Electoral Competition and Term Limits∗

Germán Gieczewski†

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Abstract

We analyze the impact of term limits in a model of electoral competition on both ability and policy platforms. Good politicians facing weak competition extract policy rents, which lowers welfare. Moreover, strong incumbents deter challenger entry, exacerbating the issue. Term limits alleviate these problems in two ways. First, they insulate future utility from the current election, which weakens the bargaining position of strong incumbents. Second, they directly create open elections which encourage challengers. However, since term limits also restrict the pool of eligible politicians, their overall welfare impact is ambiguous. Term limits are most useful when politicians are highly biased, and when challengers’ entry cost is intermediate. The model also sheds light on the optimal structure of term limits. If voters are myopic, a cap on re-elections is effective. But if voters are sophisticated, such term limits incentivize them to discriminate against challengers, and stochastic term limits may be superior.

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†Department of Politics, Princeton University.
1 Introduction

The design of presidential term limits has garnered much debate. Most democracies stipulate some form of term limits, restricting the number of times that a person can be elected president. For example, in the United States, a politician can serve as president for at most two four-year terms in total. This is a common scheme, but many countries have a limit of one or three terms instead. Alternatively, some countries employ consecutive term limits, meaning that a term-limited candidate’s cap is reset after sitting out some number of terms. In some cases, term limits also apply to other elected officials, such as governors or legislators. The tradition of term limits goes back to ancient Greece and the Roman Republic, where many public offices had such constraints: for example, Roman consuls served for one year at a time and had to wait several years to stand for election again.

Proponents of term limits often argue that such constraints curb authoritarianism. As the argument goes, frequent turnover of politicians is necessary to maintain a healthy democracy; removing term limits would lead to a powerful president becoming entrenched, which would ultimately hurt voters. This fear is present even in well-established republics where abuse of formal power or outright dictatorship are unlikely outcomes. For example, prior to the Second World War, the United States had an informal rule by which presidents did not seek to run for more than two terms. The rule was broken by Franklin D. Roosevelt who ran successfully four times, although he died a year after being elected in 1944 for the fourth time. Seeing this as a threat to democracy, Congress subsequently passed the Twenty-second Amendment, formalizing the two-term limit. In the words of 1944 Republican nominee Thomas Dewey, who campaigned in favor of the reform, “Four terms, or sixteen years, is the most dangerous threat to our freedom ever proposed” (Jordan, 2011).

However, this common wisdom clashes with standard models of dynamic electoral competition, where term limits tend to reduce welfare. Indeed, in a model where potential candidates and platforms are fixed (or determined independently of term limits), the only
effect of term limits is to restrict the choice set faced by voters, leaving them worse off. Introducing an ex post effort decision would tilt the scales further against term limits, since the prospect of reelection can serve to incentivize effort.¹

What are term limits good for, then? If they serve to prevent authoritarianism, are they useful in strong democracies? Or are they only justified in weak democracies, where a shift to non-democracy or institutional decay are real dangers?

We show that term limits can be beneficial even in a society with strong formal institutions. This results from the combination of two features included in the model: strategic challenger entry and strategic platform choice. The former means that candidates compare costs and benefits when deciding to run for office; in particular, a potential challenger has a lower incentive to run against a strong incumbent, as she is unlikely to win. The latter means that candidates choose platforms during the campaign, balancing their own desire for biased policies with the need to sway enough voters to defeat the other candidate. The losing candidate plays a disciplining role: the stronger she is, the less the winner can bias her policy and still win.

Two channels arise through which term limits improve welfare. First, term limits alleviate the deterrence effect which results from strategic challenger entry. Intuitively, a strong incumbent will sooner or later come to power; if there are no term limits, she will run for reelection indefinitely. If competition is costly, challengers will not run against her, since their chance of success is low. Hence the incumbent faces little competition, which enables her to extract rents through platform choice. Term limits break this vicious cycle by periodically removing the incumbent, which creates a fair ground for competition.

Second, term limits mitigate the outside option effect, which results from strategic platform choice combined with forward-looking voters. The key insight behind this channel is

¹For instance, in Ferejohn (1986), politicians are homogeneous but choose an effort level while in office, and voters use a retrospective rule to reward effort. Reed (1994) extends the model to the case where politicians have heterogeneous ability. In both cases, term limits would remove incentives and make such rules less effective.
that voters care about the offered ability and policy menus today, but also about the continuation utilities that each politician will generate. With strong term limits, the continuation is independent of who is elected today (because they will be replaced anyway), but with weak term limits a bad incumbent will stay in the system longer if elected, generating a bad continuation as well. As a result, weak term limits lead to bigger gaps between the attractiveness of good and bad politicians. This allows good politicians to extract more rents when they face a bad competitor, since voters have a lower outside option.

Both strategic entry and platform choice are essential to our story, as it is the combination of the two that yields the key mechanics of the model. On the one hand, in a model with strategic entry but fixed platforms, challengers would still be deterred by strong incumbents but this would not be a problem: once a good politician is found there would be no further need for entrants. On the other hand, in a model with platform choice but exogenous entry, only the outside option effect survives. While this effect may by itself justify the use of term limits, we would be led to underestimate their value. More importantly, since the outside option effect hinges on voter patience, the model would conclude that term limits are useless if voters are myopic. But, in the full model, this is precisely when term limits are needed the most, as myopic voters fail to foresee the rise of strong incumbents who will exploit them.

The combination of strategic entry and platform choice creates an intertemporal dilemma for voters: very good politicians offer high payoffs now, but they will be worse in the future, as they will take advantage of the lack of competition to bias their policy and/or engage in corruption.

However, even in the full model term limits are not necessarily beneficial, as they have two negative side effects. First, they force voters to inefficiently discard good politicians. Second, they may in their own way discourage challengers through the prize effect: if politicians care

\footnote{This is an important variant, as in reality it seems likely that many voters would not think strategically about future elections and rather compare the offered utilities in the current period. Our insights also apply if voters are rational but do not care about future generations.}
about how long they will stay in office, term limits may reduce their incentives to compete
by limiting their expected tenure. As a result, the overall welfare impact of term limits is
ambiguous in principle.

Our study of optimal term limits reflects these insights. We focus on two aspects: the
strength of term limits and their structure. Stronger term limits are always better when
politicians are more biased—so that limiting rent extraction is more important—and when
the cost of entry is intermediate. Intuitively, the higher the cost of investing, the worse the
deterrence effect will be, making term limits more desirable. But, if investing is so costly
that candidates do not even contest open elections, then it is better to relax term limits to
increase entry though the prize effect.

As for optimal term limit structure, we focus on three broad types of schemes. Classic
term limits, the ones widely used in reality, allow politicians to rule for at most \( k \) terms.
Stationary term limits are given by a probability \( p \) that an incumbent can run for reelection,
independent of seniority. Finally, block term limits force an open election every \( k \) terms. The
optimal rule depends crucially on whether voters are forward-looking. If they are, stationary
term limits are generally best. The reason is that classic and block limits create artificial
incentives that agents respond to, with perverse effects. For instance, classic term limits
can create an incumbency advantage—incentivizing voters to reelect the incumbent over an
equally qualified challenger—due to what we call the open election effect.\(^3\) The premise is
that open elections are desirable by virtue of generating more competition. Then, since
classic term limits are more binding for senior politicians, voters know that the fastest way
to get a new open election is to reelect the incumbent until his time runs out; electing a

\(^3\)This is a genuine advantage, i.e., it makes the incumbent preferable to an equally good challenger. This is
different from other mechanisms proposed in the literature, like the electoral selection effect (Gowrisankaran,
Mitchell and Moro, 2008; Zaller, 1998), whereby incumbents are better than the average politician, by virtue
of having won past elections; and the strategic challenger entry effect (Cox and Katz, 1996; Stone, Maisel and
Maestas, 2004; Gordon, Huber and Landa, 2007b), whereby challengers are deterred from running against
good incumbents. These effects—which are also encompassed by our model—would also lead to incumbents
being reelected with high probability, but not conditional on the challenger being of equal quality.
challenger would reset the clock.

On the other hand, if agents are myopic, then classic and block term limits dominate stationary limits. The mechanics behind this result are different, as myopic voters do not respond perversely to non-stationary limits. Essentially, term limits serve as a tool for raising or lowering the probability of certain continuations occurring. Classic and block limits can selectively rule out bad continuations (where incumbents extract high rents) while retaining good ones, by using seniority or calendar time as a proxy for the relevant state of the world; on the other hand, stationary limits cannot do this as they are completely random.

Finally, we study an extension that allows for regime change, an important case since most attempts to change term limits are led by an incumbent, rather than made ex ante by a social planner. We focus on an example where voters can change the scheme on the fly through referenda. Voters choose to discard bad incumbents and allow good incumbents to run for reelection, but when the cost of entry is high (i.e., there is a high potential for deterrence) they also deny very good politicians, a form of “fear of tyranny”.

Our paper ties into the theoretical literature on dynamic elections and term limits. The way we model electoral competition is closest to Ashworth and Bueno de Mesquita (2008, 2009). The former presents a two-period model with strategic entry but fixed platforms, while the latter studies a static model with strategic investment and platform choice. Although both share ingredients with this paper, it is our combination of all three elements—strategic entry and platform choice in a dynamic setting—that drives our key mechanics and enables welfare analysis. For instance, the former paper shows that incumbency advantage occurs because of electoral selection and strategic entry (a result that our model reproduces), but this has no welfare consequence without platform choice, while it is clearly detrimental in our model.

Other papers about dynamic elections yield insights on term limits (whether mentioned by the authors or not), but are based on different premises. For example, in Alesina and
Cukierman (1990), Hess (1991) and Smart and Sturm (2013), candidates have privately known preferences and policies serve as signals. In Aghion and Jackson (2014) a leader with uncertain ability must be incentivized to take risky actions, but voters will be tempted to fire him if in the process his ability is revealed to be low. In Banks and Sundaram (1998) and Ashworth (2005), politicians with uncertain ability are motivated by career concerns to exert effort. In Rogoff and Sibert (1988) and Rogoff (1990) politicians create political business and budget cycles to misrepresent their ability; in Harrington Jr (1993), the effectiveness of policy is uncertain and politicians may choose policies which are popular but inefficient. These papers share a common theme: if the politician’s ability or preferences are private information, reelection prospects lead to signaling behavior, which may be good (e.g., when it induces centrism) or bad (e.g., when it induces budget cycles). Term limits mute incentives to signal, for better or worse. Though interesting, this line of research is unrelated to our story, which does not rely on private information.

There is also a sizable empirical literature on term limits, policy choice and the incumbency advantage, which broadly supports the mechanics of our model and its predictions. For instance, Gowrisankaran et al. (2008) argue that incumbents with long tenure in the U.S. Senate deter challenger entry, and term limits would increase welfare by preventing this. Ban, Llaudet and Snyder (2016) estimate that 30 to 40% of the incumbency advantage in U.S. state legislatures is the result of deterring experienced challengers, and Carey, Niemi and Powell (2000) and Lee (2008) provide additional evidence of the incumbency advantage for U.S. Representatives and its causes. These papers support our assumption of strategic challenger entry and its logical consequence, entry deterrence. On the other hand, Gordon et al. (2007a) show that judges facing electoral competition cater to voters by being harsher; and Acemoglu, Reed and Robinson (2013) find that chiefs in Sierra Leone facing less competition generate worse economic outcomes. This parallels our prediction that politicians bias their policies more when unconstrained by challengers. Finally, we identify a novel source of
incumbency advantage: the open election effect, described above, suggests that incumbency advantage may be stronger when there are (classic) term limits. This prediction is consistent with Querubín (2011), which finds that the introduction of term limits to the Philippines has actually made incumbents safer prior to the end of their tenure.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 characterizes the equilibrium under stationary term limits, which is the simplest case. Section 4 characterizes the equilibrium under classic and block limits. Section 5 analyzes the optimal level of term limits and compares the performance of stationary, classic and block limits. Section 6 extends the model to allow for regime change. Section 7 concludes.

2 The Model

Consider a society composed of a uniformly distributed continuum \( A = [-C, C] \) of voters, where \( C \geq 1 \). Time is infinite and discrete; agents live forever and have a discount factor \( \beta < 1 \). In each period a politician is elected president through democratic elections. Voters care both about the ability of the president and the slant of her policies. The utility of voter \( \alpha \) is

\[
U_\alpha ((\theta_t)_t, (x_t)_t) = \sum_{t=0}^{\infty} \beta^t u_\alpha (\theta_t, x_t) = \sum_{t=0}^{\infty} \beta^t (\theta_t - \lambda (\alpha - x_t)^2),
\]

where \( \theta_t \) is the ability of the incumbent at time \( t \); \( x_t \) is the policy at time \( t \); and \( \lambda > 0 \) is a constant that reflects the relative importance of competence vs. ideology. All voters prefer competent politicians, but they differ in their policy preferences, with voter \( \alpha \) having bliss point \( \alpha \).

Politicians are chosen as follows. There are two parties, L and R, who field candidates

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4Alternatively, \( x_t \) can represent rent-seeking or corrupt behavior by the politician. In this case all voters have bliss point \( \alpha = 0 \) and the politicians want as much corruption as possible. Most results below can be reformulated accordingly.
with bliss points $-C$ and $C$ respectively, where $0 < C \leq \overline{C}$. Every period, each party presents a candidate to the election. If the incumbent can run for reelection, she automatically becomes her party’s candidate and there is a closed election. Otherwise there is an open election.

Parties without eligible incumbents produce challengers. To simplify matters we assume that politicians who have run in the past and lost, or been barred from running by term limits, cannot return to politics, so only new challengers enter. A prospective challenger can make an investment to improve her quality at a cost $c$. If she does, her ability $\theta$ is drawn from a distribution $F$, given by a continuous density $f$ with support $[0, 1]$. Otherwise her ability is 0. A candidate’s ability is fixed forever once determined. (The candidate may be indifferent and choose a mixed strategy.)

Excluding the cost of investment, a politician with bliss point $\alpha$ has utility given by

$$W_\alpha(I_t, x_t) = \sum_{t=0}^{\infty} \beta^t \left[ R + \gamma(- (\alpha - x_t)^2 + \alpha^2) \right] I_t,$$

where $I_t$ is 1 if the candidate is president at time $t$ and 0 if not; $R$ represents the value of being in office; $\alpha$ equals $-C$ if he belongs to party L and $C$ otherwise; and $\gamma$ reflects the relative importance of holding office vs. policy preferences. In particular, we assume that the politician only cares about policy when he is in office.

Electoral competition takes place as follows. Once each party has produced a candidate, their abilities are publicly revealed; then each candidate $i$ simultaneously picks a policy platform $x_{it} \in [-\overline{C}, \overline{C}]$ that will be implemented if $i$ wins. (This is a credible commitment, but no promises can be made about what platforms will be offered in future elections.) Given these observed abilities and platforms, each voter chooses the politician offering her

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5Assuming other cost functions, e.g. $C(q) = cq^2$ for a probability $q$ of successful investment, leads to qualitatively similar but less tractable results.

6The term $\gamma \alpha^2$ is added for simplicity so that the candidate’s net policy payoff when elected is always weakly positive, but this does not affect incentives.

7This assumption can be relaxed if $\gamma$ is relatively low. If the politician cares about policy while out of office and $\gamma$ is high, this may result in nonmonotonic incentives to run, which muddies the analysis.
the highest expected utility.

2.1 Term Limit Schemes

In general we allow for stochastic term limits, given by a sequence \((p_k)_{k \geq 0}\) of probabilities such that \(p_0 = 1\), \(0 \leq p_k \leq 1\) \(\forall k \geq 1\), and if \(p_m = 0\) then \(p_k = 0\) \(\forall k > m\). The scheme works as follows: if a politician has been elected \(k\) times before, she is allowed to run again with probability \(p_k\).

We can obtain many interesting systems as particular cases. For example, a classic \(m\)-term limit (where politicians can hold office for at most \(m\) terms) is given by \(p_k = 1\) for \(k \leq m - 1\) and \(p_m = 0\). The absence of term limits corresponds to \(p_k = 1\) for all \(k\). In \(p\)-stationary term limits, given by \(p_k = p\) \(\forall k\), a politician has a constant chance \(p\) of being allowed to run again, regardless of seniority. Later on we also consider block term limits, which condition on calendar time instead of seniority.

2.2 Timing

The structure of play in each period is shown in Figure 1. At the beginning of \(t - 1\), the newly-elected president implements the policy promised in the campaign. Then, if there are stochastic term limits, it is determined by Nature (and publicly revealed) whether the incumbent can run again (\(J_t = 1\)) to rule in period \(t\) or not (\(J_t = 0\)). After this, parties without an incumbent draw new challengers who decide whether to invest. Their abilities are realized and publicly observed. Finally, candidates make campaign promises for period \(t\) and elections are held, producing a new incumbent in period \(t\).

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8 Stochastic term limits can be taken at face value as an institutional innovation, but they can also be used to study the effect of exogenous randomness on electoral competition under deterministic term limits. A politician may be prevented from running for reelection by death, a serious scandal, criminal charges, etc.
2.3 Definition of Equilibrium

Our solution concept will be Markov Perfect Equilibrium (MPE). Hence, rather than depending on the complete history of the game, actions are assumed to depend only on the payoff-relevant components of the state. Thus, the choice of platforms depends only on the candidates’ abilities and seniority, while votes depend on abilities, seniority and platforms. In particular, this means promises about platforms can only be made for the current election.

We will further focus on symmetric MPE. This means that strategies and value functions will be equal for both parties, and so independent of \( i \). Finally, under stationary term limits, we will restrict ourselves to stationary equilibria, meaning that strategies and payoff functions are also independent of the incumbent’s seniority \( k \).

2.4 Value Functions

These value functions will serve as summary statistics for equilibrium strategies and utility. Given a Markov strategy profile, \( V \) is the ex ante utility of the median voter, i.e., \( V = E(U_0((\theta_t)_t, (x_t)_t)) \). This is also the median voter’s expected continuation utility after an open election.

\( V_k(\theta) \) (for \( k \geq 1 \)) is the median voter’s continuation utility starting at a closed election, where the incumbent has ability \( \theta \) and seniority \( k \) (i.e., the incumbent is running for her \( k + 1 \)-th term) and the challenger’s ability has not yet been realized.

\( U_k(\theta) \) (for \( k \geq 0 \)) is the median voter’s expected utility from electing a candidate of ability

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9While seniority does not affect current payoffs, it affects the incumbent’s prospect of reelection, so it is payoff-relevant.
\( \theta \) and seniority \( k \) if that candidate makes his best offer, \( x_t = 0 \), in the current election. More generally, if a politician \((\theta, k)\) offers platform \( x \), the expected utility he offers to the median voter is \( U_k(\theta) - \lambda x^2 \). We will refer to \( U_k(\theta) \) as the politician's electoral strength.

It follows that \( U_k(\theta) = \theta + \beta p_{k+1} V_{k+1}(\theta) + \beta (1 - p_{k+1}) V \) for \( k \geq 0 \). This simplifies under particular schemes. For example, under stationary limits this becomes \( U(\theta) = \theta + \beta p V(\theta) + \beta (1 - p) V \). Under an \( m \)-term classic limit, we define \( U_0(\theta), U_1(\theta), \ldots, U_{m-1}(\theta) \) and \( V_1(\theta), \ldots, V_{m-1}(\theta) \) as above, and they will satisfy \( U_k(\theta) = \theta + \beta V_{k+1}(\theta) \) for \( k = 0, \ldots, m - 2 \) and \( U_{m-1}(\theta) = \theta + \beta V \).

3 Equilibrium Analysis

In this section we study the MPE of the game. First, we solve the subgame where candidates offer platforms and voters choose between them, conditional on the candidates' abilities. The main takeaway is that the more attractive candidate—in the sense of electoral strength—wins the election, but she offers a policy that leaves the median voter indifferent between her and the loser; thus, the loser plays the role of an outside option for voters.

Next, we analyze the challenger’s incentive to invest. As expected, challengers are less willing to enter against an attractive incumbent.

Afterwards we characterize the MPE under stationary term limits, which is conceptually the simplest case. It turns out that higher-\( \theta \) candidates are always (weakly) more attractive to voters. However, since they can’t promise not to exploit voters in the future, the continuation utility they offer is hump-shaped: it is increasing in \( \theta \) as long as the candidate is not strong enough to deter future competition, but decreasing after that. This key result is driven by the combination of strategic entry and platform choice.

Finally, we characterize equilibria under classic and block term limits. Under these schemes, higher-\( \theta \) candidates may actually be less attractive in some cases; and attractiveness may depend explicitly on seniority, leading to incumbency advantage (or disadvantage).
3.1 Electoral Competition

We start with voting behavior and policy offers. Since there are only two candidates, voters sincerely choose the candidate offering them the highest expected utility. In addition, there is a way to order policy paths in terms of average policy, so that right-wing voters like right-wing policy paths better:¹⁰

**Lemma 1.** Given two (stochastic) paths \((\theta_t, x_t), (\theta'_t, x'_t)\), we say the former is more biased to the right if \(\sum_{t=0}^{\infty} \beta^t E(x_t) > \sum_{t=0}^{\infty} \beta^t E(x'_t)\). Given two voters \(\alpha > \alpha'\), voter \(\alpha\) will prefer the first path relatively more, that is,

\[
EU_\alpha(\theta_t, x_t) - EU_\alpha(\theta'_t, x'_t) > EU_{\alpha'}(\theta_t, x_t) - EU_{\alpha'}(\theta'_t, x'_t).
\]

Moreover, the Median Voter Theorem applies:

**Proposition 1** (Median Voter Theorem). *If the median voter strictly prefers candidate \(i\) over candidate \(j\) (i.e., \(U_{k_i}(\theta_i) - \lambda x^2_i > U_{k_j}(\theta_j) - \lambda x^2_j\)), \(i\) wins the election. If the median voter is indifferent, the election is tied.*

*Proof.* Assume \(U_{k_i}(\theta_i) - \lambda x^2_i > U_{k_j}(\theta_j) - \lambda x^2_j\). Consider the random paths \((\theta_t, x_t), (\theta'_t, x'_t)\) that are expected from choosing \(i\) and \(j\), respectively. If \(i\)'s path is more biased to the right (left), then all voters with \(\alpha > 0 (\alpha < 0)\) also prefer \(i\), which decides the election; if their policy bias is the same, all voters prefer \(i\). If instead \(U_{k_i}(\theta_i) - \lambda x^2_i = U_{k_j}(\theta_j) - \lambda x^2_j\), if \(i\)'s path is more biased to the right then all right-wing voters prefer \(i\) and left-wing voters prefer \(j\), while the median voter is indifferent, giving a tie; the opposite case is analogous. If bias is equal, then all voters are indifferent.

¹⁰Quadratic preferences are needed for this result. The reason is that voters have preferences over entire paths, rather than single policies, so the functional form determines how they are aggregated. However, in the corruption version of the model, all voters have the same preferences so no functional form assumption is needed.
Thus, we can collapse the votes \((v_\alpha)_{\alpha \in [-C, C]}\) into \(v_0\) for our analysis. We can then characterize the platform choices and the outcome of the election in equilibrium:

**Proposition 2 (Electoral Equilibrium).** In any MPE, if \(U_{k_i}(\theta_i) > U_{k_j}(\theta_j)\), then \(i\) wins the election. If \(C\) is high enough, \(i\) offers \(|x_i| = \sqrt{\frac{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}{\lambda}}\), while the sign of \(x_i\) is given by \(i\)’s party, and \(j\) offers \(x_j = 0\). This leaves the median voter indifferent but voting for \(i\), and she receives expected utility \(U_{k_j}(\theta_j)\).

If \(C < \sqrt{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}\), \(i\) offers \(|x_i| = C\). In this case the median voter strictly prefers \(i\), and receives expected utility \(U_{k_i}(\theta_i) - \lambda C^2\) from electing her.

**Proof.** Suppose that \(i \in L\) and \(j \in R\). First, we check that these actions are consistent with equilibrium. Assume \(C\) is high enough. Given that \(j\) is offering \(x_j = 0\), \(i\) can win by offering any \(x\) such that \(U_{k_i}(\theta_i) - \lambda x^2 \geq U_{k_j}(\theta_j)\), which translates to \(|x| \leq \sqrt{\frac{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}{\lambda}}\). Then \(i\)’s optimal offer is the one closest to her bliss point, which leaves the median voter indifferent: \(x^* = -\sqrt{\frac{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}{\lambda}}\). (This is also better than “forfeiting” the election by offering a more extremist \(x\), since losing gives the politician a payoff of 0, while winning pays at least \(R\).) As for \(j\), given \(i\)’s offer there is no way for \(j\) to win, so she is indifferent between all her actions.

Next, we check that no other action profile is compatible with equilibrium. If \(j\) offers \(x' \neq 0\), then \(i\)’s best response is \(x''\) such that \(U_{k_i}(\theta_i) - \lambda x''^2 = U_{k_j}(\theta_j) - \lambda x'^2\), implying \(x'' < x^*\). But then \(j\) could deviate and win by offering \(x' = 0\). If \(j\) offers \(x' = 0\), then \(x^*\) is \(i\)’s only best response.

When \(U_{k_i}(\theta_i) - \lambda C^2 > U_{k_j}(\theta_j)\), \(i\) chooses \(|x| = C\), as it is her bliss point and wins the election no matter what \(j\) does. Any action by \(j\) is optimal in this case, but the choice has no effect.

**Corollary 1 (Continuation Utility).** Let \(C_0 = \sqrt{\frac{\max_{k, \theta} U_k(\theta) - \min_{k, \theta} U_k(\theta')}{\lambda}}\). If \(C \geq C_0\), then in any MPE, \(V_k(\theta) = E[\min(U_k(\theta), U_0(\theta'))\theta' \sim q_k(\theta)F]\). In other words, the continuation utility from an incumbent \((\theta, k)\) is the expected value of the minimum between the incumbent and the challenger’s strengths.
Two conclusions follow from Proposition 2. First, given two candidates, the one with the higher electoral strength always wins. Formally, let \( r_k(\theta, \theta') \) be the probability that a challenger of ability \( \theta \) will beat an opponent \((\theta', k)\). Then \( U_0(\theta) > U_k(\theta') \) implies \( r_k(\theta, \theta') = 1 \) and \( U_0(\theta) < U_k(\theta') \) implies \( r_k(\theta, \theta') = 0 \). If \( U_0(\theta) = U_k(\theta') \), voter behavior is indeterminate, but we assume \( r_k(\theta, \theta') = \frac{1}{2} \).\(^{11}\)

Second, electoral competition serves to discipline winners: if one candidate is superior to the other, she needs to offer just enough utility to the voters to match the loser’s best offer. Thus, a high-ability ruler is only as good as her competitors. The only deviation from this logic is if the ability gap is so great, or the policy bias \( C \) so small, that voters are happy to give the incumbent a “blank check”.

### 3.2 Investment Decisions

In this subsection we study the challenger’s decision to enter, and find that more attractive incumbents face less competition. Take a symmetric Markov strategy profile as given. Suppose there is an incumbent with ability \( \theta' > 0 \) and seniority \( k \). A challenger \( i \)'s objective is to maximize

\[ E \left[ W^i(I_t, x_t)|q, \theta', k \right] - cq = qT(\theta', k) - cq, \]

where \( W^i \) is \( i \)'s expected utility, \( q \) is \( i \)'s chosen probability of investing in quality, and \( T \) is \( i \)'s expected reward conditional on investing (\( T \) contains both rents from office and policy payoffs; explicit expressions are in the Appendix). Note that \( W^i = 0 \) if \( q = 0 \): \( i \)'s payoff is zero if he does not invest as he has no chance of winning. Then \( i \)'s choice is \( q = 1 \) if \( T(\theta', k) > c \), \( q = 0 \) if \( T(\theta', k) < c \), and \( q \in [0,1] \) if there is equality. We can show that

**Lemma 2.** Incentives to invest, \( T(\theta', k) \), are decreasing in the incumbent’s electoral strength

\(^{11}\)This assumption can be justified by considering a perturbed game where ability suffers small shocks \( \epsilon_t \) observed after \( x_t \) is chosen. That is, a candidate of ability \( \theta \) generates payoff \( \theta + \epsilon_t \) at time \( t \). As the shocks go to 0, this game has a unique equilibrium where \( r_k(\theta, \theta') = 0 \) or 1 when electoral strengths differ, but \( r(\theta, \theta') = \frac{1}{2} \) when they are equal, because the shocks serve as tiebreakers.
The intuition behind this result is that the incumbent’s strength has two effects on the challenger: it makes the challenger less likely to win her first race, and conditional on the challenger winning, it constrains what policies she can offer. Both channels reduce the expected rents from running. On the other hand, conditional on winning the first race, rents in the continuation do not depend on the loser’s strength as she is eliminated from the game.

3.3 Equilibrium Characterization Under Stationary Term Limits

We now provide a full characterization of MPE under $p$-stationary term limits. Here, we consider symmetric stationary Markov Perfect equilibria (henceforth SSMPE) where $U_k, V_k, q_k, r_k$ are independent of $k$.

First we settle a basic question: is electoral strength $U(\theta)$ increasing in $\theta$? A naive answer would be yes, because higher-$\theta$ politicians directly produce higher utility for voters. However, stronger politicians also drive out competition and hence will offer more biased policies in the future. In principle, this could make them less attractive ex ante, i.e., it could make $U(\theta)$ nonmonotonic. It turns out that this reversal is impossible under stationary term limits:

**Proposition 3** (Increasing $U$ with Stationary Limits). In any SSMPE with $p$-stationary term limits, $U(\theta)$ is weakly increasing. In particular, $U(\theta)$ and $V(\theta)$ attain their minima at $\theta = 0$. If $C \geq C_0$, then $U(0) = V(0) = \frac{\beta (1-p) V}{1-\beta p}$.

The logic of this result is simple. If candidates with high $\theta$ were less attractive than candidates with ability $\theta'$ for $\theta > \theta'$, this would make them easier to beat in subsequent elections, which would lead to relatively more competition against them, not less; but this, in turn, would make them desirable. Thus, the inherent value of ability cannot drive out so much competition as to turn high $\theta$ into a liability.\(^{12}\) However, note that $U$ does not have

\(^{12}\)We will see that this result does not carry through with non-stationary limits.
to be strictly increasing. As it turns out, $U$ usually has a flat interval.

We now fully characterize the SSMPE of the game:

![Equilibrium under stationary limits](image)

**Figure 2: Equilibrium under stationary limits**

**Proposition 4** (Unique SSMPE). Assume that $C \geq C_0$, $c > 0$ and term limits are $p$-stationary. Then

- An SSMPE can be of type 1, 2 or 3.

- In a type 1 equilibrium, there are thresholds $0 < \theta_0 < \theta_1 \leq 1$ such that challengers enter ($q(\theta) = 1$) against incumbents with ability in $[0, \theta_0]$; are indifferent ($0 < q(\theta) < 1$) against incumbents in $(\theta_0, \theta_1)$; and never enter ($q(\theta) = 0$) against incumbents in $[\theta_1, 1]$.

- In a type 2 equilibrium, there is a threshold $0 < \theta_0 < 1$ such that challengers enter against incumbents with ability in $[0, \theta_0]$ and are indifferent against incumbents with ability in $[\theta_0, 1]$.

- In a type 3 equilibrium, challengers never enter, so that $q(\theta) \equiv V(\theta) \equiv 0$, $U(\theta) = \theta$ for all $\theta$, and $q_0 = V = 0$.

- In cases 1 and 2, for $\theta < \theta_0$, $U$ and $V$ are increasing, and $U'(\theta) = \frac{1}{1 - \beta p(1 - F(\theta))}$. 
\begin{itemize}
  \item For $\theta_0 < \theta < \min(\theta_1, 1)$, $U(\theta)$ is constant and $V, q$ are decreasing, as $V'(\theta) = \frac{1}{\beta p}$ and $q'(\theta) = \frac{1}{\beta p \bar{V}(\theta_0)}$. \footnote{We define $\bar{U}(\theta) = U(\theta) - U(0)$, and analogously $\bar{V}(\theta) = V(\theta) - U(0)$.}

Moreover, if $\frac{f(\theta)}{1 - F(\theta)} \leq \frac{\phi}{1 - \theta}$ for a fixed $\phi \geq 1$ close enough to 1; there are $\underline{f}, \bar{f} > 0$ such that $\underline{f} < f(\theta) < \bar{f}$ for all $\theta$; and $\gamma$ is low enough, then the SSMPE is unique.

The intuition is as follows. When the incumbent is weak ($\theta < \theta_0$), she always faces competition. Hence, $U$ and $V$ are increasing in $\theta$ in this region, as a stronger incumbent simply represents a better option for voters. However, when the incumbent’s ability is above $\theta_0$, challengers are increasingly deterred from running, as the potential rents from winning are harder to attain. In equilibrium, the probability of the challenger investing $q(\theta)$ declines exactly at the rate needed to make $U$ constant in this region, and challengers are indifferent about competing here, which enables mixed strategies. (Otherwise, if $q(\theta)$ declined any faster, $U$ would be decreasing, contradicting Proposition 3; or, if $q(\theta)$ declined more slowly, $U$ would be increasing and hence $T(\theta)$ would cross over $c$ at a single point, so that $q$ should jump discontinuously from 1 to 0 at $\theta_0$, a contradiction).

This yields a general intuition for why $V(\theta)$ is hump-shaped. A candidate having higher $\theta$ is valuable when that candidate is likely to be the loser (since a stronger underdog forces the winner to choose a more centrist policy), but not so much when she is likely to win (as she then captures the “rents” of her electoral strength).

Our analysis also highlights the dynamic differences across candidates with different ability levels. Even in the region where $U$ is constant, higher $\theta$ candidates offer a different path of payoffs: they can offer high flow payoffs today, but are discounted in the future because of their tendency to exploit voters given lower competition. \footnote{This is reminiscent of entrenched politicians in weak democracies: strong incumbents usually perform well at first, but over time they become more corrupt. This pattern appears in the model despite the incumbent having no power to manipulate institutions.}

Finally, an equilibrium is classified as type 1 or type 2–illustrated in Figures 2a and 2b, respectively–depending on whether $q(\theta)$ actually declines all the way to 0 within the support
of possible ability levels, $[0,1]$. In a type 1 equilibrium, incumbents with ability $\theta > \theta_1$ are never challenged and can only be removed by term limits. In a type 2 equilibrium, all incumbents are challenged with positive probability.

4 Classic and Block Term Limits

Armed with intuition from the stationary case, we now perform a parallel analysis of classic and block term limits. The former type conditions on an incumbent’s seniority, and is the most common scheme used in practice: for example, a classic 2-term limit implies that a politician can only serve two terms. The latter conditions on calendar time: for example, a block 5-term limit breaks up time into blocks of five terms, so that anyone who starts their presidential tenure during block $l$ will be unable to run after the end of block $l$. In other words, classic term limits create open elections whenever an incumbent serves the maximum number of terms; block term limits create open elections at the beginning of each block. Although block term limits are not popular in practice, we study them for two reasons. First, like stationary limits, they help elucidate the effects of classic term limits while being more tractable. Second, they sometimes yield better welfare outcomes than classic limits.

4.1 Classic Term Limits

We first discuss classic $m$-term limits. Compared to the previous section, the new feature is that electoral strength $U_k(\theta)$ depends not only on $\theta$ but also on $k$. Thus, when two candidates with the same $\theta$ compete, the incumbent may be preferred simply by virtue of being the incumbent, which we call an incumbency advantage. (For some values of $\theta$ the effect is reversed, so we may also see an incumbency disadvantage.)\(^\text{15}\) We can partially characterize

\(^{15}\)Our model contains two other features that might also be construed as incumbency advantage. First, incumbents are a selected group of politicians who have won at least one election, so they tend to have higher ability. Second, strong incumbents tend to face weak challengers. Both features are present under all term limits, and increase the chance of reelection. But they do not give a “real” advantage conditional on facing
the set of SMPEs as follows:

\[ q_k(\theta) = \begin{cases} 1 & \text{if } U_k(\theta) < U^* \vspace{1em} \text{and} \\ 0 & \text{if } U_k(\theta) > U^* \end{cases} \]

For each \( k = 0, \ldots, m-1 \) there is a set of cutoffs \( A_k = (\theta_1, \ldots, \theta_{l_k}) \) such that \( A_{k+1} \supseteq A_k \) \( \forall k \), and \( l_k \leq 2(m-k)-3 \) for \( k \leq m-2 \). Cutoffs \( \theta \in A_k \) are given by the condition \( U_{k+1}(\theta) = U^* \) for some \( \theta \in \theta_{k+1} \). Electoral strength \( U_{k+1}(\theta) \) is strictly increasing in each subinterval of \( A_k \), but has discrete jumps (up or down) at the cutoffs \( \theta_{ik} \in A_k \).

The description of equilibria becomes more straightforward for the special case of a two-term limit:

**Corollary 2** (Equilibrium Characterization for a Two-Term Limit). Consider a game with a 2-term limit. Then all SMPEs are as follows. There is a utility threshold \( U^* \) such that
\[ q_1(\theta) = \begin{cases} 1 & \text{if } U^* > U_1(\theta) \vspace{1em} \text{and} \\ 0 & \text{if } U^* < U_1(\theta) \end{cases} \]

Moreover, in the nondegenerate case a challenger with equal ability. What we call incumbency advantage here gives incumbents an edge against a challenger of equal ability, and is a unique feature of classic term limits.
where \( U_1(0) < U^* < U_1(1) \), the SMPE is described by a cutoff \( \theta_0 \in (0, 1) \) and a probability \( q_0 \) of entering open elections, such that:

- \( U^* = U_1(\theta_0) \), so that \( q_1(\theta) = 1 \) for \( \theta \in [0, \theta_0) \) and \( q_1(\theta) = 0 \) for \( \theta \in (\theta_0, 1] \).

- \( U_1(\theta) = \theta + \beta V \) is increasing everywhere. \( U_0(\theta) \) is smoothly increasing everywhere except at \( \theta_0 \), where it has a discrete drop.

- \( U_1(\theta) \geq U_0(\theta) \) for \( \theta > \theta_0 \), i.e., there is incumendency advantage for candidates above \( \theta_0 \).

- \( U'_1(\theta) < U'_0(\theta) \) for \( \theta < \theta_0 \). In addition, \( U_1(0) \geq U_0(0) \). In other words, there is also incumendency advantage for very low quality candidates, but there may be incumendency advantage or disadvantage at intermediate quality.

Two degenerate cases are possible. For \( c \) low enough, \( U^* > U_1(1) \) and challengers always enter. If \( c \) is high enough, \( U^* < U_1(0) \) and challengers never enter an open election.

Figures 3a and 3b show SMPEs in examples with a two-term limit and a three-term limit, respectively. Compared to stationary term limits, two things are amiss. First, the function \( U_0(\theta) \) is nonmonotonic, i.e., the analog of Proposition 3 is false. This means that voters may purposefully choose worse politicians due to the fear that strong incumbents will exploit them in the future. Second, there is indeed an incumendency advantage for high \( \theta \): given two identical candidates of ability \( \theta > \theta_0 \), one of which is running for reelection, voters will prefer the incumbent. Why does this happen?

Just as in the stationary case, incentives to run are a decreasing function of \( U_k(\theta) \), and there is a threshold \( U^* \) such that challengers run against incumbents with \( U < U^* \) and not against ones with \( U > U^* \). The difference is that now there cannot be intervals where \( U \) is constantly equal to \( U^* \): a term-limited incumbent’s electoral strength is always strictly increasing in \( \theta \), so it must cross \( U^* \) at a single point. This creates a discrete jump in \( U_{m-2} \), which is in turn reflected in \( U_{m-3} \), and so on. In the two-term case, the jump happens at
a challenger with $\theta > \theta_0$ is expected to suppress so much competition in her second term that she is ex ante less desirable (and a weaker challenger) than a lower-$\theta$ alternative.

The incumbency advantage also stems from the strategic effect of term limits. Suppose both candidates have the same $\theta$. However, since the incumbent is running for reelection, she will be barred from running again afterwards, which allows her to offer the continuation value $V$. On the other hand, the challenger would become a (strong) incumbent tomorrow: this would generate $q = 0$ and the continuation value $U_0(0) < V$. This asymmetry, which we call the open election effect, effectively handicaps challengers and discourages competition in closed elections.

However, for $\theta < \theta_0$ this effect may be reversed. In general, for $m > 2$, the equilibrium becomes more complicated as utilities “cycle” around the cutoffs: if $U_k(\theta) > U^*$, then $q_k(\theta) = 0$ which often implies $U_{k-1}(\theta) < U^*$, $q_{k-1}(\theta) = 1$ which in turn often implies $U_{k-2}(\theta) > U^*$ and so on, as seen in Figure 3b.

### 4.2 Block Term Limits

A block $m-$term limit divides the timeline into blocks of $m$ terms each. There are no limits to reelection within each block, but each block must begin with an open election. Like classic term limits, they become more binding over time, but there is an important difference. When a challenger enters late in a block, the clock is not reset for her; she automatically becomes a "senior" incumbent. The equilibrium under this scheme follows:

**Proposition 6** (Equilibrium Characterization for Block Term Limits). Under $m$-block term limits, there is a unique SMPE which can be found recursively. There are cutoffs $U^*_k$ for $k = 1, \ldots, m-2$ given by $T_k(U^*_k) = c$; $q_k(\theta) = 1$ if $U_k(\theta) < U^*_k$ and $q_k(\theta) = 0$ if $U_k(\theta) > U^*_k$. As with classic term limits, $U_k(\theta)$ is increasing within each interval but has discrete jumps where $U_{k'}(\theta)$ crosses $U^*_k$ for $k' > k$.

Finding the equilibrium in this case is simple. First, $U_{m-1}(\theta) = \theta + \beta V$ for some $V$. Then
\[ T_{m-1}(\theta) = R(1 - F(\theta)) \] and there is a \( U_{m-1}^* \) such that \( T_{m-1}(U_{m-1}^*) = c \). From here we find \( q_{m-1}(\theta) \), and then \( V_{m-1}(\theta) = E[\min(U_{m-1}(\theta), U_{m-1}(\theta'))] \). Given this, we have \( U_{m-2}(\theta) = \theta + \beta V_{m-1}(\theta) \). Then we can characterize \( T_{m-2}(\theta), q_{m-2}(\theta), V_{m-2}(\theta) \) and so on. Finally \( V \) can be calculated. However, as with classic limits, the shape of the functions \( U_k(\theta) \) becomes more and more complicated for large \( m \), as we can see in Figure 4.

This scheme has an upside and a downside compared to classic limits. On the one hand, challengers and incumbents are treated symmetrically, so there is no incumbency advantage: voters are not concerned that by picking a challenger they would turn her into a new, less-constrained incumbent. On the other hand, challengers are now directly discouraged from entering late in the block, as the potential reward from winning is smaller. As with classic term limits, value functions are nonmonotonic, so voters are sometimes incentivized to pick lower-\( \theta \) candidates.
5 Welfare Analysis

In this Section we show how the welfare of voters is affected by the term limit scheme chosen and other parameters of the model. As a measure of welfare, we take ex ante welfare to be average voter utility, i.e. \( \int_{C}^{C'} EU_{\alpha}(\theta_t, x_t) d\alpha \). Due to the assumptions of quadratic preferences and a uniform distribution of voters, this is equivalent to the median voter’s utility\(^{16}\), \( EU_0(\theta_t, x_t) = V \), which is easier to work with.

First, we provide a breakdown of the positive and negative effects of term limits and analyze the comparative statics of the model, focusing on the case of stationary term limits for their greater tractability. Then we compare stationary limits to classic and block limits. We find that the preferred structure of term limits depends crucially on how sophisticated voters are: in a nutshell, some combination of classic and block limits is best when voters are myopic, but stationary limits are preferable if voters are sophisticated.

5.1 Comparative Statics

We study how welfare, and the optimal level of term limits \( p \), change as we vary two elements of the model. First, we consider changes in the cost of investment, \( c \). These allow us to consider qualitatively different cases: for instance, \( c = 0 \) corresponds to the case of exogenous challenger entry. For low \( c > 0 \), the equilibrium is type 2, with low entry deterrence. As \( c \) increases, competition declines. For very high \( c \), \( q_0 \) becomes smaller than 1, i.e., competition is so costly that not even open elections are attractive.

Second, we allow the preferences of politicians to vary. So far we have focused for the sake of exposition on the extreme case \( C \geq C_0 \), where politicians are so biased that they always fully exploit any gap in electoral strength. Under this assumption strong incumbents lower welfare, making term limits attractive. In contrast, if politicians were unbiased, there

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\(^{16}\)We only need the distribution of voter bliss points to have mean 0 for this result to hold. However, quadratic preferences are necessary for voters to aggregate nicely. On the other hand, in the corruption interpretation of the model, the welfare of a representative voter can be used.
would be no rent extraction and having no limits would be optimal. So we ask what happens in the intermediate case. Concretely, we consider a variant of the model where any politician has probability $\alpha$ of being unbiased at each time $t$, and a probability $1 - \alpha$ of being biased (with $C \geq C_0$); these are i.i.d. draws. As a motivating story for this assumption, suppose each election is about a different “big issue” and platforms revolve around them; a generally biased politician may be biased on some issues but not others.\footnote{A more obvious assumption would be that $C$ is positive but small. Then, when the ability gap is large enough, the winner’s platform is given by $|x_t| = C$ and the median voter strictly prefers her offer. However, this condition $|x_t| = C$ binds only sometimes, making it impossible to obtain closed-form results.}

Proposition 4 and the results leading up to it generalize to the case with intermediate $\alpha$ (see Appendix A). The main difference is that now

$$V(\theta) = (1 - \alpha)E[\min(U(\theta), U(\theta'))] + \alpha E[\max(U(\theta), U(\theta'))],$$

$$V = (1 - \alpha)E[\min(U(\theta), U(\theta'))|\theta, \theta' \sim q_0 F] + \alpha E[\max(U(\theta), U(\theta'))|\theta, \theta' \sim q_0 F].$$

In other words, with probability $1 - \alpha$ the winner leaves voters indifferent with their outside option as before, but with probability $\alpha$ she is unbiased and simply offers her value $U(\theta)$.

From here and the analog of Proposition 4 it follows that $V$ is decreasing in $c$ and increasing in $\alpha$. This makes intuitive sense: welfare is lower if there is less competition and if politicians are more biased.

The rest of our results concern the interaction of these parameters with term limits: what is the optimal level of $p$ as a function of $(c, \alpha)$? To answer this question we need to understand the channels through which term limits affect welfare. There are four such channels:

- Term limits lower the probability of the \textit{deterrence effect} kicking in. By periodically removing strong politicians that could capture the electoral process, term limits create a fair ground for competition in the form of open elections.\footnote{This does \textit{not} mean challengers are incentivized to compete against incumbents; in general, such incentives go \textit{down} through the \textit{prize effect}. However, competition goes up on average because open elections become more frequent.}
Term limits mitigate the outside option effect, which we can explain as follows. The equilibrium policy bias is a function of the gap between winner and loser, $|U(\theta) - U(\theta')|$. Since voters are patient, this gap is not just $|\theta - \theta'|$ but also depends on the continuations $V(\theta), V(\theta')$ that candidates offer. If good candidates also offer better continuations, they become more attractive the longer they can stay in the system. Indeed, as shown in Proposition 4, when term limits are relaxed $U'(\theta)$ increases: the gap between good and bad candidates grows. This has a perverse effect on welfare: as losers become comparatively worse, the outside option goes down for voters, and winners can choose more biased policies. Conversely, stronger term limits flatten $U(\theta)$ and help constrain the winner.

Term limits sometimes inefficiently remove good politicians from office, which we call the selection effect.

Term limits shorten the expected tenure of politicians, so they reduce the potential rewards from running. Therefore, they may reduce competition through the prize effect.

Term limits increase welfare through the first two effects, but decrease it through the other two. To see which effects dominate, we derive a partially closed-form characterization of welfare:

**Proposition 7 (Welfare Effects with Limited Bias and Strategic Entry).** The median voter’s ex ante utility is given by

$$(1 - \beta)V = (\beta p a q(0) + 2\alpha q_0(1 - \beta p))W + (1 - 2\alpha)q_0^2(1 - \beta p)O,$$

$$W = \int_0^{\theta_0} \frac{1 - F(\theta)}{1 - \beta p (\alpha + (1 - 2\alpha)(1 - F(\theta)))} + \int_{\theta_1}^1 \frac{1 - F(\theta)}{1 - \beta p a},$$

$$O = \int_0^{\theta_0} \frac{(1 - F(\theta))^2}{1 - \beta p (\alpha + (1 - 2\alpha)(1 - F(\theta)))} + \int_{\theta_1}^1 \frac{(1 - F(\theta))^2}{1 - \beta p a}.$$
The first term measures what voter utility would be if in every election voters were to randomly receive the $U(\theta)$ of one of the candidates, as is the case when $\alpha = \frac{1}{2}$. Hence it depends directly on the expected ability distribution, $F$. The second term measures welfare when voters only get the loser’s value, i.e., $\min(U(\theta), U(\theta'))$, as is always the case when $\alpha = 0$, whence it depends on $F^2$, the distribution of a minimum of two abilities.

Several conclusions follow from here. First, consider the benchmark case $c = 0$, where challenger entry is exogenous (in other words $\theta_0 = 1$, $q(\theta) = q_0 = 1$, so challengers always have an ability distribution $\theta' \sim F$). In this case, it can be shown that $V$ is decreasing in $p$ if $\alpha < \frac{1}{2}$, constant if $\alpha = \frac{1}{2}$ and increasing if $\alpha > \frac{1}{2}$. In other words, if politicians are biased more than half the time then $p = 0$ (no reelection) is optimal; else $p = 1$ (unlimited reelection) is optimal.

This result may seem counterintuitive. The obvious channel through which term limits improve welfare is by preventing incumbents from exploiting uncontested elections. So shouldn’t term limits be welfare-reducing when there is no deterrence? Not necessarily, because the outside option effect may be substantial. Indeed, in this case there are no deterrence or prize effects, so the outside option effect competes with the selection effect only. If politicians are biased more than half the time, reducing bias is more important than keeping good politicians, and vice-versa.

Next, we ask how this result changes for positive $c$: are term limits more valuable in a model with strategic entry? The answer is yes, but with one caveat. Term limits are more useful the higher $c$ is, as long as it’s low enough that challengers want to enter open elections. Formally

**Corollary 3** (Optimal Term Limits With Strategic Entry). Let $c_0 = \frac{R}{2}$ be the highest cost for which $q_0 = 1$ regardless of $p$. Then, for $c \in (0, c_0)$, there are thresholds $\frac{1}{2} < \alpha^*(c) < 1$ such that $p = 0$ is optimal for all $\alpha < \alpha^*(c)$.

On the other hand, let $\overline{c}$ be the lowest cost of investing for which the equilibrium is type
3 regardless of $p$. There is $c_0 < c_1 < \bar{c}$ such that $p = 1$ is optimal for $c \in [c_1, \bar{c}]$, regardless of $\alpha$.

The first part of the Corollary says that, compared to the exogenous entry case, the scales tilt further in favor of term limits when $c$ is positive but small. The reason is that, so long as $q_0 = 1$ (candidates still run in open elections), full term limits eliminate the deterrence effect (incumbents can’t run for reelection so they can’t deter entrants) and give the same welfare as when when $c = 0$. But, under any other scheme $p > 0$, the deterrence effect lowers welfare compared to the exogenous entry case. If we took prize effects out of the model, e.g., by assuming that politicians only care about becoming president and not the length of their tenure, then this conclusion would extend to any $c$: the existence of entry deterrence would always the value of term limits.\(^\text{19}\)

Different forces are at work for high $c$, as implied in the second part of the result. When $c$ is high enough that challengers drop out of open elections, weak term limits are needed to incentivize entry through the prize effect. In fact, the optimal $p$ for high $c$ may be bigger than the optimum under exogenous entry, when the prize effect dominates the deterrence effect.

Figure 5 shows numerical results which extend Corollary 3, illustrating how welfare $V$ varies as a function of $p$ for different values of $c \in [0, \bar{c}]$, and $\alpha = 0$. First, for low $\alpha$, full term limits ($p = 0$) can be optimal even without strategic entry ($c = 0$), but full term limits become even more desirable when $c$ is intermediate (compare $V^{c=0}$ to $V^{c=1}$). Second, even for $\alpha = 0$, a high $p$ (all the way up to $p = 1$) may be optimal when $c$ is very high, as challengers must be enticed to enter open elections.

Finally, note that the Corollary and much of the intuition translate trivially to the case

\(^{19}\) The size of prize effects is hard to bound in general. However, there are good reasons to study the case where they are small. Consider a model in which candidates get a flow payoff $R$ while in office and some other $0 < R' < R$ per period after leaving: for example, they may hold a governorship, retain influence within the party, or profit from book royalties or speaker fees. This would make the total reward less dependent on the length of tenure.
of classic and block limits, though not the expressions in Proposition 7. For instance, when 
\((c, \alpha) = (0, 1)\), \(m = \infty\) (no term limits) is optimal; when \((c, \alpha) = (0, 0)\), \(m = 1\) (no reelection) is optimal. In addition, so long as \(q_0 = 1\), \(m = 1\) is relatively better for \(c > 0\) than for \(c = 0\).

5.2 Optimal Structure of Term Limits

How do classic, block and stationary term limits compare to each other? The answer depends crucially on whether agents are patient or not, and the results are driven by different mechanics in each case. We consider two cases: when \(\beta > 0\) (agents are strategic) and when \(\beta = 0\) (all agents, voters and politicians, are myopic).\(^{20}\)

Simulations show that, in the former case, stationary limits are best.\(^{21}\) This is shown in Figure 6, which measures comparable limits of all three kinds against each other for different values of \(c\).\(^{22}\) This is no surprise given the artificial incentives created by classic and block

\(^{20}\)When agents are myopic, we evaluate long-term welfare from the point of view of a social planner with discount factor \(\beta \approx 1\). Assuming that voters are myopic but politicians are patient does not substantially change the results.

\(^{21}\)For stationary and block limits, the numerical solution can be computed using the analytical results in the paper. For classic limits, the simulation constructs a solution as in Proposition 5 and iterates on \(U^*, V\) and \(U_i\) until convergence is reached. For all parameters used the limit is unique.

\(^{22}\)Stationary limits are plotted by mapping \(p\) to \(\sum_k p^k\), the expected allowed tenure. To fill in the graph
Figure 6: Welfare of stationary, block and classic term limits with patient agents; $R = 2$, $\alpha = 0; c \in \{0.5, 1.5\}; F \sim U[0, 1]$
limits. With the former, competition in closed elections is deterred as voters discriminate against challengers; with the latter, challengers are discouraged from entering late in the block. On top of that, both schemes sometimes incentivize voters to choose lower-quality challengers in open elections, since $U_0(\theta)$ is nonmonotonic. Welfare is lower as a result. In contrast, stationary limits sidestep all these problems by making seniority and calendar time irrelevant.

Figure 7: Long-term welfare of block, classic and stationary limits with myopic agents; $R = 2$, $c = 0.037$, $\alpha = 0$, $f(\theta) = 5(1 - \theta)^4$

In the second case, in contrast, block and classic limits outperform stationary limits, as seen in Figure 7. We can show this analytically for block limits:

**Proposition 8** (Optimal Term Limits With Myopic Agents). If $\beta = 0$, then generically there is a unique $m^*$ that maximizes $V_{LT}^{m^* - \text{block}}$, and $V_{LT}^{m^* - \text{block}} > V_{LT}^{p - \text{stationary}}$ for any $p$.

With myopic agents, the incentive distortions that made classic and block limits inferior become irrelevant. So what force still differentiates between these schemes? To understand for classic and block limits, we use "fractional" $(m + p)$-limits where $m$ terms are allowed, followed by an $(m + 1)$-th term with some probability $p$. 
the result, we have to think about term limits as a way of reshaping the probability of certain continuations. Denote the expected flow payoffs of each period after an open election by $v_0, v_1, \ldots$ and let $w_t = \sum_{s=0}^{t-1} v_s$ be their partial sums. Block term limits reset the model every $m$ periods; this truncates the sequence, giving long-term welfare $V = \frac{w_m}{m}$. The optimal $m^*$ is the value that maximizes this partial average. Stationary term limits put exponential weights on these flow payoffs, yielding $V = (1 - p) \sum p^t v_t = (1 - p)^2 \sum p^t w_t$, which is worse than putting all the weight on the optimal $m^*$.

Intuitively, suppose the sequence $(v_t)_t$ is hump-shaped ($v_t$ increases as good candidates enter and decreases as a strong incumbent becomes entrenched). Block limits allow us to cut off the right tail at the optimal point; stationary limits only let us discount the right tail by also discounting intermediate $v_t$’s, which are desirable. At a deeper level, the true state variable we’d want to condition on is the incumbent’s $\theta$; block limits use time since an open election as a proxy for this, so they can selectively cut off bad continuations, while stationary limits neglect this information. Classic limits are also somewhat selective, and hence are also superior to stationary limits, although they use a different proxy for $\theta$ (the incumbent’s seniority).

6 Flexible Term Limits and Regime Change

So far, we have focused on the ex ante choice of fixed term limits. This is akin to assuming that voters (or a social planner) gather to choose a constitution before the game begins, and before the likely incumbents are known; afterwards, the constitution is unchangeable.

However, many attempts to change term limits occur in a different context. The push for reform is often driven by a popular incumbent that would need a constitutional amendment to run again. For example, Hugo Chávez came to power in Venezuela in 1999; in 2009, during his purported last term, he abolished term limits by means of a referendum, after which he was narrowly reelected in 2013. In Argentina, prior to 1994, presidents were restricted to a
single six-year term. Elected in 1989, Carlos Menem engineered a constitutional reform that changed the maximum to two four-year terms, allowing him to run again in 1995. During his second term he campaigned unsuccessfully to extend the limit to three terms. Similarly, Fernando Cardoso in Brazil and Álvaro Uribe in Colombia extended the limit from one four-year term to two in their respective countries.\footnote{For an in-depth discussion of the Latin American case, see Carey (2003).} The common theme in these examples is that voters know who the incumbent will be if the term limits are relaxed.

In this section, we extend our model to answer the question: should this sort of discretionary change of rules be allowed, or should term limits be set in stone? We discuss two opposites of fixed term limits, which yield very different results: flexible term limits and persistent regime change.

In the former case, the default rule is $p = 0$ but, before each election, voters can choose in a referendum to override the rule and let the incumbent run for reelection this time; the incumbent’s right to run again sets no precedent and must be renewed every time. In the latter, the default rule is $p = 0$ but the incumbent in each period can call for a referendum to change the rule to $p = 1$ permanently; if she succeeds, there are no further changes.\footnote{We could consider semi-flexible schemes where there is a default rule and voters can override it only if they have a supermajority, or hybrid systems where one-time overrides and rule changes require different supermajorities, etc. A systematic exploration of such schemes is beyond the scope of this paper.}

Consider flexible term limits first. The way voters use these limits is straightforward. Since they cannot commit, they will vote to keep politicians with $V(\theta) > V$ and discard the rest. This leads to the following equilibrium:

**Proposition 9 (Equilibrium With Flexible Term Limits).** Consider a game with flexible term limits and $\alpha = 0$. Then the MPE may be of type 1, 2, or 3. In a type 1 equilibrium, there are $0 < \theta < \theta_0 < \theta_1 < 1$ such that $q(\theta) = 1$ for $0 \leq \theta \leq \theta_0$, $q(\theta)$ is decreasing between $\theta_0$ and $\theta_1$, and drops discontinuously to $q(\theta) = 0$ for $\theta > \theta_1$. Candidates are allowed to run again if $\theta \in [\theta, \theta_1]$. In a type 2 equilibrium, there are $0 < \theta < \theta_0 < 1$ such that $0 < q(\theta) < 1$ for $\theta > \theta_0$ and candidates can run again if $\theta \in [\theta, 1]$. Type 3 equilibria have $q \equiv 0$ as before.
In general voters discard bad politicians (with $\theta < \theta$) and keep good ones. However, when competition against strong candidates is low, the best candidates (with $\theta > \theta_1$) are also discarded as they are, in effect, dangerous. If they were allowed to run, such candidates would deter entrants and thus leave voters no choice but to reelect them; hence, given the choice, voters will bar their reelection bid in the first place.\footnote{This is reminiscent of Athenian ostracism, whereby citizens could preemptively exile other citizens that might become too powerful and pose a threat to the state.} This is illustrated in Figure 8.

How does this scheme compare to the fixed limits from Section 3 in terms of welfare? Again the answer depends on whether agents are strategic. If they are, flexible limits are better if $c$ is low, and worse if it is high (as shown in Figure 9):

**Proposition 10** (Welfare Effect of Flexible Term Limits). Let $\alpha = 0$ and $c > 0$ close enough to $c$. Then the MPE under flexible term limits generates $q_0 = 0$ and $V = 0$, which is inferior to fixing $p = 1$. On the other hand, if $c = 0$ then flexible term limits generate a higher $V$ than $p = 0$ (the optimal fixed rule).

These results are driven by two differences between flexible and fixed term limits. On the one hand, making term limits dependent on $\theta$ is informationally more efficient: it allows
voters to target undesirable incumbents while keeping the good ones. On the other hand, voters—being unable to commit— decide who to keep based on the “greedy” rule $V(\theta) > V$, ignoring that such decisions affect challengers’ incentives to enter. Thus, when weak term limits would be ex ante optimal (because the prize must be increased to generate entry), voters under flexible limits will still be tempted to kick out entrenched incumbents. But challengers will anticipate this and choose not to enter in the first place.\footnote{Term limits could theoretically consist of a committed $\theta$-dependent rule. This would be better than both flexible and fixed limits. However, this seems unimplementable, because voters must reveal their information about $\theta$ for it to work—but if the scheme doesn’t obey the “greedy” rule, voters will game it ex post by misreporting $\theta$.} If anything, the sin of flexible limits is to give voters too much freedom to avoid exploitation, which runs against the common wisdom on relaxing term limits.\footnote{Flexible term limits could still prove dangerous in a model where incumbents can build up de facto power over time, so that eventually they can corrupt the referendum, etc.}

If agents are myopic, on the other hand, flexible limits are optimal in a very broad sense: they not only beat stationary limits but also classic and block limits and even the optimal hybrid rule $J^*$ we found above. The reason is that, in terms of selecting good continuations, flexible term limits use the most information, and this does not hurt incentives when agents
are myopic.

Next, we discuss persistent regime change. Equilibrium behavior is similar: voters obtain $V$ from any continuation if the rule $p = 0$ persists, and $V^1(\theta)$ if it changes to $p = 1$, where $\theta$ is the current incumbent’s ability and $V^1(\theta)$ is simply $V(\theta)$ from the game with term limits fixed at $p = 1$. Hence voters will allow regime change when $V^1(\theta) > V$. Since $V(\theta)$ is hump-shaped, this will happen for good incumbents, but not for very good ones if competition is low against high $\theta$.

If agents are strategic, this system is weakly better than fixing either $p = 0$ or $p = 1$: if voters choose to change the regime, it must be beneficial for them. However, if voters are myopic, allowing persistent regime change is disastrous: voters will relax term limits for an incumbent that is good in the near future, but this will yield a long-term welfare of 0 since the electoral process will be captured sooner or later. Hence, in deciding whether term limits should be malleable, the distinction between allowing exceptions and persistently changing the rules is critical.

7 Conclusions

We study the optimal design of term limits in a world where politicians are tempted to choose policies disliked by the median voter, and where challengers are less likely to enter if their probability of winning is low. Although the interplay between term limits and welfare outcomes is complex, the model highlights a few general themes.

First, term limits may be valuable as a tool for disciplining incumbents, even without strategic entry: just by bringing the continuation values of different politicians closer together, term limits can reduce opportunities for exploitation and improve welfare.

Second, the presence of strategic entry usually increases the need for term limits. In particular, when the cost of investing is intermediate, so that challengers are willing to enter open elections but unwilling to face a strong incumbent (a plausible case), full term limits are
more likely to be optimal. However, the result is reversed when the cost is very high: in that case term limits should be relaxed so as to entice politicians to compete for a bigger prize. In turn, these conclusions change substantially if agents are myopic. In that case, strong term limits become comparatively better when there is high entry deterrence, because the social planner is now more concerned about incumbents becoming entrenched, and voters do not protect against this.

Third, the optimal structure of term limits also depends on whether agents are strategic. If they are, stationary limits are optimal, because other schemes—such as classic and block limits—generate complex incentives that lead to perverse behavior. For the popular two-term limit, in particular, an incumbency advantage for good politicians is created: strong incumbents will be unbeatable in their reelection bid, even by equally qualified challengers, which in turn discourages entry and lowers welfare. If agents are indeed strategic, this means that the popularity of classic term limits may be misguided.

In contrast, when agents are myopic, these incentive problems disappear. In this case, the best schemes are simply those that let the good continuations go on while cutting the bad ones short. For this purpose, flexible limits are best because they use the most information, being based on voters’ knowledge of the incumbent’s $\theta$. Block and classic limits are in between: using calendar time or seniority as proxies for $\theta$ is less efficient, but better than nothing. Stationary limits are poor, as they ignore this information. Finally, allowing regime change is the worst option, as voters will fall for it and politicians will never have an incentive to re-establish term limits afterwards.

Our model also sheds light on the welfare effects of making term limits flexible. If voters are impatient, whether relaxing term limits is likely good or bad in practice may depend on whether the change is persistent. According to this logic, exceptions to the rule, like FDR’s four-term tenure in the United States, or the occasional multi-year stints allowed to Roman consuls in times of emergency, are unlikely to be harmful. The opposite is true when
incumbents manage to permanently transform institutions, in a way that increases their bargaining power at the expense of voters.
A Appendix

Proofs

Proof of Proposition 3. For simplicity we consider the case $C \geq C_0$. First note that $V(\theta) \leq U(\theta)$ because $V(\theta) = E[\min(U(\theta), U(\theta'))]$. Then $U(0) = \beta p V(0) + \beta (1 - p) V \leq \beta p U(0) + \beta (1 - p) V$, which implies $U(0) \leq \frac{\beta (1 - p) V}{1 - \beta p}$. On the other hand, let $m = U(\theta^*) = \min U(\theta)$. Then $m = V(\theta^*)$ as well, since $V(\theta^*) = E[\min(m, U(\theta'))] = m$. Then $m = \theta^* + \beta p m + \beta (1 - p) V$, implying $m \geq \frac{\beta (1 - p) V}{1 - \beta p}$. Hence $\theta^* = 0$ and $U(0) = V(0) = \frac{\beta (1 - p) V}{1 - \beta p}$.

Next we show that $U$ is nondecreasing. Suppose that there are $\theta < \theta'$ such that $U(\theta) > U(\theta')$. Then $r(\theta'', \theta) \leq r(\theta'', \theta')$ for all $\theta''$, strictly for some $\theta''$, so $T(\theta) < T(\theta')$ and $q(\theta) \leq q(\theta')$. Hence

$$
\theta + \beta p V(\theta) + \beta (1 - p) V > \theta' + \beta p V(\theta') + \beta (1 - p) V
$$

$$
\implies 0 < \theta' - \theta < \beta p (V(\theta) - V(\theta')) =
$$

$$
= \beta p (E[\min(U(\theta), U(\theta''))|\theta'' \sim q(\theta) F] - E[\min(U(\theta'), U(\theta''))|\theta'' \sim q(\theta') F]) \leq
$$

$$
\leq \beta p (E[\min(U(\theta), U(\theta''))|\theta'' \sim q(\theta) F] - E[\min(U(\theta'), U(\theta''))|\theta'' \sim q(\theta) F]) \leq
$$

$$
\leq \beta p (U(\theta) - U(\theta')) < 0,
$$

a contradiction. \hfill \Box

Proof of Proposition 4. The proof proceeds in two steps. First, we argue that any SSMPE must be of the general form described in the Proposition, for some values of $\theta_0$, $q_0$ and $V$. The second step is to pin down the parameters $\theta_0$, $q_0$ and $V$. 

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Characterization

Denote \( \tilde{U}(\theta) = U(\theta) - U(0) \), \( \tilde{V}(\theta) = V(\theta) - U(0) \), \( W(\theta) = E[\min(U(\theta), U(\theta'))|\theta' \sim F] \) and \( \tilde{W}(\theta) = W(\theta) - U(0) \). Note that \( \tilde{U}(\theta) = \theta + \beta p \tilde{V}(\theta) \).

By Proposition 3, \( U(\theta) \) is nondecreasing. Then, by Lemma 2, \( T(\theta) \) is weakly decreasing in \( \theta \). Furthermore, if \( 0 < q(\theta) < 1 \) within an interval \( A \), then \( T(\theta) = c \) for \( \theta \in A \), so that challengers are indifferent about investing. In turn, this implies that \( U \) is constant in \( A \), for otherwise \( r(\cdot, \theta) \) would be decreasing in \( \theta \), and hence \( T(\theta) \) would be decreasing too.

Since \( U'(\theta) = 1 + \beta p V'(\theta) \), this implies that \( V(\theta) \) and \( q(\theta) \) must decrease linearly within \( A \) at a specific rate: indeed, we must have \( V'(\theta) = \frac{-1}{\beta p} \). Hence

\[
V(\theta) = E[\min(U(\theta), U(\theta'))|\theta' \sim q(\theta)F]
\]
\[
\tilde{V}(\theta) = E[\min(\tilde{U}(\theta), \tilde{U}(\theta'))|\theta' \sim q(\theta)F] = q(\theta)E[\min(\tilde{U}(\theta), \tilde{U}(\theta'))|\theta' \sim F] = q(\theta)\tilde{W}(\theta),
\]

where \( \tilde{W}(\theta) \) is constant in \( \theta \) because \( U(\theta) \) is constant. Thus \( q'(\theta) = \frac{-1}{\beta p V'(\theta_0)} \) for \( \theta \in A \).

Define

\[
\theta_0 = \sup \{ \theta : T(\theta) > c \},
\]

so \( q(\theta) = 1 \) for \( \theta \leq \theta_0 \). Assume \( \theta_0 > 0 \). Then \( q(\theta) \to 1 \) when \( \theta \to \theta_0 \) (otherwise \( q \) would jump down discontinuously, which would make \( U \) decreasing). Hence \( 0 < q(\theta) < 1 \) in some interval \( A = (\theta_0, \theta_1) \) or \( A = (\theta_0, 1] \).

Now, if indeed \( \theta_0 > 0 \) then there are three possible cases. In the first case \( A = [\theta_0, \theta_1] \) with \( \theta_1 < 1 \), so that for \( \theta > \theta_1 \) we will have \( T(\theta) < c \) and thus \( q(\theta) = 0 \), \( \tilde{V}(\theta) = 0 \), \( \tilde{U}(\theta) = \theta \); this gives a type 1 equilibrium. In the second case \( A = [\theta_0, 1] \) and the equilibrium is of type 2. In the third case \( q(\theta) = 1 \) for all \( \theta \), but we will argue later that this is only possible if \( c = 0 \).
Next we show how to calculate $U$ for $\theta < \theta_0$. Note first that

$$W(\theta) = E \left[ \min(U(\theta), U(\theta')) | \theta' \sim F \right] = \int_0^{\theta} U(\theta') f(\theta') d\theta' + U(\theta)(1 - F(\theta))$$

$$\Rightarrow W'(\theta) = U'(\theta)(1 - F(\theta)).$$

For $\theta < \theta_0$, $q(\theta) = 1$ so $W(\theta) = V(\theta)$. Thus

$$U(\theta) = \theta + \beta p V(\theta) + \beta (1 - p)V$$

$$\Rightarrow U'(\theta) = 1 + \beta p U'(\theta)(1 - F(\theta)) \Rightarrow U'(\theta) = \frac{1}{1 - \beta p(1 - F(\theta))}.$$

Then we can calculate $\tilde{U}(\theta)$ by integrating.

In particular, in the case when $q(\theta) = 1$ everywhere, this would imply that $U$ is strictly increasing, which would mean that an incumbent with $\theta = 1$ is invincible, i.e. $T(1) = 0 < c$, a contradiction unless $c = 0$.

Finally, if $\theta_0 = 0$, then $q(\theta) \equiv 0$. Suppose otherwise that $A = [0, \theta_1]$ with $\theta_1 > 0$. Then, as above, $U$ must be constant in $A$, so $\tilde{U}(\theta) = 0$ for all $\theta \in A$. But this is impossible since $\tilde{U}(\theta) \geq \theta$. Thus $q(\theta) \equiv 0$. Moreover, $q_0 = 0$ because incentives for running in an open election are at most as high as for running against an incumbent with $\theta = 0$, and $q(0) = 0$.

Then the equilibrium is type 3: $V = 0$ and $V(\theta) = 0$ for all $\theta$. Note that type 3 equilibria and type 1 or 2 equilibria cannot coexist for given parameters: in a type 3 equilibrium $T(\theta)$ is maximized because no competition is expected in the continuation, and yet running is still not worth it, i.e. $T(0) < c$, while in a type 1 or 2 equilibrium $T(0) \geq c$ even though $T$ is lower because of expected competition.
Pinning down $\theta_0$

Given a value of $\theta_0$, we can calculate an equilibrium candidate $\tilde{U}$, $\tilde{V}$, $q$ and $T$ as above. For this to work, the resulting $T$ must satisfy $T = c$, where $T = T(\theta)$ for $\theta \in A$. Under the conditions provided in the Proposition, $T$ is a decreasing function of $\theta_0$—an intuitive fact since $\theta_0$ parameterizes the level of competition, and expected rents are lower when future competition is high. Hence, there is a unique value of $\theta_0$ such that $T(\theta_0) = c$.

The reason why uniqueness of equilibrium requires parameter conditions is that increasing $\theta_0$ has three effects on $T$, one of which has the "wrong" sign. These effects are:

- It increases future competition (by shifting the function $q(\theta)$ upwards), which lowers expected tenure and rents, reducing $T$.

- It shifts the set $A = (\theta_0, \theta_1)$ upwards, making incumbents with $\theta \in A$ electorally stronger. This reduces $T$ since challengers have a lower chance of winning their first election against someone in $A$.

- Yet, conditional on winning the first election, these challengers-turned-incumbents will be electorally stronger themselves by the same logic, which increases $T$.

In particular, $T$ will be increasing if the third effect dominates. As a result, multiple equilibria may exist when $F$ is badly behaved, when $\gamma$ is high (because policy rents increase with $U(\theta)$), and when $\beta p$ is close to 1 (because the third effect operates through future rents, so it becomes stronger when candidates are patient). However, this is not an issue for reasonable calibrations of the model. For the details see Appendix B.
Pinning down $q_0$ and $V$

To determine $q_0$, we consider the incentive to invest in an open election: if the other party is investing with probability $q_0$, the expected payoff from investing with probability $q$ is

$$q_0 T_{\max}^2 + (1 - q_0)qT(0) + (1 - q_0)(1 - q)\frac{R(0)}{2} - qc.$$  

Here $\frac{T_{\max}^2}{2}$ is the expected reward from investing when the other player also invests, i.e.,

$$T_{\max}^2 = E[R(\bar{\theta}) + \gamma Q(\bar{\theta})]\frac{1}{\gamma} = (\max\{\theta, \theta'\}, \min\{\theta, \theta'\}) : \theta, \theta' \sim F; T(0)$$

is the reward from investing against a challenger that doesn’t; and $R(0)$ are the rents from winning without investing. The derivative of the expected rents with respect to $q$ is

$$q_0 T_{\max}^2 + (1 - q_0)T(0) - (1 - q_0)\frac{R(0)}{2} - c,$$

which we can show is decreasing in $q_0$ (i.e., investing is less profitable when the other challenger invests). To see this, let $T_{\min}^2 = E[R(\bar{\theta}) + \gamma Q(\bar{\theta})]$. Then $T(0) \geq \frac{T_{\max}^2 + T_{\min}^2}{2} > \frac{T_{\max}^2 + R(0)}{2}$.

Then, in equilibrium, there are three possible cases. Either $\frac{T_{\max}^2}{2} > c$ and $q_0 = 1$, or $T(0) - \frac{R(0)}{2} < c$ and $q_0 = 0$, or $\frac{T_{\max}^2}{2} < c < T(0) - \frac{R(0)}{2}$ and there is a unique $q_0$ such that

$$q_0 T_{\max}^2 + (1 - q_0)T(0) - (1 - q_0)\frac{R(0)}{2} = c.$$  

Finally, we can find $V$ explicitly like so, using that $U(0) = \frac{\beta(1-p)}{1-\beta p} V$:

$$V = q_0^2 E[\min(U(\theta), U(\theta'))]\theta, \theta' \sim F] + (1 - q_0)^2 U(0)$$

$$V - U(0) = q_0^2 E[\min(U(\theta'), U(\theta'))]\theta, \theta' \sim F]$$

$$V = \frac{1 - \beta p}{1 - \beta} q_0^2 E[\min(U(\theta), U(\theta'))]\theta, \theta' \sim F] \implies V = \frac{1 - \beta}{1 - \beta p} q_0^2 E[\bar{U}(\theta)\theta \sim H],$$

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where $H = 1 - (1 - F)^2$. Then, integrating by parts,

$$
E \left[ \tilde{U}(\theta) \mid \theta \sim H \right] = \int_0^1 \tilde{U}(\theta)h(\theta)d\theta = \tilde{U}(1) - \int_0^1 U'(\theta)H(\theta)d\theta
$$

$$
= \int_0^1 U'(\theta)(1 - H(\theta))d\theta = \int_0^1 U'(\theta)(1 - F(\theta))^2d\theta
$$

$$
= \int^{\theta_0}_0 \frac{(1 - F(\theta))^2}{1 - \beta p(1 - F(\theta))}d\theta + \int_1^{1 - F(\theta)}d\theta.
$$

Proof of Proposition 5. $T_k(\theta)$ is a decreasing function of $U_k(\theta)$. Then there is a threshold $U^*$ such that (abusing notation) $T(U^*) = c$, so that $q_k(\theta) = 1$ for $U_k(\theta) < U^*$ and $q_k(\theta) = 0$ for $U_k(\theta) > U^*$.

Given $U^*$, $V$ and a candidate function $U_0(\theta)$, we can solve for $U_k(\theta)$ with a form of backward induction. First, $U_{m-1}(\theta) = \theta + \beta V$, which is linear and increasing. If $\theta_{1(m-2)} + \beta V = U^*$ for some $\theta_{1(m-2)}$, then $U_{m-2}(\theta) = \theta + \beta E[\min(U_{m-1}(\theta), U_0(\theta'))]$, $q_{m-2}(\theta) = 1$ for $\theta < \theta_{1(m-2)}$ and $U_{m-2}(\theta) = \theta + \beta U_0(0)$, $q_{m-2}(\theta) = 0$ for $\theta > \theta_{1(m-2)}$.\footnote{This assumes that $U_k(\theta) \geq U^* \Rightarrow U_k(\theta) \geq U_0(0)$, but that can be easily shown. Note that $U_k(0) = \beta V_{k+1}(0) \leq \beta U_{k+1}(0)$. If $U_0(0) > U^*$, then $U_k(0) > U^*$ for all $k$, so $V_{k+1}(0) = U_0(0)$, implying $U_0(0) = 0 \Rightarrow U^* < 0$, a type 3 equilibrium. But in that case $U_k(0) \geq 0 = U_0(0)$ still.} We can proceed like this for $k = m - 3, \ldots, 0$.

This argument doubles as an algorithm for equilibrium construction: pick candidate values of $U^*$, $V$ and $U_0(\theta)$ and solve for an equilibrium as above. Then the resulting strategy profile is an equilibrium if the generated $U^*$, $V$ and $U_0(\theta)$ match the conjectures.

We also obtain the set of cutoffs $A_k$ from this construction. To show that $l_k \leq 2(m - k) - 3$ we argue as follows. Let $h(U)$ be a function given by $h(U) = E[\min(U, U_0(\theta'))\mid \theta' \sim F]$ if $U < U^*$ and $h(U) = U_0(0)$ if $U > U^*$. Define also $h_\theta(U) = \theta + \beta h(U)$. Then $U_{k-1}(\theta) = h_\theta(U_k(\theta))$; in other words the sequence $(U_{m-1}(\theta), U_{m-2}(\theta), \ldots)$ is recursive. Let $a_\theta \geq 2$ be the smallest natural number such that $U_{m-a+1}(\theta) > U^*$, and $b_\theta \geq 3$ the smallest such that $U_{m-b+1}(\theta) > U^*$ and $b_\theta > a_\theta$. These numbers characterize the sequence $(U_{m-1}(\theta), U_{m-2}(\theta), \ldots)$ because, after
$U_k(\theta) > U^*$ occurs, we get $U_{k-1}(\theta) = \theta + \beta U_0(0)$ regardless of the exact value of $U_k(\theta)$; in other words, the sequence is periodic. Furthermore, let $a^k_\theta = \min(a_{\theta}, k+1)$, $b^k_\theta = \min(b_{\theta}, k+1)$. We can check that discontinuities in $U_{m-k}(\theta)$ correspond to changes in $a^k_\theta$ and/or $b^k_\theta$. So we have to argue that $(a^k_\theta, b^k_\theta)$ can only change $2k - 3$ times within $[0, 1]$.

To do this, note that $a^k_\theta$, $b^k_\theta$ are non-increasing in $\theta$ and the values that $(a^k_\theta, b^k_\theta)$ can take are bounded by $(2, 3)$ and $(k+1, k+1)$. Then there can be at most $(k+1-3) + (k+1-2) = 2k-3$ jumps.

\[ \square \]

**Proof of Proposition 7.** When $\alpha > 0$, we can show an analogous equilibrium characterization as in the case $\alpha = 0$. $U(\theta)$ is still nondecreasing, so $T(\theta)$ is still nonincreasing, and there is an interval $A$ of the form $(\theta_0, \theta_1)$ or $(\theta_0, 1)^{20}$ within which $T(\theta) = c$, $U$ is constant and $V'(\theta) = -\frac{1}{\beta p}$. However, there are some differences:

\[
V(\theta) = (1 - \alpha)E[\min(U(\theta), U(\theta'))|\theta' \sim q(\theta)F] + \alpha E[\max(U(\theta), U(\theta'))|\theta' \sim q(\theta)F]
\]

\[
\iff V(\theta) = (1 - 2\alpha)E[\min(U(\theta), U(\theta'))|\theta' \sim q(\theta)F] + \alpha U(\theta) + \alpha E(U(\theta')|\theta' \sim q(\theta)F)
\]

\[
\iff \tilde{V}(\theta) = (1 - 2\alpha)q(\theta)E[\min(\tilde{U}(\theta), \tilde{U}(\theta'))] + \alpha \tilde{U}(\theta) + \alpha q(\theta)E(\tilde{U}(\theta')),
\]

where $\tilde{X}(\theta) = X(\theta) - U(0)$ as before. This implies that, within $A$, $q'(\theta) = -\frac{1}{\beta p(V(\theta_0) - \alpha U(\theta_0))}$. In addition, for $\theta < \theta_0$, now $U'(\theta) = 1 + \beta pV'(\theta) = 1 + \beta p[(1 - 2\alpha)(1 - F(\theta)) + \alpha]U'(\theta)$, so $U'(\theta) = \frac{1}{1 - \beta p[(1 - 2\alpha)(1 - F(\theta)) + \alpha]}. \text{ For } \theta > \theta_1, \tilde{U}'(\theta) = \frac{1}{1 - \beta p}.$

Finally, $U(0) = \beta pV(0) + \beta (1 - p)V$, where $V(0) = \alpha q(0)E(\tilde{U}(\theta')) + U(0)$; hence $U(0) = \frac{\beta p q(0) E(\tilde{U}(\theta')) + \beta (1 - p)V}{1 - \beta p}$.

With all this, finding an expression for $V$ is similar to the case $\alpha = 0$, albeit messier.

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\[20\] For $\alpha > 0$ there is actually a fourth possible type of equilibrium, where $A = (0, \theta_1)$; it is not significantly different from the other cases.
Now
\[
\tilde{V} = (1 - 2\alpha)q_0^2 E\left(\min(\tilde{U}(\theta), \tilde{U}(\theta'))\right) + 2\alpha q_0 E\left(\tilde{U}(\theta)\right)
\]
\[
V = (1 - 2\alpha)q_0^2 E\left(\min(\tilde{U}(\theta), \tilde{U}(\theta'))\right) + 2\alpha q_0 E\left(\tilde{U}(\theta)\right) + \frac{\beta pq_0(0)E(\tilde{U}(\theta'))}{1 - \beta p} + \frac{\beta(1 - p)V}{1 - \beta p}
\]

\[
(1 - \beta)V = (\beta pq_0(0) + 2\alpha q_0(1 - \beta p))\tilde{W} + (1 - 2\alpha)q_0^2(1 - \beta p)\tilde{O},
\]

\[
\tilde{W} = E(\tilde{U}(\theta')|\theta' \sim F) = \int_0^{\theta_0} \frac{1 - F(\theta)}{1 - \beta p(\alpha + (1 - 2\alpha)(1 - F(\theta)))} + \int_{\theta_1}^{1} \frac{1 - F(\theta)}{1 - \beta p\alpha},
\]
\[
\tilde{O} = E(\tilde{U}(\theta')|\theta' \sim 1 - (1 - F)^2) = \int_0^{\theta_0} \frac{(1 - F(\theta))^2}{1 - \beta p(\alpha + (1 - 2\alpha)(1 - F(\theta)))} + \int_{\theta_1}^{1} \frac{(1 - F(\theta))^2}{1 - \beta p\alpha}.
\]

**Proof of Corollary 3.** Note that if \( c < c_0 = \frac{R}{2} \) then \( q_0 = 1 \). The reason is that investing in an open election increases the chance of winning by at least \( \frac{1}{2} \), which is worth at least \( \frac{R}{2} \). Now take some \( c \in (0, c_0) \) and consider \( V(p, c) \). Then \( V_{0,c} = V_{0,0} \) because, when \( p = 0 \), only \( q_0 \) matters for welfare. However, for any \( p_0 > 0 \), \( V_{p_0,c} < V_{p_0,0} \) since \( c > 0 \) results in a lower \( q(\theta) \). Then, since (when \( c = 0 \)) \( p = 0 \) is weakly optimal for any \( \alpha \leq \frac{1}{2} \), when \( c > 0 \) it must be strictly optimal for such \( \alpha \). Thus, by continuity, \( p = 0 \) must be optimal for a bigger interval \([0, \alpha^*(c)] \) where \( \alpha^*(c) > \frac{1}{2} \).

To show the existence of \( c_1 \), we argue that \( \frac{\partial q_0}{\partial p} \) dominates all the other terms in \( \frac{\partial V}{\partial p} \) when \( c \) is high enough. Consider the "worst" case \( \alpha = 0 \). We first show that \( \frac{\partial q_0}{\partial p} \) is bounded below for all \( c \) close enough to \( \bar{c} \) and all \( p \) such that \( q_0 > 0 \). From Proposition 4, we know

\[
q_0 = \frac{T(0) - c - \frac{R(0)}{2}}{T(0) - \frac{T_{\text{max}}}{2} + \frac{R(0)}{2}} \approx \frac{\frac{R}{1 - \beta p} - \frac{R}{2}}{\frac{R}{2(1 - \beta p)} + \frac{R}{2}} \Rightarrow \frac{\partial q_0}{\partial p} \approx \frac{3\beta + 2\beta \frac{c}{R}}{(2 - \beta p)^2}
\]

when \( c \) is close to \( \bar{c} \) (since in that case no future competition is expected). Thus \( \frac{\partial q_0}{\partial p} \) is bounded below by \( \frac{3}{4}\beta + \frac{1}{2}\beta \frac{c}{R} \). In addition, \( q_0 \to 0 \) as \( c \to \bar{c} \). Thus \( \frac{1}{q_0} \frac{\partial q_0}{\partial p} \) becomes arbitrarily
large as $c \to \bar{c}$, for any $p$ such that $q_0 > 0$. To finish, we can check that the other terms in 
\[ \frac{1}{V} \frac{\partial V}{\partial p} \] are bounded.
B Supplementary Appendix – For Online Publication

Proof of Lemma 1. $EU_\alpha(\theta_t, x_t)$ can be written as

$$E \left[ \sum_{t=0}^{\infty} \beta^t \theta_t - \lambda \sum_{t=0}^{\infty} \beta^t (\alpha - x_t)^2 \right] = E \left[ \sum_{t=0}^{\infty} \beta^t \theta_t \right] - \lambda \sum_{t=0}^{\infty} \beta^t [(\alpha - E(x_t))^2 + V(x_t)].$$

Then, given one path $(\theta_t, x_t)_t$, and two voters $\alpha > \alpha'$,

$$EU_\alpha(\theta_t, x_t) - EU_{\alpha'}(\theta_t, x_t) = -\lambda \sum_{t=0}^{\infty} \beta^t [(\alpha - E(x_t))^2 + V(x_t)] + \lambda \sum_{t=0}^{\infty} \beta^t [(\alpha' - E(x_t))^2 + V(x_t)]$$

$$= -\lambda \sum_{t=0}^{\infty} \beta^t (\alpha^2 - \alpha'^2) + 2\lambda (\alpha - \alpha') \sum_{t=0}^{\infty} \beta^t E(x_t).$$

Then the result follows from $\sum_{t=0}^{\infty} \beta^t E(x_t) > \sum_{t=0}^{\infty} \beta^t E(x').$

Proof of Lemma 2. Let $R(\theta)$ be $i$’s expected rents from office conditional on winning his first election, and conditional on his ability being $\theta$. Similarly $Q(\theta)$ are future policy rents, and $p(\theta, \theta', k)$ are policy rents earned in the current election. Then

$$T(\theta', k) = \int_0^1 (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta', k)) r_k(\theta, \theta') f(\theta) d\theta,$$

$$R(\theta) = R \sum_{0}^{\infty} \beta^t y_t, \quad Q(\theta) = \sum_{1}^{\infty} \beta^t y_{t-1}[(1 - q(\theta, t)) x_{0t}(\theta) + q(\theta, t) x_{1t}(\theta)],$$

$$y_t = \prod_{1}^{t} p_s (1 - q(\theta, s) \kappa_s(\theta)), \quad \kappa_s(\theta) = \int_0^1 r_s(\theta, \theta') d\theta.$$

Here $y_t$ $(t \geq 1)$ is the probability that $i$ will win his first $t$ reelection bids; $\kappa_s(\theta)$ is the probability that an incumbent $(\theta, s)$ will lose against a challenger who invests; and $x_{1t}(\theta)$, $x_{0t}(\theta)$ are expected policy rents against a challenger who invests or does not invest, respectively. Note that $R(\theta)$ and $Q(\theta)$ are independent of $(\theta', k)$: the challenger’s rents depend only on $\theta$, conditional on defeating the incumbent.
We want to show that $T(\theta', k)$ is decreasing in $U_k(\theta')$. From Proposition 2, $r_k(\theta, \theta')$ is decreasing in $U_k(\theta')$. Then all that remains is to show that $p(\theta, \theta', k)$ is also decreasing in $U_k(\theta')$. If $U_0(\theta) < U_k(\theta')$ then $p(\theta, \theta', k) = 0$. If $U_0(\theta) > U_k(\theta')$ and $\sqrt{\frac{U_0(\theta) - U_k(\theta')}{\lambda}} < C$,

$$p(\theta, \theta', k) = 2C \sqrt{\frac{U_0(\theta) - U_k(\theta')}{\lambda}} - \frac{U_0(\theta) - U_k(\theta')}{\lambda},$$

which is decreasing in $U_k(\theta')$ and goes from 0 to $C^2$. If $\sqrt{\frac{U_0(\theta) - U_k(\theta')}{\lambda}} \geq C$ then $p(\theta, \theta', k) = C^2$, so the result holds.

**Proof of Proposition 4—Pinning down $\theta_0$.** Under stationary limits, the expressions for $R$ and $Q$ simplify to

$$R(\theta) = \frac{R}{1 - \beta p(1 - q(\theta)\kappa(\theta))} = \frac{R}{1 - \beta p + \beta pq(\theta)\kappa(\theta)},$$

$$Q(\theta) = [(1 - q(\theta))C^2 + q(\theta)(2C x(\theta) - x^2(\theta))] \frac{\beta p}{1 - \beta p + \beta pq(\theta)\kappa(\theta)},$$

where $\kappa(\theta) = \int_0^1 r(\theta', \theta)f(\theta')d\theta'$. Remember also that

$$\bar{T}_{\theta_0} = \int_0^1 (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta_0)) r(\theta, \theta_0)f(\theta) d\theta.$$

Suppose first that the equilibrium is of type 1, and let $\theta_1 = \theta_1(\theta_0)$. Then $r(\theta, \theta_0) = 0$ for $\theta < \theta_0$, $r(\theta, \theta_0) = \frac{1}{2}$ for $\theta \in [\theta_0, \theta_1]$ and $r(\theta, \theta_0) = 1$ for $\theta > \theta_1$:

$$\bar{T}_{\theta_0} = \frac{1}{2} \int_{\theta_0}^{\theta_1} (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta_0)) f(\theta) d\theta + \int_{\theta_1}^1 (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta_0)) f(\theta) d\theta.$$

Then

$$\frac{\partial T_{\theta_0}}{\partial \theta_0} = \frac{1}{2} \int_{\theta_0}^{\theta_1} (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta_0)) f(\theta) d\theta + \int_{\theta_1}^1 (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta_0)) f(\theta) d\theta$$

$$- \frac{1}{2} (R(\theta_0) + \gamma Q(\theta_0) + \gamma p(\theta_0, \theta_0)) f(\theta_0) - \frac{1}{2} \theta'_1(\theta_0)(R(\theta_1) + \gamma Q(\theta_1) + \gamma p(\theta_1, \theta_0)) f(\theta_1)$$

$$49$$
where $R_* = \frac{\partial R}{\partial \theta_0}$ and so on. We want to show that $\frac{\partial T_{\theta_0}}{\partial \theta_0} < 0$.

The third and fourth terms are negative, since $\theta'_1(\theta_0) > 0$ (because $q'_* > 0$). Note also that: $R_*(\theta) = 0$ for $\theta > \theta_1$ (because $q(\theta)\kappa(\theta) \equiv 0$); $Q_*(\theta) = 0$ for $\theta > \theta_1$ (because raising $\theta_0$ has no effect on $U(\theta) - U(0)$); $p_*(\theta, \theta_0) < 0$ for $\theta > \theta_1$ (because raising $\theta_0$ increases $U|_A$ but not $U(\theta)$); and $p_*(\theta, \theta_0) = 0$ for $\theta \in A$ (because $\theta, \theta' \in A$ implies $U(\theta) = U(\theta')$). Thus, it is enough to show that

$$\int_{\theta_0}^{\theta_1} (R_*(\theta) + \gamma Q_*(\theta)) f(\theta) \, d\theta < (R(\theta_0) + \gamma Q(\theta_0)) f(\theta_0) + \theta'_1(\theta_0) (R(\theta_1) + \gamma Q(\theta_1) + \gamma p(\theta_1, \theta_0)) f(\theta_1)$$

We can calculate

$$R_*(\theta) = -\frac{R}{(1 - \beta p + \beta pq(\theta)\kappa(\theta))^2}\beta pq(\theta)\kappa(\theta) \left( \frac{q_*(\theta)}{q(\theta)} + \frac{\kappa_*(\theta)}{\kappa(\theta)} \right),$$

where

$$q_*(\theta) \geq \frac{1}{\theta_1 - \theta_0} \geq \frac{1}{1 - \theta_0}; \kappa(\theta) = 1 - \frac{F(\theta_0) + F(\theta_1)}{2} \implies \kappa_*(\theta) = -\frac{f(\theta_0) + f(\theta_1)\theta'_1(\theta_0)}{2}.$$

Here the bound on $q_*$ follows from $\theta'_1(\theta_0) > 1$, which is easy to show. We can also bound

$$-\frac{\kappa_*(\theta)}{\kappa(\theta)} \leq \frac{f(\theta_0) + f(\theta_1)\theta'_1(\theta_0)}{1 - F(\theta_0)}.$$

Then the integral of the second term is bounded by $\theta'_1(\theta_0) R(\theta_1)$, i.e.

$$\int_{\theta_0}^{\theta_1} \frac{R}{(1 - \beta p + \beta pq(\theta)\kappa(\theta))^2}\beta pq(\theta)\kappa(\theta) \frac{f(\theta_1)\theta'_1(\theta_0)}{(1 - F(\theta_0))} f(\theta) < \theta'_1(\theta_0) R(\theta_1) f(\theta_1),$$

50
because $\frac{\beta pq(\theta)\kappa(\theta)}{1-\beta p+\beta pq(\theta)\kappa(\theta)} < 1$. As for policy payoffs,

$$Q(\theta) = [q(\theta)x_1 + (1-q(\theta))x_0] \frac{1}{1-\beta p + \beta pq(\theta)\kappa(\theta)}$$

$$\implies Q_*(\theta) = \frac{q(\theta)x_1 + (1-q(\theta))x_0}{(1-\beta p + \beta pq(\theta)\kappa(\theta))^2} \beta pq(\theta)\kappa(\theta) \left( \frac{q_* (\theta)}{q(\theta)} + \frac{\kappa_* (\theta)}{\kappa(\theta)} \right) - \frac{q_* (\theta)(x_0 - x_1)}{1-\beta p + \beta pq(\theta)\kappa(\theta)} + \frac{q(\theta)x_1 + (1-q(\theta))x_0}{1-\beta p + \beta pq(\theta)\kappa(\theta)},$$

where

$$x_0 = 2C \sqrt{\frac{\bar{U}(\theta_0)}{\lambda} - \frac{\bar{U}(\theta_0)}{\lambda}}, \quad x_1 = \int_0^{\theta_0} \left( 2C \sqrt{\frac{\bar{U}(\theta_0) - \bar{U}(\theta)}{\lambda} - \frac{\bar{U}(\theta_0) - \bar{U}(\theta)}{\lambda}} \right) f(\theta) d\theta,$$

$$x_{0*} = U'(\theta_0) \left[ C \sqrt{\frac{1}{U(\theta_0)\lambda} - \frac{1}{\lambda}} \right], \quad x_{1*} = U'(\theta_0) \left[ \int_0^{\theta_0} \left( C \sqrt{\frac{1}{(U(\theta_0) - \bar{U}(\theta))\lambda} - \frac{1}{\lambda}} \right) f(\theta) d\theta \right].$$

To bound the first term of $Q_*(\theta)$, note that $\frac{q_* (\theta)}{\kappa(\theta)}$ breaks down in the same way as before, and the second part can be handled analogously. Moreover, we can bound $x_0, x_1 \leq C^2$. The second term of $Q_*(\theta)$ is negative so we can ignore it. Finally, we can bound $x_{0*}$ and $x_{1*}$ like so: $\bar{U}(\theta_0) \geq \bar{V}(\theta_0) = -\frac{1}{\beta pq(\theta)} = \frac{\theta_1 - \theta_0}{\beta p}$ and $U'(\theta) \leq \frac{1}{1-\beta p}$, hence

$$x_{0*} \leq \frac{C}{1-\beta p} \sqrt{\frac{\beta p}{\theta_1 - \theta_0} \lambda - \frac{1}{(1-\beta p)\lambda}} < \frac{C}{(1-\beta p)\sqrt{\lambda}} \sqrt{\frac{\beta p}{\theta_1 - \theta_0}}.$$

On the other hand, $U$ is concave so $\bar{U}(\theta_0) - \bar{U}(\theta) \geq U'(\theta_0)(\theta_0 - \theta) \geq \theta_0 - \theta$, thus

$$(1-\beta p)x_{1*} \leq \int_0^{\theta_0} \left( C \sqrt{\frac{1}{(\theta_0 - \theta)\lambda} - \frac{1}{\lambda}} \right) f(\theta) d\theta \leq \int_0^{\theta_0} C \sqrt{\frac{1}{(\theta_0 - \theta)\lambda} \bar{f} d\theta} = \frac{2C^2 \sqrt{\theta_0}}{\sqrt{\lambda}} \leq \frac{2C^2}{\sqrt{\lambda}}.$$

So we have to show that
\[
\int_{\theta_0}^{\theta_1} \left[ (R + \gamma C^2) \frac{\beta pq(\theta) \kappa(\theta)}{(1 - \beta p + \beta pq(\theta) \kappa(\theta))^2} \left( \frac{1}{1 - \theta(\theta)} f(\theta) + \frac{\frac{\gamma}{1 - \theta} \kappa(\theta)}{1 - \theta + \beta pq(\theta) \kappa(\theta)} \right) \right. \\
\left. + \frac{\gamma C q(\theta) 2 \bar{f} + (1 - q(\theta)) \sqrt{\frac{\beta p}{\theta_1 - \theta_0}}}{(1 - \beta p) \sqrt{\lambda}} \right] \bigg| f(\theta) < R(\theta_0) f(\theta_0).
\]

Using that \( f(\theta_0) \leq \phi \frac{1 - F(\theta_0)}{1 - \theta_0} \), it is enough to show that for any \( 0 \leq q \leq 1 \)

\[
\left[ \frac{(R + \gamma C^2) \beta pq \kappa}{(1 - \beta p + \beta pq \kappa)^2} \left( 1 - \frac{1}{\phi q} \right) \frac{f(\theta_0)}{1 - F(\theta_0)} + \frac{\gamma C q 2 \bar{f} + (1 - q) \sqrt{\frac{\beta p}{\theta_1 - \theta_0}}}{\sqrt{\lambda} (1 - \beta p)(1 - \beta p + \beta pq \kappa)} \right] < \\
\frac{1}{F(\theta_1) - F(\theta_0)} \frac{R f(\theta_0)}{1 - \beta p + \beta pq \kappa}
\]

\[
\frac{(R + \gamma C^2) \beta pq \kappa}{(1 - \beta p + \beta pq \kappa)^2} \left( 1 - \frac{1}{\phi q} \right) + \frac{\gamma C}{\sqrt{\lambda}} \frac{F(\theta_1) - F(\theta_0)}{f(\theta_0)} \frac{q^2 \bar{f} + (1 - q) \sqrt{\frac{\beta p}{\theta_1 - \theta_0}}}{(1 - \beta p)^2} < \frac{R}{1 - \beta p + \beta pq \kappa},
\]

\[
\frac{(R + \gamma C^2) \beta pq \kappa}{(1 - \beta p + \beta pq \kappa)^2} \left( 1 - \frac{1}{\phi q} \right) + \frac{\gamma C \beta p \frac{q^2 \bar{f} + (1 - q) \sqrt{\beta p \bar{f}}}{f \sqrt{\lambda}}}{(1 - \beta p)^2} < \frac{R}{1 - \beta p + \beta pq \kappa}.
\]

Now \( \frac{\beta pq \kappa}{(1 - \beta p + \beta pq \kappa)^2} \left( 1 - \frac{1}{\phi q} \right) \) is single peaked in \( q \) with a maximum at \( q^* = \frac{1 - \beta p + \beta pq \kappa}{\phi} \). If this \( q^* \) is greater than 1, then we need

\[
\frac{(R + \gamma C^2) \beta pq \kappa}{(1 - \beta p + \beta pq \kappa)^2} \left( 1 - \frac{1}{\phi} \right) + \frac{\gamma C}{\sqrt{\lambda}} \frac{2 \bar{f} + \sqrt{\beta p \bar{f}}}{f \sqrt{\lambda} (1 - \beta p)^2} < \frac{R}{1 - \beta p + \beta pq \kappa},
\]

which holds for any \( \phi \) if \( \gamma \) is small enough. If \( 0 < q^* < 1 \), then the maximized value is

\[
\frac{1}{4(1 - \beta p + \beta pq \kappa)},
\]

and we need

\[
\frac{(R + \gamma C^2)}{4 \left( 1 - \beta p + \frac{\beta pq \kappa}{\phi} \right)} + \frac{\gamma C}{f \sqrt{\lambda}} \frac{2 \bar{f} + \sqrt{\beta p \bar{f}}}{(1 - \beta p)^2} < \frac{R}{1 - \beta p + \beta pq \kappa}.
\]
which holds for small enough \( \gamma \) if

\[
\beta p < \frac{1}{1 + \frac{\phi}{3} \left( 1 - \frac{4}{\phi} \right)},
\]

in particular if \( \phi < 4 \). If the equilibrium is type 2, then

\[
\overline{T}_{\theta_0} = \frac{1}{2} \int_{\theta_0}^{1} (R(\theta) + \gamma Q(\theta) + \gamma p(\theta, \theta_0)) f(\theta) d\theta
\]

\[
\frac{\partial \overline{T}_{\theta_0}}{\partial \theta_0} = \frac{1}{2} \int_{\theta_0}^{1} (R_*(\theta) + \gamma Q_*(\theta) + \gamma p_*(\theta, \theta_0)) f(\theta) d\theta - \frac{1}{2} (R(\theta_0) + \gamma Q(\theta_0) + \gamma p(\theta_0, \theta_0)) f(\theta_0)
\]

And we have to show that

\[
\int_{\theta_0}^{1} (R_*(\theta) + \gamma Q_*(\theta)) f(\theta) d\theta < (R(\theta_0) + \gamma Q(\theta_0)) f(\theta_0).
\]

We now have

\[
q_*(\theta) \geq \frac{1 - q(1)}{1 - \theta}, \quad \kappa(\theta) = \frac{1 - F(\theta_0)}{2} \implies \kappa_*(\theta) = -\frac{1}{2} f(\theta_0), \quad -\frac{\kappa_*(\theta)}{\kappa(\theta)} \leq \frac{f(\theta_0)}{1 - F(\theta_0)}.
\]

Note that the bound on \( q_* \) is weaker in this case. Arguing as before, we have to show

\[
\left[ \frac{(R + \gamma C^2) \beta pq \kappa}{(1 - \beta p + \beta pq)q} \left( 1 - \frac{1 - q(1)}{\phi q} \right) \frac{f(\theta_0)}{1 - F(\theta_0)} + \frac{\gamma C q^2 f + (1 - q) \sqrt{\beta p}}{1 - \beta p} \right] < R f(\theta_0)
\]

subject to \( q \geq q(1) \). It is enough to show

\[
\frac{(R + \gamma C^2) \beta pq \kappa}{(1 - \beta p + \beta pq)q} \left( 1 - \frac{1 - q(1)}{\phi q} \right) + \frac{\gamma C q^2 f + \sqrt{\beta p f}}{f \sqrt{\lambda} (1 - \beta p)^2} < \frac{R}{1 - \beta p + \beta pq}.
\]
Again $\frac{\beta pq \kappa}{(1-\beta p + \beta p q \kappa)^2} \left(1 - \frac{1-q(1)}{\phi q} \right)$ is single peaked in $q$ with a maximum at $q^* = \frac{1-\beta p}{\beta p k} + \frac{2(1-q(1))}{\phi}$. There are three cases. If $q^* > 1$, then we need

$$(R + \gamma C^2) \frac{\beta pk}{(1-\beta p + \beta pk)^2} \left(1 - \frac{1-q(1)}{\alpha} \right) + \frac{\gamma C}{f \sqrt{\lambda}} \frac{2f + \sqrt{\beta p f}}{1-\beta p} < \frac{R}{1-\beta p + \beta pk},$$

which holds for any $\phi$ if $\gamma$ is small enough. If $1 > q^* > q(1)$, then $q^* > \frac{1-\beta p}{\beta p k} + \frac{2}{\phi} > q(1)$, and

$$\frac{\beta pq^* \kappa}{(1-\beta p + \beta pq^* \kappa)^2} \left(1 - \frac{1-q(1)}{\phi} \right) =$$

$$= \frac{1}{4 \left(1-\beta p + \frac{\beta pk}{\phi} (1-q(1)) \right)} < \frac{1}{4 \left(1-\beta p + \frac{\beta pk}{\phi} \left(1 - \frac{1-\beta p}{\phi + 2} + \frac{3}{\phi + 2} \right) \right)} =$$

$$= \frac{1}{4 \left((1-\beta p) \left(1 - \frac{1}{\phi + 2} \right) + \frac{\beta pk}{\phi} \left(1 - \frac{2}{\phi + 2} \right) \right)}$$

which is smaller than $\frac{1}{1-\beta p + \beta pk}$ if $\phi < 2$, so the main result holds for small enough $\gamma$.

Finally, if $q(1) > q^*$, then for small enough $\gamma$ we need

$$\frac{\beta pq(1) \kappa}{(1-\beta p + \beta pq(1) \kappa)^2} \left(1 - \frac{1-q(1)}{\phi q(1)} \right) < \frac{1}{1-\beta p + \beta pk}.$$
This quadratic inequality has roots
\[ q_{1,2} = \frac{\phi + 1}{2\phi} (z + 1) - z \pm \sqrt{\left( \frac{\phi + 1}{2\phi} (z + 1) - z \right)^2 - \left( z^2 + \frac{z + 1}{\phi} \right)} \]

It can be shown that these roots are either complex, or real and in \([0, 1]\).\(^{30}\) To make sure the above inequality holds without knowing \(q(1)\), we need complex solutions, i.e.

\[ \left( \frac{\phi + 1}{2\phi} (z + 1) - z \right)^2 < z^2 + \frac{z + 1}{\phi} \iff \phi^2(1 - 3z^2 - 2z) < (2\phi - 1)(z + 1)^2 \]

Let \(K = 1 - 3z^2 - 2z, \quad L = (z + 1)^2\). Then the inequality holds for \(\phi < \phi^*\), where

\[ \phi^* = \frac{L}{K} + \sqrt{\frac{L^2}{K^2} - \frac{L}{K}} > 1. \]

\[ \text{Proof of Corollary 2.} \quad \]

First, we characterize the solution from Proposition 5 as a function of \(U^*\) and \(V\). \(U_1(\theta) = \theta + \beta V\) is increasing. If \(U^*\) is between \(U_1(0)\) and \(U_1(1)\), there is a cutoff \(\theta_0\), given by \(U^* = U_1(\theta_0)\), such that \(q_1(\theta) = 1\) for \(\theta \in [0, \theta_0)\) and \(q_1(\theta) = 0\) for \(\theta \in (\theta_0, 1]\).

Hence \(V_1(\theta) = E(\min(U_1(\theta), U_0(\theta'))|\theta' \sim F)\) for \(\theta < \theta_0\) (note that this expression increases in \(\theta\)) since \(U_1\) is increasing), but \(V_1(\theta) = E(\min(U_1(\theta), U_0(0)))\) for \(\theta > \theta_0\). We now argue that \(U_0(0) < U_1(\theta)\) for all \(\theta\). This follows since \(U_0(0) = \beta V_1(0) \leq V_1(0)\) and \(V_1(0) = E(\min(U_1(0), U_0(\theta'))|\theta' \sim F) \leq U_1(0)\), and \(U_1\) is increasing. (Note that \(V, U_0, U_1, V_1 \geq 0\) everywhere, since electing the weaker candidate always yields a nonnegative flow payoff.) Hence \(V_1(\theta) = U_0(0)\) for \(\theta > \theta_0\). Thus \(U_0\) is increasing in \([0, \theta_0)\) and \((\theta_0, 1]\). It also follows that \(U_0(0) \leq V\), as \(U_0(0) = \beta V_1(0) \leq \beta U_1(0) = \beta^2 V\). Hence \(U_1(\theta) \geq U_0(\theta)\) for \(\theta > \theta_0\), as

\(^{30}\)If \(\left( \frac{\phi + 1}{2\phi} (z + 1) - z \right)^2 < \left( z^2 + \frac{z + 1}{\phi} \right)\) then the solutions are complex. If not, the solutions are real. They are positive because, if \(\frac{\phi + 1}{2\phi} (z + 1) - z < 0\), then \(\left( \frac{\phi + 1}{2\phi} (z + 1) - z \right)^2 > \left( z^2 + \frac{z + 1}{\phi} \right)\) is impossible. The small solution is smaller than \(\frac{\phi + 1}{2\phi} (z + 1) - z\), and this is \(\leq 1\) for \(\phi \geq 1\); then the other solution is also less than 1 because the inequality always holds at 1; thus they are both in \([0, 1]\).
\( V \geq V_1(\theta) = U_0(0) \) for \( \theta > \theta_0 \). In other words, there is incumbency advantage above \( \theta_0 \); note that the inequality is strict unless \( V = 0 \), which happens iff \( q_0 = 0 \).

Next, we show that \( V_1(\theta) \) has a discrete drop at \( \theta_0 \) (and hence \( U_1(\theta) \) does as well). To see this, suppose for the sake of contradiction that \( U_0(\theta_0 - \epsilon) \leq U_0(\theta_0 + \epsilon) \) for arbitrarily small \( \epsilon > 0 \). Then, since \( U_0 \) is increasing elsewhere, it follows that \( U_0 \) is increasing everywhere, so it is minimized at 0. Hence

\[
V_1(\theta_0 - \epsilon) = E(\min(U_1(\theta_0 - \epsilon), U_0(\theta_0 + \epsilon) | \theta' \sim F)) > E(\min(U_1(\theta_0 - \epsilon), U_0(0))) \approx E(\min(U_1(\theta_0 + \epsilon), U_0(0))) = V_1(\theta_0 + \epsilon),
\]
a contradiction.

Finally, \( U'_0(\theta) > U'_1(\theta) \) for \( \theta < \theta_0 \), since \( V_1(\theta) \) is increasing for \( \theta < \theta_0 \), while the continuation value of an incumbent is constant at \( V \). This may result in a region of incumbency disadvantage \( (\theta, \theta_0) \) or incumbency advantage everywhere, depending on parameters.

There are two degenerate cases. If \( U^* \) is above \( U_1(1) \), there always is competition. This is possible in under classic limits if \( c \) is low enough, since in an open election there is always a positive probability of winning, and in a closed election the challenger can always defeat the incumbent with significant probability when \( \theta \) is just above \( \theta_0 \). If \( U^* \) is below \( U_1(0) \), there never is competition in a closed election. This is possible if \( c \) is high enough.

\( \square \)

Proof of Proposition 8. When \( \beta = 0 \), \( \bar{U}(\theta) = \theta \) and there is a cutoff \( \theta_0 \) such that \( q(\theta) = 1 \) for \( \theta < \theta_0 \), \( q(\theta) = 0 \) for \( \theta > \theta_0 \) (this is a degenerate case of Proposition 4). \( \theta_0 \) is given by
\[
R(1 - F(\theta_0)) = c. \quad q_0 = 1 \text{ if } R > 2c \text{ and } 0 \text{ if } R < 2c.
\]

We first find the flow payoff \( v(\theta) \) of an election where the incumbent has ability \( \theta \):
\[
v(\theta) = E[\min(\theta, \theta') | \theta' \sim F] = \int_0^\theta (1 - F(\theta')) \text{ if } \theta < \theta_0, \text{ and } v(\theta) = 0 \text{ if } \theta > \theta_0.
\]

Then the flow payoffs \( v_t \) are given by \( v_t = \int_0^\theta f_t(\theta) v(\theta) \), where \( f_t \) is the density of the incumbent’s \( \theta \) at time \( t \). Equivalently, this can be written as \( v_t = \int_0^\theta (F_t(\theta_0) - F_t(\theta)) v'(\theta) = \)

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\[ f_0^\theta (F_t(\theta_0) - F_t(\theta))(1 - F(\theta)) \] and \( v_0 = q_0^2 \int_0^1 (1 - F(\theta))^2 \). Finally, note that \( F_t(\theta) = F^{t+1}(\theta) \) for \( \theta < \theta_0 \).

This is enough to characterize the payoffs of stationary and block limits. Moreover, it can be shown that \((v_t)_{t \geq 1}\) is a quasiconcave sequence, so we can choose the optimal \( m^* \) for block limits as shown in Figure 7. Finally, block limits outdo stationary ones by the argument in Subsection 5.2.

**Proposition 11 (Generalized Term Limits).** If \( \overline{\beta} = 1 \) and \( \beta = 0 \), there is an optimal \( J^* \) which only conditions on seniority and time since an open election. In other words, \( J^* \) is given by \( m^*(t) \) such that \( J^*_t(h_t) = 0 \) if the incumbent’s seniority at the end of \( h_t \) is \( m^*(t) \) or greater, and \( 1 \) otherwise.

**Proof of Proposition 11.** First note that non-deterministic rules are weakly dominated. Informally, given a non-deterministic \( J \), and a history \( H \) such that \( 0 < J(H) < 1 \), increase \( J(H) \) to \( 1 \) if the continuation after \( H \) has flow payoffs greater than \( V \), and decrease \( J(H) \) to \( 0 \) otherwise; repeat for all such \( H \). This yields a deterministic \( \tilde{J} \) that pays at least \( V \). Thus we can focus on deterministic rules. For a deterministic \( J \),

\[
V = \frac{\int_0^1 (1 - F(\theta))^2 + \int_0^\theta (\sum_H f_H(\theta)) v(\theta)}{1 + \int_0^1 (\sum_H f_H(\theta))},
\]

where \( H \) spans the histories allowed to continue by \( J \), and \( f_H(\theta) \) is the density of the incumbent’s ability after history \( H \), assuming no term limits \((f_H \) is not normalized, i.e., \( \int_0^1 f_H \) is the probability that \( H \) is realized and the incumbent is allowed to run again). Next we aim to find the densities \( f_H \). Let \( t = |H| \) (time since an open election) and \( s \) be the number of \( R \)'s at the end of \( H \) (i.e., the incumbent’s seniority plus one). Then we can check
\[ f_0(\theta) = 2F(\theta)f(\theta), \quad F_0(\theta) = F^2(\theta), \]

\[
f_{HR}(\theta) = \begin{cases} 
   f_H(\theta)F(\theta) & \text{if } \theta < \theta_0 \\
   f_H(\theta) & \text{if } \theta > \theta_0 
\end{cases}
\]

\[
f_{HN}(\theta) = \begin{cases} 
   F_H(\theta)f(\theta) & \text{if } \theta < \theta_0 \\
   F_H(\theta_0)f(\theta) & \text{if } \theta > \theta_0 
\end{cases}
\]

Hence we can find the densities \( f_H \) recursively:

\[
f_H(\theta) = \begin{cases} 
   \frac{2}{\prod_{H_k=N(k+1)} F^{t+1}(\theta_0)f(\theta)} & \text{if } \theta < \theta_0 \\
   \frac{2}{\prod_{H_k=N(k+1)} F^{t-l+1}(\theta_0)f(\theta)} & \text{if } \theta > \theta_0 \text{ and } s < t \\
   2F(\theta)f(\theta) & \text{if } \theta > \theta_0 \text{ and } s = t 
\end{cases}
\]

\[
F_H(\theta) = \begin{cases} 
   \frac{2}{(t+2)\prod_{H_k=N(k+1)} F^{t+2}(\theta)} & \text{if } \theta < \theta_0 \\
   \frac{2}{(t+2)\prod_{H_k=N(k+1)} (F^{t+2}(\theta_0) + (t + 2)F^{t-l+1}(\theta_0)(1 - F(\theta_0)))} & \text{if } \theta = 1 \text{ and } s < t \\
   \frac{2F^{t+2}(\theta_0)}{t+2} + 1 - F(\theta_0)^2 & \text{if } \theta = 1 \text{ and } s = t 
\end{cases}
\]

We can see that \( \frac{f_H(\theta)}{F_H(1)} \) depends only on \( t \) and \( s \), not on the entire history \( H \). In other words, histories \( H, H' \) with equal \( t \) and \( s \) have proportional densities and generate the same normalized payoffs. Note that the optimal \( J \) must satisfy the rule: if the normalized payoffs on the continuation after \( H \) are greater than \( V \), then \( J(H) = 1 \); otherwise \( J(H) = 0 \). Then \( J(H) = 1 \) iff \( J(H') = 1 \), i.e., \( J \) only conditions on \( t \) and \( s \).
References


Aristotle, Athenian Constitution, Chapter 22.


