Agency Business Cycles*

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Abstract

We develop a theory of endogenous and stochastic fluctuations in aggregate economic activity. Individual firms choose to randomize over firing or keeping workers who performed poorly in the past to give them an ex-ante incentive to perform. Different firms choose to correlate the outcome of their randomization to reduce the probability with which they fire non-performing workers. Correlated randomization leads to aggregate fluctuations. Aggregate fluctuations are endogenous—they emerge because firms choose to randomize and they choose to randomize in a correlated fashion—and they are stochastic—they are the manifestation of a randomization process. The hallmark of a theory of endogenous and stochastic fluctuations is that the stochastic process for aggregate "shocks" is an equilibrium object.

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1 Introduction

What causes cyclical fluctuations in economic activity? This is one of the central questions in macroeconomics and naturally it has attracted a great deal of attention from both theorists and empiricists. One explanation for cyclical fluctuations is that the economy is subject to aggregate shocks to its fundamentals (see, e.g, Kydland and Prescott 1983). These can be economy-wide shocks to current productivity, to the productivity expected in the future, to the preferences of market participants, to the tax system, etc... A second explanation for cyclical fluctuations is that the economic system admits multiple equilibria and there are shocks to the equilibrium selected by market participants (see, e.g., Benhabib and Farmer 1994). Equilibrium selection shocks can often be described as shocks to the markets' expectations about the future value of some equilibrium outcome such as aggregate demand, unemployment, etc... According to these two theories, cyclical fluctuations are *exogenous and stochastic*, in the sense that they are driven by an exogenously given stochastic process for the shocks to fundamentals or to the selection of equilibrium. The main criticism to these theories is that they leave the driving force of business cycles completely unexplained. A rather different view of cyclical fluctuations is that the economic system does not tend towards a state of stasis—where the extent of economic activity remains constant over time—but it naturally oscillates between periods of high and low activity (see, e.g., Boldrin and Woodford, 1990). According to the third theory, cyclical fluctuations are *endogenous and deterministic*. The main criticism to this theory is that business cycles do not appear to follow a deterministic pattern.

In this paper, we develop a theory of *endogenous and stochastic* business cycles. The structure of our theory is simple: individual agents find it optimal to randomize over some choice in order to overcome a non-convexity in their decision problem, and different agents find it optimal to correlate the outcome of their randomization. Aggregate fluctuations are endogenous because they are an equilibrium outcome: individual agents *choose* to randomize over some decision (which endogenously creates individual uncertainty) and they *choose* to correlate the outcome of their randomization (which endogenously generates aggregate uncertainty from individual uncertainty). Aggregate fluctuations are stochastic because they are the manifestation of aggregate uncertainty. The unique feature of our theory—and more generally the hallmark of a theory of endogenous and stochastic fluctuations—is that the *stochastic process for "shocks" is endogenous* and determined by fundamentals and by the state of the economy.

While the structure of our theory is quite general, we illustrate it in the context of a search-theoretic model of the labor market in the spirit of Pissarides (1985) and Mortensen and Pissarides (1994). We consider a labor market populated by identical risk-averse workers and by identical risk-neutral firms. Unemployed workers and vacant firms come together through a frictional search process which is described by a matching function. Once matched, firm-worker pairs Nash bargain over the terms of an employment contract and produce output. Production is subject to moral hazard, as the firm does not observe the worker's effort but only his output, which is a noisy measure of effort. The employment contract allocates the gains from trade between the firm and the worker and tries to overcome the moral hazard problem. In particular, the employment contract specifies the level of effort recommended to the worker, the wage paid to the worker, and the probability with which the worker is fired conditional on the output of the worker and possibly on the realization of a publicly observed sunspot.

Two assumptions are critical for our theory. First, we need employment contracts to be incomplete enough that firms need to threaten non-performing workers with a firing lottery in order to give them an incentive to provide effort. Specifically, we assume that firms pay the current period's wages before observing the worker's output and that firms and workers renegotiate future wages in every period. Under these assumptions, firms need to randomize over firing or keeping non-performing workers in order to give them any incentive to exert effort. Second, we need the value to a worker from being unemployed is decreasing in the unemployment rate. Specifically, we assume that the matching process between unemployed workers and vacant firms features decreasing returns to scale.

In the first part of the paper, we characterize the properties of the optimal employment contract. We show that the optimal contract is such that the worker is fired only when the output of the worker is low and the realization of the sunspot is such that the continuation gains from trade accruing to the worker are highest relative to the continuation gains from trade accruing to the firm. The result is intuitive. Firing is costly—as it destroys a valuable firm-worker relationship—but it is necessary—as it is the only way for the firm to give the worker an incentive to exert effort. However, when firing takes place, it is only the value of the destroyed relationship that would have accrued to the worker that gives incentives. The value of the destroyed relationship that would have accrued to the firm is just collateral damage. The optimal contract minimizes the collateral damage by loading all the firing probability on the realizations of the sunspot for which the worker's continuation gains from trade are highest relative to the firm's. In other words, the optimal contract loads the firing probability on the states of the world where the cost to the worker from losing the job is highest relative to the cost to the firm from losing the worker.

In the second part of the paper, we characterize the equilibrium relationship between the realization of the sunspot and firing. We find that there exists an equilibrium in which every firm fires non-performing workers for a given subset of realization of the sunspot and every firm keeps non-performing workers for the other realizations of the sunspot. The measure of realizations of the sunspot for which there is firing is not exogenous but it is uniquely pinned down by the workers' incentive compatibility constraint. In this equilibrium, individual firms perfectly correlate the outcome of their firing lottery. There is a simple logic behind this finding. Suppose that firms load up the firing probability on some realizations of the sunspot. In those states of the world, unemployment is higher and, because of decreasing returns to matching, the job-finding probability of unemployed workers is lower. In turn, if the job-finding probability is lower, the workers have a worse outside option when bargaining with the firms and their wage is lower. If the wage is lower, the workers' marginal utility of consumption relative to the firms' is lower and, according to Nash bargaining, the workers' gains from trade relative to the firms' are higher. Hence, if the other firms in the market load the firing probability on some realizations of the sunspot, an individual firm has the incentive to load the firing probability on the very same states of the world. In other words, firms want to correlate the outcome of the randomization between firing and keeping their non-performing workers, and the sunspot allows them to achieve correlation.

Naturally, alongside the equilibrium in which firms use the sunspot to correlate the outcome of the lottery over firing or keeping their non-performing workers, there are also equilibria in which firms (fully or partially) ignore the sunspot and randomize on firing or keeping non-performing workers in an uncorrelated fashion. These equilibria are an artifact of the simplifying assumption that all firms randomize simultaneously and, hence, correlation relies on a common interpretation of the inherently meaningless sunspot. In a version of the model where firms fire sequentially, correlation can be achieved without a sunspot and only the equilibrium with perfect correlation survives.

In the equilibrium where firms perfectly correlate the outcome of the lottery over firing or keeping non-performing workers, the economy experiences aggregate fluctuations. We refer to these aggregate fluctuations as *Agency Business Cycles* (ABCs). These fluctuations are endogenous. They are not caused by exogenous shocks to current or future fundamentals, nor by exogenous shocks to the selection of the equilibrium played by market participants. Instead, these aggregate fluctuations are caused by *correlated randomization*—i.e. individual firms choose to randomize on firing or keeping its non-performing workers, and different firms choose to correlate the outcomes of their randomization. Moreover, the aggregate fluctuations in our model are stochastic. The economy does not follow a deterministic cycle, but a random process in which the probability of a firing burst and, hence, of a recession is endogenous.

ABCs have three distinctive features. First, in ABCs, recessions start with a burst of firings and, hence, with an increase in the rate at which employed workers become unemployed (EU rate). The increase in the EU rate leads to an increase in unemployment and, because of decreasing returns to matching, to a fall in the rate at which unemployed workers become employed (UE rate). Even though the increase in the EU rate is short-lived, the unemployment rate (and, consequently, the UE rate) recovers slowly because it takes time to rebuild the stock of firm-worker matches. Moreover, these fluctuations in the labor market outcomes are uncorrelated with fluctuations in productivity. Second, in ABCs, the probability of a recession is endogenous and depends on the aggregate state of the economy. Specifically, the lower is the unemployment rate, the higher is the probability with which firms need to fire their non-performing workers in order to give them an incentive to exert effort and, hence, the higher is the probability of a recession. Third, in stark contrast with Real Business Cycle theory, in ABCs a recession is a time when the value of a worker in the market is unusually *high* relative to the value of a worker at home (and, for this reason, a time when firms find it optimal to fire their non-performing workers). We find empirical evidence that is broadly consistent with all three features of ABCs.

The main contribution of the paper is to develop the first theory of endogenous and stochastic business cycles. We show that endogenous and stochastic cycles emerge in equilibrium when individual agents want to randomize over some economic decision and different agents find it optimal to correlate the outcome of their randomization. The hallmark of a theory of endogenous and stochastic business cycles is a stochastic process for aggregate "shocks" that is determined endogenously. Conceptually, a theory of endogenous and stochastic cycles is useful because it explains why the economy is subject to shocks, rather than simply assuming the existence of shocks. This implies that the theory has something to say about what determines the frequency of shocks, the magnitude of shocks and what policies may affect the stochastic process of shocks. Empirically, a theory of endogenous and stochastic business cycles is useful because it helps explain why economic activity seems (at first blush) to be more volatile than its fundamentals, and it does so without resorting to unobserved shocks to equilibrium selection.

Intellectually, a theory of endogenous and stochastic cycles adds to the class of theories that we can use to understand macroeconomic fluctuations. Some of the existing theories of aggregate fluctuations are based on exogenous shocks to fundamentals. These can be shocks to the current value of economy-wide fundamentals (e.g., Kydland and Prescott 1982 or Mortensen and Pissarides 1994), to the future value of fundamentals (e.g., Beaudry and Portier 2004 or Jaimovich and Rebelo 2009), to the stochastic process of fundamentals (e.g., Bloom 2009), or to higher-order beliefs (e.g., Angeletos and La'O 2013). Relatedly, there are granular theories of business cycles, in which aggregate fluctuations are driven by shocks to the fundamentals of individual agents who are large enough or connected enough to others to cause aggregate swings in economic activity (e.g., Jovanovic 1987 or Gabaix 2011). Other theories of business cycles are driven by exogenous shocks to equilibrium selection (e.g., Heller 1986, Cooper and John 1988, Benhabib and Farmer 1994, Kaplan and Menzio 2016). Finally, there are theories of endogenous and deterministic aggregate fluctuations, where the economy converges to a limit cycle (e.g., Diamond 1982, Diamond and Fudenberg 1989, Mortensen 1999 or Beaudry, Galizia and Portier 2015) or follows chaotic dynamics (e.g., Boldrin and Montrucchio 1986 or Boldrin and Woodford 1990). Our theory is most closely related to chaotic theories (as chaotic dynamics may appear stochastic to the observer) and to granular theories (as the outcome of the randomization of some firms may act as a correlation device for other firms).

The particular illustration of our theory contributes to the literature on labor market fluctuations. Shimer (2005) showed that the basic search-theoretic model of the labor market implies very small fluctuations in unemployment in response to the observed fluctuations in labor productivity. In response to this observation, many papers have identified channels through which labor productivity shocks can lead to sizeable movements in unemployment (e.g., wage rigidity in Hall 2005, Menzio 2005, Kennan 2010, Menzio and Moen 2010, small gap between home productivity and market productivity in Hagedorn and Manovskii 2008, match heterogeneity in Menzio and Shi 2011). These papers typically generate a perfect negative correlation between labor productivity and unemployment. Yet, since 1984, this correlation has vanished. Recent work has thus focused on identifying different sources of unemployment fluctuations (e.g., Farmer 2013, Galí and Van Rens 2014, Kaplan and Menzio 2016, Beaudry, Galizia and Portier 2016, Hall 2016). Our model offers a novel explanation for why unemployment is so volatile and why its volatility is uncorrelated with productivity. A distinguishing feature of our explanation relative to others is that it implies a positive correlation between the net value of employment and unemployment. We find preliminary evidence of this positive correlation in the data.

2 Environment and Equilibrium

2.1 Environment

Time is discrete and continues forever. The economy is populated by a measure 1 of identical workers. Every worker has preferences described by $\sum \beta^t \left[\upsilon(c_t) - \psi e_t \right]$, where $\beta \in (0,1)$ is the discount factor, $\upsilon(c_t)$ is the utility of consuming c_t units of output in period t, and ψe_t is the disutility of exerting e_t units of effort in period t. The utility function $\upsilon(\cdot)$ is strictly increasing and strictly concave, with a first derivative $\upsilon'(\cdot)$ such that $\upsilon'(\cdot) \in [\underline{\upsilon}', \overline{\upsilon}']$, and a second derivative $\upsilon''(\cdot)$ such that $-\upsilon''(\cdot) \in [\underline{\upsilon}'', \overline{\upsilon}'']$, with $\overline{\upsilon}' > \underline{\upsilon}' > 0$ and $\overline{\upsilon}'' > \underline{\upsilon}'' > 0$. The consumption c_t is equal to the wage w_t if the worker is employed in period t, and to the value of home production b if the worker is unemployed in period t.[1] The coefficient ψ is strictly positive, and the effort e_t is equal to either 0 or 1. Every worker is endowed with

[1] As the reader can infer from the notation, we assume that workers are banned from the credit market and, hence, they consume their income in every period. The assumption is made only for the sake of simplicity.

one indivisible unit of labor.

The economy is also populated by a positive measure of identical firms. Every firm has preferences described by $\sum \beta^t c_t$, where $\beta \in (0,1)$ is the discount factor and c_t is the firm's profit in period t. Every firm operates a constant returns to scale production technology that transforms one unit of labor (i.e. one employee) into y_t units of output, where y_t is a random variable that depends on the employee's effort e_t. In particular, y_t takes the value y_h with probability $p_h(e)$ and the value y_ℓ with probability $p_\ell(e) = 1 - p_h(e)$, with $y_h > y_\ell \geq 0$ and $0 < p_h(0) < p_h(1) < 1$. Production suffers from a moral hazard problem, in the sense that the firm does not directly observe the effort of its employee, but only the output.

Every period t is divided into five stages: sunspot, separation, matching, bargaining and production. At the first stage, a random variable, z_t, is drawn from a uniform distribution with support $[0,1]$.[2] The random variable is aggregate, in the sense that it is publicly observed by all market participants. The random variable is a sunspot, in the sense that it does not directly affect technology, preferences or any other fundamentals, although it may help correlate the outcome of the lotteries played by different market participants.

At the separation stage, some employed workers become unemployed. An employed worker becomes unemployed for exogenous reasons with probability $\delta \in (0,1)$. In addition, an employed worker becomes unemployed because he is fired with probability $s(y_{t-1}, z_t)$, where $s(y_{t-1}, z_t)$ is determined by the worker's employment contract and it is allowed to depend on the output of the worker in the previous period, y_{t-1}, and on realization of the sunspot in the current period, z_t. For the sake of simplicity, we assume that a worker who becomes unemployed in period t can search for a new job only starting in period $t+1$.

At the matching stage, some unemployed workers become employed. Firms decide how many job vacancies v_t to create at the unit cost $k > 0$. Then, the u_{t-1} workers who were unemployed at the beginning of the period and the v_t vacant jobs that were created by the firms search for each other. The outcome of the search process is described by a decreasing return to scale matching function, $M(u_{t-1}, v_t)$, which gives the measure of bilateral matches formed between unemployed workers and vacant firms. We denote v_t/u_{t-1} as θ_t, and we refer to θ_t as the *tightness of the labor market*. We denote as $\lambda(\theta_t, u_{t-1})$ the probability that an unemployed worker meets a vacancy, i.e. $\lambda(\theta_t, u_{t-1}) = M(u_{t-1}, \theta_t u_{t-1})/u_{t-1}$. Similarly, we denote as $\eta(\theta_t, u_{t-1})$ the probability that a vacancy meets an unemployed worker, i.e. $\eta(\theta_t, u_{t-1}) = M(u_{t-1}, \theta_t u_{t-1})/\theta_t u_{t-1}$. We assume that the job-finding probability $\lambda(\theta_t, u_{t-1})$ is strictly increasing in θ_t and strictly decreasing in u_{t-1} and that the job-filling probability $\eta(\theta_t, u_{t-1})$ is strictly decreasing in both θ_t and u_{t-1}. That is, the higher is the labor market tightness, the higher is the job-finding probability and the lower is the job-filling probability. However, for a given labor market tightness, both the job-finding and the job-filling

[2] Assuming that the sunspot is drawn from a uniform with support $[0,1]$ is without loss in generality.

probabilities are strictly decreasing in unemployment.[3]

At the bargaining stage, each firm-worker pair negotiates the terms of a one-period employment contract x_t. The contract x_t specifies the effort e_t recommended to the worker in the current period, the wage w_t paid by the firm to the worker in the current period, and the probability $s(y_t, z_{t+1})$ with which the firm fires the worker at the next separation stage, conditional on the output of the worker in the current period and on the realization of the sunspot at the beginning of next period. We assume that the outcome of the bargain between the firm and the worker is the Axiomatic Nash Bargaining Solution.

At the production stage, an unemployed worker home-produces and consumes b units of output. An employed worker chooses an effort level, e_t, and consumes w_t units of output. Then, the output of the worker, y_t, is realized and observed by both the firm and the worker.

A few comments about the environment are in order. We assume that the employment contract cannot specify a wage that depends on the current realization of the worker's output. Hence, the firm cannot use the current wage to give the worker an incentive to exert effort. We also assume that the employment contract is re-bargained every period. Hence, the firm cannot use future wages to give the worker an incentive to exert effort. Overall, a firing lottery is the only tool that the firm can use to incentivize the worker. These restrictions on the contract space are much stronger than what we need. Our theory of business cycles only requires that firms threaten non-performing workers with a lottery that includes firing as a possible outcome and that this threat is carried out along the equilibrium path.[4] As we know from Clementi and Hopenhayn (2006), if workers are protected by some form of limited liability, a firing lottery may be offered and firing may be carried out along the equilibrium path even under complete contracts.

We assume that the matching function has decreasing returns to scale. Empirically, it is hard to tell whether the matching process between firms and workers has decreasing, constant or increasing returns to scale. The main difficulty in estimating the returns to scale of the matching process is that–while unemployed, employed and out-of-the-labor force workers all participate to the matching process–we do not have reliable measures of the difference in search effort between these groups. So, while the literature has typically assumed constant returns in the matching process, there is no compelling evidence that this is indeed the case.[5] More importantly, our theory of business cycles only requires the worker's value from being

[3] Given any constant returns to scale matching function, the job-finding and the job-filling probabilities are only functions of the market tightness. Given any decreasing returns to scale matching function, the job-finding and the job-filling probabilities are also (decreasing) functions of unemployment.

[4] Shapiro and Stiglitz (1984) study a moral hazard problem between firms and workers under incomplete contracts. In their model, firms use firing as a threat to incentivize workers. However, the threat is never carried out along the equilibrium path.

[5] Petrongolo and Pissarides (2000) and Menzio and Shi (2011) show that, if the actual matching process involves vacancies and both employed and unemployed workers, the estimates of a matching function $M(u, v)$ with only vacancies and unemployed workers are generally biased.

unemployed to be decreasing in the unemployment rate. This condition is satisfied when the matching function M has decreasing returns to scale, when the consumption b enjoyed by an unemployed worker is decreasing in the unemployment rate[6] (because, e.g., it is harder to obtain transfers from either the government or from relatives), or when the cost k of an additional vacancy is increasing in the aggregate measure of vacancies v (so that, when unemployment is higher, the labor market tightness and the job-finding rate are lower).

2.2 Equilibrium

We now derive the conditions for an equilibrium in our model economy. Let u denote the measure of unemployed works at the beginning of the bargaining stage. Let $W_0(u)$ denote the lifetime utility of a worker who is unemployed at the beginning of the production stage. Let $W_1(x,u)$ denote the lifetime utility of a worker who is employed under the contract x at the beginning of the production stage. Let $W(x,u)$ denote the difference between $W_1(x,u)$ and $W_0(u)$. Let $F(x,u)$ denote the present value of profits for a firm that, at the beginning of the production stage, employs a worker under the contract x. Let $x^*(u)$ denote the equilibrium contract between a firm and a worker when unemployment is u. Finally, let $\theta(u,\hat{z})$ denote the labor market tightness at the matching stage of next period, when the current unemployment is u and next period's sunspot is $\hat{z}$. Similarly, let $h(u,\hat{z})$ denote the unemployment at the bargaining stage of next period, when the current unemployment is u and next period's sunspot is $\hat{z}$.

The lifetime utility $W_0(u)$ of an unemployed worker is such that

$$W_0(u) = \upsilon(b) + \beta E_{\hat{z}} \left[W_0(h(u,\hat{z})) + \lambda(\theta(u,\hat{z}),u) W(x^*(h(u,\hat{z})), h(u,\hat{z})) \right]. \tag{1}$$

In the current period, the worker home-produces and consumes b units of output. At the matching stage of next period, the worker finds a job with probability $\lambda(\theta(u,\hat{z}),u)$ in which case his continuation lifetime utility is $W_0(h(u,\hat{z}))+W(x^*(h(u,\hat{z})),h(u,\hat{z}))$. With probability $1-\lambda(\theta(u,\hat{z}),u)$, the worker does not find a job and his continuation lifetime utility is $W_0(h(u,\hat{z}))$.

The lifetime utility $W_1(x,u)$ of a worker employed under the contract $x=(e,w,s)$ is such that

$$\begin{aligned} W_1(x,u) &= \upsilon(w) - \psi e + \\ &+ \beta E_{y,\hat{z}} \left[W_0(h(u,\hat{z})) + (1-\delta)(1-s(y,\hat{z})) W(x^*(h(u,\hat{z})), h(u,\hat{z})) | e \right]. \end{aligned} \tag{2}$$

In the current period, the worker consumes w units of output and exerts effort e. At the separation stage of next period, the worker keeps his job with probability $(1-\delta)(1-s(y,\hat{z}))$,

[6] Chodorow-Reich and Karabarbounis (2016) construct an empirical measure of b and find it to be negatively correlated with unemployment.

in which case his continuation lifetime utility is $W_0(h(u,\hat{z}))+W(x^*(h(u,\hat{z})),h(u,\hat{z}))$. With probability $1-(1-\delta)(1-s(y,\hat{z}))$, the worker loses his job and his continuation lifetime utility is $W_0(h(u,\hat{z}))$.

The difference $W(x,u)$ between $W_1(x,u)$ and $W_0(u)$ represents the gains from trade to a worker employed under the contract x. From (1) and (2), it follows that $W(x,u)$ is such that

$$\begin{aligned} W(x,u) \quad &= \upsilon(w)-\upsilon(b)-ce+ \\ &+\beta E_{y,\hat{z}}\left\{[(1-\delta)(1-s(y,\hat{z}))-\lambda(\theta(\hat{z},u),u)]W(x^*(h(u,\hat{z})),h(u,\hat{z}))|e\right\}. \end{aligned} \tag{3}$$

We find it useful to denote as $V(u)$ the gains from trade for a worker employed under the equilibrium contract $x^*(u)$, i.e. $V(u)=W(x(u),u)$. We refer to $V(u)$ as the *equilibrium gains from trade accruing to the worker.*

The present value of profits $F(x,u)$ for a firm that employs a worker under the contract $x=(e,w,s)$ is such that

$$F(x,u)=E_y[y|e]+\beta E_{y,\hat{z}}\left[(1-\delta)(1-s(y,\hat{z}))F(x^*(h(u,\hat{z})),h(u,\hat{z}))|e\right] \tag{4}$$

In the current period, the firm enjoys a profit equal to the expected output of the worker net of the wage. At the separation stage of next period, the firm retains the worker with probability $(1-\delta)(1-s(y,\hat{z}))$, in which case the firm's continuation present value of profits is $F(x^*(h(u,\hat{z})),h(u,\hat{z}))$. With probability $1-(1-\delta)(1-s(y,\hat{z}))$, the firm loses the worker, in which case the firm's continuation present value of profits is zero. We find it useful to denote as $J(u)$ the present value of profits for a firm that employs a worker at the equilibrium contract $x^*(u)$, i.e. $J(u)=F(x^*(u),u)$. We refer to $J(u)$ as the *equilibrium gains from trade accruing to the firm.*

The equilibrium contract $x^*(u)$ is the Axiomatic Nash Solution to the bargaining problem between the firm and the worker. That is, $x^*(u)$ is such that

$$\max_{x=(e,w,s)} W(x,u)F(x,u), \tag{5}$$

subject to the logical constraints

$$e\in\{0,1\} \text{ and } s(y,\hat{z})\in[0,1],$$

and the worker's incentive compatibility constraints

$$\begin{aligned} \psi &\leq \beta(p_h(1)-p_h(0))E_{\hat{z}}\left[(1-\delta)(s(y_\ell,\hat{z})-s(y_h,\hat{z}))V(h(u,\hat{z}))\right], \quad \text{if } e=1, \\ \psi &\geq \beta(p_h(1)-p_h(0))E_{\hat{z}}\left[(1-\delta)(s(y_\ell,\hat{z})-s(y_h,\hat{z}))V(h(u,\hat{z}))\right], \quad \text{if } e=0. \end{aligned}$$

In words, the equilibrium contract $x^*(u)$ maximizes the product between the gains from trade accruing to the worker, $W(x,u)$, and the gains from trade accruing to the firm, $F(x,u)$, among all contracts x that satisfy the worker's incentive compatibility constraints. The first incentive compatibility constraint states that, if the contract specifies $e = 1$, the cost to the worker from exerting effort must be smaller than the benefit. The second constraint states that, if the contract specifies $e = 0$, the cost to the worker from exerting effort must be greater than the benefit. The cost of effort is ψ. The benefit of effort is given by the effect of effort on the probability that the realization of output is high, $p_h(1) - p_h(0)$, times the effect of a high realization of output on the probability of keeping the job, $(1-\delta)(s(y_\ell, \hat{z}) - s(y_h, \hat{z}))$, times the value of the job to the worker, $\beta V(h(u, \hat{z}))$.

The equilibrium market tightness $\theta(u, \hat{z})$ must be consistent with the firm's incentives to create vacancies. The cost to the firm from creating an additional vacancy is k. The benefit to the firm from creating an additional vacancy is given by the job-filling probability, $\eta(\theta(u, \hat{z}), u)$, times the value to the firm of filling a vacancy, $J(h(u, \hat{z}))$. The market tightness is consistent with the firm's incentives to create vacancies if $k = \eta(\theta(u, \hat{z}), u) J(h(u, \hat{z}))$ when $\theta(u, \hat{z}) > 0$, and if $k \geq \eta(\theta(u, \hat{z}), u) J(h(u, \hat{z}))$ when $\theta(u, \hat{z}) = 0$. Overall, the market tightness is consistent with the firm's incentives to create vacancies iff

$$k \geq \eta(\theta(u, \hat{z}), u) J(h(u, \hat{z})) \text{ and } \theta(u, \hat{z}) \geq 0, \tag{6}$$

where the two inequalities hold with complementary slackness.

The equilibrium law of motion for unemployment, $h(u, \hat{z})$, must be consistent with the equilibrium firing probability $s^*(y, \hat{z}, u)$ and with the job-finding probability $\lambda(\theta(u, \hat{z}), u)$. Specifically, $h(u, \hat{z})$ must be such that

$$h(u, \hat{z}) = u - u\mu(J(h(u, \hat{z}), u) + (1-u) E_y \left[\delta + (1-\delta) s^*(y, \hat{z}, u)\right], \tag{7}$$

where

$$\mu(J, u) = \lambda\left(\eta^{-1}\left(\min\{k/J, 1\}, u\right), u\right),$$

and $\eta^{-1}(\min\{k/J, 1\}, u)$ is the labor market tightness that solves (6). The first term on the right-hand side of (7) is unemployment at the beginning of the bargaining stage in the current period. The second term in (7) is the measure of unemployed workers who become employed during the matching stage of next period, which is given by unemployment u times the probability that an unemployed worker becomes employed $\mu(J(h(u, \hat{z}), u)$. The last term in (7) is the measure of employed workers who become unemployed during the separation stage of next period. The sum of the three terms on the right-hand side of (7) is the unemployment at the beginning of the bargaining stage in the next period.

We are now in the position to define a recursive equilibrium for our model economy.

Definition 1: *A Recursive Equilibrium is a tuple* (W, F, V, J, x^*, h) *such that: (i) The gains from trade accruing to the worker,* $W(x, u)$*, and to the firm,* $F(x, u)$*, satisfy (3) and (4) and*

$V(u) = W(x^*(u), u)$, $J(u) = F(x^*(u), u)$; *(ii) The employment contract* $x^*(u)$ *satisfies (5); (iii) The law of motion* $h(u, \hat{z})$ *satisfies (7).*

Over the next two sections, we will characterize the properties of the recursive equilibrium. We are going to carry out the analysis under the maintained assumptions that the equilibrium gains from trade are strictly positive, i.e. $J(u) > 0$ and $V(u) > 0$, and that the equilibrium contract requires the worker to exert effort, i.e. $e^*(u) = 1$. The first assumption guarantees that firms and workers trade in the labor market, and the second assumption guarantees that firms and workers find it optimal to solve the moral hazard problem.

3 Optimal Contract

In this section, we characterize the properties of the Axiomatic Nash Solution to the bargaining problem between the firm and the worker. That is, we characterize the properties of the employment contract that maximizes the product of the gains from trade accruing to the worker and the gains from trade accruing to the firm subject to the worker's incentive compatibility constraint. We refer to such contract as the optimal employment contract. Our key finding is that the worker is fired if and only if the realization of output is low and the realization of the state of the world is such that the cost to the worker from losing the job relative to the cost to the firm from losing the worker is sufficiently high.

We carry out the characterization of the optimal employment contract in four lemmas, all proved in Appendix A. In order to lighten up the notation, and without risk of confusion, we will drop the dependence of the gains from trade to the worker, W, and to the firm, F, as well as the dependence of the optimal contract, x^*, on unemployment in the current period, u. We will also drop the dependence of the continuation gains from trade to the worker and to the firm on unemployment in the current period and write $V(h(u, \hat{z}))$ as $V(\hat{z})$ and $J(h(u, \hat{z}))$ as $J(\hat{z})$.

Lemma 1: *Any optimal contract* x^* *is such that the worker's incentive compatibility holds with equality. That is,*

$$\psi = \beta(p_h(1) - p_h(0))E_{\hat{z}}\left[(1 - \delta)(s^*(y_\ell, \hat{z}) - s^*(y_h, \hat{z}))V(\hat{z})\right]. \tag{8}$$

To understand Lemma 1 consider a contract x such that the worker's incentive compatibility constraint is lax. Clearly, this contract prescribes that the worker is fired with some positive probability after a low realization of output, i.e. $s(y_\ell, \hat{z})$. If we lower $s(y_\ell, \hat{z})$ by some small amount, the worker's incentive compatibility constraint is still satisfied. Moreover, if we lower $s(y_\ell, \hat{z})$, the survival probability of the match increases. Since the continuation value of the match is strictly positive for both the worker and the firm, an increase in the survival probability of the match raises the gains from trade accruing to the worker, W, the gains from trade accruing to the firm, F, and the Nash product WF. Therefore, the contract x cannot be optimal.

Lemma 2: *Any optimal contract x^* is such that, if the realization of output is high, the worker is fired with probability* 0*. That is, for all $\hat{z} \in [0,1]$,*

$$s^*(y_h, \hat{z}) = 0. \tag{9}$$

To understand Lemma 2 consider a contract x such that the worker is fired with positive probability when the realization of output is high, i.e. $s(y_h, \hat{z}) > 0$. If we lower the firing probability $s(y_h, \hat{z})$, the incentive compatibility constraint of the worker is relaxed. Moreover, if we lower the firing probability $s(y_h, \hat{z})$, the survival probability of the match increases. In turn, the increase in the survival probability of the match raises the gains from trade accruing to the worker, W, the gains from trade accruing to the firm, F, and the Nash product WF. Thus, the contract x cannot be optimal.

Lemma 3: *Let $\phi(\hat{z}) \equiv V(\hat{z})/J(\hat{z})$. Any optimal contract x^* is such, if the realization of output is low, there exists a ϕ^* with the property that the worker is fired with probability* 1 *if $\phi(\hat{z}) > \phi^*$, and the worker is fired with probability* 0 *if $\phi(\hat{z}) < \phi^*$. That is, for all $\hat{z} \in [0,1]$,*

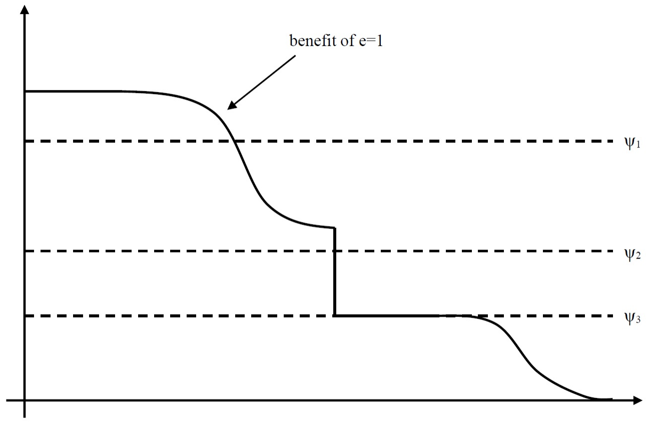
$$s^*(y_\ell, \hat{z}) = \begin{cases} 1, & \text{if } \phi(\hat{z}) > \phi^*, \\ 0, & \text{if } \phi(\hat{z}) < \phi^*. \end{cases} \tag{10}$$

Lemma 3 is one of the main results of the paper. It states that any optimal contract x^* is such that, if the realization of output is low, the worker is fired with probability 1 in states of the world $\hat{z}$ in which the continuation gains from trade to the worker, $V(\hat{z})$, relative to the continuation gains from trade to the firm, $J(\hat{z})$, are above some cutoff, and the worker is fired with probability 0 in states of the world in which the ratio $V(\hat{z})/J(\hat{z})$ is below the cutoff. There is a simple intuition behind this result. Firing is costly—as it destroys a valuable relationship—but also necessary—as it is the only tool to provide the worker with an incentive to exert effort. However, only the value of the destroyed relationship that would have accrued to the worker serves the purpose of providing incentives. The value of the destroyed relationship that would have accrued to the firm is *collateral damage*. The optimal contract minimizes the collateral damage by loading the firing probability on states of the world in which the value of the relationship to the worker would have been highest relative to the value of the relationship to the firm. In other words, the optimal contract minimizes the collateral damage by loading the firing probability on states of the world in which the cost to the worker from losing the job, $V(\hat{z})$, is highest relative to the cost to the firm from losing the worker, $J(\hat{z})$.

In any optimal contract x^*, the firing cutoff ϕ^* is such that

$$\psi = \beta(p_h(1) - p_h(0))(1-\delta)\left[\int_{\phi(\hat{z})>\phi^*} V(\hat{z})d\hat{z} + \int_{\phi(\hat{z})=\phi^*} s^*(y_\ell, \hat{z})V(\hat{z})d\hat{z}\right], \tag{11}$$

Figure 1: Equilibrium cutoff $\phi*$.



The above equation is the worker's incentive compatibility constraint (8) written using the fact that $s^*(y_h, \hat{z})$ is given by (9) and $s^*(y_\ell, \hat{z})$ is given by (10). Figure 1 plots the right-hand side of (11), which is the worker's benefit from exerting effort, as a function of the firing cutoff ϕ^*. On any interval $[\phi_0, \phi_1]$ where the distribution of the random variable $\phi(\hat{z})$ has positive density, the right-hand side of (11) is strictly decreasing in ϕ^*. On any interval $[\phi_0, \phi_1]$ where the distribution of $\phi(\hat{z})$ has no density, the right-hand side of (11) is constant. At any value ϕ where the distribution of $\phi(\hat{z})$ has a mass point, the right-hand side of (11) can take on an interval of values, as the firing probability $s^*(y_\ell, \hat{z})$ for $\hat{z}$ such that $\phi(\hat{z}) = \phi$ varies between 0 and 1. Overall, the right-hand side of (11) is a weakly decreasing function of the firing cutoff ϕ^*.

In any optimal contract x^*, the firing cutoff ϕ^* is such that the right-hand side of (11) is equal to the worker's cost ψ from exerting effort. There are three cases to consider. First, consider the case in which the right-hand side of (11) equals ψ at a point where the right-hand side of (11) is strictly decreasing in ϕ^*. In this case, the equilibrium firing cutoff is uniquely pinned down. Moreover, since the right-hand side of (11) is strictly decreasing in ϕ^*, the random variable $\phi(\hat{z})$ has no mass point at the equilibrium firing cutoff. Hence, in this case, the firm either fires the worker with probability 0 or with probability 1. This is the case of ψ_1 in Figure 1. Second, consider the case in which the right-hand side of (11) equals ψ at a point where the right-hand side of (11) can take on a range of values. In this case, the equilibrium firing cutoff is uniquely pinned down. However, the random variable $\phi(\hat{z})$ has a mass point at the equilibrium firing cutoff. Hence, in this case, for any realization of $\phi(\hat{z})$ equal to the equilibrium firing cutoff, the firm fires the worker with the probability $s^*(y_\ell, \hat{z})$ that satisfies (11). This is the case of ψ_2 in Figure 1. Finally, consider the case in which there is an interval of values of $\phi(\hat{z})$ such that the right-hand side of (11) equals ψ. In this case, the equilibrium firing cutoff can take on any value in the interval. However, the choice of the cutoff is immaterial, as the probability that the random variable $\phi(\hat{z})$ falls in the interval is zero. This is the case of ψ_3 in Figure 1. In any of the three cases, the equilibrium firing cutoff is effectively unique and so is the firing probability for any realization of the random variable $\phi(\hat{z})$.

Lemma 4: *Any optimal contract x^* is such that the wage w^* satisfies*

$$\frac{W(x^*)}{F(x^*)} = \frac{\upsilon'(w^*)}{1}. \tag{12}$$

Lemma 4 states that any optimal contract x^* prescribes a wage such that the ratio of the marginal utility of consumption to the worker to the marginal utility of consumption to the firm, $\upsilon'(w^*)/1$, is equal to the ratio of the equilibrium gains from trade accruing to the worker to the equilibrium gains from trade accruing to the firm, $W(x^*)/F(x^*) = V/J$. This is the standard optimality condition for the wage in the Axiomatic Nash Solution. Lemma 4 is important, as it tells us that the relative gains from trade accruing to the worker are higher in states of the world in which the worker's wage is lower. Hence, it follows from Lemma 3 that the optimal contract is such that the firm fires the worker if and only if the realization of output is low and the realization of the state of the world is such that the worker's wage next period would have been sufficiently low.

We are now in the position to summarize the characterization of the optimal contract.

Theorem 1: (Contracts) *Any optimal contract x^* is such that: (i) the worker is paid the wage w^* given by (12); (ii) If the realization of output is high, the worker is fired with probability $s^*(y_h, \hat{z})$ given by (9); (iii) If the realization of output is low, the worker is fired with probability $s^*(y_\ell, \hat{z})$ given by (10), where the ϕ^* is uniquely pinned down by (11).*

4 Properties of Equilibrium

In this section, we characterize the properties of the equilibrium. First, we characterize the role of the sunspot within a period. We show that there exists a *Perfect Correlation Equilibrium* in which all firms fire their non-performing workers with probability 1 for some realization of the sunspot and with probability 0 for the other realizations of the sunspot. In this equilibrium, firms use the sunspot to randomize over keeping or firing their non-performing workers in a perfectly correlated fashion. There is also a *No Correlation Equilibrium* in which firms fire workers with the same probability independently of the realization of the sunspot. In this equilibrium, firms randomize over firing or keeping their non-performing workers independently from each other. However, we show that the No Correlation Equilibrium only exists because of the firms randomize simultaneously and have to rely on an inherently meaningless signal to correlated outcomes. Indeed, we show that, in a version of the model where firms randomize sequentially, the unique equilibrium is the one with perfect correlation. Finally, we establish the existence and characterize the properties of the Recursive Equilibrium of the economy.

4.1 Stage Equilibrium

In any Recursive Equilibrium, the probability $s^*(y_\ell, \hat{z})$ with which firms fire non-performing workers and the worker's relative gains from trade $\phi(\hat{z})$ must simultaneously satisfy two conditions. For any $\hat{z}$, the firing probability $s^*(y_\ell, \hat{z})$ must be part of the optimal employment contract given the worker's relative gains from trade $\phi(\hat{z})$ and the probability distribution of the worker's relative gains from trade across realizations of the sunspot. Moreover, for any $\hat{z}$, the worker's relative gains from trade $\phi(\hat{z})$ must be those implied by the evolution of unemployment, given the firing probability $s^*(y_\ell, \hat{z})$. Formally, in any equilibrium, the functions $\phi(\hat{z})$ and $s^*(y_\ell, \hat{z})$ must be a fixed-point of the mapping we just described. Borrowing language from game theory, we refer to such a fixed-point as the *Stage Equilibrium*, as it describes the key outcomes of the economy within one period.

We first characterize the effect of the firm's firing probability $s^*(y_\ell, \hat{z})$ on the worker's relative gains from trade $\phi(\hat{z})$. Given that unemployment at the beginning of the period is u and that the firing probability at the separation stage is $s(\hat{z}) = s^*(y_\ell, \hat{z})$, the law of motion (7) implies that unemployment at the bargaining stage is $\hat{u}(s(\hat{z}))$ such that

$$\hat{u}(s(\hat{z})) = u - u\mu(J(\hat{u}(s(\hat{z}))), u) + (1-u)(\delta + (1-\delta)p_\ell(1)s(\hat{z})). \tag{13}$$

We conjecture that the gains from trade accruing to the firm are a strictly increasing function of unemployment, i.e. $J(\hat{u}(s(\hat{z})))$ is strictly increasing in $\hat{u}(s(\hat{z}))$. Under this conjecture, there is a unique $\hat{u}(s(\hat{z}))$ that satisfies (13) and $\hat{u}(s(\hat{z}))$ is strictly increasing in the firing probability $s(\hat{z})$.

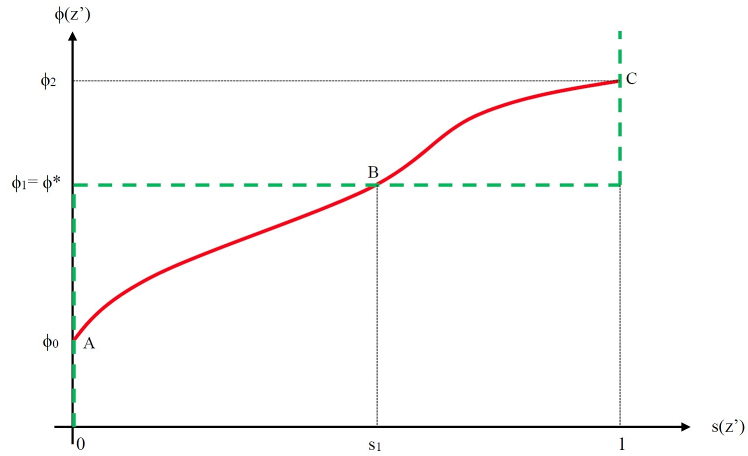
Given that unemployment at the bargaining stage is $\hat{u}(s(\hat{z}))$, the worker's wage is $w^*(\hat{u}(s(\hat{z})))$. Then, it follows from the optimality condition (12) that the worker's relative gains from trade are such that

$$\phi(\hat{z}) = \frac{V(\hat{u}(s(\hat{z})))}{J(\hat{u}(s(\hat{z})))} = \frac{v'(w^*(\hat{u}(s(\hat{z}))))}{1}. \tag{14}$$

We conjecture that the wage is a strictly decreasing function of unemployment, i.e. $w^*(\hat{u}(s(\hat{z})))$ is strictly decreasing in $\hat{u}(s(\hat{z}))$. Under this conjecture, the worker's relative gains from trade $\phi(\hat{z})$ are strictly increasing in the unemployment $\hat{u}(s(\hat{z}))$ and, since $\hat{u}(s(\hat{z}))$ is strictly increasing in $s(\hat{z})$, they are also strictly increasing in the firing probability $s(\hat{z})$. The solid red line in Figure 2 illustrates the effect of the firing probability on the worker's relative gains from trade.

The conjectures that the worker's wage is decreasing in unemployment and the firm's gains from trade are increasing in unemployment are natural and will be verified in Section

Figure 2: Stage Equilibrium



4.3. Intuitively, when the matching function has decreasing returns to scale, an increase in unemployment tends to lower the job-finding probability of unemployed workers and, hence, the value of unemployment to a worker. A decline in the value of unemployment tends to increase the gains from trade between the firm and the worker. Therefore, it is natural to conjecture that the gains from trade accruing to the firm are increasing in unemployment. A decline in the value of unemployment also lowers the worker's outside option in bargaining and, hence, the equilibrium wage. Therefore, it is natural to conjecture that the wage falls with unemployment.

Next, we characterize the effect of the worker's relative gains from trade $\phi(\hat{z})$ on the probability $s(\hat{z}) = s^*(y_\ell, \hat{z})$ with which firms fire their non-performing workers. From the optimality condition (10), it follows that the firing probability $s(\hat{z})$ is such that

$$s^*(y_\ell, \hat{z}) = \begin{cases} 0, & \text{if } \phi(\hat{z}) < \phi^*, \\ \in [0,1], & \text{if } \phi(\hat{z}) = \phi^*, \\ 1, & \text{if } \phi(\hat{z}) > \phi^*, \end{cases} \tag{15}$$

where ϕ^* is implicitly defined by the worker's incentive compatibility constraint (11). As explained in the previous section, the higher are the worker's relative gains from trade in $\hat{z}$, the stronger is the firm's incentive to fire its non-performing workers in that state of the world. The dashed green line in Figure 2 illustrates the effect of the worker's relative gains from trade $\phi(\hat{z})$ on the firing probability $s(\hat{z})$.

For any realization $\hat{z}$ of the sunspot, the firing probability must be optimal given the worker's gains from trade (i.e. we must be on the dashed green line) and the worker's gains from trade must be consistent with the firing probability (i.e. we must be on the solid red line). As it is clear from Figure 2, for any realization of $\hat{z}$, only three outcomes are possible: points A, B and C. The first outcome, point A, is such that the firm's firing probability $s(\hat{z})$

is zero, and the worker's relative gains from trade $\phi(\hat{z})$ are smaller than ϕ^*. The second outcome, point B, is such that the firm's firing probability $s(\hat{z})$ is greater than zero and smaller than one, and the worker's relative gains from trade $\phi(\hat{z})$ are equal to ϕ^*. The third outcome, point C, is such that the firm's firing probability $s(\hat{z})$ is one, and the worker's relative gains from trade $\phi(\hat{z})$ are greater than ϕ^*. The coexistence of multiple outcomes is a consequence of the fact that firms have a desire to correlate the result of the randomization over firing or keeping their workers. If other firms are more likely to fire their workers in one state of the world than in another, an individual firm wants to do the same, because, in the state of the world where other firms are more likely to fire their workers, unemployment is higher and so are the worker's relative gains from trade.

Let Z_0 denote the realizations of the sunspot for which firms fire non-performing workers with probability 0, and let π_0 denote the measure of Z_0. Let Z_1 denote the realizations of the sunspot for which firms fire non-performing workers with a probability $s^*(y_\ell, \hat{z}) = s_1 \in (0,1)$, and let π_1 denote the measure of Z_1. Similarly, let Z_2 denote the realizations of the sunspot for which firms fire non-performing workers with probability 1, and let π_2 denote the measure of Z_2.

Depending on the value of π_1, we can identify three different types of stage equilibria. If $\pi_1 = 1$, we have a *No Correlation Equilibrium.* In this equilibrium, firms fire the non-performing workers with probability $s^*(y_\ell, \hat{z}) = s_1 \in (0,1)$ for all $\hat{z} \in [0,1]$. Basically, firms ignore the sunspot and randomize over firing or keeping their non-performing workers independently from each other. In a No Correlation Equilibrium, the worker's incentive compatibility constraint (11) becomes

$$\psi = \beta(p_h(1) - p_h(0))(1-\delta)s_1 V(\hat{u}(s_1)). \tag{16}$$

When we solve the constraint with respect to s_1, we find that the constant probability with which firms fire their non-performing workers is

$$s_1 = \frac{\psi}{\beta(p_h(1) - p_h(0))(1-\delta)V(\hat{u}(s_1))}. \tag{17}$$

If $\pi_1 = 0$, we have a *Perfect Correlation Equilibrium.* In this equilibrium, firms fire their non-performing workers with probability 0 for the realizations of the sunspot $\hat{z} \in Z_0$, and with probability 1 for the other realization of the sunspot. Basically, firms use the sunspot to randomize over firing or keeping their non-performing workers in a perfectly correlated fashion. In a Perfect Correlation Equilibrium, the worker's incentive compatibility constraint (11) becomes

$$\psi = \beta(p_h(1) - p_h(0))(1-\delta)\pi_2 V(\hat{u}(1)). \tag{18}$$

When we solve the constraint with respect to π_2, we find that the probability with which firms fire their non-performing workers is given by

$$\pi_2 = \frac{\psi}{\beta(p_h(1) - p_h(0))(1-\delta)V(\hat{u}(1))}. \tag{19}$$

If $\pi_1 \in (0,1)$, we have a *Partial Correlation Equilibrium.* In this equilibrium, firms fire workers with probability $s^*(y_\ell, \hat{z}) = s_1 \in (0,1)$ for all $\hat{z} \in Z_1$. Hence, when $\hat{z} \in Z_1$, firms randomize over firing or keeping their non-performing workers independently from each other. However, if $\hat{z} \notin Z_1$, firms fire their workers with probability 0 if $\hat{z} \in Z_0$ and with probability 1 if $\hat{z} \in Z_2$. Hence, when $\hat{z} \notin Z_1$, firms use the sunspot to randomize over firing or keeping their non-performing workers in a correlated fashion. Overall, a Partial Correlation Equilibrium is a combination of a No Correlation and a Perfect Correlation Equilibrium. In a Partial Correlation Equilibrium, the worker's incentive compatibility constraint (11) becomes

$$\psi = \beta(p_h(1) - p_h(0))(1-\delta)\left[\pi_1 s_1 V(\hat{u}(s_1)) + \pi_2 V(\hat{u}(1))\right]. \tag{20}$$

When we solve the constraint with respect to s, we find that the constant probability with which firms fire their non-performing workers for $\hat{z} \in Z_1$ is

$$s_1 = \frac{\psi - \beta(p_h(1) - p_h(0))(1-\delta)\pi_2 V(\hat{u}(1))}{\beta(p_h(1) - p_h(0))(1-\delta)\pi_1 V(\hat{u}(s_1))}. \tag{21}$$

The above results are summarized in Theorem 2.

Theorem 2: (Stage Equilibrium). *Three Stage Equilibria exist: (i) No Correlation Equilibrium where $s^*(y_\ell, \hat{z}) = s_1$ for all $\hat{z} \in Z_1$, where Z_1 has probability measure $\pi_1 = 1$ and s_1 is given by (17); (ii) Perfect Correlation Equilibrium where $s^*(y_\ell, \hat{z}) = 0$ for all $\hat{z} \in Z_0$ and $s^*(y_\ell, \hat{z}) = 1$ for all $\hat{z} \in Z_2$, where Z_0 has probability measure $1 - \pi_2$ and Z_2 has probability measure π_2 and $\pi_2 \in (0,1)$ is given by (19); (iii) Partial Correlation Equilibrium where $s^*(y_\ell, \hat{z}) = 0$ for all $\hat{z} \in Z_0$, $s^*(y_\ell, \hat{z}) = s_1$ for all $\hat{z} \in Z_1$, and $s^*(y_\ell, \hat{z}) = 1$ for all $\hat{z} \in Z_2$, where Z_1 has probability measure $\pi_0 \in (0,1)$ and s_1 is given by (21).*

The Perfect Correlation Equilibrium exists because firms have an incentive to correlate the outcome of the randomization over firing and keeping their non-performing workers in order to minimize the collateral damage involved in providing workers with incentives. Moreover, firms are able to correlate the outcome of the randomization because of the sunspot. The No Correlation Equilibrium and the Partial Correlation Equilibrium exist because the sunspot is inherently meaningless and, hence, there always exist an equilibrium in which it is ignored. However, if firms do not need to rely on an inherently meaningless signal to correlate, they will always be able to do so and the No and Partial Correlation Equilibria will disappear. Indeed, in the next subsection, we consider a version of the stage game in which firms fire sequentially and show that its unique equilibrium is the one with perfect correlation.

4.2 Equilibrium Refinement

As we discussed above, the existence of No and Partial Correlation Equilibria is an artifact of the simplifying assumption that all firms randomize over firing or keeping non-performing workers simultaneously. When firms randomize simultaneously, they need to rely on the sunspot to correlate the outcome of their randomization. However, since the sunspot is inherently meaningless, there is always an equilibrium in which the sunspot is ignored and firms cannot correlate the outcome of their randomization. In this subsection, we consider a version of the environment in which firms randomize over firing or keeping their non-performing workers sequentially. We show that, firms moving later can always condition their randomization on the realization of the lottery played by the firms moving first and, hence, the unique equilibrium is the one with perfect correlation. This version of the model also shows that the sunspot is not essential to our theory of business cycles.

Here is a formal description of the modified environment. Let $1-u$ denote the measure of employed workers at the bargaining stage of the current period. The measure of employed workers is equally divided into a large number NK firms, each employing one worker of "measure" $(1-u)/NK$. Firms are clustered into a large number K of groups, each comprising a large number N of firms. Firms and workers bargain over the terms of the one-period employment contract knowing the group to which they belong. At the separation stage of next period, firm-worker pairs in different groups decide to break up or stay together sequentially. First, the firm-worker pairs in group 1 decide to separate or not. Second, after observing the outcomes of group 1, the firm-worker pairs in group 2 decide to separate or not. Third, after observing the outcomes of groups 1 and 2, the firm-worker pairs in group 3 decide to separate or not. The process continues until the firm-worker pairs in group K decide to separate or not, after having observed the outcomes of groups 1 through $K-1$. Naturally, in this version of the model, we assume that there is no sunspot.

Let T_i denote the measure of workers separating from firms in groups 1 through i. We assume that each firm in group i takes as given the probability distribution of T_i conditional on T_{i-1}, which we denote as $P_i(T_i|T_{i-1})$. The assumption means that each firm views itself as small compared to its group. The assumption is reasonable when N is large. We also assume that $P_i(T_i|T_{i-1})$ is increasing, in the sense of first-order stochastic dominance, in T_{i-1}. The assumption means that firms in a group view themselves as small compared to the whole economy. The assumption is reasonable when K is large. We also approximate the worker's gains from trade, $V(\hat{u})$, and the firm's gains from trade, $J(\hat{u})$, with linear functions. The approximation implies that the expectation of the worker's relative gains from trade over next period's unemployment, $E[V(\hat{u})]/E[J(\hat{u})]$, is equal to the worker's relative gains from trade evaluated at the expectation of next period's unemployment, $V(E[\hat{u}])]/J(E[\hat{u}]) = \phi(E[\hat{u}])$.

We can now characterize the optimal contract between a worker and a firm in group

$i = 2, 3, \ldots K$. The contract can condition the firing probability $s_i(y, T_{i-1})$ on the realization of the worker's output y and on the measure T_{i-1} of workers separating from firms in groups 1 through $i-1$. As in Section 3, it is easy to show that the optimal contract is such that: (i) the worker's incentive compatibility constraint holds with equality; (ii) if the realization of output is high, the worker is fired with probability 0, i.e. $s_i(y_h, T_{i-1}) = 0$ for all T_{i-1}; (iii) if the realization of output is low, the worker is fired with probability 0 if the worker's relative gains from trade are below a cutoff ϕ_i^*, and with probability 1 if they are above the cutoff, i.e. $s_i(y_\ell, T_{i-1}) = 0$ if $\phi(E[\hat{u}|T_{i-1}]) < \phi_i^*$, and $s_i(y_\ell, T_{i-1}) = 1$ if $\phi(E[\hat{u}|T_{i-1}]) > \phi_i^*$. The optimal contract between a worker and a firm in group 1 can only condition the firing probability on the realization of the worker's output y. In this case, the optimal contract is such that $s_1(y_h) = 0$ and $s_1(y_\ell) = s_1$, where s_1 is such that the worker's incentive compatibility constraint holds with equality.

The following lemma shows that the probability $s_i(y_\ell, T_{i-1})$ with which firms in group i fire their non-performing workers has a threshold property with respect to T_{i-1}, the measure of workers separating from firms in groups 1 through $i-1$. This property of equilibrium is intuitive, as a higher T_{i-1} leads to a higher expectation for unemployment and, in turn, to a higher expectation for the worker's relative gains from trade.

Lemma 5: *For $i = 2, 3, \ldots K$, the firing probability $s_i(y_\ell, T_{i-1})$ equals 0 for all $T_{i-1} < T_{i-1}^*$, and it equals 1 for all $T_{i-1} > T_{i-1}^*$.*

Proof: In Appendix B. ■

We can now compute the equilibrium probability distribution of the measure t_i of workers separating from firms in group $i = 1, 2, \ldots K$, conditional on the measure T_{i-1} of workers separating from firms in groups 1 through $i-1$. Any firm-worker pair in group 1 separates with probability $\tau_1 = \delta + (1-\delta)p_\ell(1)s_1$. Since N is large, we can use the Central Limit Theorem to approximate the measure t_1 of workers separating from firms in group 1 with a Normal distribution with mean $E[t_1] = \tau_1(1-u)/K$ and variance $Var[t_1] = \tau_1(1-\tau_1)[(1-u)/K]^2/N$. Conditional on T_{i-1}, a firm-worker pair in group $i = 2, 3, \ldots .K$ separates with probability $\tau_i(T_{i-1}) = \delta + (1-\delta)p_\ell(1)s_i(y_\ell, T_{i-1})$. Since N is large, we can approximate the measure t_i of workers separating from firms in group i with a Normal distribution with mean $E[t_i|T_{i-1}] = \tau_i(T_{i-1})(1-u)/K$ and variance $Var[t_i|T_{i-1}] = \tau_i(T_{i-1})(1-\tau_1(T_{i-1}))[(1-u)/K]^2/N$. We find it convenient to define $t_\ell = \delta(1-u)/K$, and $t_h = [\delta + (1-\delta)p_\ell(1)](1-u)/K$.

The following lemma shows that, for $N \to \infty$, there exists an equilibrium in which the firms in groups 2 through K either all fire or all keep their non-performing workers depending on the measure of workers separating from firms in group 1. Let us give some intuition for this result in the case of $K = 3$. If the firms in group 1 happen to break up with more than T_1^* workers, firms in group 2 fire their non-performing workers with probability 1. If the firms in group 1 happen to break up with less than T_1^* workers, firms fire in group 2 fire their non-performing workers with probability 0. Since the variance of t_1 and t_2 is vanishing as $N \to \infty$, $T_2 = E[t_1] + t_h$ with probability $1 - P_1(T_1^*)$ and $T_2 = E[t_1] + t_\ell$ with probability $P_1(T_1^*)$. Suppose that $T_2^* = E[t_1] + (t_h + t_\ell)/2$. Then, firms in group 3 fire their non-performing workers with probability 1 if $T_2 = E[t_1] + t_h$ and with probability 0 if $T_2 = E[t_1] + t_\ell$. Since the variance of t_3 is vanishing as $N \to \infty$, $T_3 = E[t_1] + 2t_h$ with probability $1 - P_1(T_1^*)$ and $T_3 = E[t_1] + 2t_\ell$ with probability $P_1(T_1^*)$. Overall, firms in group 2 fire their non-performing workers with probability $1 - P_1(T_1^*)$ and, in that case, expect total separations $T_3 = E[t_1] + 2t_h$. Firms in group 3 fire their non-performing workers with probability $1 - P_1(T_1^*)$ and, in that case, they expect total separations $T_3 = E[t_1] + 2t_h$. Therefore, if T_1^* satisfies the incentive compatibility constraint of workers in group 2, then T_2^* satisfies the incentive compatibility constraint of workers in group 3. This confirms the existence of the desired equilibrium for $K = 3$. Clearly, for $K \to \infty$, this equilibrium converges to the Perfect Correlation Equilibrium.

Lemma 6: *For $N \to \infty$, there is an equilibrium in which firms in groups 2 through K fire their non-performing workers with probability 0 if $T_1 < T_1^*$, and with probability 1 if $T_1 > T_1^*$, where T_1^* is such that*

$$\psi = \beta(1-\delta)(p_h(1) - p_h(0))(1 - P_1(T_1^*))V(E[\hat{u}|T_K = E[t_1] + (K-1)t_h]). \tag{22}$$

Proof: In Appendix B. ■

Next, we rule out the existence of other equilibria. To this aim, notice that, in any equilibrium, firms in group 2 fire their non-performing workers with probability 0 if $T_1 < T_1^*$, and they fire them with probability 1 if $T_1 > T_1^*$, where T_1^* is such that the worker's incentive compatibility constraint is satisfied, i.e.

$$\psi = \beta(1-\delta)(p_h(1) - p_h(0))(1 - P_1(T_1^*))V(E[\hat{u}|T_1 > T_1^*]) \tag{23}$$

For $N \to \infty$, T_2 is approximately equal to $E[t_1] + t_\ell$ with probability $P_1(T_1^*)$ and, it is approximately equal to $E[t_1] + t_h$ with probability $1 - P_1(T_1^*)$.

Now, suppose that the threshold T_2^* is such that $P_2(T_2^*) > P_1(T_1^*)$. Then, conditional on any T_2 approximately equal to $E[t_1] + t_\ell$, firms in group 3 fire their non-performing workers with probability 0. Conditional on T_2 being approximately equal to $E[t_1]+t_h$, firms in group 3 do not fire their non-performing workers with probability $(P_2(T_2^*)-P_1(T_1^*))/(1-P_1(T_1^*))$ and they do with probability $(1 - P_2(T_2^*))/(1 - P_1(T_1^*))$. The incentive compatibility constraint for workers employed by firms in group 3 is thus given by

$$\psi = \beta(1-\delta)(p_h(1) - p_h(0))(1 - P_2(T_2^*))V(E[\hat{u}|T_2 > T_2^*]) \tag{24}$$

However, the incentive compatibility constraints (23) and (24) cannot hold simultaneously and, hence, there cannot be an equilibrium in which $P_2(T_2^*) > P_1(T_1^*)$. To see why this is the case, notice that $E[\hat{u}|T_1 > T_1^*]$ is equal to $E[\hat{u}|T_2 > T_2^*] + E[\hat{u}|T_2 < T_2^*, T_1 > T_1^*]$ and $1 - P_1(T_1^*) > 1 - P_2(T_2^*)$. Therefore, the right-hand side of (23) is strictly greater than the right-hand side of (24). Following a similar argument, we can rule also out equilibria in which $P_2(T_2^*) < P_1(T_1^*)$. Hence, in any equilibrium, $P_2(T_2^*) = P_1(T_1^*)$, which implies that firms in group 3 fire their non-performing workers if and only if firms in group 2 do. Repeating the above argument for $i = 4, 5, \ldots .K$, we can show that, in any equilibrium, firms in group i fire their non-performing workers if and only if firms in group $i-1$ do. Therefore, the only equilibrium of the modified environment is the one described in Lemma 6.

Finally, by taking the limit for $K \to \infty$, the following result obtains.

Theorem 3: (Equilibrium Refinement) *For $K \to \infty$, $N \to \infty$, the unique equilibrium of the environment with K groups of N firms firing sequentially is the Perfect Correlation Equilibrium.*

4.3 Recursive Equilibrium

In the previous subsections, we established the existence of a Perfect Correlation Equilibrium of the stage game under the conjectures that the firm's gains from trade is increasing in unemployment and that the wage is decreasing in unemployment. We also argued that the Perfect Correlation Equilibrium is the only equilibrium of the stage game that is robust to a natural perturbation of the environment. In this subsection, we show that, given a Perfect Correlation Equilibrium of the stage game, there exists a Recursive Equilibrium such that the firm's and worker's gains from trade are increasing in unemployment, while the wage is decreasing (and, hence, the worker's relative gains from trade are increasing) in unemployment.

The existence proof is an application of Schauder's fixed point theorem. The proof is lengthy and relegated in Appendix C. Here we outline the structure of the proof. Denote as Ω the set of bounded functions (V_+, J_+), $V_+ : [0,1] \longrightarrow \mathbb{R}$ and $J_+ : [0,1] \to \mathbb{R}$, such that for all u_0 and u_1, $0 \leq u_0 \leq u_1 \leq 1$, $V_+(u_1) - V_+(u_0)$ is greater than $\underline{D}_{V_+}(u_1 - u_0)$ and smaller than $\overline{D}_{V_+}(u_1 - u_0)$, and $J_+(u_1) - J_+(u_0)$ is greater than $\underline{D}_{J_+}(u_1 - u_0)$ and smaller than $\overline{D}_{J_+}(u_1 - u_0)$, where $\overline{D}_{V_+} > \underline{D}_{V_+} > 0$, $\overline{D}_{J_+} > 0 \geq \underline{D}_{J_+}$. That is, Ω is the set of functions (V_+, J_+) that are bounded and Lipschitz continuous, with Lipschitz bounds respectively given by $\underline{D}_{V_+}$ and $\overline{D}_{V_+}$, and $\underline{D}_{J_+}$ and $\overline{D}_{J_+}$.[7] Also, we denote as $\underline{\mu}_u$ and $\overline{\mu}_u$ the upper and the lower bound of the partial derivative of the job-finding probability $\mu(J, u)$ with respect to u, and as $\underline{\mu}_J$ and $\overline{\mu}_J$ the upper and the lower bound of the partial derivative of $\mu(J, u)$ with respect to J.

We take an arbitrary pair of functions $V_+(u)$ and $J_+(u)$ from the set Ω. We let V_+ be the worker's expected gains from trade at the end of the production stage, and we let J_+ be the firm's expected gains from trade at the end of the production stage. First, given (V_+, J_+),

[7] The reader can find the expression for the Lipschitz in Appendix B.

we use the fact that $W(w,u) = \upsilon(w) - \upsilon(b) - \psi + V_+(u)$ and $F(w,u) = E[y] - w + J_+(u)$ and condition (12) for the optimal contract to compute the equilibrium wage function $w(u)$. We prove that $w(u)$ is strictly decreasing in u. Intuitively, given the choice for the bounds $\underline{D}_{V_+}$ and $\overline{D}_{J_+}$, an increase in unemployment leads to a larger increase in $W(w,u)$ than in $\upsilon'(w)F(w,u)$ for any given wage w. For this reason, the wage must fall to make sure that the worker's relative gains from trade, $W(w,u)/F(w,u)$, are equated to the worker's relative marginal utility of consumption $\upsilon'(w)$.

Second, given the functions $V_+(u)$ and $J_+(u)$ and the wage function $w(u)$, we use the fact that $V(u) = W(w(u),u)$ and $J(u) = F(w(u),u)$ to compute the equilibrium gains from trade accruing to the worker and to the firm. We prove that $J(u)$ is strictly increasing in u. Intuitively, given the choice for the bound $\underline{D}_{J_+}$, an increase in unemployment leads to a decline in the wage that more than compensates the largest possible decline in $J_+(u)$. Similarly, we prove that $V(u)$ is strictly increasing in u. Intuitively, this is because $V(u)$ is equal to $J(u)\upsilon'(w(u))$ and both $J(u)$ and $\upsilon'(w(u))$ are strictly increasing in u.

Third, given the function $J(u)$, we use (7) to compute the law of motion for unemployment $h(u,\hat{z})$. We prove that, as long as $(1-\delta)p_h(1) - \mu(J,u) > 0$, next period's unemployment $h(u,\hat{z})$ is strictly increasing in current period's unemployment. Moreover, we prove that next period's unemployment $h(u,\hat{z})$ is strictly greater for the realization of the sunspot for which firms fire their non-performing workers, i.e. for $\hat{z} \in Z_2$, than for the realizations for which firms keep their non-performing workers, i.e. for $\hat{z} \in Z_0$.

Finally, given the functions $V(u)$, $J(u)$ and $h(u,\hat{z})$, we compute updates $\mathcal{F}V_+(u)$ and $\mathcal{F}J_+(u)$ for the worker's and firm's expected gains from trade at the end of the production process. More precisely, we compute $\mathcal{F}V_+(u)$ and $\mathcal{F}J_+(u)$ as

$$\begin{aligned} \mathcal{F}V_+(u) &= \beta E_{\hat{z}} \left\{ [(1-\delta)(1-p_\ell(1)s(y_\ell,\hat{z})) - \mu(J(h(u,\hat{z})),u)] V(h(u,\hat{z})) \right\}, \\ \mathcal{F}J_+(u) &= \beta E_{\hat{z}} \left[(1-\delta)(1-p_\ell(1)s(y_\ell,\hat{z})) J(h(u,\hat{z})) \right], \end{aligned} \tag{25}$$

where the firing probability $s(y_\ell,\hat{z}) = 1$ for $\hat{z} \in Z_2$ and $s(y_\ell,\hat{z}) = 0$ for $\hat{z} \in Z_0$, while the probability that $\hat{z} \in Z_2$ is given by π_2 in (19) and the probability that $\hat{z} \in Z_2$ is $\pi_0 = 1 - \pi_2$. We prove that, as long as $\underline{\mu}_u - \overline{\mu}_J(1-\delta+\overline{\mu}_u) > 0$ and β and ψ are not too large, $\mathcal{F}V_+(u)$ is bounded and such that $\mathcal{F}V_+(u_1) - \mathcal{F}V_+(u_0)$ is greater than $\underline{D}_{V_+}(u_1 - u_0)$ and smaller than $\overline{D}_{V_+}(u_1 - u_0)$ for all u_0, u_1 with $0 \le u_0 \le u_1 \le 1$. Similarly, $\mathcal{F}J_+(u)$ is bounded and such that $\mathcal{F}J_+(u_1) - \mathcal{F}J_+(u_0)$ is greater than $\underline{D}_{J_+}(u_1 - u_0)$ and smaller than $\overline{D}_{J_+}(u_1 - u_0)$ for all u_0, u_1 with $0 \le u_0 \le u_1 \le 1$.

The above observations imply that the operator $\mathcal{F}$ is a self-map, in the sense that it maps pairs of functions in the set Ω into pairs of functions that also belong to the set Ω. The set Ω is a non-empty, bounded, closed convex subset of the space of bounded continuous functions with the sup norm (see Lemma A.1 in Menzio and Shi, 2010). We also establish that the operator $\mathcal{F}$ is continuous. Finally, we establish that the family of functions $\mathcal{F}(\Omega)$ is equicontinuous. Equicontinuity, which is typically rather difficult to establish, here follows immediately because the functions $\mathcal{F}V_+$ and $\mathcal{F}J_+$ have the same Lipschitz bounds for all $(V_+, J_+) \in \Omega$.

The properties of the operator $\mathcal{F}$ are the conditions of Schauder's fixed point theorem (see Theorem 17.4 in Stokey, Lucas and Prescott 1989). Thus, there exists a pair of functions $(V_+^*, J_+^*) \in \Omega$ such that $\mathcal{F}(V_+^*, J_+^*) = (V_+^*, J_+^*)$. Given the functions V_+^* and J_+^*, we construct the associated wage function, w^*, the gains from trade to the worker and to the firm, V^* and J^*, and the law of motion for unemployment, h^*. These objects, together with an optimal contract $x^*(u)$ that prescribes the wage $w^*(u)$ and the firing probabilities $s^*(y_h, \hat{z}, u) = 0$, $s^*(y_h, \hat{z}, u) = 1$ for $\hat{z} \in Z_2$ and $s^*(y_h, \hat{z}, u) = 0$ for $\hat{z} \in Z_0$ constitute a Recursive Equilibrium in which, for all u, the equilibrium stage game is such that firms perfectly correlate the outcome of the lottery over firing or keeping their non-performing workers.

This completes the proof of the following theorem.

Theorem 4: (Recursive Equilibrium) (i) *Assume* $(1-\delta)p_h(1) - \mu(u, J) > 0$ *and* $\underline{\mu}_u - \overline{\mu}_J(1 - \delta + \overline{\mu}_u) > 0$. *For all* (β, ψ) *such that* $\beta \in (0, \beta^*)$ *and* $\psi \in (0, \psi^*)$, *where* $\beta^* > 0$ *and* $\psi^* > 0$, *there is a Recursive Equilibrium in which firms play the Perfect Correlation Equilibrium at the stage game. (ii) The Recursive Equilibrium is such that: (a)* $V(u_0) < V(u_1)$ *and* $J(u_0) < J(u_1)$ *for all* u_0, u_1 *with* $0 \leq u_0 < u_1 \leq 1$*; (b)* $w(u_0) > w(u_1)$ *and* $V(u_0)/J(u_0) < V(u_1)/J(u_1)$ *for all* u_0, u_1 *with* $0 \leq u_0 < u_1 \leq 1$; (c) $h(u_0, \hat{z}) < h(u_1, \hat{z})$ *for all* u_0, u_1 *with* $0 \leq u_0 < u_1 \leq 1$ *and* $\hat{z} \in [0, 1]$, *and* $h(u, \hat{z}_0) < h(u, \hat{z}_2)$ *for all* $\hat{z}_0 \in Z_0$, $\hat{z}_2 \in Z_2$ *and* $0 \leq u < 1$.

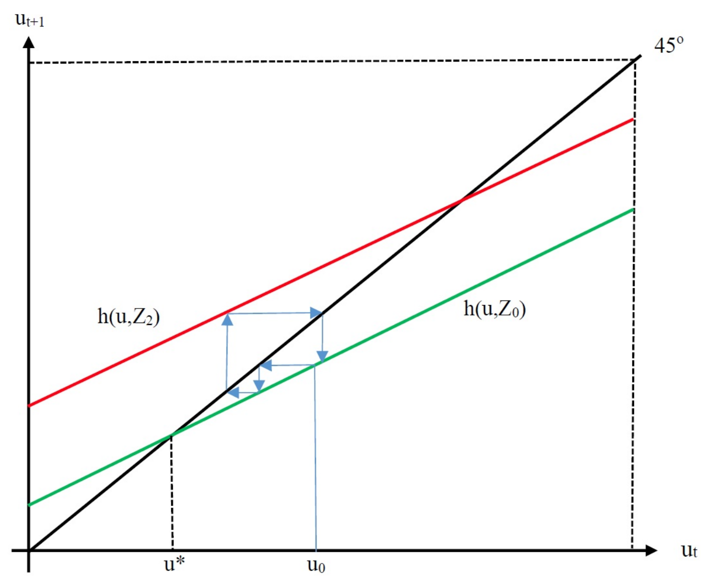
4.4 Agency Business Cycles

In the previous subsection, we have established the existence of a Recursive Equilibrium where firms randomize over firing or keeping their non-performing workers in a perfectly correlated fashion. Here, we characterize the aggregate behavior of the economy in this equilibrium.

First, notice that the economy faces aggregate uncertainty. Given the current unemployment rate $u \in [0, 1)$, all firms their non-performing workers if $\hat{z} \in Z_2$, an event that happens with probability π_2, and all firms keep their non-performing workers if $\hat{z} \in Z_0$, an event that happens with probability $\pi_0 = 1 - \pi_2$, where π_2 is given by (19). In the first case, the rate at which employed workers become unemployed is $\delta + (1-\delta)p_\ell(1)$ and next period's unemployment rate is $h(u, Z_2)$. In the second case, the rate at which employed workers become unemployed is δ and next period's unemployment rate is $h(u, Z_0)$, where $h(u, Z_0) < h(u, Z_2)$.

Aggregate uncertainty leads to aggregate economic fluctuations, which we name *Agency Business Cycles* (or ABCs). These cycles are illustrated in Figure 3. Imagine an economy in which the unemployment rate is u_0. As long as firms keep their non-performing workers,

Figure 3: Agency Business Cycles



the unemployment rate evolves according to the policy function $h(u, Z_0)$. The rate at which employed workers become unemployed remains constant at δ and the unemployment rate falls towards u^*. When firms eventually fire their non-performing workers, unemployment evolves according to the policy function $h(u, Z_2)$. The rate at which employed workers become unemployed increases to $\delta + (1 - \delta)p_\ell(1)$ and so does the unemployment rate. After the firing episode, the rate at which employed workers become unemployed returns to δ and the unemployment rate starts falling back towards u^*.

Second, notice that the aggregate fluctuations described above are endogenous. They are not driven by exogenous shocks to fundamentals. In fact, all fundamentals are constant. Fluctuations are not driven by exogenous shocks to the future value of fundamentals. In fact, all fundamentals are known to be constant. Fluctuations are not driven by exogenous shocks to equilibrium selection. In fact, firms always play the Perfect Correlation Equilibrium in the stage game. Moreover, the firms' randomization over firing and keeping non-performing workers is not a randomization over different equilibria. There is no equilibrium in which firms keep non-performing workers with probability 1, as this violates the incentive compatibility constraint of workers. Similarly, there is no equilibrium in which firms keep their non-performing workers with probability 0. Fluctuations in our model emerge from a feature of the equilibrium: *correlated randomization*. Specifically, every firm wants to randomize over keeping or firing a non-performing workers in order to provide incentives, and the various firms in the economy want to correlate the outcome of their randomization.

Third, notice that the economic fluctuations emerging in equilibrium are stochastic. The time at which a firing burst takes place is random and so are the dynamics of the unemployment rate, the rate at which employed workers become unemployed and the rate at which unemployed workers become employed. Specifically, the probability of a firing burst is

$$\pi_2 = \frac{\psi}{\beta(p_h(1) - p_h(0))(1-\delta)V(h(u, Z_2))}.$$

The probability of a firing burst is endogenous, showing very clearly that there is no exogenous force driving aggregate fluctuations in our model. The probability of a firing episode depends positively on the disutility of effort, ψ, negatively on the difference between successful production if the worker does and does not put effort, $p_h(1) - p_h(0)$, and on the cost to the worker from losing his job, $V(h(u, Z_2))$. The first two terms are parameters and are constant over time. The last term is a value function that depends on the current unemployment rate. In particular, since $h(u, Z_2)$ is a strictly increasing function of u and V is a strictly increasing function, it follows that the probability of a firing burst is strictly decreasing in the unemployment rate u. Therefore, as the economy moves from u_0 to u^* in Figure 3, the probability of a firing burst grows larger and larger. When the firing burst takes place, the unemployment rate goes up and the probability of a firing episode goes down.

Finally, notice in our model of endogenous fluctuations, recessions have a different nature than in models where cycles are driven by fluctuations in productivity (e.g., Kydland and Prescott 1983 and Mortensen and Pissarides 1994). In those models, a recession is a time when the gains from trade in the labor market are unusually low because of some negative shock to productivity. For this reason, firms and workers do not search very hard and unemployment is high. In our model, a recession is a time when the gains from trade in the labor market are unusually high because the value of being unemployed is unusually low. This the time when firms find it optimal to fire their non-performing workers and, hence, unemployment is high. Thus, while models of cyclical fluctuations driven by productivity shocks imply a negative correlation between gains from trade in the labor market and unemployment, our model implies a positive correlation between gains from trade and unemployment. Formally, in our model, the workers' gains from trade, $V(u)$, and the firm's gains from trade, $J(u)$, are increasing in the unemployment rate u.

The above observations are summarized in the following theorem.

Theorem 5: (Agency Business Cycles) *(i) Aggregate uncertainty: For any $u \in [0, 1)$, next period's unemployment rate is $h(u, Z_2)$ with probability π_2 and $h(u, Z_0)$ with probability $\pi_0 = 1 - \pi_2$, where $h(u, Z_0) < h(u, Z_2)$ and π_2 is given by (19). (ii) Probability of a recession and unemployment: The probability π_2 is strictly decreasing in u. (iii) Gains from trade and unemployment: The net value of employment to a worker, $V(u)$, and the net value of a worker to a firm, $J(u)$, are strictly increasing in u.*

5 Quantifying the Theory

We do not believe that Agency Business Cycles are the only cause of cyclical fluctuations in the US economy—certainly fluctuations in unemployment benefits, in productivity, and shocks to the financial sector are important drivers of recessions and expansions. We also realize that our theory is not rich enough to say anything about important features of the business cycles, such as the dynamics of investment and inventories. Nevertheless, we think it is worthwhile to calibrate our theory to get a sense of the magnitude of ABCs relative to the cyclical fluctuations of the US labor market. We find that ABCs can generate fluctuations in unemployment, in the rate at which employed workers become unemployed (EU rate) and in the rate at which unemployed workers become employed (the UE rate) that are of the same order of magnitude as those observed in the US economy. Moreover, ABCs generate labor market fluctuations that are uncorrelated with labor productivity, as they have been in the US since 1984.

We then examine three qualitative features of ABCs. First, in ABCs, a recession starts with an increase in the EU rate which drives up unemployment. In turn, the increase in unemployment lowers the UE rate because of decreasing returns to matching. Even though the increase in the EU rate is short-lived, the unemployment rate (and, consequently, the UE rate) recovers slowly because it takes time to rebuild the stock of firm-worker matches. We find that, in the US economy, labor market variables follow the same pattern of leads and lags that is implied by our theory. Second, in ABCs, the probability of a recession is endogenous and, in particular, it becomes higher when unemployment is lower. We find that, in the US economy as in our theory, the probability of a recession increases when unemployment is lower. Third, in ABCs, a recession is a period when the value of workers in the market relative to the value of workers at home is abnormally high. We find preliminary evidence that this is also true for the US economy.

5.1 Calibration

We start by calibrating the primitives of the model. Preferences are described by the worker's periodical utility function, $\upsilon(c) - \psi e$, and by the discount factor, β. Market production is described by the realizations of output, y_h and y_ℓ, by the probability that output is high given the worker's effort, $p_h(1)$ and $p_h(0)$, and by the exogenous job destruction probability δ. Home production is described by the output of an unemployed worker, b. The search and matching process is described by the vacancy cost, k, and the matching function, $M(u, v)$. We specialize the utility function for consumption to be of the form $\upsilon(c) = c^{1-\sigma}/(1-\sigma)$, where σ is the coefficient of relative risk aversion. We specialize the matching function to be of the form $M(u, v) = A(u)m(u, v)$, where $m(u, v) = uv(u^\xi + v^\xi)^{-1/\xi}$ is a constant returns to scale matching function with an elasticity of substitution ξ, and $A(u) = \exp(-\rho u)$ is a matching efficiency function with a semi-elasticity with respect to unemployment of $-\rho$.

We calibrate the parameters of the model to match some key statistics of the US labor market between 1951 and 2014, such as the average unemployment rate, the average UE rate, and the average EU rate. We measure the US unemployment rate as the CPS civilian unemployment rate. We measure the UE and the EU rates using the civilian unemployment and short-term unemployment rates from the CPS, following the same methodology as in Shimer (2005). We measure labor productivity as output per worker in the non-farm sector.

We calibrate the basic parameters of the model as is now standard in the literature. We choose the model period to be one month. We set the discount factor, β, so that the annual real interest rate, $(1/\beta)^{1/12} - 1$, is 5 percent. We choose the vacancy cost, k, and the exogenous job destruction probability, δ, so that the model matches the average UE and EU rates in the US economy (respectively, 44% and 2.6%). We normalize the average value of market production, $p_h(1)y_h + (1 - p_h(1))y_\ell$, to 1. We choose the value of home production, b, to be 70% of the average value of market production, which Hall and Milgrom (2008) argue is a reasonable estimate for the US economy.

We calibrate the parameters of the model that determine the extent of the agency problem as follows. The probability that the realization of output is high when the worker exerts effort, $p_h(1)$, affects the number of non-performing workers and, hence, the magnitude of firing bursts. The disutility of effort, ψ, affects the frequency at which firms need to fire non-performing workers, hence, the frequency of firing bursts. Therefore, we choose y_h so that the model generates the same standard deviation in the cyclical component of the EU rate as in the US economy (approximately 9%). We choose ψ so that, on average, firms fire their non-performing workers once every 50 months. The parameters y_h and y_ℓ and $p_h(0)$ cannot be uniquely pinned down. Given that average output is 1, the realizations of output y_h and y_ℓ do not affect the equilibrium, as long as it is optimal for firms to require effort from their workers. Similarly, the probability $p_h(0)$ only affects the equilibrium through the ratio $\psi/(p_h(1) - p_h(0))$. Therefore, we choose some arbitrary values for y_h, y_ℓ and $p_h(0)$ such that firms find it optimal to require effort.

Finally, we need to choose values for the parameters in the utility and the matching functions. We set the coefficient σ of relative risk aversion in the utility function υ to 1. This is a standard value from micro-estimates of risk aversion. We set the elasticity ξ of substitution between unemployment and vacancy in the matching function m to 1.24. This is the value estimated by Menzio and Shi (2011).[8] We tentatively set the parameter ρ in the

[8] Correctly estimating a matching function requires taking into account the fact that unemployed and employed workers all search for vacancies to some degree. Using a model of search off and on the job, Menzio and Shi (2011) estimate the elasticity of substitution between searching workers and vacant jobs in the matching function to be 1.24.

Table 1: Calibrated Parameters

	Description	(a)	(b)	(c)
y_h	high output	1.03	1.03	1.03
$p_h(1)$	prob. y_h given $e = 1$	.967	.967	.967
$p_h(0)$	prob. y_h given $e = 0$	.467	.467	.467
b	UI benefit/leisure	.700	.700	.700
σ	relative risk aversion	1	1	1
ψ	disutility of effort	.007	.006	.007
δ	exogenous destruction	.025	.025	.025
k	vacancy cost	.257	.342	.126
ξ	elasticity of sub. u and v	1.24	1.24	1.24
ρ	semi-elasticity A wrt u	6	3	10

Notes: Calibrated parameters for $\rho = 6$ (column a), $\rho = 3$ (column b) and $\rho = 10$ (column c). In all calibrations, the low realization of output is set to zero.

matching efficiency function A to 6, but we also report our findings for $\rho = 3$ and for $\rho = 10$.

5.2 Magnitude and Properties of ABCs

Table 1 reports the calibrated values of the parameters of the model. Table 2 reports summary statistics for the cyclical component of the unemployment rate, the UE rate, the EU rate and of average labor productivity generated by the calibrated model. The first take-away from Table 2 is that ABC can account for a significant fraction of the volatility of the US labor market. For $\rho = 6$ (our benchmark calibration), the model generates 55% of the empirical volatility of unemployment, 33% of the empirical volatility of the UE rate and all of the empirical volatility of the EU rate. For $\rho = 10$, the model generates 75% of the empirical volatility of unemployment, 60% of the empirical volatility of the UE rate and all of the empirical volatility of the EU rate. For $\rho = 3$, the model generates less volatility. Naturally, these are upper bounds on the magnitude of ABCs, as the model is calibrated to account for all of the volatility of the EU rate.

The second take-away from Table 2 is that ABCs display the same pattern of comovement between the unemployment rate, the UE rate and the EU rate as in the data. That is, ABCs display positive comovement between the unemployment and the EU rate and negative comovement between the unemployment and the UE rate. ABCs display a positive comovement between unemployment and vacancies, while in the data unemployment and vacancies are negatively correlated. This discrepancy is an artifact of the simplifying and counterfactual assumption that workers search the labor market only when they are unemployed. Indeed, under this assumption, an increase in unemployment causes an increase in the number of workers searching the labor market which, in turn, gives firms an incentive to create more vacancies. Under the more realistic assumption that workers search both off

Table 2: Agency Business Cycles

		u rate	UE rate	EU rate	APL
Model: $\rho = 6$	std	9.34	4.09	9.11	0
	cor. wrt u	1	-.98	.32	-
Model: $\rho = 3$	std	7.89	1.92	9.12	0
	cor. wrt u	1	-.97	.39	-
Model: $\rho = 10$	std	12.3	7.05	9.15	0
	cor. wrt u	1	-.99	.25	-
Data: 1951-2014	std	16.9	12.9	9.7	1.98
	cor. wrt u	1	-.94	.80	-.37
Data: 1984-2014	std	17.3	13.8	6.91	1.38
	cor. wrt u	1	-.96	.70	.09

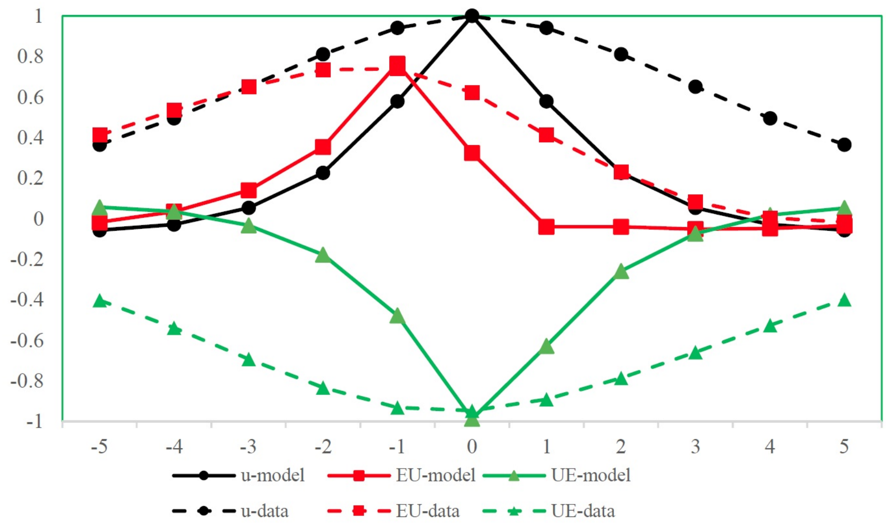
Notes: For each variable, we construct quarterly time-series by taking 3-month averages. We then compute the cyclical component of each variable as the percentage deviation of its quarterly value from a Hodrick-Prescott trend constructed using a smoothing parameter of 100,000.

and on the job, an increase in unemployment does not cause an increase in the number of workers searching the labor market and, hence, does not give firms a clear incentive to create more vacancies.

Finally, Table 2 shows that ABCs display relatively large fluctuations in unemployment that are uncorrelated with fluctuations in labor productivity. This is an important feature of the model. The empirical correlation between unemployment and labor productivity—which was significantly negative for the period 1951-1984—has become basically zero for the period 1984-2014. Therefore, over the period 1951-1984, fluctuations in labor productivity may have driven the cyclical movements of the US labor market. In contrast, over the period 1984-2014, fluctuations in labor productivity seem an unlikely driver of cycles in the US labor market. Our theory provides an explanation for the recent lack of comovement between labor productivity and unemployment by identifying a novel, non-technological source of aggregate fluctuations.

Overall, Table 2 shows that ABCs can be large. Now, we turn to examine three qualitative features of ABCs. First, in ABCs, a recession starts with an increase in the EU rate. The increase in the EU rate quickly drives up the unemployment rate which, because of decreasing returns to matching, lowers the UE rate. Even though the EU rate falls back immediately to its pre-recession levels, the unemployment rate recovers slowly because it takes time to rebuild the stock of firm-worker matches. Because of decreasing returns to matching, also the UE rate recovers slowly. The timing of ABCs squares well with the data. In the US, a typical recession starts with a sharp short-lived increase in the EU rate (and, especially, a sharp increase in the firing rate), which causes the unemployment rate to increase very quickly. The recovery of the unemployment rate is much slower and so is the recovery in the UE rate.

Figure 4: Leads and Lags in UE and EU



Notes: Correlation between UE, EU, unemployment rates in quarter $t+x$ with unemployment in quarter t.

Figure 4 formalizes these observations by displaying the correlation between unemployment in quarter t and other labor market variables in quarter $t+x$. It is immediate to see that the EU rate leads the unemployment rate by a quarter—in the sense that the absolute value of the correlation between unemployment in quarter t and the EU rate in quarter $t+x$ is highest for $x=-1$—while the UE rate is contemporaneous with unemployment– in the sense that the correlation between unemployment in quarter t and the UE rate in quarter $t+x$ is highest for $x=0$.[9] Moreover, one can see that the correlation between current unemployment and future EU rate dies off much more rapidly than the correlation between current unemployment and future unemployment and UE rates. The US labor market displays exactly the same pattern of leads and lags, as can be seen from the dashed lines in Figure 4. The main difference between the correlation functions in the model and in the data is that, in the model, all correlations die off more quickly than in the data. This shortcoming of the model is due to the fact that the driving mechanism of ABCs (i.e. the correlated randomization over firing or keeping non-performing workers) must be i.i.d. over time.[10]

Next, we turn to the hallmark property of our theory. In ABC theory (as in any theory of endogenous and stochastic business cycles), the probability of a recession is an endogenous object which depends on fundamentals and the state of the economy. In particular, in ABC theory, the probability of a recession depends negatively on the unemployment rate (see

[9]Fujita and Ramey (2009) were the first to point out that the EU rate leads the unemployment rate, while the UE rate is contemporaneous with the unemployment rate. Our theory explains this pattern as a causal link from the EU rate to the unemployment rate and from the unemployment rate to the UE rate.

[10]We believe we could create persistence in firings by assuming that firm and workers observe output in a staggered fashion—e.g., half in odd periods and half in even periods.

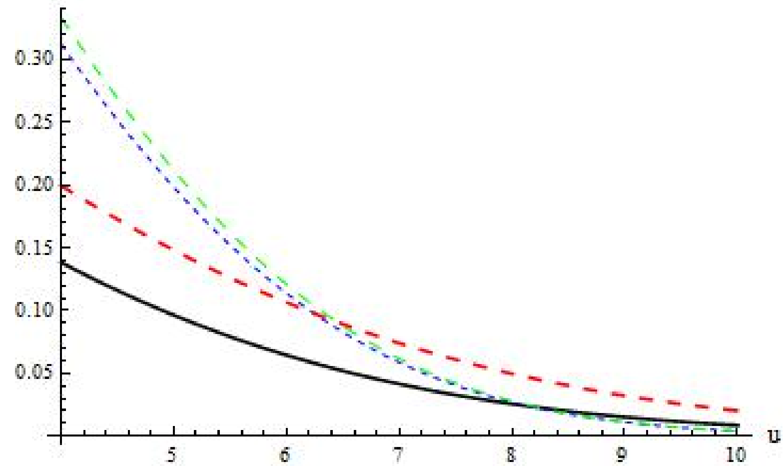
Theorem 5). Intuitively, the higher is the unemployment rate, the higher is the cost to the worker from losing his job and, hence, the lower is the probability with which firms need to fire non-performing workers in order to give them an incentive to perform. In order to find out whether this feature of ABCs is borne out in the data, we take the time-series for the unemployment rate and define the start of a recession as a quarter in which the unemployment rate turns from decreasing to increasing and keeps growing for at least two consecutive quarters.[11] We then estimate a probit model for the probability of the start of a recession as a function of the unemployment rate. The estimated coefficient on the unemployment rate is -.21, with a standard deviation of 11%. The estimated coefficient implies that the probability of a recession increases from 8% to 11% as the unemployment falls from 5.5% to 4.5%. Clearly, it is not possible to precisely estimate the effect of unemployment on the probability that a recession starts because recessions are relatively rare events. In order to gather more observations, we take the time-series of unemployment for Australia, Canada, Italy, Japan, France and the UK.[12] After taking out the average unemployment from the time-series of each country, we merge these data to those for the US and re-estimate the probit model. The estimated coefficient on the unemployment rate is -.20 with a standard deviation of 4%. Figure 5 plots the estimated probability of a recession as a function of unemployment.

Finally, we examine the cyclicality of the gains from trade in the labor market. In ABC theory, a recession is a time when the gains from trade in the labor market are unusually high because the value of being unemployed is unusually low. This the time when firms find it optimal to fire their non-performing workers and, hence, unemployment is high. In ABC theory, gains from trade and unemployment are positively correlated (see Theorem 5). In the Real Business Cycle (RBC) theory of Kydland and Prescott (1982), in Mortensen and Pissarides (1994) or in any other theory where cycles are driven by productivity shocks, a recession is a time when the gains from trade in the labor market are unusually low (due to low productivity) and, hence, unemployment is high. In RBC theory, gains from trade and unemployment are negatively correlated. To paint a picture, in ABC theory, a recession is a time when the TV set is broken at home. In RBC theory, a recession is a time when it is raining on the marketplace. Given the striking difference between these theories, it is then natural to wonder whether the gains from trade in the labor market are indeed pro or countercyclical. In order to address this question, we construct a rudimentary empirical time-series of the workers' gains from trade, i.e. the difference between the value of being

[11] The estimates are robust to alternative definitions of a recession.

[12] These are countries for which the OECD provides sufficiently long time-series for unemployment. We drop Germany from the sample because of the large movements in unemployment related to the unification. When estimated for each country separately, the coefficient on unemployment in the probit model is always negative.

Figure 5: Probability of a Recession



Notes: Estimated probability of a recession next quarter as a function of current unemployment for US (black solid line), UK (long-dashed blue line), Australia (short-dashed green line) and for a set of countries pooled together (thick dashed line).

employed and the value of being unemployed.[13]

We measure the value of employment to a worker, $W_{1,t}$, and the value of unemployment to a worker, $W_{0,t}$, as

$$\begin{aligned} W_{1,t} &= w_t + \beta \left[h^{EU}_{t+1} W_{0,t+1} + (1 - h^{EU}_{t+1}) W_{1,t+1} \right], \\ W_{0,t} &= b_t + \beta \left[h^{UE}_{t+1} W_{1,t+1} + (1 - h^{UE}_{t+1}) W_{0,t+1} \right], \end{aligned} \tag{26}$$

where w_t is a measure of the real wage in month t, b_t is a measure of unemployment benefit/value of leisure in month t, h^{EU}_{t+1} is a measure of the EU rate in month $t+1$, h^{UE}_{t+1} is a measure of the UE rate in month $t+1$, and $W_{1,t+1}$ and $W_{0,t+1}$ are respectively the value of employment and unemployment in month $t+1$. We measure the net value of employment to a worker, V_t, in month t as the difference between $W_{1,t}$ and $W_{0,t}$.

We measure w_t using the time-series for the hourly wage that have been constructed by Haefke, Sonntag and van Rens (2013). We consider two alternative time-series: the average hourly wage in the cross-section of all employed workers, and the average hourly wage in the cross-section of newly hired workers after controlling for the composition of new hires. The first time-series may be more appropriate when we want to interpret V_t as the cost of losing a job to a worker, the second-time series may be more appropriate when we want to interpret V_t as the benefit of finding a job to a worker. As in Shimer (2005), we measure h^{UE}_t and h^{EU}_t

[13] We only attempt to measure the gains from trade accruing to the workers as measuring the gains from trade accruing to the firms requires the difficult task of constructing a time-series for profit per employee. When we try to measure profit per employee as labor productivity minus wages, we find that the firm's gains from trade are countercyclical as well.

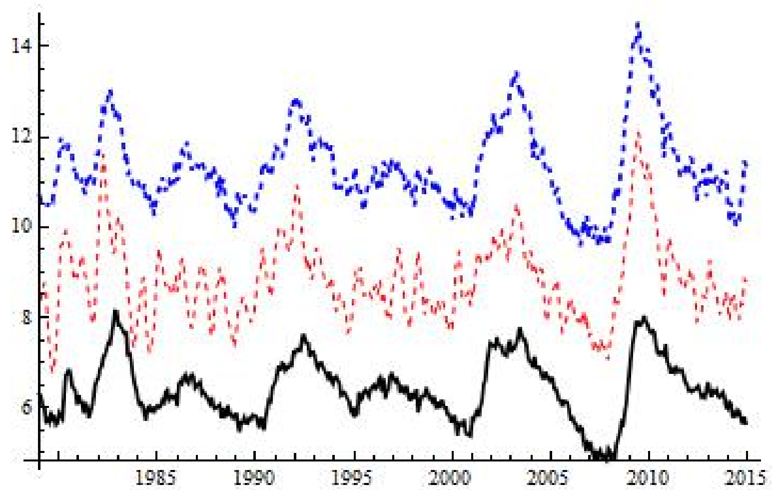
using, respectively, the values for the EU rate and UE rates implied by the time-series for unemployment and short-term unemployment. In order to make the time-series for w_t, h_t^{UE} and h_t^{EU} stationary, we construct their Hodrick-Prescott trend using a smoothing parameter of 10^5. We then take the difference between the value of each variable and its trend and add this difference to the time-series average for that variable. Since b_t is not directly observable, we follow Hall and Milgrom (2008) and set it equal to 70% of the average of w_t.

We are now in the position to construct time-series for the value of employment to a worker, $W_{1,t}$, and the value of unemployment to a worker, $W_{0,t}$, over the period going from January 1979 to December 2014. We compute the values for W_1 and W_0 in December 2014 by assuming that, from January 2015 onwards, w, h^{UE}, and h^{EU} are equal to their historical averages. Given the values for W_1 and W_0 in December 2014, we compute the values for W_1 and W_0 from November 2014 back to January 1979 by using equation (26) and the time-series for w, b, h^{UE}, and h^{EU}. The reader should notice that the values of W_1 and W_0 thus computed differ from their theoretical counterpart because they are constructed using the realizations of future w, h^{UE}, and h^{EU}, rather than the expectation of these variables.

Figure 6 presents the result of our calculations. Figure 6 displays the time-series for the net value of employment to a worker, V_t, computed using the average wage of all employed workers (solid black line) and the average wage of newly hired workers (dashed black line). Figure 4 also displays the time-series for the detrended unemployment rate (solid grey line). The figure clearly shows that V_t is countercyclical, in the sense that V_t moves together with the unemployment rate. This is the case whether we measure V_t using the average wage of all employed workers—in which the correlation between V_t and unemployment is 80%—or whether we measure V_t using the average wage of newly hired workers—in which case the correlation between V_t and unemployment is 71%. Mechanically, V_t is countercyclical because, when unemployment increases, the decline in the value of being unemployed caused by the large decline in the UE rate is larger than the decline in the value of being employed cause by the small decline in the wage and by the short-lived increase in the EU rate. The countercyclicality of V_t is a very robust finding. The measure of V_t becomes even more countercyclical if b_t is assumed to be constant fraction of w_t rather than a constant. The correlation between V_t and u_t remains practically unchanged if we do not filter the data.

The finding that V_t is countercyclical means that recessions are times when unemployed workers find it especially valuable to find a job, and when employed workers find it especially costly to lose a job. The finding is in stark contrast with the view of recessions as "days of rain in the marketplace" advanced by Kydland and Prescott (1982) or by Mortensen and Pissarides (1994). In contrast, the finding is supportive of the view of recessions as "days of no TV at home" advanced by our theory. Notice that our finding that V_t should not be entirely surprising. Using a different, more sophisticated approach and richer data, Davis

Figure 6: Net Value of Employment



Notes: This figure reports the time series of unemployment (thick black solid line), the net value of employment computed using average wages (thick dashed blue line) and computed using the wage of new hires (thin dashed red line).

and von Wachter (2011) show that the lifetime earning cost of losing a job is much higher in recessions than in expansions. Even if one is skeptical about our theory of recessions, the finding that V_t is countercyclical represents a challenge for many existing theories of business cycles.

6 Conclusions

We developed a theory of endogenous and stochastic fluctuations in aggregate economic activity. The structure of the theory is simple. Individual agents find it optimal to randomize over some choice in order to overcome a non-convexity in their decision problem and different agents across the economy find it optimal to correlate the outcome of their randomization. Correlated randomization leads to aggregate fluctuations. These fluctuations are endogenous—as they emerge from the agents' decision to randomize in a correlated fashion—and they are stochastic—as they are the manifestation of a randomization process. The unique feature of the theory is to generate a stochastic process for "shocks" that is determined as an equilibrium outcome. We exemplified this theoretical structure with a model where firms randomize over firing or keeping non-performing workers to provide them with an incentive to exert effort and where different firms correlate the outcome of the randomization to economize on the probability of firing. Other applications of the theoretical structure seem promising. A particularly interesting application is to the credit market. As in Clementi and Hopenhayn (2006), individual lenders may find it optimal to randomize over continuing or terminating credit lines to firms that have performed poorly in the past. Different lenders may find it optimal to correlate the outcome of their randomization in order to minimize agency costs. In this context, correlated randomization might generate endogenous and stochastic credit crunches.

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Online Appendix

A Proof of Lemmas 1-4

A.1 Proof of Lemma 1

Let $\rho \geq 0$ denote the Lagrange multiplier on the worker's incentive compatibility constraint, let $\overline{\nu}(y, \hat{z}) \geq 0$ denote the multiplier on the constraint $1 - s(y, \hat{z}) \geq 0$ and let $\underline{\nu}(y, \hat{z})$ denote the multiplier on the constraint $s(y, \hat{z}) \geq 0$.

The first order condition with respect to the firing probability $s(y_\ell, \hat{z})$ is given by

$$(1-\delta)\left[F(x)V(\hat{z}) + W(x)J(\hat{z})\right] = \rho\beta(1-\delta)(p_h(1) - p_h(0))V(\hat{z}) + \underline{\nu}(y_\ell, \hat{z}) - \overline{\nu}(y_\ell, \hat{z}), \quad \text{(A1)}$$

together with the complementary slackness conditions $\overline{\nu}(y_\ell, \hat{z}) \cdot (1 - s(y_\ell, \hat{z})) = 0$ and $\underline{\nu}(y_\ell, \hat{z}) \cdot s(y_\ell, \hat{z}) = 0$. The left-hand side of (A1) is the marginal cost of increasing $s(y_\ell, \hat{z})$. This cost is given by the decline in the product of the worker's and firm's gains from trade caused by a marginal increase in the firing probability $s(y_\ell, \hat{z})$. The right-hand side of (A1) is the marginal benefit of increasing $s(y_\ell, \hat{z})$. This benefit is given by the value of relaxing the worker's incentive compatibility and the $s(y_\ell, \hat{z}) \geq 0$ constraints net of the cost of tightening the $s(y_\ell, \hat{z}) \leq 1$ constraint by marginally increasing the firing probability $s(y_\ell, \hat{z})$.

Similarly, the first order condition with respect to the firing probability $s(y_h, \hat{z})$ is given by

$$(1-\delta)\left[F(x)V(\hat{z}) + W(x)J(\hat{z}) + \rho\beta(p_h(1) - p_h(0))V(\hat{z})\right] = \underline{\nu}(y_h, \hat{z}) - \overline{\nu}(y_h, \hat{z}), \quad \text{(A2)}$$

together with the complementary slackness conditions $\overline{\nu}(y_h, \hat{z}) \cdot (1 - s(y_h, \hat{z})) = 0$ and $\underline{\nu}(y_h, \hat{z}) \cdot s(y_h, \hat{z}) = 0$. The left-hand side of (A2) represents the marginal cost of increasing $s(y_h, \hat{z})$. The rigth-hand side of (A2) represents the marginal benefit of increasing $s(y_h, \hat{z})$. Notice that increasing the firing probability $s(y_h, \hat{z})$ tightens the worker's incentive compatibility constraint and, hence, the term in ρ is now on the left-hand side of (A2).

Suppose $\rho = 0$. First, notice that the left-hand side of (A1) is strictly positive as $V(\hat{z}) > 0$, $J(\hat{z}) > 0$ by assumption, and $W(x) > 0$, $F(x) > 0$ at the optimum x^*. The right-hand side of (A1) is strictly positive only if $\underline{\nu}(y_\ell, \hat{z}) > 0$. Hence, if $\rho = 0$, the only solution to the first order condition with respect to the firing probability $s(y_\ell, \hat{z})$ is 0. Next, notice that the left-hand side of (A2) is strictly positive and the right-hand side is strictly positive only if $\underline{\nu}(y_h, \hat{z}) > 0$. Hence, if $\rho = 0$, the only solution to the first order condition with respect to the firing probability $s(y_h, \hat{z})$ is 0. However, if $s(y_\ell, \hat{z}) = s(y_h, \hat{z}) = 0$, the worker's incentive compatibility constraint is violated. Therefore, $\rho > 0$ and the worker's incentive compatibility constraint holds with equality.

A.2 Proof of Lemma 2

The first order condition with respect to $s(y_h, \hat{z})$ is given by (A2) together with the complementary slackness conditions $\overline{\nu}(y_h, \hat{z})(1 - s(y_h, \hat{z})) = 0$ and $\underline{\nu}(y_h, \hat{z})s(y_h, \hat{z}) = 0$. The left-hand side of (A2) is strictly positive. The right-hand side of (A2) is strictly positive only if $\underline{\nu}(y_h, \hat{z}) > 0$. Therefore, the first order condition is satisfied only if $\underline{\nu}(y_h, \hat{z}) > 0$ and, hence, only if $s(y_h, \hat{z}) = 0$.

A.3 Proof of Lemma 3

Using the definition of $\phi(\hat{z})$, we can rewrite the first order condition with respect to the firing probability $s(y_\ell, \hat{z})$ as

$$(1 - \delta)V(\hat{z})\left[F(x) + W(x)/\phi(\hat{z}) - \rho\beta(p_h(1) - p_h(0))\right] = \underline{\nu}(y_\ell, \hat{z}) - \overline{\nu}(y_\ell, \hat{z}), \tag{A3}$$

together with $\overline{\nu}(y_\ell, \hat{z}) \cdot (1 - s(y_\ell, \hat{z})) = 0$ and $\underline{\nu}(y_\ell, \hat{z}) \cdot s(y_\ell, \hat{z}) = 0$. The left-hand side of (A3) is strictly decreasing in $\phi(\hat{z})$. The right-hand side of (A3) is strictly positive if $\underline{\nu}(y_\ell, \hat{z})$ is strictly positive and it is strictly negative if $\overline{\nu}(y_\ell, \hat{z})$ is strictly positive. Therefore, there exists a ϕ^* such that if $\phi(\hat{z}) > \phi^*$, the left-hand side is strictly negative and the solution to (A3) requires $\overline{\nu}(y_\ell, \hat{z}) > 0$. In this case, the solution to the first order condition for $s(y_\ell, \hat{z})$ is 1. If $\phi(\hat{z}) < \phi^*$, the left-hand side is strictly positive and the solution to (A3) requires $\underline{\nu}(y_\ell, \hat{z}) > 0$. In this case, the solution to the first order condition for $s(y_\ell, \hat{z})$ is 0.

A.4 Proof of Lemma 4

The first order condition with respect to the wage w is given by

$$F(x)\upsilon'(w) - W(x) = 0. \tag{A4}$$

The left-hand side of (A4) is the increase in the product of the worker's and firm's gains caused by a marginal increase in the worker's wage w. A marginal increase in w, increases the worker's gains from trade by $\upsilon'(w)$ and decreases the firm's gains from trade by 1. Therefore, a marginal increase in w, increases the product of the worker's and firm's gains from trade by $F(x)\upsilon'(w) - W(x)$. The first order condition for w states that the effect of a marginal increase in w is zero.

B Proof of Lemmas 5-6

B.1 Proof of Lemma 5

We first consider a firm-worker pair in group K. Since $P_K(T_K|T_{K-1})$ is strictly increasing in T_{K-1} and $\hat{u}$ is strictly increasing in T_K, $E[\hat{u}|T_{K-1}]$ is strictly increasing in T_{K-1}. In turn, since

$E[\hat{u}|T_{K-1}]$ is strictly increasing in T_{K-1} and $\phi(\hat{u})$ is strictly increasing in $\hat{u}$, $\phi(E[\hat{u}|T_{K-1}])$ is strictly increasing in T_{K-1}. It then follows from property (iii) of the optimal contract that there exists a T^*_{K-1} such that $s_K(y_\ell, T_{K-1}) = 0$ for all $T_{K-1} < T^*_{K-1}$, and $s_K(y_\ell, T_{K-1}) = 1$ for all $T_{K-1} > T^*_{K-1}$. This establishes that the firing probability $s_K(y_\ell, T_{K-1})$ has the desired threshold property. The threshold property implies that the measure t_K of workers separating from firms in group K is increasing in the measure T_{K-1} of workers separating from firms in groups 1 through $K-1$.

Next, consider a firm-worker pair in group $K-1$. Since $P_{K-1}(T_{K-1}|T_{K-2})$ is strictly increasing in T_{K-2} and t_K is increasing in T_{K-1}, it follows that $E[\hat{u}|T_{K-2}]$ and, in turn, $\phi(E[\hat{u}|T_{K-2}])$ are strictly increasing in T_{K-2}. Then property (iii) of the optimal contract implies that there exists a T^*_{K-2} such that $s_{K-1}(y_\ell, T_{K-2}) = 0$ for all $T_{K-2} < T^*_{K-2}$, and $s_{K-1}(y_\ell, T_{K-2}) = 1$ for all $T_{K-2} > T^*_{K-2}$. This establishes that the firing probability $s_{K-1}(y_\ell, T_{K-2})$ has the desired threshold property. The threshold property implies that the measure t_{K-1} of workers separating from firms in group $K-1$ is increasing in the measure T_{K-2} of workers separating from firms in groups 1 through $K-2$. By repeating the above argument for firm-worker pairs in groups $K-i$, with $i = 2, 3, \ldots K-1$, we can establish that the firing probability $s_{K-i}(y_\ell, T_{K-i-1})$ has the threshold property and that the measure t_{K-i} is increasing in T_{K-i-1}.

B.2 Proof of Lemma 6

Let T^*_1 be given as in (22) and let $T^*_i = E[t_1] + (i-1)(t_h - t_\ell)/2$ for $i = 2, 3, \ldots\ K-1$. Given the thresholds $\{T^*_1, T^*_2, ...T^*_{K-1}\}$, we can compute the unconditional probability distribution of the random variable T_i. For $N \to \infty$, T_2 is approximately equal to $E[t_1]+t_\ell$ if $T_1 < T^*_1$, and it is approximately equal to $E[t_1]+t_h$ if $T_1 > T^*_1$. Since $E[t_1]+t_\ell < T^*_2$ and $E[t_1]+t_h > T^*_2$, T_3 is approximately equal to $E[t_1]+2t_\ell$ if $T_1 < T^*_1$, and T_3 is approximately equal to $E[t_1]+2t_h$ if $T_1 > T^*_1$. Similarly, for $i = 4, 5, \ldots K$, T_i is approximately equal to $E[t_1] + (i-1)t_\ell$ if $T_1 < T^*_1$, and it is approximately equal to $E[t_1] + (i-1)t_h$ if $T_1 > T^*_1$. Hence, if $T_1 < T^*_1$, firms in groups 2 through K fire their non-performing workers with probability 0 and the total measure of workers separating from firms is $T_K = E[t_1] + (K-1)t_\ell$. If $T_1 > T^*_1$, firms in groups 2 through K fire their non-performing workers with probability 1 and the total measure of workers separating from firms is $T_k = E[t_1] + (K-1)t_h$.

The above observations imply that benefit from exerting effort for a worker employed at a firm in group $i = 2, 3, \ldots K$ is given by

$$\beta(1-\delta)(p_h(1) - p_h(0))(1 - P_1(T^*_1))V(E[\hat{u}|T_K = E[t_1] + (K-1)t_h]). \tag{B1}$$

The benefit in (B1) is equal to the cost ψ of exerting effort given the choice of T^*_1. Thus, the incentive compatibility for workers employed by firms in groups $i = 2, 3, \ldots\ K$ is satisfied.

The incentive compatibility constraint for workers employed by firms in group 1 is satisfied given the choice of s_1.

C Proof of Theorem 4

The existence proof is based on an application of Schauder's fixed point theorem. In particular, we are going to take arbitrary value functions $V^+(u)$ and $J^+(u)$ denoting, respectively, the worker's gains from trade at the end of the production stage and the firm's gains from trade at the end of the production stage. Then, we are going to use the conditions for a Recursive Equilibrium with perfect correlation to construct a mapping $\mathcal{F}$ that returns updates for $V^+(u)$ and $J^+(u)$. Using Schauder's fixed point theorem, we are going to prove that the mapping $\mathcal{F}$ admits a fixed point and we are going to show that this fixed point is a Recursive Equilibrium with perfect correlation in the stage game.

The functions $V^+(u)$ and $J^+(u)$ are chosen from a set Ω of Lipschitz continuous functions with fixed Lipschitz bounds. These property of the set Ω is critical to establish that the operator $\mathcal{F}$ satisfies the conditions for Schauder's fixed point theorem (namely, that the family of functions $\mathcal{F}(\otimes)$ is equicontinuous). Formally, let Ω denote the set of bounded and continuous functions $\omega(u,i) = iV^+(u) + (1-i)J^+(u)$, with $\omega : [0,1] \times \{0,1\} \to \mathbb{R}$, and such that: (i) for all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, $V^+(u_1) - V^+(u_0)$ is greater than $\underline{D}_{V_+}(u_1 - u_0)$ and smaller than $\overline{D}_{V_+}(u_1 - u_0)$; (ii) for all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, $J^+(u_1) - J^+(u_0)$ is greater than $\underline{D}_{J_+}(u_1 - u_0)$ and smaller than $\overline{D}_{J_+}(u_1 - u_0)$. In other words, Ω is the set of bounded and Lipschitz continuous functions $\omega = (V^+, J^+)$ with fixed Lipschitz bounds $\underline{D}_{V_+}$, $\overline{D}_{V_+}$, $\underline{D}_{J_+}$ and $\overline{D}_{J_+}$. We choose the Lipschitz bounds to satisfy $\overline{D}_{V_+} > \underline{D}_{V_+} > 0$, $\overline{D}_{J_+} > 0 \geq \underline{D}_{J_+}$, and $\underline{D}_{V_+} > \overline{v}'(\overline{D}_{J_+} - \underline{D}_{J_+})$.

Starting from $V^+(u)$ and $J^+(u)$ in Ω, we use the equilibrium conditions to construct the equilibrium wage, the equilibrium gains from trade accruing to the worker and the firm, the equilibrium law of motion for unemployment and an update for $V^+(u)$ and $J^+(u)$. This process implicitely defines the operator $\mathcal{F}$. In order to make sure that $\mathcal{F}$ maps functions in Ω into functions in Ω (which is a condition of Schauder's fixed point theorem), we need to verify that all equilibrium objects are Lipschitz continuous with fixed Lipschitz bounds. In order to make sure that $\mathcal{F}$ is continuous (which is also a condition of Schauder's fixed point theorem), we need to verify that all equilibrium objects are continuous. To carry out these tasks, we need a few more pieces of notation. In particular, we use $\underline{V}$, $\overline{V}$, $\underline{J}$ and $\overline{J}$ to denote lower and upper bounds on the worker's and firm's gains from trade constructed as in (**??**). Also, we use $\underline{\mu}_u$ and $\overline{\mu}_u$ to denote the minimum and the maximum of the (absolute value) of the partial derivative of the job-finding probability $\mu(J,u)$ with respect to $i = u$. That is, $\underline{\mu}_u$ denotes $\min |\partial\mu(J,u)/\partial u|$ for $(J,u) \in [\underline{J}, \overline{J}] \times [0,1]$, and $\overline{\mu}_u$ denotes $\max |\partial\mu(J,u)/\partial u|$ for

$(J, u) \in [\underline{J}, \overline{J}] \times [0, 1]$. Similarly, we use $\underline{\mu}_J$ and $\overline{\mu}_J$ to denote the minimum and the maximum on the (absolute value) of the partial derivative of $\mu(J, u)$ with respect to J.

C.1 Wage

Take an arbitrary pair of value functions $\omega = (V^+, J^+) \in \Omega$. For any u, the equilibrium wage function $w(u)$ takes a value w such that

$$\upsilon(w) - \upsilon(b) - \psi + V^+(u) = \upsilon'(w) \left[E[y|e = 1] - w + J^+(u)\right]. \tag{C1}$$

For any u, there is a unique wage that satisfies (C1). In fact, the right-hand side of (C1) is strictly increasing in w, as $\upsilon(w) - \upsilon(b) - \psi + V^+(u)$ is strictly increasing in w. The left-hand side of (C1) strictly decreasing in w, as $\upsilon'(w)$ and $E[y|e = 1] - w + J^+(u)$ are both strictly decreasing in w. Therefore, there exists a unique wage w that solves (C1) for any u.

The next lemma proves that the equilibrium wage function $w(u)$ is Lipschitz continuous in u with fixed Lipschitz bounds. Moreover, the lemma proves that $w(u)$ is strictly decreasing in u.

Lemma C1: *For all u_0, $u_1 \in [0, 1]$ with $u_0 < u_1$, the wage function $w(u)$ is such that*

$$\underline{D}_w(u_1 - u_0) \leq w(u_0) - w(u_1) \leq \overline{D}_w(u_1 - u_0), \tag{C2}$$

where the bounds $\underline{D}_w$ and $\overline{D}_w$ are defined as

$$\underline{D}_w = \frac{\underline{D}_{V_+}/\overline{\upsilon}' - \overline{D}_{J_+}}{2 + \overline{J}\overline{\upsilon}''/\underline{\upsilon}'}, \quad \overline{D}_w = \overline{D}_{V_+}/\underline{\upsilon}' - \underline{D}_{J_+}. \tag{C3}$$

Proof: To alleviate notation, let w_0 denote $w(u_0)$ and w_1 denote $w(u_1)$. First, we establish that $w_0 > w_1$. To this aim, notice that

$$\begin{aligned} & \upsilon'(w_0) \left[E[y|e = 1] - w_0 + J^+(u_1)\right] \\ \leq \quad & \upsilon'(w_0)E[y|e = 1] - w_0 + J^+(u_0)] + \overline{\upsilon}'\overline{D}_{J+}(u_1 - u_0). \end{aligned} \tag{C4}$$

Also, notice that

$$\begin{aligned} & \upsilon(w_0) - \upsilon(b) - c + V^+(u_1) \\ \geq \quad & \upsilon(w_0) - \upsilon(b) - c + V^+(u_0) + \underline{D}_{V+}(u_1 - u_0) \\ = \quad & \upsilon'(w_0)E[y|e = 1] - w_0 + J^+(u_0) + \underline{D}_{V+}(u_1 - u_0) \\ > \quad & \upsilon'(w_0)E[y|e = 1] - w_0 + J^+(u_0) + \overline{\upsilon}'\overline{D}_{J+}(u_1 - u_0), \end{aligned} \tag{C5}$$

where the third line follows from (C1) and the last line follows from $\underline{D}_{V_+} > \overline{\upsilon}'\overline{D}_{J_+}$. Taken together (C4) and (C5) imply that $\upsilon(w_0) - \upsilon(b) - c + V^+(u_1)$ is strictly greater than $\upsilon'(w_0) \left[E[y|e = 1] - w_0 + J^+(u_1)\right]$. Therefore, the left-hand side of (C1) is strictly smaller than the right-hand side of (C1) when evaluated at $w = w_0$ and $u = u_1$. Since the left-hand of (C1) is strictly increasing in w and the left-hand side is strictly decreasing in w, w_1 is strictly smaller than w_0.

Next, we derive lower and upper bounds on $w_0 - w_1$. From (C1), it follows that

$$\begin{aligned} & \upsilon(w_0) - \upsilon(w_1) + V^+(u_0) - V^+(u_1) \\ = & \ \upsilon'(w_0)\left[E[y|e=1] - w_0 + J^+(u_0)\right] - \upsilon'(w_1)\left[E[y|e=1] - w_0 + J^+(u_0)\right] \\ + & \ \upsilon'(w_1)\left[E[y|e=1] - w_0 + J^+(u_0)\right] - \upsilon'(w_1)\left[E[y|e=1] - w_1 + J^+(u_1)\right] \end{aligned} \tag{C6}$$

The above equation can be rewritten as

$$\begin{aligned} & w_0 - w_1 \\ = & \left[\frac{V^+(u_1) - V^+(u_0)}{\upsilon'(w_1)}\right] - \left[J^+(u_1) - J^+(u_0)\right] \\ - & \left[\frac{\upsilon(w_0) - \upsilon(w_1)}{\upsilon'(w_1)}\right] - \left[\frac{\upsilon'(w_1) - \upsilon'(w_0)}{\upsilon'(w_1)}\left(E[y|e=1] - w_0 + J^+(u_0)\right)\right]. \end{aligned} \tag{C7}$$

The first term in square brackets on the right-hand side of (C6) is greater than $(\underline{D}_{V+}/\overline{\upsilon}')(u_1 - u_0)$ and smaller than $(\overline{D}_{V+}/\underline{\upsilon}')(u_1 - u_0)$. The second term in square brackets is greater than $\underline{D}_{J+}(u_1 - u_0)$ and smaller than $\overline{D}_{J+}(u_1 - u_0)$. The third term in square brackets is greater than zero and smaller than $w_1 - w_0$. The last term on the right-hand side of (C6) is greater than zero and smaller than $\overline{\upsilon}''\overline{D}_w\overline{J}/\underline{\upsilon}'(w_1 - w_0)$. From the above observations, it follows that

$$w_0 - w_1 \leq \left[\frac{\overline{D}_{V+}}{\underline{\upsilon}'} - \underline{D}_{J+}\right](u_1 - u_0). \tag{C8}$$

Similarly, we have

$$w_0 - w_1 \geq \left[2 + \frac{\overline{\upsilon}''}{\underline{\upsilon}'}\overline{D}_w\overline{J}\right]^{-1}\left[\frac{\underline{D}_{V+}}{\overline{\upsilon}'} - \overline{D}_{J+}\right](u_1 - u_0). \tag{C9}$$

The inequalities (C8) and (C9) represent the desired bounds on $w_0 - w_1$. ■

The next lemma proves that the equilibrium wage function w is continuous with respect to the value functions V^+ and J^+. Specifically, consider $\omega_0 = (V_0^+, J_0^+)$ and $\omega_1 = (V_1^+, J_1^+)$ with $\omega_0, \omega_1 \in \Omega$. Denote as w_i the wage function computed using ω_i in (C1) for $i = \{0, 1\}$. If the distance between ω_0 and ω_1 goes to 0, so does the distance between w_0 and w_1.

Lemma C2: *For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $||\omega_0$- $\omega_1|| < \kappa$, we have*

$$||w_0 - w_1|| < \alpha_w \kappa, \quad \alpha_w = 1 + 1/\underline{\upsilon}'. \tag{C10}$$

Proof: Take an arbitrary $u \in [0, 1]$. To alleviate notation, let w_0 denote $w_0(u)$ and w_1 denote $w_1(u)$. From (C10), it follows that

$$\begin{aligned} \upsilon(w_0) - \upsilon(b) - c + V_0^+(u) - \upsilon'(w_0)\left[E[y|e=1] - w_0 + J_0^+(u)\right] = 0, \\ \upsilon(w_1) - \upsilon(b) - c + V_1^+(u) - \upsilon'(w_1)\left[E[y|e=1] - w_1 + J_1^+(u)\right] = 0. \end{aligned} \tag{C11}$$

Subtracting the second equation from the first, we obtain

$$\begin{aligned} & V_0^+(u) - V_1^+(u) + \upsilon(w_0) - \upsilon(w_1) \\ + \quad & \upsilon'(w_0)\left[J_1^+(u) - J_0^+(u) + w_0 - w_1\right] \\ + \quad & [\upsilon'(w_1) - \upsilon'(w_0)]\left[E[y|e=1] - w_1 + J_1^+(u)\right] = 0. \end{aligned} \tag{C12}$$

Suppose without loss in generality that $w_0 \geq w_1$. We can rewrite the above equation as

$$\begin{aligned} w_0 - w_1 \quad &= \left[J_0^+(u) - J_1^+(u)\right] + \left[\frac{V_1^+(u) - V_0^+(u)}{\upsilon'(w_0)}\right] + \left[\frac{\upsilon(w_1) - \upsilon(w_0)}{\upsilon'(w_0)}\right] \\ &+ \left[E[y|e=1] - w_1 + J_1^+(u)\right]\left[\frac{\upsilon'(w_0) - \upsilon'(w_1)}{\upsilon'(w_0)}\right]. \end{aligned} \tag{C13}$$

The first term in square brackets on the right-hand side of (C13) is strictly smaller than κ. The second term in square brackets is strictly smaller than $\kappa/\underline{\upsilon}'$. The third term in square brackets is strictly negative. The last term is strictly negative. Hence, we have

$$0 \leq w_0 - w_1 < (1 + 1/\underline{\upsilon}')\kappa. \tag{C14}$$

Since the above inequality holds for any $u \in [0,1]$, we conclude that $\|w_0 - w_1\| < \alpha_w \kappa$ where $\alpha_w = (1 + 1/\underline{\upsilon}')$. ■

C.2 Gains from trade to worker and firm

Given V^+, J^+ and w, the equilibrium gains from trade accruing to the firm and to the worker, J and V, are respectively given by

$$\begin{aligned} J(u) &= E[y|e=1] - w(u) + J^+(u), \\ V(u) &= \upsilon(w(u)) - \upsilon(b) - \psi + V^+(u) \end{aligned} \tag{C15}$$

The next lemma proves that the equilibrium gains from trade $J(u)$ and $V(u)$ are Lipschitz continuous in u with fixed Lipschitz bounds. Moreover, the lemma proves that $J(u)$ and $V(u)$ are strictly increasing in u.

Lemma C3: *For all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, the equilibrium gains from trade accruing to the firm, $J(u)$, and to the worker, $V(u)$, are such that*

$$\begin{aligned} \underline{D}_J(u_1 - u_0) \leq J(u_1) - J(u_0) \leq \overline{D}_J(u_1 - u_0), \\ \underline{D}_V(u_1 - u_0) \leq V(u_1) - V(u_0) \leq \overline{D}_V(u_1 - u_0), \end{aligned} \tag{C16}$$

where the bounds are defined as

$$\begin{aligned} \underline{D}_J &= \underline{D}_w + \underline{D}_{J^+} > 0, \quad \overline{D}_J = \overline{D}_w + \overline{D}_{J^+}, \\ \underline{D}_V &= \underline{\upsilon}'(\underline{D}_w + \underline{D}_{J^+}) > 0, \quad \overline{D}_V = \overline{\upsilon}'(\overline{D}_w + \overline{D}_{J^+}) + \overline{\upsilon}''\overline{D}_w\overline{J}. \end{aligned} \tag{C17}$$

Proof: To simplify notation, let w_0 denote $w(u_0)$ and w_1 denote $w(u_1)$. Then $J(u_1) - J(u_0)$

is given by

$$J(u_1) - J(u_0) = w_0 - w_1 + J^+(u_1) - J^+(u_0).$$

From the above expression, it follows that

$$\begin{aligned} J(u_1) - J(u_0) &\geq (\underline{D}_w + \underline{D}_{J+})(u_1 - u_0), \\ J(u_1) - J(u_0) &\leq (\overline{D}_w + \overline{D}_{J+})(u_1 - u_0). \end{aligned} \tag{C18}$$

The difference $V(u_1) - V(u_0)$ is given by

$$\begin{aligned} V(u_1) - V(u_0) &= \upsilon'(w_1)J(u_1) - \upsilon'(w_0)J(u_0) \\ &= [\upsilon'(w_1)J(u_1) - \upsilon'(w_1)J(u_0)] + [\upsilon'(w_1)J(u_0) - \upsilon'(w_0)J(u_0)] \end{aligned}$$

From the above expression, it follows that

$$\begin{aligned} V(u_1) - V(u_0) &\geq \underline{\upsilon}'(\underline{D}_w + \underline{D}_{J+})(u_1 - u_0), \\ V(u_1) - V(u_0) &\leq \left[\overline{\upsilon}'(\overline{D}_w + \overline{D}_{J+}) + \overline{\upsilon}''\overline{D}_w\overline{J}\right](u_1 - u_0). \end{aligned} \tag{C19}$$

Lemma 7 follows directly from the inequalities in (C18) and (C19). ■

The next lemma proves that the firm's and worker's gains from trade are continuous with respect to V^+ and J^+. Specifically, consider $\omega_0 = (V_0^+,\ J_0^+)$ and $\omega_1 = (V_1^+,\ J_1^+)$ with ω_0, ω_1 in Ω. For $i \in \{0, 1\}$, denote as w_i the equilibrium wage computed using ω_i, and as J_i and V_i the equilibrium gains from trade computed using ω_i and w_i in (C15). If the distance between ω_0 and ω_1 goes to 0, so does the distance between J_0 and J_1 and between V_0 and V_1.

Lemma C4: *For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $||\omega_0$- $\omega_1|| < \kappa$, we have*

$$\begin{aligned} ||J_0 - J_1|| &< \alpha_J\kappa, \quad \alpha_J = 1 + \alpha_w, \\ ||V_0 - V_1|| &< \alpha_V\kappa, \quad \alpha_V = 1 + \overline{\upsilon}'\alpha_w. \end{aligned} . \tag{C20}$$

Proof: Take an arbitrary $u \in [0, 1]$. The difference $J_0(u) - J_1(u)$ is such that

$$\begin{aligned} |J_0(u) - J_1(u)| &\leq |w_0(u) - w_1(u)| + \left|J_0^+(u) - J_1^+(u)\right| \\ &< (\alpha_w + 1)\kappa. \end{aligned} \tag{C21}$$

The difference $V_0(u) - V_1(u)$ is such that

$$\begin{aligned} |V_0(u) - V_1(u)| &\leq |\upsilon(w_0(u)) - \upsilon(w_1(u))| + \left|V_0^+(u) - V_1^+(u)\right| \\ &< (\overline{\upsilon}'\alpha_w + 1)\kappa. \end{aligned} \tag{C22}$$

Since the inequalities (C21) and (C22) hold for all $u \in [0, 1]$, we conclude that $\|J_0 - J_1\| < \alpha_{J'}\kappa$ and $\|V_0 - V_1\| < \alpha_{V'}\kappa$, where $\alpha_J = \alpha_w + 1$ and $\alpha_V = \overline{\upsilon}'\alpha_w + 1$. ■

C.3 Law of motion for unemployment

Given J, the law of motion for unemployment $h(u, \hat{z})$, is such that—for any current period's unemployment u and any realization of the sunspot $\hat{z}$—next period's unemployment takes on a value $\hat{u}$ such that

$$\hat{u} = u - u\mu(J(\hat{u}), u) + (1-u)[\delta + (1-\delta)p_\ell(1)s(y_\ell, \hat{z})], \tag{28}$$

Since we are trying to construct a Recursive Equilibrium in which there is perfect correlation in the stage game, we set $s(y_\ell, \hat{z}) = 0$ if $\hat{z} \in Z_0$ and $s(y_\ell, \hat{z}) = 1$ if $\hat{z} \in Z_2$.

Next period's unemployment $\hat{u}$ is uniquely determined by (28). In fact, the left-hand side of (28) equals zero for $\hat{u} = 0$, it is strictly increasing in $\hat{u}$ and it equals one for $\hat{u} = 1$. The right-hand side of (28) is strictly positive for $\hat{u} = 0$ and it is strictly decreasing in $\hat{u}$, as the worker's job finding probability μ is strictly increasing in the firm's gains from trade J, and J is strictly increasing in $\hat{u}$. Therefore, there exists one and only one $\hat{u}$ that satisfies (28). Next period's unemployment $\hat{u}$ is strictly increasing in u. In fact, the left-hand side of (28) is independent of u. The right-hand side of (28) is strictly increasing in u, as its derivative with respect to u is greater than $(1-\delta)p_h(1) - \mu$, which we assume to be strictly positive. Therefore, the $\hat{u}$ that solves (28) is strictly increasing in u. Moreover, next period's unemployment $\hat{u}$ is strictly higher if $\hat{z} \in Z_2$ than if $\hat{z} \in Z_0$. To see this, it is sufficient to notice that the left-hand side of (28) is independent of $s(y_\ell, \hat{z})$, while the right hand side is strictly increasing in it.

The next lemma proves that $h(u, \hat{z})$ is Lipschitz continuous in u with fixed Lipschitz bounds.

Lemma C5: For all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, $h(u, \hat{z})$ is such that

$$\underline{D}_h(u_1 - u_0) < h(u_1, \hat{z}) - h(u_0, \hat{z}) \leq \overline{D}_h(u_1 - u_0), \tag{C24}$$

where the bounds $\underline{D}_h$ and $\overline{D}_h$ are defined as

$$\underline{D}_h = 0, \quad \overline{D}_h = 1 - \delta + \overline{\mu}_u. \tag{C25}$$

Proof: Let $\hat{u}_1$ denote the solution to (28) for $u = u_1$ and let $\hat{u}_0$ denote the solution to (28) for $u = u_0$, i.e.

$$\hat{u}_1 = u_1 - u_1\mu(J(\hat{u}_1), u_1) + (1-u_1)[\delta + (1-\delta)p_\ell(1)s(y_\ell, \hat{z}')],$$
$$\hat{u}_0 = u_0 - u_0\mu(J(\hat{u}_0), u_0) + (1-u_0)[\delta + (1-\delta)p_\ell(1)s(y_\ell, \hat{z}')].$$

Subtracting the second equation from the first one, we obtain

$$\begin{aligned} \hat{u}_1 - \hat{u}_0 \quad &= (u_1 - u_0)(1-\delta)[1 - p_\ell(1)s(y_\ell, \hat{z}')] \\ &+ u_0\left[\mu\left(J(\hat{u}_0), u_0\right) - \mu\left(J(\hat{u}_1), u_0\right)\right] \\ &+ u_0\left[\mu\left(J(\hat{u}_1), u_0\right) - \mu\left(J(\hat{u}_1), u_1\right)\right] \\ &+ u_0\mu\left(J(\hat{u}_1), u_1\right) - u_1\mu\left(J(\hat{u}_1), u_1\right). \end{aligned} \tag{C26}$$

The first line on the right-hand side of (C26) is positive. Since $\mu(J,u)$ is increasing in J, J is increasing in u and, as established in the main text, $\hat{u}_1 > \hat{u}_0$, the second line on the right-hand side of (C26) is negative. Since $\mu(J,u)$ is decreasing in u and $u_1 > u_0$, the third line on the right-hand side of (C26) is positive. The fourth line on the right-hand side of (C26) is obviously negative. Hence, an upper bound on $\hat{u}_1 - \hat{u}_0$ is given by

$$\begin{aligned} \hat{u}_1 - \hat{u}_0 \quad & \leq (u_1 - u_0)(1-\delta)[1 - p_\ell(1)s(y_\ell, \hat{z}')] + u_0 \left[\mu\left(J(\hat{u}_1), u_0\right) - \mu\left(J(\hat{u}_1), u_1\right)\right] \\ & \leq (u_1 - u_0)\left[1 - \delta + \overline{\mu}_u\right]. \end{aligned} \tag{C27}$$

Combining the above inequality with $\hat{u}_1 < \hat{u}_0$, we obtain

$$\underline{D}_h \left(u_1 - u_0\right) < \hat{u}_1 - \hat{u}_0 \leq \overline{D}_h \left(u_1 - u_0\right), \tag{C28}$$

where

$$\underline{D}_h = 0, \quad \overline{D}_h = 1 - \delta + \overline{\mu}_u. \quad \blacksquare$$

Next, we prove that $h(u, \hat{z})$ is continuous with respect to V^+ and J^+. Formally, we consider $\omega_0 = (V_0^+,\ J_0^+)$ and $\omega_1 = (V_1^+,\ J_1^+)$ with ω_0, ω_1 in Ω. For $i = \{0, 1\}$, we denote as J_i the firm's equilibrium gains from trade computed using ω_i in (C15), and as h_i the equilibrium law of motion for unemployment computed using J_i in (28). Then, we show that, if the distance between ω_0 and ω_1 goes to 0, so does the distance between h_0 and h_1.

Lemma C6: *For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $||\omega_0$- $\omega_1|| < \kappa$, we have*

$$||h_0 - h_1|| < \alpha_h \kappa, \quad \alpha_h = \overline{\mu}_J \alpha_J. \tag{C29}$$

Proof: Take an arbitrary $u \in [0, 1]$ and $\hat{z} \in \{Z_0, Z_2\}$. Let $\hat{u}_0$ denote $h_0(u, \hat{z})$ and $\hat{u}_1$ denote $h_1(u, \hat{z})$. From (28), it follows that

$$\begin{aligned} \hat{u}_0 &= u - u\mu(J_0(\hat{u}_0), u) + (1-u)[\delta + (1-\delta)p_\ell(1)s(y_\ell, \hat{z})], \\ \hat{u}_1 &= u - u\mu(J_1(\hat{u}_1), u) + (1-u)[\delta + (1-\delta)p_\ell(1)s(y_\ell, \hat{z})]. \end{aligned}$$

Without loss in generality suppose that $\hat{u}_0 \geq \hat{u}_1$. In this case,

$$\hat{u}_0 - \hat{u}_1 = u\left\{[\mu(J_1(\hat{u}_1), u) - \mu(J_0(\hat{u}_1), u)] + [\mu(J_0(\hat{u}_1), u) - \mu(J_0(\hat{u}_0), u)]\right\}. \tag{C30}$$

The term $\mu(J_0(\hat{u}_1), u) - \mu(J_0(\hat{u}_0), u)$ on the right-hand side of (C30) is negative as $\mu(J, u)$ is increasing in J, J_0 is increasing in u and $\hat{u}_0 \geq \hat{u}_1$. Therefore, we have

$$\hat{u}_0 - \hat{u}_1 \leq u\left[\mu(J_1(\hat{u}_1), u) - \mu(J_0(\hat{u}_1), u)\right] < \overline{\mu}_J \alpha_J \kappa. \tag{C31}$$

Since the above inequality holds for any $u \in [0, 1]$ and any $\hat{z}' \in \{B, G\}$, we conclude that $\|h_0 - h_1\| < \alpha_h \kappa$, where $\alpha_h = \overline{\mu}_J \alpha_J$. $\blacksquare$

C.4 Updated value functions

Given J, V and h, we can construct an update, $V^{+\prime}$, for the worker's gains from trade at the end of the production stage as

$$V^{+\prime}(u) = \beta E_{\hat{z}} \left\{ [(1-\delta)(1-p_\ell(1)s(y_\ell,\hat{z})) - \mu(J(h(u,\hat{z})),u)]V(h(u,\hat{z})) \right\} \tag{C32}$$

Similarly, we can construct an update, $J^{+\prime}$, for the firm's gains from trade at the end of the production stage as

$$J^{+\prime}(u) = \beta E_{\hat{z}} \left[(1-\delta)(1-p_\ell(1)s(y_\ell,\hat{z}))J(h(u,\hat{z})) \right] \tag{C33}$$

Since we are looking for a Recursive Equilibrium with perfect correlation in the stage game, we set $s(y_\ell,\hat{z}) = 0$ for $\hat{z} \in Z_0$ and $s(y_\ell,\hat{z}) = 1$ for $\hat{z} \in Z_2$. Also, we set the probability π_0 that $\hat{z} \in Z_0$ and the probability π_2 that $\hat{z} \in Z_2$ so that $\pi_0 = 1 - \pi_2$ and

$$\pi_2 = \frac{\psi}{\beta(p_h(1) - p_h(0))(1-\delta)V(h(u,Z_2))}. \tag{C34}$$

In the next lemma, we prove that $V^{+\prime}(u)$ and $J^{+\prime}(u)$ are Lipschitz continuous in u with fixed Lipschitz bounds.

Lemma C7: *For all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, $V^{+\prime}$ and $J^{+\prime}$ are such that*

$$\begin{aligned} \underline{D}'_{V+}(u_1 - u_0) < V^{+\prime}(u_1) - V^{+\prime}(u_0) \le \overline{D}'_{V+}(u_1 - u_0), \\ \underline{D}'_{J+}(u_1 - u_0) < J^{+\prime}(u_1) - J^{+\prime}(u_0) \le \overline{D}'_{J+}(u_1 - u_0). \end{aligned} \tag{C35}$$

where the bounds $\underline{D}'_{V+}$ and $\overline{D}'_{V+}$ are defined as

$$\begin{aligned} \underline{D}'_{V+} &= \beta\underline{V}(\underline{\mu}_u - \overline{\mu}_J\overline{D}_J\overline{D}_h) - \frac{2\psi\overline{D}_V\overline{D}_h\overline{V}}{(p_h(1)-p_h(0))(1-\delta)\underline{V}^2}, \\ \overline{D}'_{V+} &= \beta\left[\overline{V}\overline{\mu}_u + (1-\delta)\overline{D}_V\overline{D}_h\right], \end{aligned} \tag{C36}$$

and the bounds $\underline{D}'_{J+}$ and $\overline{D}'_{J+}$ are defined as

$$\begin{aligned} \underline{D}'_{J+} &= -\frac{2c\overline{D}_V\overline{D}_h\overline{J}}{(p_h(1)-p_h(0))(1-\delta)\underline{V}^2}, \\ \overline{D}'_{J+} &= \beta(1-\delta)\overline{D}_J\overline{D}_h \end{aligned} \tag{C37}$$

Proof: Take arbitrary $u_0, u_1 \in [0,1]$ with $u_1 > u_0$. To simplify notation, let $\hat{u}_{j,0}$ denote $h(u_0,\hat{z})$ for $\hat{z} \in Z_j$ and let $\hat{u}_{j,1}$ denote $h(u_1,\hat{z})$ for $\hat{z} \in Z_j$. Similarly, let $\pi_{j,0}$ denote the probability that the realization of the sunspot is $\hat{z} \in Z_j$ for u_0, and let $\pi_{j,1}$ denote the probability that the realization of the sunspot is $\hat{z} \in Z_j$ for u_1. Using this notation and (C32), we can write $V^{+\prime}(u_1) - V^{+\prime}(u_0)$ as

$$\begin{aligned} & V^{+\prime}(u_1) - V^{+\prime}(u_0) \\ = \quad & \beta \textstyle\sum_j \{(\pi_{j,1} - \pi_{j,0}) \left[(1-\delta)(1 - p_\ell(1)s(y_\ell, \hat{z})) - \mu(J(\hat{u}_{j,1}, u_1)\right] V(\hat{u}_{j,1}) \\ + \quad & \pi_{j,0}[(1-\delta)(1 - p_\ell(1)s(y_\ell, \hat{z})) - \mu(J(\hat{u}_{2,1}, u_1)] \left[V(\hat{u}_{j,1}) - V(\hat{u}_{j,0})\right] \\ + \quad & \pi_{j,0} V(\hat{u}_{j,0})[\mu(J(\hat{u}_{2,0}, u_0) - \mu(J(\hat{u}_{2,0}, u_1)]. \\ + \quad & \pi_{j,0} V(\hat{u}_{j,0})[\mu(J(\hat{u}_{2,0}, u_1) - \mu(J(\hat{u}_{2,1}, u_1)]\}. \end{aligned} \tag{C38}$$

The first term on the right-hand side of (C38) is negative. In absolute value, this term is smaller than $c\overline{D}_V \overline{D}_h \overline{V}(u_1 - u_0)/[\beta(p_h(1) - p_h(0))(1-\delta)\underline{V}^2]$. The second term on the right-hand side of (C38) is geater than 0 and smaller than $\pi_{j,0}(1-\delta)\overline{D}_V \overline{D}_h(u_1 - u_0)$. The third term on the right-hand side of (C38) is positive, greater than $\pi_{j,0}\underline{V}\underline{\mu}_u(u_1 - u_0)$ and smaller than $\pi_{j,0}\overline{V}\overline{\mu}_u(u_1 - u_0)$. The last term on the right-hand side of (C38) is negative. In absolute value, this term is smaller than $\pi_{j,0}\overline{V}\overline{\mu}_J \overline{D}_J \overline{D}_h(u_1 - u_0)$. Overall, we have

$$V^{+\prime}(u_1) - V^{+\prime}(u_0) \geq \left[\beta \underline{V}(\underline{\mu}_u - \overline{\mu}_J \overline{D}_J \overline{D}_h) - \frac{2c\overline{D}_V \overline{D}_h \overline{V}}{(p_h(1) - p_h(0))(1-\delta)\underline{V}^2}\right](u_1 - u_0), \tag{C39}$$

and

$$V^{+\prime}(u_1) - V^{+\prime}(u_0) \leq \beta \left[\overline{V}\overline{\mu}_u + (1-\delta)\overline{D}_V \overline{D}_h\right](u_1 - u_0). \tag{C40}$$

Using (C33), we can write $J^{+\prime}(u_1) - J^{+\prime}(u_0)$ as

$$\begin{aligned} & J^{+\prime}(u_1) - J^{+\prime}(u_0) \\ = \quad & \beta \textstyle\sum_j \{(\pi_{j,1} - \pi_{j,0}) \left[(1-\delta)(1 - p_\ell(1)s(y_\ell, \hat{z}))\right] J(\hat{u}_{j,1}) \\ + \quad & \pi_{j,0}[(1-\delta)(1 - p_\ell(1)s(y_\ell, \hat{z}))] \left[J(\hat{u}_{j,1}) - J(\hat{u}_{j,0})\right]. \end{aligned} \tag{C41}$$

The first term on the right-hand side of (C41) is negative. In absolute value, this term is smaller than $\psi\overline{D}_V \overline{D}_h \overline{J}(u_1 - u_0)/[\beta(p_h(1) - p_h(0))(1-\delta)\underline{V}^2]$. The second term on the right-hand side of (C41) is geater than zero and smaller than $\pi_{j,0}(1-\delta)\overline{D}_J \overline{D}_h(u_1 - u_0)$. Overall, we have

$$J^{+\prime}(u_1) - J^{+\prime}(u_0) \geq -\frac{2\psi\overline{D}_V \overline{D}_h \overline{J}}{(p_h(1) - p_h(0))(1-\delta)\underline{V}^2}(u_1 - u_0), \tag{C42}$$

and

$$J^{+\prime}(u_1) - J^{+\prime}(u_0) \leq \beta(1-\delta)\overline{D}_J \overline{D}_h(u_1 - u_0). \tag{C43}$$

Lemma C7 follows directly from the inequalities in (C39)-(C40) and (C42)-(C43). ■

The next lemma shows that, under some parametric conditions, there exists a fixed point for the Lipschitz bounds on the firm's and worker's gains from trade at the end of the production stage.

Lemma C8: *Assume $\underline{\mu}_u - \overline{\mu}_J(1 - \delta + \overline{\mu}_u) > 0$. Then there exist $\beta^* > 0$ and $\psi^* > 0$ such that, if $\beta \in (0, \beta^*)$ and $\psi \in (0, \psi^*)$, there are Lipschitz bounds $\underline{D}_{V+}$, $\overline{D}_{V+}$, $\underline{D}_{J+}$, $\overline{D}_{J+}$ such* that: (i) $\underline{D}'_{V+} = \underline{D}_{V+}$, $\overline{D}'_{V+} = \overline{D}_{V+}$, $\underline{D}'_{J+} = \underline{D}_{J+}$, and $\overline{D}'_{J+} = \overline{D}_{J+}$; (ii) $\overline{D}_{V_+} > \underline{D}_{V_+} > 0$, $\overline{D}_{J_+} > 0 \geq \underline{D}_{J_+}$ and $\underline{D}_{V_+} > \overline{v}'(\overline{D}_{J_+} - \underline{D}_{J_+})$.

Proof: Set $\underline{D}'_{V+}$ and $\overline{D}'_{V+}$ in (C36) equal to $\underline{D}_{V+}$ and $\overline{D}_{V+}$, and $\underline{D}'_{J+}$ and $\overline{D}'_{J+}$ in (C37) equal to $\underline{D}_{J+}$ and $\overline{D}_{J+}$. Then solve for $\underline{D}_{V+}$, $\overline{D}_{V+}$, $\underline{D}_{J+}$ and $\overline{D}_{J+}$. It is immediate to verify that, since $\underline{\mu}_u - \overline{\mu}_J(1-\delta+\overline{\mu}_u) > 0$, $\overline{D}_{V_+} > \underline{D}_{V_+} > 0$, $\overline{D}_{J_+} > 0 \geq \underline{D}_{J_+}$ and $\underline{D}_{V_+} > \overline{v}'(\overline{D}_{J_+} - \underline{D}_{J_+})$ for β and ψ small enough. ■

In the last lemma, we prove that $V^{+\prime}$ and $J^{+\prime}$ are continuous with respect to V^+ and J^+. Specifically, consider $\omega_0 = (V_0^+,\ J_0^+)$ and $\omega_1 = (V_1^+,\ J_1^+)$ with $\omega_0,\ \omega_1 \in \Omega$. For $i \in \{0,1\}$, denote as $V_i^{+\prime}$ and $J_i^{+\prime}$ the continuation value functions computed using V_i^+, J_i^+ and h_i in (C32) and (C33). If the distance between ω_0 and ω_1 goes to 0, so does the distance between V_0^+ and V_1^+ and between J_0^+ and J_1^+.

Lemma C9: *For any $\kappa > 0$ and any $\omega_0,\ \omega_1 \in \Omega$ such that $||\omega_0$- $\omega_1|| < \kappa$, we have $||V_0^{+\prime} - V_1^{+\prime}|| < \alpha_{V'_+}\kappa$ and $||J_0^{+\prime} - J_1^{+\prime}|| < \alpha_{J'_+}\kappa$, where*

$$\begin{aligned}\alpha_{V'_+} &= \left[\frac{2\psi\overline{V}}{(p_h(1)-p_h(0))(1-\delta)\underline{V}^2}+1\right]\left(\alpha_V + \overline{D}_V\alpha_h\right) + \overline{V}\overline{\mu}_J\left(\alpha_J + \overline{D}_J\alpha_h\right), \\ \alpha_{J'_+} &= \frac{2\psi\overline{J}\left(\alpha_V + \overline{D}_V\alpha_h\right)}{(p_h(1)-p_h(0))(1-\delta)\underline{V}^2} + \left(\alpha_J + \overline{D}_J\alpha_h\right).\end{aligned} \tag{C44}$$

Proof: Take an arbitrary $u \in [0,1]$. To simplify notation, let $\hat{u}_{j,0}$ denote $h_0(u,\hat{z})$ for $\hat{z} \in Z_j$, and let $\hat{u}_{j,1}$ denote $h_1(u,\hat{z})$ for $\hat{z} \in Z_j$. Similarly, let $\pi_{j,0}$ denote the probability that the realization of the sunspot is $\hat{z} \in Z_j$ defined by using V_0 and h_0 in (C34), and let $\pi_{j,1}$ denote the probability that the realization of the sunspot is $\hat{z} \in Z_j$ defined by using V_1 and h_1 in (C34). Using this notation and (C32), we can write $V_0^{+\prime}(u) - V_1^{+\prime}(u)$ as

$$\begin{aligned} & V_0^{+\prime}(u) - V_1^{+\prime}(u) \\ = \ & \beta\textstyle\sum_j\{(\pi_{j,0}-\pi_{j,1})\left[(1-\delta)(1-p_\ell(1)s(y_\ell,\hat{z})) - \mu(J_0(\hat{u}_{j,0},u)\right]V_0(\hat{u}_{j,0}) \\ + \ & \pi_{j,1}[(1-\delta)(1-p_\ell(1)s(y_\ell,\hat{z})) - \mu(J_0(\hat{u}_{j,0},u)]\left[V_0(\hat{u}_{j,0}) - V_1(\hat{u}_{j,1})\right] \\ + \ & \pi_{j,1}V_1(\hat{u}_{j,1})[\mu(J_1(\hat{u}_{j,1},u) - \mu(J_0(\hat{u}_{j,0},u)]\}. \end{aligned} \tag{C45}$$

From (C45), it follows that

$$\begin{aligned} & \left|V_0^{+\prime}(u) - V_1^{+}(u)\right| \\ \leq \ & \frac{2c\overline{V}}{(p_h(1)-p_h(0))(1-\delta)\underline{V}^2}\left(\alpha_V + \overline{D}_V\alpha_h\right)\kappa + \left(\alpha_V + \overline{D}_V\alpha_h\right)\kappa + \overline{V}\overline{\mu}_J\left(\alpha_J + \overline{D}_J\alpha_h\right)\kappa, \end{aligned} \tag{C46}$$

where the first term on the right-hand side of (C46) is an upper bound on the absolute value on the first line on the right-hand side of (C45), the second term on the right-hand side of (C46) is an upper bound on the absolute value on the second line on the right-hand side of (C45), and the last term on the right-hand side of (C46) is an upper bound on the absolute value on the last line on the right-hand side of (C45).

Using (C33) we can write $J_0^{+\prime}(u) - J_1^{+\prime}(u)$ as

$$\begin{aligned} & J_0^{+\prime}(u) - J_1^{+\prime}(u) \\ = \quad & \beta \sum_j \{(\pi_{j,0} - \pi_{j,1}) (1-\delta)(1 - p_\ell(1) s(y_\ell, \hat{z})) J_0(\hat{u}_{j,0}) \\ + \quad & \pi_{j,1}(1-\delta)(1 - p_\ell(1) s(y_\ell, \hat{z})) \left[J_0(\hat{u}_{j,0}) - J_1(\hat{u}_{j,1}) \right]. \end{aligned} \tag{C47}$$

From (C47), it follows that

$$\begin{aligned} & \left| J_0^{+\prime}(u) - J_1^{+\prime}(u) \right| \\ \leq \quad & \frac{2\psi \overline{J}}{(p_h(1) - p_h(0))(1-\delta)\underline{V}^2} \left(\alpha_V + \overline{D}_V \alpha_h \right) \kappa + \left(\alpha_J + \overline{D}_J \alpha_h \right) \kappa, \end{aligned} \tag{C48}$$

where the first term on the right-hand side of (C48) is an upper bound on the absolute value on the first line on the right-hand side of (C47), and the second term on the right-hand side of (C48) is an upper bound on the absolute value on the second line on the right-hand side of (C47). Since the inequalities (C46) and (C48) hold for any $u \in [0,1]$, we conclude that $\left\| V_0^{+\prime} - V_1^{+\prime} \right\| < \alpha_{V+}\kappa$ and $\left\| J_0^{+\prime} - J_1^{+\prime} \right\| < \alpha_{J+}\kappa$. ■

C.5 Existence

In the previous subsections, we have taken a pair of continuation gains from trade V^+ and J^+ and, using the conditions for a perfect correlation equilibrium, we have constructed an updated pair of continuation gains from trade $V^{+\prime}$ and $J^{+\prime}$. We denote as $\mathcal{F}$ the operator that takes $\omega(u,i) = (1-i)V^+(u) + iJ^+(u)$ and returns $\omega'(u,i) = (1-i)V^{+\prime}(u) + iJ^{+\prime}(u)$.

The operator $\mathcal{F}$ has three key properties. First, the operator $\mathcal{F}$ maps functions that belong to the set Ω into functions that also belong to the set Ω. In fact, for any $\omega = (V^+, J^+) \in \Omega$, $\omega' = (V^{+\prime}, J^{+\prime})$ is bounded and continuous and, as established in Lemma C8, it is such that: (i) for all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, the difference $V^{+\prime}(u_1) - V^{+\prime}(u_0)$ is greater than $\underline{D}_{V_+}(u_1 - u_0)$ and smaller than $\overline{D}_{V_+}(u_1 - u_0)$; (ii) for all u_0, $u_1 \in [0,1]$ with $u_0 < u_1$, the difference $J^{+\prime}(u_1) - J^{+\prime}(u_0)$ is greater than $\underline{D}_{J_+}(u_1 - u_0)$ and smaller than $\overline{D}_{J_+}(u_1 - u_0)$. Second, the operator $\mathcal{F}$ is continuous, as established in Lemma C9. Third, the family of functions $\mathcal{F}(\Omega)$ is equicontinuous. To see that this is the case, let $||.||_E$ denote the standard norm on the Euclidean space $[0,1] \times \{0,1\}$. For any $\epsilon > 0$, let $\kappa_\epsilon = \min\{(\max\{\overline{D}_{V+}, \overline{D}_{J+}, |\underline{D}_{J+}|\})^{-1}, 1\}$. Then, for all (u_0, i_0), $(u_1, i_1) \in [0,1] \times \{0,1\}$ such that $||(u_0, i_0) - (u_1, i_1)||_E < \kappa_\epsilon$, we have

$$|(\mathcal{F}\omega)(u_0, i_0) - (\mathcal{F}\omega)(u_1, i_1)| < \epsilon, \text{ for all } \omega \in \Omega. \tag{29}$$

Since $\mathcal{F} : \Omega \longrightarrow \Omega$, $\mathcal{F}$ is continuous and $\mathcal{F}(\Omega)$ is equicontinuous, the operator $\mathcal{F}$ satisfies the conditions of Schauder's fixed point theorem. Therefore there exists a $\omega^* = (V^{+*}, J^{+*}) \in \mathcal{F}$ such that $\mathcal{F}\omega^* = \omega^*$.

Now, we compute the equilibrium wage function w^* using V^{+*} and J^{+*}. We compute the equilibrium gains from trade accruing to the worker, V^*, and to the firm, J^*, using w^*, V^{+*} and J^{+*}. We compute the equilibrium law of motion for unemployment h^* using J^*. Finally, we construct the equilibrium employment contract $x^*(u)$ as the effort $e^* = 1$, the wage function w^* and the firing probabilities $s^*(y_h, \hat{z}) = 0$, $s^*(y_\ell, \hat{z}) = 0$ for $\hat{z} \in Z_0$ and $s^*(y_h, \hat{z}) = 1$ for $\hat{z} \in Z_2$. Since the wage function w^* is strictly decreasing in u and $h^*(u, Z_2)$ is strictly greater than $h^*(u, Z_0)$, these objects constitute a Recursive Equilibrium in which there is perfect correlation at the stage game.