma \left( \frac{\epsilon - 1}{\epsilon} \right) \left( \frac{\alpha^*_t}{\zeta_{nit}} \right)^\frac{1}{\epsilon}. $$

### D.8.2 Capital Accumulation Across Periods

Landlords in each location choose their consumption and investment to maximize their intertemporal utility subject to their budget constraint. Landlords’ intertemporal utility equals the expected present discounted value of their flow utility:

$$v^k_{nt} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left( \frac{c^k_{nit+s}}{1 - 1/\psi} \right)^{1-1/\psi},$$

where we use the superscript $k$ to denote landlords; $c^k_{nit}$ is the consumption index; $\beta$ is the discount rate; $\psi$ is the elasticity of intertemporal substitution. Since landlords are geographically immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

We assume that the investment technology in each location uses the varieties from all locations with the same functional form as consumption. Therefore, landlords in each location can produce one unit of capital using one unit of the consumption index in that location, where this unit of capital can be allocated to any location, as characterized in the previous subsection. We interpret capital as buildings and structures, which depreciate at the constant rate $\delta$, and we allow
for the possibility of negative investment. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($R_{nt}k_{nt}$) equals the total value of their consumption ($p_{nt}c_{nt}^k$) plus the total value of net investment ($p_{nt}(k_{nt+1} - (1 - \delta)k_{nt})$):

$$R_{nt}k_{nt} = p_{nt}(c_{nt}^k + k_{nt+1} - (1 - \delta)k_{nt})$$  \hspace{1cm} (D.309)

where we have used the property established in the previous subsection that the realized rate of return is the same across all host locations for a given source location ($R_{nt} = R_{nt}$). We use $R_{nt} = 1 - \delta + R_{nt}/p_{nt}$ to denote the gross realized return on capital. Following the same line of argument as in Section 2 of the paper, the optimal consumption of landlords in location $n$ satisfies $c_{nt} = \zeta_{nt}R_{nt}k_{nt}$, where $\zeta_{nt}$ is defined recursively as:

$$\zeta_{nt}^{-1} = 1 + \beta^\psi \left( \mathbb{E}_t \left[ \frac{\psi + 1}{\psi} R_{nt+1} \zeta_{t+1}^{\psi - \frac{1}{\psi}} \right] \right)^{\frac{1}{\psi}}.$$  \hspace{1cm} (D.310)

Landlords’ corresponding optimal saving and investment decisions satisfy $k_{nt+1} = (1 - \zeta_{nt})R_{nt}k_{nt}$. Therefore, the landlords in each location have a linear saving rate $(1 - \zeta_{nt})$ out of current period wealth $R_{nt}k_{nt}$, as in Angeletos (2007) and our baseline specification. The remainder of our quantitative analysis goes through as in our baseline specification in Section 2 of the paper, modifying the capital market clearing condition to take into account that capital from each location is allocated to all locations, as characterized in the previous subsection.

## E Tradeable and Non-Tradeable Sector

We consider an economy with many locations indexed by $i \in \{1, \ldots, N\}$. Time is discrete and is denoted by $t$. There are two sectors: tradeable and non-tradeable. There are two types of infinitely-lived agents: workers and landlords. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile subject to migration costs. Workers do not have access to an investment technology and hence live hand to mouth, as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital.

We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate $\delta$.

### E.1 Worker Migration Decisions

Worker migration decisions are modelled in exactly the same way as in our baseline Armington model with a single sector. We assume that workers have idiosyncratic preferences across locations and face bilateral migration costs in moving between locations. We assume perfect labor
mobility across sectors within each location, such that there is a common wage across sectors within each location.

E.2 Worker Consumption

Worker preferences are defined over both traded and non-traded goods. The traded sector is modelled as in the standard Armington model of trade with constant elasticity of substitution (CES) preferences. The non-traded sector consists of a single local non-traded good. The indirect utility function each period depends on worker’s wage ($w_{nt}$), the cost of living ($p_{nt}$) and amenities ($b_{nt}$):

$$\ln u^w_{nt} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}, \quad (E.1)$$

where amenities ($b_{nt}$) capture characteristics of a location that make it a more attractive place to live regardless of goods consumption (e.g. climate and scenic views). In this section of the online appendix, we assume that amenities are exogenous. The cost of living ($p_{nt}$) in location $n$ depends on the price index for traded goods ($p^T_{it}$) and the price of the non-traded good ($p^N_{it}$):

$$p_{it} = \left( p^T_{it} \right)^\gamma \left( p^N_{it} \right)^{1-\gamma}. \quad (E.2)$$

The price index for traded goods depends on the price of the variety sourced from each location $i$ ($p^e_{nit}$):

$$p^T_{it} = \left[ \sum_{m=1}^N p^{-\theta}_{imt} \right]^{-1/\theta}, \quad (E.3)$$

where $\sigma > 1$ is the constant elasticity of substitution and $\theta = \sigma - 1 > 0$ is the constant trade elasticity.

Using the properties of these CES preferences, the share of expenditure in importer $n$ on the goods supplied by exporter $i$ in the traded sector takes the standard form:

$$S_{nit} = \frac{(p_{nit})^{-\theta}}{\sum_{m=1}^N (p_{nimt})^{-\theta}}. \quad (E.4)$$

E.3 Production

Production in each sector uses labor and capital. Production is assumed to occur under conditions of perfect competition and using a constant returns to scale Cobb-Douglas production technology. For simplicity, we assume the same factor intensity and productivity in the traded and non-traded sectors. Profit maximization and zero profits implies the following equilibrium prices in the two sectors:

$$p_{nit} = \frac{\tau_{nit} \mu^{1-\mu}}{z_{it}}, \quad (E.5)$$
where \( z_{it} \) denotes productivity in location \( i \) at time \( t \). In this section of the online appendix, we assume that productivity is exogenous.

E.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords’ intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

\[
v^k_{it} = \sum_{t=0}^{\infty} \beta^t \ln c^k_{it},
\]

(E.7)

where \( c^k_{it} = \left( c^{T,k}_{it}\right)^{\gamma} \left( c^{NT,k}_{it}\right)^{1-\gamma} \) is the overall consumption for landlords, which depends on the consumption index for tradeables \( c^{T,k}_{it} \) and the consumption index for non-tradeables \( c^{NT,k}_{it} \).

Landlords in a given location can produce one unit of capital in that location using one unit of the overall consumption index in that location. We assume that capital is geographically immobile once installed and depreciates at a constant rate \( \delta \). The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital \( (r_{it} k_{it}) \) equals the total value of their consumption \( (p_{it} c^k_{it}) \) plus the total value of net investment \( (p_{it} (k_{it+1} - (1 - \delta) k_{it})) \):

\[
r_{it} k_{it} = p_{it} c^k_{it} + p_{it} (k_{it+1} - (1 - \delta) k_{it}).
\]

(E.8)

Combining the landlords’ intertemporal utility (E.7) and budget constraint (E.8), the landlords’ intertemporal optimization problem is:

\[
\max \{ c^k_{it}, k_{it+1} \} \sum_{t=0}^{\infty} \beta^t \ln c^k_{it},
\]

subject to \( p_{it} c^k_{it} + p_{it} (k_{it+1} - (1 - \delta) k_{it}) = r_{it} k_{it} \).

We can write this problem as the following Lagrangian:

\[
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c^k_{it} - \xi_t \left[ p_{it} c^k_{it} + p_{it} (k_{it+1} - (1 - \delta) k_{it}) - r_{it} k_{it} \right].
\]

(E.10)

The first-order conditions are:

\[
\left\{ c^k_{it} \right\} \quad \frac{\beta^t}{c^k_{it}} - p_{it} \xi_t = 0,
\]
Together these first-order conditions imply the familiar Euler equation linking the marginal utility of consumption between any two time periods:

\[
\frac{c_{it+1}^k}{c_{it}^k} = \beta \frac{p_{it+1}}{p_{it+1}\mu_{t+1}} = \beta \left( \frac{r_{it+1}}{p_{it+1}} + (1 - \delta) \right),
\]

(E.11)

where the transversality condition implies:

\[
\lim_{t \to \infty} \beta^t \frac{k_{it+1}}{c_{it}^k} = 0.
\]

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords’ optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

\[
p_{it+1}^k = (1 - \beta) \left( r_{it} + p_{it+1} (1 - \delta) \right) k_{it},
\]

(E.12)

\[
k_{it+1} = \beta \left( \frac{r_{it}}{p_{it+1}} + (1 - \delta) \right) k_{it}.
\]

(E.13)

Substituting the consumption policy function (E.12) into the Euler equation (E.11), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

\[
\frac{c_{it+1}^j}{c_{it}^j} = \beta^j \frac{k_{it+1}}{c_{it}^j} = \beta \left( \frac{r_{it+1}}{p_{it+1}} + (1 - \delta^j) \right) k_{it+1}^j.
\]

(E.14)

\[
\beta \left( \frac{r_{it+1}}{p_{it+1}} + (1 - \delta^j) \right) k_{it+1}^j = \beta \left( \frac{r_{it+1}}{p_{it+1}} + (1 - \delta^j) \right) k_{it}^j.
\]

(E.15)

E.5 Market Clearing

Income equals expenditure implies that the sum of the income of workers and landlords in each location is equal to expenditure on the goods produced by that location:

\[
\begin{align*}
(w_{it}^T + r_{it}k_{it}^T) &+ (w_{it}^NT + r_{it}k_{it}^NT) \\
&= \gamma \sum_{n=1}^{N} S_{nit} (w_{nt}\ell_{nt} + r_{nt}k_{nt}) + (1 - \gamma) (w_{nt}\ell_{it} + r_{nt}k_{it}) \\
\end{align*}
\]

(E.16)

Non-traded goods market clearing implies that income in the non-traded sector is equal to local expenditure on non-traded goods:

\[
(w_{it}\ell_{it}^{NT} + r_{it}k_{it}^{NT}) = (1 - \gamma) [(w_{nt}\ell_{it} + r_{nt}k_{it})].
\]

(E.15)

Using this non-traded goods market clearing condition (E.15), our equality between income and expenditure in equation (E.14) simplifies to:

\[
(w_{it}^T + r_{it}k_{it}^T) = \gamma \sum_{n=1}^{N} S_{nit} (w_{nt}\ell_{nt} + r_{nt}k_{nt}).
\]

(E.16)
Now note that factor market clearing implies:

\[ (w_{it}\ell_{it}^T + r_{it}k_{it}^T) + (w_{it}\ell_{it}^{NT} + r_{it}k_{it}^{NT}) = (w_{it}\ell_{it} + r_{it}k_{it}) . \]  

(E.17)

Combining this factor market clearing condition (E.17) with non-traded goods market clearing (E.15), total payments for factors of production in the traded sector are also a constant share of total factor payments:

\[ (w_{it}\ell_{it}^T + r_{it}k_{it}^T) = \gamma (w_{nt}\ell_{nt} + r_{nt}k_{nt}) \]  

(E.18)

Using this result, the goods market clearing condition (E.16) can be re-written as:

\[ (w_{it}\ell_{it} + r_{it}k_{it}) = \sum_{n=1}^{N} S_{nit} (w_{nt}\ell_{nt} + w_{nt}\ell_{nt}) . \]  

(E.19)

Additionally, from profit maximization and zero-profits, capital payments are the same constant multiple of labor payments in each sector:

\[ r_{it}k_{it}^T = \frac{1 - \mu}{\mu} w_{it}\ell_{it}^T; \]  

(E.20)
\[ r_{it}k_{it}^{NT} = \frac{1 - \mu}{\mu} w_{it}\ell_{it}^{NT}. \]  

(E.21)

Combining these results with factor market clearing, we obtain:

\[ r_{it}k_{it} = \frac{1 - \mu}{\mu} w_{it}\ell_{it}. \]  

(E.22)

Using this property that capital payments are a constant multiple of labor payments, the goods market clearing condition (E.19) simplifies to:

\[ w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt}\ell_{nt}. \]  

(E.23)

Finally, combining the relationships between capital and labor payments in each sector in equations (E.20) and (E.21), with our earlier results in equations (E.15) and (E.18) that total factor payments in each sector are a constant multiple of total factor payments, we find that a constant share of each location’s labor and capital is allocated to the traded and non-traded sectors:

\[ \ell_{it}^T = \gamma\ell_{it}, \quad \ell_{it}^{NT} = (1 - \gamma)\ell_{it}, \]  

(E.24)
\[ k_{it}^T = \gamma k_{it}, \quad k_{it}^{NT} = (1 - \gamma) k_{it}. \]  

(E.25)
E.6 General Equilibrium

Given the state variables \( \{ \ell_{it}, k_{it} \} \), the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables \( \{ \ell_{it}, k_{it}, w_{it}, v_{it} \} \). All other endogenous variables of the model can be recovered as a function of these variables. In particular, we immediately recover the sectoral allocation of labor and capital from equations (E.24) and (E.25).

**Capital Accumulation:** Using capital market clearing (E.22), the price index (E.2), the price index for traded goods (E.3), and the equilibrium pricing rules (E.5) and (E.6), the capital accumulation equation (E.13) becomes:

\[
k_{it+1} = \beta \frac{1 - \mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1 - \delta) k_{it},
\]

\((E.26)\)

\[
p_{it} = \left( p_{it}^T \right)^\gamma \left( p_{it}^{NT} \right)^{1-\gamma},
\]

\((E.27)\)

\[
p_{it}^T = w_{it} \left[ \frac{1 - \mu}{\mu} \left( \frac{\ell_{it}}{k_{it}} \right)^{1-\mu} \left( \frac{\tau_{ni}}{z_i} \right) \right]^{-\theta} \frac{1}{1/\theta},
\]

\((E.28)\)

\[
p_{nt}^{NT} = w_{it} \left[ \frac{1 - \mu}{\mu} \left( \frac{\ell_{it}}{k_{it}} \right)^{1-\mu} \right],
\]

\((E.29)\)

**Goods Market Clearing:** Using the equilibrium pricing rule in the traded sector (E.5), the expenditure share (E.4) and capital market clearing condition (E.22) in the goods market clearing condition (E.23), we obtain:

\[
w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt},
\]

\((E.30)\)

\[
S_{nit} = \frac{\left( w_{it} \left( \frac{\ell_{it}}{k_{it}} \right)^{1-\mu} \tau_{mi}/z_i \right)^{-\theta}}{\sum_{m=1}^{N} \left( w_{mt} \left( \frac{\ell_{mt}}{k_{mt}} \right)^{1-\mu} \tau_{nm}/z_m \right)^{-\theta}},
\]

\((E.31)\)

\[
T_{int} = \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},
\]

\((E.31)\)

where \( S_{nit} \) is the expenditure share of importer \( n \) on exporter \( i \) at time \( t \); we have defined \( T_{int} \) as the corresponding income share of exporter \( i \) from importer \( n \) at time \( t \); and note that the order of subscripts switches between the expenditure share (\( S_{nit} \)) and the income share (\( T_{int} \)), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.
Population Flow: Using the analogous derivations for migration decisions as in our baseline Armington model, the population flow condition for the evolution of the population distribution over time is given by:

\[ \ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \]  

\[ D_{igt} = \frac{\exp \left( \beta \mathbb{E}_t v_{gt+1}^w \right)}{\sum_{m=1}^{N} \left( \exp \left( \beta \mathbb{E}_t v_{mt+1}^w \right) \right) / \kappa_{mt}^{1/\rho}}; \quad E_{git} = \frac{\ell_{it} D_{igt}}{\ell_{gt+1}}, \]  

where \( D_{igt} \) is the outmigration probability from location \( i \) to location \( g \) between time \( t \) and \( t+1 \); we have defined \( E_{git} \) as the corresponding immigration probability to location \( g \) from location \( i \) between time \( t \) and \( t+1 \); and again note that the order of subscripts switches between the outmigration probability \( (D_{igt}) \) and the immigration probability \( (E_{git}) \), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the analogous derivations for migration decisions as in our baseline Armington model, the expected value from living in location \( n \) at time \( t \) can be written as:

\[ v_{nt}^w = \ln b_{nt} + \ln \left( \frac{w_{nt}}{P_{nt}} \right) + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_t v_{gt+1}^w \right) / \kappa_{gt} \right)^{1/\rho}. \]  

E.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path. In the interests of brevity, we focus on differences from the specification in our baseline single-sector Armington model.

Prices Totally differentiating the pricing rules in the traded and non-traded sectors, we have:

\[ d \ln p_{nt} = d \ln \tau_{nt} + d \ln w_{nt} - (1 - \mu) d \ln \chi_{nt} - d \ln z_{nt}. \]  

\[ d \ln p_{nt}^{NT} = d \ln \tau_{nt} + d \ln w_{nt} - (1 - \mu) d \ln \chi_{nt} - d \ln z_{nt}. \]  

Price Indices Totally differentiating the consumption goods price index in equation (E.2), we have:

\[ d \ln p_{nt} = \gamma d \ln p_T^T + (1 - \gamma) d \ln p_{nt}^{NT}. \]
\[ d \ln p_{nt}^T = \sum_{m=1}^{N} S_{nmt} d \ln p_{nmt}^T \]

**Real Income.** Totally differentiating real income we have:

\[
d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - d \ln p_{it},
\]

\[
d \ln \left( \frac{w_{it}}{p_{it}} \right) = d \ln w_{it} - \gamma \sum_{m=1}^{N} S_{nmt} d \ln p_{nmt} - (1 - \gamma) \ d \ln p_{nt}^{NT},
\]

\[
d \ln \left( \frac{w_{it}}{p_{it}} \right) = \left[ d \ln w_{it} - \gamma \sum_{m=1}^{N} S_{nmt} \left[ d \ln \tau_{nt} + d \ln w_{nt} - (1 - \mu) \ d \ln \chi_{nt} - d \ln z_{nt} \right] - (1 - \gamma) \left[ d \ln \tau_{it} + d \ln w_{it} - (1 - \mu) \ d \ln \chi_{it} - d \ln z_{it} \right] \right], \quad (E.38)
\]

**Goods Market Clearing** Totally differentiating the goods market clearing condition (E.30), we obtain the same expression as in our baseline single-sector Armington model:

\[
\left[ \frac{d \ln w_{it}}{d \ln \varepsilon_{it}} \right] = \left[ +\theta \sum_{m=1}^{N} \sum_{n=1}^{N} T_{nt} |S_{nmt}| \left( d \ln \tau_{nt} + d \ln w_{nt} - (1 - \mu) \ d \ln \chi_{nt} - d \ln z_{nt} \right) \right].
\]

**Value Function.** Note that the value function (E.34) can be re-written using the following results:

\[ v_{it} = \ln \left( \frac{w_{it}}{(p_{it}^T)^{\gamma} (p_{nt}^{NT})^{1-\gamma}} \right) + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_{g} v_{gt+1} / \kappa_{git} \right) \right)^{1/\rho}, \]

\[ p_{it}^T = \left( \sum_{m=1}^{N} p_{imt}^{\theta} \right)^{-1/\theta} = \left( \frac{p_{it}^T}{S_{it}} \right)^{-1/\theta}, \quad \tau_{it} = 1, \]

\[
\sum_{g=1}^{N} \left( \exp \left( \beta \mathbb{E}_{g} v_{gt+1} / \kappa_{git} \right) \right)^{1/\rho} = \frac{\exp \left( \beta \mathbb{E}_{g} v_{gt+1} / \kappa_{git} \right)^{1/\rho}}{D_{it}}, \quad \kappa_{git} = 1, \]

\[ v_{it} = \ln w_{it} - \gamma \ln S_{it} - \gamma \ln p_{it} - (1 - \gamma) \ln p_{nt}^{NT} + \ln b_{it} + \beta \mathbb{E}_{g} v_{gt+1} - \rho \ln D_{it}. \]

Totally differentiating this expression for the value function, we have:

\[ dv_{it} = d \ln w_{it} - \frac{\gamma}{\theta} d \ln S_{it} - \gamma d \ln p_{it} - (1 - \gamma) d \ln p_{nt}^{NT} + d \ln b_{it} + \beta dv_{it+1} - \rho d \ln D_{it}, \]

where

\[ d \ln S_{it} = -\theta d \ln p_{it} + \theta \left[ \sum_{m=1}^{N} S_{imt} d \ln p_{imt} \right], \]

\[ d \ln D_{it} = \frac{1}{\rho} \left[ \beta \mathbb{E}_{g} dv_{gt+1} - d \ln \kappa_{git} - \sum_{m=1}^{N} D_{imt} \left( \beta \mathbb{E}_{g} v_{mt+1} - d \ln \kappa_{mit} \right) \right]. \]
Using these results for $\ln S_{iit}$ and $\ln D_{iit}$ in the expression for $\ln v_{iil}$ above, we have:

$$
\begin{align*}
\ln v_{iil} & = \begin{bmatrix}
\ln w_{iil} - \gamma \sum_{m=1}^{N} S_{iitm} (\ln p_{itm} - (1 - \gamma) \ln p_{iit}^{NT}) \\
+ \ln b_{iit} + \sum_{m=1}^{N} D_{iitm} (\beta E_{l} \ln v_{iil}^{l+1} - \ln \kappa_{mit})
\end{bmatrix},
\end{align*}
$$

where we have used $\ln \kappa_{iil} = 0$. Using the total derivative of the pricing rules (E.35) and (E.36), we can re-write this derivative of the value function as follows:

$$
\begin{align*}
\ln v_{iil} & = \begin{bmatrix}
\ln w_{iil} - \gamma \sum_{m=1}^{N} S_{iitm} (\ln \tau_{itm} + \ln w_{itm} - (1 - \mu) \ln \chi_{itm} - \ln z_{itm}) \\
(1 - \gamma) (\ln w_{itm} - (1 - \mu) \ln \chi_{itm} - \ln z_{itm}) \\
+ \ln b_{iit} + \sum_{m=1}^{N} D_{iitm} (\beta E_{l} \ln v_{iil}^{l+1} - \ln \kappa_{mit})
\end{bmatrix},
\end{align*}
$$

(E.40)

### E.8 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{iit+1} = k_{iit} = k_{iit}^{*}$, $\ell_{iit+1} = \ell_{iit} = \ell_{iit}^{*}$, $w_{iit+1} = w_{iit} = w_{iit}^{*}$ and $v_{iit+1} = v_{iit} = v_{iit}^{*}$, where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity ($\ln z$) and amenities ($\ln b$) in each location, holding constant the economy’s aggregate labor endowment ($\ln \overline{b} = 0$), trade costs ($\ln \tau = 0$) and commuting costs ($\ln \kappa = 0$).

**Capital Accumulation.** From the capital accumulation equation (E.26), the steady-state stock of capital solves:

$$(1 - \beta (1 - \delta)) \chi_{iit}^{*} = (1 - \beta (1 - \delta)) \frac{k_{iit}^{*}}{\ell_{iit}^{*}} = \frac{1 - \mu}{\mu} \frac{w_{iit}^{*}}{p_{iit}^{*}}.$$ 

Totally differentiating, we have:

$$
\begin{align*}
\ln \chi_{iit}^{*} & = \ln \left( \frac{w_{iit}^{*}}{p_{iit}^{*}} \right).
\end{align*}
$$

Using the total derivative of real income (E.38) above, this becomes:

$$
\begin{align*}
\ln \chi_{iit}^{*} = \begin{bmatrix}
\ln w_{iit}^{*} - \gamma \sum_{m=1}^{N} S_{iitm}^{*} (\ln w_{m}^{*} - (1 - \mu) \ln \chi_{m}^{*} - \ln z_{m}) \\
- (1 - \gamma) (\ln w_{m}^{*} - (1 - \mu) \ln \chi_{m}^{*} - \ln z_{m})
\end{bmatrix},
\end{align*}
$$

where we have used and $\ln \tau_{im} = 0$. This relationship has the matrix representation:

$$
\begin{align*}
\ln \chi^{*} = \begin{bmatrix}
\ln w^{*} - [\gamma S + (1 - \gamma) I] \ln w^{*} \\
+ (1 - \mu) [\gamma S + (1 - \gamma) I] \ln \chi^{*} + [\gamma S + (1 - \gamma) I] \ln z
\end{bmatrix},
\end{align*}
$$

$$
\begin{align*}
(I - (1 - \mu) [\gamma S + (1 - \gamma) I]) \ln \chi^{*} = \begin{bmatrix}
(I - [\gamma S + (1 - \gamma) I]) \ln w^{*} \\
+ [\gamma S + (1 - \gamma) I] \ln z
\end{bmatrix}.
\end{align*}
$$

(E.41)
**Goods Market Clearing.** The total derivative of the goods market clearing condition (E.39) has the following matrix representation:

\[
d\ln w_t + d\ln \ell_t = T (d\ln w_t + d\ln \ell_t) + \theta (TS - I) (d\ln z + (1 - \mu) d\ln z),
\]

where we have used \(d\ln \tau = 0\). We can re-write this relationship as:

\[
[I - T + \theta (I - TS)] d\ln w_t = -(I - T) \ d\ln \ell_t + \theta (I - TS) (d\ln z + (1 - \mu) d\ln \chi_t).
\]

In steady-state we have:

\[
[I - T + \theta (I - TS)] d\ln w^* = -(I - T) d\ln \ell^* + \theta (I - TS) (d\ln z + (1 - \mu) d\ln \chi^*). \tag{E.42}
\]

**Population Flow.** The total derivative of the population flow condition has the same matrix representation as in our baseline single-sector Armington model:

\[
d\ln \ell_{t+1} = E d\ln \ell_t + \frac{\beta}{\rho} (I - ED) \mathbb{E}_t d\nu_{t+1}.
\]

In steady-state, we have:

\[
d\ln \ell^* = E d\ln \ell^* + \frac{\beta}{\rho} (I - ED) d\nu^*. \tag{E.43}
\]

**Value function.** The total derivative of the value function (E.40) has the following matrix representation:

\[
d\nu_t = (I - [\gamma S + (1 - \gamma) I]) d\ln w_t + [\gamma S + (1 - \gamma) I] (d\ln z + (1 - \mu) d\ln \chi_t) + d\ln b + \beta D E_t d\nu_{t+1},
\]

where we have used \(d\ln \tau = d\ln \kappa = 0\). In steady-state, we have:

\[
d\nu^* = (I - [\gamma S + (1 - \gamma) I]) d\ln w^* + [\gamma S + (1 - \gamma) I] (d\ln z + (1 - \mu) d\ln \chi^*) + d\ln b + \beta D d\nu^*. \tag{E.44}
\]

**System of Steady-State Equations.** Collecting together the system of steady-state equations, we have:

\[
d\ln \chi^* = [I - (1 - \mu) [\gamma S + (1 - \gamma) I]]^{-1} \left( (I - [\gamma S + (1 - \gamma) I]) d\ln w^* + [\gamma S + (1 - \gamma) I] d\ln z \right). \tag{E.45}
\]

\[
d\ln w^* = [I - T + \theta (I - TS)]^{-1} \left[ -(I - T) d\ln \ell^* + \theta (I - TS) (d\ln z + (1 - \mu) d\ln \chi^*) \right]. \tag{E.46}
\]

\[
d\ln \ell^* = \frac{\beta}{\rho} (I - E)^{-1} (I - ED) d\nu^*. \tag{E.47}
\]

\[
d\nu^* = (I - \beta D)^{-1} \left[ -[\gamma S + (1 - \gamma) I] (d\ln w^* + d\ln b) + d\ln w^* - d\ln z - (1 - \mu) d\ln \chi^* \right]. \tag{E.48}
\]

As the expenditure shares (\(S\)) and income shares (\(T\)) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income
of all locations as our numeraire \((\sum_{i=1}^{N} w_i^* \ell_i^* = \sum_{i=1}^{N} q_i^* = \bar{q} = 1)\), which implies that the log changes in incomes satisfy \(Q^* d \ln q^* = \sum_{i=1}^{N} q_i^* d \ln q_i^* = \sum_{i=1}^{N} q_i^* \frac{dq_i^*}{q_i^*} = \sum_{i=1}^{N} d q_i^* = 0\), where \(Q\) is a row vector of the income of each location. Similarly, the outmigration shares \((D)\) and immigration shares \((E)\) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: \(\sum_{i=1}^{N} \ell_{it} = \bar{\ell} = 1\), which implies \(L^* d \ln \ell^* = \sum_{i=1}^{N} \ell_i^* d \ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{d \ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} d \ell_i^* = 0\), where \(L\) is a row vector of the population of each location.

### E.9 Sufficient Statistics for Transition Dynamics

We suppose that the economy starts from an initial steady-state distribution of economic activity \([k_i^*, \ell_i^*, w_i^*, v_i^*]\). We consider small shocks to productivity \((d \ln z)\) and amenities \((d \ln b)\) in each location, holding constant the economy’s aggregate labor endowment \((d \ln \bar{\ell}_t)\), trade costs \((d \ln \tau = 0)\) and commuting costs \((d \ln \kappa = 0)\). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that \(\tilde{\chi}_{it+1} = \ln \chi_{it+1} - \ln \chi_i^*\), for all variables except for the worker value function \(v_{it}\); with a slight abuse of notation we use \(\tilde{v}_{it} \equiv v_{it} - v_i^*\) to denote the deviation in levels for the worker value function.

#### Capital Accumulation.

Following the same line of argument in our baseline single-sector Armington model, the log deviation of the capital-labor ratio from steady-state can be written as:

\[
\ln \left( \frac{\chi_{it+1}}{\chi_i^*} \right) + \ln \left( \frac{\ell_{it+1} / \ell_i^*}{\ell_{it} / \ell_i^*} \right) = \beta (1 - \delta) \ln \left( \frac{\chi_{it}}{\chi_i^*} \right) + (1 - \beta (1 - \delta)) \ln \left( \frac{w_{it}/w_i^*}{p_{it}/p_i^*} \right),
\]

which can be re-written as:

\[
\tilde{\chi}_{it+1} = \beta (1 - \delta) \tilde{\chi}_{it} + (1 - \beta (1 - \delta)) (\tilde{w}_{it} - \tilde{p}_{it}) - \tilde{\ell}_{it+1} + \tilde{\ell}_t,
\]

We can re-write this relationship in matrix form as:

\[
\tilde{\chi}_{t+1} = \beta (1 - \delta) \tilde{\chi}_{t} + (1 - \beta (1 - \delta)) (\tilde{w}_{t} - \tilde{p}_{t}) - \tilde{\ell}_{t+1} + \tilde{\ell}_t.
\]

Taking the total derivative of real income relative to the initial steady-state, we have:

\[
\tilde{w}_{it} - \tilde{p}_{it} = \left[ \tilde{w}_{it} - \gamma \sum_{m=1}^{N} S_{m} \tilde{\chi}_{mt} - \tilde{z}_m \right] / \left( 1 - \gamma \right) \tilde{w}_{it} - \left( 1 - \mu \right) \tilde{\chi}_{it} - \tilde{z}_i
\]

where we have used \(d \ln \tau_{nm} = 0\). We can re-write this relationship in matrix form as:

\[
\tilde{w}_t - \tilde{p}_t = (I - [\gamma S + (1 - \gamma) I]) \tilde{w}_t + (1 - \mu) [\gamma S] \tilde{\chi}_t + S\tilde{z}.
\]

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

\[
\tilde{\chi}_{t+1} = \begin{bmatrix}
\beta (1 - \delta) \tilde{\chi}_t + (1 - \beta (1 - \delta)) (1 - \mu) \gamma S + (1 - \gamma) \gamma S + (1 - \gamma) I \tilde{w}_t \\
+ (1 - \beta (1 - \delta)) (1 - \mu) \gamma S + (1 - \gamma) I \tilde{w}_t \\
+ (1 - \beta (1 - \delta)) \gamma S + (1 - \gamma) I \tilde{z} - \tilde{\ell}_{t+1} + \tilde{\ell}_t
\end{bmatrix}.
\]

(E.50)
Goods Market Clearing. The total derivative of the goods market clearing condition (E.39) relative to the initial steady-state has the following matrix representation:

\[
\tilde{\bar{w}}_t + \tilde{\ell}_t = T \left( \tilde{\bar{w}}_t + \tilde{\ell}_t \right) + \theta \left( TS - I \right) \left( \tilde{\bar{w}}_t - (1 - \mu) \tilde{\chi}_t - \tilde{z} \right),
\]

where we have used \( d \ln \tau = 0 \). We can re-write this relationship as:

\[
\tilde{\bar{w}}_t = \left[ I - T + \theta \left( I - TS \right) \right]^{-1} \left[ - (I - T) \tilde{\ell}_t + \theta \left( I - TS \right) \left( \tilde{z} + (1 - \mu) \tilde{\chi}_t \right) \right]. \tag{E.51}
\]

Population Flow. Following the same line of argument in our baseline single-sector Armington model, the total derivative of the population flow condition relative to the initial steady-state has the following matrix representation:

\[
\tilde{\ell}_{t+1} = E \tilde{\ell}_t + \frac{\beta}{\rho} \left( I - ED \right) E \tilde{v}_{t+1}. \tag{E.52}
\]

Value Function. The total derivative of the value function (E.40) relative to the initial steady-state has the following matrix representation:

\[
\tilde{\bar{v}}_t = \left[ \left( I - [\gamma S + (1 - \gamma) I] \right) \tilde{\bar{w}}_t + \gamma S + (1 - \gamma) I \left[ \tilde{z} \right] \right] + (1 - \mu) \left[ \gamma S + (1 - \gamma) I \right] \tilde{\chi}_t + \tilde{\bar{v}} + \beta D E \tilde{v}_{t+1}, \tag{E.53}
\]

where we have used \( d \ln \tau = d \ln \kappa = 0 \).

System of Equations for Transition Dynamics Relative to the Initial Steady-State. Collecting together the capital accumulation equation (E.50), the goods market clearing condition (E.51), the population flow condition (E.52), and the value function (E.53), the system of equations for the transition dynamics relative to the initial steady-state takes the following form:

\[
\tilde{\bar{v}}_{t+1} = \left[ \left( I - [\gamma S + (1 - \gamma) I] \right) \tilde{\bar{w}}_t + \gamma S + (1 - \gamma) I \left[ \tilde{z} \right] \right] + (1 - \mu) \left[ \gamma S + (1 - \gamma) I \right] \tilde{\chi}_t + \tilde{\bar{v}} + \beta D E \tilde{v}_{t+1}, \tag{E.54}
\]

\[
\tilde{\bar{w}}_t = \left[ I - T + \theta \left( I - TS \right) \right]^{-1} \left[ - (I - T) \tilde{\ell}_t + \theta \left( I - TS \right) \left( \tilde{z} + (1 - \mu) \tilde{\chi}_t \right) \right]. \tag{E.55}
\]

\[
\tilde{\ell}_{t+1} = E \tilde{\ell}_t + \frac{\beta}{\rho} \left( I - ED \right) E \tilde{v}_{t+1}. \tag{E.56}
\]

\[
\tilde{\bar{v}}_t = \left[ \left( I - [\gamma S + (1 - \gamma) I] \right) \tilde{\bar{w}}_t + \gamma S + (1 - \gamma) I \left[ \tilde{z} \right] \right] + (1 - \mu) \left[ \gamma S + (1 - \gamma) I \right] \tilde{\chi}_t + \tilde{\bar{v}} + \beta D E \tilde{v}_{t+1}. \tag{E.57}
\]
F Additional Empirical Results

In this section of the online appendix, we report additional empirical results that are discussed in the paper. Subsection F.1 shows that individual U.S. states differ substantially in terms of the dynamics of their capital-labor ratios, highlighting the empirical relevance of capital accumulation for income convergence. Subsection F.2 provides evidence of substantial net migration between U.S. states, highlighting the empirical salience of migration for the population dynamics of U.S. states. Subsection F.3 show that the model’s gravity equation predictions provide a good approximation to the observed data on trade and migration flows.

Subsection F.4 reports additional evidence on the predictive power of convergence to the initial steady-state for the observed population growth of U.S. states. Subsection F.5 provides further information about the implied fundamentals from inverting the non-linear model. Subsection F.6 presents additional details about the solution algorithm used to solve for the economy’s transition path in the non-linear model.

F.1 Capital Dynamics

In this section of the online appendix, we show that U.S. states differ substantially in terms of the dynamics of their capital-labor ratios over our sample period, highlighting the empirical relevance of capital accumulation for income convergence.

We provide evidence on the dynamics of capital-labor ratios for both individual states and four geographic groupings of states. Following Alder, Legakos and Ohanian (2019), we define the Rust Belt as the states of Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin, and the Sun Belt as the states of Arizona, California, Florida, New Mexico and Nevada. We group the remaining states into two categories to capture longstanding differences between the North and South: Other Southern States, which includes all former members of the Confederacy, except those in the Sun Belt; and Other Northern States, which comprises all the Union states from the U.S. Civil War, except those in the Rust Belt or Sun Belt.7

In Figure F.1, we show the capital-labor over time for each U.S. state using the solid gray lines. We also show the population-share weighted average of these capital-labor ratios for our four geographical groupings using the black dashed lines. The capital-labor ratio is measured as the ratio of the real capital stock to population. While all U.S. states experience an increase in the capital-labor ratio, the rate of increase differs substantially across states, implying that capital

accumulation plays an important role in regional income convergence. We find substantial heterogeneity in the trends in the capital-labor ratio across states within all four of our geographical groupings, with this heterogeneity greatest for the Other Northern and Other Southern states.

Figure F.1: Capital-Labor Ratios for U.S. States over Time

![Graph showing capital-labor ratios for U.S. states over time.](image)

Note: Gray lines show capital-labor ratios for each U.S. state and year; black dashed lines show the population-weighted average of these steady-state gaps for the four geographical regions of the Rust Belt, Sun Belt, Other Northern States and Other Southern States, as defined in the main text; capital-labor ratios measured as the ratio of the real capital stock to population.

F.2 Migration Flows

In this section of the online appendix, we provide evidence on the role of internal migration as a source of population changes for the four groups of states. Internal migration is measured as movements of people between states within the United States and excludes international migration. We focus for brevity on in-migrants, measured as inflows of internal migrants (in thousands) into each state, separated out by origin state.

In Figure F.2, we show in-migration flows for our four geographical groupings of states. Three features are noteworthy. First, geographical proximity matters for migration flows, such that other Rust Belt states are one of the leading sources of in-migrants for the Rust Belt (top-left panel), consistent with our model’s gravity equation predictions. Second, all groups of states receive non-negligible in-migration flows, such that gross migration flows are larger than net migration flows, in line with the idiosyncratic mobility shocks in our model. Third, despite the role for geography, the Rust Belt and Other Northern states are the two largest sources of in-migrants for the Sun Belt, consistent with internal migration contributing to the observed reorientation of population shares.
Finally, although not shown in these figures, we find a modest decline in rates of internal migration between states in the later years of our sample, which is in line with the findings of a number of studies, including Kaplan and Schulhofer-Wohl (2017) and Molloy et al. (2011). Consistent with the comparison of several different sources of administrative data in Hyatt et al. (2018), we find that this decline in rates of internal migration between states is smaller in the population census data than in Current Population Survey (CPS) data.

Figure F.2: Internal In-migrants for each Destination Region by Origin Region over Time

Rust Belt Destination
Sun Belt Destination
Other North Destination
Other South Destination

Notes: Internal in-migration to each destination region by source region from 1960-2000; internal migration includes all movements of people between states within the United States and excludes international migration.

F.3 Gravity in Trade and Migration

In this section of the online appendix, we show that bilateral flows of goods and migrants between U.S. states both exhibit strong gravity equations, as predicted by our theoretical framework.

We begin with the gravity equation for the bilateral value of trade in goods, where we model bilateral trade costs as a constant elasticity function of bilateral geographical distance, measured as the Great Circle distance between the population centers of states. First, we regress the log bilateral value of trade on origin and destination fixed effects, and generate the residuals. Second, we regress log bilateral geographical distance on origin and destination fixed effects, and generate the residuals. Third, we display the two sets of residuals against one another and the linear regression relationship between them, using the Frisch-Waugh-Lovell theorem. As shown in Figure F.3, we find a strong, negative and statistically significant and approximately log linear
conditional correlation between the bilateral value of trade and bilateral geographical distance, consistent with our model’s gravity equation predictions for goods trade.

Figure F.3: Gravity Equation for the Bilateral Value of Goods Trade in 2017

![Gravity Equation for the Bilateral Value of Goods Trade in 2017](image)

Note: Slope coefficient: -1.2478; standard error: 0.0210; R-squared: 0.6891.

Notes: Conditional correlation between the log bilateral value of goods trade between U.S. states and the log of bilateral distance between the population centers of U.S. states; residual log trade value and residual log distance from conditioning on origin and destination fixed effects.

We next turn to the gravity equation for bilateral migration flows, where we again model bilateral migration costs as a constant elasticity function of bilateral geographical distance. First, we regress log bilateral migration on origin and destination fixed effects, and generate the residuals. Second, we regress log bilateral geographical distance on origin and destination fixed effects, and generate the residuals. Third, we display the two sets of residuals against one another and the linear regression relationship between them, using the Frisch-Waugh-Lovell theorem. As shown in Figure F.4, we find a strong, negative and statistically significant and approximately log linear conditional correlation between bilateral migration and bilateral geographical distance, consistent with our model’s gravity equation predictions for migration flows.
Notes: Conditional correlation between log bilateral migration flows between U.S. states and the log of bilateral distance between the population centers of U.S. states; residual log migrants and residual log distance from conditioning on origin and destination fixed effects.

Taken together, these results confirm that the gravity equation is a strong empirical feature of both bilateral goods trade and bilateral migration flows between U.S. states, as predicted by our theoretical framework.

F.4 Convergence to Steady-state

In Section 5.3 of the paper, we provide evidence that much of the observed decline in the rate of income convergence is explained by initial conditions at the beginning of our sample period rather than by any subsequent fundamental shocks. In this section of the online appendix, we provide further evidence on the role of initial conditions in explaining subsequent growth, by regressing actual population growth on predicted population growth based on convergence towards an initial steady-state with unchanged fundamentals. Importantly, predicted population growth is calculated using only the initial values of the labor and capital state variables and the initial trade and migration share matrices, and uses no information about subsequent population growth.

In Figure F.5a, we display actual population growth from 1965-2015 against predicted population growth based on convergence to an initial steady-state with 1965 fundamentals. The predictions based on convergence to an initial steady-state with unchanged fundamentals use only the 1964 and 1965 values of the state variables (population and the capital stock in each location) and the 1965 values of the trade and migration share matrices. Each circle in the figure corresponds
to a different US state and the sizes of the circles are proportional to the initial population size of each state. The red line shows the linear fit between the two variables. We find a strong positive and statistically significant relationship between actual and predicted population growth, with a regression slope (standard error) of 0.64 (0.18) and R-squared of 0.19.

As discussed in the paper, we find the largest contribution from shocks to fundamentals to the evolution of state population shares over time at the beginning of our sample period. From 1975 onwards, we find that predicted population growth based on convergence to an initial steady-state with unchanged fundamentals has even greater predictive power for actual population growth. In Figure F.5b, we display actual population growth from 1975-2015 against predicted population growth based on convergence to an initial steady-state with 1975 fundamentals. We find an even stronger positive and statistically significant relationship between actual and predicted population growth, with a regression slope (standard error) of 0.99 (0.095) and R-squared of 0.82. We find a similar pattern of results for later periods, such as 1985-2015 and 1995-2015.

Figure F.5: Actual Growth in Population Shares Versus Predicted Growth in Population Shares Based on Convergence to an Initial Steady-State with Unchanged Fundamentals

(a) 1965-2015

(b) 1975-2015

Note: Vertical axis is actual log population growth; horizontal axis is predicted log population growth based on convergence to the implied initial steady-state assuming no further changes in fundamentals; left-panel shows results for 1965-2015; right-panel shows results for 1975-2015; size of circles for each US state is proportional to initial population size.

In Table F.1 below, we show that these results are robust to controlling for the initial level and growth of economic activity. In Column (1), we augment the regression between actual and predicted population growth from 1965-2015 in Figure F.5a with the initial log population in 1965, initial log capital stock in 1965 and the initial growth in population from 1965-6. We continue to find a positive and statistically significant relationship between actual and predicted population growth, with the inclusion of these additional control variables having relatively little impact on the estimated coefficient and regression R-squared. In Columns (2)-(4), we show that
we find the same pattern of results for 1975-2015, 1985-2015 and 1995-2015, with somewhat larger slope coefficients and R-squared, which reflects the smaller residual contributions from shocks to fundamentals for these later time periods. Therefore, the predictive power of initial convergence towards steady-state does not simply reflect mean reversion, because we find substantial independent information in this variable, even after controlling for initial levels of population and the capital stock and initial population growth.

Table F.1: Predictive Power of Convergence Towards Initial Steady-State with Unchanged Fundamentals for Population Growth

<table>
<thead>
<tr>
<th>Outcome: Base-year - 2015 pop. log growth</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base-year - 2015 predicted pop. growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.521**</td>
<td>0.892***</td>
<td>1.179***</td>
<td>0.735**</td>
</tr>
<tr>
<td></td>
<td>(0.258)</td>
<td>(0.163)</td>
<td>(0.224)</td>
<td>(0.346)</td>
</tr>
<tr>
<td>Log base-year population</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.113**</td>
<td>0.0235</td>
<td>-0.0180</td>
<td>0.00155</td>
</tr>
<tr>
<td></td>
<td>(0.0556)</td>
<td>(0.0204)</td>
<td>(0.0197)</td>
<td>(0.00818)</td>
</tr>
<tr>
<td>Log base-year K-L ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.194</td>
<td>-0.108</td>
<td>-0.119**</td>
<td>0.0590**</td>
</tr>
<tr>
<td></td>
<td>(0.165)</td>
<td>(0.0800)</td>
<td>(0.0584)</td>
<td>(0.0281)</td>
</tr>
<tr>
<td>Base-year pop. growth rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.77</td>
<td>5.573</td>
<td>1.952</td>
<td>4.396</td>
</tr>
<tr>
<td></td>
<td>(6.649)</td>
<td>(5.505)</td>
<td>(3.025)</td>
<td>(3.613)</td>
</tr>
<tr>
<td>N</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>R²</td>
<td>0.341</td>
<td>0.783</td>
<td>0.722</td>
<td>0.826</td>
</tr>
</tbody>
</table>

Note: Dependent variable is actual log population growth between each base year and 2015; base years include 1965, 1975, 1985 and 1995 (columns 1-4, respectively); predicted log population growth is predicted based on convergence to the implied initial steady-state at the beginning of each base year using equation (25) in the paper; log base-year population is log population at each base-year; log base-year K-L ratio is the log capital-labor ratio at each base-year; pop. growth rate is the rate of population growth between each base-year and the subsequent year.

F.5 Implied Fundamentals

In this section of the online appendix, we provide further evidence on the implied location fundamentals \((z_{it}, b_{it}, \tau_{nit}, k_{git})\) from inverting the full non-linear model.

**Empirical Distribution of Fundamental Shocks** We make the conventional assumption of perfect foresight and use our extension of existing dynamic exact-hat algebra approaches to incorporate forward-looking capital investments from Proposition 2 in the paper. We solve for the unobserved changes in fundamentals from the general equilibrium conditions of the model, using the observed data on bilateral trade and migration flows, population, capital stock and labor income per capita, as discussed in Section B.9 above. We recover these unobserved fundamentals,
without making assumptions about where the economy lies on the transition path or the specific trajectory of future fundamentals.

In the left and right panels of Figure F.6, we show the recovered empirical distributions of relative changes in productivity \( \tilde{z}_i = z_{i2000}/z_{i1990} \) and amenities \( \tilde{b}_i = b_{i2000}/b_{i1990} \) across U.S. states from 1990-2000, where both variables are normalized to have a geometric mean of one. We find that relative changes in productivity and amenities are clustered around their geometric mean of one, but individual states can experience substantial decadal changes in relative productivity and amenities from around -30 to 30 percent.

**Figure F.6: Relative Productivity and Amenity Shocks from 1990-2000 from our Model Inversion**

(a) Productivity Shocks \( \tilde{z}_i = z_{i2000}/z_{i1990} \)  
(b) Amenity Shocks \( \tilde{b}_i = b_{i2000}/b_{i1990} \)

Note: Histograms of the distributions of relative changes in productivity \( \tilde{z}_i = z_{i2000}/z_{i1990} \) and amenities \( \tilde{b}_i = b_{i2000}/b_{i1990} \) from 1990-2000 from our model inversion, as discussed in Section B.9 of the online appendix. Relative changes in productivity \( \tilde{z}_i = z_{i2000}/z_{i1990} \) and amenities \( \tilde{b}_i = b_{i2000}/b_{i1990} \) are both normalized to have a geometric mean of one.

In Figure F.7, we display relative productivity and amenities for our four geographical groupings of states, where the values for each group are population-weighted averages of those for each state within that region. We find substantial changes in both relative productivity and amenities over time. From the top-left panel, the Rust Belt experiences a substantial decline in its relative productivity in the 1960s and 1970s, consistent with the argument in Holmes (1988) and Alder et al. (2019) that high unionization in these states during this time period could have retarded productivity growth relative to “right to work” states in Other Southern States. From the top-right panel, the rise in the population and income shares of the Sun Belt in previous figures is largely driven by an increase in its relative amenities. In contrast, relative productivity in the Sun Belt falls over time. From the bottom-right panel, the Other Southern States experience the largest increases in relative productivity over time, consistent with technological catch-up as well as potentially with more pro-business policy environments. Finally, from the bottom-left panel, both relative productivity and amenities are comparatively flat in the Other Northern States.
Figure F.7: Relative Productivity and Amenities over Time by Region

Notes: Productivity and amenities for each state are recovered from the inversion of the full non-linear model in Section B.9 of this online appendix; productivity and amenities are measured in relative terms and are normalized to have a geometric mean of one across U.S. states in each year; productivity and amenities for each group of states are the population-weighted average of their values for each state within that group.

In Figure F.8, we show the relationship between our solutions for bilateral trade and migration frictions and bilateral geographical distance. We find a strong positive, statistically significant and approximately log linear relationship between these variables, consistent with the model’s gravity equation predictions. In the interests of brevity, we show results for the year 2000, but we find the same pattern of results for all years of our sample period.
Figure F.8: Recovered Trade and Migration Frictions

(a) Bilateral Trade Frictions Versus Distance  
(b) Bilateral Migration Frictions Versus Distance

Note: Bilateral trade and migration frictions recovered from the inversion of the full non-linear model in Section B.9 of this online appendix; distance corresponds to the geographical (Great Circle) distance between the centroids of bilateral pairs of US states.

F.6 Non-Linear Model Solution Algorithm

In Section 5.3 of the paper, we use our spectral analysis to provide an analytical characterization of the speed of convergence to steady-state and the interaction between the capital and labor adjustment margins. Although this spectral analysis uses a linearization of the model, we show that this linearization provides a good approximation to the transition path of the full non-linear model. In this section of the online appendix, we provide further details on the solution algorithm used to solve for the economy’s transition path in the non-linear model.

Solving for the Sequential Competitive Equilibrium  Consider an economy on a transition path to some unknown steady-state starting from an initial allocation \( \left\{ \mathbf{I}_0 \right\}_{i=1}^N, \left\{ \mathbf{K}_1 \right\}_{i=1}^N, \left\{ \mathbf{K}_2 \right\}_{i=1}^N, \left\{ \mathbf{S}_{n0} \right\}_{n,i=1}^N, \left\{ \mathbf{D}_{ni,1} \right\}_{n,i=1}^N \), given an anticipated sequence of changes in fundamentals, \( \left\{ \mathbf{\dot{z}}_{it} \right\}_{i=1}^N, \left\{ \mathbf{\dot{b}}_{it} \right\}_{i=1}^N, \left\{ \mathbf{\dot{r}}_{ijt} \right\}_{i,j=1}^N, \left\{ \mathbf{\dot{k}}_{ijt} \right\}_{i,j=1}^N \) \( \mathbf{1}_{t=1}^\infty \).

The strategy to solve the sequential competitive equilibrium is as follows:

1. Initiate the algorithm at \( t = 0 \): guess the path of relative changes in transformed expected utility \( \left\{ \mathbf{u}_t^{(0)} \right\}_{t=1}^{T+1} \), where \( \mathbf{u}_t = \exp \left( \frac{\mathbf{\dot{u}}_{it}}{\mathbf{\dot{p}}_{it}} \right) \), and the path of capitalist consumption rates, \( \left\{ \mathbf{\psi}_t^{(0)} \right\}_{t=1}^{T+1} \), for a sufficient large \( T \). The path should converge by period \( T+1 \), i.e. \( \mathbf{\dot{u}}_{iT+1}^{(0)} = 1 \).

2. Set the rental rates in period \( t = 1 \) in accordance to the guessed consumption rates and the observed allocation

\[
R_{ii} = \left( \frac{\chi_{ii} \ell_{ii}}{\chi_{i0} \ell_{i0}} \right) \left( 1 - \mathbf{\psi}_1^{(0)} \right) \quad \forall i.
\]
3. Use the path of transformed expected utility \( \{ u_t^{(0)} \}_{t=1}^{T+1} \) to get migration rates \( \{ D_t \}_{t=1}^{T+1} \):

\[
D_{igt+1} = \frac{D_{igt} u_{igt+2}^{(0)} / \kappa_{igt+1}^{1/\rho}}{\sum_{m=1}^{N} D_{imt} u_{imt+2}^{(0)} / \kappa_{imt+1}^{1/\rho}}.
\]

4. Use the migration rates to get employment levels in all periods \( t > 2 \):

\[
\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}
\]

5. For each period \( t > 0 \):

(a) Use \( \ell_t, \ell_{t-1}, \chi_t, \chi_{t-1} \) and \( S_{t-1} \) to solve for the relative changes in wages \( \hat{\omega}_{t+1} \) and the new expenditure shares \( S_{t+1} \), by solving the system of non-linear equations

\[
\hat{\omega}_{it+1} \hat{\ell}_{it+1} = \sum_{n=1}^{N} \frac{S_{nit+1} w_{nt} \ell_{nt}}{\sum_{k=1}^{N} S_{knt} w_{kt} \ell_{kt}} \hat{\omega}_{nt+1} \hat{\ell}_{nt+1}
\]

and

\[
S_{nit+1} \equiv \frac{\left( \hat{\tau}_{nit+1} \hat{\omega}_{it+1} \left( \hat{\chi}_{it+1} \right)^{\mu-1} / \hat{z}_{it+1} \right)^{-\theta}}{\sum_{k=1}^{N} S_{nknt} \left( \hat{\tau}_{nknt+1} \hat{\omega}_{kt+1} \left( \hat{\chi}_{mt+1} \right)^{\mu-1} / \hat{z}_{kt+1} \right)^{-\theta}}.
\]

Note that the initial level of wages \( w_t \) can be recovered from the expenditure share matrix \( S_t \) and the population levels \( \ell_t \). Also note that this system can be solved through an iterative procedure after guessing a vector of relative changes in wages \( \hat{\omega}_{t+1} \).

(b) Solve for the implied relative changes in price indices \( \hat{p}_{t+1} \) from

\[
\hat{p}_{it+1} = \left( \sum_{m=1}^{N} S_{imt} \left( \hat{\tau}_{imt+1} \hat{\omega}_{mt+1} \left( \hat{\chi}_{mt+1} \right)^{\mu-1} / \hat{z}_{mt+1} \right)^{-\theta} \right)^{-1/\theta}.
\]

(c) Solve for the new rental rates \( \hat{R}_{t+1} \) using

\[
R_{it+1} = (1 - \delta) + \frac{\hat{w}_{it+1}}{\hat{p}_{it+1} \hat{\chi}_{it+1}} \left( R_{it} - (1 - \delta) \right).
\]

(d) Finally, update the capital labor ratios \( \hat{\chi}_{t+2} \) if \( t < T \) using

\[
\chi_{it+2} \ell_{it+2} = (1 - \zeta_{it+1}) R_{it+1} \chi_{it+1} \ell_{it+1},
\]

where recall that we have already solved for the path of population levels in all periods.
6. For each $t$, solve backwards for $\left\{ \dot{u}^{(1)}_t \right\}_{t=1}^{T+1}$ using

$$\dot{u}_{it+1} = \left( \frac{\dot{w}^{(1)}_{it+1}}{\dot{p}^{(1)}_{it+1}} \right)^{\frac{1}{N}} \left( \sum_{g=1}^{N} \frac{\dot{u}_{git+2}}{k^{1/\rho}_{git+1}} \right)^{\beta}. $$

7. For each $t$, solve backwards for $\left\{ s^{(1)}_t \right\}_{t=1}^{T+1}$ using

$$s_{it} = \frac{s_{it+1}}{s_{it+1} + \beta^{\gamma}R_{it+1}^{\gamma}};$$

imposing $R_{T+1} = 1/\beta$. 

8. Take the new paths for $\left\{ \dot{u}^{(1)}_t \right\}_{t=1}^{T+1}$ and $\left\{ s^{(1)}_t \right\}_{t=1}^{T+1}$ as the new initial conditions, and return to step 2.

9. Continue until convergence of $\left\{ \dot{u}^{(1)}_t \right\}_{t=1}^{T+1}$ and $\left\{ s^{(1)}_t \right\}_{t=1}^{T+1}$.

G Data Appendix

In this section of the online appendix, we report further details about the data sources and definitions. In Section G.1, we discuss the data used for the quantitative analysis of our baseline single-sector model. In Section G.2, we discuss the data used for the quantitative analysis of our multi-sector extension.

G.1 State Data

In the single-sector version of the model, we consider the 48 contiguous U.S. states (excluding Alaska and Hawaii) plus Washington DC.

State-to-State Migration Data. The decennial population censuses for 1960, 1970, 1980, 1990 and 2000 ask respondents their current state of residence and their state of residence five years ago. From the reported responses, we obtain bilateral five-year migration flows between U.S. states for 1960, 1970, 1980, 1990 and 2000. We construct an analogous bilateral five-year migration flows for 2010 using the American Community Survey (ACS) data for the years 2008-2012. The ACS provides data only on annual migration flows, so we take the 5th power of the annual outmigration shares matrix. The state-to-state migration data are reported for the population over 5 years in age. We construct own-state-to-own-state migration flows as total population over 5 years in age minus total immigrants from other states. We interpolate between years to
obtain five-year bilateral migration flows for each sample year from 1965-2015. We use these bilateral migration flows to construct our outmigration matrix \((D)\) and our immigration matrix \((E)\). To take account of international migration to each state and fertility/mortality differences across states, we adjust these migration flows by a scalar for each origin and destination state, such that origin population in year \(t\) pre-multiplied by the migration matrix equals destination population in year \(t + 1\), as required for internal consistency.

**State-to-State Trade Data.** The Commodity Flow Survey (CFS) reports the value of state-to-state shipments for the years 1993, 1997, 2002, 2007, 2012 and 2017. The CFS covers business establishments in mining, manufacturing, wholesale trade, and selected retail and services trade industries. The survey also covers selected auxiliary establishments (e.g., warehouses) of inscope, multi-unit, and retail companies. Industries not covered by the CFS include transportation, construction, most retail and services industries, farms, fisheries, foreign establishments, and most government-owned establishments. The CFS collects data on shipments originating from within-scope industries, including exports. Imports are not included until the point that they leave the importer’s initial domestic location for shipment to another location. The survey does not cover shipments originating from business establishments located in Puerto Rico and other U.S. possessions and territories.

The predecessor of the CFS was the Commodity Transportation Survey (CTS), which covers the manufacturing sector alone. The 1977 CTS reports the value of shipments from each state of origin to each census division of destination: New England, Middle Atlantic, East North Central, West North Central, South Atlantic, East South Central, West South Central, Mountain, and Pacific. We allocate the value of shipments across destination states within these destination census divisions according to their shares of the value of shipments in the CFS in 1993. We interpolate the value of shipments between years to obtain annual data on the value of shipments for each year of our sample from 1977-2015. We estimate the value of shipments between states for years before 1977 by assuming the following gravity equation:

\[
X_{nis} = X_{is}X_{ns}\tau_{nis}, \quad s \leq t = 1977,
\]

where \(X_{nis}\) is the value of bilateral shipments from exporter \(i\) to importer \(n\) in year \(s\); \(X_{is}\) is exporter gross domestic product (GDP); \(X_{ns}\) is importer GDP; and \(\tau_{nis}\) captures observed and unobserved bilateral trade costs. Assuming that bilateral trade costs remain constant, the value of bilateral shipments in any previous year \(s < t\) can be expressed in the following exact-hat algebra form:

\[
X_{nis} = \hat{X}_{is}\hat{X}_{ns}X_{nit}, \quad s \leq t = 1977,
\]

where a hat above a variable denotes a relative change between years \(s\) and \(t\), such that \(\hat{X}_{is} = \)
We observe these relative changes in exporter and importer GDP for each year back to the beginning of our sample period in 1965.

We thus obtain the bilateral value of shipments between states for each year of our sample from 1965-2015. We use these bilateral shipments data to construct our expenditure share matrix \((S)\) and income share matrix \((T)\). For our baseline quantitative analysis, we abstract from direct shipments to and from foreign countries, because of the relatively low level of U.S. trade openness, particularly towards the beginning of our sample period.

**Gross Domestic Product, Population and Capital Stock.** The Regional Economic Accounts of the Bureau of Economic Analysis (BEA) report population and state gross domestic product (GDP) for each state and year of our sample from 1965-2015. Estimates of real GDP at the state level are available starting from 1977. For the years 1965-1977, we deflate nominal GDP by splicing the national GDP deflator from the years 1965-1977 to the state-level deflator from 1977 onwards. To obtain state-level real capital stocks, we first compute state-industry level nominal gross operating surplus by subtracting labor compensation from GDP. We then deflate each observation by a corresponding estimate of the national capital income deflator, taken from the USA World KLEMS Database. Finally, we sum across industries to obtain a state-level measure of the real capital stock.

**G.2 Region-Sector Data**

In the multi-sector, multi-country version of the model, we consider the 48 contiguous US states, 43 other countries and 23 economic sectors, yielding a total of 2,093 region-sector combinations, where a region is either a US state or a foreign country. We allow for trade across all regions, and for migration across all states and sectors within the US.

**State-Sector Migration Data.** To implement our multi-sector application, we require a state-sector to state-sector migration matrix for the year 2000. To this end, we first recover state to state worker flows from the 2001 American Community Survey, which includes questions on current state of residence and the state of residence one year ago. We then use data from the Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC), which includes questions on current and past industry of employment, to divide these flows across origin and destination sectors, pooling together data from 1998-2002 to enlarge the sample size. Specifically, we first compute sector to sector transition rates for each state separately, and then assume that these transition probabilities are constant across destination states conditional on the state of origin. We multiply these sectoral transition probabilities by the state to state transition rates to get our matrix of state-sector to state-sector transition rates. Note that due to the small sample
size of the CPS, we cannot use it to directly compute transition probabilities across all state-sector combinations, amounting to close to 1,000,000 cells in the migration matrix.

**Region-Sector Production, Employment and Capital Stock.** For each country-sector combination, we take employment data and nominal value-added, gross-output and capital stocks from the Socio-Economic Accounts of the World Input-Output Tables (WIOT) 2016 release. To obtain real capital stocks, we take country-level estimates from the International Monetary Fund (IMF) Investment and Capital Stock Dataset, which provides private capital stocks in 2005 international dollars for most countries in the WIOT data over the period 1960-2013. We allocate national real capital stocks across sectors according to country-level shares of nominal capital stocks from WIOT. We allocate US aggregates across individual states using states shares in national GDP, gross output and capital using the BEA’s Regional Accounts.

**State-Sector Foreign Imports and Exports.** We use data on exports by state of the origin of movement (OM) and imports by state of destination (SD) from the Foreign Trade Division of the U.S. Census Bureau. The origin of movement (OM) export data are based on the state from which the shipment starts its journey to the port of export. Therefore, the data reflect the transportation origin of an export shipment, which need not correspond to the state in which the good was produced. The state of destination (SD) import data are based on the U.S. state, U.S. territory or U.S. possession where the merchandise is destined, as known at the time of customs filing at the port of entry. If the contents of the shipment are destined to more than one state, territory, or possession, or if the entry summary represents a consolidated shipment, the state of destination is reported as the state with the greatest aggregate value. If in either case, this information is unknown, the state of the ultimate consignee, or the state where the customs entry is filed are reported, in that order. However, before either of those alternatives is used, a good faith effort is required of the customs filer to ascertain the state where the imported merchandise will be delivered.

Therefore, these export origin of movement (OM) and import state of destination (SD) data differ from measures of exports and imports by port of exit and entry. The export data also differ from the exports of manufacturing enterprises (EME) data from the Annual Survey of Manufactures (ASM), which are restricted to manufacturing and based on the state of production. In contrast, our export and import data cover all traded sectors, and are collected by origin of movement (for exports) and destination of shipment (for imports).
References


