War with Sovereign Debt*

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Abstract

Current thinking on the causes of war focuses on bargaining strategies, resources on hand, and the ability to make transfers as drivers of conflict. This view ignores the important role access to credit can play in shaping war and peace. We explore how financial markets impact the potential for interstate war. We demonstrate that access to capital markets increase the possibility of peace, but preventive war remains possible. The effects of the market on crisis outcomes is through the price of debt and that prices are determined by market conditions like the risk free interest rate and state specific conditions like the likelihood of default.

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If a state expects to experience a large shift in its military power, it faces a problem with its rivals. Everyone knows once the rising state grows stronger, it will demand more resources or a larger say in policy outcomes. Furthermore, it is unclear why such arising state would honor any previous agreements with the relatively declining state that did not reflect the rising state’s new found power. As a result, when the rising state is bargaining today current thinking argues that it can only freely give up all of today’s resources in order to avoid war. If such a sacrifice is insufficient from the view of the rivals to compensate for the series of losses they see coming in the future, the rivals may find preventive war a better option. Here war gives the declining rivals hope of avoiding the consequences of the shifts in power and locking the their access to resources in the long run.

Thus, current theories of preventive war are at their core wars resulting from a liquidity problem arising from a state’s inability to commit future distributions of resources that do not reflect the underlying distribution of power. But more accurately we can think of the rising power’s bargaining position as illiquid, but, in some sense, not insolvent. The rising state could credibly make transfers today if it could borrow against future streams of resources.\textsuperscript{1}

In fact, there is strong reason to believe that borrowing from a credit market solves a portion of this credibility problem. Loans that are not paid back will limit access to credit markets and hurt the rising state in the future. For instance, a state that defaults on its sovereign debt and loses access to credit markets will no longer be able smooth consumption or tap credit market to smooth over the potential for preventive war caused by future shifts in power.

To demonstrate how significant borrowing can be, consider the following reasonable parameterization of a resource consumption problem between two states. Normalize the size

\textsuperscript{1}For instance, Baliga and Sjostrom (2013) comment that commitment problems as a cause of war arise when transfers are limited to current output. They note that transfers are limited in this manner when international bankers are unwilling to lend to states.
of each period’s resource pie to 1. Think of each period as a year, so a discount rate of future periods 0.95 is reasonable. Assume the cost of war is 0.5 or half of all the resources available in a period. After a shift in power, assume that the rising power will win a war with the declining power with 0.5 probability. In this case, we can calculate that by going to war after the shift is complete the rising power will be able to guarantee itself the following expected value: the present value of the entire pie in every future period with probability 0.5 minus the cost of war. This value is approximately 9 pies in present value. If the rising state could borrow against this future value, then this is a source of transfers that is nine times the size of today’s pie. That is, the traditional model only captures a tenth of the resources potentially available for bargaining. Of course, this is just an illustrative example, but there is nothing pathological about the parameterization. In fact, one could easily imagine realistic parameterizations with far more extreme outcomes.

If credit market access drives a great deal of the resources available for bargaining, this fundamentally alters our understanding of preventive war, bargaining, and the connections between conflict and international finance. In this paper, we derive four immediate results from this change in approach, but this just scratches the surface of what future research might find.

First, one might ask if accessing credit marks always allows states to avoid war. The answer is no. The commitment problem due to power shifts remains a vital mechanism for interstate war. In fact, the extent of its impact is broadened by allowing for sovereign borrowing. Borrowing allows states to avoid a number of wars that would occur if credit market access was entirely shut down. The bar for a shift to cause war is increased, but it is still the case that increasing the likelihood, size, rapidity, and persistence of a power shift will make war more likely. Now though, significant power shifts that do not cause war, but do cause borrowing bring about a new kind of inefficiency. Namely, borrowing in order to prevent war is non-productive and inherently inefficient. The more a state is forced to borrow in order to avoid preventive war, the more inefficiency a power shift introduces. This
inefficiency is increasing in the size of the power shift until, ultimately, a significant enough
power shift will bring about the full inefficiency of war.

Second, if credit market access is so critical to conflict, what drives credit market access?
One might imagine a number of factors, but here we take on the most obvious – price. States must, at minimum, pay the risk-free rate the bond market could achieve through some other loan. In addition to this, states must pay an endogenously determined premium on any loans they may default on. When a state’s potential shift in power occurs with less than certainty, they may default exactly when they fail to grow. It’s important to note that price may constrain borrowing through two different mechanisms: (1) from the rising power’s perspective, loans may be too expensive to borrow relative to war today; (2) the bond market may be unwilling to lend enough to prevent war because the rising state is not sufficiently likely to be able to pay back the loan in the future.

Third, understanding that the bond market cares deeply about risk when making lending decisions allows us to derive a novel implication about the types of power shifts that are particularly dangerous. As in standard models, shifts with large expected values are more dangerous. In our model, given the size of an expected shift, more extreme, but less likely shifts are more dangerous than highly likely but only moderately sized shifts. In these cases, the bond market is unwilling to lend because of the default risk in cases where the rising power fails to grow in strength.

Fourth, by connecting conflict and sovereign borrowing in our model, we are able to immediately see some obvious ways in which international economics and conflict intertwine. In our model, economics impacts international relations in that higher real rates reduce the mitigation effect of borrowing on preventive war therefore increasing the likelihood conflict. On the other, international relations impacts economics in that shift in power may increase a state’s borrowing for non-economic reasons driving higher premiums on debt and potential defaults.

To explore the effects of borrowing on war, we build a simple model of preventive war
where sovereign lending is possible. A rising state may sell bonds to a profit-maximizing market generating resources that they may then transfer to a declining state. Our model considers a bond market where states are allowed to default on these loans but do not do so if they have resources to pay their debt. We also simplify away from traditional consumption smoothing motives for state borrowing and instead focus on how borrowing may help states avoid preventive war due to commitment problems arising from stochastic shifts in power.

We find that borrowing from an outside lender or market against future bargaining gains allows the rising state to avoid war in certain circumstances. However, in some cases, preventive war is still unavoidable. This is due to two reasons. First, states must pay interest on their loans and, depending on market conditions, this interest may be high enough that states cannot credibly borrow enough today to fully alleviate the threat of preventive war. Second, states only achieve gains in power with some probability. When shifts fail to occur, states may default. When this risk of default is significant enough, lenders and states may be unwilling to agree to loans that would have avoided preventive war. This means both the nature of the power-shift and market conditions can determine when commitment problems cause war.

In addition to this result, three empirical predictions arise quite naturally from our simple setup. First, for future shifts of the same expected size, less probable but more extreme shifts are especially dangerous. In this case, markets are less able to provide liquidity given the high probability of default. This indicates that preventive war should be especially likely in situations where low probability, high impact changes are expected. For example, a nuclear weapons program delivers a relatively low probability of success in any given period, but very high impact and would be difficult to resolve through borrowing.

Second, even when states can successfully borrow against uncertain future power shifts, they will often pay a premium on their debt. Markets will demand higher rates in order to cover themselves in the event that the state fails to grow more powerful and is forced to default. This effect may address why rapidly growing states in adverse security environ-
ments, like South Korea, pay a premium on their debt versus states in more benign security situations.\textsuperscript{2}

Third, all else equal, when the world real risk-free interest rate is high, war is more likely through the commitment problem mechanism. The higher the risk-free rate, the better the outside option for the bond market relative to loaning to a rising power in our model. This makes bond holders less willing to lend and those loans that do occur are burdensome. Under these circumstances, potential borrowers may even prefer risking war to the very high rate loans they are offered.\textsuperscript{3} Moreover, while the risk-free rate is exogenous in our model, it allows us to draw a direct connection between events like the worldwide depression before World War II that increased the cost of capital, and the subsequently heightened dangers of war due to commitment problems. For example, Romer (1992) demonstrates that real interest rates in the United States skyrocketed in the early part of the Depression and then again in 1937. Moreover, U.S. lending to Europe dropped significantly, from 598 million dollars in 1928, to 142 million dollars in 1929 (Kindleberger 1973, pg 56). The economic implications were particularly serious for Germany, which depended on U.S. loans to make reparations payments.\textsuperscript{4} A particularly interesting connection between financial markets and war is the implication that significant conflict in one part of the world may raise the risk-free rate and serve as a contagion channel for war in other parts of the world.

\textsuperscript{2}Several studies, including recently Coudert and Mignon (2013), demonstrate that the carry trade with South Korea can produce excess returns in normal economic times. Rare economic disasters have been put forth as an explanation for why excess returns in the carry trade persist (Farhi and Gabaix 2016). Barro (2006) explicitly links rare economic disasters with the possibility of warfare.

\textsuperscript{3}In an empirical paper, Chapman and Reinhardt (2013) find that higher costs of foreign capital increase the likelihood of civil conflict.

\textsuperscript{4}A full discussion of this highly complex situation is beyond the scope of this paper. See Kindleberger (1973) and Tooze (2006) for in-depth analyses of this case.
Commitment Problems and Sovereign Debt

This paper connects two distinct literatures. The first studies how dynamic changes in the international power structure lead to commitment problems that potentially cause war. The second studies how shifting economic fortunes may lead states to strategically default on their debt.

The international relations literature on commitment problems begins with Fearon (1995) and is further theoretically developed in several subsequent papers (Powell 1999, 2004, 2006, 2012, 2013; Fearon 1996, 2004; Leventoglu and Slantchev 2007; Chassang and Padro i Miquel 2010; Bas and Coe 2012; Debs and Monteiro 2014; Krainin and Wiseman 2016; Krainin 2017; Krainin and Slinkman 2017; Wiseman 2017). Commitment problem models have been recently utilized to understand a number of applied issues including civil wars (Paine 2016) and how domestic politics affects the potential for interstate war (Chapman, McDonald, and Moser 2015). Moreover, new techniques have been developed to empirically test commitment problem models (Bell and Johnson 2015; Bas and Schub 2017). However, thus far no paper has addressed how borrowing against the future may impact the liquidity constraint that lies at the heart of the commitment problem.

In order to address this question, we connect this international relations literature on commitment problems to the economics literature on sovereign default. The default side of this paper’s model is most closely connected to the one developed in Arellano (2008). Eaton and Gersovitz (1981) provides a classic contribution to this literature while Chatterjee et al. (2007) makes important recent theoretical advances in the context of strategic consumer default. The literature on sovereign default includes a vast number of papers that make theoretical and empirical contributions. However, no papers in this literature model lending in a strategic security context, and it therefore cannot address how the specter of preventive war may lead to debt build-ups and subsequent defaults.

Some previous work has studied the militarization of sovereign debt collection. Both Finnemore (2003) and Mitchener and Weidenmier (2010) find that militarized debt collection
was a common tool in the pre-World War I era. On the other hand, Tomz (2007) argues that creditor governments rarely used or threatened to use military force in the event of sovereign default. While these studies engage with the incentive to militarize debt collection, our paper focuses on how sovereign lending interacts with the incentives for preventive war.

Another recent literature has focused on debt financing war efforts. McDonald (2011) demonstrates how sovereign lending allows states to maintain arms races without having to renegotiate their society’s basic social contract. Slantchev (2012) builds a model where states may borrow unlimited amounts of debt to finance mobilization efforts, and default occurs when a state is defeated in war. Slantchev establishes that the incentives states have to borrow can endogenously induce conflict. In his model, borrowing can endogenously increase the cost of preserving a peaceful status quo relative to war because war lowers the burden of debt by allowing the defeated state to default. In contrast to work emphasizing war as a driver of sovereign default, Shae and Poast (2017) finds that states are unlikely to default after losing a war. The reasoning is similar to an effect present in our model – lenders will strategically limit loans to amounts that receiving states can pay back with sufficient probability. Finally, Poast (2015) notes how states that possess central banks are better able to secure debt financing, especially in times of war.

North and Weingast (1989) develops a compelling theory that constitutional limits increase the credibility of sovereign debt. Schultz and Weingast (2003) argues that therefore, democratic states have greater ability to commit to repaying sovereign debts over autocratic states. More democratic states thus have an advantage in long-run hegemonic competition with less democratic states, since the more democratic states will be better able to finance wars and arms races. While some empirical studies have failed to support the “democratic advantage” thesis (Saiegh 2005; Archer, Biglaiser, and DeRouen 2007), several subsequent studies have refined the theory and found supporting empirical evidence (Stasavage 2007, 5Powell (2006, pg. 192-194) analyzes a different context where the cost of maintaining the status quo leads to war.
2011; Dincecco 2009; Beaulieu, Cox, and Saiegh 2012).

Our study does not directly address the democratic advantage. However, our results suggest that in a context where strategic default is possible, more democratic states would similarly have an advantage in maintaining peace by utilizing greater debt market access to supply peaceful transfers in order to avoid the possibility of preventive war. This effect may speak to the peaceful rise of the United States vis-à-vis Great Britain in the period between the American Civil War and World War I.

Another long-running literature has argued that financial interests work to create peace. Polanyi (1944) argues that powerful, cartel-like financial interests actively pushed the international system toward peace in the 19th century. Recently, Flandreau and Flores (2012) has refined Polanyi’s argument, suggesting that “prestigious” financial certification intermediaries act to avoid war in order to avoid the possibility of sovereign default and the consequent damage to their reputations. Alternatively, Kirshner (2007) proposes a preference-based argument emphasizing that financial communities are averse to war due to its deleterious impact on macrэкономic stability.

Similarly, in our model, financial interests effectively work to help states avoid preventive war. However, they do so as a direct consequence of their profit-maximizing activity. Purely through their economic self-interest of seeking the most profitable investments, financial interests may help states maintain peaceful international bargains. One advantage of our analysis is that the limits of this incentive are clear, and we are able to identify when wars will occur despite the effect of financial interests. Moreover, our argument does not preclude that the peaceful effects others have pointed to in this literature may also motivate financiers beyond our proposed mechanism.
Model

To model the bargaining problem with power shifts and a financial market, we start with the canonical model of bargaining and war and then add a profit maximizing bond market. As we will see, the bond market has important effects on the probability of conflict arising from the commitment problem.

Players and Resources

There are two states, Home (H) and Foreign (F), that interact over two periods, \( t \in \{1, 2\} \). Home can borrow against the future through the bond market or lender, L, for an amount that it can use in bargaining with Foreign.\(^6\) Namely, in period 1, H can sell one-period discount bonds, \( B \), at a “discount price” \( q < 1 \). The countries must bargain over an international flow of benefits each period, normalized to size 1, plus \( qB \) if Home chooses to borrow knowing they will need to pay back \( B \) in the future. Foreign makes a take-it-or-leave-it proposal \( x_t \), in each of two periods where Home receives \( x_t \) and Foreign receives the remainder of the benefits, as well as any amount borrowed by H from the bond market.

Future periods are discounted at the common rate \( \beta \in (0, 1) \). We think of period 2 as representing the entire future, therefore payoffs in period 2 are valued at \( \beta / (1 - \beta) \) times the value of payoffs in period 1. Hence, H’s total utility for a peaceful sequence of bargain offers is

\[
V_H = x_1 + \frac{\beta x_2}{1 - \beta},
\]

and F’s total peaceful utility without borrowing is

\[
V_F = (1-x_1) + \frac{\beta}{1 - \beta} (1-x_2).
\]

\(^6\)Alternatively, the lender may be another state. In this case, the lender may be strategic, a possibility we consider below.
With borrowing by $H$, $F$ additionally gains a one-time transfer of $qB$.

**The Bond Market**

We start by considering a financial market consisting of profit-maximizing traders, collectively acting as the non-strategic lender $L$, who are willing to buy bonds from $Home$. $L$ can alternatively lend money at an international interest rate $r > 0$. If feasible, $Home$ commits to pay back $L$ when $Home$ borrows. Therefore, $Home$ does not default strategically and only defaults when it lacks the ability to pay back its bonds. This happens with probability $\delta$, a value that will be determined endogenously and described in a subsequent section. Other than maximizing its return, $L$ has no further interest in outcomes for $Home$ and $Foreign$. We also assume $Home$ places a sufficient value on the future, $\beta > \frac{1}{1+r}$, so that it does not have an incentive to borrow against future wealth in order to consume more today purely due to impatience. For simplicity, we also do not allow other states, like $Foreign$, to buy $Home$’s bonds.\(^7\)

Our commitment assumption that $Home$ does not default strategically is a stark simplification, but could be justified in two ways in a more general model. One, $Home$ may want to borrow for a variety of reasons (such as consumption smoothing) and loses access to markets after defaulting. Two, the model could be extended to the case where shifts in power happen repeatedly and $Home$ must preserve a good reputation with bond traders in order to preserve liquidity in case of future shifts.

Empirically countries are borrowing for many reasons, and bond income goes into general funds that are used for many things like building roads, social welfare programs, and foreign aid. While our model considers the single war oriented motivation for borrowing, its real world implication is an association between the price of public debt and peace.

\(^7\)In this sense the financial resources are coming from outside the strategic interaction. We refer to this later as outside money. The case for inside money is considered in the extensions.
War

In our model war is represented by a costly lottery. Each state wins the war with some probability and pays costs $\kappa > 0$. The winning state captures the value of the international flow of benefits in both periods. The losing state can still consume any domestic resources, but can no longer challenge the winning state for a portion of the international pie. We can think of the losing state as being disarmed.

In this model, states win a war with an exogenously determined probability. $H$’s probability of winning a war, or strength, in period 1 is $s$, and Foreign’s probability of victory at time 1 is $1 - s$. If war occurs in period 1, then the value of war to $H$ is

$$\frac{1}{1 - \beta} s - \kappa.$$

We assume this value is positive to avoid uninteresting cases.

In the event that $H$ wins the war, $H$ captures the entire international pie of size 1 today and in the future. This is multiplied by $\frac{1}{1 - \beta}$ to account for period 2 representing the entire future. $F$’s value for war in period 1 and the value of war for both states in period 2 can be similarly defined.

Exogenous Power Shifts

Now consider the impact of a potential exogenous power shift, which occurs at the end of period 1 with probability $\rho \in (0, 1)$. If no shift occurs, $H$’s probability of victory remains the same as in period 1, at $s$. If a shock does occur, $H$’s probability of victory increases to $\theta s$ where $\theta \in (1, \frac{1}{2})$. Note that for simplicity we only consider positive shocks to $H$’s exogenous probability of victory.

Timing

Putting it all together, period 1 proceeds as follows:
1. Home and Foreign both learn the values of $\rho$ and $\theta$.

2. Home chooses how much to borrow, $qB$, this period.

3. Foreign makes a take-it-our-leave-it offer to Home of $x_1$, where Foreign gets the remainder of the international pie and whatever Home borrowed or Foreign declares war.

4. Home either accepts or rejects the offer. If Home accepts, the states peacefully consume their allocations. If Home rejects, war occurs, and states receive their war payoffs.

5. Power shifts occur with probability $\rho$.

Period 2 proceeds in the same way except that steps 1, 2, and 5 are skipped, and that if war has occurred, the winner receives the whole international pie. Our solution concept is Subgame Perfect Equilibrium (SPE).

**Analysis**

**Power Shifts without Borrowing**

In this portion of the analysis, we explore how Home behaves when borrowing is not a possibility. We can see how our problem relates to the classic explorations of the commitment problem in Fearon (1995; 2004) and Powell (1999; 2004; 2006). This relationship is easiest to see when $\rho = 1$, such that a power shift will occur with certainty. When this is the case, $H$ has an initial war value in period 1 of

$$\frac{1}{1-\beta}s - \kappa,$$
which, in period 2, increases to 
\[
\frac{1}{1-\beta} \theta s - \kappa
\]
after the power shift takes place.

Anticipating this shift in power, Foreign prefers war versus any bargain when its period 1 war value is greater than the largest bargain \(H\) can credibly commit to in the future. This amount is the entire pie today, plus the entire future bargain value, minus the discounted value of Home’s period 2 war value. That is,
\[
\frac{1}{1-\beta} (1 - s) - \kappa > 1 + \frac{\beta}{1-\beta} - \beta \left[ \frac{1}{1-\beta} \theta s - \kappa \right].
\]

After some rearrangement, we can solve for the minimum size \(\theta\) that leads to war. This result is presented as Lemma 1.

**Lemma 1.** When there is no borrowing and \(\rho = 1\), then war occurs if and only if \(\theta > \frac{1-\beta}{\beta s} (1 + \beta) \kappa + \frac{1}{\beta}\).

This result is essentially identical in form to the conditions on war found previously in the literature for one period, exogenous shifts in power (Powell, 2006). The size of the shift necessary to cause war is increasing in the costs of war, decreasing in the discount rate, and decreasing in the initial probability of victory for the positively shifting power.

The first modification to this canonical case is that we allow for the possibility that \(\rho \neq 1\). Hence, with probability \(\rho\), a shift as above will occur, while with probability \(1 - \rho\), a shift will not occur. This leads to a modification of the condition in Lemma 1, which is stated in Lemma 2.

**Lemma 2.** When there is no borrowing, then war occurs if and only if

\[
\theta > 1 + \frac{1-\beta}{\beta \rho} \left( 1 + \frac{1+\beta}{s} \kappa \right).
\]
Figure 1: War with uncertain power shifts.

(The proof of all results can be found in the appendix.)

As can be seen, when the probability of the shift goes to one, smaller changes in Home’s military capabilities, \((\theta)\), can lead to war as a result of the commitment problem. This can be seen in the graph in Figure 1. The condition for war for a probabilistic shift otherwise behaves in the same manner as when the shift is certain, with the the size of shift needing to increase in the costs of war, and decrease in the discount rate and initial probability of victory.

**The Effect of Borrowing**

Lemma 2, represented by the shaded region of Figure 1 defines the circumstances where Home may choose to borrow from the bond market in this model. Home only desires to borrow in order to avoid war and therefore only borrows when inequality (1) is satisfied. There are two further conditions on borrowing explored in detail below. First, the buyer of the bond must prefer to loan \(H\) a sufficient amount of money to avoid war to its outside option for that money. Here, that outside option is the exogenously given global risk-free interest rate, \(r\). Second, \(H\) must prefer to borrow a sufficient amount of money to avoid war.
to the payoff from war.

Borrowing increases the possibility of peace, by transferring money from $H$’s future income to today where it can be credibly be transferred to $F$. The amount of borrowing necessary to prevent war can be derived directly from inequality (1), since $F$’s utility from fighting in period 1 must be at least as much as its utility from getting all of the pie in period 1, its portion of the pie in period 2, and any amount $H$ borrows to appease $F$. Borrowing amount $qB$ today is therefore just sufficient to prevent war when

$$\frac{1}{1-\beta}(1-s) - \kappa = 1 + \frac{\beta}{1-\beta} + \beta\kappa - \rho \left( \frac{\beta}{1-\beta} \theta s \right) - (1 - \rho) \left( \frac{\beta}{1-\beta} s \right) + qB^*.$$ 

We can solve for the minimum loan $qB^*$ required to prevent war, presented in Lemma 3. Note that if $qB^* < 0$, then no borrowing is required.

**Lemma 3.** When there is borrowing, $F$ requires at least $qB^* = \frac{s\beta \rho}{1-\beta} (\theta - 1) - s \left( 1 + \frac{1+\beta}{s} \kappa \right)$ to opt for peace.

$H$ome has to borrow more when the probability of a power shift is higher, since Foreign will be more worried about the possibility of power transition, as well as when the size of the possible shift increases. $H$ome borrows less when the cost of war is higher, since $F$ is less worried about its willingness to fight wars. The effect of the discount rate and the initial probability of victory are ambiguous on the borrowed amount.

**The Bond Market’s Perspective**

$H$ome defaults on its loans in period 1 when it lacks the resources to pay back bond holders. Let the probability of default be $\delta$ (this will be determined endogenously later). Before calculating the probability of default for various parameters, first consider the Lender’s bond purchasing decision. By borrowing from $L$, $H$ome promises to pay back $B$ to bond holders in period 2, while receiving $qB$ today. However, $H$ome only pays $L$ back with endogenous probability $1-\delta$. Alternatively, market actors could lend $qB$ on the international market.
and receive back \((1 + r) q B\) for sure next period. By no arbitrage, the expected return must be equivalent under both investment schemes, so that

\[
(1 - \delta) B = (1 + r) q B
\]

\[
q = \frac{1 - \delta}{1 + r}.
\]

Hence, we can calculate the price of the bond, \(q\), as a function of the default rate \(\delta\) and the going risk-free rate of \(r\).\(^8\)

Now consider Home’s borrowing decision. Home pays a premium on borrowed money and, therefore, only borrows when either \(q\) is low relative to future value of consumption—that is when Home would prefer to consume tomorrow’s income today because interest rates are low and it is impatient. We rule out this possibility in our model with the assumption that \(\beta > \frac{1}{1+r}\)—or when liquidity is constrained, so transfers are needed in order to avoid war.\(^9\)

Therefore, Home does not borrow when it can buy peace without borrowing, meaning when inequality (1) does not hold.

When inequality (1) does hold, Home prefers to borrow and avoid war so long as borrowing and avoiding war gives a higher payoff than fighting. In order to calculate this, we must first calculate the value of borrowing. If borrowing is sufficient to prevent war in the initial period, two things may happen in period 2. Either Home experiences a positive shift with probability \(\rho\) and can borrow, and pay-back, up to the entire present value of future

\(^8\)The assumption of total default is based on the idea that the inability to pay back the bond makes future borrowing for any purpose impossible so there is no point in paying back part of the debt. If instead we assumed that there was partial default, it would have the natural effect of making risky lending more attractive and risky borrowing more costly, but not fundamentally change any result.

\(^9\)Note that in traditional macro models, \(H\) would borrow in order to smooth consumption across periods. This incentive does not come into play in this model since we have assumed linear utility in consumption.
payoffs determined by $H$’s war value in this case, i.e.

$$\beta \left( \frac{1}{1-\beta}s - \kappa \right),$$

or a shift does not happen and it can only pay bond holders back with

$$\beta \left( \frac{1}{1-\beta}s - \kappa \right).$$

As determined above, $B^*$ is the smallest bond necessary to prevent war, with bond price $qB^* = \frac{1 - \delta}{1 + r} B^*$. Therefore, there exists three borrowing regions. In the region where $B^* > \beta \left( \frac{1}{1-\beta}s - \kappa \right)$, Home cannot commit to ever repaying and default is assured, so $\delta = 1$. In the region where $B^* < \beta \left( \frac{1}{1-\beta}s - \kappa \right)$, then $\delta = 0$ and default never occurs. If $B^*$ is between these two values, then $\delta$ is equal to the probability of no shift, or $1 - \rho$. These regions imply a $q$ of 0, $\frac{1}{1+r}$, and $\frac{\rho}{1+r}$, respectively. The bond market will only lend to $H$ in the latter two cases.

**The Borrower’s Perspective**

Although borrowing can reduce the likelihood of war, war may still occur if Lenders will not lend as much as Home needs, or if Home does not think borrowing is a better deal than going ahead and fighting a war. First, consider solutions where $\delta = 0$, which is only the case when

$$\frac{1}{1 + r} B^* = \frac{s \beta \rho}{1 - \beta}(\theta - 1) - s \left( 1 + \frac{1 + \beta}{s} \kappa \right)$$

and

$$B^* < \beta \left( \frac{1}{1 - \beta}s - \kappa \right).$$
The bond market will not offer such a loan when

\[ s \left( 1 + r \right) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] > \beta \left( \frac{1}{1 - \beta} s - \kappa \right) \]

In any situation where borrowing is considered, \( H \) cannot appease \( F \) in period 1 simply with the existing international pie, so it will either go to war in the first period, or it will give up all of the pie and a borrowed amount in the first period while gaining part of the pie in the second period. Substituting the above determined value of \( B^* \), this inequality indicates that war is still preferred to no-default borrowing when:

\[ \frac{1}{1 - \beta} s - \kappa > \beta \left[ \rho \left( \frac{1}{1 - \beta} \theta s - \kappa \right) + (1 - \rho) \left( \frac{1}{1 - \beta} s - \kappa \right) - s \left( 1 + r \right) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] \right]. \]

Analogously, a risky loan with \( \delta = 1 - \rho \) gives the bond amount

\[ \frac{\rho}{1 + r} B^* = \frac{s \beta \rho}{1 - \beta} (\theta - 1) - s \left( 1 + \frac{1 + \beta}{s} \kappa \right), \]

such that

\[ \beta \left( \frac{1}{1 - \beta} s - \kappa \right) < B^* < \beta \left( \frac{1}{1 - \beta} \theta s - \kappa \right). \]

In other words, the bond market will still not lend if

\[ \frac{s}{\rho} \left( 1 + r \right) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] \]

\[ > \beta \left( \frac{1}{1 - \beta} \theta s - \kappa \right). \]

Here there is a \( 1 - \rho \) chance of default, in which case \( H \) does not have to pay off the loan, but the effect on \( H \)'s finances is canceled out by the difference in pricing from \( q = \frac{1}{1 + r} \) to \( q = \frac{\rho}{1 + r} \). \( H \) therefore prefers war to either kind of borrowing when

\[ \frac{1}{1 - \beta} s - \kappa > \beta \left[ \rho \left( \frac{1}{1 - \beta} \theta s - \kappa \right) + (1 - \rho) \left( \frac{1}{1 - \beta} s - \kappa \right) - \rho \left( \frac{s}{\rho} \right) \left( 1 + r \right) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] \right]. \]
Figure 2: Peace bonds and the incentives to borrow and lend.

These conditions are summarize graphically in Figure 2, war occurs if (1) holds and Foreign prefers preventive war to peace without borrowing, and either (2) Home prefers war to the amount of borrowing that prevents war, or (3) the bond market is not willing to offer a loan for the amount Home needs to prevent war either the lender’s lending constraint or Home’s borrowing constraint may be violated. When they do it is a consequence of both the exogenous costs of borrowing, \( r \), and the endogenous elements, \( \delta \) and \( q \). But clearly access to credit markets still leaves room for war. We express these latter two conditions in Lemmas 4 and 5, where the condition for Home being willing to fight rather than borrow is rearranged and simplified:

**Lemma 4.** Home prefers fighting to borrowing when

\[
\theta \rho > \rho + \frac{1-\beta}{\beta} - \frac{\kappa}{s} \left( \beta + \frac{1-\beta(1-r)}{\beta(1+\beta s)} \right).
\]  

**Lemma 5.** The bond market will not offer a loan if

\[
\frac{\kappa}{\rho} \left( 1 + r \right) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \frac{\beta}{s} \right] > \beta \left( \frac{1}{1-\beta} \theta s - \kappa \right).
\]  

In the following section, we analyze the comparative statics of inequalities (1), (2), and
Figure 3: Borrowing, war and the risk of default.

(3) from Lemmas 2, 4, and 5 respectively.

Figure 3 shows the different conditions one might encounter in a given crisis. In this figure the borrower or lender’s constraint might bind, but we show the lender’s constraint in this example. As can be seen the market is often willing to make risky loans that will avoid war, but this is not always the case.

Potential for Conflict Despite the Possibility of Borrowing

Recall inequalities (1), (2), and (3). War occurs whenever (1) is satisfied and borrowing fails to occur because either $H$ prefers paying the cost of war to borrowing at a high premium (as in equation (2)) or (from equation (3)), $L$ will not lend because $H$ would default at any interest rate that $L$ is otherwise willing to offer. Proposition 1 shows that increasing either the probability of the shift $\rho$ or the size of the potential shift $\theta$ increases the likelihood of a power transition war.

**Proposition 1.** There exist shifts that cause war even with borrowing. Increasing the expected size of a shift increases in the potential for conflict.

The proof follows fairly directly from the inequalities. Qualitatively, this proposition
suggests that borrowing does not upend our thinking about power shifts, and that the
dynamics seen in the non-borrowing case still hold. As illustrated in Figure 3, borrowing
does not prevent all wars as \( \theta \) increases, and the argument for \( \rho \) is analogous. Another way
to phrase this proposition is that if a power shift described by \((\theta, \rho)\) first-order stochastically
dominates the shift \((\theta', \rho')\), then \((\theta, \rho)\) has a higher potential for conflict. So when shifts
are more likely or more extreme, war becomes more likely. Moreover, even as \( \beta \to 1 \), some
power shifts may cause war. Increasing \( \beta \) makes condition (1) easier to satisfy. Both (2) and
(3) are always satisfied as \( \beta \to 1 \): the lower bound constraint of (2) converges to \( \rho \) which is
always less than the lefthand side value of \( \theta \rho \) since it assumed that \( \theta \in (1, 1/s) \) and \( s < 1 \);
(3) is always satisfied since the lefthand side goes to positive infinity while the righthand
side is finite.

We next look at instances in which changes in the type of shift increase the potential for
conflict by increasing the range of parameters for which war occurs. Two shifts, \((\theta, \rho)\) and
\((\theta', \rho')\), have the same expected size if

\[
\rho \left( \frac{\beta}{1-\beta} \theta s \right) + (1 - \rho) \left( \frac{\beta}{1-\beta} s \right) = \rho' \left( \frac{\beta}{1-\beta} \theta' s \right) + (1 - \rho') \left( \frac{\beta}{1-\beta} s \right) = \rho s \theta s + s - \rho s = \rho' \theta' s + s - \rho' s \]
\[
(\theta - 1) \rho s = (\theta' - 1) \rho' s.
\]

Note that if two shocks have the same expected size, then \( F \) expects to offer \( H \) the same
size pie in the second period, so it can be bought off with the same transfer amount:

\[ qB^* = q'B'. \]

Because traders on the bond market care as much about the likelihood of repayment as
about the rate of repayment, they may refuse to lend in situations where \( H \)'s expected power
shift is extreme but improbable. Even when \( H \) and \( F \)'s calculations are not affected because
there is no change in the expected size of the shift, \( L \) may prefer not to lend and leave \( H \)
Figure 4: Mean preserving lotteries and the solving the commitment problem through debt with no recourse but war.

**Proposition 2.** For shifts of the same expected size, lower probability but more extreme shifts increase the potential for conflict.

In other words, if a power shift \((\theta, \rho)\) second-order stochastically dominates a shift \((\theta', \rho')\), then \((\theta', \rho')\) has a higher potential for conflict, even though the shift \((\theta', \rho')\) is a mean-preserving spread of shift \((\theta, \rho)\). This conclusion can be generalized to a continuous distribution of shifts.

Figure 4 illustrates this proposition for shifts that could induce risky borrowing. Shifts that are more extreme, but lower probability, are more dangerous for peace because it is harder for bond markets to provide the necessary liquidity. For example, to the lower right a shift of expected size \(\gamma\) can be compensated for by selling a risky bond, but a riskier shift of the same expected size leads to war. This has qualitative implications for interstate conflict. For instance, this may explain differences in how countries respond to purchases of conventional military equipment (low \(\theta\), high \(\rho\)), as opposed to investments in nuclear weapons technology (high \(\theta\), low \(\rho\)). Proposition 2 suggests that the latter would more frequently cause conflict. Similar effects can be seen in Figure 5 where the graph is in terms...
Shifts of the Same Expected Size

Consider an initial shift of \((\theta, \rho)\). Proposition 2 states that if a shift \((\theta', \rho')\) is of the same expected size, then the more extreme/less likely \((\theta', \rho')\) is, the greater the potential for conflict. The left panel plots out how \(\theta'\) changes in \(\rho'\). The right panel plots how the left hand and right hand sides of inequality (3) change in \(\rho'\). The left hand side (the blue line) indicates the necessary amount of bonds to avoid war. The right hand side (the green line) indicates the maximum amount of bonds \(L\) is willing to buy. When the blue line is above the green line, war results.

Figure 5: The Effect of Changing \(\rho'\) on Shifts of the Same Expected Size

The War Threshold
of the demand for bonds.

Finally, \( H \) may not be able to borrow at rates that it can afford if \( L \) has other, safer lending opportunities elsewhere because \( r \) is high.

**Proposition 3.** Increasing the world risk-free interest rate, \( r \), increases the potential for conflict.

From this proposition, we conclude that exogenous shocks to the world economy that increase the cost of capital will also increase the likelihood of war. Such exogenous shocks could come in many forms. Burgeoning conflict in other parts of the world may cause increasing rates, which in turn causes bond traders to pick and choose where they lend. This could serve as a contagion channel for war to spread.

**Strategic Lenders and the Existence of Preventive War**

The model in this paper contains a number of simplifying assumptions. It is reasonable to ask how relaxing some of these assumptions might impact the main result of the paper – namely, Proposition 1 which states that sovereign borrowing may alleviate, but not wholly eliminate the possibility of preventive war. We make three key assumptions in our model: (1) full commitment to repay loans, (2) loans are “outside money”, and (3) loans are made by a “non-strategic” bond market.

To understand these assumptions clearly, first imagine that we reversed assumption (3).\(^{10}\) There are two countries, \( H \) and \( F \) as in our baseline model, but no “non-strategic” bond market to provide loans. Instead, the only source of loans for \( H \) is actually \( F \) who will certainly be strategic in its loan making decisions. In this setting, \( F \) may have an added incentive to provide loans to \( H \) beyond what a non-strategic bond market is willing to supply since \( F \) internalizes the costs and risks of war while the bond market only cares about the

\(^{10}\)Note that all else equal, relaxing assumptions (1) or (2) just makes the result that preventive war exists in this setting easier to obtain.
Figure 6: Inter-state system with the potential for inside money

return and risk to its loan. However, having reversed (3), does it then make sense to maintain assumptions (1) and (2)?

The answer to both is clearly “no.” Assumption (2) is now nonsensical due to physical constraints. The model is about two countries bargaining and fighting over the entire pie between them. If there were some outside source of money that countries have internal access to, then why can they not use this money for transfers directly? Why can this money be tapped for loans, but cannot be captured in war? In fact, if countries have access to unlimited internal funds, preventive war in the sense of Powell (2004) is immediately ruled out. If there is some limited source of internal funds, then it is unclear why these should not be included in the initial pie at issue between the two countries.

Moreover, maintaining assumption (1) in this setting would be to say that $H$ cannot commit to making transfers after a power shift, but can fully commit to paying back a loan. When dealing with an outside bond market, commitment could make sense for the reasons presented in the base line model – maintaining access to bond market will allow for future consumption smoothing and future loans to avoid war in the event of future power shifts. $H$ may face some of these same incentives when receiving loans from $F$. However, its unclear why $H$ defaulting on promised future loan payments would be any different from defaulting on future transfers. To the extent that we assume $H$ cannot commit to transfers to $F$ in the future, we must also assume that $H$ cannot commit to future loan payments to $F$ in the
That is, if we reverse assumption (3), we must also reverse assumptions (1) and (2). The natural thing to ask next, would be what if there are other strategically connected countries, countries $A$, $B$, $C$, ... that may want to provide $F$ a loan for strategic reasons that is “inside money” in the sense that it is part of some country’s resources within a strategically connected system?

Consider the international system depicted in Figure 6. First, we must clarify what it means for a country other than $H$ and $F$ to be strategically connected to $H$ and $F$. One definition would that this other country $i$’s security is strategically impacted by a war between $H$ and $F$. That is, either $H$ or $F$ must be capable of going to war with $i$ either today or at some point in the future, possibly contingent on other wars occurring. That is, $H$, $F$, and $i$ are in a connected network where there is some path by which conflict can spread from $H$ and $F$ to $i$.

In this example we see that country $C$ is connected to both $H$ and $F$ on some path and therefore is strategically relevant. Countries $A$ and $B$ are not strategic connected, though they might be connected through market changes we discuss below. In this system we can think of the bond market $L$ as being an isolated actor who cannot be “attacked" or invaded by any other state. In this sense, $L$ is providing money outside of strategic military considerations and hence what we call “outside money.”

So what if a current development between $F$ and $H$ has the potential for a power shift and conflict where $C$ can use its resources to lend to $H$ to prevent a war that might bring $H$ to $C$’s doorstep? Fortunately, it is straight forward to show that there is no way for countries to transfer or loan each other “inside money" in a connected network of states in order to avoid war. This gives us Proposition 4, whose technical details can be found in the appendix.

**Proposition 4.** There always exists values for power shifts, cost of war, and discount factors in the international system game such that preventive war occurs in a subgame perfect
equilibrium when strategic lenders can loan each other inside money.

However, what if there exists both a connected network of strategically interested countries and some outside source of money (a group of unconnected or non-strategic countries)? Clearly, by Proposition 4, some amount of outside money will be necessary to maintain peace for some power shifts. However, we have demonstrated in this paper that the amount of outside money from a nonstrategic source will always be limited by a combination of the going interest rate \( r \) and endogenous default. In other words, preventive war always exists in this environment. Moreover, shifts in power will be more dangerous exactly as described in this paper – when they are greater in expected value, when \( r \) is large, and, for a fixed expected value, when they are low in probability, but potentially extreme in size.

We might imagine that countries are strategically connected to \( H \) and \( F \) in some other way that is not related to security. Perhaps there is something economically or culturally unique about \( H \) and \( F \) that war would undermine. Or for humanitarian reasons, these outside countries internalize to some degree the costs of war to the population of \( H \) and \( F \). Our results clarify that it is only in these types of settings that outside countries would be willing to make strategic loans that potentially violate the preventive war logic of this paper. In this setting, the only limits on the terms of loans that could prevent war would be the extent of the cultural or economic interest in avoiding war or the level of internalization of harm to other countries. While important, these considerations are far afield from what we hope to accomplish in this paper and are equally relevant to traditional transfers as well as loans. We leave it to future research to consider these questions in depth.

Finally, in an extended model with an endogenously determined risk-free rate of return and many states all acting as potential borrowers and lenders, power shifts and war have the potential to transmit economic implications throughout the system. Power shifts that do not cause war, but require borrowing to remain peaceful, act as a positive demand shock on credit, as illustrated in Figure 7, while full wars act as a negative supply shock as the waring states lose resources to loan to international markets. As mentioned before, this may lead
to a contagion effect of conflict as it causes $r$ to rise endogenously pushing other peaceful settlements out of reach. However, short of war, the increase in $r$ would also impact the distribution of the economic pie. Suppliers of capital would benefit from the improved terms, while demanders of capital would suffer from increased rates. However, higher rates may also hurt lenders if it causes borrowers to default on their loans.

**Conclusion**

In this paper we built a model of commitment problems and sovereign lending. We were able to demonstrate that war due to commitment problems may still occur in this setting, and that the potential for conflict is increasing in the expected size of a shift. Moreover, significant empirical implications result from our simple model. First, we can characterize that extreme but unlikely shifts are more dangerous than moderate but likely shifts. Second, borrowing rates in growing countries facing relatively declining adversaries will be heightened. Third, exogenous increases in the real risk-free rate will increase the potential for conflict due to commitment problems.

The model we pursued here is highly simplified. A number of immediate extensions
would provide further insight on the connections between international finance and conflict. Allowing for endogenous military spending may add an interesting dimension where potentially rising states avoid preventive war by actively constraining future military spending by building debt and spending it on non-military social programs. Additionally, more general approaches to modeling power shifts, fighting, and sovereign borrowing may identify a number of more nuanced results.

Particularly relevant extensions would bring a greater level of sophistication to the economic side of the model. First, in this paper, we have set the risk-free rate exogenously. However, in reality this rate will vary with the endogenous demand on capital. When states demand bonds for non-productive reasons such as avoiding war, this demand shock for capital raises the risk-free rate for all other borrowers. This means less productive economic activity will be financed and any other states needing to borrow due to shifts in power will face a higher price for capital – perhaps even causing a war that would have been avoided when the risk-free rate was lower. Second, persistent shifts in power arise naturally in this setting as the result of long-term economic growth. A full macroeconomic model of the relevant countries would provide a fuller understanding of how power shifts due to economic growth and access to credit relate. Indeed, both declining powers and potential financiers will be deeply concerned about the duration of a positive economic shock in almost directly opposing ways. Third, directly building in consumption smoothing and other reputational concerns over access to credit markets would allow the model to endogenously determine the credibility of paying back loans.

An historical analysis of cases on this topic would be of great interest. However, we caution that this is potentially complex. Modern states issue and carry debt all the time. Money is fungible. Tying a particular loan to a particular international crisis is often quite difficult and potentially counterproductive to the more general point of this paper. Our paper argues that access to credit markets relieves the liquidity constraint states face when bargaining. It does not say that states issue loans specifically tied to transfers. Consider the
following analogy of Bob who has $100,000 and needs to buy a home that minimally costs $100,000 in order to shelter his family, but would also like to buy a car. Without access to credit, Bob pays $100,000 for the house and goes without the car. With access to credit, Bob could take out a mortgage for $80,000 and purchase the home with only $20,000 of cash. This means that Bob has $80,000 left over to spend on a car or other items. Not observing Bob take out a car loan is not evidence that credit did not make purchasing a car possible. Similarly, many states issue large amounts of debt while mandatory spending often makes up a majority of the state budget (for instance the United States). In this context, not observing a loan specifically earmarked for transfer to another state is not evidence against credit market access making the transfer possible. In some sense every rising power, like say China, who both has debt and peace is evidence, if not very convincing evidence, for our theory. Instead, a viable empirical test for our model might study the impact on conflict outcomes of the real risk free-rate of return and other factors that affect the cost of sovereign borrowing. But as there are also many economic and domestic political forces driving borrowing decisions, a careful version of that empirical analysis is an article of its own.

There are few, if any, models connecting the bargaining model of war with the deep literature studying international macroeconomics and finance. Even from the simple model presented in this paper, a number of non-obvious empirical connections arise between macroeconomic indicators and the potential for conflict. Beyond the results presented here, this paper hopes to contribute to both literatures by providing a framework on which to build more sophisticated models at the intersection of these two literatures.
Appendix

Lemma 1

Proof. $F$ prefers war to peace exactly when

$$\frac{1}{1-\beta} (1-s) - \kappa > 1 + \frac{\beta}{1-\beta} - \beta \left[ \frac{1}{1-\beta} \theta s - \kappa \right]$$

$$\frac{1}{\beta} (1-s) - \frac{1-\beta}{\beta} \kappa > \frac{1-\beta}{\beta} (1 + \beta \kappa) + (1 - \theta s)$$

$$1 - s - \beta + \beta \theta s > (1 - \beta) (1 + (1 + \beta) \kappa)$$

$$\beta \theta s > (1 - \beta) (1 + \beta) \kappa + s$$

$$\theta > \frac{1-\beta}{\beta s} (1 + \beta) \kappa + \frac{1}{\beta}$$

\[\Box\]

Lemma 2

Proof. $F$ prefers war to peace exactly when

$$\frac{1}{1-\beta} (1-s) - \kappa > 1 + \frac{\beta}{1-\beta} + \beta \kappa - \rho \left( \frac{\beta}{1-\beta} \theta s \right) - (1 - \rho) \left( \frac{\beta}{1-\beta} s \right)$$

$$\frac{1}{1-\beta} (1-s) - \kappa > 1 + \frac{\beta}{1-\beta} + \beta \kappa - \frac{\beta}{1-\beta} [\rho \theta s + (1 - \rho) s]$$

$$\frac{1}{\beta} (1-s) > 1 + \left( \frac{1-\beta}{\beta} \right) (1 + (1 + \beta) \kappa) - s [\rho \theta + (1 - \rho)]$$

$$\frac{1}{\beta} [1 - s - \beta + \beta s (\rho \theta + 1 - \rho)] > \left( \frac{1-\beta}{\beta} \right) (1 + (1 + \beta) \kappa)$$

$$s [\beta (\rho \theta + 1 - \rho) - 1] > (1 - \beta) (1 + \beta) \kappa$$

$$\beta \rho \theta > \frac{1-\beta}{s} (1 + \beta) \kappa + 1 - \beta + \beta \rho$$

$$\beta \rho \theta > (1 - \beta) \left( \frac{1+s}{s} \kappa + 1 \right) + \beta \rho$$

$$\theta > 1 + \frac{1-\beta}{\beta \rho} \left( 1 + \frac{1+s}{s} \kappa \right)$$

\[\Box\]
Lemma 3

Proof. $F$ is indifferent between war and peace when

$$\frac{1}{1 - \beta} (1 - s) - \kappa = 1 + \frac{\beta}{1 - \beta} + \kappa - \rho \left( \frac{\beta}{1 - \beta} \theta s \right) - (1 - \rho) \left( \frac{\beta}{1 - \beta} s \right) + qB^*$$

$$\frac{1}{\beta} (1 - s) = 1 + \left( \frac{1 - \beta}{\beta} \right) (1 + (1 + \beta) \kappa) - s [\rho \theta + (1 - \rho)] + \frac{1 - \beta}{\beta} qB^*$$

$$s [\beta (\rho \theta + 1 - \rho) - 1] = (1 - \beta) (1 + \beta) \kappa + (1 - \beta) qB^*$$

$$\beta \rho \theta = \frac{1 - \beta}{s} (1 + \beta) \kappa + 1 - \beta + \beta \rho + \frac{1 - \beta}{s} qB^*$$

$$\theta - 1 - \frac{1 - \beta}{\beta \rho} (1 + 1 + \beta) \kappa = \frac{1 - \beta}{s \beta \rho} qB^*$$

$$qB^* = \frac{s \beta \rho}{1 - \beta} (\theta - 1) - s (1 + 1 + \beta) \kappa$$

Lemma 4

Proof. $H$ prefers war to borrowing exactly when

$$\frac{1 - \beta}{1 - \beta} s - \kappa > \beta \left[ \rho \left( \frac{1 - \beta}{1 - \beta} \theta s - \kappa \right) + (1 - \rho) \left( \frac{1 - \beta}{1 - \beta} s - \kappa \right) - s (1 + r) \left[ \frac{\beta \rho (1 - 1)}{1 - \beta} - (1 + 1 + \beta) \kappa \right] \right]$$

$$\frac{1 - \beta}{1 - \beta} s - \kappa > \beta \left[ \rho \left( \frac{1 - \beta}{1 - \beta} \theta s \right) + (1 - \rho) \left( \frac{1 - \beta}{1 - \beta} s \right) - \kappa - s \beta \rho (\theta - 1) (1 + r) + s (1 + r) (1 + 1 + \beta) \kappa \right]$$

$$\frac{s}{\beta} - \frac{1 - \beta}{\beta} \kappa > \rho \theta s + (1 - \rho) s - (1 - \beta) \kappa > s \beta \rho (\theta - 1) (1 + r) + (1 - \beta) (1 + r) \left( s + (1 + \beta) \kappa \right)$$

$$\frac{s}{\beta} - \frac{1 - \beta}{\beta} \kappa > \rho \theta s [1 - \beta (1 + r)] + (1 + r) s + s \beta \rho (1 + r) + (1 - \beta) [s (1 + r) + (1 + r + \beta + r \beta) \kappa - \kappa]$$

$$\frac{s}{\beta} - \frac{1 - \beta}{\beta} \kappa - (1 - \rho) s - s \beta \rho (1 + r) - (1 - \beta) [s (1 + r) + (r + \beta + r \beta) \kappa] > \rho \theta s [1 - \beta (1 + r)]$$

$$s \left( \frac{1}{\beta} - 1 \right) + \rho - s (1 + r) (1 + \beta + \beta \rho) - \kappa (1 - \beta) \left[ r + \beta (1 + r) + \frac{1}{\beta} \right] > \rho \theta s [1 - \beta (1 + r)]$$

$$\theta > \frac{1}{1 - \beta (1 + r)} \left[ 1 + \frac{1 - \beta}{\beta \rho} - \frac{1 + r}{\rho} (1 - \beta + \beta \rho) - \frac{\kappa}{s \rho} (1 - \beta) \left[ r + \beta (1 + r) + \frac{1}{\beta} \right] \right]$$

$$\theta > \frac{1 - \beta (1 + r)}{1 - \beta (1 + r)} \left[ 1 - \beta (1 + r) + \frac{1 - \beta}{\beta \rho} \left[ 1 - \beta (1 + r) \right] - \frac{\kappa}{s \rho} \left( r - r \beta + \beta - \beta^2 + r \beta - r \beta^2 + \frac{1}{\beta} - 1 \right) \right]$$

$$\theta > 1 + \frac{1 - \beta}{\beta \rho} - \frac{\kappa}{s \rho [1 - \beta (1 + r)]} \left[ \beta (1 - \beta (1 + r)) + \frac{1}{\beta} (1 - \beta (1 - r)) \right]$$

$$\theta > \frac{1 - \beta (1 + r)}{s \rho [1 - \beta (1 + r)]} \left[ \beta (1 - \beta (1 + r)) + \frac{1}{\beta} (1 - \beta (1 - r)) \right]$$
Note that the sign changes in the middle of the solving since $\beta > \frac{1}{1+r}$.

Proposition 1

Proof. The first statement holds since parameters exist where both (1) and either (2) or (3) is satisfied. For instance, fix a $\theta$ that satisfies (1). Then (3) holds when the bracketed value is positive (this is assured when $\kappa$ is small and $\beta$ is close to 1) and $r$ is large.

For the second statement, there are three ways expected shift size can increase. Either $\theta$ increases, $\rho$ increases, or both increase.

**$\theta$ increases:** Increasing $\theta$ directly increases the LHS of (1), while the RHS is unaffected, which directly increases the range of parameters that satisfy inequality (1). Increasing $\theta$ directly increases the LHS of (2), while the RHS, is unaffected which directly increases the range of parameters that satisfy inequality (2). Finally, increasing $\theta$ causes the LHS of (3) to increase faster than the RHS of (3), since taking the derivative with respect to $\theta$ of both sides gives

$$s(1+r)\frac{\beta}{1-\beta} > s\frac{\beta}{1-\beta},$$

since $r > 0$ by definition. Therefore, increasing $\theta$ increases the range of values that satisfy (3). For all three inequalities, increasing $\theta$ increases the potential for conflict.

**$\rho$ increases:** Increasing $\rho$ lowers the RHS of (1) without affecting the LHS, which increases the range of parameters that satisfy inequality (1).

For (2), subtract $\rho$ from each side to get

$$(\theta - 1) \rho > \frac{1-\beta}{\beta} - \frac{\kappa}{s} \left(\beta + \frac{1-\beta(1-r)}{\beta(1-\beta(1+r))}\right).$$

Since $\theta > 1$ by definition, the LHS is increasing in $\rho$ while the RHS is unaffected. Hence, increasing $\rho$ increases the range of parameters that satisfy (2).
For (3), the RHS is unaffected by $\rho$.  Rearranging the LHS gives

$$s (1 + r) \left[ \frac{\beta (\theta - 1)}{1 - \beta} - \frac{1}{\rho} \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right].$$

Since the second term in the brackets is negative and decreasing in $\rho$, the LHS is increasing in $\rho$. Hence (3) will be satisfied for a larger range of parameters.

Finally, if expected shifts increases due to increase in both $\theta$ and $\rho$, the above two cases demonstrates that the range of parameters satisfying (1), (2), and (3) will all increase.

**Proposition 2**

*Proof.* For two shifts of the same expected size, it is clear from the equation immediately above Lemma 3 that the amount borrowed to prevent war remains the same. Label the first shift $(\theta, \rho)$ and the second shift $(\theta', \rho')$ where $\rho' < \rho$ and $\theta' > \theta$, since the shifts have the same expected size. Assume that a positive amount must be borrowed to prevent war under $(\theta, \rho)$, and label the borrowed amount necessary to prevent war, $qB^*$. Similarly, let $q'B'$ be the amount necessary to prevent war under $(\theta', \rho')$.

Since the expected size of the shifts are the same, it must be that, if there is any risk of default at all,

$$qB^* = q'B'$$

$$\frac{\rho}{1+r} B^* = \frac{\rho'}{1+r} B'$$

$$B' = \frac{\rho}{\rho'} B^*.$$

That is, $B' > B^*$ since $\rho' < \rho$. Specifically, plugging in for $B^*$ we get

$$B' = \frac{s}{\rho'} (1 + r) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right].$$

Note that it cannot be the case that $(\theta, \rho)$ leads to a no-default loan, but $(\theta', \rho')$ leads to
a risky loan. The bond market offers a no-default loan for \((\theta, \rho)\) when

\[
s (1 + r) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] < \beta \left( \frac{1}{1 - \beta} s - \kappa \right).
\]

Since \((\theta - 1) \rho s = (\theta' - 1) \rho' s\), the market should also offer a no-default loan for \((\theta', \rho')\). However, if these values lead to a no-default loan, then inequality (3) cannot be satisfied, so for this inequality in isolation, the potential for conflict is invariant in \(\rho'\).

Inequality (3) is satisfied and war occurs under \((\theta', \rho')\) when

\[
\frac{s}{\rho'} (1 + r) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] > \beta \left( \frac{1}{1 - \beta} \theta' s - \kappa \right).
\]

From the definition of shifts of the same expected size, we have

\[
(\theta - 1) \rho s = (\theta' - 1) \rho' s
\]

\[
\theta' = \frac{\theta - 1}{\rho'} \rho + 1.
\]

Plugging this in to the RHS gives

\[
\frac{s}{\rho'} (1 + r) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] > \beta \left( \frac{1}{1 - \beta} \left[ \frac{\theta - 1}{\rho'} \rho + 1 \right] s - \kappa \right).
\]

Multiplying through by \(\rho'\) results in

\[
s (1 + r) \left[ \frac{\beta \rho (\theta - 1)}{1 - \beta} - \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right] > \beta \left( \frac{1}{1 - \beta} \left[ \theta - 1 \rho \right] s + \rho' \left( \frac{s}{1 - \beta} - \kappa \right) \right).
\]

Thus, since \(\frac{s}{1 - \beta} > \kappa\), giving a positive war value in the first period, the RHS is increasing in \(\rho'\), whereas decreasing \(\rho'\) (shift becomes more extreme, lower probability) lowers the RHS. This makes (3) easier to satisfy.
For (1), we can manipulate the inequality to get

$$(\theta - 1) \rho s > \frac{1 - \beta}{\beta} (s + (1 + \beta) \kappa).$$

The LHS is invariant by definition of equal expected shifts, and the RHS is invariant since it does not depend on $\theta$ or $\rho$. Similarly for (2), we can manipulate the inequality to get

$$(\theta - 1) \rho s > \frac{1 - \beta}{\beta} s - \kappa \left( \beta + \frac{1 - \beta (1 - r)}{\beta (1 - \beta (1 + r))} \right).$$

Once again, the LHS is invariant by definition of equal expected shifts, and the RHS is invariant since it does not depend on $\theta$ or $\rho$. Thus, only inequality (3) sees a change.

**Proposition 3**

**Proof.** $r$ is not present in inequality (1), so this inequality is unaffected by changes to $r$.

For (2), $r$ is not present in the LHS. Take the derivative of the RHS with respect to $r$:

$$-\kappa s \left( \frac{\beta}{\beta (1 - \beta (1 + r))} - \frac{1 - \beta (1 - r)}{[\beta (1 - \beta (1 + r))]^2} (-\beta^2) \right)$$

$$-\kappa s \left( \frac{\beta^2 (1 - \beta (1 + r))}{[\beta (1 - \beta (1 + r))]^4} + \frac{\beta^2 (1 - \beta (1 - r))}{[\beta (1 - \beta (1 + r))]^2} \right)$$

$$-\kappa s \left( \frac{2 - 2 \beta}{[1 - \beta (1 + r)]^2} \right)$$

The term in parenthesis is positive, so increasing $r$ is negative for the RHS. So, overall, increasing $r$ decreases $H$’s willingness to borrow.

For (3), $r$ is not present in the RHS, and the derivative of the LHS with respect to $r$ is

$$s \left[ \frac{\beta (\theta - 1)}{1 - \beta} - \frac{1}{\rho} \left( 1 + \frac{1 + \beta}{s} \kappa \right) \right],$$

which is positive so long as the bracketed amount is positive. Since this amount must be positive for (3) to be satisfied, increasing $r$ increases the LHS and makes (3) easier to
satisfy.

Strategic lending with inside money

Consider the following strategic game with many states. Suppose there are \( N > 1 \) countries that interact at discrete times \( t \in \{0, 1, 2, \ldots\} \). All countries discount the future at a common rate \( \delta \in (0, 1) \) per period. The total amount of resources available for consumption in each period is \( X > 0 \), and the amount controlled by country \( i \) (which may vary over time) is denoted \( x_i \). In addition to its resource level, each country \( i \) is characterized by its current military strength, \( s_i \). There is a finite set \( \hat{S} = S \cup \{0\} \) of possible states, where an element \( s \) of \( S = \{1, \ldots, K\} \), \( K > 1 \) represents an active country’s strength in war, and a country in state 0 is disarmed and becomes inactive. Let \( s \in \hat{S}^N \) denote a vector of military strengths for all countries, let \( I(s) \equiv \{i: s_i \neq 0\} \) denote the set of active countries, and let \( S(n) \) denote the set of strength vectors where \( n \leq N \) countries are active.

In each period, each active country can either choose transfers of resources to each other active country or initiate a war with any or all of the countries with which it shares a (undirected) link. Each node represents a country, and two countries can go to war with each other only if they share a link. The set of links is represented by \( l \subseteq N^2 \), with a typical element \( ij \in l \) representing a link between country \( i \) and country \( j \). Thus, a country can fight a directed war, but the set of countries that it can fight with may be limited. We assume that the network is **connected** – that is, that there is a path of links between any two countries. (Connected actors are potential strategic lenders in the context of the current paper.) Let \( L \subseteq N^2 \) denote the set of all possible connected networks with undirected links.

When a country starts a war with another country to which it is linked, the link has become **engaged** for that period. The term **general war** denotes the case where all active countries have an engaged link. In any period in which one or more countries choose war, consumption is 0 for all countries possessing an engaged link. In the absence of war, net
transfers from \( i \) to \( j \) are labeled \( \tau_{ij} \), so that consumption in country \( i \) is

\[
c_i = x_i - \sum_{j \in N \setminus i} \tau_{ij}.
\]

We restrict transfers so that \( c_i \geq 0 \) for all \( i \). The total payoff to a country that receives consumption stream \( \{c_t\}_{t \geq 1} \) is

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} c_t.
\]

Being disarmed \((s = 0)\) is an absorbing state, and a disarmed country receives a continuation payoff of 0. When a country \( i \) becomes disarmed, all active countries with engaged links to \( i \) receive equal shares of \( i \)'s resources \( x_i \). Additionally, all active countries that have an engaged link with \( i \) at the time of its disarmament inherit all of \( i \)'s links.

Resources for each country are specified by a nonnegative vector \( \mathbf{x} = (x_1, \ldots, x_n) \), where \( x_i > 0 \) if country \( i \) is active, \( x_i = 0 \) if country \( i \) is disarmed, and \( \sum_{i=1}^{N} x_i = X \). Let \( \mathcal{X}(n) \) denote the set of such resource vectors where \( n \leq N \) countries are active. If there are two or more active countries in a period, not all of them can be disarmed. (If all active countries transition to \( s = 0 \) in the same period, then one is randomly selected to remain active, and the others are disarmed.) If and when a single country remains, the game ends, and the survivor receives the entire stream of available consumption \((1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} X = X\).

Let \( \gamma(s'; c, s) \) denote the probability that a country in state \( s \in S \) that consumes \( c \) will transition to state \( s' \in S \). A country in state 1, the weakest military position, may become disarmed in a period in which war occurs. Let \( \gamma^W(s'; s) \) denote the probability that a country transitions from \( s \) to \( s' \) after a period of war. State transitions are independent across countries. The following assumptions on transition probabilities are maintained throughout:

**Assumption (A).**

1. **One step at a time with full support:** there exists \( \underline{\gamma} > 0 \) such that for all \( c \in [0, X] \)

   (a) for all \( 1 < s \leq K \), \( \gamma(s'; c, s) \geq \gamma \) for all \( s' \in \{s - 1, s, s + 1\} \cap S \) and \( \gamma(s'; c, s) = 0 \)
for all other $s'$, and

$$(b) \gamma(s'; c, 1) \geq \gamma$$ for $s' \in \{1, 2\}$.

2. War is necessary for disarmament:

$$(a) \gamma(0; c, 1) = 0.$$

3. War transitions:

$$(a) \gamma^w(s'; s) = 0$$ unless $s' \in \{s - 1, s, s + 1\}$,

$$(b) \gamma^w(0; 1) > 0,$$ and

$$(c) \gamma^w(s'; s) \geq \gamma(s'; c, s)$$ if $s' < s$ and $\gamma^w(s'; s) \leq \gamma(s'; c, s)$ if $s' > s$.

We also make the following assumption:

**Assumption (B).** *Either the number of military strength states $K$ is at least 3 or the network is complete.*

Transfers and war decisions are publicly observed, as are military strengths and resource levels.

**Proposition 4**

**Proof.** Without “loans” Krainin and Wiseman (2016) demonstrate the following theorem:

**Theorem 1.** *In the baseline model, there exists $\delta < 1$ such that the following holds: for any $\delta \geq \delta$ and any initial military strength vector $s \in \hat{S}$, network $l \in L$, and resource vector $x \in X(N)$, in any SPE war occurs, and eventually only a single country remains active.*

How do loans affect this conclusion?

In the main paper we assume that countries have (1) full commitment to repay loans, (2) a source of outside money, and (3) loans are made by a nonstrategic actor. In the current
extension, we are interested in the case where many strategically connected (linked) countries might loan each other inside money in order to avoid preventive war.

The first thing to note, is that there is nothing the dynamic network model of bargaining that rules out states lending each other resources to make transfers. Theorem 1 applies to this possibility. However, Theorem 1 assumes a particular notion of the commitment technology to loans: there is no way to commit to not going to war in the future, therefore there is no way to commit to repaying loans that imply a game value less than the minmax achieved through war in any given period.

To make it as hard as possible to demonstrate that preventive war still occurs with loans in the main body of the text, we made assumption (1), complete commitment to repay a loan. That is, the setup is implicitly partial equilibrium. Countries are willing to repay loans that violate their minmax within the game we study, because their is implicitly some other source of value in the game. Does this assumption still make sense in the current context? In other words, could a country be better off paying back a loan today that it received in the past, even if it violates their current minmax condition? The answer is “no.” By only considering inside money made by strategically involved actors, there is no other source of value which players can lose out on if they violate the terms of their loan.

Imagine this were not the case. One can show that certain states are achieved where some country, say country 1, achieves a minmax value greater than an even split of all the resources [the probability of victory in that state times the total resources, \( \alpha_i(s^i) X \) is greater than \( X/N \)]. Definitionally, this value cannot be brought lower by any punishment strategy other countries may enact. Therefore, if country 1 chooses to continue to repay the loan in such a state, it must be that 1’s expected return to this strategy is greater than or equal to the minmax value achieved through war. Call this value \( Z_i(s^i) \geq \alpha_i(s^i) X \) where the inequality follows from the argument above.

Therefore, 1 achieves a value greater than the even split of all resources in this state since \( Z_i(s^i) \geq \alpha_i(s^i) X > X/N \). For such states, replace \( \alpha_i(s^i) X \) with \( Z_i(s^i) \) in the proof
of Theorem 1 and the proof goes through as before.

Since Proposition 4 is a direct corollary of Theorem 1 with loans, we have demonstrated the proposition. □
References


