# Exchange Rate Disconnect in General Equilibrium*

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**Abstract**

We propose a dynamic general equilibrium model of exchange rate determination which simultaneously accounts for all major exchange rate puzzles. This includes the Meese-Rogoff disconnect puzzle, the PPP puzzle, the terms-of-trade puzzle, the Backus-Smith puzzle, and the UIP puzzle. We build on a standard international real business cycle model with *home bias* in consumption, augmented with a *segmented* international financial market featuring noise traders and risk-averse intermediaries, which results in equilibrium UIP deviations due to limits to arbitrage. We show that shocks in the financial market result in a volatile and persistent near-martingale process for the exchange rate, and ensure empirically relevant comovement properties between exchange rates and macro variables, including inflation, consumption, output and interest rates. In contrast, conventional productivity and monetary shocks, while successful in explaining the international business cycle comovement, result in counterfactual exchange rate dynamics with insufficient volatility. Combining financial and macro shocks allows the model to reproduce the exchange rate disconnect properties without compromising the fit of the business cycle moments. Nominal rigidities improve the quantitative performance of the model, yet are not essential in explaining the disconnect.

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# 1 Introduction

Equilibrium exchange rate dynamics is a foundational topic in international macroeconomics (Dornbusch 1976, Obstfeld and Rogoff 1995). At the same time, *exchange rate disconnect* remains among the most challenging and persistent puzzles (Obstfeld and Rogoff 2001). The term disconnect narrowly refers to the lack of correlation between exchange rates and other macro variables, but the broader puzzle is more pervasive and nests a number of additional empirical patterns, which stand at odds with conventional international macro models. We define the broader exchange rate disconnect to include:

1. Meese and Rogoff (1983) puzzle: the *nominal* exchange rate follows a random-walk-like process, which is *not* robustly correlated, even contemporaneously, with macroeconomic fundamentals (see also Engel and West 2005). Furthermore, the exchange rate is an order of magnitude more volatile than macroeconomic aggregates such as consumption, output and inflation.
2. Purchasing Power Parity (PPP) puzzle (Rogoff 1996): the *real* exchange rate closely tracks the nominal exchange rate at most frequencies and, in particular, exhibits a similarly large persistence and volatility as the nominal exchange rate. Mean reversion, if any, takes a very long time, with half-life estimates in the range of 3-to-5 years, much in excess of conventional durations of price stickiness (see also Chari, Kehoe, and McGrattan 2002, henceforth CKM).
3. *Terms of trade* are weakly positively correlated with the real exchange rate, yet exhibit a markedly lower volatility, in contrast with the predictions of standard models (Atkeson and Burstein 2008).
4. Backus and Smith (1993) puzzle: the international risk-sharing condition that relative consumption across countries should be strongly positively correlated with the real exchange rates (implying high relative consumption in periods of low relative prices) is sharply violated in the data, with a mildly negative correlation and a markedly lower volatility of relative consumption (see Kollmann 1995 and also Corsetti, Dedola, and Leduc 2008).
5. Forward premium puzzle (Fama 1984), or the violation of the *uncovered interest rate parity* (UIP): cross-currency interest rate differentials are not balanced out by expected depreciations, and instead predict exchange rate appreciations (albeit with a very low $R^2$), resulting in positive expected returns on *currency carry trades* (see also Engel 1996, McCallum 1994).

We summarize the above puzzles as a set of moments characterizing comovement between exchange rates and macro variables in *developed* countries under a *floating* regime, and use them as quantitative goals in our analysis (see Table 1).

Existing general equilibrium international macro models either feature these puzzles, or attempt to address one puzzle at a time, often at the expense of aggravating the other puzzles, resulting in a lack of a unifying framework that exhibits satisfactory exchange rate properties. This is a major challenge for the academic and policy discussion, since exchange rates are the core prices in any international model, and failing to match their basic properties jeopardizes the conclusions one can draw from the analysis. In particular, would the conclusions in the vast literatures on currency unions, international policy spillovers and international transmission of shocks survive in a model with realistic exchange rate properties? Furthermore, what are the implications of such a model for the numerous micro-level empirical studies that treat exchange rate shocks as a source of exogenous variation?

The goal of this paper is to offer a unifying theory of exchange rates that can simultaneously account for all stylized facts introduced above. Our theory builds on a standard international macro model, with a conventional transmission mechanism, and emphasizes shocks in the financial market. Common productivity and monetary shocks, while successful in explaining the business cycle comovement, result in counterfactual exchange rate dynamics as reflected in the puzzles above. In contrast, we show that shocks in the financial market simultaneously resolve all exchange rate puzzles and deliver the empirically relevant comovement properties between exchange rates and macro variables, including a large gap in their volatilities. Furthermore, when combined together, the two sets of shocks allow the model to reproduce the exchange rate disconnect behavior together with the standard international business cycle comovement of the macro variables (as in e.g. Backus, Kehoe, and Kydland 1992, henceforth BKK).[1] In other words, our multi-shock model does *not* compromise the fit of the international business cycle moments in order to reproduce the empirical properties of the exchange rates.[2]

Our disconnect mechanism relies on two essential ingredients: *home bias* in the product market (following Obstfeld and Rogoff 2001) and an *imperfect* financial market featuring equilibrium UIP violations. In particular, we introduce a *segmented* financial market with noise traders and limits to arbitrage, following De Long, Shleifer, Summers, and Waldmann (1990), Jeanne and Rose (2002) and Gabaix and Maggiori (2015), into an otherwise conventional international business cycle model. In the model, households can trade only local bonds, and their net foreign asset positions need to be intermediated by arbitrageurs, who take on the exchange rate risk. The arbitrageurs are risk averse, and demand a risk premium for their intermediation of the positions of both households and noise traders, resulting in equilibrium UIP deviations, without implying equivalent violations of the covered interest parity (CIP).[3]

Furthermore, our analysis underscores that exchange rate disconnect is a *robust* implication of a standard international macro model augmented with financial shocks, without relying on other specific assumptions or a particular parameterization. The baseline model does *not* feature nominal rigidities or other sources of incomplete exchange rate pass-through, emphasizing that the empirically relevant

[1]The relative volatilities of the two types of shocks are identified by the Backus-Smith correlation between real exchange rate and relative consumption growth, which results in financial shocks dominating the variance decompositions of the exchange rates, while consumption and output are still largely determined by conventional macroeconomic shocks.

[2]Our framework is related to the international DSGE models in Devereux and Engel (2002) and Kollmann (2001, 2005), as well as to the open-economy wedge accounting (see Guillén 2013, Eaton, Kortum, and Neiman 2015). What sets our analysis apart is the emphasis on *simultaneously* resolving a broad range of exchange rate disconnect puzzles using a concise and tractable framework. In particular, we show that a *small-scale* model with just two shocks — financial and productivity — robustly explains the comovement of all international macro variables. Our work is also related to the VAR literature which studies impulse responses of international macro variables to structural shocks (Eichenbaum and Evans 1995, Erceg, Guerrieri, and Gust 2006, Scholl and Uhlig 2008, Stavrakeva and Tang 2015, Schmitt-Grohé and Uribe 2018). In view of large uncertainty around *conditional* moments in the data, e.g. the existence of delayed overshooting (see Kim and Roubini 2000), we focus on the *unconditional* moments as our primary targets (see discussion in Nakamura and Steinsson 2018).

[3]*Exogenous* UIP shocks are commonly used in the international macro literature (see e.g. Devereux and Engel 2002, Kollmann 2005, Farhi and Werning 2012), and can be viewed to emerge from exogenous asset demand, as in the literature following Kouri (1976, 1983). Alternative models of *endogenous* UIP deviations include models with incomplete information, expectational errors and heterogeneous beliefs (Evans and Lyons 2002, Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006, Burnside, Han, Hirshleifer, and Wang 2011), financial frictions (Adrian, Etula, and Shin 2015, Camanho, Hau, and Rey 2018), habits, long-run risk and rare disasters (Verdelhan 2010, Colacito and Croce 2013, Farhi and Gabaix 2016), and alternative formulations of segmented markets (Alvarez, Atkeson, and Kehoe 2009). We show that the disconnect mechanism requires that UIP deviations are not associated with *large contemporaneous* shocks to productivity or money supply.

extent of home bias is sufficient to mute the transmission of exchange rate volatility into macroeconomic aggregates, even in small *open* economies. We then show how various sources of incomplete pass-through, including pricing to market and foreign-currency price stickiness, can improve the quantitative fit of the model.

We supplement the quantitative analysis with analytical results in the context of a simplified version of the model, which admits a tractable closed-form solution, yet maintains the main disconnect properties of a richer quantitative environment. This analytical framework delivers three main conceptual insights. First, we characterize equilibrium exchange rate dynamics, which emerges as an interplay between forces in the financial and in the real (product and factor) markets. In particular, we show that a demand shock for the foreign-currency bonds results in a UIP violation and a slow but persistent expected appreciation of the home currency — to ensure equilibrium in the financial market. In turn, this needs to be balanced out by an *unexpected* depreciation on impact — to satisfy the intertemporal budget constraint. The more persistent is the shock, the larger is the initial depreciation, and thus the closer is the behavior of the nominal exchange rate to a random walk. Other persistent shocks, including productivity, result in a similar near-martingale behavior (see also Engel and West 2005). However, unlike productivity and other macro shocks, financial shocks generate *excess volatility* in exchange rates relative to macro aggregates, which is an essential feature of exchange rate disconnect (see also Devereux and Engel 2002). As the economy becomes less open to international trade, financial shocks result in more volatile exchange rate fluctuations with vanishingly small effects on the rest of the economy.[4]

Second, we address the equilibrium properties of the real exchange rate, and in particular the PPP puzzle, which is often viewed as the prime evidence in support of long-lasting real effects of nominal rigidities (as surveyed in Rogoff 1996). However, in view of the moderate empirical durations of nominal prices, calibrated sticky price models are incapable of generating persistent PPP deviations observed in the data (see CKM). The baseline assumption in this analysis is that monetary shocks are the main drivers of the nominal exchange rate, and that nominal rigidity is the key part of the transmission mechanism into the real exchange rate. We suggest an entirely different perspective, which deemphasizes nominal rigidities, and instead shifts focus to the nature of the shock process. We argue that the behavior of the real exchange rate — both in the time series (see e.g. Blanco and Cravino 2018) and in the cross-section (see e.g. Kehoe and Midrigan 2008) — is evidence neither in favor *nor* against sticky prices, but instead suggests that monetary shocks *cannot* be the key source of exchange rate fluctuations. We show that financial shocks drive both nominal and real exchange rates in concert, resulting in volatile and persistent behavior of both variables, thus reproducing the PPP puzzle. The only two essential ingredients of the transmission mechanism for this result are the monetary policy rule, which stabilizes domestic inflation, and home bias in consumption, which limits the response of aggregate prices to the exchange rate.[5]

[4]Intuitively, consider the extreme case of a demand shock for foreign bonds in an economy which cannot trade goods internationally. The full equilibrium adjustment in this case is achieved exclusively by means of exchange rate movements.

[5]Consumer price stabilization can account for the PPP puzzle even in response to productivity shocks (*cf.* Eichenbaum, Johannsen, and Rebelo 2018); however, this results in counterfactual predictions for alternative measures of the real exchange rate, in particular those based on relative wages, as well as in the other exchange rate puzzles.

Third, we address the Backus-Smith puzzle, namely the comovement between consumption and the real exchange rate. Our approach crucially shifts focus from risk sharing (in the financial market) to expenditure switching (in the goods market) as the key force shaping this comovement. We show that expenditure switching robustly implies a negative correlation between relative consumption and the real exchange rate, as is the case in the data. An exchange rate depreciation increases global demand for domestic goods, which in light of home bias requires a reduction in domestic consumption.[6] Indeed, this force is present in all models with expenditure switching and goods market clearing, yet it is usually dominated by the direct effect of shocks on consumption.[7] With a financial shock as the key source of exchange rate volatility, there is no direct effect, and thus expenditure switching is the only force affecting consumption — and weakly so under home bias — resulting in the empirically relevant comovement properties. Put differently, in order to reproduce the empirical Backus-Smith comovement, real depreciations must be mostly triggered by relative demand shocks for foreign-currency assets rather than supply shocks of domestically-produced goods.

In addition, we show that the model with financial shocks reproduces the comovement properties of the exchange rate with interest rates, and in particular the forward premium puzzle. Indeed, a demand shock for foreign-currency bonds is compensated in equilibrium with a lower return (a UIP deviation) due to both an increase in the relative home interest rate and an expected home-currency appreciation. This leads to a negative *Fama coefficient* in the regression of exchange rate changes on interest rate differentials, albeit with a vanishingly small $R^2$ as financial shocks become more persistent and the exchange rate becomes closer to a random walk. The disconnect mechanism further ensures that interest rates, just like consumption, are an order of magnitude less volatile than the exchange rate.[8]

The rest of the paper is organized as follows. In Section 2, we describe our baseline modeling framework, and in particular the model of the financial sector, which is our only departure from a conventional international business cycle environment. Section 3 explores the disconnect mechanism with a sequence of analytical results, addressing each of the exchange rate puzzles. Section 4 presents the quantitative results, emphasizing the fit of both exchange rate moments and conventional international business cycle moments. This section starts with the outline of the full quantitative environment and our calibration strategy, and concludes with the additional analysis of incomplete exchange rate pass-through and small open economies. Section 5 offers closing remarks and the online appendix provides detailed derivations and proofs.

[6]Perhaps more intuitively, the same general equilibrium mechanism can be restated as follows: financial shocks that lead home households to delay their consumption, which is biased towards domestically-produced goods, require an exchange rate depreciation to shift global expenditure towards these goods in order to clear the goods market.

[7]For example, productivity shocks (or also expansionary monetary shocks) increase the supply of domestic goods, reducing their prices (hence depreciating the real exchange rate) and increasing consumption, which induces a counterfactual correlation pattern. Alternative mechanisms in the literature (see e.g. Corsetti, Dedola, and Leduc 2008, Benigno and Thoenissen 2008, Kocherlakota and Pistaferri 2007, Colacito and Croce 2013, Karabarbounis 2014) either mute the direct effect of productivity shocks on consumption or reverse the sign of the exchange rate response, as we discuss further below.

[8]While the fact that a financial shock can match the forward premium moments is, perhaps, least surprising, the contribution of the paper is to show how the same shock simultaneously accounts for the other exchange rate puzzles, which the literature has typically viewed as distinct and often unrelated.

## 2 Modeling Framework

We build on a standard international real business cycle (IRBC) model with *home bias* in consumption and productivity shocks, and without capital in the simple *baseline* model. Monetary policy is conducted according to a conventional Taylor rule targeting inflation, resulting in a floating nominal exchange rate. The baseline specification features no nominal rigidities — all prices and wages are flexible. The only departure from a conventional IRBC model is a segmented international financial market, which features noise traders and risk-averse arbitrageurs, who intermediate the bond holdings of the households by taking carry trade positions. Online Appendix A.2 sets up our general quantitative model which additionally features capital and investment with adjustment costs, domestic and foreign intermediate inputs, Kimball (1995) demand resulting in variable markups and pricing to market, and sticky wages and prices in either producer, destination or dominant currency.

### 2.1 Model setup

There are two symmetric countries — home (Europe) and foreign (US, denoted with a $*$) — each with its own nominal unit of account in which the local prices are quoted: for example, the home wage rate is $W_t$ euros and the foreign is $W_t^*$ dollars. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, hence an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the euro (the home currency). In our description, we focus on the home country. In Section 4.4, we extend the analysis to accommodate asymmetric large and *small open* economies.

**Households** A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right),$$

where $\sigma$ is the relative risk aversion and $1/\varphi$ is the Frisch elasticity of labor supply (the results are robust to alternative utility specifications, e.g. GHH preferences). The flow budget constraint is given by:

$$P_t C_t + \frac{B_{t+1}}{R_t} \leq W_t L_t + B_t + \Pi_t, \tag{1}$$

where $P_t$ is the consumer price index and $W_t$ is the nominal wage rate, $B_t$ is the quantity of the local-currency bond paying out one unit of the home currency next period, $R_t$ is the gross nominal interest rate (and, thus, $1/R_t$ is the price of the bond), and $\Pi_t$ are dividends from domestic firms. Household optimization results in the standard labor supply condition and Euler equation for bonds:

$$C_t^{\sigma} L_t^{1/\nu} = W_t/P_t, \tag{2}$$

$$1 = \beta R_t \, \mathbb{E}_t \left\{ (C_{t+1}/C_t)^{-\sigma} P_t/P_{t+1} \right\}. \tag{3}$$

We assume that households trade only local-currency bonds, as well as own home firms.

The foreign households are symmetric, with their behavior characterized by parallel optimality conditions. In particular, they trade only foreign-currency bonds $B_{t+1}^*$, which pay nominal interest $R_t^*$ in foreign currency, and own foreign firms which pay $\Pi_t^*$ as dividends.

The domestic households allocate their within-period consumption expenditure $P_tC_t$ between home and foreign varieties of the final good $C_t$, defined by a CES aggregator featuring home bias:

$$C_t = \left(\int_0^1 \left[(1-\gamma)^{\frac{1}{\theta}} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{Ft}(i)^{\frac{\theta-1}{\theta}}\right] \mathrm{d}i\right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution and $\gamma \in [0, \frac{1}{2})$ is the *trade openness* parameter, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see Obstfeld and Rogoff 2001).[9] The households minimize expenditure, $P_tC_t = \int_0^1 \left[P_{Ht}(i)C_{Ht}(i) + P_{Ft}(i)C_{Ft}(i)\right]\mathrm{d}i$, resulting in the conventional constant elasticity demand schedules:

$$C_{Ht}(i) = (1-\gamma)\left(\frac{P_{Ht}(i)}{P_t}\right)^{-\theta} C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma \left(\frac{P_{Ft}(j)}{P_t}\right)^{-\theta} C_t, \tag{4}$$

where the consumer price level is given by $P_t = \left(\int_0^1 \left[(1-\gamma)P_{Ht}(i)^{1-\theta} + \gamma P_{Ft}(i)^{1-\theta}\right]\mathrm{d}i\right)^{1/(1-\theta)}$.

The expenditure allocation of the foreign households is characterized by a symmetric demand system. In particular, the demand for home and foreign goods by foreign households is given by:

$$C^*_{Ht}(i) = \gamma \left(\frac{P^*_{Ht}(i)}{P^*_t}\right)^{-\theta} C^*_t \qquad \text{and} \qquad C^*_{Ft}(j) = (1-\gamma)\left(\frac{P^*_{Ft}(j)}{P^*_t}\right)^{-\theta} C^*_t, \tag{5}$$

where $P^*_{Ht}(i)$ and $P^*_{Ft}(j)$ are the foreign-currency prices of the home and foreign goods in the foreign market, and $P^*_t$ is the foreign consumer price level with a respective home bias.

The *real exchange rate*, $\mathcal{Q}_t \equiv (P^*_t\mathcal{E}_t)/P_t$, is the relative consumer price level in the two countries. An increase in $\mathcal{Q}_t$ corresponds to a real *depreciation*, i.e. a decrease in the relative price of the home consumption basket.

**Producers** Home output is produced by a given pool of identical firms (hence we omit indicator $i$) using a linear technology in labor:

$$Y_t = e^{a_t} L_t \qquad \text{with} \qquad a_t = \rho_a a_{t-1} + \sigma_a \varepsilon^a_t, \tag{6}$$

where $a_t$ is the log total factor productivity following an AR(1) process with persistence $\rho_a \in [0, 1]$ and volatility of the innovation $\sigma_a \geq 0$. Given the wage rate $W_t$, the associated marginal cost of production is $MC_t = e^{-a_t}W_t$. Our results below do not depend on constant returns to scale, a constant elasticity production function, or a single production input, which we adopt solely to simplify exposition.

Every firm faces a downward sloping demand for its variety in each market. The firm maximizes profits from serving the home and foreign markets:

$$\Pi_t(i) = \left(P_{Ht}(i) - MC_t\right) C_{Ht}(i) + \left(P^*_{Ht}(i)\mathcal{E}_t - MC_t\right) C^*_{Ht}(i), \tag{7}$$

by setting $P_{Ht}(i)$ and $P^*_{Ht}(i)$, expressed in home and foreign currency respectively, and producing $Y_t = C_{Ht}(i) + C^*_{Ht}(i)$ to accommodate the resulting demand (4) and (5) in the two markets. The aggre-

[9] The aggregate implications of the model do not depend on whether home bias emerges on the extensive margin due to non-tradables or on the intensive margin due to trade costs or preferences; hence, we do not explicitly introduce non-tradables.

gate profits of the domestic firms, $\Pi_t = \int_0^1 \Pi_t(i)\mathrm{d}i$, are then distributed to the domestic households. We assume no entry or exit of firms, and therefore our model captures the short and the medium run — namely the horizons from *one month* up to *five years*, where empirically extensive margins of firm entry and exit play a negligible role in the aggregate (see e.g. Bernard, Jensen, Redding, and Schott 2009).

The firms set prices $P_{Ht}(i)$ and $P^*_{Ht}(i)$ flexibly by maximizing profits in (7) state-by-state. With CES demand, this results in constant-markup pricing with a common price across all domestic firms and the *law of one price* (LOP) holding across the two markets:

$$P_{Ht}(i) = P_{Ht} = \frac{\theta}{\theta-1} e^{-a_t} W_t \qquad \text{and} \qquad P^*_{Ht}(i) = P^*_{Ht} = P_{Ht}/\mathcal{E}_t, \tag{8}$$

for all $i \in [0,1]$. An equivalent price setting problem characterizes the behavior of the foreign firms:

$$P^*_{Ft}(i) = P^*_{Ft} = \frac{\theta}{\theta-1} e^{-a^*_t} W^*_t \qquad \text{and} \qquad P_{Ft}(i) = P_{Ft} = P^*_{Ft}\mathcal{E}_t, \tag{9}$$

in the foreign and home markets respectively. Price setting in (8)–(9) features complete pass-through of shocks and no pricing to market — assumptions that we relax in our quantitative analysis.

**Market clearing** The labor market clearing requires that $L_t$ equals simultaneously the labor supply of the households in (2) and the labor demand of the firms in (6), and equivalently for $L^*_t$ in foreign. The wage rates, $W_t$ and $W^*_t$, adjust to clear this market.

Goods market clearing requires that total production by the home firms is split between supply to the home and foreign markets respectively and satisfies the local demand in each market:

$$Y_t = C_{Ht} + C^*_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\theta} C_t + \gamma \left(\frac{P^*_{Ht}}{P^*_t}\right)^{-\theta} C^*_t, \tag{10}$$

where $Y_t$ is the aggregate home production and $C_{Jt}(i) = C_{Jt}$ for all $i \in [0,1]$ and $J \in \{H, F\}$, due to symmetry in price setting across firms. A symmetric condition holds for the foreign production $Y^*_t$.

Lastly, we combine the household budget constraint (1) with profits (7), aggregated across all home firms, as well as the market clearing conditions above, to obtain the home country budget constraint:

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{with} \qquad NX_t = \mathcal{E}_t P^*_{Ht} C^*_{Ht} - P_{Ft} C_{Ft}, \tag{11}$$

where $NX_t$ denotes net exports expressed in units of the home currency. The relative price of imports, $\mathcal{S}_t \equiv P_{Ft}/(\mathcal{E}_t P^*_{Ht})$, is the *terms of trade*. The foreign country faces a symmetric budget constraint, which is redundant by Walras law.

**Monetary policy** The government is present in the economy only by means of the monetary policy rule, as the fiscal authority is fully passive. This is without loss of generality as, in view of Ricardian equivalence, the net foreign asset position of the country $B_{t+1}$ should be regarded as the consolidated position of the public and the private sectors. The monetary policy in both countries is implemented via a conventional Taylor rule. In the baseline model, which features no nominal rigidities, we consider the limiting case with fully stable consumer prices, or zero inflation: $\pi_t \equiv \Delta \log P_t = 0$ and $\pi^*_t = 0$ for all $t$.

While this offers a useful simplification for the analytical analysis, we also view it as a reasonable point of approximation to a low-inflation environment in the developed countries.

## 2.2 Segmented financial market

The remaining block of the model concerns the international financial market and the resulting equilibrium risk-sharing between home and foreign households, which constitutes the only departure from an otherwise conventional IRBC model. The financial market is incomplete and *segmented*, as the home and foreign households cannot directly trade any assets with each other, and their international asset positions are intermediated by the financial sector. The equilibrium in the financial market results in a modified interest rate parity condition, which is subject to *financial shocks*, as we now describe.

Our model of the financial sector builds on Jeanne and Rose (2002) and Gabaix and Maggiori (2015), and features three types of agents: households, noise traders and professional intermediaries.[10] The home and foreign households trade the local-currency bonds only, and hence cannot directly trade assets with each other. In particular, the home households demand a quantity $B_{t+1}$ of the home-currency bond at time $t$, while the foreign households demand a quantity $B^*_{t+1}$ of the foreign-currency bond. Both $B_{t+1}$ and $B^*_{t+1}$ can take positive or negative values, depending on whether the households save or borrow.

In addition to the household *fundamental* demand for currency (bonds), the financial market features a *liquidity* currency demand — independent of the expected currency return and the other macroeconomic fundamentals — from a measure $n$ of symmetric *noise* traders. In particular, noise traders follow a zero-capital strategy by taking a long position in the foreign currency and shorting equal value in the home currency, or vice versa if they have an excess demand for the home currency. The overall position of the noise traders is $\frac{N^*_{t+1}}{R^*_t}$ dollars invested in the foreign-currency bond, matched by $\frac{N_{t+1}}{R_t} = -\frac{\mathcal{E}_t N^*_{t+1}}{R^*_t}$ euros invested in the home-currency bond, and we model it as an exogenous process:

$$\frac{N^*_{t+1}}{R^*_t} = n\left(e^{\psi_t} - 1\right) \qquad \text{with} \qquad \psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon^\psi_t. \tag{12}$$

We refer to the noise-trader demand shock $\psi_t$ as the *financial shock*, with $\rho_\psi \in [0,1]$ and $\sigma_\psi \geq 0$ parametrizing its persistence and volatility, respectively.

The trades of the households and the noise traders are intermediated by a measure $m$ of symmetric risk-averse arbitrageurs, or market makers. These intermediaries adopt a zero-capital *carry trade* strategy by taking a long position in the foreign-currency bond and a short position of equal value in the home-currency bond, or vice versa. The return on the carry trade is given by:

$$\tilde{R}^*_{t+1} = R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{13}$$

per dollar invested in the foreign-currency bond and $\mathcal{E}_t$ euros sold of the home-currency bond at time $t$.

[10] We follow Jeanne and Rose (2002) in modeling the financial intermediaries, who take limited asset positions due to exposure to the exchange rate risk, rather than due to financial constraints as in Gabaix and Maggiori (2015). In contrast, we follow Gabaix and Maggiori (2015) in modeling the segmented participation of the households. Lastly, the exogenous liquidity needs of the noise trader are akin to the exogenous 'portfolio flows' in Gabaix and Maggiori (2015), but can equally emerge from biased expectations about the exchange rate, $\mathbb{E}^n_t \mathcal{E}_{t+1} \neq \mathbb{E}_t \mathcal{E}_{t+1}$, as in Jeanne and Rose (2002).

We denote the size of individual position by $d^*_{t+1}$, which may take positive or negative values, and assume that intermediaries maximize the CARA utility of the real return in units of the foreign good:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left( -\omega \frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} \right) \right\}, \tag{14}$$

where $\omega \geq 0$ is the risk aversion parameter.[11] In aggregate, all $m$ intermediaries invest $\frac{D^*_{t+1}}{R^*_t} = m \frac{d^*_{t+1}}{R^*_t}$ dollars in foreign-currency bond, and take an offsetting position of $\frac{D_{t+1}}{R_t} = -\frac{\mathcal{E}_t D^*_{t+1}}{R^*_t}$ euros in home-currency bond, resulting in a zero-capital portfolio at time $t$.

Both currency bonds are in zero net supply, and therefore financial market clearing requires that the positions of the households, noise traders and intermediaries balance out:

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \qquad \text{and} \qquad B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0. \tag{15}$$

In equilibrium, the intermediaries absorb the demand for home and foreign currency of both households and noise traders. If intermediaries were risk neutral, $\omega = 0$, they would do so without a risk premium, resulting in the *uncovered interest parity* (UIP), or equivalently a zero expected real return, $\mathbb{E}_t\{\tilde{R}^*_{t+1}/P^*_{t+1}\} = 0$. Risk-averse intermediaries, however, require an appropriate compensation for taking a risky carry trade, which results in equilibrium risk premia and deviations from the UIP.

**Lemma 1** *The equilibrium condition in the financial market, log-linearized around a symmetric steady state with $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{\mathcal{Q}} = 1$, and a finite nonzero $\omega\sigma_e^2/m$, is given by:*

$$i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \tag{16}$$

*where $i_t - i^*_t \equiv \log(R_t/R^*_t)$, $b_{t+1} \equiv B_{t+1}/\bar{Y}$, and the coefficients $\chi_1 \equiv \frac{n}{\beta}\frac{\omega\sigma_e^2}{m}$ and $\chi_2 \equiv \bar{Y}\frac{\omega\sigma_e^2}{m}$, with $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ denoting the volatility of the log nominal exchange rate, $e_t \equiv \log \mathcal{E}_t$.*

The equilibrium condition (16) is the *modified UIP* in our model with imperfect financial intermediation, where the right-hand side corresponds to the departures from the UIP. Condition (16) arises from the combination of the financial market clearing (15) with the solution to the portfolio choice problem of the intermediaries (14), as we formally show in Online Appendix A.4. Intuitively, the optimal portfolio of the intermediaries $D^*_{t+1}$ is proportional to the expected log return on the carry trade, $i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1}$, scaled by the risk absorption capacity of the intermediary sector, $\omega\sigma_e^2/m$, i.e. the product of their effective risk aversion ($\omega/m$, the price of risk) and the volatility of the carry trade return ($\sigma_e^2$, the exchange rate risk). As $\omega\sigma_e^2/m \to 0$, the risk absorption capacity of the intermediaries increases, and the UIP deviations disappear in the limit, $\chi_1, \chi_2 \to 0$. With $\omega\sigma_e^2/m > 0$, the UIP deviations remain first order and hence affect the first-order equilibrium dynamics. Note that both $\psi_t > 0$ and $b_{t+1} < 0$ correspond to the excess demand for the foreign-currency bond — by noise traders and households, respectively — resulting in a negative expected return on the foreign currency bond.

[11] CARA utility provides tractability, as it results in a portfolio choice that does not depend on the level of wealth of the intermediaries, thus avoiding the need to carry it as an additional state variable; the tradeoff of working with CARA-utility, however, is that intermediaries need to be short-lived, maximizing the one-period return on their investment.

**International risk sharing** What are the implications of this financial market equilibrium for the risk sharing between the home and foreign households? First, as both noise traders and intermediaries hold zero-capital positions, financial market clearing (15) implies a balanced position for the home and foreign households combined, $\frac{B_{t+1}}{R_t} + \mathcal{E}_t \frac{B^*_{t+1}}{R^*_t} = 0$. In other words, even though the home and foreign households do not trade any assets directly, the financial market acts to intermediate the intertemporal borrowing between them. However, this intermediation is *frictional*, as there is a wedge between the interest rates faced by the home and foreign households, $R_t$ and $R^*_t$, namely the (expected) departures from the UIP in (16). If interest rate parity held, the equilibrium would correspond to a conventional bonds-only *incomplete-market* IRBC model. The UIP wedge limits further the extent of international risk sharing.[12] The exogenous noise-trade shock $\psi_t$ plays the key role as the driver of the deviations from the interest rate parity and international risk-sharing. The specific nature of this shock is, however, less important for the resulting macroeconomic and exchange rate dynamics.[13]

**Covered interest parity** We consider briefly the equilibrium pricing of the currency forwards by the financial sector, and the resulting *covered interest parity* (CIP). Consider a period $t$ forward price $\mathcal{F}_t$ of one unit of foreign currency at $t+1$, in units of home currency. The financial sector prices it at $\mathcal{F}_t = \mathcal{E}_t R_t / R^*_t$, as any other price would lead the intermediaries to take unbounded positions buying or selling such forwards (see Online Appendix A.4). Therefore, CIP holds in equilibrium, even though UIP is generally violated, as the imperfection in the financial market is due to market segmentation and limited risk absorption by the risk-averse intermediaries. Profiting from the UIP deviations requires taking a carry-trade risk, which commands an equilibrium risk premium, while the departures from CIP generate riskless arbitrage opportunities. This is in contrast with models of financial constraints, where the departures from both UIP and CIP are due to binding constraints on the balance sheet of the financial intermediaries. We opt in favor of the former modeling approach as empirically the size of the CIP deviations is at least an order of magnitude smaller than the expected departures from the UIP.[14]

# 3 The Disconnect Mechanism

In this section, we explore the baseline model, which admits a tractable analytical solution without compromising the exchange rate *disconnect mechanism* of the more general framework. This allows us to fully dissect the mechanism, and show in particular that two ingredients play the central role in the model's ability to explain the exchange rate puzzles — namely, the financial shock $\psi_t$ as the leading

[12] The positions of intermediaries and noise traders also generate income (or losses), which for concreteness we assume is returned to the foreign households at the end of each trading period, as a lump-sum payment. This, however, is inconsequential for the first-order dynamics of the model, as this transfer is second order (see Appendix A.4).

[13] In Itskhoki and Mukhin (2017), we discuss various alternative microfoundations for the UIP shock $\psi_t$, ranging from complete-market models of risk-premia (e.g., Verdelhan 2010, Colacito and Croce 2013, Farhi and Gabaix 2016) to models of heterogeneous beliefs and expectational errors (e.g., Evans and Lyons 2002, Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006). All models resulting in a variant of (16) with $\chi_1 > 0$ (and $\chi_2 \geq 0$) produce similar equilibrium exchange rate dynamics, and can only be differentiated by additional financial moments (e.g., covered interest parity, comovement of exchange rates with additional asset classes). Consistent with the recent work of Lustig and Verdelhan (2018), we emphasize the importance of financial frictions in explaining exchange rates, and in particular focus on the role of segmented markets.

[14] Du, Tepper, and Verdelhan (2018) document that the average annualized CIP deviations were a negligible 2 basis points prior to 2007, and since then increased tenfold to 20 basis points, yet this is still an order of magnitude smaller than the expected UIP deviations, which are around 200 basis points, or 2%.

driver of the exchange rates, and the low trade openness $\gamma$, which ensures a muted pass-through of the exchange rate volatility into the macro aggregates. Section 4 confirms that the same two features of the model remain key in a full quantitative environment, which additionally features domestic and imported intermediate inputs, pricing to market, and nominal rigidities. These additional ingredients reinforce home bias to further mute the exchange rate transmission, helping to quantitatively match a rich set of moments describing the comovement between exchange rates and macro variables, given the empirical degree of openness of various countries, as we explore in Section 4.4.

For concreteness, we focus here on just two shocks — the relative productivity shock $\tilde{a}_t \equiv a_t - a_t^*$ and the financial shock $\psi_t$ — with the same persistence parameter $\rho$. Yet, our results generalize for a variety of macro-fundamental shocks following general statistical processes, including monetary, government spending, price-markup and labor-wedge shocks, so long as this set features the financial shock $\psi_t$, as we show in Itskhoki and Mukhin (2017). To further streamline exposition, we consider the limiting case with $\chi_2 = 0$ and a normalization $\chi_1 = 1$ in the modified UIP condition (16), which simplifies the dynamic roots of the equilibrium system without changing its quantitative properties.[15]

We solve the model by log-linearization, and write all expressions in terms of log deviations from a symmetric steady state, denoted with corresponding small letters (e.g., $y_t \equiv \log Y_t - \log \bar{Y}$ is the log deviation of GDP). The model admits a block-recursive structure, which allows for a sequential analysis of its equilibrium properties. We begin here by postulating the equilibrium near-random-walk property of the nominal exchange rate. We then proceed sequentially with the analysis of the PPP puzzle (the price block), the Backus-Smith puzzle (the quantity block), and the Forward Premium puzzle (the intertemporal block). Finally, we conclude with the general equilibrium properties of the model, and in particular the Meese-Rogoff disconnect puzzle, circling back to the result that we now introduce:

**Proposition 1** *The equilibrium nominal exchange rate, $e_t \equiv \log \mathcal{E}_t$, follows a volatile near-random-walk process; in particular, when both discount factor $\beta$ and shock persistence $\rho \approx 1$ then* $\mathrm{corr}(\Delta e_t, \Delta e_{t+1}) \approx 0$.

An important property for our analysis is that the exchange rate follows a volatile and persistent process, as it does in the data. We take this general equilibrium result as given in Section 3.1–3.3, and return to its detailed analysis in Section 3.4, where we focus both on the near-random-walk properties of the exchange rate and its excess volatility relative to fundamental macroeconomic variables, such as GDP, for realistic values of the model parameters.

## 3.1 The Purchasing Power Parity (PPP) puzzle

We start our analysis with the equilibrium relationship between real and nominal exchange rates, and the associated purchasing power parity (PPP) puzzle, which we broadly interpret as the *close comovement* between the nominal and the real exchange rates. In the data, all notions of the real exchange rate — consumer price, producer price and wage-based — closely comove with the nominal exchange rate, exhibiting strong persistence with very long half-lives. The close comovement involves both nearly

[15] We present the general analytical solution with endogenous $\chi_1, \chi_2 > 0$ in Online Appendix A.6. From Lemma 1, the limit with $\chi_1 > 0$ and $\chi_2 \to 0$ emerges when $\bar{Y}/m \to 0$ as $n/m$ stays bounded away from zero, that is when the size of the financial sector (number of both noise traders $n$ and arbitrageurs $m$) increases relative to the size of the real economy. The further normalization of $\chi_1 = 1$ is simply a rescaling of the volatility units of the shock $\psi_t$.

perfect correlations at various horizons and nearly equal volatilities for all exchange rate series, as we illustrate in Figure 1. Such properties may, perhaps, be expected of the nominal exchange rate if it is viewed as a financial variable; they are, however, puzzling from the point of the real exchange rate — a key international relative price with an important role in the product market, especially in light of the relative stability of the macro aggregates, including price levels, consumption and output.

Our disconnect mechanism immediately delivers a close comovement between nominal and real exchange rates. A monetary policy that ensures stable price levels, or $p_t = p_t^* \equiv 0$ in log-deviation terms, implies the identical dynamics for the real and nominal exchange rates:

$$q_t \equiv p_t^* + e_t - p_t = e_t, \tag{17}$$

where $q_t \equiv \log \mathcal{Q}_t$. In the absence of nominal frictions, the monetary policy rule simply selects a path of the nominal exchange rate, which tracks the real exchange rate by keeping the consumer price levels stable. In other words, if the monetary authority is successful at stabilizing consumer prices, as is the case in developed countries, this immediately explains the comovement between the nominal and the real exchange rates. It remains to show how in general equilibrium both exchange rates follow a volatile and persistent near-random-walk process, as predicted by Proposition 1 (see Section 3.4).[16]

Next we explore the properties of the other real exchange rates. Combining the price setting equations (8)–(9) with the log-linearized expression for the price level $P_t$, we solve for the real wage rate:[17]

$$w_t - p_t = a_t - \frac{\gamma}{1-2\gamma} q_t. \tag{18}$$

The real wage increases with the productivity of the economy $a_t$ and with its relative purchasing power, captured by the real exchange rate in proportion with the openness of the economy $\gamma$. This allows us to characterize the producer-price and the wage-based real exchange rates as follows:

$$q_t^P \equiv p_{Ft}^* + e_t - p_{Ht} = \frac{1}{1-2\gamma} q_t, \tag{19}$$

$$q_t^W \equiv w_t^* + e_t - w_t = \frac{1}{1-2\gamma} q_t - (a_t - a_t^*). \tag{20}$$

From (19), which holds independently of the source of the shocks, we immediately see that the model reproduces a close comovement between consumer and producer relative prices. Furthermore, as openness $\gamma$ decreases, $q_t$ and $q_t^P$ are not just tightly correlated, but also have approximately the same volatility. Intuitively, a monetary policy that stabilizes domestic consumer prices also stabilizes domestic producer prices, as the difference between the two is small in economies with home bias.

This logic does *not* hold for the wage-based real exchange rate $q_t^W$ in (20). In particular, productivity shocks $a_t$ and $a_t^*$ drive a wedge between price and wage inflation. In other words, a monetary policy that stabilizes prices in response to productivity shocks results in volatile wages, as real wages (18)

[16]In particular, if one were to fit an AR(1) process for the real exchange rate in our model, as is conventionally done in the PPP puzzle literature (see Rogoff 1996), one would be challenged to find evidence of mean reversion and would infer very long *half lives* in finite samples (see Online Appendix A.6), quantitatively consistent with the 3-to-5 year empirical range.

[17]The log deviation of the price index $P_t$ around a symmetric equilibrium is given by $p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}$, with $p_{Ht} = w_t - a_t$ and $p_{Ft} = w_t^* + e_t - a_t^*$, and symmetric expressions holding in foreign.

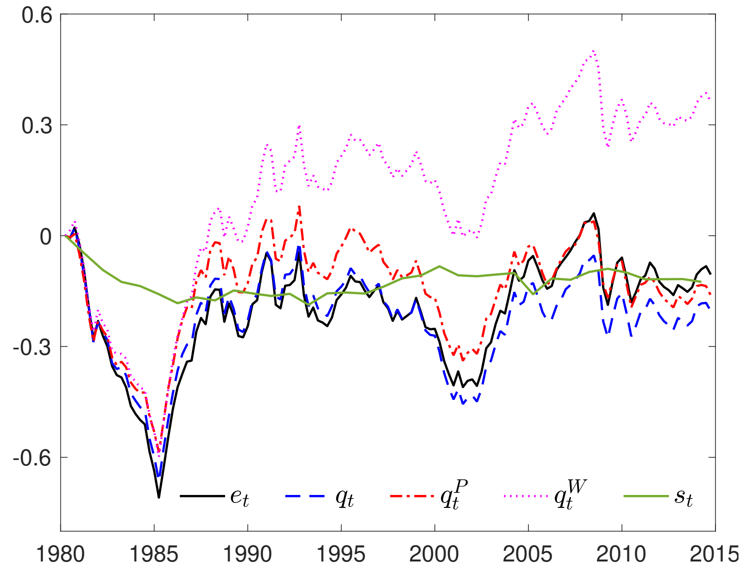


Figure 1: Nominal and real exchange rates

Note: The figure plots quarterly NER $e_t$, CPI-based RER $q_t$, PPI-based RER $q_t^P$, wage-based RER $q_t^W$ for the US against the PPP-weighted sum of France, Germany, and the UK, as well as the annual terms of trade $s_t$ for the US against the rest of the world (see Online Data Appendix A.1). All series are in logs and normalized to zero in 1980:Q1.

reflect productivity.[18] This tradeoff is not present, however, if financial shocks are the key drivers of the exchange rates. Indeed, financial shocks generate a volatile and persistent nominal exchange rate, which — under a monetary policy that stabilizes price levels — translates into equally volatile and persistent real exchange rates, independently of whether they are measured using consumer prices, producer prices, or wages. We summarize these results in:

**Lemma 2** *Home bias and monetary policy that stabilizes consumer prices ensure a near-perfect comovement between $e_t$, $q_t$ and $q_t^P$. Financial shocks, in addition, result in a near-perfect comovement between $q_t$ and $q_t^W$.*

Lemma 2, combined with Proposition 1, has the following equilibrium implication:

**Proposition 2** *With financial shocks, home bias, and $\beta, \rho \approx 1$, all real exchange rates exhibit a volatile near-random-walk behavior, with arbitrarily large half lives, closely tracking the nominal exchange rate.*

In other words, the combination of (i) a conventional monetary policy, (ii) significant home bias, and (iii) financial shocks allows the model to reproduce the empirical behavior of all exchange rate series. The absence of the direct effect of the financial shock on the product and labor markets, translates price stability into both nominal and real wage stability. As a result, the international relative prices and wages comove closely with the nominal exchange rate, exhibiting a high degree of persistence. Greater openness of the economies leads to larger feedback effects of the exchange rate into domestic relative prices, as the foreign value added plays a bigger role in the domestic consumption basket (see (18)). Importantly, these properties of the model arise under flexible prices and wages, and hence do not rely

[18] Empirically, in Figure 1, while $q_t^W$ and $q_t$ are highly correlated in changes, their low-frequency movements differ, reflecting the differential productivity and real-wage growth in the US and Europe, consistent with the role of $(a_t - a_t^*)$ in (20).

on nominal rigidities. While wage and price stickiness is arguably a salient feature of the real world, Proposition 2 shows that the empirical behavior of exchange rates is well captured to a first order by a flexible-price model, provided that financial shocks account for a considerable portion of exchange rate volatility.

**Relationship to the PPP puzzle literature** In view of this result, a natural question is why the PPP puzzle posed such a challenge to the literature, as summarized in a seminal survey by Rogoff (1996). From the definition of the real exchange rate, the close comovement between $q_t$ and $e_t$ implies that price levels $p_t$ and $p_t^*$ must, in turn, move little with the nominal exchange rate $e_t$. The PPP puzzle literature has largely focused on one conceptual possibility, namely that price levels move little due to nominal rigidities, assuming monetary shocks are the main drivers of the nominal exchange rate. The issue with this approach is that monetary shocks necessarily imply cointegration between relative nominal variables — $(w_t - w_t^*)$, $(p_t - p_t^*)$ and $e_t$ — resulting in mean reversion in the real exchange rate $q_t$. The speed of this mean reversion is directly controlled by the duration of nominal stickiness, which is empirically insufficient to generate long half lives, characteristic of the real exchange rate (see e.g. CKM).

We focus on the other conceptual possibility, namely that prices are largely disconnected from exchange rates, or in other words the low exchange rate pass-through into CPI inflation, even in the long run, due to substantial home bias (small $\gamma$). Importantly, this mechanism requires that the main drivers of the exchange rate are not productivity or monetary shocks, which introduce a wedge between nominal and real exchange rates independently of the extent of the home bias. In contrast, financial shocks do *not* drive a wedge between nominal and real exchange rates, even in the long run, and thus our mechanism does not need to rely on short-run nominal rigidities to induce their comovement.[19]

### 3.2 The Backus-Smith puzzle

We now study the relationship between aggregate consumption and the real exchange rate, emphasizing the role of *expenditure switching* in the product market, as opposed to international risk sharing in the financial market (the Backus-Smith condition).[20] The expenditure switching mechanism relies on the standard equilibrium conditions in international macro models, namely the labor and product market clearing, which derive from (2), (6) and (10). We rewrite these conditions in log deviation terms:

$$\begin{aligned} \sigma c_t + \varphi \ell_t &= w_t - p_t, \\ y_t &= a_t + \ell_t, \\ y_t &= (1-\gamma)\big[-\theta(p_{Ht} - p_t) + c_t\big] + \gamma\big[-\theta(p^*_{Ht} - p^*_t) + c^*_t\big]. \end{aligned}$$

[19] As a result, our model is consistent with the recent cross-sectional and time-series evidence, which poses a challenge for the conventional monetary model: Kehoe and Midrigan (2008) show a missing correlation between price durations and the volatility and persistence of the *sectoral* real exchange rates (see also Imbs, Mumtaz, Ravn, and Rey 2005, Carvalho and Nechio 2011), while Blanco and Cravino (2018) show that *reset-price RER* is as volatile and persistent as the conventional RER. Neither of this is evidence against price stickiness *per se*, but rather it is evidence against monetary shocks as the key driver of the nominal exchange rate. Furthermore, our model is also in line with the empirical findings in Engel (1999) and Betts and Kehoe (2008), as it is the *tradable* component that drives the volatility of the overall RER (see Online Appendix A.6).

[20] With complete markets, the efficient international risk-sharing requires $\sigma(c_t - c_t^*) = q_t$, implying a perfect positive correlation between relative consumption and RER. Our model, instead, features incomplete markets and a shock to *risk-sharing* $\psi_t$, resulting in $\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\} = \psi_t$, which does *not* impose a particular correlation pattern.

Using the solution for prices and wages from the previous subsection, and solving out equilibrium employment $\ell_t$, we arrive at the two equilibrium loci for the labor and product markets respectively:

$$\sigma c_t + \varphi y_t = (1+\varphi)a_t - \frac{\gamma}{1-2\gamma}q_t, \tag{21}$$

$$y_t = (1-\gamma)c_t + \gamma c_t^* + 2\theta\frac{\gamma(1-\gamma)}{1-2\gamma}q_t. \tag{22}$$

Consider first the labor market clearing condition (21). A real depreciation (an increase in $q_t$) reduces the real wage (recall (18)) and hence labor supply, resulting in lower output $y_t$. In turn, higher productivity increases output both directly and indirectly (due to increased labor supply), while higher consumption reduces labor supply via the income effect. We turn next to the goods market clearing condition (22), which in the closed economy with $\gamma = 0$ reduced to $y_t = c_t$. In an open economy, home production $y_t$ is split between domestic and foreign consumption of the home good, which increases with overall consumption levels in the two countries ($c_t$ and $c_t^*$), as well as with the real depreciation of the home currency. This latter force is the expenditure switching effect that we emphasize in our analysis, and its quantitative magnitude is proportional to $\gamma\theta$ — a product of the economy's openness $\gamma$ and the elasticity of substitution between home and foreign goods $\theta$.

Combining (21)–(22) with their foreign counterparts, we obtain the equilibrium relationship between relative consumption and the real exchange rate, implied by the labor and product market clearing:

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t, \tag{23}$$

where the derived parameters $\kappa_a \equiv \frac{1+\varphi}{\sigma+\varphi(1-2\gamma)}$ and $\gamma\kappa_q \equiv \frac{2\gamma}{1-2\gamma}\frac{1+2\theta\varphi(1-\gamma)}{\sigma+\varphi(1-2\gamma)}$ are both positive. This relationship immediately implies:

**Proposition 3** *An equilibrium response to a financial shock $\psi_t$ implies a negative comovement between the real exchange rate $q_t$ and relative consumption $c_t - c_t^*$, with the relative volatility of the consumption response, $\mathrm{var}_t(\Delta c_{t+1} - \Delta c_{t+1}^*)/\mathrm{var}_t(\Delta q_{t+1})$, declining to zero as $\gamma \to 0$.*

Proposition 3 emphasizes two important properties of the financial shocks. First, they result in a negative correlation between relative consumption and the real exchange rate, both in levels and in growth rates. In other words, consumption is low when prices are low, in relative terms across countries. This violates the pattern of efficient international risk sharing predicted by the Backus-Smith condition, yet it is in line with the robust empirical patterns observed in the data for rich countries — the Backus-Smith puzzle. Second, the proposition also shows that in economies with home bias, the relative volatility of the consumption response to financial shocks can be an order of magnitude smaller than the response of the real exchange rate — an essential property for the model to be consistent with the empirical exchange rate disconnect.

What is most striking about this simple resolution of the celebrated Backus-Smith puzzle is that it derives from conventional labor and product market clearing conditions, which are ubiquitous in international general equilibrium models. Indeed, the negative relationship between consumption and the real exchange rate is a *robust* feature of the expenditure switching mechanism. A real exchange rate depreciation switches expenditure towards home goods, and in order to clear the markets, home output needs to rise and home consumption needs to fall because of the home bias.[21] Furthermore, this effect persists regardless of the other parameters of the model, including the relative risk aversion $\sigma$ and the inverse Frisch elasticity of labor supply $\varphi$.

**Relationship to the conventional models** We have just argued that the expenditure switching effect is a robust property of nearly every international macro model. Why is it, then, that the Backus-Smith puzzle proved to be such a challenge for both the productivity-driven IRBC models and the monetary New Keynesian models, even when these models feature incomplete asset markets? Equilibrium condition (23) sheds light on this question as well. It implies the following variance decomposition:

$$\frac{\operatorname{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t)}{\operatorname{var}(\Delta q_t)} = -\gamma\kappa_q + \kappa_a \frac{\operatorname{cov}(\Delta a_t - \Delta a_t^*, \Delta q_t)}{\operatorname{var}(\Delta q_t)}, \tag{24}$$

where the last term is the contribution of the product-market shocks to the overall equilibrium volatility of the real exchange rate.[22] While the expenditure switching force is generally present, its effect on *aggregate* consumption is weak due to home bias (small $\gamma$). In contrast, the direct effect of domestic goods supply on domestic consumption is strong, particularly so under home bias, and generally produces a counterfactual comovement between the relative consumption and the real exchange rate.

In other words, models in which real depreciations are mostly driven by product market expansions — whether due to a positive productivity shock or an expansionary monetary shock — generally predict a simultaneous expansions in consumption, resulting in the Backus-Smith puzzle independently of the asset market completeness. In contrast, financial shocks that cause a real depreciation and have no direct effect on the supply of goods exert *only* an indirect expenditure switching effect on the real economy, which results in lower consumption consistent with the empirical patterns. Therefore, a successful resolution of the Backus-Smith puzzle must limit the role of product-market shocks in the unconditional variance of the real exchange rate (the last term in (24)).

Simply put, real depreciations must largely reflect an increased demand for foreign assets rather than an increased supply of home goods. Note that the mechanism arises under the news shocks about future productivity or the long-run risk shocks, as in Colacito and Croce (2013). Such shocks, just like a financial shock $\psi_t$, trigger large exchange rate movements without significantly affecting the *contemporaneous* supply and consumption of domestic output. Similarly, persistent productivity growth coupled with increased investment demand, as in Corsetti, Dedola, and Leduc (2008), can also generate increased demand for foreign financing and a currency appreciation, akin to a financial shock, without a significant direct effect on output available for current domestic consumption.[23]

[21] This equilibrium logic can be traced backwards as well: a financial shock that makes home households postpone their consumption, results in a lower relative demand for home goods, which requires an exchange rate depreciation to shift relative demand towards the home good world-wide in order to clear the market — a version of *Keynes' transfer effect* (see e.g. Pavlova and Rigobon 2008, Caballero, Farhi, and Gourinchas 2008).

[22] Here, we focus on the productivity shock as the only product-market shock, yet the results generalize to other shocks including shifts in markups induced by monetary shocks under sticky prices. One way to see this is to use the product market clearing condition (22) directly, without specifying the supply side. Combining it with its foreign counterpart yields $c_t - c_t^* = \frac{1}{1-2\gamma}(y_t - y_t^*) - 2\theta\frac{2\gamma(1-\gamma)}{1-2\gamma}q_t$, where $y_t - y_t^*$ is the equilibrium relative supply of the home and foreign goods.

[23] Alternatively, the Backus-Smith puzzle can be resolved if the real exchange rate *appreciates* with a positive productivity shock, either due to Balassa-Samuelson forces, as in Benigno and Thoenissen (2008), or due to a low elasticity of substitution between home and foreign goods ($\theta < 1$), as in the second mechanism considered by Corsetti, Dedola, and Leduc (2008). These alternative mechanisms are, however, at odds with the other exchange rate puzzles, including Meese-Rogoff and PPP.

### 3.3 The Forward Premium puzzle

We now turn to the equilibrium properties of the interest rates and their comovement with the nominal exchange rate. Log-linearization of the home household Euler equation (3) leads to a conventional relationship for the nominal interest rate, $i_t = \mathbb{E}_t\{\sigma\Delta c_{t+1} + \pi_{t+1}\}$. Combining it with the foreign counterpart and our assumption on monetary policy stabilizing consumer prices $\pi_{t+1} = \pi^*_{t+1} = 0$ which results in $\Delta e_t = \Delta q_t$, we can write the interest rate differential as:

$$i_t - i^*_t = \sigma\mathbb{E}_t\{\Delta c_{t+1} - \Delta c^*_{t+1}\} = \sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1} - \gamma\sigma\kappa_q\mathbb{E}_t\Delta e_{t+1}, \tag{25}$$

where the second equality substitutes in the relationship between consumption and the exchange rate (23). The interest rate differential reflects the intertemporal substitution in consumption, which is in part due to the expected depreciation and in part due to the mean reversion in productivity shocks.

In turn, the modified UIP condition (16) ensures equilibrium in the international financial market. In particular, it implies that a financial shock $\psi_t$ — a relative demand shock for foreign currency — must be accommodated by some combination of a positive interest rate differential for home-currency bonds and an expected appreciation of the home currency. Both of these effects make holding foreign-currency bonds less attractive, returning the financial market to equilibrium.

We now combine (25) with the modified UIP condition (16) under our simplifying assumption $\chi_2 = 0$ to solve for the equilibrium interest rate differential and the expected nominal depreciation:

$$i_t - i^*_t = -\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}(1-\rho)(a_t - a^*_t) + \frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\psi_t, \tag{26}$$

$$\mathbb{E}_t\Delta e_{t+1} = -\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}(1-\rho)(a_t - a^*_t) - \frac{1}{1+\gamma\sigma\kappa_q}\psi_t, \tag{27}$$

where we expressed $\mathbb{E}_t\Delta\tilde{a}_{t+1} = -(1-\rho)(a_t - a^*_t)$. Note that these relationships imply that both $i_t - i^*_t$ and $\mathbb{E}_t\Delta e_{t+1}$ follow AR(1) processes with persistence $\rho$.

Using these results, we can now characterize the joint properties of the interest rates and the nominal exchange rate, and in particular the *Fama regression*, $\mathbb{E}\{\Delta e_{t+1}|i_t - i^*_t\} = \hat{\beta}_F(i_t - i^*_t)$. We prove:

**Proposition 4** (a) *Conditional on a productivity shock, the Fama coefficient* $\hat{\beta}_{F|a} = 1$. *Conditional on a financial shock, the Fama coefficient is negative,* $\hat{\beta}_{F|\psi} = -\frac{1}{\gamma\sigma\kappa_q} < 0$. (b) *Unconditionally, as* $\beta\rho \to 1$*:*

i. $\hat{\beta}_F \to -\frac{1}{\gamma\sigma\kappa_q} < 0$ *and the* $R^2$ *in the* Fama regression *becomes arbitrarily small;*

ii. *the volatility (persistence) of* $(i_t - i^*_t)$ *relative to* $\Delta e_{t+1}$ *becomes arbitrarily small (large);*

iii. *the Sharpe ratio of the* Carry trade *becomes arbitrarily small.*

Note that the first part of Proposition (4) follows immediately from (26)–(27), while the second part also relies on the equilibrium properties of the nominal exchange rate (Proposition 1 and Lemma 3 below). We provide a formal proof in Online Appendix A.6 and offer here an intuitive discussion of the results.

Conditional on a financial shock, positive interest rate differentials predict expected exchange rate appreciations — a pattern of *UIP deviations* known as the *Forward Premium puzzle* (Fama 1984). The reason for this is increased demand for foreign-currency bonds, resulting in an equilibrium positive expected return on home-currency bonds.[24] The productivity shocks are unable to reproduce this empirical pattern, implying a Fama coefficient of one. However, if financial shocks play an important role in the dynamics of the exchange rate, the model reproduces a negative unconditional Fama coefficient. At the same time, the predictive ability of the interest rate differential for future exchange rate changes is very weak in the data (see e.g. Valchev 2016), and our model captures this with a vanishingly small $R^2$ in the Fama regression, as shocks become more persistent and the exchange rate becomes closer to a pure random walk. The model also captures the pronounced differences in the statistical properties of $i_t - i_t^*$ and $\Delta e_{t+1}$, with the former following a smooth and persistent process and the latter being close to a volatile white noise. Importantly, the financial shock in our model does *not* produce expected UIP deviations with counterfactually *large* associated carry trade returns, which allows the quantitative model in the next section to match the size of its Sharpe ratio.

The presence of a UIP shock $\psi_t$ makes it, perhaps, unsurprising that the model can match the empirical patterns of the UIP deviations. Nonetheless, we point out that the rich patterns of comovement between interest rates and the exchange rate, summarized in Proposition 4, are reproduced in the model using a single-parameter AR(1) process for $\psi_t$. Our main emphasis, however, is that a simple financial shock, disciplined with the properties of the UIP deviations in the data, accounts for the other exchange rate puzzles, which the literature conventionally viewed as *not* directly related with each other.

### 3.4 Equilibrium exchange rate dynamics

We provide now a complete analytical characterization of equilibrium exchange rate dynamics, which are shaped by the interplay between financial and macroeconomic forces. The equilibrium in the financial market requires that the modified UIP condition (16) holds, which in turn results in (27), and imposes discipline on the expected future appreciations and depreciations of the nominal exchange rate. Indeed, the equilibrium in the financial market is not affected by the *level* of the exchange rate, only by its *expected changes*. This can be seen formally by solving (27) forward to express the exchange rate $e_t$ as a function of the expected future shocks and its long-run expectation $\mathbb{E}_t e_\infty$, which remains indeterminate from the perspective of the financial market alone.

In contrast to the financial market, the product market equilibrium depends on the *level* of the exchange rate. In particular, a depreciated (real) exchange rate causes expenditure switching towards home-produced goods, as we have seen in Section 3.2. To characterize the equilibrium level of the exchange rate, we need to appeal to the country's intertemporal budget constraint (11), which upon log-linearization can be written as:[25]

$$\beta b_{t+1} - b_t = nx_t = \gamma\big[\lambda q_t - \kappa_a \tilde{a}_t\big], \tag{28}$$

where $\lambda \equiv \frac{2\theta(1-\gamma)-1}{1-2\gamma} + \gamma\kappa_q > 0$, and $\kappa_q$ and $\kappa_a$ are defined in (23). The left-hand side of (28) is the

[24] In contrast with the macro-finance literature, surveyed in Engel (2014), our mechanism is *not* a partial asset-pricing result, but a general-equilibrium outcome of joint interest rate and exchange rate determination to clear product and asset markets.

[25] Net exports, like net foreign assets, are zero in a symmetric steady state, and we denote $nx_t \equiv NX_t/\bar{Y}$, in parallel with $b_t$ introduced in Lemma 1. It follows from (11) that $nx_t = \gamma[c_{Ht}^* - c_{Ft} - s_t] = \gamma\big[\theta q_t + (\theta-1)s_t - (c_t - c_t^*)\big]$, where terms of trade $s_t = q_t^P = \frac{1}{1-2\gamma}q_t$ (from (19)); solving out relative consumption using (23), yields (28). Also note that $\lambda > 0$ in (28) is the general equilibrium version of the *Marshall-Lerner condition*, which holds in the model, as $\theta > 1$.

evolution of the net foreign assets (NFA), while the right-hand side is equilibrium net exports, which *decrease* with relative domestic demand $c_t - c_t^*$, and hence $\tilde{a}_t$, which increases imports. Net exports *increase* with the expenditure switching towards home goods induced by a real devaluation (higher $q_t$).

The intertemporal budget constraint, $b_t+\sum_{j=0}^{\infty}\beta^j nx_{t+j}=0$, is obtained from (28) by rolling it forward and imposing the No-Ponzi-Game Condition (NPGC), $\lim_T \beta^T b_{t+T}\geq 0$, which holds with equality in equilibrium. Given the expected path of future exchange rate changes, which equilibrate the financial market, the intertemporal budget constraint pins down the equilibrium level of the exchange rate. Finally, note that net exports $nx_t$ depend on the real exchange rate $q_t$, while monetary policy, which stabilizes domestic price levels in (17), ties together the nominal and the real exchange rates, $e_t = q_t$.

Formally, the interplay between the two dynamic conditions, (27) and (28), shapes the equilibrium dynamics of both exchange rates. In particular, the *unique* solution to this dynamic system implies the following equilibrium relationship between the exchange rate $e_t$ and the state variable $b_t$ (NFA):[26]

$$e_t = -\frac{1-\beta}{\gamma\lambda}b_t + \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta}{1-\beta\rho}\left[\psi_t + \left((1-\rho)+\frac{1+\gamma\sigma\kappa_q}{\sigma\lambda}\frac{1-\beta}{\beta}\right)\sigma\kappa_a\tilde{a}_t\right]. \tag{29}$$

Intuitively, the exchange rate is stronger (lower $e_t$) the greater are the net foreign assets $b_t$, and it *depreciates* with the financial shock $\psi_t$, which shifts demand to the foreign currency, as well as with the relative productivity shock $\tilde{a}_t$, which results in additional supply of home goods. This pins down the unique equilibrium path of the exchange rate, as deviations from (29) shift the entire expected path of the exchange rate, and hence all expected trade balances $nx_{t+j}$, violating the intertemporal budget constraint. Combining (29) with (28), we can solve for the equilibrium dynamic process for $e_t$:

**Lemma 3** *The equilibrium exchange rate $e_t$ follows an ARIMA(1,1,1) process, or equivalently $\Delta e_t$ follows an ARMA(1,1), with the autoregressive root $\rho$, and given by:*

$$\Delta e_t = \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta}{1-\beta\rho}\left[\Big(1-\frac{1}{\beta}L\Big)\psi_t + \Big[(1-\rho)\Big(1-\frac{1}{\beta}L\Big)+\frac{1+\gamma\sigma\kappa_q}{\sigma\lambda}\frac{1-\beta}{\beta}(1-\rho L)\Big]\sigma\kappa_a\tilde{a}_t\right], \tag{30}$$

*where $L$ is the lag operator such that $L\psi_t=\psi_{t-1}$ and $L\tilde{a}_t=\tilde{a}_{t-1}$.*

Lemma 3 is the basis for Proposition 1, which postulates the equilibrium properties of the exchange rate. In particular, the lemma shows that the dynamics of the exchange rate are shaped by the parameters $\beta$ and $\rho$, which determine the ARMA roots in (30), while the other parameters of the model affect the proportional volatility scalers of the shocks. Interestingly, the exchange rate volatility is higher in more closed economies, and is maximized in the autarky limit, as $\gamma\to 0$ (see Appendix Figure A1a). This is in line with the data, where the more open economies have indeed less volatile exchange rates, even after controlling for country size and other characteristics (see e.g. Hau 2002). Intuitively, a more open economy cannot sustain the same amount of equilibrium exchange rate volatility without it causing more volatile behavior of the macro variables, as we discuss further in Section 4.4.

[26]Solving (27) and (28) forward and using the fact that shocks are AR(1), results in $e_t = \mathbb{E}_t e_\infty + \frac{1}{1+\gamma\sigma\kappa_q}\left[\frac{1}{1-\rho}\psi_t + \sigma\kappa_a\tilde{a}_t\right]$ and $b_t + \gamma\lambda\sum_{j=0}^{\infty}\beta^j\mathbb{E}_t e_{t+j} - \frac{\gamma\kappa_a}{1-\beta\rho}\tilde{a}_t = 0$ which together yield (29). Online Appendix A.6 offers an alternative derivation, using the Blanchard and Kahn (1980) technique, for the general case with $\chi_1,\chi_2\geq 0$ that are endogenously determined in equilibrium; with $\chi_2>0$, the equilibrium process becomes a stationary mean-reverting ARMA(2,1) instead of an integrated ARIMA(1,1,1), yet the two have indistinguishable finite-sample properties for small $\chi_2>0$.

We now focus on the two essential equilibrium properties of the nominal exchange rate:[27]

**Proposition 5** *As $\beta\rho \to 1$:* (i) *the exchange rate process* (30) *becomes indistinguishable from a random walk, with $\mathbb{E}_t \Delta e_{t+1} \to 0$ and $\operatorname{corr}(\Delta e_t, \Delta e_{t-1}) \to 0$;* (ii) *the volatility of the exchange rate becomes unboundedly large relative to the volatility of the financial shock, $\frac{\operatorname{var}(\Delta e_t)}{\operatorname{var}(\psi_t)} \to \infty$, with the contribution of $\psi_t$ (relative to $\tilde{a}_t$) dominating the variance of the exchange rate; furthermore, under strong home bias ($\gamma \approx 0$), the* relative *volatility of macroeconomic aggregates, such as GDP, is arbitrarily small ($\frac{\operatorname{var}(\Delta y_t)}{\operatorname{var}(\Delta e_t)} \approx 0$).*

Proposition 5 describes the qualitative *order-of-magnitude* properties of the nominal exchange rate, namely that it can follow a process arbitrarily close to a random walk and with an arbitrarily large volatility relative to fundamental macroeconomic variables, such as GDP. Figure 2 illustrates these properties quantitatively for the empirically relevant values of $\beta$, $\rho$ and $\gamma$ (see calibration in Section 4).

Figure 2a plots the impulse response of the exchange rate to the financial shock $\psi_t$. A shift in demand towards the foreign currency, $\psi_t > 0$, results in an instantaneous depreciation of the home currency, while also predicting an expected appreciation, consistent with (27), akin to the celebrated *overshooting* dynamics in Dornbusch (1976). This exchange rate path ensures both equilibrium in the financial market (via expected appreciation) and a balanced country budget (via instantaneous depreciation). The impulse response to the relative productivity shock $\tilde{a}_t$ is quantitatively similar (see Appendix Figure A1b).[28] In both cases, a large instantaneous depreciation is followed by small but persistent expected future appreciations. Furthermore, as shocks become more persistent, the impulse response of $e_t$ becomes closer to a step function of a random walk for both types of shocks. Thus, the financial shock is not unique in delivering near-random-walk behavior for the exchange rate; in fact, any persistent fundamental shock achieves this.[29]

The essential *difference* between financial and productivity shocks is emphasized in the second part of Proposition 5 and concerns the relative volatility implications for the equilibrium exchange rate. As shocks become more persistent, the effect of the financial shock on the exchange rate increases without bound, while the effect of the productivity shock remains bounded.[30] The differential implications of the two types of shocks become increasingly apparent when we consider the comovement between the exchange rate and macro variables, such as GDP (earlier subsections address the comovement with other macro variables). In response to a financial shock, the volatility of the exchange rate relative to that of GDP is arbitrarily large provided the economy is sufficiently closed. Formally, this means that $\operatorname{std}(\Delta y_t)/\operatorname{std}(\Delta e_t)$ has the order of magnitude of $\gamma$, when $\gamma$ is small. In contrast, conditional on a productivity shock $a_t$, the volatility of GDP is of the same order of magnitude as that of the ex-

[27] The results in Proposition 5 follow from the exchange rate process in Lemma 3, combined with the equilibrium solution for GDP, $y_t = \frac{1+\varphi}{\sigma+\varphi}\left[a_t - \frac{2\gamma\sigma}{\sigma+\varphi(1-2\gamma)}\tilde{a}_t\right] + \frac{\gamma}{1-2\gamma}\frac{2\sigma\theta(1-\gamma)-(1-2\gamma)}{\sigma+\varphi(1-2\gamma)}q_t$, which follows from the equilibrium conditions in Section 3.2.

[28] An increase in productivity and resulting supply of home goods lead to an instantaneous depreciation needed to equilibrate the intertemporal budget constraint (in view of the increased domestic production and consumption), with the currency gradually appreciating thereafter as the productivity shock wears out.

[29] This is reminiscent of the Engel and West (2005) result, which however derives from the partial equilibrium in the financial market. In contrast, our solution relies on the full general equilibrium, and in particular endogenizes the real exchange rate; thus, our results are not nested by their Theorem (see the earlier draft, Itskhoki and Mukhin 2017, for details).

[30] Formally, this is the case because random-walk productivity shocks have no direct effect on the intertemporal optimality condition (25), and thus on the expected exchange rate changes (27). In contrast, rare-disaster, long-run-risk or productivity-news shocks are more similar in their properties to the financial shock $\psi_t$ from the point of intertemporal optimality.

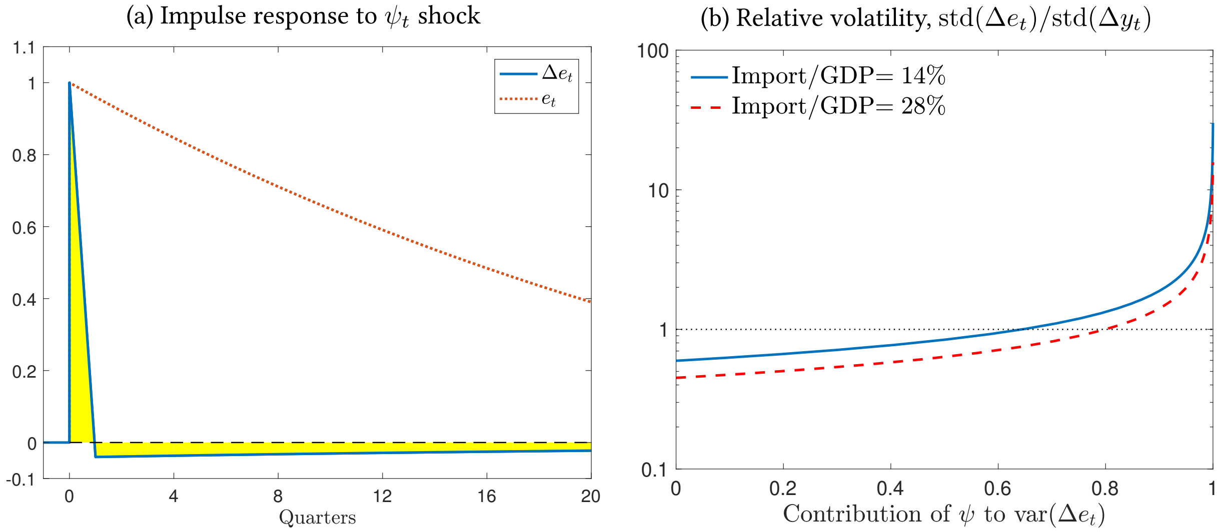


Figure 2: Properties of the equilibrium exchange rate process

Note: Calibrated IRBC model (see Section 4). Panel (a): impulse response of $\Delta e_t$ and $e_t$ to a $\psi_t$-shock innovation (see Appendix Figure A1b for $\tilde{a}_t$ shock). Panel (b): Ratio of the unconditional standard deviations, $\text{std}(\Delta e_t)/\text{std}(\Delta y_t)$, on a *logarithmic* scale, against the variance contribution (share) of the financial shock, $\frac{\text{var}(\Delta e_t|\psi_t)}{\text{var}(\Delta e_t)}$, plotted by varying the volatility of the financial shock, $\sigma_\psi \in [0, \infty)$; the two curves correspond to different value of $\gamma$ (matching the indicated import-to-GDP ratios).

change rate, independently of $\gamma$: for example, around $\gamma \approx 0$ and with persistent shocks $\rho \approx 1$, we have $\text{std}(\Delta y_t)/\text{std}(\Delta e_t) \approx (2\theta - 1) > 1$. That is, in response to productivity shocks, GDP is counterfactually *more* volatile than the exchange rate. This is a stark negative result for product-market shocks, which generally cannot deliver exchange rate *disconnect in volatilities*, even in limiting economies.

Figure 2b provides a quantitative illustration of both the negative result for product-market shocks and the positive result for the financial shock, using our calibrated model. In particular, we plot the equilibrium volatility of the exchange rate relative to GDP, $\text{std}(\Delta e_t)/\text{std}(\Delta y_t)$, as a function of the share of the exchange rate volatility, $\text{var}(\Delta e_t)$, explained by the financial shock (with the remained accounted for by the productivity shocks). We do so for two values of openness, $\gamma$ — one typical of large relatively-closed economies, such as the US, Japan and the EU, and the other typical of smaller open economies, such as the UK, South Korea and New Zealand. The figure illustrates how the model reproduces the significantly greater volatility of the exchange rate relative to GDP, but *only* when financial shocks dominate the variance decomposition of the exchange rate. This gap in volatility is indeed greater in less open economies, a pattern that we document in the data in Section 4.4.

**Predictability** Equation (27) suggests departures from martingale behavior and implies predictability of the nominal exchange rate. Indeed, there exists empirical evidence on the departure of the exchange rate process from a pure random walk (see e.g. Bacchetta and van Wincoop 2006, Engel 2016, Lustig, Stathopoulos, and Verdelhan 2016). In a recent paper, Eichenbaum, Johannsen, and Rebelo (2018) emphasize the predictability of future nominal exchange rate changes with the current value of the real exchange rate, $\mathbb{E}\{e_{t+h} - e_t|q_t\} = \alpha_h + \beta_h q_t$, which becomes stronger with the horizon $h$. We now explore the properties of our model for this predictive regression. In Appendix Figure A1c, we plot the projection coefficients $\hat{\beta}_h$ and the corresponding $R^2$s from the simulated paths of exchange rates in our baseline model, reproducing closely the empirical findings of Eichenbaum, Johannsen, and Rebelo (2018): (i) the projection coefficients $\hat{\beta}_h$ are about zero for small $h$ and become increasingly more negative as $h$ increases, crossing $-1$ after about 6 years; and (ii) $R_h^2$ also starts around zero and increases towards $0.6$ for large $h$. This pattern holds similarly for financial and productivity shocks, and does not rely on stationarity of the nominal or real exchange rate. Therefore, our model reproduces simultaneously the near-random-walk behavior and the subtle departures from a pure random walk in the nominal exchange rate observed in the data.

# 4 Quantitative Analysis

In this section, we turn to the full model to study its quantitative properties. The goal is threefold. First, we show that our baseline exchange rate disconnect results in Section 3 are robust to the introduction of capital, nominal rigidities, and a conventional Taylor-rule monetary policy. Second, we show that a multi-shock version of the model matches not only the relative volatilities, but also the empirical correlations between exchange rates and macro variables. Finally, we show that matching the exchange rate moments comes at no cost in terms of the model's ability to match the standard international business cycle moments. In particular, while financial shocks account for much of the exchange rate volatility, standard productivity and monetary shocks remain the key drivers of consumption, investment and output. As a result, the relative volatilities and correlations of macro aggregates in our model are similar to those in the international business cycle literature following BKK. We first focus on large open economies, such as the US and the Euro Area, and then consider small open economies in Section 4.4.

## 4.1 Full quantitative model and calibration

We first outline five additional ingredients of our quantitative model relative to the baseline model presented in Section 2. The full model setup can be found in Online Appendix A.2. The production function now additionally features capital $K_t$ and intermediate inputs $X_t$:

$$Y_t = \left(e^{a_t} K_t^{\vartheta} L_t^{1-\vartheta}\right)^{1-\phi} X_t^{\phi}, \tag{31}$$

where $\vartheta$ is the elasticity of the value added with respect to capital and $\phi$ is the elasticity of output with respect to intermediates, which determines the equilibrium expenditure share on intermediate goods. Capital is accumulated according to $K_{t+1} = (1-\delta)K_t + \left[Z_t - \frac{\kappa}{2}\frac{(\Delta K_{t+1})^2}{K_t}\right]$, where the term in square brackets is investment $Z_t$ net of quadratic adjustment costs. Both intermediate inputs $X_t$ and investment goods $Z_t$ are bundles of domestic and foreign varieties, in parallel with finite consumption $C_t$.

Monetary policy is now implemented by means of a Taylor rule for the nominal interest rate:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\phi_\pi \pi_t + \sigma_m \varepsilon_t^m, \tag{32}$$

where $\pi_t = \Delta \log P_t$ is the inflation rate and $\varepsilon_t^m$ is the monetary shock. Parameters $\sigma_m \geq 0$ and $\rho_m \in [0, 1)$ characterize the volatility of monetary shocks and the persistence of the monetary policy rule, while $\phi_\pi > 1$ is the Taylor-rule coefficient, which ensures that the *Taylor principle* is satisfied.

The final two ingredients are variable markups, arising from Kimball (1995) demand, and sticky wages and prices following Calvo (1983). The *desired* price of the firm depends on both its marginal cost and the average price of its local competitors, reflecting *strategic complementarities* in price setting. In particular, the home- and foreign-market desired prices of the home firm are given by:

$$p_{Ht} = (1-\alpha)mc_t + \alpha p_t \qquad \text{and} \qquad p^*_{Ht} = (1-\alpha)(mc_t - e_t) + \alpha p^*_t, \tag{33}$$

where $\alpha \in [0,1)$ is the elasticity of strategic complementarities and $(1-\alpha)$ is the (incomplete) cost pass-through elasticity, both arising from variable markups (see Amiti, Itskhoki, and Konings 2019). Desired prices of the foreign firms are defined by symmetric equations. Lastly, we introduce wage and price stickiness in a conventional way, as described in Galí (2008) and Online Appendix A.7. We denote with $\lambda_p$ and $\lambda_w$ the Calvo probabilities of price and wage non-adjustment respectively, and we assume that the prices are sticky in the destination (local) currency (LCP, as in CKM). Therefore, the model features both sources of the *law-of-one-price* deviations — due to variable markups and pricing to market (PTM) and due to local currency price stickiness (LCP) — which we explore in detail below, in Section 4.3.

**Empirical moments** The first column of Table 1 shows the empirical moments that are the focus of our quantitative analysis. For comparability, we follow CKM and estimate the moments for the US relative to the PPP-weighted sum of France, Germany, Italy and the UK, using quarterly data from 1973–94. The empirical moments are similar for the longer period that we extend to 2017 (see Online Data Appendix A.1). Additionally, for the moments that involve interest rates, we rely on the estimates in Hassan and Mano (2014) and Valchev (2016). Finally, due to the high variability of the Backus-Smith correlation across countries and periods, we use the average estimate from Corsetti, Dedola, and Leduc (2008), which is representative of the conventional value in the literature.

**Calibration** Our calibration, for the most part, does *not* target the empirical moments in Table 1, and instead adopts conventional values for the model parameters following the broader macro literature, as summarized in Appendix Table A1. In particular, we set the *imports-in-expenditure* ratio $\gamma = 0.07$, to be consistent with the $0.28$ trade-to-GDP ratio of the United States, provided the intermediate input share $\phi = 0.5$.[31] This value of the trade-to-GDP ratio is also characteristic of the other large developed economies (Japan and the Euro Area), and we explore a small-open-economy calibration in Section 4.4.

We use the estimate of Amiti, Itskhoki, and Konings (2019) for the elasticity of strategic complementarities, $\alpha = 0.4$, which is in line with the exchange rate pass-through literature, corresponding to the pass-through elasticity of $1-\alpha = 0.6$ (see survey in Gopinath and Itskhoki 2011). We follow the estimates of Feenstra, Luck, Obstfeld, and Russ (2018) and set the elasticity of substitution $\theta = 1.5$, which is also the number used in the original calibrations of Backus, Kehoe, and Kydland (1994) and CKM.[32]

For the other parameters, we use conventional values of the relative risk aversion $\sigma = 2$, the Frisch elasticity of labor supply $1/\varphi = 1$, the *quarterly* discount factor $\beta = 0.99$, the capital share in value

[31] In a symmetric steady state, imports are half of trade (imports+exports); and GDP (final consumption) is roughly half of expenditure in the data, with the other half allocated to intermediate inputs. The US trade-to-GDP ratio increased from 20% in 1980 to 28% in 2018, corresponding to an increase in $\gamma$ from 0.05 to 0.07 (see Table 4 below for further details).

[32] The estimates of the *micro* elasticity at more disaggregated levels are typically larger, around 3 or 4, yet it is the *macro* elasticity of substitution between the aggregates of home and foreign goods that is the relevant elasticity for our analysis.

added $\vartheta = 0.3$, and the quarterly capital depreciation rate $\delta = 0.02$. For each specification of the model, we calibrate the capital adjustment cost parameter $\kappa$ to match the relative volatility of investment, $\frac{\text{std}(\Delta z_t)}{\text{std}(\Delta gdp_t)} = 2.5$. We set the Taylor-rule parameter $\phi_\pi = 2.15$ and the interest-rate smoothness parameter $\rho_m = 0.95$, following the estimates in Clarida, Galí, and Gertler (2000). In the sticky-price version of the model, we assume that prices adjust on average once a year, and thus set $\lambda_p = 0.75$, while wages adjust on average every six quarters, $\lambda_w = 0.85$, following standard calibrations in the literature (Galí 2008). Thus, the range of the models that we consider includes both the flexible-price benchmark and specifications with a considerable extent of price and wage stickiness.

Finally, we discuss the calibration of the shock processes. The model features three exogenous shocks — two country-specific productivity shocks $(a_t, a_t^*)$ and a financial shock $\psi_t$ — for which we need to calibrate the covariance matrix. We assume that $\psi_t$ is orthogonal to $(a_t, a_t^*)$, while $a_t$ and $a_t^*$ are correlated and have a common variance. We choose the relative volatility of the productivity shock, $\sigma_a/\sigma_\psi$, to match the Backus-Smith correlation, $\text{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, while the cross-country correlation of productivity shocks is calibrated to match $\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.35$. In addition, we consider a version of the sticky-price model with monetary shocks $(\varepsilon_t^m, \varepsilon_t^{m*})$, instead of productivity shocks, and we discipline their relative volatility $\sigma_m/\sigma_\psi$ and cross-country correlation in the same way. Lastly, we assume that all shocks follow AR(1) processes with the same autoregressive coefficient $\rho = 0.97$, which is consistent with the observed persistence of both macroeconomic variables, such as GDP and interest rates, as well as risk premia in international financial markets.[33]

## 4.2 Main quantitative results

Before studying the full quantitative model, we start by evaluating partial single-shock specifications, in order to dissect how individual shocks account for the fit of specific moments in the full model. Our quantitative results are reported in Table 1, with the exchange rate related moments summarized in panels A–C and the international business cycle moments in panel D.

**Single-shock models** Columns 2–5 of Table 1 report the results from four single-shock specifications of the model: a financial shock under both flexible and sticky prices; productivity shocks in a standard flexible-price IRBC model; and monetary shocks in a NKOE model with sticky prices and wages. We start with the exchange rate moments, which confirm the various analytical results of Section 3.

First, all four single-shock specifications match the near random-walk behavior of the nominal exchange rate. Consistent with Proposition 5, it is the persistence of the shock rather than its type that ensures that the exchange rate is a near-martingale. At the same time, it is *only* the financial shock that has the ability to replicate the empirical disconnect in volatilities of the exchange rate and the macro variables. In the data, exchange rates are about 5 times more volatile than GDP and 6 times more

[33] The *ex-ante* risk premium $\psi_t$ is not directly observable in the data, and hence cannot be readily used to calibrate the volatility and persistence of the financial shock. Our calibration is, nonetheless, consistent with the statistical properties of the estimated $\hat{\psi}_t = i_t - i_t^* - \hat{\mathbb{E}}_t \Delta e_{t+1}$ using the econometric forecasts of $\hat{\mathbb{E}}_t \Delta e_{t+1}$ (see e.g. Bekaert 1995, Kollmann 2005). We also note that the relative volatilities of the shocks could be calibrated to match the relative volatilities of exchange rates and macro variables, instead of targeting the negative Backus-Smith correlation. As we show below, our model can fit these moments without targeting them directly, suggesting that there is no conflict between these alternative calibration targets. Finally, when we calibrate the level of volatility in the model to match the 10% annualized standard deviation of the nominal exchange rate, the implied volatility of the TFP innovations is 1.4% annually, consistent with the CKM calibration.

Table 1: Quantitative Models

| Moments | Data | Single-type shocks: Financial shock | | Single-type shocks: IRBC | Single-type shocks: NKOE | Multi-shock models: IRBC | Multi-shock models: IRBC$^+$ | Multi-shock models: NKOE |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| A. Exchange rate disconnect: | | | | | | | | |
| $\rho(\Delta e)$ | $\approx 0$ | $-0.02$ (0.09) | $-0.03$ (0.09) | 0.01 (0.09) | $-0.06$ (0.09) | $-0.02$ (0.09) | $-0.03$ (0.09) | $-0.03$ (0.09) |
| $\sigma(\Delta e)/\sigma(\Delta gdp)$ | 5.2 | 17.7 | 9.8 | 0.58 | 1.31 | 2.9 | 3.4 | 3.8 |
| $\sigma(\Delta e)/\sigma(\Delta c)$ | 6.3 | 7.5 | 17.5 | 0.96 | 2.18 | 3.9 | 5.8 | 6.6 |
| B. Real exchange rate and the PPP: | | | | | | | | |
| $\rho(q)$ | 0.94 | 0.93 (0.04) | 0.91 (0.05) | 0.94 (0.04) | 0.82 (0.06) | 0.93 (0.04) | 0.91 (0.05) | 0.90 (0.05) |
| $\sigma(\Delta q)/\sigma(\Delta e)$ | 0.99 | 0.74 | 0.98 | 1.40 | 0.95 | 0.83 | 0.98 | 0.97 |
| $\mathrm{corr}(\Delta q, \Delta e)$ | 0.99 | 1.00 | 1.00 | 1.00 | 0.99 | 0.99 | 1.00 | 1.00 |
| $\sigma(\Delta q^W)/\sigma(\Delta e)$ | 1.01 | 1.04 | 1.00 | 0.56 | 0.95 | 1.07 | 1.00 | 1.00 |
| $\mathrm{corr}(\Delta q^W, \Delta e)$ | 0.99 | 1.00 | 1.00 | $-0.99$ | 0.99 | 0.96 | 1.00 | 1.00 |
| C. Backus-Smith and Forward premium: | | | | | | | | |
| $\mathrm{corr}(\Delta q, \Delta c - \Delta c^*)$ | $-0.40$ | $-1.00$ | $-0.95$ (0.01) | 1.00 | 1.00 | $-0.40$ (0.08) | $-0.40$ (0.08) | $-0.40$ (0.08) |
| Fama $\beta$ | $< 0$ | $-2.0$ (1.4) | $-3.4$ (2.7) | 1.6 (0.8) | 1.4 (0.8) | $-1.7$ (1.5) | $-2.2$ (2.1) | $-1.9$ (1.3) |
| Fama $R^2$ | 0.02 | 0.03 (0.02) | 0.03 (0.02) | 0.08 (0.03) | 0.03 (0.02) | 0.02 (0.02) | 0.02 (0.02) | 0.02 (0.02) |
| Carry trade $SR$ | 0.20 | 0.23 (0.04) | 0.22 (0.04) | 0.01 (0.10) | $-0.03$ (0.09) | 0.23 (0.04) | 0.22 (0.04) | 0.22 (0.04) |
| $\sigma(i - i^*)/\sigma(\Delta e)$ | 0.06 | 0.08 | 0.05 | 0.17 | 0.12 | 0.08 | 0.06 | 0.06 |
| $\rho(i - i^*)$ | 0.90 | 0.93 | 0.98 | 0.95 | 0.82 | 0.94 | 0.98 | 0.94 |
| $\rho(i)$ | 0.97 | 0.93 | 0.98 | 0.95 | 0.83 | 0.94 | 0.98 | 0.89 |
| D. International business cycle moments: | | | | | | | | |
| $\sigma(\Delta c)/\sigma(\Delta gdp)$ | 0.82 | 2.37 | 0.56 | 0.60 | 0.60 | 0.74 | 0.59 | 0.57 |
| $\mathrm{corr}(\Delta c, \Delta gdp)$ | 0.64 | $-1.00$ | $-0.82$ | 1.00 | 1.00 | 0.85 | 0.80 | 0.75 |
| $\mathrm{corr}(\Delta z, \Delta gdp)$ | 0.81 | $-1.00$ | $-0.20$ | 1.00 | 1.00 | 0.81 | 0.86 | 0.86 |
| $\mathrm{corr}(\Delta gdp, \Delta gdp^*)$ | 0.35 | $-1.00$ | $-1.00$ | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 |
| $\mathrm{corr}(\Delta c, \Delta c^*)$ | 0.30 | $-1.00$ | $-1.00$ | 0.40 | 0.38 | 0.14 | 0.39 | 0.40 |
| $\mathrm{corr}(\Delta z, \Delta z^*)$ | 0.27 | $-1.00$ | $-1.00$ | 0.45 | 0.39 | 0.10 | 0.54 | 0.55 |
| $\frac{\sigma_a}{\chi_1 \sigma_\psi}$ or $\frac{\sigma_m}{\chi_1 \sigma_\psi}$ | | – | – | – | – | 3.3 | 2.5 | 0.38 |
| $\rho_{a,a^*}$ or $\rho_{m,m^*}$ | | – | – | 0.28 | 0.34 | 0.30 | 0.37 | 0.56 |
| Nominal rigidities | | – | ✓ | – | ✓ | – | ✓ | ✓ |

Note: Panels A–D report the simulation results, where each entry is the median value of moments across 10,000 simulations of 120 quarters, and brackets reporting (when relevant) the standard deviations across simulations. The bottom panel describes the model specifications. Columns 2, 4, 6 feature flexible prices and wages; columns 3, 5, 7, 8 feature both sticky wages and LCP sticky prices. Shocks: financial $\psi_t$ in columns 2–3, 6–8; productivity $(a_t, a_t^*)$ in columns 4, 6–7; monetary $(\varepsilon_t^m, \varepsilon_t^{m*})$ in columns 5, 8. In columns 4–8, correlation of shocks matches $\mathrm{corr}(\Delta gdp, \Delta gdp^*) = 0.35$; in columns 6–8, in addition, the relative volatility of shocks matches $\mathrm{corr}(\Delta q, \Delta c - \Delta c^*) = -0.4$ (see bottom panel). See data description in the text.

volatile than consumption. Both versions of the model with the $\psi_t$ shock consistently reproduce this gap in the volatility. In fact, they predict that macro variables are an order of magnitude less volatile than exchange rates. As we explain in Section 3, home bias in the goods market allows to sustain large equilibrium exchange rate swings without passing excessive volatility through to the macro variables. In contrast, the effect of productivity and monetary shocks on macro aggregates is of the same order of magnitude as on the exchange rates, *in*consistent with the disconnect in volatilities.

Second, we consider the properties of the real exchange rate. Note that the Taylor rule that targets inflation ensures a close comovement of the CPI-based real exchange rate with the nominal exchange rate, *independently* from the type of the shock or price stickiness. However, in line with Proposition 2, only the models with the financial shock are consistent with a broader set of the PPP moments. In particular, consistent with the PPP puzzle literature, the monetary model (NKOE) *cannot* match the persistence of the real exchange rate — predicting a short half-life under one year, considerably below the empirical estimates of about three years. While this is not an issue for the IRBC model, this model produces a wedge between the CPI-based and the wage-based real exchange rates, which moves with the productivity shocks. Specifically, the IRBC model predicts a negative correlation between the nominal and the wage-based real exchange rates, in contrast with the data (recall Figure 1). The models with the $\psi_t$ shock, on the other hand, have no difficulty in simultaneously matching the persistence of the real exchange rate and nearly perfect comovement between both measures of the real exchange rate and the nominal exchange rate. In contrast to the conventional wisdom, price stickiness is *not* crucial to resolve the PPP puzzle, but it *does* help to increase the relative volatility of the real exchange rate towards one.

Third, as anticipated by Propositions 3 and 4, only the financial shock models are consistent with a negative Backus-Smith correlation and a negative Fama regression coefficient, again independently of the presence of nominal rigidities. In contrast, and despite the segmented asset market, the correlation between relative consumption and the real exchange rate and the Fama coefficient are both close to one for productivity and monetary shocks alike. These properties again favor the models with financial shocks, which additionally have a good fit of the other financial moments — the positive but small *Sharpe ratio* of the carry trade, the low volatility and high persistence of the interest rates, and the close-to-zero $R^2$ in the Fama regression.[34]

While the financial shock model is highly successful in matching exchange rate moments, it is clearly dominated by the productivity and monetary shock models in terms of the standard international business cycle moments, as we report in panel D of Table 1. In particular, financial shocks counterfactually induce negative correlations between GDP and its domestic components (consumption and investment), as well as negative correlations between macro variables across countries. In contrast, and consistent with the earlier literature, this is not an issue for either IRBC or NKOE models, which reproduce the empirical positive comovement of macro aggregates within and across countries.

[34]We follow Lustig and Verdelhan (2011) in specifying the carry trade strategy (see Online Appendix A.6). Its unconditional Sharpe ratio in the data is about 0.5, but at least half of it is due to the cross-sectional country fixed effects, not modeled in our framework, which focuses on the time-series properties. Our empirical target for the Sharpe ratio of 0.2 corresponds to the "forward premium trade" in Hassan and Mano (2014). In the earlier draft, Itskhoki and Mukhin (2017), we also show how a multi-shock version of the model matches the additional moments on the intertemporal comovement of interest rates and exchange rates documented by Bacchetta and van Wincoop (2010), Engel (2016) and Valchev (2016).

Table 2: Contribution of $\psi_t$ to macroeconomic volatility

| | | IRBC (6) | IRBC$^+$ (7) | NKOE (8) |
|---|---|---|---|---|
| Nominal exchange rate | $\mathrm{var}(\Delta e)$ | 96% | 98% | 94% |
| Real exchange rate | $\mathrm{var}(\Delta q)$ | 87% | 97% | 94% |
| Consumption | $\mathrm{var}(\Delta c)$ | 20% | 10% | 12% |
| GDP | $\mathrm{var}(\Delta gdp)$ | 1% | 11% | 14% |

Note: Variance decompositions correspond to the quantitative models in columns (6)–(8) of Table 1. The entries are the % contributions of the financial shock $\psi_t$ to the unconditional variances of the macro variables, with the remaining shares accounted for by the other shocks in each model specification.

**The full model** We finally turn to the full quantitative model. In column 6 of Table 1, we report the results for the IRBC model with productivity and financial shocks, and no nominal rigidities. Column 7 adds sticky wages and sticky prices to the same specification, and we label it IRBC$^+$. Finally, column 8 replaces productivity shocks with monetary shocks, keeping nominal rigidities as in column 7. The bottom line is that all three specifications are successful at *simultaneously* matching the exchange rate moments in panels A–C and the international business cycle moments in panel D.

Indeed, multi-shock models inherit the ability of the financial-shock model to match the exchange rate moments and the capacity of standard IRBC and NKOE models in matching the international business cycle moments. In particular, multi-shock models generate volatile and persistent nominal and real exchange rates, which all comove nearly perfectly together, a negative Backus-Smith correlation and a negative Fama coefficient, while still allowing the main macro aggregates (GDP, consumption and investment) to be positively correlated with each other and across countries. Therefore, the multi-shock model faces no trade-off in matching the exchange-rate and business-cycle moments simultaneously, despite the failure of all single-shock models in one or the other task.[35] Recall that our only explicit calibration target is the negative Backus-Smith correlation, which identifies the relative contribution of the financial and macro-fundamental shocks (see equation (24) in Section 3.2).

To provide an intuitive explanation for these, perhaps surprising, findings, Table 2 describes the variance contribution of the shocks to various macroeconomic variables. In particular, the table reports the contribution of the financial shock to the unconditional variance of the exchange rates, consumption and GDP, while the remaining shares are accounted for by the other shocks. Across specifications, financial shocks account for almost all of the nominal exchange rate volatility and about 90% of the real exchange rate volatility. At the same time, these shocks account for around 10% of the consumption and output volatility. Home bias in the goods market, coupled with incomplete pass-through and low substitutability of home and foreign products, limits the transmission of large exchange rate fluctuations into the macro variables. As a result, the macro variables are both stable in comparison with exchange rates and are primarily driven by productivity and monetary shocks, which exert strong direct effects

[35]Note also that the presence of the segmented financial market allows our multi-shock model to resolve two prominent international business cycle puzzles — the weak cross-country correlation of consumption (Obstfeld and Rogoff 2001) and the positive cross-correlation of investment (Kehoe and Perri 2002) — without targeting these moments in the calibration.

on these variables, yet contribute relatively little to the exchange rate volatility.

Returning to the main quantitative results in columns 6–8 in Table 1, what we find particularly surprising is that the quantitative success of the model is not sensitive, to a first approximation, to the presence or absence of nominal rigidities and to the nature of the shocks, provided that the financial shock is included in the mix. This emphasizes the robustness of the disconnect mechanism laid out in Section 3, as well as the reason why the earlier literature was unable to explain the equilibrium behavior of exchange rates. Specifically, it is not the failures of the flexible-price or sticky-price transmission mechanisms, but rather the focus on productivity and monetary shocks as the key drivers of exchange rates. Instead, we argue that these shocks are fundamentally inconsistent with the *disconnect*, which calls for the financial shock as the key ingredient in a model of exchange rates.

### 4.3 The role of incomplete pass-through

We have emphasized so far the role of the financial shock and home bias as the two main ingredients of the disconnect mechanism. Home bias alone goes a long way in muting the transmission of exchange rate volatility into the macro aggregates. Nonetheless, incomplete exchange rate pass-through at the border — due to both variable markups and foreign-currency price stickiness — acts to reinforce home bias in limiting the transmission of exchange rate volatility into macroeconomic prices and quantities. We now evaluate the quantitative contribution of incomplete pass-through to the disconnect mechanism, focusing on the exchange rate transmission via the terms of trade and net exports.

Variable-markup price setting in (33) results in deviations from the *law of one price* (LOP), that is $p^*_{Ht} + e_t - p_{Ht} = \alpha q_t \neq 0$ whenever $q_t \neq 0$ and strategic complementarity elasticity $\alpha > 0$, and similarly for foreign goods. Therefore, if prices are flexible, the terms of trade (ToT), which measure the relative price of imports and exports $s_t = p_{Ft} - p^*_{Ht} - e_t$, are related to RER as follows:[36]

$$s_t = \frac{1 - 2\alpha(1-\gamma)}{1-2\gamma} q_t. \tag{34}$$

Without strategic complementarities ($\alpha = 0$), the ToT are more volatile than the consumer-price RER, $s_t = q_t/(1-2\gamma)$, as consumption bundles are more similar across countries than exported production bundles. This is empirically counterfactual, since in the data ToT are substantially more stable than RER (see Figure 1 and Table 3). *Pricing to market* (PTM), when $\alpha > 0$, mutes the transmission of RER into the ToT, thus reconciling the model with the data (see Atkeson and Burstein 2008).

When prices are sticky, relationship (34) no longer holds in the short run, when the transmission from RER into ToT is instead shaped by the specific pattern of border price stickiness. In particular, when prices are sticky in the producer currency (PCP), ToT depreciate together with RER, as foreign imports become cheaper. In contrast, under local currency price stickiness (LCP), ToT appreciate with a real depreciation, as home export prices increase (Obstfeld and Rogoff 2000). In fact, both of these patterns are at odds with the data, where the correlation between ToT and RER is weak, even if slightly

[36] This equation results from two relationships: (i) $q_t = (1-\gamma)q^P_t - \gamma s_t$ states that the relative consumer prices $q_t$ differ from the relative producer prices $q^P_t$ by the relative price of imports $s_t$; and (ii) $s_t = q^P_t - 2\alpha q_t$ states that ToT reflect the relative producer prices adjusted for the law of one price deviations, generalizing our analysis in Section 3.1 with $\alpha = 0$.

Table 3: Terms of trade and net exports

| Moments | Data | IRBC | IRBC$^+$ | | | NKOE | | |
|---|---|---|---|---|---|---|---|---|
| | | | PCP | LCP | DCP | PCP | LCP | DCP |
| $\sigma(\Delta q)/\sigma(\Delta e)$ | 0.99 | 0.83 | 0.85 | 0.98 | 0.91 | 0.83 | 0.97 | 0.90 |
| $\sigma(\Delta s)/\sigma(\Delta e)$ | 0.25 | 0.22 | 0.86 | 0.81 | 0.08 | 0.85 | 0.81 | 0.06 |
| $\mathrm{corr}(\Delta s, \Delta e)$ | $\approx$0.20 | 0.98 | 0.98 | $-0.94$ | 0.59 | 0.97 | $-0.94$ | 0.63 |
| $\sigma(\Delta nx)/\sigma(\Delta q)$ | 0.10 | 0.26 | 0.27 | 0.17 | 0.22 | 0.25 | 0.16 | 0.20 |
| $\mathrm{corr}(\Delta nx, \Delta q)$ | $\approx 0$ | 0.97 | 0.99 | 0.99 | 0.99 | 0.97 | 0.97 | 0.97 |

Note: Additional moments and model specifications; IRBC and the two LCP columns correspond to columns 6–8 in Table 1; PCP, LCP and DCP stand for producer, local and dominant currency price stickiness, respectively.

positive (Gopinath et al. 2020). Instead, under dominant currency pricing (DCP), whereby a single currency is used in pricing both exports and imports, ToT are uncorrelated with RER in the short run.

We now contrast the quantitative implications of various pricing assumptions in the context of our calibrated model. In particular, we compare a flexible-price model with PTM against the three alternative versions of the sticky-price model — namely PCP, LCP and DCP. In all these cases, we still let the PTM mechanism operate in the background, along with sticky wages, which improve the quantitative fit of the sticky price specifications. We consider both the IRBC$^+$ model with productivity shocks and the NKOE model with monetary shocks, as in Table 1. While all versions of the model are comparable in their fit of the exchange rate and business cycle moments in Table 1, their ability to match the behavior of the terms of trade and net exports varies across specifications.

We report the results in Table 3, which identifies the two PCP specifications as clear losers, as they lag in matching all three types of moments — the volatility of the real exchange rate and the behavior of the terms of trade and the net exports. The flexible price IRBC model comes in second-to-last, with a good fit of the terms of trade volatility due to the pricing-to-market mechanism. The LCP and DCP specifications compete for the first place, with LCP being more successful in matching the volatility of the real exchange rate and net exports, while DCP having a clear lead in its ability to match the behavior of the terms of trade.[37] The DCP mechanism captures both the stability of the terms of trade, as well as their imperfect correlation with the real exchange rate — properties that both PCP and LCP lack. This, however, leads the DCP specification to yield volatile relative prices of imported to domestically-produced goods, resulting in an insufficiently volatile real exchange rate and excessively volatile net exports. Both of these issues are addressed under LCP, which produces stable relative prices of imported and domestically produced goods, and thus stable import demand. In order to match all these moments simultaneously, it is likely necessary to generalize the model with either mixed-currency pricing at the border (see Amiti, Itskhoki, and Konings 2020), or combine DCP at the border with local distribution margin and LCP retail-price stickiness for imported goods (see Auer, Burstein, and Lein 2018).

[37]Note that all specifications imply a counterfactually high correlation between the real exchange rate and net exports, which is nearly zero in the data, suggesting the need either for additional home-bias or trade-cost shocks, or for slow adjustment in the trade quantities (the $J$-*curve*; see e.g. Fitzgerald, Yedid-Levi, and Haller 2019), as net exports are more closely correlated with the real exchange rate over the long run (see Alessandria and Choi 2019).

### 4.4 Small Open Economy

While all large developed economies — the US, Japan and the Euro Area — exhibit a strong home bias, it is much less pronounced in other countries, especially small open economies. What are the implications of greater trade openness for the exchange rate disconnect mechanism proposed in this paper? We now study the data from six developed countries of very different size and openness — the US, Japan, the UK, South Korea, Sweden and New Zealand — and show how our model, calibrated to match the size and openness of individual countries, captures both the differences and the similarities in the exchange rate moments observed across these countries in the data.

Even the US, with the largest economy in the world, still represents less than one quarter of the world economy. Therefore, the baseline assumption that US openness to imports from the rest of the world, $\gamma$, equals the openness of the rest of the world to imports from the US, $\gamma^*$, is a stretch; in reality, $\gamma^* < \gamma$, and roughly in proportion to the relative size of the US and the rest of the world. For all other countries, this gap is even bigger, and for the truly small open economies, such as New Zealand, an accurate approximation requires a high $\gamma$ and $\gamma^* \approx 0$, as such countries are open, yet account for a negligible share of global trade. Our two-country model can be readily adjusted to accommodate small open economies by breaking the implicitly imposed symmetry between $\gamma$ and $\gamma^*$ and allowing for $\gamma > \gamma^*$ as is consistent with the data on size and openness of individual countries.[38]

The left portion of Table 4 describes the empirical patterns we observe across countries, ranked by their size from the US to New Zealand. Smaller countries are systematically more open, with a larger share of imports in their final expenditure (GDP). For example, according to this measure, South Korea and Sweden are 3 times more open than the US, which is about 20 times bigger than these countries. New Zealand is 100 times smaller than the US, and about 2.5 times more open (on par with the much larger UK), reflecting its relative remoteness. We now explore how such vast differences in country size and openness are reflected in exchange rate moments across these countries.

Perhaps surprisingly, there is no systematic relationship between country openness and the time-series comovement between the nominal and real exchange rates. In particular, Table 4 shows that the persistence of the real exchange rate and its volatility relative to the nominal exchange rate are essentially the same across the six countries that we study, and the same applies to a nearly perfect correlation between RER and NER (not reported). In this sense, the PPP puzzle is equally pronounced in large closed and small open economies. The same is true of the Backus-Smith puzzle, in the sense that there is no systematic relationship between country openness and the sign or the magnitude of the correlation between real exchange rate and relative consumption growth, which is small for all countries, and typically negative.[39]

[38] That is, we still consider a country versus the rest of the world and treat the moments in the data accordingly. The log-linearized equilibrium system remains the same, only with an added asymmetry in market clearing (10), which now features $\gamma$ at home and $\gamma^*$ abroad, with $\gamma^*/(\gamma+\gamma^*)$ equal to the steady-state share of the home economy in world GDP, and the consumer price indexes $p_t$ and $p_t^*$ reflect these changes accordingly.

[39] Our results in Section 3.2, in particular the variance decomposition (24), provide insight as to why this is likely the case. While the relationship between relative consumption and RER depends on openness $\gamma$, the contribution of RER to the overall volatility of consumption is not very large, even as $\gamma$ increases. Therefore, the variation in the Backus-Smith correlation across countries largely reflects the composition of shocks over a given time interval, rather than the degree of country openness.

Table 4: Small open economy

| Moments | Data | | | | | | Model (IRBC$^+$) | |
|---|---|---|---|---|---|---|---|---|
| | US | Japan | UK | Korea | Sweden | NZ | US | NZ |
| Size (% world GDP), $\frac{\gamma^*}{\gamma+\gamma^*}$ | 23.7% | 10.5% | 4.1% | 1.3% | 0.8% | 0.2% | | |
| Import/GDP (%), $2\gamma$ | 12.1% | 11.5% | 24.4% | 33.4% | 33.0% | 22.8% | 12.1% | 22.8% |
| Export/GDP* (%), $2\gamma^*$ | 2.9% | 1.2% | 1.0% | 0.5% | 0.3% | 0.1% | 2.9% | 0.1% |
| $\rho(q)$ | 0.96 | 0.96 | 0.93 | 0.92 | 0.97 | 0.96 | 0.92 (0.04) | 0.91 (0.04) |
| $\sigma(\Delta q)/\sigma(\Delta e)$ | 0.97 | 1.03 | 1.04 | 0.95 | 0.99 | 1.01 | 0.99 | 0.99 |
| $\text{corr}(\Delta q, \Delta c - \Delta c^*)$ | $-0.22$ | 0.12 | $-0.03$ | $-0.50$ | $-0.17$ | 0.01 | $-0.40$ (0.08) | $-0.40$ (0.08) |
| $\sigma(\Delta e)/\sigma(\Delta gdp)$ | 4.5 | 4.5 | 4.9 | 3.3 | 2.9 | 2.1 | 5.3 (0.4) | 3.6 (0.2) |
| $\sigma(\Delta nx)/\sigma(\Delta q)$ | 0.06 | 0.09 | 0.16 | 0.24 | 0.25 | 0.26 | 0.15 | 0.28 |

Note: The empirical moments calculated using quarterly data for 1981–2017 from the Worldbank WITS database; exchange rates are trade-weighted bilateral exchange rates with major trading partners (see Online Data Appendix A.1).

The only two exchange rate moments that are clearly sensitive to the openness of the economy are the volatility of the nominal exchange rate relative to macroeconomic aggregates and the volatility of net exports relative to the real exchange rate. While net exports is relatively more volatile, the nominal exchange rate is relatively less volatile in more open economies. Note that this is not driven by the changing volatility of GDP, which is somewhat larger in small open economies, but instead is driven by lower volatility of exchange rate and greater volatility of net exports.

We now verify how our model accommodates these empirical patterns. To this end, we focus on our preferred IRBC$^+$ version of the quantitative model with nominal rigidities (as in column 7 of Table 1), and recalibrate it to feature $\gamma > \gamma^*$. In particular, we consider two calibrations — to the size and openness of the US and New Zealand, which lie at the two extremes. For transparency of the comparison, we keep all other parameters unchanged, and adjust the relative volatility of shocks to keep the Backus-Smith correlation and the cross-country GDP correlation unchanged, as we described above.[40]

The last two columns of Table 4 show that, in the model, significant differences in the openness of the economies do not change the time-series comovement between the real and nominal exchange rates, reproducing the 'PPP puzzle' moments observed in the data. While the exchange rate pass-through into *domestic* CPI inflation increases due to a higher openness of the economy $\gamma$, consistent with empirical evidence, the pass-through into *foreign* prices falls because of a correspondingly lower $\gamma^*$. As a result, the effect of the nominal exchange rate on the relative prices across countries remains largely unchanged, and so does the comovement between the nominal and real exchange rates.

The changing openness of the economy, however, affects the equilibrium relative volatility of the exchange rates and macroeconomic aggregates. Indeed, a small open economy with a high $\gamma$, features a smaller volatility of the exchange rates and a greater volatility of net exports, relative to the volatility

[40]This constitutes a conservative approach, as the composition of shocks and business cycle moments are somewhat different in small open economies, which allows to further improve the fit of the exchange rate moments in these economies, as we discuss in the end of this section. Also recall that we target a negative value of the Backus-Smith correlation, which is conventional in the literature; our results change little when we target a lower absolute value for this correlation (e.g., –0.2).

of GDP. Intuitively, the transmission of exchange rate volatility into domestic output and net exports depends primarily on the home import share $\gamma$, and hence is significantly higher under the small open economy calibration. In general equilibrium, this results in a mildly higher GDP volatility (with GDP, as before, largely determined by domestic productivity), a notably lower exchange rate volatility (recall the role of $\gamma$ in (30)), and a substantially higher volatility of net exports, consistent with the empirical patterns we document in Table 4.

**Developing countries** In our analysis we target the moments from developed OECD countries under floating exchange rate regimes. In non-OECD countries, the volatility contribution of monetary, productivity and commodity-price shocks is arguably more pronounced (see e.g. Aguiar and Gopinath 2007) and, in addition, pegs and partial pegs are more ubiquitous (see e.g. Reinhart and Rogoff 2004).[41] In the model, this makes the exchange rate puzzles less pronounced, as it reduces the relative role of the financial shock in shaping the dynamics of the exchange rate. This is in line with the empirical evidence that the Meese-Rogoff disconnect, the PPP, the Backus-Smith and the UIP conditions fare less badly in developing countries (see e.g. Rogoff 1996, Bansal and Dahlquist 2000) and under pegged regimes (see Itskhoki and Mukhin 2019, for the analysis of a peg). Therefore, accommodating these differences in the combination of shocks and in the exchange rate policies allows our quantitative model to capture the properties of exchange rates in rich and developing countries of different sizes and openness.

## 5 Conclusion

We propose a parsimonious general equilibrium model of exchange rate determination, which offers a unifying resolution to the main exchange rate puzzles in international macroeconomics. In particular, we show that introducing a financial shock into an otherwise standard international business cycle model allows it to match a rich set of moments describing the comovement between exchange rates and macro variables without compromising the model's ability to explain the main international business cycle properties. We take advantage of the analytical tractability of the model to dissect the underlying exchange rate disconnect mechanism, which we show is robust and requires only an empirically relevant degree of home bias in consumption. Additional sources of incomplete pass-through, including pricing to market and foreign-currency price stickiness, improve the quantitative fit of the model without changing its qualitative properties.

With this general equilibrium model, one can reconsider the conclusions in the broad international macro literature, which has been plagued by exchange rate puzzles. In particular, our analysis shows that these puzzles do not necessarily invalidate the standard international transmission mechanism for monetary and productivity shocks, including international spillovers from monetary policy (see e.g. Corsetti, Dedola, and Leduc 2010, Egorov and Mukhin 2019). We emphasize instead that these conventional shocks *cannot* be the main drivers of the *unconditional* behavior of exchange rates. In contrast, our findings likely challenge the conventional normative analysis in open economies, and in

[41] On the role of commodity prices in shaping the exchange rates see e.g. Chen and Rogoff (2003), Casas, Díez, Gopinath, and Gourinchas (2016) and Ayres, Hevia, and Nicolini (2019).

particular the studies of the optimal exchange rate regimes and capital controls. Pegging the exchange rate may simultaneously reduce monetary policy flexibility (Friedman 1953), yet improve international risk-sharing by offsetting the noise-trader risk (Jeanne and Rose 2002, Devereux and Engel 2003). Furthermore, a microfoundation of financial shocks is essential, as they may endogenously interact with or arise from monetary policy (see Alvarez, Atkeson, and Kehoe 2007, Itskhoki and Mukhin 2019).

In addition, our framework can be used as a theoretical foundation for the vast empirical literature that relies on exchange rate variation for identification (see e.g. Burstein and Gopinath 2012). Similarly, it can serve as a point of departure for the equilibrium analysis of the international price system (Gopinath 2016, Mukhin 2017) and the global financial cycle (Rey 2013). The model also offers a simple general equilibrium framework for nesting the financial sector in an open economy environment. This may prove particularly useful for future explorations into the nature of financial shocks, which can be disciplined by additional comovement properties between exchange rates and financial variables (e.g., see recent work by Jiang, Krishnamurthy, and Lustig 2018, Engel and Wu 2019).

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# A Appendix: Additional Figures and Tables

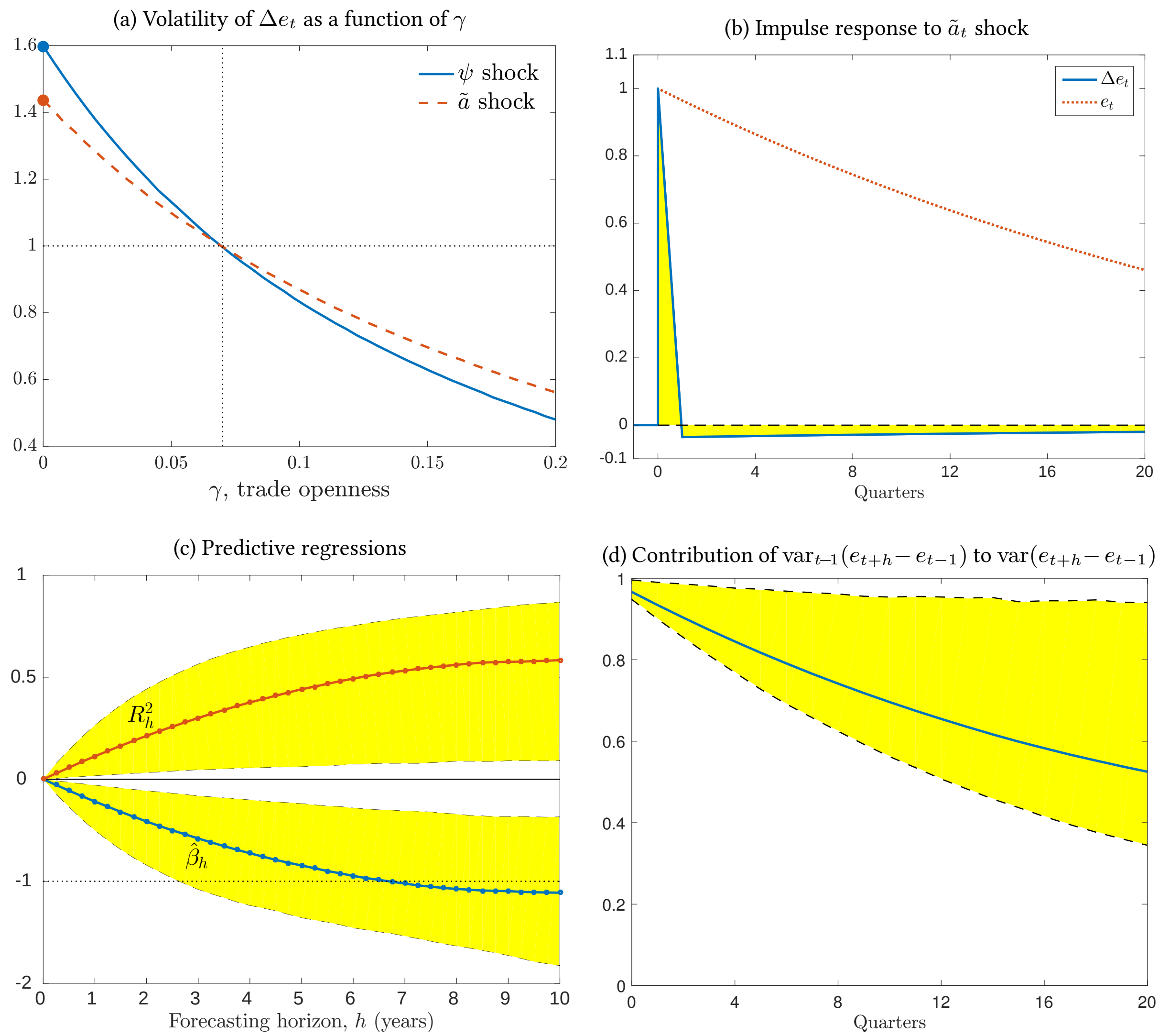


Figure A1: Additional properties of the equilibrium exchange rate process

Note: Panel (a): plots $\text{std}(\Delta e_t)$ as a function of openness $\gamma$, conditional on $\psi_t$ and $\tilde{a}_t$ shocks, normalizing to one the calibrated value of $\text{std}(\Delta e_t)$ for $\gamma = 0.07$; (b) impulse response of $\Delta e_t$ and $e_t$ to a productivity shock $\tilde{a}_t$ innovation (*cf.* Figure 2a); (c): $\hat{\beta}_h$ and $R_h^2$ from the predictive regression $\mathbb{E}\{e_{t+h} - e_t | q_t\} = \alpha_h + \beta_h q_t$, at different horizons $h \geq 1$; (d): variance contribution of the unexpected component, $e_{t+h} - \mathbb{E}_{t-1} e_{t+h}$, to the overall variance of $e_{t+h} - e_{t-1}$ for different horizons $h \geq 0$. Lines in (c) and (d) plot medians across 10,000 simulations with 120 quarters each and shaded areas provide the 5%–95% range across simulations.

Table A1: Model parameters

| Parameter | | Value | Comment |
|---|---|---|---|
| **Conventional parameters** | | | |
| Discount factor, quarterly | $\beta$ | 0.99 | |
| Relative risk aversion | $\sigma$ | 2 | |
| Macro Frisch elasticity | $\nu$ | 1 | |
| Intermediate share | $\phi$ | 0.5 | |
| Capital share in value added | $\vartheta$ | 0.3 | |
| Depreciation rate, quarterly | $\delta$ | 0.02 | |
| **Transmission mechanism** | | | |
| Trade openness | $\gamma$ | 0.07 | Trade-to-GDP ratio $= 0.28$ |
| Elasticity of substitution | $\theta$ | 1.5 | Feenstra et al. (2018) |
| Strategic complementarity | $\alpha$ | 0.4 | Amiti et al. (2019) |
| **Monetary parameters** | | | |
| Taylor rule coefficient | $\phi_\pi$ | 2.15 | CGG |
| Interest rate smoothing | $\rho_m$ | 0.95 | $\rho(i_t), \rho(i_t - i_t^*) \approx 0.95$ |
| Calvo probability for prices | $\lambda_p$ | 0.75 | average duration of 4 quarters |
| Calvo probability for wages | $\lambda_w$ | 0.85 | average duration of 6 quarters |
| **Calibrated parameters** | | | |
| Persistence of shocks | $\rho$ | 0.97 | $\rho(gdp_t), \rho(i_t), \rho(\psi_t)$ |
| Coefficient on NFA in UIP (16) | $\chi_2$ | 0.001 | $\rho(\Delta b_t) \approx 0.95$ |
| Std. of the UIP shock | $\chi_1\sigma_\psi$ | 1 | normalization |
| Std. productivity (monetary) shocks | $\sigma_a$ ($\sigma_m$) | $\longrightarrow$ | $\text{corr}(\Delta c_t - \Delta c_t^*, \Delta q_t) = -0.4$ |
| Corr. productivity (monetary) shocks | $\varrho_{a,a^*}$ ($\varrho_{m,m^*}$) | $\longrightarrow$ | $\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.35$ |
| Capital adjustment cost parameter | $\kappa$ | $\longrightarrow$ | $\text{std}(\Delta z_t)/\text{std}(\Delta gdp_t) = 2.5$ |

Note: See calibration details in Sections 4. The values of $\kappa$, $\sigma_a$, $\sigma_m$, $\varrho_{a,a^*}$ and $\varrho_{m,m^*}$ vary across specifications to keep the targeted moments unchanged. We normalize the effective volatility of the financial shock $\chi_1\sigma_\psi = 1$, as our results focus on the relative volatilities of the variables; scaling $\sigma_a$ and $\chi_1\sigma_\psi$ proportionally does not effect the results reported in Table 1, and allows us to match the volatility of the nominal exchange rate in any specification of the model. In addition, we use a small positive $\chi_2 = 0.001$ in the modified UIP (16), which endogenously ensures long-run stationarity of the model (*cf.* Schmitt-Grohé and Uribe 2003), and is also consistent with the high persistence of the US current account $\Delta b_t$ in the data.

# Online Appendix

Itskhoki and Mukhin: "Exchange Rate Disconnect in General Equilibrium"

## A.1 Data appendix

As explained in Section 4, for comparability, we estimate most empirical moments in Tables 1 and 3 following CKM. In particular, we use their quarterly data from 1973–94 available on Ellen McGrattan's website and estimate the moments for the US against the PPP-weighted sum of France, Germany, Italy and the UK. While we use the first differences of the variables, rather than the HP-filtered series, our estimates are very close to the ones in CKM.

As a robustness exercise, we also collected quarterly data for a longer time period from 1981–2017 from FRED database. In particular, we use seasonally-adjusted GDP, consumption and gross capital formation in constant prices and aggregate them across the same countries using PPP-adjusted nominal GDP in 2000 (from OECD database). Table A2 shows that the estimates for the two periods are very similar. The only exception are the moments for net exports in Table 3: in contrast to CKM, who focus on bilateral trade between the US and the European countries, we use total imports and exports of the US, consistent with our model. Indeed, the former includes only a part of international trade of the US and underestimates the openness of the US economy.

We use seasonally-adjusted hourly earnings in manufacturing as a proxy for nominal wages to compute the wage-based real exchange rate. Given the limited availability of the terms-of-trade data at the quarterly frequency, we use the estimates for terms-of-trade moments from Obstfeld and Rogoff (2001) and Gopinath et al. 2020. We also compute the same moments using FRED annual data on import and export price indices for the US. For our baseline period from 1973–1994, we obtain $\sigma(\Delta s)/\sigma(\Delta e) = 0.29$ and $\text{corr}(\Delta s, \Delta e) = 0.20$, in line with the estimates in the literature (see Table 3).

Finally, we borrow several financial moments from the previous literature. The slope coefficient $\beta$ and $R^2$ in the Fama regression are from the survey by Engel (1996) and recent estimates by Burnside, Han, Hirshleifer, and Wang (2011, Table 1) and Valchev (2016, Table B.1). The estimates for the Sharpe ratio correspond to the forward premium trade from Hassan and Mano (2014, Table 2). We estimate the volatility and persistence of the interest rates using quarterly data for the US versus the UK, France, Germany and Japan from 1979–2009.

We calculate additional moments for the six countries in Table 4 against the rest of the world using quarterly data from 1981–2017 and focus on a sample of developed economies (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the UK and the US). After computing the log changes of variables for each individual country, we aggregate series into the RoW using the average import and export shares for 1991-2017 from the Worldbank WITS database. Despite the fact that our sample does not include such import trade partners as Mexico and China, and we use time-invariant weights, the resulting exchange rates mimic very closely the "narrow effect" exchange rates calculated by the BIS.

Table A2: Empirical Moments

| Moments | CKM | IM | Moments | CKM | IM |
|---|---|---|---|---|---|
| A. Exchange rate disconnect: | | | D. International business cycle moments: | | |
| $\rho(\Delta e)$ | 0.3 | 0.3 | $\sigma(\Delta c)/\sigma(\Delta gdp)$ | 0.82 | 0.81 |
| $\sigma(\Delta e)/\sigma(\Delta gdp)$ | 5.2 | 6.5 | $\text{corr}(\Delta c, \Delta gdp)$ | 0.64 | 0.63 |
| $\sigma(\Delta e)/\sigma(\Delta c)$ | 6.3 | 8.0 | $\text{corr}(\Delta z, \Delta gdp)$ | 0.81 | 0.75 |
| B. Real exchange rate and the PPP: | | | $\text{corr}(\Delta gdp, \Delta gdp^*)$ | 0.35 | 0.42 |
| $\rho(q)$ | 0.96 | 0.94 | $\text{corr}(\Delta c, \Delta c^*)$ | 0.30 | 0.40 |
| $\sigma(\Delta q)/\sigma(\Delta e)$ | 0.99 | 0.97 | $\text{corr}(\Delta z, \Delta z^*)$ | 0.27 | 0.32 |
| $\text{corr}(\Delta q, \Delta e)$ | 0.99 | 0.99 | E. Terms of trade and net exports moments: | | |
| C. Backus-Smith correlation: | | | $\sigma(\Delta nx)/\sigma(\Delta q)$ | 0.01 | 0.09 |
| $\text{corr}(\Delta q, \Delta c - \Delta c^*)$ | $-0.20$ | $-0.17$ | $\text{corr}(\Delta nx, \Delta q)$ | $-0.01$ | 0.35 |

Note: CKM and IM correspond respectively to the estimates obtained for the periods 1973–94 and our estimates for 1981–2017.

## A.2 Full quantitative model

We focus on home households and firms with the understanding that the problem of foreign agents is symmetric. The equation below generalize the equations given in the text in the context of a simplified baseline model.

**Households** A representative home household maximizes the expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+1/\nu} L_t^{1+1/\nu} \right), \tag{A1}$$

where $\nu \equiv 1/\varphi$ is the Frisch elasticity, subject to the flow budget constraint:

$$P_t C_t + P_t Z_t + \frac{B_{t+1}}{R_t} \leq W_t L_t + R_t^K K_t + B_t + \Pi_t, \tag{A2}$$

where $R_t^K$ is the nominal rental rate of capital and $Z_t$ is the gross investment into the domestic capital stock $K_t$, which accumulates according to a standard rule with depreciation $\delta$ and quadratic capital adjustment costs:

$$K_{t+1} = (1-\delta)K_t + \big[Z_t - \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t}\big]. \tag{A3}$$

The domestic households allocate their within-period consumption expenditure $P_t C_t$ between home and foreign varieties of the goods

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft} = \int_0^1 \Big[ P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \Big] \mathrm{d}i \tag{A4}$$

to minimize expenditure on aggregate consumption, defined implicitly by a Kimball (1995) aggregator:[42]

$$\int_0^1 \left[ (1-\gamma) g\left( \frac{C_{Ht}(i)}{(1-\gamma)C_t} \right) + \gamma g\left( \frac{C_{Ft}(i)}{\gamma C_t} \right) \right] \mathrm{d}i = 1, \tag{A5}$$

where the aggregator function $g(\cdot)$ in (A5) has the following properties: $g'(\cdot) > 0$, $g''(\cdot) < 0$ and $-g''(1) \in (0,1)$, and two normalizations: $g(1) = g'(1) = 1$. The solution to the optimal expenditure allocation results in the following homothetic demand schedules:

$$C_{Ht}(i) = (1-\gamma) h\left( \frac{P_{Ht}(i)}{\mathcal{P}_t} \right) C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma h\left( \frac{P_{Ft}(j)}{\mathcal{P}_t} \right) C_t, \tag{A6}$$

where $h(\cdot) = g'^{-1}(\cdot) > 0$ and satisfies $h(1) = 1$ and $h'(\cdot) < 0$. The function $h(\cdot)$ controls the curvature of the demand schedule, and we denote its point elasticity with $\theta \equiv -\frac{\partial \log h(x)}{\partial \log x}\big|_{x=1} = -h'(1) > 1$. The consumer price level $P_t$ and the auxiliary variable $\mathcal{P}_t$ in (A6) are two alternative measures of average prices in the home market, which are defined implicitly by (A4) and (A5) after substituting in the demand schedules (A6).

**Production** Home output is produced according to a Cobb-Douglas technology in labor $L_t$, capital $K_t$ and intermediate inputs $X_t$:

$$Y_t = \left( e^{a_t} K_t^{\vartheta} L_t^{1-\vartheta} \right)^{1-\phi} X_t^{\phi}, \tag{A7}$$

where $\vartheta$ is the elasticity of the value added with respect to capital and $\phi$ is the elasticity of output with respect to intermediates. Intermediates are the same bundle of home and foreign varieties as the final consumption bundle (A5). The marginal cost of production is thus:

$$MC_t = \tfrac{1}{\varpi} \big[ e^{-a_t} (R_t^K)^{\vartheta} W_t^{1-\vartheta} \big]^{1-\phi} P_t^{\phi}, \quad \text{where } \varpi \equiv \phi^{\phi} \big[ (1-\phi) \vartheta^{\vartheta} (1-\vartheta)^{1-\vartheta} \big]^{1-\phi}. \tag{A8}$$

The aggregate *value-added productivity* follows an AR(1) process in logs:

[42] The CES demand is nested as a special case of the Kimball aggregator (A5) with $g(z) = 1 + \frac{\theta}{\theta-1}\big(z^{1-1/\theta} - 1\big)$, resulting in the demand schedule $h(x) = x^{-\theta}$ and price index $P_t = \mathcal{P}_t = \big( \int_0^1 \big[ (1-\gamma) P_{Ht}(i)^{1-\theta} + \gamma P_{Ft}(i)^{1-\theta} \big] \mathrm{d}i \big)^{1/(1-\theta)}$.

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \qquad \varepsilon_t^a \sim iid(0,1), \tag{A9}$$

where $\rho_a \in [0,1]$ is the persistence parameter and $\sigma_a \geq 0$ is the volatility of the innovation.

**Profits and price setting** The firm maximizes profits from serving the home and foreign markets:

$$\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \Theta_t \Pi_t(i), \quad \text{where} \quad \Pi_t(i) = (P_{Ht}(i) - MC_t)\, Y_{Ht}(i) + (P^*_{Ht}(i)\mathcal{E}_t - MC_t)\, Y^*_{Ht}(i), \tag{A10}$$

where $\Theta_t \equiv \beta^t \frac{C_t^{-\sigma}}{P_t}$ is the nominal stochastic discount factor. In the absence of nominal frictions, this results in the markup pricing rules, with a common price across all domestic firms $i \in [0,1]$ in a given destination market and expressed in the destination currency:

$$P_{Ht}(i) = P_{Ht} = \mu\left(\frac{P_{Ht}}{\mathcal{P}_t}\right) \cdot MC_t \qquad \text{and} \qquad P^*_{Ht}(i) = P^*_{Ht} = \mu\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right) \cdot \frac{MC_t}{\mathcal{E}_t},$$

where $\mu(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$ is the markup function and $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x}$ is the demand elasticity schedule derived from demand (A6) (see also (A11) below).

**Nominal rigidities** We introduce Calvo sticky prices and wages in a conventional way (see e.g. Clarida, Galí, and Gertler 2002, Galí 2008, as we further discuss in Appendix A.7 below). Denote with $\epsilon$ the elasticity of substitution between varieties of labor, and let $\lambda_p$ and $\lambda_w$ be the Calvo probability of price and wage non-adjustment. Then the resulting New Keynesian Phillips Curves (NKPC) for nominal wages inflation and domestic prices inflation can be written respectively as:

$$\pi^w_t = k_w\left[\sigma c_t + \frac{1}{\nu}\ell_t + p_t - w_t\right] + \beta\mathbb{E}_t\pi^w_{t+1}, \quad \text{where} \quad k_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)},$$

$$\pi_{Ht} = k_p\Big[(1-\alpha)mc_t + \alpha p_t - p_{Ht}\Big] + \beta\mathbb{E}_t\pi_{Ht+1}, \quad \text{where} \quad k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}.$$

The NKPC for export prices depends on the currency of invoicing and is given by:

$$\pi^*_{Ht} = k_p\Big[(1-\alpha)(mc_t - e_t) + \alpha p^*_t - p^*_{Ht}\Big] + \beta\mathbb{E}_t\pi^*_{Ht+1} \quad \text{under LCP},$$

$$(\pi^*_{Ht} + \Delta e_t) = k_p\Big[(1-\alpha)mc_t + \alpha(p^*_t + e_t) - (p^*_{Ht} + e_t)\Big] + \beta\mathbb{E}_t\left(\pi^*_{Ht+1} + \Delta e_{t+1}\right) \quad \text{under PCP}.$$

Notice that the DCP case with all international trade invoiced in Foreign currency can be expressed as a mix of the two other regimes — Home exporters use LCP and Foreign exporters use PCP.

**Good and factor market clearing** The labor market clearing requires that $L_t$ equals simultaneously the labor supply of the households and the labor demand of the firms, and equivalently for $L^*_t$ in foreign. Similarly, equilibrium in the capital market requires that $K_t$ (and $K^*_t$) equals simultaneously the capital supply of the households and the capital demand of the local firms. The goods market clearing requires that the total production by the home firms is split between supply to the home and foreign markets respectively, $Y_t = Y_{Ht} + Y^*_{Ht}$, and satisfies the local demand in each market for the final, intermediate and capital goods:

$$Y_{Ht} = C_{Ht} + X_{Ht} + Z_{Ht} = (1-\gamma)h\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)\big[C_t + X_t + Z_t\big], \tag{A11}$$

$$Y^*_{Ht} = C^*_{Ht} + X^*_{Ht} + Z^*_{Ht} = \gamma h\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)\big[C^*_t + X^*_t + Z^*_t\big]. \tag{A13}$$

Lastly, we combine the household budget constraint (A2) with profits (A10), aggregated across all home firms, as well as the market clearing conditions above to obtain the home country budget constraint:

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{with} \qquad NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft}, \tag{A14}$$

where $NX_t$ denotes net exports expressed in units of the home currency.

**Financial sector** The structure of the financial markets is exactly the same as described in the text for the baseline model (see Section 2.2).

**Monetary policy rule** The monetary policy is implemented by means of a conventional Taylor rule:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\phi_\pi \pi_t + \sigma_m \varepsilon_t^m, \tag{A15}$$

where $i_t \equiv \log R_t$ is the log nominal interest rate, $\pi_t = \Delta \log P_t$ is the inflation rate, and $\varepsilon_t^m \sim iid(0,1)$ is the monetary policy shock with volatility parameter $\sigma_m \geq 0$; parameter $\rho_m$ captures the persistence of monetary policy.

## A.3 Derivation of the equilibrium conditions

This section derives the optimality conditions of firms and households in goods and asset markets. We focus on a generalized version of the model with Kimball demand and capital and intermediate goods in production, but keep flexible prices as in the baseline model (see Appendix A.7).

**Household optimization** Substituting the capital accumulation equation into the budget constraint (A2), the Lagrangian for household utility maximization (A1) is:

$$\begin{aligned}\max_{\{C_t,L_t,B_{t+1},K_{t+1}\}} \sum_{t,s^t} \beta^t \pi(s^t) \Bigg\{ & \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+1/\nu}}{1+1/\nu} \\ & + \lambda_t \left[ W_t L_t - P_t C_t + B_t - \frac{B_{t+1}}{R_t} + P_t K_t \left( \frac{R_t^K}{P_t} - \delta - \frac{\Delta K_{t+1}}{K_t} - \frac{\kappa}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 \right) \right] \Bigg\},\end{aligned}$$

where $\pi(s^t)$ is probability of state $s^t = (s_0, s_1, \dots, s_t)$ at time $t$ and $\beta^t \pi(s^t)\lambda_t$ is the Lagrange multiplier on the flow budget constraint in state $s^t$ at time $t$ (note that we suppress the dependence of variables on $s^t$ for brevity). The optimality conditions are:

$$\begin{aligned} C_t^{-\sigma} &= \lambda_t P_t, \\ L_t^{1/\nu} &= \lambda_t W_t, \\ \lambda_t &= \beta R_t \mathbb{E}_t \lambda_{t+1}, \\ 1 + \kappa \frac{\Delta K_{t+1}}{K_t} &= \beta \mathbb{E}_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[ \frac{R_{t+1}^K}{P_{t+1}} + (1-\delta) + \kappa \frac{\Delta K_{t+2}}{K_{t+1}} + \frac{\kappa}{2} \left( \frac{\Delta K_{t+2}}{K_{t+1}} \right)^2 \right]. \end{aligned}$$

Solving out $\lambda_t = C_t^{-\sigma}/P_t$, we arrive at the optimality conditions (2)–(3) in the text, as well as the Euler equation for capital:

$$\eta_t = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[ \frac{R_{t+1}^K}{P_{t+1}} - \delta + \eta_{t+1} + \frac{(\eta_{t+1}-1)^2}{2\kappa} \right] \right\}, \tag{A16}$$

where $\eta_t \equiv 1 + \kappa \frac{\Delta K_{t+1}}{K_t}$ is the ($q$-theory) market price of one unit of capital in units of the home consumption good, with the last term in (A16) arising from the quadratic adjustment costs.

**Expenditure minimization and the price level** The household expenditure minimization problem for a given consumption level $C_t$ is given by

$$\min_{\{C_{Ht}(i), C_{Ft}(i)\}} P_t C_t = \int_0^1 \Big[ P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \Big] \mathrm{d}i$$

subject to (A5). The optimality conditions are:

$$P_{Jt}(i) = \mathcal{R}_t g' \left( \frac{C_{Jt}(i)}{\gamma_J C_t} \right) \frac{1}{C_t}, \quad J \in \{H, F\},$$

where $\gamma_H \equiv 1-\gamma$ and $\gamma_F \equiv \gamma$, and $\mathcal{R}_t$ is the Lagrange multiplier on the consumption aggregator constraint.

Denoting $\mathcal{P}_t = \mathcal{R}_t / C_t$ and $h(\cdot) \equiv g'^{-1}(\cdot)$, we obtain the demand schedules (A6):

$$C_{Jt}(i) = \gamma_J \, h\left(\frac{P_{Jt}(i)}{\mathcal{P}_t}\right) C_t, \tag{A17}$$

where $\mathcal{P}_t$ and $P_t$ are defined implicitly by the following system:

$$1 = \int_0^1 \left[ (1-\gamma) g\left( h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right)\right) + \gamma g\left( h\left(\frac{P_{Ft}(i)}{\mathcal{P}_t}\right)\right)\right] \mathrm{d}i, \tag{A18}$$

$$P_t = \int_0^1 \left[ (1-\gamma) P_{Ht}(i) h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) + \gamma P_{Ft}(i) h\left(\frac{P_{Ft}(i)}{\mathcal{P}_t}\right)\right] \mathrm{d}i. \tag{A19}$$

Equation (A18) arises from the definition of the Kimball consumption aggregator (A5), and uniquely determines $\mathcal{P}_t$, as $h'(\cdot) < 0$. Given $\mathcal{P}_t$, equation (A19) determines the value of $P_t$, which ensures that the sum of all market shares is one, given that $P_t C_t$ is total expenditure. Indeed,

$$S_{Jt}(i) = \frac{C_{Jt}(i) P_{Jt}(i)}{P_t C_t} = \gamma_J \frac{P_{Jt}(i)}{P_t} h\left(\frac{P_{Jt}(i)}{\mathcal{P}_t}\right)$$

is the market share of domestic (for $J = H$) or foreign (for $J = F$) variety $i$ in the home market.

When all $P_{Ht}(i) = P_{Ft}(j) = \mathcal{P}_t$ for all $i, j$ and for some $\mathcal{P}_t$, then $P_t = \mathcal{P}_t$, given our normalization that $g(1) = g'(1) = 1$, which implies $h(1) = 1$. More generally, $P_t$ and $\mathcal{P}_t$ offer two alternative generalized averages of prices of the varieties in the domestic market, and the difference between $P_t$ and $\mathcal{P}_t$ is second order in the dispersion of prices. Indeed, taking the first order approximation to (A18)–(A19) around a symmetric equilibrium described above, we have:

$$p_t \equiv \mathrm{d}\log P_t = \int_0^1 \left[(1-\gamma) p_{Ht}(i) + \gamma p_{Ft}(i)\right] \mathrm{d}i \quad \text{and} \quad \mathrm{d}\log \mathcal{P}_t = \int_0^1 \left[(1-\gamma) p_{Ht}(i) + \gamma p_{Ft}(i)\right] \mathrm{d}i = p_t,$$

where $\mathrm{d}\log P_t$ and $\mathrm{d}\log \mathcal{P}_t$ denote the log-deviations from some symmetric steady state. In the derivations, we used the facts that $g'(1) = h(1) = 1$ and $h'(1) = -\theta$. This confirms that $P_t$ and $\mathcal{P}_t$ differ at most by a second-order term in the dispersion of the vector $\left(\{p_{Ht}(i) - p_t\}_i, \{p_{Ft}(j) - p_t\}_j\right)$, which is an identical zero in a symmetric steady state. Lastly, note that the expenditure share on foreign goods in a symmetric equilibrium is given by $\int_0^1 S_{Ft}(i) \mathrm{d}i = \gamma$.

**Price setting** Monopolistically competitive firms set prices flexibly to maximize profits (A10) subject to the demand schedule (A6). The price setting problem of a representative home firm $i$ partitions into price setting in the home and foreign markets separately:

$$P_{Ht}(i) = \arg\max_{P_{Ht}(i)} \left\{ (P_{Ht}(i) - MC_t)(1-\gamma) h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) C_t \right\} = \mu\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) \cdot MC_t, \tag{A20}$$

$$P^*_{Ht}(i) = \arg\max_{P^*_{Ht}(i)} \left\{ (P^*_{Ft}(i)\mathcal{E}_t - MC_t)\gamma h\left(\frac{P^*_{Ht}(i)}{\mathcal{P}^*_t}\right) C^*_t \right\} = \mu\left(\frac{P^*_{Ht}(i)}{\mathcal{P}^*_t}\right) \cdot \frac{MC_t}{\mathcal{E}_t}, \tag{A21}$$

where $\mu(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x) - 1}$ is the optimal markup function and $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x}$ is the elasticity of the demand schedule, and $\mathcal{P}^*_t$ is the auxiliary average price in the foreign market. Note that all home firms charge the same price in each of the markets, $P_{Ht}$ and $P^*_{Ht}$ repsectively, yet the prices may differ across markets, $P_{Ht} \neq P^*_{Ht}\mathcal{E}_t$, violating the law of one price. This happens *iff* $\mathcal{P}_t \neq \mathcal{P}^*_t$.

Note that $P_{Ht}$ and $P^*_{Ht}$ also correspond to the price indexes of the home good aggregator in the home and foreign markets respectively, defined in parallel with the overall price index $P_t$ in (A19). Given the same prices, we also have the same quantities across domestic firms, in particular $P_{Ht} C_{Ht} = P_{Ht}(i) C_{Ht}(i)$ for all $i$, where $C_{Ht}$ is the aggregate consumption index of all home goods in the home market defined analogously to the aggregate consumption $C_t$ in (A5). The same property applies in the foreign market, and for foreign goods in both markets. Finally, we have the aggregate expenditure in the home and foreign markets given by $P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft}$ and $P^*_t C^*_t = P^*_{Ht} C^*_{Ht} + P^*_{Ft} C^*_{Ft}$ respectively.

Next, consider the full log-differential of the optimal price setting equations around a symmetric equilibrium in both markets:

$$
\begin{aligned}
p_{Ht} &= -\Gamma(p_{Ht}-p_t)+mc_t,\\
p^*_{Ht} &= -\Gamma(p^*_{Ht}-p^*_t)+(mc_t-e_t),
\end{aligned}
$$

where small letters denote log-deviations from the symmetric equilibrium and we used the fact that $\mathrm{d}\log\mathcal{P}_t=p_t$ (and same in foreign) and that

$$
\Gamma\equiv-\frac{\partial\log\mu(x)}{\partial\log x}\Big|_{x=1}=\frac{\epsilon}{\theta-1},
$$

where we used the properties $\mu(x)=\frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$ and $\theta=\tilde{\theta}(1)$, and we defined the super elasticity of demand $\epsilon=\frac{\partial\log\tilde{\theta}(x)}{\partial\log x}\big|_{x=1}$. From the definition of $\tilde{\theta}(x)$, it follows that

$$
\epsilon=\left[1-\frac{h'(x)x}{h(x)}+\frac{h''(x)x}{h'(x)}\right]\Big|_{x=1}=1+\theta+\frac{h''(x)x}{h'(x)}\Big|_{x=1}=1+\theta-h''(1)/\theta,
$$

and therefore, given the slope of demand $\theta$, $\epsilon$ characterizes the curvature (i.e. the second derivative, $h''(1)$) of the demand schedule. We assume that the demand schedule $h(\cdot)$ is log-concave, that is $\epsilon\geq0$, and therefore $\Gamma\geq0$, that is the markup decreases with the relative price of the firm, and hence increases with its market share. Note that the $\theta>1$ requirement corresponds to the second-order condition for the optimal price.

Solving the equations above for $p_{Ht}$ and $p^*_{Ht}$, we arrive at

$$
p_{Ht}(i)\equiv p_{Ht}=(1-\alpha)mc_t+\alpha p_t, \tag{A22}
$$

$$
p^*_{Ht}(i)\equiv p^*_{Ht}=(1-\alpha)(mc_t-e_t)+\alpha p^*_t, \tag{A23}
$$

where the coefficient $\alpha=\frac{\Gamma}{1+\Gamma}=\frac{\epsilon}{\epsilon+\theta-1}\in[0,1)$, as $\theta>1$ and $\epsilon\geq0$. In particular, $\alpha=0$ *iff* $\epsilon=0$, which also implies $\tilde{\theta}(x)\equiv\theta=const$, that is a constant elasticity (CES) demand.

**Example 1: CES** Consider the case of CES demand, which obtains when $g(z)=1+\frac{\theta}{\theta-1}\left(z^{\frac{\theta-1}{\theta}}-1\right)$, which is normalized to satisfy $g(1)=g'(1)=1$. In this case, $g'(z)=z^{-1/\theta}$ and $h(x)=x^{-\theta}$, so that $g(h(x))=-\frac{1}{\theta-1}+\frac{\theta}{\theta-1}x^{1-\theta}$. As a result, equations (A18)–(A19) can be solved to yield:

$$
P_t=\mathcal{P}_t=\left[\int_0^1\left[(1-\gamma)P_{Ht}(i)^{1-\theta}+\gamma P_{Ft}(i)^{1-\theta}\right]\mathrm{d}i\right]^{1/(1-\theta)}.
$$

Note that in the CES model $\tilde{\theta}(x)=-\frac{\partial\log h(x)}{\partial\log x}\equiv\theta=const$, implying $\epsilon=0$, and therefore $\mu(x)\equiv\frac{\theta}{\theta-1}=const$ and $\Gamma=\alpha=0$.

**Example 2: Klenow and Willis (2016)** Consider the demand structure implicitly defined by the demand schedule $h(x)=[1-\epsilon\log(x)]^{\theta/\epsilon}$ for some elasticity parameter $\theta>1$ and super-elasticity parameter $\epsilon>0$. This demand structure has been originally developed by Klenow and Willis (2016) and was later used in Gopinath and Itskhoki (2010) in the context of exchange rate transmission. Note that it is indeed the case that $h(1)=1$, $h'(1)=-\theta$, $\tilde{\theta}(x)=-\frac{\partial\log h(x)}{\partial\log x}=\frac{\theta}{1-\epsilon\log x}$ and $\frac{\partial\log\tilde{\theta}(x)}{\partial\log x}\big|_{x=1}=\frac{\epsilon}{1-\epsilon\log x}\big|_{x=1}=\epsilon$. Therefore, parameter $\theta$ controls the local slope of the demand schedule, while parameter $\epsilon$ controls its local curvature, and the two can be chosen independently. As $\varepsilon\to0$, the demand schedule converges to CES demand, $h(x)\to x^{-\theta}$. The preference aggregator $g(\cdot)$ corresponding to the Klenow-Willis demand schedule $h(\cdot)$ is well-defined, but is not an analytical function. Therefore, there is no analytical characterization of $\mathcal{P}_t$ and $P_t$ in the general case, yet the general result that $P_t$ and $\mathcal{P}_t$ are both first-order equivalent to the sales-weighted average industry price still holds.[43] With this demand structure, we have $\mu(x)=\frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}=\frac{\theta}{\theta-1+\epsilon\log x}$, which results in $\Gamma=-\frac{\partial\log\mu(x)}{\partial\log x}\big|_{x=1}=\frac{\epsilon}{\theta-1}>0$. Therefore, indeed, $\Gamma$ and $\alpha$ are shaped by the primitive parameters of demand $\epsilon$ and $\theta$, and any value of $\alpha=$

[43] A special case with a tractable analytical solution obtains when $\theta=\epsilon$. In this case, $\mathcal{P}_t$ is a simple weighted average of prices $\bar{P}_t$, while $P_t$ equals to $\bar{P}_t$ adjusted downwards by $\epsilon$ times the measure of price dispersion (namely, the Theil index), as consumers benefit from a greater price dispersion holding the average price constant.

$\frac{\epsilon}{\epsilon+\theta-1} > 0$ can be obtained independently of the value of $\theta$ by setting $\epsilon = \frac{\alpha}{1-\alpha}(\theta - 1) > 0$. In the CES limit, as $\epsilon \to 0$, we have $\alpha \to 0$.

**Country budget constraint and Walras law.** Aggregating firm profits (A10) across all domestic firms:

$$\begin{aligned}\Pi_t &= (P_{Ht} - MC_t)\, Y_{Ht} + (P^*_{Ht}\mathcal{E}_t - MC_t)\, Y^*_{Ht} \\ &= P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - MC_t Y_t \\ &= P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - W_t L_t - R^K_t K_t - P_t X_t,\end{aligned}$$

where we used in the second line the fact that total output $Y_t = Y_{Ht} + Y^*_{Ht}$ and in the third line the expressions for the aggregate production demand for labor

$$W_t L_t = (1-\vartheta)(1-\phi) MC_t\, Y_t, \tag{A24}$$

and analogous conditions for capital and intermediates:

$$R^K_t K_t = (1-\phi)\vartheta MC_t\, Y_t \qquad \text{and} \qquad P_t X_t = \phi MC_t\, Y_t. \tag{A25}$$

We next substitute $\Pi_t$ into the household budget constraint (1), resulting after rearranging in:

$$\frac{B_{t+1}}{R_t} - B_t = P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - P_t(C_t + X_t + Z_t).$$

Finally, we use the fact that total domestic expenditure can be split into the expenditure on the home and the foreign goods, $P_t(C_t + X_t + Z_t) = P_{Ht}Y_{Ht} + P_{Ft}Y_{Ft}$, as ensured by expenditure minimization (A4) and market clearing (A11) and the foreign counterpart to (A12), which implies $Y_{Ft} = C_{Ft} + X_{Ft} + Z_{Ft}$. As a result, we can rewrite:

$$\frac{B_{t+1}}{R_t} - B_t = \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - P_{Ft}Y_{Ft} \equiv NX_t, \tag{A26}$$

which yields (11) in the text.

A parallel expression to (A26) for foreign yields:

$$\frac{B^*_{t+1}}{R^*_t} - B^*_t = NX^*_t + \tilde{R}^*_t \frac{D^*_t + N^*_t}{R^*_{t-1}},$$

where $NX^*_t = -\frac{NX_t}{\mathcal{E}_t} = \frac{P_{Ft}}{\mathcal{E}_t}Y_{Ft} - P^*_{Ht}Y^*_{Ht}$ and the last term is the period $t$ realized income (or loss) from the aggregate carry trade position of the financial sector, i.e. the intermediaries and the noise traders combined, transferred lump-sum to the foreign households. Indeed, note that $\frac{D^*_t + N^*_t}{R^*_{t-1}}$ is the dollar exposure of the financial sector from $t-1$ to $t$ and $\tilde{R}^*_t = R^*_{t-1} - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}$ is the realized return at $t$ per one dollar invested in a carry trade at $t-1$.

Using the market clearing condition in the financial sector (15), we have $D^*_t + N^*_t = -B^*_t$, and therefore we can rewrite the foreign country budget constraint as

$$NX^*_t = \frac{B^*_{t+1}}{R^*_t} - B^*_t + \frac{B^*_t}{R^*_{t-1}}\tilde{R}^*_t = \frac{B^*_{t+1}}{R^*_t} - B^*_t \frac{R_{t-1}}{R^*_{t-1}}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t},$$

where the second equality substitutes in the definitions of the carry trade return $\tilde{R}^*_t$ from (13).

Lastly, since the financial sector holds a zero-capital position, this implies a zero-capital position for the home and foreign households combined, that is $\frac{B_{t+1}}{R_t} + \mathcal{E}_t\frac{B^*_{t+1}}{R^*_t} = 0$ at all $t$. Applying this market clearing condition at $t-1$ and $t$ to the foreign budget constraint yields after rearranging:

$$\frac{B_{t+1}}{R_t} - B_t = -\mathcal{E}_t NX^*_t = NX_t,$$

which is exactly equivalent to the home country budget constraint (11). Note that this represents a version of *Walras Law* in our economy with the financial sector, making the foreign budget constraint a redundant equation in the equilibrium system.

## A.4 Financial sector

This appendix proves the results in Section 2.2; we make use of equations (12)–(16) displayed in the text.

**Proof of Lemma 1** The proof of the lemma follows two steps. First, it characterizes the solution to the portfolio problem (14) of the intermediaries. Second, it combines this solution with the financial market clearing (15) to derive the equilibrium condition (16).

**(a) Portfolio choice**: *The solution to the portfolio choice problem (14) when the time periods are short is given by:*

$$\frac{d^*_{t+1}}{P^*_t} = -\frac{i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2}, \tag{A27}$$

*where* $i_t - i^*_t \equiv \log(R_t/R^*_t)$, $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ *and* $\sigma_{e\pi^*} = \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$.

**Proof:** The proof follows Campbell and Viceira (2002, Chapter 3 and Appendix 2.1.1). Consider the objective in the intermediary problem (14) and rewrite it as:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left( -\omega\big(1 - e^{x^*_{t+1}}\big)e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t} \right) \right\}, \tag{A28}$$

where we used the definition of $\tilde{R}^*_{t+1}$ in (13) and the following algebraic manipulation:

$$\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}}\frac{d^*_{t+1}}{R^*_t} = \frac{\tilde{R}^*_{t+1}/R^*_t}{P^*_{t+1}/P^*_t}\frac{d^*_{t+1}}{P^*_t} = \frac{1 - \frac{R_{t+1}}{R^*_t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}}{e^{\pi^*_{t+1}}}\frac{d^*_{t+1}}{P^*_t} = \left(1 - e^{x^*_{t+1}}\right)e^{-\pi^*_{t+1}}\frac{d^*_{t+1}}{P^*_t}$$

and defined the log Carry trade return and foreign inflation rate as

$$x^*_{t+1} \equiv i_t - i^*_t - \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta\log\mathcal{E}_{t+1} \qquad \text{and} \qquad \pi^*_{t+1} \equiv \Delta\log P^*_{t+1}.$$

When time periods are short, $(x^*_{t+1}, \pi^*_{t+1})$ correspond to the increments of a vector normal diffusion process $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ with time-varying drift $\boldsymbol{\mu}_t$ and time-invariant conditional variance matrix $\boldsymbol{\sigma}$:

$$\begin{pmatrix} \mathrm{d}\mathcal{X}^*_t \\ \mathrm{d}\mathcal{P}^*_t \end{pmatrix} = \boldsymbol{\mu}_t\mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}B_t, \tag{A29}$$

where $B_t$ is a standard two-dimensional Brownian motion. Indeed, as we show below, in equilibrium $x^*_{t+1}$ and $\pi^*_{t+1}$ follow stationary linear stochastic processes (ARMAs) with correlated innovations, and therefore

$$(x^*_{t+1}, \pi^*_{t+1})\,\big|\,\mathcal{I}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\sigma}^2),$$

where $\mathcal{I}_t$ is the information set at time $t$, and the drift and variance matrix are given by:

$$\boldsymbol{\mu}_t = \mathbb{E}_t\begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1} \\ \mathbb{E}_t\pi^*_{t+1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\sigma}^2 = \mathrm{var}_t\begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} \sigma_e^2 & -\sigma_{e\pi^*} \\ -\sigma_{e\pi^*} & \sigma^2_{\pi^*} \end{pmatrix},$$

where $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$, $\sigma^2_{\pi^*} \equiv \mathrm{var}_t(\Delta p^*_{t+1})$ and $\sigma_{e\pi^*} \equiv \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$ are time-invariant (annualized) conditional second moments. Following Campbell and Viceira (2002), we treat $(x^*_{t+1}, \pi^*_{t+1})$ as discrete-interval differences of the continuous process, $(\mathcal{X}^*_{t+1} - \mathcal{X}^*_t, \mathcal{P}^*_{t+1} - \mathcal{P}^*_t)$.

With short time periods, the solution to (A28) is equivalent to

$$\max_{d^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left( -\omega\big(1 - e^{\mathrm{d}\mathcal{X}^*_t}\big)e^{-\mathrm{d}\mathcal{P}^*_t} \frac{d^*}{P^*_t} \right) \right\}, \tag{A30}$$

where $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ follow (A29). Using Ito's Lemma, we rewrite the objective as:

$$
\begin{aligned}
&\mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left( -\omega\Big( -\mathrm{d}\mathcal{X}_t^* - \tfrac{1}{2}(\mathrm{d}\mathcal{X}_t^*)^2\Big)\Big(1 - \mathrm{d}\mathcal{P}_t^* + \tfrac{1}{2}(\mathrm{d}\mathcal{P}_t^*)^2\Big)\frac{d^*}{P_t^*}\right)\right\} \\
&= \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left( -\omega\Big( -\mathrm{d}\mathcal{X}_t^* - \tfrac{1}{2}(\mathrm{d}\mathcal{X}_t^*)^2 + \mathrm{d}\mathcal{X}_t^*\mathrm{d}\mathcal{P}_t^*\Big)\frac{d^*}{P_t^*}\right)\right\} \\
&= -\frac{1}{\omega} \exp\left( \left[ \omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}\big)\frac{d^*}{P_t^*} + \frac{\omega^2\sigma_e^2}{2}\left(\frac{d^*}{P_t^*}\right)^2\right]\mathrm{d}t\right),
\end{aligned}
$$

where the last line uses the facts that $(\mathrm{d}\mathcal{X}_t^*)^2 = \sigma_e^2\mathrm{d}t$ and $\mathrm{d}\mathcal{X}_t^*\mathrm{d}\mathcal{P}_t^* = -\sigma_{e\pi^*}\mathrm{d}t$, as well as the property of the expectation of an exponent of a normally distributed random variable; $\mu_{1,t}$ denotes the first component of the drift vector $\boldsymbol{\mu}_t$. Therefore, maximization in (A30) is equivalent to:

$$
\max_{d^*}\left\{ -\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}\big)\frac{d^*}{P_t^*} - \tfrac{1}{2}\omega^2\sigma_e^2\left(\frac{d^*}{P_t^*}\right)^2\right\} \quad \text{w/solution} \quad \frac{d^*}{P_t^*} = -\frac{\mu_{1,t} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2}.
$$

This is the portfolio choice equation (A27), which obtains under CARA utility in the limit of short time periods, but note is also equivalent to the exact solution under mean-variance preferences. The extra terms in the numerator correspond to Jensen's Inequality corrections to the expected real log return on the carry trade. ■

**(b) Equilibrium condition**: To derive the modified UIP condition (16), we combine the portfolio choice solution (A27) with the market clearing condition (15) and the noise-trader currency demand (12) to obtain:

$$
B_{t+1}^* + R_t^* n\big(e^{\psi_t} - 1\big) - mP_t^*\frac{i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2} = 0. \tag{A31}
$$

The market clearing conditions in (15) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1}+N_{t+1}}{R_t} = -\mathcal{E}_t\frac{D_{t+1}^*+N_{t+1}^*}{R_t^*}$, results in the equilibrium balance between home and foreign household asset positions, $\frac{B_{t+1}}{R_t} = -\mathcal{E}_t\frac{B_{t+1}^*}{R_t^*}$. Therefore, we can rewrite (A31) as:

$$
\frac{i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2/m} = \frac{R_t^*}{P_t^*}n\big(e^{\psi_t} - 1\big) - \frac{R_t^*}{R_t}\frac{Y_t}{\mathcal{Q}_t}\frac{B_{t+1}}{P_tY_t},
$$

where we normalized net foreign assets by nominal output $P_tY_t$ and used the definition of the real exchange rate $\mathcal{Q}_t$. We next log-linearize this equilibrium condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{B} = \bar{B}^* = 0$, $\mathcal{Q} = 1$, and $\bar{P} = \bar{P}^* = 1$ and some $\bar{Y}$. As shocks become small, the (co)variances $\sigma_e^2$ and $\sigma_{e\pi^*}$ become second order and drop out from the log-linearization. We adopt the asymptotics in which as $\sigma_e^2$ shrinks $\omega/m$ increases proportionally leaving the risk premium term $\omega\sigma_e^2/m$ constant, finite and nonzero in the limit. As a result, the log-linearized equilibrium condition is:

$$
\frac{1}{\omega\sigma_e^2/m}\Big(i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1}\Big) = \frac{n}{\beta}\psi_t - \bar{Y}b_{t+1}, \tag{A32}
$$

where $b_{t+1} = \frac{1}{\bar{P}\bar{Y}}B_{t+1} = -\frac{1}{\bar{P}\bar{Y}}B_{t+1}^*$. This corresponds to the modified UIP condition (16) in Lemma 1, which completes the proof of the lemma.[44] ■

[44] Note that $\sigma_e^2/m$ is the quantity of risk per intermediary and $\omega$ is their aversion to risk; alternatively, $\omega/m$ can be viewed as the effective risk aversion of the whole sector of intermediaries who jointly hold all exchange rate risk. Our approach follows Hansen and Sargent (2011) and Hansen and Miao (2018), who consider the continuous-time limit in the models with ambiguity aversion. The economic rationale of this asymptotics is not that second moments are zero and effective risk aversion $\omega/m$ is infinite, but rather that risk premia terms, which are proportional to $\omega\sigma_e^2/m$, are finite and nonzero. Indeed, the first-order dynamics of the equilibrium system results in well-defined second moments of the variables, including $\sigma_e^2$, as in Devereux and Sutherland (2011) and Tille and van Wincoop (2010); an important difference of our solution concept is that it allows for

**Income and losses in the financial market** Consider the income and losses of the non-household participants in the financial market — the intermediaries and the noise traders:

$$\frac{D_{t+1}^{*}+N_{t+1}^{*}}{R_{t}^{*}}\tilde{R}_{t+1}^{*}=\big(md_{t+1}^{*}+R_{t}^{*}n(e^{\psi_t}-1)\big)\big(1-e^{x_{t+1}}\big),$$

where we used the definition of $\tilde{R}_{t+1}^{*}$ in (13) and the log Carry trade return $x_{t+1}\equiv i_t-i_t^{*}-\mathbb{E}_t\Delta e_{t+1}=\log(R_t/R_t^{*})-\Delta\log\mathcal{E}_{t+1}$. Using the same steps as in the proof of Lemma 1, we can approximate this income as:

$$\left(-m\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2}+\frac{n}{\beta}\psi_t\right)\big(-x_{t+1}\big)=m\left(\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2}-\frac{n}{\beta m}\psi_t\right)x_{t+1}=-\bar{Y}b_{t+1}x_{t+1},$$

where the last equality uses (A32). Therefore, while the UIP deviations (realized $x_{t+1}$ and expected $\mathbb{E}_t x_{t+1}$) are first order, the income and losses in the financial markets are only second order, as $B_{t+1}=\bar{P}\bar{Y}b_{t+1}$ is first order around $\bar{B}=0$. Intuitively, the income and losses in the financial market are equal to the realized UIP deviation times the gross portfolio position — while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

**Covered interest parity** Consider the extension of the portfolio choice problem (14) of the intermediaries with the additional option to invest in the CIP deviations:

$$\max_{d_{t+1}^{*},d_{t+1}^{F*}}\ \mathbb{E}_t\left\{-\frac{1}{\omega}\exp\left(-\omega\left[\frac{\tilde{R}_{t+1}^{*}}{P_{t+1}^{*}}\frac{d_{t+1}^{*}}{R_t^{*}}+\frac{R_t^{F*}}{P_{t+1}^{*}}\frac{d_{t+1}^{F*}}{R_t^{*}}+\frac{R_t^{*}}{P_{t+1}^{*}}\mathcal{W}_t^{*}\right]\right)\right\},$$

where the return on one dollar invested in the CIP deviation (long foreign-currency bond, short foreign-currency bond, plus a forward) is:

$$R_t^{F*}=R_t^{*}-\frac{\mathcal{E}_t}{\mathcal{F}_t}R_t,$$

since 1 dollar at $t$ buys $R_t^{*}$ units of foreign-currency bonds and $\mathcal{E}_tR_t$ units of home-currency bonds, and hence $\frac{d_{t+1}^{F*}}{R_t^{*}}\gtrless 0$ is the period-$t$ dollar size of this position. Note that we also allowed for nonzero dollar wealth $\mathcal{W}_t^{*}$ of the intermediaries, which is by default invested into the 'riskless' foreign-currency bond. Both CIP investment and wealth investment are subject to the foreign inflation risk only, but no risk of nominal return, unlike the carry trade $d_{t+1}^{*}$. Note that the CIP investment, just like the carry trade, requires no capital at time $t$. Lastly, note that intermediaries may be pricing the forward without trading it, or trading it with the noise traders; as long as the households have access to the home-currency bond only, and not the forward, this does not change the macro equilibrium outcomes of the model.

The first order optimality condition of the intermediaries with respect to the CIP investment is:

$$R_t^{F*}\cdot\mathbb{E}_t\left\{\frac{1}{P_{t+1}^{*}R_t^{*}}\exp\left(-\omega\left[\frac{\tilde{R}_{t+1}^{*}}{P_{t+1}^{*}}\frac{d_{t+1}^{*}}{R_t^{*}}+\frac{R_t^{F*}}{P_{t+1}^{*}}\frac{d_{t+1}^{F*}}{R_t^{*}}+\frac{R_t^{*}}{P_{t+1}^{*}}\mathcal{W}_t^{*}\right]\right)\right\}=0.$$

However, since the expectation term is strictly positive for any $d_{t+1}^{F*}\in(-\infty,\infty)$, this condition can be satisfied only if $R_t^{F*}=0$. If $R_t^{F*}>0$, the intermediaries will take an unbounded position in the CIP trade, $d_{t+1}^{F*}=\infty$, and vice versa.

---

a non-zero first-order component of the return differential, namely a non-zero expected Carry trade return. We characterize the equilibrium $\sigma_e^2$ in Appendix A.5.

## A.5 Equilibrium system

We summarize here the equilibrium system of the full flexible-price model by breaking it into blocks. The version of the model with sticky prices and wages is described in Appendix A.7.

1. **Labor market**: Labor supply (2) and its exact foreign counterpart. Labor demand in (A24), used together with the definition of the marginal cost (A8), and its exact foreign counterparts. Labor market clearing ensures that $L_t$ ($L_t^*$ respectively) satisfies simultaneously labor demand and labor supply at the equilibrium wage rate $W_t$ ($W_t^*$ respectively).
2. **Capital market**: Euler equation for capital (A16) determines supply of capital and the firm capital demand is given by the first-order condition:
$$R_t^K K_t = (1-\phi)\vartheta MC_t Y_t, \tag{A33}$$
where marginal cost $MC_t$ is defined in (A8). The equilibrium rental rate of capital $R_t^K$ ensures that $K_t$ satisfies simultaneously the demand and supply of capital. Identical equations characterize equilibrium in the foreign capital market. The home gross investment $Z_t$ obtains from the capital dynamics equation, which we rewrite here as:
$$Z_t = \Big[K_{t+1} - (1-\delta)K_t\Big] + \frac{\kappa}{2}\frac{(\Delta K_{t+1})^2}{K_t}, \tag{A34}$$
where the first term is net investment and the second term is adjustment cost. The foreign investment $Z_t^*$ satisfies a symmetric equation.
3. **Goods prices**: Price setting is characterized by (A20) and (A21) for home firms in the two markets, and symmetric equations characterize price setting by foreign firms. The price indexes $P_t$ and $\mathcal{P}_t$ are defined implicitly by (A19)–(A18) respectively. As a result, equilibrium prices of all varieties supplied from a given country to a given market are the same: $P_{Jt}(i) = P_{Jt}$ and $P_{Jt}^*(i) = P_{Jt}^*$ for all $i \in [0,1]$ and $J \in \{H, F\}$.
4. **Goods market**: As a result of price setting, the quantities supplied by all firms from a given country to a given market are also the same: $Y_{Jt}(i) = Y_{Jt}$ and $Y_{Jt}^*(i) = Y_{Jt}^*$ for all $i \in [0,1]$ and $J \in \{H, F\}$. The total demand for home and foreign goods satisfies:
$$Y_t = Y_{Ht} + Y_{Ht}^* \qquad \text{and} \qquad Y_t^* = Y_{Ft} + Y_{Ft}^*, \tag{A35}$$
where the sources of demand for home goods are given in (A11) and (A12), and the counterpart sources of demand for foreign goods are given by:
$$Y_{Ft} = C_{Ft} + X_{Ft} + Z_{Ft} = \gamma h\left(\frac{P_{Ft}}{\mathcal{P}_t}\right)\big[C_t + X_t + Z_t\big], \tag{A36}$$
$$Y_{Ft}^* = C_{Ft}^* + X_{Ft}^* + Z_{Ft}^* = \gamma h\left(\frac{P_{Ft}^*}{\mathcal{P}_t^*}\right)\big[C_t^* + X_t^* + Z_t^*\big], \tag{A37}$$
where $X_t$ is the intermediate good demand by the home firms:
$$P_t X_t = \phi MC_t Y_t, \tag{A38}$$
with $MC_t$ defined in (A8), and a symmetric equation characterizes the intermediate good demand by the foreign firms $X_t^*$. The total supply (production) of the home goods $Y_t$ satisfies the production function (A7), with log productivity $a_t$ that follows an exogenous shock process (A9).[45] A symmetric equation and foreign productivity process $a_t^*$ characterize foreign production $Y_t^*$.
5. **Asset market**: The only traded assets are home- and foreign-currency bonds, which are in zero net supply according to market clearing (15). The demand for home-currency bonds by home households $B_{t+1}$ satisfies the Euler equation (3) given the nominal interest rate $R_t$. Similarly, the demand for foreign-currency bonds by foreign households $B_{t+1}^*$ satisfies a symmetric Euler equation given foreign nominal

[45]Note that the input demand equations (A24), (A33) and (A38) together with the definition of the marginal cost (A8) imply the production function in (A7).

interest rate $R_t^*$. The demand for bonds by noise traders and arbitrageurs are characterized by Lemma 1 respectively. The noise trader shock follows an exogenous process (12). No other assets are traded.

Nominal interest rates are set by the monetary authorities according to the Taylor rule (A15) — where $i_t \equiv \log R_t$, $\pi_t \equiv \Delta \log P_t$ and an exogenous random shock $\varepsilon_t^m$ — and its foreign counterpart.

6. **Country budget constraint**: The home-country flow budget constraint (A26) derives from the combination of the household budget constraint and firm profits. The flow budget constraint (A26), together with the household Euler equation (3) and its foreign counterpart, establishes a condition on the path of consumption and nominal exchange rate, $\{C_t, C_t^*, \mathcal{E}_t\}$. The foreign flow-budget constraint is redundant by Walras Law (see Appendix A.3).

**Symmetric steady state** In a symmetric steady state, exogenous shocks $a_t = a_t^* = \varepsilon_t^m = \varepsilon_t^{m*} = \psi_t \equiv 0$, and state variables $\bar{B} = \bar{B}^* = \bar{NX} = 0$. This is the unique steady state in a model with $\chi_2 > 0$ in (16), which also ensures stationarity of the model around this steady state. We also for concreteness normalize $\bar{P} = \bar{P}^* = 1$. Then, from Euler equations (3) and (A16) and their foreign counterparts, we have:

$$\bar{R} = \bar{R}^* = \bar{R}^K + 1 - \delta = \bar{R}^{K*} + 1 - \delta = \frac{1}{\beta}.$$

By symmetry, the exchange rates and terms of trade satisfy

$$\bar{\mathcal{E}} = \bar{\mathcal{Q}} = \bar{\mathcal{S}} = 1,$$

and all individual prices are equal 1 (the price level). Denote the steady state markup with $\bar{\mu} \geq 1$, so that the steady state marginal costs $\bar{MC} = \bar{MC}^* = 1/\bar{\mu}$, which allows to solve for $\bar{W} = \bar{W}^*$ given $\bar{R} = 1/\beta$ and $\bar{P} = 1$ from (A8) as a function of model parameters.

Next, product and factor market clearing in a symmetric steady state requires:

$$\begin{aligned} \bar{Y} &= \bar{C} + \bar{X} + \delta \bar{K}, \\ \bar{Y} &= \left(\bar{K}^{\vartheta} \bar{L}^{1-\vartheta}\right)^{1-\phi} \bar{X}^{\phi}, \end{aligned}$$

$$\begin{aligned} \bar{X} &= \tfrac{\phi}{\bar{\mu}} \bar{Y}, \\ \left(\frac{1-\beta}{\beta} + \delta\right) \bar{K} &= \tfrac{(1-\phi)\vartheta}{\bar{\mu}} \bar{Y}, \\ \bar{C}^{\sigma} \bar{L}^{1/\nu} &= \tfrac{(1-\phi)(1-\vartheta)}{\bar{\mu}} \frac{\bar{Y}}{\bar{L}}, \end{aligned}$$

since $\bar{Z} = \delta \bar{K}$. These equations allow to solve for $(\bar{Y}, \bar{C}, \bar{L}, \bar{K}, \bar{X})$ and their symmetric foreign counterparts as a function of the model parameters.

Lastly, we define the following useful ratios in a symmetric steady state:

$$\zeta \equiv \frac{\text{GDP}}{\text{Output}} = \frac{P(C+Z)}{PY} = \frac{\bar{C} + \delta \bar{K}}{\bar{Y}} = \frac{\bar{Y} - \bar{X}}{\bar{Y}} = 1 - \frac{\phi}{\bar{\mu}}, \tag{A39}$$

$$\gamma \equiv \frac{\text{Import}}{\text{Expenditure}} = \frac{P_F Y_F}{P_H Y_H + P_F Y_F} = \frac{\bar{Y}_F}{\bar{Y}} = \gamma, \tag{A40}$$

$$\frac{\text{Import+Export}}{\text{GDP}} = \frac{\mathcal{E} P_H^* Y_H^* + P_F Y_F}{P(C+Z)} = \frac{2\bar{Y}_F}{\bar{Y} - \bar{X}} = \frac{2\gamma}{\zeta}. \tag{A41}$$

The steady state markup is $\bar{\mu} = \frac{\theta}{\theta-1}(1-\varsigma)$, where $\varsigma$ is the subsidy that offsets the markup distortion, conventional in the normative macro literature. To avoid the need to calibrate an extra parameter, we assume $\varsigma = 1/\theta$, so that $\bar{\mu} = 1$ and $\zeta = 1 - \phi$, or in words the share of intermediates in output equals the elasticity of the production function with respect to intermediate inputs. The qualitative and quantitative results in our analysis are not sensitive to the departures from $\zeta = 1 - \phi$.

## Log-linearized system

We describe here the log-linearized equilibrium system in the model without capital or nominal rigidities and in the limiting case of monetary policy fully stabilizing the price levels, but allowing for Kimball demand and intermediate inputs. The simplified model studied in Sections 2–3 of the paper is the special case with $\alpha = \phi = 0$ (and recall that $\nu \equiv 1/\varphi$). We log-linearize the equilibrium system around the symmetric steady state. We take advantage of the block-recursive structure of the equilibrium system, and characterize the solution in blocks.

**Exchange rates and prices** The price block contains definitions of the price index at home and abroad:

$$p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}, \tag{A42}$$
$$p_t^* = \gamma p_{Ht}^* + (1-\gamma)p_{Ft}^*, \tag{A43}$$

as well as the price setting equations:

$$p_{Ht} = (1-\alpha)(1-\phi)(w_t - p_t - a_t) + p_t, \tag{A44}$$
$$p_{Ht}^* = (1-\alpha)[(1-\phi)(w_t - p_t - a_t) + p_t - e_t] + \alpha p_t^*, \tag{A45}$$
$$p_{Ft}^* = (1-\alpha)(1-\phi)(w_t^* - p_t^* - a_t^*) + p_t^*, \tag{A46}$$
$$p_{Ft} = (1-\alpha)[(1-\phi)(w_t^* - p_t^* - a_t^*) + p_t^* + e_t] + \alpha p_t. \tag{A47}$$

Note that small letters denote log-deviations from steady state, and therefore constant terms drop out from equations (A44)–(A47). In addition, we use the logs of the real exchange rate (RER) and the terms of trade (ToT):

$$q_t = p_t^* + e_t - p_t, \tag{A48}$$
$$s_t = p_{Ft} - p_{Ht}^* - e_t, \tag{A49}$$

as well as the wage-based and PPI-based real exchange rates:

$$q_t^W = w_t^* + e_t - w_t, \tag{A50}$$
$$q_t^P = p_{Ft}^* + e_t - p_{Ht}. \tag{A51}$$

We solve (A42)–(A51) for equilibrium prices and exchange rates. In particular, we have:

$$s_t = q_t^P - 2\alpha q_t \qquad \text{and} \qquad q_t = (1-\gamma)q_t^P - \gamma s_t,$$

where $\alpha q_t$ equals the equilibrium LOP deviation for both home- and foreign-produced goods:

$$\alpha q_t = p_{Ht}^* + e_t - p_{Ht} = p_{Ft}^* + e_t - p_{Ft},$$

as follows from (A44)–(A47). Intuitively, ToT equals PPI-RER adjusted for LOP deviations; and CPI-RER equals PPI-RER adjusted for ToT. Using these relationships, we solve for $s_t$ and $q_t^P$ as a function of $q_t$:

$$s_t = \frac{1-2\alpha(1-\gamma)}{1-2\gamma} q_t, \tag{A52}$$
$$q_t^P = \left[1 + (1-\alpha)\frac{2\gamma}{1-2\gamma}\right] q_t. \tag{A53}$$

Finally, we combine (A42)–(A47) to derive the relationship between $q_t^W$ and $q_t$:

$$q_t^W = \left[1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right] q_t - (a_t - a_t^*), \tag{A54}$$

and in addition we have the expressions for the equilibrium real wages:

$$w_t - p_t = a_t - \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma} q_t \qquad \text{and} \qquad w_t^* - p_t^* = a_t^* + \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma} q_t. \tag{A55}$$

Intuitively, the real wage reflects the country productivity level adjusted by the international purchasing power of the country, which is proportional to the strength of its RER.

**Real exchange rate and quantities** The labor supply (2) and labor demand (A24) equations (together with the marginal cost (A8)) can be written as:

$$\sigma c_t + \tfrac{1}{\nu}\ell_t = w_t - p_t,$$
$$\ell_t = -(1-\phi)a_t - \phi(w_t - p_t) + y_t.$$

Combining the two to solve out $\ell_t$, and using (A55) to solve out $(w_t - p_t)$, we obtain:

$$\nu\sigma c_t + y_t = (1+\nu)a_t - \tfrac{\nu+\phi}{1-\phi}\tfrac{\gamma}{1-2\gamma}q_t.$$

Subtracting a symmetric equation for foreign yields:

$$\nu\sigma\tilde{c}_t + \tilde{y}_t = (1+\nu)\tilde{a}_t - \tfrac{\nu+\phi}{1-\phi}\tfrac{2\gamma}{1-2\gamma}q_t, \tag{A56}$$

where $\tilde{x}_t \equiv x_t - x_t^*$ for any pair of variables $(x_t, x_t^*)$. This characterizes the supply side.

The demand side is the goods market clearing (A35) together with (A11)–(A12), which log-linearize as:

$$y_t = (1-\gamma)y_{Ht} + \gamma y_{Ht}^*,$$
$$y_{Ht} = -\theta(p_{Ht} - p_t) + (1-\phi)c_t + \phi\big[(1-\phi)(w_t - p_t - a_t) + y_t\big],$$
$$y_{Ht}^* = -\theta(p_{Ht}^* - p_t^*) + (1-\phi)c_t^* + \phi\big[(1-\phi)(w_t^* - p_t^* - a_t^*) + y_t^*\big],$$

where $\phi = 1-\zeta \equiv \bar{X}/\bar{Y}$, and we used expression (A38) and (A8) to substitute for $X_t$ (and correspondingly for $X_t^*$). Combining together, we derive:

$$y_t = \phi[y_t - \gamma\tilde{y}_t] + (1-\phi)[c_t - \gamma\tilde{c}_t] + \gamma\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi\right]q_t$$

where we have solved out $(p_{Ht} - p_t)$ and $(p_{Ht}^* - p_t^*)$ using (A44)–(A47) and $(w_t - p_t - a_t)$ and $(w_t^* - p_t^* - a_t^*)$ using (A55). Adding and subtracting the foreign counterpart, we obtain:

$$[1-(1-2\gamma)\phi]\tilde{y}_t = (1-2\gamma)(1-\phi)\tilde{c}_t + 2\gamma\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi\right]q_t. \tag{A57}$$

Combining (A56) and (A57) we can solve for $\tilde{y}_t$ and $\tilde{c}_t$. For example, the expression for $\tilde{c}_t$ is:

$$\tilde{c}_t = \kappa_a\tilde{a}_t - \gamma\kappa_q q_t, \qquad \text{where} \tag{A58}$$

$$\kappa_a \equiv \frac{(1+\nu)(1+\varkappa)}{1+\nu\sigma(1+\varkappa)} \quad\text{and}\quad \kappa_q \equiv \frac{\varkappa}{\gamma}\frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu(1+\varkappa) + \phi\varkappa}{1+\nu\sigma(1+\varkappa)}, \quad\text{with}\quad \varkappa \equiv \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}.$$

Note that $\kappa_a, \kappa_q > 0$ independently of the values of the parameters, and in the autarky limit as $\gamma \to 0$ we have $\kappa_a \to \frac{1+\nu}{1+\nu\sigma}$ and $\kappa_q \to \frac{2}{1-\phi}\frac{2\theta(1-\alpha)+\nu}{1+\nu\sigma}$, since $\frac{\varkappa}{\gamma} \to \frac{2}{1-\phi}$. Therefore, $(\kappa_a, \kappa_q)$ are positive derived parameters separated from zero even as $\gamma \to 0$.

Lastly, we log-linearize the flow budget constraint (11) as:

$$\beta b_{t+1} - b_t = nx_t = \gamma\Big(y_{Ht}^* - y_{Ft} - s_t\Big), \tag{A59}$$

where $\beta = 1/\bar{R}$ and since $\bar{B} = \bar{NX} = 0$ in a symmetric steady state, we define $b_{t+1} = B_{t+1}/\bar{Y}$ and $nx_t = NX_t/\bar{Y}$, so that $\gamma$ represents the steady-state share of imports (and also exports) in output, which is the relevant coefficient in the log-linearization (A59). Next we use the expression for export quantity $y_{Ht}^*$ above and a symmetric counterpart for $y_{Ft}$, together with the solution for prices and quantities, to derive:[46]

$$nx_t = \gamma\big[\lambda_q q_t - \lambda_a \tilde{a}_t\big], \tag{A60}$$

where the second equality substitutes in the solution for $(\tilde{y}_t, \tilde{c}_t)$ from (A56)–(A57), and we define:

$$\lambda_a \equiv \frac{1}{1-2\gamma}\frac{1+\nu}{1+\nu\sigma(1+\varkappa)},$$
$$\lambda_q \equiv \frac{1}{1-2\gamma}\left(\frac{1+\nu\sigma}{1+\nu\sigma(1+\varkappa)}\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}+\phi\varkappa+\frac{\nu\varkappa}{1+\nu\sigma}\right]-[1-2\alpha(1-\gamma)]\right).$$

Note that $\lambda_a \equiv \frac{1}{1-2\gamma}\frac{\kappa_a}{1+\varkappa} > 0$, and as $\gamma \to 0$ we have $\lambda_a - \kappa_a \to 0$. Furthermore, $\lambda_q > 0$ is equivalent to the generalized *Marshall-Lerner condition* in our general-equilibrium model, and $\theta > 1$ is sufficient to ensure this independently of the values of other parameters $(\gamma, \alpha, \phi, \nu, \sigma)$. In the limit $\gamma \to 0$, we have $\lambda_q \to 1+2(\theta-1)(1-\alpha)$, which is in general different from $\kappa_q$.

**Exchange rate and interest rates** We log-linearize the household Euler equation (3) and its foreign counterpart:

$$i_t = \mathbb{E}_t\left\{\sigma\Delta c_{t+1} + \Delta p_{t+1}\right\} \quad \text{and} \quad i_t^* = \mathbb{E}_t\left\{\sigma\Delta c_{t+1}^* + \Delta p_{t+1}^*\right\},$$

where $i_t \equiv \log R_t – \log \bar{R}$ and similarly for $i_t^*$. Taking the difference, we can express the interest rate differential as:

$$i_t - i_t^* = \mathbb{E}_t\left\{\sigma\Delta\tilde{c}_{t+1} + \Delta\tilde{p}_{t+1}\right\}, \tag{A61}$$

Subtracting the expected inflation differential, $\mathbb{E}_t\Delta\tilde{p}_{t+1}$, on both sides allows to characterize the equilibrium real interest rate differential as

$$i_t - i_t^* - \mathbb{E}_t\Delta\tilde{p}_{t+1} = \sigma\mathbb{E}_t\Delta\tilde{c}_{t+1} = \sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1} - \gamma\sigma\kappa_q\mathbb{E}_t\Delta q_{t+1}, \tag{A62}$$

where we substituted the solution for $\Delta\tilde{c}_{t+1}$ from (A58). To solve for the equilibrium nominal interest rate differential we combine (A61) with the Taylor rule (A15) and its foreign counterpart. Since $\Delta\tilde{p}_{t+1} \equiv 0$, the nominal interest rate (differential) tracks the real interest rate (differential), equal $\sigma$ times the expected (relative) consumption growth.

Next, subtracting $\mathbb{E}_t\Delta e_{t+1}$ on both sides of (A61) and combining with the modified UIP equation (16), we have:

$$\mathbb{E}_t\left\{\sigma\Delta\tilde{c}_{t+1} - \Delta q_{t+1}\right\} = i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \chi_1\psi_t - \chi_2 b_{t+1}, \tag{A63}$$

which amounts to the international risk-sharing condition in this economy. Combining (A63) with the solution for the equilibrium consumption differential (A58), we arrive at the condition for the expected change in the real

[46] This is a rather tedious deviations, which relies on the previous equilibrium relationships. We start with:

$$\begin{aligned} y_{Ht}^* - y_{Ft} - s_t &= -\theta(p_{Ht}^* - p^*) + \theta(p_{Ft} - p) - \phi(1-\phi)(\tilde{w}_t - \tilde{p}_t - \tilde{a}_t) - [\phi\tilde{y}_t + (1-\phi)\tilde{c}_t] - s_t \\ &= -[\theta(1-\alpha)+\phi](1-\phi)(\tilde{w}_t - \tilde{p}_t - \tilde{a}_t) + 2\theta(1-\alpha)q_t - \tfrac{1-2\alpha(1-\gamma)}{1-2\gamma}q_t - [\phi\tilde{y}_t + (1-\phi)\tilde{c}_t] \\ &= \left([\theta(1-\alpha)+\phi]\frac{2\gamma}{1-2\gamma} + 2\theta(1-\alpha) - \tfrac{1-2\alpha(1-\gamma)}{1-2\gamma}\right)q_t - [\phi\tilde{y}_t + (1-\phi)\tilde{c}_t], \end{aligned}$$

where the first line substitutes the expressions for $y_{Ht}^*$ and $y_{Ft}$, the second equality uses (A45), (A47) and (A52), and the third equality uses (A55). Next we use (A57) to solve for:

$$\begin{aligned} \phi\tilde{y}_t + (1-\phi)\tilde{c}_t &= \frac{1}{1-2\gamma}\tilde{y}_t - \frac{2\gamma}{1-2\gamma}\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi\right]q_t \\ &= \frac{1}{1-2\gamma}\frac{1+\nu}{1+\nu\sigma(1+\varkappa)}\tilde{a}_t - \tfrac{2\gamma}{1-2\gamma}\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi + \tfrac{1}{1-2\gamma}\tfrac{\nu+\phi}{1-\phi} - \frac{\nu\sigma\kappa_q}{2}\right]q_t, \end{aligned}$$

where the second line uses (A56) to solve out $\tilde{y}_t$ and then (A58) to solve out $\tilde{c}_t$. Finally, substituting in the expression for $\kappa_q$ from (A58) and rearranging terms yields the resulting (A60).

exchange rate:

$$(1+\gamma\sigma\kappa_q)\mathbb{E}_t\Delta q_{t+1} = \chi_2 b_{t+1} - \chi_1\psi_t + \sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1}. \tag{A64}$$

Substituting this into (A62) yields the solution for the interest rate differential:

$$i_t - i_t^* - \mathbb{E}_t\Delta\tilde{p}_{t+1} = -\frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}[\chi_2 b_{t+1} - \chi_1\psi_t] + \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\mathbb{E}_t\Delta\tilde{a}_{t+1}. \tag{A65}$$

## A.6 Derivation of the analytical results in Section 3

**Equilibrium exchange rate dynamics (Lemma 3)** We first characterize the generalized version of the exchange rate dynamics in (30) in corresponding to the modified UIP condition (16) with endogenous coefficients $\chi_1$ and $\chi_2$. We combine (A64) with (A59)–(A60) and the exogenous shock processes (6) and (12) assuming for concreteness $\rho_a = \rho_\psi = \rho$ (and analogous derivations apply in the general case with $\rho_a \neq \rho_\psi$). This system corresponds to the generalized versions of (17)—(28) in the text.[47] We write the equilibrium dynamic system in matrix form:

$$\begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbb{E}_t q_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\beta \end{pmatrix}\begin{pmatrix} q_t \\ \hat{b}_t \end{pmatrix} - \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix},$$

where for brevity we made the following substitution of variables:

$$\hat{b}_t \equiv \frac{\beta}{\gamma\lambda_q}b_t, \quad \hat{a}_t \equiv \frac{\lambda_a}{\lambda_q}\tilde{a}_t, \quad \hat{\chi}_1 \equiv \frac{\chi_1}{1+\gamma\sigma\kappa_q}, \quad \hat{\chi}_2 \equiv \frac{\gamma\lambda_q/\beta}{1+\gamma\sigma\kappa_q}\chi_2, \quad k \equiv \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\lambda_q}{\lambda_a}. \tag{A66}$$

Diagonalizing the dynamic system, we have:

$$\mathbb{E}_t z_{t+1} = Bz_t - C\begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix}, \quad \text{where} \quad B \equiv \begin{pmatrix} 1+\hat{\chi}_2 & \hat{\chi}_2/\beta \\ 1 & 1/\beta \end{pmatrix}, \quad C \equiv \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k + \hat{\chi}_2 \\ 0 & 1 \end{pmatrix},$$

and we denoted $z_t \equiv (q_t, \hat{b}_t)'$. The eigenvalues of $B$ are:

$$\mu_{1,2} = \frac{1}{2}\Big[(1+\hat{\chi}_2+1/\beta) \mp \sqrt{(1+\hat{\chi}_2+1/\beta)^2 - 4/\beta}\Big] \qquad \text{such that} \qquad 0 < \mu_1 \le 1 < \frac{1}{\beta} \le \mu_2,$$

and $\mu_1 + \mu_2 = 1 + \hat{\chi}_2 + 1/\beta$ and $\mu_1 \cdot \mu_2 = 1/\beta$. Note that when $\chi_2 = 0$, and hence $\hat{\chi}_2 = 0$, the two roots are simply $\mu_1 = 1$ and $\mu_2 = 1/\beta$.

The left eigenvalue associated with $\mu_2 > 1$ is $v = (1, 1/\beta - \mu_1)$, such that $vB = \mu_2 v$. Therefore, we can pre-multiply the dynamic system by $v$ and rearrange to obtain:

$$vz_t = \frac{1}{\mu_2}\mathbb{E}_t\{vz_{t+1}\} + \frac{1}{\mu_2}\hat{\chi}_1\psi_t + \left[\frac{(1-\rho)k + \hat{\chi}_2}{\mu_2} + \frac{1/\beta - \mu_1}{\mu_2}\right]\hat{a}_t.$$

Using the facts that $\hat{\chi}_2 + 1/\beta - \mu_1 = \mu_2 - 1$ and $1/\mu_2 = \beta\mu_1$, we solve this dynamic equation forward to obtain the equilibrium cointegration relationship:

$$vz_t = q_t + (1/\beta - \mu_1)\hat{b}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{1-\beta\mu_1+\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}\hat{a}_t. \tag{A67}$$

Combining this with the second dynamic equation for $\hat{b}_{t+1}$, we solve for:

$$\hat{b}_{t+1} - \mu_1\hat{b}_t = \overbrace{q_t + \big(\tfrac{1}{\beta} - \mu_1\big)\hat{b}_t}^{=vz_t} - \hat{a}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t, \tag{A68}$$

Note that $\hat{b}_{t+1}$ in (A68) follows a stationary AR(2) with roots $\rho$ and $\mu_1$. Recall that as $\chi_2 \to 0$, $\mu_1 \to 1$, and the

[47] Note that, while monetary policy results in $e_t = q_t$, it does not affect the equilibrium system for RER $q_t$ in the flexible-price version of the model we consider here.

process for $\hat{b}_{t+1}$ becomes an ARIMA(1,1,0), which corresponds to the solution in footnote 26 in the text (after reverse substitution of variables).

Finally, we apply lag operator $(1-\mu_1 L)$ to (A67) and use (A68) to solve for:

$$\begin{aligned}(1-\mu_1 L)q_t &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t\right] + (1-\mu_1 L)\hat{a}_t \\ &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k\hat{a}_t\right] + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\hat{a}_t, \qquad \text{(A69)}\end{aligned}$$

where $L$ is the lag operator such that $Lq_t = q_{t-1}$. Therefore, equilibrium RER $q_t$ follows a stationary ARMA(2,1) with autoregressive roots $\mu_1$ and $\rho$. Again, in the limit $\chi_2 \to 0$, $\mu_1 \to 1$, and this process becomes an ARIMA(1,1,1), which corresponds to (30) in the text.[48] ■

**Equilibrium variance of the exchange rate and Lemma 1** Solution (A69) characterizes the behavior of $q_t$ for given values of $\chi_1$ and $\chi_2$ (and hence $\mu_1$, $\mu_2$), which from (16) themselves depend on $\sigma_e^2 = \mathrm{var}_t(\Delta e_{t+1})$. Since the monetary policy stabilizes inflation, ensuring $e_t = q_t$, we also have $\sigma_e^2 = \mathrm{var}_t(\Delta q_{t+1})$, and we now solve for the equilibrium value of $\sigma_e^2$, and hence $(\chi_1, \chi_2, \mu_1, \mu_2)$.

Using (A69), we calculate $\sigma_e^2 = \mathrm{var}_t(\Delta q_{t+1})$ for given $\chi_1$ and $\chi_2$:

$$\sigma_e^2 = \mathrm{var}_t(\Delta q_{t+1}) = \left(\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\right)^2\sigma_\psi^2 + \left(\frac{\beta(1-\rho)\mu_1 k + (1-\beta\mu_1)}{1-\beta\rho\mu_1}\right)^2\hat{\sigma}_a^2 = \frac{\hat{\chi}_1^2\sigma_\psi^2 + \big((1-\rho)k + (\mu_2-1)\big)^2\hat{\sigma}_a^2}{(\mu_2-\rho)^2},$$

where $\hat{\sigma}_a$ is the standard deviation of the innovation to $\hat{a}_t$, and the second line uses the fact that $\beta\mu_1 = 1/\mu_2$. In addition, recall that:

$$\hat{\chi}_1 = \frac{n/\beta}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m}, \qquad \hat{\chi}_2 \equiv \frac{\gamma\lambda_q\bar{Y}/\beta}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m} \quad \text{and} \quad \mu_2 = \frac{(1+\beta\hat{\chi}_2+\beta) + \sqrt{(1+\beta\hat{\chi}_2+\beta)^2 - 4\beta}}{2\beta}.$$

We therefore can rewrite the fixed point equation for $\sigma_e^2 > 0$ as follows:

$$F(x,\tilde{\omega}) = \big(\mu_2(\tilde{\omega}x) - \rho\big)^2 x - b(\tilde{\omega}x)^2 - c = 0, \qquad \text{(A70)}$$

where we used the following notation:

$$x \equiv \sigma_e^2 \geq 0, \qquad \tilde{\omega} = \frac{\omega}{m}, \qquad b \equiv \left(\frac{n/\beta}{1+\gamma\sigma\kappa_q}\right)^2\sigma_\psi^2, \qquad c \equiv \big((1-\rho)k + (\mu_2-1)\big)^2\hat{\sigma}_a^2 \geq 0,$$

and $\mu_2(\cdot)$ is a function which gives the equilibrium values of $\mu_2$ defined above as a function of $\tilde{\omega}\sigma_e^2$ for given values of the model parameters. Note that for any given $\tilde{\omega} > 0$:

$$\begin{aligned}&\lim_{x\to 0} F(x,\tilde{\omega}) = -c \leq 0, \\ &\lim_{x\to\infty}\frac{F(x,\tilde{\omega})}{x^3} = \lim_{x\to\infty}\left(\frac{\mu_2(\tilde{\omega}x)}{x}\right)^2 = \left(\frac{\beta\hat{\chi}_2^2}{\sigma_e^2}\right) = \left(\frac{\gamma\lambda_q\bar{Y}}{1+\gamma\sigma\kappa_q}\tilde{\omega}\right)^2 > 0.\end{aligned}$$

Therefore, by continuity at least one fixed-point $F(\sigma_e^2,\omega) = 0$ with $\sigma_e^2 \geq 0$ exists, and all such $\sigma_e^2 > 0$ whenever $c > 0$ (that is, when $\hat{\sigma}_a > 0$). One can further show that when $\hat{\sigma}_a/\sigma_\psi$ is not too small, this equilibrium is unique.[49]

[48] Note that we prefer the specification in the second line of (A69) since it separates the exchange rate effects of $\hat{a}_t$ via the budget constraint (latter term) and via the modified UIP condition (the former term with factor $k$ in front of $\hat{a}_t$). In the limit of $\rho \to 1$, the effect through the UIP condition vanishes, while the effect through the budget constraint results in a random walk response of $q_t$ to $\hat{a}_t$.

[49] For $\hat{\sigma}_a/\sigma_\psi \approx 0$, there typically exist three equilibria. In particular, when $\hat{\sigma}_a = 0$, there always exists an equilibrium with $\sigma_e^2 = \chi_1 = 0$, in addition to two other potential equilibria with $\sigma_e^2 > 0$, which exist when $\sigma_\psi$ is not too small (see Itskhoki and Mukhin 2017).

Finally, we consider the limit of log-linearization in Lemma 1, where $(\hat{\sigma}_a, \sigma_\psi) = \sqrt{\xi}\cdot(\bar{\sigma}_a, \bar{\sigma}_\psi) = \mathcal{O}(\sqrt{\xi})$ as $\xi \to 0$, where $(\bar{\sigma}_a, \bar{\sigma}_\psi)$ are some fixed numbers. Then in (A70), $(b, c) = \mathcal{O}(\xi)$, as $(b, c)$ are linear in $(\hat{\sigma}_a^2, \sigma_\psi^2)$. This implies that for any given fixed point $(\bar{\sigma}_e^2, \bar{\omega})$, with $F(\bar{\sigma}_e^2, \bar{\omega}; \bar{\sigma}_a^2, \bar{\sigma}_\psi^2) = 0$, there exists a sequence of fixed points $F(\xi\bar{\sigma}_e^2, \bar{\omega}/\xi; \xi\bar{\sigma}_a^2, \xi\bar{\sigma}_\psi^2) = 0$ as $\xi \to 0$, for which $\sigma_e^2 = \xi\bar{\sigma}_e^2 = \mathcal{O}(\xi)$, $\tilde{\omega} = \bar{\omega}/\xi = \mathcal{O}(1/\xi)$ and $\tilde{\omega}\sigma_e^2 = \bar{\omega}\bar{\sigma}_e^2 = const$. To verify this, one can simply divide (A70) by $\xi$ and note that, for a given $\tilde{\omega}x$, $F(x, \tilde{\omega})$ is linear in $(x, b, c)$, which means that the fixed point $x$ scales with $(b, c)$ provided that $\tilde{\omega}x$ stays constant. This confirms the conjecture used in the proof of Lemma 1. ■

**Proofs of Propositions** In the remainder of the proofs, we specialize to the case of $\chi_2 = 0$ and normalization $\chi_1 = 1$, which we adopted in the text of Section 3. The results, however, obtain under the general case with endogenous nonzero $\chi_1$ and $\chi_2$, as we considered until now. Throughout the proofs we use the notation for $\hat{\chi}_1$, $k$ and $\hat{a}_t$ introduced in (A66) above.

**Proof of Lemma 3** The equilibrium process for the nominal exchange rate (30) follows directly from the solution (A69), as $\mu_1 = 1$ when $\chi_2 = 0$, and considering the fact that the assumed monetary policy ensures $e_t = q_t$ (see (17)). This is the unique path of the exchange rate that simultaneously satisfies the modified UIP condition (27) and the country budget constraint (28). Indeed, (27) determines $e_t$ and $\mathbb{E}_t e_{t+j}$ for all $j > 0$ up to a long-run expectation $\mathbb{E}_t e_\infty \equiv \lim_{j\to\infty} \mathbb{E}_t e_{t+j}$, which in turn is uniquely pinned down by the budget constraint, as any departures from $\mathbb{E}_t e_\infty$ result in expected violations of the intertemporal budget of the country.

**Proofs of Proposition 1 and 5** As $\beta, \rho \to 1$, we have $\big[(1-\beta^{-1}L) - (1-\rho L)\big]x_t \to 0$ for any stationary $x_t$, and therefore we can use (30) to show that in this limit:

$$\lim_{\beta,\rho\to 1}\Big\{\Delta e_t - \Big[\overbrace{\tfrac{\beta\hat{\chi}_1}{1-\beta\rho}\sigma_\psi\varepsilon_t^\psi + \big(\tfrac{\beta(1-\rho)}{1-\beta\rho}k + \tfrac{1-\beta}{1-\beta\rho}\big)\hat{\sigma}_a\hat{\varepsilon}_t^a}^{\sim iid}\Big]\Big\}$$

$$= \lim_{\beta,\rho\to 1}\Big\{\big[(1-\beta^{-1}L) - (1-\rho L)\big]\Big(\tfrac{\beta\hat{\chi}_1}{1-\beta\rho}\psi_t + \tfrac{\beta(1-\rho)}{1-\beta\rho}k\hat{a}_t\Big)\Big\} = 0,$$

where we used the notation $\hat{\chi}_1$, $k$ and $\hat{a}_t$ from (A66) and denoted with $\hat{\sigma}_a\hat{\varepsilon}_t^a$ the innovation of the $\hat{a}_t$ shock process, so that $\hat{\sigma}_a\hat{\varepsilon}_t^a = (1-\rho L)\hat{a}_t$ and similarly $\sigma_\psi\varepsilon_t^\psi = (1-\rho L)\psi_t$. Therefore, in the limit $\Delta e_t$ is $iid$, and hence $e_t$ follows a random walk.[50]

We next prove related limiting results characterizing the second moments of the exchange rate process, by rewriting (30) as follows:

$$\Delta e_t = \tfrac{\beta\hat{\chi}_1}{1-\beta\rho}\Big[\sigma_\psi\varepsilon_t^\psi - \tfrac{1-\beta\rho}{\beta}\psi_{t-1}\Big] + \tfrac{\beta(1-\rho)k}{1-\beta\rho}\Big[\hat{\sigma}_a\hat{\varepsilon}_t^a - \tfrac{1-\beta\rho}{\beta}\hat{a}_{t-1}\Big] + \tfrac{1-\beta}{1-\beta\rho}\hat{\sigma}_a\hat{\varepsilon}_t^a,$$

where again we use notation $\hat{\chi}_1$, $k$ and $\hat{a}_t$ from (A66). We can then calculate:

$$\begin{aligned}
\mathrm{var}(\Delta e_t) &= \tfrac{1+\beta^2-2\beta\rho}{(1-\beta\rho)^2(1-\rho^2)}\big[\hat{\chi}_1^2\sigma_\psi^2 + (1-\rho)^2k^2\hat{\sigma}_a^2\big] + \Big(\tfrac{1-\beta}{1-\beta\rho}\Big)^2\Big[1 + \tfrac{2\beta(1-\rho)k}{1-\beta}\Big]\hat{\sigma}_a^2,\\
\mathrm{cov}(\Delta e_t, \Delta e_{t-1}) &= -\tfrac{\beta-\rho}{(1-\beta\rho)(1-\rho^2)}\big[\hat{\chi}_1^2\sigma_\psi^2 + (1-\rho)^2k^2\hat{\sigma}_a^2\big] - \tfrac{(1-\rho)(1-\beta)}{1-\beta\rho}k\hat{\sigma}_a^2,\\
\mathrm{var}_t(\Delta e_{t+1}) &= \Big(\tfrac{\beta}{1-\beta\rho}\Big)^2\hat{\chi}_1^2\sigma_\psi^2 + \Big(\tfrac{\beta(1-\rho)k}{1-\beta\rho} + \tfrac{1-\beta}{1-\beta\rho}\Big)^2\hat{\sigma}_a^2.
\end{aligned}$$

[50] Two remarks are in order. First, as $\beta\rho \to 1$, we should ensure that $\psi_t/(1-\beta\rho)$ does not explode in order to keep the volatility of $\Delta e_t$ finite; a natural approach is to require that $\sigma_\psi/(1-\beta\rho)$ remains finite in this limit. Second, note that the limiting process for $\Delta e_t$ depends on the relative speed of convergence of $\beta$ and $\rho$ to 1, as depending on it $\frac{1-\rho}{1-\beta\rho}, \frac{1-\beta}{1-\beta\rho} \in [0,1]$. For example, if we first take $\beta \to 1$, as will be our approach below, then $\Delta e_t \to \frac{\hat{\chi}_1}{1-\rho}\Delta\psi_t + k\Delta\hat{a}_t$, as the last term in (30) disappears already in this first limit, prior to taking $\rho \to 1$.

Next, taking first the $\beta \to 1$ limit, we can characterize the following ratios for $\rho < 1$ and as $\rho \to 1$:[51]

$$\text{corr}(\Delta e_t, \Delta e_{t-1}) = \frac{\text{cov}(\Delta e_t, \Delta e_{t-1})}{\text{var}(\Delta e_t)} = -\frac{1-\rho}{2} \to 0,$$

$$\frac{\text{var}_t(\Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = \frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = \frac{1+\rho}{2} \to 1,$$

$$\frac{\text{var}(\Delta e_t)}{\text{var}(\hat{\chi}_1 \psi_t)} = \frac{2}{1-\rho} \frac{\hat{\chi}_1^2 \sigma_\psi^2 + (1-\rho)^2 k^2 \hat{\sigma}_a^2}{\hat{\chi}_1^2 \sigma_\psi^2} \to \infty,$$

$$\frac{\text{var}(\Delta e_t | \psi_t)}{\text{var}(\Delta e_t)} = \frac{\hat{\chi}_1^2 \sigma_\psi^2}{\hat{\chi}_1^2 \sigma_\psi^2 + k^2 \hat{\sigma}_a^2 (1-\rho)^2} \to 1,$$

where we used the fact that $\text{var}(\psi_t) = \sigma_\psi^2/(1-\rho^2)$. These are the results summarized in Proposition 1 and in the text following it. The last result in particular confirms that as $\beta\rho \to 1$, the contribution of the $\psi_t$ shock to the variance of $\Delta e_t$ fully dominates the contribution of the productivity shock. The second-to-last result confirms that as $\beta\rho \to 1$, an arbitrarily small volatility of the UIP shock $\chi_1 \psi_t$ results in an arbitrarily large volatility of $\Delta e_t$. The first two results are the confirmation of the limiting random-walk behavior of the exchange rate as $\beta\rho \to 1$.

Lastly, we prove that as $\gamma$ *decreases* or $\beta\rho$ *increases*, the volatility of the exchange rate response to the UIP shock $\chi_1 \psi_t$ *increases*. We denote $\sigma_{e|\psi}^2 \equiv \text{var}(\Delta e_t | \psi_t) = \frac{1+\beta^2-2\beta\rho}{(1-\beta\rho)^2(1-\rho^2)} \hat{\chi}_1^2 \sigma_\psi^2$ the variance of the exchange rate conditional on the financial shock $\psi_t$, or equivalently when we switch off the other shock, $\sigma_a = 0$. We characterize:

$$\frac{\text{var}(\Delta e_t | \psi_t)}{\text{var}(\chi_1 \psi_t)} = \frac{1+\beta^2-2\beta\rho}{(1-\beta\rho)^2} \frac{1}{(1+\gamma\sigma\kappa_q)^2},$$

where we used the notation for $\hat{\chi}_1$ in (A66) and the fact that $\text{var}(\psi_t) = \sigma_\psi^2/(1-\rho^2)$, which already adjusts for the persistence of the shock.[52] It is immediate to see that the volatility of the exchange rate response always increases in $\beta$ and increases in $\rho$ iff $\rho < \beta$, which we take as the empirically relevant case. Note that the overall unscaled volatility of the exchange rate, $\sigma_{e|\psi}^2 \equiv \text{var}(\Delta e_t | \psi_t)$, always increases in both $\beta$ and $\rho$. Finally, we establish that $\gamma\kappa_q$ is increasing in $\gamma$, where $\kappa_q$ is defined in (A58), reducing the volatility of the exchange rate response. We have:

$$\gamma\kappa_q = \varkappa \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu(1+\varkappa) + \phi\varkappa}{1+\nu\sigma(1+\varkappa)}, \qquad \text{where} \qquad \varkappa \equiv \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}.$$

Since $\varkappa$ is increasing in $\gamma$ on its range $[0, 1/2)$, we only need to establish that the right-hand side is increasing in $\varkappa$, which can be immediately verified by differentiation. This completes the proof of Proposition 1.

To finish the proof of Proposition 5, we make use of (A57)–(A58), which in the limit of closed economy $\gamma \to 0$ becomes:

$$y_t = \frac{1+\nu}{1+\nu\sigma} a_t + \frac{2\gamma}{1-\phi} \frac{\nu\sigma 2\theta(1-\alpha) - \nu - (1+\nu\sigma)\phi}{1+\nu\sigma} q_t,$$

where the term in front of $q_t$ may be positive or negative and is of the order of $\gamma$, that is vanishes in the limit if the volatility of $q_t$ is finite. As a result, the volatility of $y_t$ in this case is shaped by productivity shocks $a_t$ alone. Using the results above, it is easy to verify that in the presence of the financial shock, as $\rho$ increases towards 1, the volatility of $\Delta e_t$ increases without bound relative to the volatility of $\Delta a_t$, completing the proof. ■

[51] We focus on this sequence of taking limits as the variance of $\Delta e_t$ is not well-defined when $\rho = 1$, as this results in a double-integrated process for the exchange rate. In contrast, all second moments are well-defined for $\beta = 1$, and we can then study their properties as $\rho$ increases towards 1. In addition, this sequence of limits provides a better approximation to our calibrated model, in which we have $\rho < \beta < 1$, namely $\rho = 0.97$ and $\beta = 0.99$.

[52] We could alternatively characterize the response to the primitive noise-trader shock $\psi_t$ with an endogenous coefficient $\chi_1$, rather than the overall endogenous UIP shock $\chi_1 \psi_t$; this however simply introduces an amplification loop, by which an initial increase in $\sigma_{e|\psi}$ gets amplified by an endogenous increase in $\chi_1$, leaving the qualitative result unchanged.

**Proofs of Lemma 2 and Proposition 2** The relationships between nominal exchange rate, real exchange rates and terms of trade are derived above — see (A52)–(A54). The results follow immediately from these equations. In particular, in response to $\psi_t$, all real exchange rates — $q_t$, $q_t^P$ and $q_t^W$ — comove perfectly with the nominal exchange rate $e_t$, which under the consumer-price-stabilizing monetary policy equals RER, $e_t = q_t$. Smaller $\gamma$ and larger $\alpha$ ensure that the ratios of volatility of $q_t^W$ and $q_t^P$ to that of $q_t$ are closer to 1. In contrast, higher $\alpha$ reduces the relative volatility of the terms of trade $s_t$ relative to $q_t$.

The proof of Proposition 1 already establishes that, as $\beta\rho \to 1$, the process for $q_t = e_t$ converges to a random walk with an arbitrarily large volatility of $\Delta q_t = \Delta e_t$ (when scaled by the exogenous volatility of the UIP shock $\psi_t$, that is arbitrarily small volatility of $\psi_t$ can result in arbitrarily large volatility of $\Delta q_t$). We now show that the small-sample persistence of the real exchange also increases without bound, and hence so does the measured half-live of the RER process. Specifically, we calculate the finite-sample autocorrelation of the real exchange rate in levels, that is the coefficient from a regression of $q_t$ on $q_{t-1}$ (with a constant) in a sample with $T+1$ observations:

$$\hat{\rho}_q(T) = \frac{\frac{1}{T}\sum_{t=1}^T (q_t - \bar{q})(q_{t-1} - \bar{q})}{\frac{1}{T}\sum_{t=1}^T (q_{t-1} - \bar{q})^2} = 1 + \frac{\frac{1}{T}\sum_{t=1}^T \Delta q_t q_{t-1}}{\frac{1}{T}\sum_{t=1}^T (q_{t-1} - \bar{q})^2}.$$

Note that the denominator is positive and finite for any finite $T$, but diverges as $T \to \infty$, since $q_t$ is an integrated process. The numerator, however, has a finite limit (conditional on a given initial value $q_0$, and due to stationarity of $\Delta q_t$):

$$\underset{T\to\infty}{p\lim}\, \frac{1}{T}\sum_{t=1}^T \Delta q_t q_{t-1} = \mathrm{cov}(\Delta q_t, q_{t-1}) = \sum_{j=1}^{\infty} \mathrm{cov}(\Delta q_t, \Delta q_{t-j}) = -\frac{\beta - \rho}{(1-\rho)(1-\beta\rho)} \frac{\chi_1^2 \sigma_\psi^2}{(1+\gamma\sigma\kappa_q)^2},$$

where we used the fact that for process (30) $\mathrm{cov}(\Delta q_t, \Delta q_{t-j}) = \rho^{j-1}\mathrm{cov}(\Delta q_t, \Delta q_{t-1})$ for $j \geq 1$, and the expression for $\mathrm{cov}(\Delta q_t, \Delta q_{t-1}) = \mathrm{cov}(\Delta e_t, \Delta e_{t-1})$ calculated above (conditional on $\psi_t$ shocks, i.e. when $\sigma_a = 0$). This implies that the finite sample autocorrelation of $q_t$: (a) tends to 1 asymptotically as samples size increases (and hence the associated half-life tends to infinity); and (b) is smaller than 1 in large but finite samples, provided that $\rho < \beta$. The associated half-live of $q_t$ is given by $\log(0.5)/\log\hat{\rho}_q(T)$, is finite in finite samples, and increases unboundedly with $T$. This completes the proof of Proposition 2. ■

**Proof of Proposition 3** The equilibrium condition (23) and the coefficients $\kappa_a, \kappa_q$ are derived above: see (A58).

Consider a $\psi_t$ shock first. From (30) and Proposition 1, it results in exchange rate depreciation, $\Delta q_t = \Delta e_t > 0$. From (23), this depreciation is associated with a reduction in consumption, since $\gamma\kappa_q > 0$. The Backus-Smith correlation in response to $\psi_t$ derives from:

$$\mathrm{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t | \psi_t) = -\gamma\kappa_q \mathrm{var}(\Delta q_t | \psi_t) < 0,$$

which follows directly from (23). The relative volatility of consumption in response to $\psi_t$ is:

$$\frac{\mathrm{var}(\Delta\tilde{c}_t|\psi_t)}{\mathrm{var}(\Delta q_t|\psi_t)} = (\gamma\kappa_q)^2,$$

which decreases as $\gamma$ decreases (see the Proof of Proposition 1), and converges to zero in the limit $\gamma \to 0$ (as $\lim_{\gamma\to 0}\kappa_q = \frac{2}{1-\phi}\frac{2\theta(1-\alpha)+\nu}{1+\nu\sigma}$ if finite; see (A58)). ■

**Proof of Proposition 4** (a) The proof of this part follows from the equilibrium relationship (A62) between expected (real) devaluation and the interest rate differential, which combines the household Euler equations with the market-clearing relationship between consumption and the real exchange rate (23). In view of perfect price level stabilization by the monetary authority, implying in particular $e_t = q_t$, we rewrite (A62) as:

$$i_t - i_t^* = -(1-\rho)\sigma\kappa_a\tilde{a}_t - \gamma\sigma\kappa_q \mathbb{E}_t \Delta e_{t+1},$$

where we also made use of the AR(1) assumption for $\tilde{a}_t$ to simplify the expression.

The Fama coefficient, $\beta_F$, is a projection coefficient of $\Delta e_{t+1}$ on $i_t - i_t^*$, and since $i_t - i_t^*$ is known at time $t$, it is equivalent to the projection of $\mathrm{E}_t \Delta e_{t+1}$ instead of $\Delta e_{t+1}$. Using the expression above, we have:

$$\beta_F \equiv \frac{\mathrm{cov}(\mathbb{E}_t \Delta e_{t+1}, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)} = -\frac{1}{\gamma\sigma\kappa_q} - (1-\rho)\frac{\sigma\kappa_a}{\gamma\sigma\kappa_q}\frac{\mathrm{cov}(\tilde{a}_t, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)}.$$

In the absence of productivity shocks, $\sigma_a = 0$, the second term is zero, and $\beta_F = -1/(\gamma\sigma\kappa_q) < 0$. Note that this result does not rely on the assumption $\chi_2 = 0$ in (16), since it derives from equilibrium conditions other than (16) (akin to our resolution of the Backus-Smith puzzle in Proposition 3).

Furthermore, under the restriction that $\chi_2 = 0$, we can use the solution for the interest rate differential (26) to calculate:

$$\frac{\mathrm{cov}(\tilde{a}_t, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)} = -\frac{1+\gamma\sigma\kappa_q}{(1-\rho)\sigma\kappa_a}\frac{(1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2}{(\gamma\sigma\kappa_q)^2\chi_1^2\sigma_\psi^2 + (1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2},$$

where the second term in the product on the right-hand side is the share of the productivity shock in the variance decomposition of $\mathrm{var}(i_t - i_t^*)$. When $\sigma_\psi = 0$, this share is one, and it follows that $\frac{\mathrm{cov}(\tilde{a}_t, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)} = -\frac{1+\gamma\sigma\kappa_q}{(1-\rho)\sigma\kappa_a}$, resulting in $\beta_F = -\frac{1}{\gamma\sigma\kappa_q} + (1-\rho)\frac{\sigma\kappa_a}{\gamma\sigma\kappa_q}\cdot\frac{1+\gamma\sigma\kappa_q}{(1-\rho)\sigma\kappa_a} = 1$. $\beta_F = 1$ could also be obtained directly from (16) after imposing $\chi_2 = 0$ and $\sigma_\psi = 0$. This completes the proof of part (i).

We note, in addition, that the qualitative result still applies when $\chi_2 > 0$; in this case one can calculate $\beta_F$ using (A65) and the equilibrium solution for the dynamics of $b_{t+1}$ in (A68). As already shown above, this does not affect the value of $\beta_F = -1/(\gamma\sigma\kappa_q)$ conditional on $\psi_t$ shocks. It does, however, affect the value of $\beta_F$ conditional on productivity shocks, which can be below or above 1 depending on whether $k$ defined in (A66) is above or below 1. Nevertheless, the departure of $\beta_F$ from 1 in this case is quantitatively small, as it is of the order of $\gamma^2$. We omit the derivation for brevity.

(b) The proof here relies on the modified UIP condition (16) and the results in Proposition 1. For simplicity, we make use of $\chi_2 = 0$. Note from the solutions (27) and (26) that as $\rho \to 1$, the variance contribution of $\tilde{a}_t$ to both $\mathrm{var}(\mathbb{E}_t\Delta e_{t+1})$ and $\mathrm{var}(i_t - i_t^*)$ goes to zero, and the variance of both is fully shaped by the $\psi_t$ shock, independently of the value of $\sigma_a, \sigma_\psi > 0$. Therefore, from our characterization in the proof of part (i), $\beta_F \to -1/(\gamma\sigma\kappa_q) < 0$ in this case. In order to characterize $R^2$ in the Fama regression, we additionally use the solution (30), which we rewrite as follows:

$$\Delta e_{t+1} = \mathbb{E}_t \Delta e_{t+1} + \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta\chi_1}{1-\beta\rho}\sigma_\psi\varepsilon_t^\psi + \left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\right]\tilde{\varepsilon}_t^a.$$

Since $i_t - i_t^*$ is orthogonal to the innovation $\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1}$, we can establish an upper bound:

$$R^2 \leq \frac{\mathrm{var}(\mathbb{E}_t\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} = 1 - \frac{\mathrm{var}_t(\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} \to 0$$

as $\beta\rho \to 1$, since the proof of Proposition 1 establishes that in this case $\frac{\mathrm{var}_t(\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} \to 1$.

By the same token, using the expression for $\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1}$ above and (26), we can evaluate $\frac{\mathrm{var}(i_t - i_t^*)}{\mathrm{var}_t(\Delta e_{t+1})}$. As in Proposition 1, we interpret the $\beta\rho \to 1$ limit sequentially with $\beta \to 1$ first and $\rho \to 1$ next (see the argument in footnote 51), and the results apply more generally with $\beta$ and $\rho$ converging to 1 simultaneously with $\rho \leq \beta < 1$ along the limit sequence. We have:

$$\begin{aligned}\lim_{\beta\to 1}\frac{\mathrm{var}(i_t - i_t^*)}{\mathrm{var}_t(\Delta e_{t+1})} &= \lim_{\beta\to 1}\frac{\left(\frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\right)^2\frac{\chi_1^2\sigma_\psi^2}{1-\rho^2} + (1-\rho)^2\left(\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\right)^2\frac{\sigma_a^2}{1-\rho^2}}{\left(\frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta}{1-\beta\rho}\right)^2\chi_1^2\sigma_\psi^2 + \left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\right]^2\sigma_a^2} \\ &= \frac{1-\rho}{1+\rho}\frac{(\gamma\sigma\kappa_q)^2\chi_1^2\sigma_\psi^2 + (1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2}{\chi_1^2\sigma_\psi^2 + (1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2} \to 0,\end{aligned}$$

as $\rho \to 1$. Since $\mathrm{var}_t(\Delta e_{t+1}) \leq \mathrm{var}(\Delta e_{t+1})$, the variance of $i_t - i_t^*$ is indeed arbitrarily smaller than that of $\Delta e_{t+1}$ in the limit as $\beta\rho \to 1$.

Proposition 1 has already established that the autocorrelation of $\Delta e_{t+1}$ becomes arbitrarily close to zero as

$\beta\rho \to 1$. At the same time, from (26), the autocorrelation of $i_t - i_t^*$ is equal to $\rho$, and hence it becomes arbitrarily close to 1 as $\beta\rho \to 1$.

Lastly, we characterize the Carry trade return and its Sharpe ratio. Consider a zero-capital strategy of buying a (temporarily) high-interest bond and selling short low-interest bond, with return $R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_t^*$ per dollar of gross investment, and the size of the gross position determined by the expected return, $\mathbb{E}_t\big\{R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_t^*\big\}$, which can be positive or negative.[53] Therefore, a log approximation to the return on this trade is:

$$r_{t+1}^C = (i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1})(i_t - i_t^* - \Delta e_{t+1}) = \chi_1\psi_t\big[\chi_1\psi_t - (\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1})\big], \tag{A71}$$

where the second equality uses (16) under $\chi_2 = 0$. The (unconditional) Sharp ratio associated with this trade is given by $SR^C = \frac{\mathbb{E}\, r_{t+1}^C}{\mathrm{std}(r_{t+1}^C)}$. We calculate:

$$\mathbb{E}\, r_{t+1}^C = \mathbb{E}\big\{\chi_1\psi_t\, \mathbb{E}_t\{\chi_1\psi_t - (\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1})\}\big\} = \chi_1^2\mathbb{E}\psi_t^2 = \chi_1^2\mathrm{var}(\psi_t),$$

$$\begin{aligned}\mathrm{var}(r_{t+1}^C) &= \mathbb{E}(r_{t+1}^C)^2 - (\mathbb{E}r_{t+1}^C)^2 = \mathbb{E}\big\{\chi_1^2\psi_t^2\mathbb{E}_t\{\chi_1\psi_t - (\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1})\}^2\big\} - \big[\chi_1^2\mathrm{var}(\psi_t)\big]^2 \\ &= \chi_1^4\mathbb{E}\psi_t^4 + \mathbb{E}\left\{\chi_1^2\psi_t^2\mathrm{var}_t(\Delta e_{t+1})\right\} - \big[\chi_1^2\mathrm{var}(\psi_t)\big]^2 = 2\big[\chi_1^2\mathrm{var}(\psi_t)\big]^2 + \sigma_e^2\chi_1^2\mathrm{var}(\psi_t),\end{aligned}$$

where the last line uses the fact that $\sigma_e^2 = \mathrm{var}_t(\Delta e_{t+1})$ does not depend on $t$ (and in particular on the realization of $\psi_t$, that is the unexpected component of $\Delta e_{t+1}$ is homoskedastic, as we proof in Proposition 1), as well as the facts that $\mathbb{E}\{\psi_t^3(\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1})\} = \mathbb{E}\{\psi_t^3\mathbb{E}_t\{\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1}\}\} = 0$ and $\mathbb{E}\psi_t^4 = 3(\mathbb{E}\psi_t^2)^2$ due to the normality of the shocks. With this, we calculate:

$$SR^C = \frac{\chi_1^2\mathrm{var}(\psi_t)}{\sqrt{2[\chi_1^2\mathrm{var}(\psi_t)]^2 + \sigma_e^2\chi_1^2\mathrm{var}(\psi_t)}} = \left(2 + \frac{\sigma_e^2}{\chi_1^2\mathrm{var}(\psi_t)}\right)^{-1/2},$$

which declines to zero as $\beta\rho \to 1$, since in this limit $\frac{\sigma_e^2}{\chi_1^2\mathrm{var}(\psi_t)} \to \infty$, as we proved in Proposition 1. Indeed, our derivations show that the expected Carry trade return is proportional to the volatility of the UIP shock, $\chi_1\psi_t$, while the standard deviation of the return increases additionally with the volatility of the exchange rate, which in the limit becomes arbitrarily larger than the volatility of the UIP shock. ■

**Engel decomposition and Balassa-Samuelson effect** Engel (1999) decomposes $q_t = q_t^T + q_t^N$ into the tradable RER $q_t^T$ and the relative price of non-tradables to tradables in the two countries $q_t^N$, and shows that $q_t^T$ dominates the variance decomposition of $q_t$ at all horizons. Our model reproduces this pattern. Since our model does not incorporate any asymmetry between domestically produced tradables and non-tradables (i.e., no Balassa-Samuelson force), the price index for non-tradables at home is $p_{Nt} = p_{Ht}$, i.e. the same as the price of domestically produced tradables shipped to the domestic market. Assuming $\omega$ is the share of tradable sectors in expenditure, we have:

$$p_t = \omega p_{Tt} + (1-\omega)p_{Nt} \qquad \Rightarrow \qquad p_{Tt} = \tfrac{1}{\omega}[p_t - (1-\omega)p_{Ht}] = \tfrac{\gamma}{\omega}p_{Ft} + \big(1 - \tfrac{\gamma}{\omega}\big)p_{Ht},$$

where we used the definition of $p_t$. Note that our model does not need to take a stand whether $\gamma$ is due to home bias in tradables ($\omega = 1$), or due to non-tradables ($\omega = \gamma$), or a combination of the two ($\gamma < \omega < 1$, where $1-\omega$ is the expenditure share of non-tradables and $\gamma/\omega$ is the home bias in the tradable sector). For concreteness, and in line with the empirical patterns in Engel (1999), we assume that $\omega \approx 1/2$, so that $\gamma \ll \omega$, and there exists a considerable home bias in tradables (on the role of the local distribution margin in tradable prices, see e.g. Burstein, Eichenbaum, and Rebelo 2005).

[53] Note that in a symmetric steady state $\bar{R} = \bar{R}^* = 1/\beta$, and there is no low or high interest rate bond in the long run, yet over time the relative interest rates on bonds fluctuate according to (26), allowing for a temporary Carry trade (or 'forward premium trade' in the terminology of Hassan and Mano 2014).

The tradable RER is defined as $q_t^T = p_{Tt}^* + e_t - p_{Tt}$. We then calculate the non-tradable RER:[54]

$$\begin{aligned} q_t^N &\equiv q_t - q_t^T = (1-\omega)\big[(p_{Nt}^* - p_{Tt}^*) - (p_{Nt} - p_{Tt})\big] \\ &= \gamma\tfrac{1-\omega}{\omega}\big[(p_{Ft}^* - p_{Ht}^*) - (p_{Ht} - p_{Ft})\big] = \gamma\tfrac{1-\omega}{\omega}\big[q_t^P + s_t\big] = \tfrac{1-\omega}{\omega}(1-\alpha)\tfrac{2\gamma}{1-2\gamma}q_t \end{aligned}$$

Therefore, the contribution of the non-tradable component to the volatility of RER is:

$$\frac{\mathrm{cov}(\Delta q_t^N, \Delta q_t)}{\mathrm{var}(\Delta q_t)} = \frac{1-\omega}{\omega}(1-\alpha)\frac{2\gamma}{1-2\gamma},$$

which is small whenever $\gamma$ is small, even if $\alpha = 0$. LOP deviations, due to PTM ($\alpha > 0$) or LCP, further reduce the contribution of the non-tradable component, but are conceptually not necessary to replicate the patterns documented in Engel (1999) and Betts and Kehoe (2008), as home bias in tradables is sufficient.

It is straightforward to extend the model and allow for the relative tradable-nontradable productivity shocks in order to address the evidence on the Balassa-Samuelson effect (see Rogoff 1996). In the data, Balassa-Samuelson forces are difficult to detect under a floating exchange rate regime, yet they become considerably more pronounced under a peg (see Mendoza 2005, Berka, Devereux, and Engel 2018). This is exactly in line with the prediction of the disconnect model, as we explore in Itskhoki and Mukhin (2019), where we use the model to study a switch from a peg to a float. Intuitively, to the extent that relative productivity shocks are considerably less volatile than the nominal exchange rate under a float, Balassa-Samuelson effect would be difficult to detect, yet when nominal exchange rate volatility is switched off under a peg, the relative productivity shocks become important drivers of the (much less volatile) real exchange rate. To summarize, the seminal evidence in Engel (1999) should not be interpreted as against the importance of nontradables in understanding the real exchange rate, but rather against the importance of (relative) productivity shocks as the key driver of the real exchange rate under a floating regime.

## A.7 Nominal rigidities

This section outlines the details of the monetary model with nominal wage and price rigidities. As before, we focus on Home and symmetric relationships hold in Foreign. We consider a standard New Keynesian two-country model in a cashless limit, as described in Galí (2008).

**Wages** The aggregate labor input is a CES aggregate of individual varieties with elasticity of substitution $\epsilon$, which results in labor demand:

$$L_{it} = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon} L_t, \quad \text{where} \quad L_t = \left(\int L_{it}^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{and} \quad W_t = \left(\int W_{it}^{1-\epsilon}\mathrm{d}i\right)^{\frac{1}{1-\epsilon}},$$

and the rest of the model production structure is unchanged. Households set wages à la Calvo and supply as much labor as demanded at a given wage rate. The probability of changing wage in the next period is $1-\lambda_w$. The first-order condition for wage setting is:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\lambda_w)^{s-t}\frac{C_s^{-\sigma}}{P_s} W_s^{\epsilon} L_s \left(\bar{W}_t^{1+\epsilon/\nu} - \frac{\kappa\epsilon}{\epsilon-1}P_s C_s^{\sigma} L_s^{1/\nu} W_s^{\epsilon/\nu}\right) = 0.$$

Substituting in labor demand and log-linearizing, we obtain:

$$\hat{w}_t = \frac{1-\beta\lambda_w}{1+\epsilon/\nu}\left(\sigma c_t + \frac{1}{\nu}\ell_t + p_t + \frac{\epsilon}{\nu}w_t\right) + \beta\lambda_w \mathbb{E}_t \hat{w}_{t+1},$$

where $\hat{w}_t$ denotes the log deviation from the steady state of the wage rate reset at $t$. Note that the wage inflation can be expressed as $\pi_t^w \equiv \Delta w_t = (1-\lambda_w)(\hat{w}_t - w_{t-1})$. Aggregate wages using these equalities and express

[54] We can also evaluate the tradable $q_t^T = p_{Tt}^* + e_t - p_{Tt} = \frac{\gamma}{\omega}p_{Ht}^* + (1-\frac{\gamma}{\omega})p_{Ft}^* + e_t - \frac{\gamma}{\omega}p_{Ft} - (1-\frac{\gamma}{\omega})p_{Ht} = (1-\frac{\gamma}{\omega})q_t^P - \frac{\gamma}{\omega}s_t$.

the wage process in terms of cross-country differences to obtain the NKPC for wages:

$$\tilde{\pi}_t^w = k_w \left[ \sigma \tilde{c}_t + \frac{1}{\nu} \tilde{\ell}_t + \tilde{p}_t - \tilde{w}_t \right] + \beta \mathbb{E}_t \tilde{\pi}_{t+1}^w,$$

where $k_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)}$.

**Prices** We assume that firms set prices à la Calvo with a probability of changing price next period equal to $1-\lambda_p$. There are two Phillips curves, one for domestic sales and one for exports. The first-order conditions for reset prices in log-linearized form are

$$\hat{p}_{Ht} = (1-\beta\lambda_p)\,\mathbb{E}_t \sum\nolimits_{j=t}^{\infty} (\beta\lambda_p)^{j-t} \left[(1-\alpha)\, mc_j + \alpha p_j\right],$$
$$\hat{p}_{Ht}^* = (1-\beta\lambda_p)\,\mathbb{E}_t \sum\nolimits_{j=t}^{\infty} (\beta\lambda_p)^{j-t} \left[(1-\alpha)\,(mc_j - (1-\iota)e_j) + \alpha(p_j^* + \iota e_j)\right],$$

where $mc_t = (1-\phi)\left(\vartheta r_t^K + (1-\vartheta)w_t - a_t\right) + \phi p_t$ are the marginal costs of Home firms, and $\iota \in \{0,1\}$ with $\iota = 1$ corresponding to the case of PCP (producer currency pricing) and $\iota = 0$ to the case of LCP (local currency pricing). The optimal reset prices are equal to an expected discounted sum of future optimal static prices, which in turn are a weighted average of marginal costs and competitor prices. Given the law of motion for home prices, $\pi_{Ht} = (1-\lambda_p)\left(\hat{p}_{Ht} - p_{Ht-1}\right) = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}_{Ht} - p_{Ht}\right)$, the resulting NKPC is:

$$\pi_{Ht} = k_p \Big[(1-\alpha)mc_t + \alpha p_t - p_{Ht}\Big] + \beta \mathbb{E}_t \pi_{Ht+1}, \quad \text{where} \quad k_p = \frac{(1-\beta\lambda_p)\,(1-\lambda_p)}{\lambda_p}.$$

The law of motion for export prices depends on the currency of invoicing. Under LCP, the law of motion for prices is, $\pi_{Ht}^* = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}_{Ht}^* - p_{Ht}^*\right)$, and the export NKPC can be expressed as:

$$\pi_{Ht}^* = k_p \Big[(1-\alpha)(mc_t - e_t) + \alpha p_t^* - p_{Ht}^*\Big] + \beta \mathbb{E}_t \pi_{Ht+1}^*.$$

Under PCP, the law of motion for the export price index, $\pi_{Ht}^* = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}_{Ht}^* - p_{Ht}^*\right) - \Delta e_t$, implies that the NKPC is

$$(\pi_{Ht}^* + \Delta e_t) = k_p \Big[(1-\alpha)mc_t + \alpha(p_t^* + e_t) - (p_{Ht}^* + e_t)\Big] + \beta \mathbb{E}_t \left(\pi_{Ht+1}^* + \Delta e_{t+1}\right).$$

Finally, notice that the DCP case with all international trade invoiced in Foreign currency can be expressed as a mix of the two other regimes — Home exporters use LCP and Foreign exporters use PCP. The rest of the model is as described in Section 2.1.