Key Takeaways

- Real vs. Nominal Debt/Cashless vs. Cash
  - Inflation risk can improve risk sharing
- Intertemporal unit of account
  - State-contingent MoPo $\sigma^B$ as
- Equivalence of capital vs. risk allocation setting ($\kappa$ vs. $\chi$)
- Liquidity and Disinflationary Spiral

- Policy
  - Fiscal Policy
  - Monetary Policy
    - Stealth recapitalization of intermediaries
    - Macroprudential Policy

- Technical Takeaways
  - Two sector money models
One sector model with idio risk - “The I Theory without I” (steady state focus)
- Store of value
  - Insurance role of money within sector
- Money as bubble or not
- Fiscal Theory of the Price Level
- Medium of Exchange Role ⇒ SDF-Liquidity multiplier ⇒ Money bubble

2 sector/type model with money and idio risk
- Generic Solution procedure (compared to lecture 03)
- Equivalence btw experts producers and intermediaries
- Real debt vs. nominal debt/money
  - Implicit insurance role of money across sectors
- I Theory

Welfare analysis

Optimal Monetary Policy and Macroprudential Policy

International Monetary Model
The 4 Roles of Money

- **Unit of account**
  - Intratemporal: Numeraire
  - Intertemporal: Debt contract

- **Store of value**
  - “I Theory of Money without I”
    - Less risky than other “capital” – no idiosyncratic risk
  - Fiscal theory of the price level

- **Medium of exchange**
  - Overcome double-coincidence of wants problem

- **Record keeping device – money is memory**
  - Virtual ledger
Safe Assets $\supseteq$ (Narrow) Money

- **Asset Price** = $E[PV(cash flows)] + E[PV(service flows)]$
  - dividends/interest
- **Service flows/convenience yield**
  1. **Collateral**: relax constraints (Lagrange multiplier)
  2. **Safe asset**: [good friend analogy]
     - When one needs funds, one can sell at stable price... since others buy
     - Partial insurance through re-trading - market liquidity!
  3. **Money (narrow)**: relax double-coincidence of wants
    - Higher Asset Price = lower expected return
- **Problem**: safe asset + money status might burst like a bubble
  - Multiple equilibria: [safe asset tautology]
## Models on Money as Store of Value

<table>
<thead>
<tr>
<th>Friction</th>
<th>OLG</th>
<th>Incomplete Markets + idiosyncratic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>deterministic</td>
<td>endowment risk borrowing constraint</td>
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<tr>
<td></td>
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<tr>
<td>Only money</td>
<td>Samuelson</td>
<td>Bewley</td>
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<tr>
<td>With capital</td>
<td>Diamond</td>
<td>Aiyagari</td>
</tr>
</tbody>
</table>
## (New) Keynesian Demand Management

<table>
<thead>
<tr>
<th></th>
<th>I Theory of Money Risk (Premium) Management</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stimulate aggregate consumption</strong></td>
<td><strong>Alleviate balance sheet constraints</strong></td>
</tr>
<tr>
<td>Woodford (2003)</td>
<td>Tobin (1982), HANK</td>
</tr>
<tr>
<td>Tobin (1982), HANK</td>
<td>BruSan</td>
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<tr>
<td><strong>Price stickiness</strong> &amp; ZLB</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial frictions</td>
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<tr>
<td></td>
<td>Incomplete markets</td>
</tr>
<tr>
<td><strong>Representative Agent</strong></td>
<td><strong>Heterogeneous Agents</strong></td>
</tr>
<tr>
<td><strong>Cut</strong> ( i )</td>
<td><strong>Cut</strong> ( i )</td>
</tr>
<tr>
<td><strong>Reduces</strong> ( r ) due to price stickiness</td>
<td><strong>Changes bond prices</strong></td>
</tr>
<tr>
<td><strong>Consumption</strong> ( c ) rises</td>
<td><strong>Redistributes from low MPC to high MPC consumers</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Cut</strong> ( i ) or QE</td>
</tr>
<tr>
<td></td>
<td><strong>Changes asset prices</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Ex-post: Redistributes to balance sheet impaired sector</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Price of Risk Dynamics</strong></td>
</tr>
</tbody>
</table>
“Money and Banking” (in macro-finance)

- Money: store of value/safe asset/Gov. bond
- Banking: "diversifier" holds risky assets, issues inside money

- Amplification/endogenous risk dynamics
- Value of capital declines due to fire-sales liquidity spiral
- Flight to safety
- Value of money rises disinflation spiral a la Fisher
- Demand for money rises – less idiosyncratic risk is diversified
- Supply for inside money declines – less creation by intermediaries
- Endogenous money multiplier = f(capitalization of critical sector)

- Paradox of Thrift (in risk terms)
- Monetary Policy (redistributive)

Watch “Money and Banking”
YouTube Video Channel: “markus.economicus”
https://www.youtube.com/channel/UCY8DKGTKyJxk4rJu6RYIoA/videos?pbireload=10
“Money and Banking” (in macro-finance)

- Money store of value/safe asset/Gov. bond
- Banking “diversifier” holds risky assets, issues inside money

Amplification/endogenous risk dynamics

- Value of capital declines due to fire-sales Liquidity spiral
  - Flight to safety
- Value of money rises Disinflation spiral a la Fisher
  - Demand for money rises – less idiosyncratic risk is diversified
  - Supply for inside money declines – less creation by intermediaries
    - Endogenous money multiplier = f(capitalization of critical sector)

Paradox of Prudence

Paradox of Thrift (in risk terms)

- Monetary Policy (redistributive)
Roadmap

- Intro
- Equivalence btw experts producers and intermediaries
- Real vs. Nominal Debt
- I Theory of Money
- Policy
- **Frictions:**
  - Household cannot diversify idio risk
  - Limited risky claims issuance
Equivalence

\[ a^e = a^h \]
\[ \tilde{\sigma}^e < \tilde{\sigma}^h \]
Equivalence

- Why equivalence btw. Intermediaries $\chi$-risk allocation model and experts $\kappa$-capital allocation model?

Poll 13: Why are both models equivalent?
- a) Since $a^e = a^h$.
- b) Intermediary sector doesn’t produce any output
- c) Risk $\chi$ and capital allocation $\kappa$ are fundamentally different.

- Next: Contrast Real Debt with Nominal Debt/Money Model
  - solve generic model and highlight the differences
Roadmap

- Intro
- Equivalence btw experts producers and intermediaries
- Real vs. Nominal Debt
- I Theory of Money
- Policy
Model with Intermediary Sector

Intermediary sector

- Hold equity up to $\tilde{\chi} \leq 1$
- Diversify idio risk to $\phi \tilde{\sigma}$
- Consumption rate: $c_t^l$
- $E_0[\int_0^\infty e^{-\rho t} \log c_t^l \, dt]$

- Friction: Can only issue debt
  - 2 Models:
    1. **Real** debt issuance only (and money has no value)
    2. **Nominal** debt issuance
- Bond/money supply $\frac{dB_t}{B_t} = (\bar{\mu}_t^B + i_t) \, dt + \sigma_t^B \, dZ_t$
- seigniorage distribution as in Lecture 05 (no fiscal impact – per period balanced budget)

Household sector

- Output: $y_t^h = a^h k_t^h$
- Investment rate: $\iota_t^h$
  
  $\frac{dk_t^{h,l}}{k_t^h} = (\Phi(\iota_t^h) - \delta^h) \, dt + \sigma^h d\tilde{Z}_t + d\Delta_{k,h,l}$
- Consumption rate: $c_t^h$
- $E_0[\int_0^\infty e^{-\rho t} \log c_t^h \, dt]$
Solving MacroModels Step-by-Step

0. Postulate aggregates, price processes & obtain return processes

1. For given $C/N$-ratio and SDF processes for each $i$ finance block
   a. Real investment $\iota$ + Goods market clearing (static)
      • Toolbox 1: Martingale Approach, HJB vs. Stochastic Maximum Principle Approach
   b. Portfolio choice $\theta$ + Asset market clearing or Asset allocation $\kappa$ & risk allocation $\chi$
      • Toolbox 2: “price-taking social planner approach” – Fisher separation theorem
   c. “Money evaluation equation” $\vartheta$
      • Toolbox 3: Change in numeraire to total wealth (including SDF)

2. Evolution of state variable $\eta$ (and $K$)

3. Value functions
   a. Value fcn. as fcn. of individual investment opportunities $\omega$
      • Special cases: log-utility, constant investment opportunities
   b. Separating value fcn. $V^i(n^i; \eta, K)$ into $v^i(\eta)u(K)(n^i/n^i)^{1-\gamma}$
   c. Derive $\rho = C/N$-ratio and $\zeta, \xi$ prices of risks

4. Numerical model solution
   a. Transform BSDE for separated value fcn. $v^i(\eta)$ into PDE
   b. Solve PDE via value function iteration

5. KFE: Stationary distribution, Fan charts
0. Postulate Aggregates and Processes

- $q_t^K K_t$ value of physical capital
- $q_t^B K_t$ value of nominal capital/outside money/gov. debt
  - $\wp_t := B_t / q_t^B K_t$ price level (inverse of “value of money”)
- $N_t := (q_t^K + q_t^B)K_t$ is total wealth in the economy
- $\vartheta_t := \frac{q_t^B}{q_t^K + q_t^B}$ fraction of nominal wealth
0. Postulate Aggregates and Processes

- \( q_t^K K_t \) value of physical capital
- \( q_t^B K_t \) value of nominal capital/outside money/gov. debt
  - \( \psi_t := B_t / q_t^B K_t \) price level (inverse of “value of money”)
- \( N_t := (q_t^K + q_t^B) K_t \) is total wealth in the economy
- \( \vartheta_t := \frac{q_t^B}{q_t^K + q_t^B} \) fraction of nominal wealth

0. Postulate in the \( N_t \)-numeraire!

- \( \vartheta \)-price process
  \[
d\vartheta_t / \vartheta_t = \mu_t^\vartheta dt + \sigma_t^\vartheta dZ_t,
\]
- SDF for each \( \tilde{i} \) agent
  \[
  \frac{d\xi_t^{\tilde{i}}}{\xi_t^{\tilde{i}}} = -r_t^{\tilde{i}} dt - \zeta_t^{\tilde{i}} dZ_t - \tilde{\zeta}_t^{\tilde{i}} d\tilde{Z}_t
  \]
  - Change of notation compared to Lectures 02-05!
0. Postulate Aggregates and Processes

- $q_t^K K_t$ value of physical capital
- $q_t^B K_t$ value of nominal capital/outside money/gov. debt
  - $\varphi_t := B_t / q_t^B K_t$ price level (inverse of “value of money”)
- $N_t := (q_t^K + q_t^B) K_t$ is total wealth in the economy
- $\vartheta_t := \frac{q_t^B}{q_t^K + q_t^B}$ fraction of nominal wealth

0. Postulate in the $N_t$-numeraire!

- $\vartheta$-price process
  \[ d\vartheta_t / \vartheta_t = \mu^\vartheta_t dt + \sigma^\vartheta_t dZ_t, \]
- SDF for each $\tilde{t}$ agent
  \[ \frac{d\xi_t}{\xi_t} = -r_t^{\tilde{t}} dt - \zeta_t^{\tilde{t}} dZ_t - \tilde{\zeta}_t^{\tilde{t}} d\tilde{Z}_t \]

- Change of notation (dropped “hat”) compared to Lectures 02-05!

Poll 19: Why is the drift $-r_t^{\tilde{t}}$ and not simply $-r_t^f$?

a) With only nominal debt a real risk-free rate might not be in asset span.
b) Negative drift of the SDF in $N_t$-numeraire is not risk-free rate.
1a. Optimal $\iota +$ Goods Market

- Use optional real investment $\iota$ and goods market clearing
- Same as in Lecture 05
- Price of physical capital
  \[ q^K_t = (1 - \vartheta_t) \frac{1 + \phi \alpha}{(1 - \vartheta_t) + \phi \rho} \]
- Price of nominal capital
  \[ q^B_t = \vartheta_t \frac{1 + \phi \alpha}{(1 - \vartheta_t) + \phi \rho} \]
- Optimal investment rate
  \[ \iota_t = \frac{(1 - \vartheta_t) \alpha - \rho}{(1 - \vartheta_t) + \phi \rho} \]
- Moneyless equilibrium with $q^B_t = 0 \Rightarrow \vartheta_t = 0 \Rightarrow q^K_t = \frac{1 + \phi \alpha}{1 + \phi \rho}$
1b. Price-taking Planner’s Allocation

\[
\max_{\{\kappa_t, \chi_t, \tilde{\chi}_t\}} E_t[dr_t^N(\kappa_t)] - \zeta_t \sigma(\psi_t, \chi_t) - \tilde{\zeta}_t \tilde{\sigma}(\psi_t, \tilde{\chi}_t)
\]

In our model(s):

\(\kappa = 0\) (households manage all physical capital)

\(\tilde{\chi}_t = \chi_t\)

\(E_t[dr_t^N(\kappa_t)] = 0\)

Poll 21: Why is \(E_t[dr_t^N(\kappa_t)] = 0\)?

a) Because capital is not reallocated, i.e. \(\kappa = 0\) all the time.

b) In the \(N_t\)-numeraire return of total wealth \(dr_t^N = 0\).
1b. Price-taking Planner’s Allocation

\[
\max_{\{\psi_t, x_t, \tilde{x}_t\}} E_t [dr^N_t (\kappa_t)] - \zeta_t \sigma(\psi_t, x_t) - \tilde{\zeta}_t \tilde{\sigma}(\psi_t, \tilde{x}_t)
\]

In our model(s):

- \(\kappa = 0\) (households manage all physical capital)
- \(\tilde{x}_t = x_t\)
- \(E_t [dr^N_t (\kappa_t)] = 0\)
- \(\sigma = (x_t \sigma_t^{xK}, (1 - x_t) \sigma_t^{xK})\),
  where \(\sigma_t^{xK}\) = Risk of the excess return of capital beyond benchmark asset
- \(\tilde{\sigma} = (x_t \varphi \tilde{\sigma}, (1 - x_t) \tilde{\sigma})\)
  - \(\varphi < 1\)
1b. Price-taking Planner’s Allocation

- Minimize weighted average cost of financing

\[
\min_{\chi_t \leq \bar{\chi}} \left( \zeta^l_t x_t + \zeta^h_t (1 - x_t) \right) \sigma^{xK}_t + \left( \tilde{\zeta}^l_t \varphi x_t + \tilde{\zeta}^h_t (1 - x_t) \right) \tilde{\sigma}
\]

- FOC: (equality if \( x_t < \bar{\chi} \))

\[
\zeta^l_t \sigma^{xK}_t + \tilde{\zeta}^l_t \phi \tilde{\sigma} \leq \zeta^h_t \sigma^{xK}_t + \tilde{\zeta}^h_t \tilde{\sigma}
\]

- Real debt model:

- Nominal debt model

- Risk of capital

\[
\sigma + \sigma_t^{qK} + \vartheta_t \sigma_t^B / (1 - \vartheta_t) - \sigma_t^N
\]

- Risk of bond/money

\[
\sigma + \sigma_t^{qB} + \sigma_t^B - \sigma_t^N
\]
“Benchmark Asset Evaluation Equation”

- In $N_t$-numeraire $\eta_t^i$ takes on role of sector net worth $N_t^i$
- Return on individual agent’s net worth return (in $N_t$-numeraire)
\[
\frac{d\eta_t^i}{\eta_t^i} + \frac{d\tilde{\eta}_t^i}{\tilde{\eta}_t^i} + \rho dt = dt
\]
- Martingale condition relative to benchmark asset is
\[
\mu_t^i + \rho - r_t^{bm} = \zeta_t^i \left( \sigma_t^i - \sigma_t^{bm} \right) + \tilde{\zeta}^i_t \tilde{\sigma}_t^i
\]
- Take $\eta_t^i$-weighted sum (across 2 types $i = I, h$ here)
\[
\rho - r_t^{bm} = \eta_t \zeta_t^I \left( \sigma_t^I - \sigma_t^{bm} \right) + (1-\eta_t) \zeta_t^h \left( -\frac{\eta_t}{1-\eta_t} \sigma_t^I - \sigma_t^{bm} \right) + \eta_t \tilde{\zeta}_t^I \tilde{\sigma}_t^I + (1-\eta_t) \tilde{\zeta}_t^h \tilde{\sigma}_t^h
\]
- For log utility:
\[
\zeta_t^I = \sigma_t^I \tilde{\sigma}_t^I, \zeta_t^h = -\frac{\eta_t}{1-\eta_t} \sigma_t^h \tilde{\sigma}_t^h
\]
\[
\rho - r_t^{bm} = \eta_t \left( \sigma_t^I \right)^2 + (1-\eta_t) \left( -\frac{\eta_t}{1-\eta_t} \sigma_t^I \right)^2 + \eta_t \left( \tilde{\sigma}_t^I \right)^2 + (1-\eta_t) \left( \tilde{\sigma}_t^h \right)^2
\]
“Benchmark Asset Evaluation Equation”

- **Real** debt = benchmark asset \( bm \)
  - Redundant equation for allocation just useful for deriving risk-free rate in \( c \)-numeraire \( r_t^f \) (expressed in \( N_t \)-numeraire)

- **Nominal** debt/money = benchmark asset \( bm \)
  - Money evaluation equation (bubble)
  - Replace \( r_t^{bm} = \mu_t^{\vartheta/B} := \mu_t^\vartheta - \mu_t^B - \sigma_t^B (\sigma_t^\vartheta - \sigma_t^B) \) (and \( \sigma_t^{bm} = \sigma_t^\vartheta \))

\[
\rho - \mu_t^{\vartheta/B} = \eta_t (\sigma_t^\eta)^2 + (1 - \eta_t) \left( -\frac{\eta_t}{1 - \eta_t} \sigma_t^\eta \right)^2 + \eta_t \left( \tilde{\eta} \tilde{\eta} \right)^2 + (1 - \eta_t) \left( \tilde{\eta} \tilde{\eta} \right)^2
\]

excess return = (required) “net worth weighted risk premium”

of \( N_t \) (for holding risk in excess of money risk)
“Benchmark Asset Evaluation Equation”

- **Nominal** debt/money = benchmark asset \( bm \)
  - Money evaluation equation
  - Replace \( r^{bm}_t = \mu_t^{\vartheta/B} : = \mu_t^{\vartheta} - \mu_t^B - \sigma_t^B (\sigma_t^{\vartheta} - \sigma_t^B) \) (and \( \sigma_t^{bm} = \sigma_t^{\vartheta} \))

\[
\rho - \mu_t^{\vartheta/B} = \eta_t (\sigma_t^\eta)^2 + (1 - \eta_t) \left( -\frac{\eta_t}{1 - \eta_t} \sigma_t^\eta \right)^2 + \eta_t \left( \tilde{\sigma}_t^{\eta_I} \right)^2 + (1 - \eta_t) \left( \tilde{\sigma}_t^{\tilde{\eta}_R} \right)^2
\]

- Integrate

\[
\vartheta_t = E_t \left[ \int_t^\infty e^{-\rho(s-t)} \left( \eta_s \left( \sigma_s^\eta \right)^2 + (1 - \eta_s) \left( \sigma_s^{\tilde{\eta}_I} \right)^2 + \eta_s \left( \tilde{\sigma}_s^{\eta_I} \right)^2 + (1 - \eta_s) \left( \tilde{\sigma}_s^{\tilde{\eta}_R} \right)^2 - \mu_s^B - \sigma_s^B (\sigma_s^{\vartheta} - \sigma_s^B) \right) - \vartheta_s \right] ds \]
2. $\eta$-Evolution: Drift $\mu^\eta_t$ (in $N_t$-numeraire)

- Take difference from two earlier equations

\[
\begin{align*}
\mu^\eta_t + \rho - r^bm_t &= \zeta^l_t (\sigma^\eta_t - \sigma^bm_t) + \tilde{\zeta}^l_t \tilde{\eta}^l_t \\
\rho - r^bm_t &= \eta_t \zeta^l_t (\sigma^\eta_t - \sigma^bm_t) + (1 - \eta_t) \zeta^h_t \left( \frac{-\eta_t}{1 - \eta_t} \sigma^\eta_t - \sigma^bm_t \right) + \eta_t \tilde{\zeta}^l_t \tilde{\eta}^l_t + (1 - \eta_t) \tilde{\zeta}^h_t \tilde{\eta}^h_t
\end{align*}
\]

- \[
\mu^\eta_t = (1 - \eta_t) \left[ \zeta^l_t (\sigma^\eta_t - \sigma^bm_t) - \zeta^h_t \left( \frac{-\eta_t}{1 - \eta_t} \sigma^\eta_t - \sigma^bm_t \right) + \tilde{\zeta}^l_t \tilde{\eta}^l_t - \tilde{\zeta}^h_t \tilde{\eta}^h_t \right]
\]

- **Real** Debt
  - $\sigma^bm_t = -\sigma^N_t = -\sigma$ (Recall $\sigma^q_t = 0$)

- **Nominal** Debt/Money
  - $\sigma^bm_t = \sigma^q_t - \sigma^B$
2. $\eta$-Evolution: $\eta$-Aggregate Risk

- $\sigma_t^\eta = \sigma_t^{r,bm} + (1 - \theta_t^I) (\sigma_t^{r,K} - \sigma_t^{r,bm})$
  - Where portfolio share $1 - \theta_t^I = \frac{x_t}{\eta_t} (1 - \vartheta_t)$

- **Real Debt**
  - Note $\sigma_t^{r,K} = 0$ given $N_t = q_t^K K_t$-numeraire
  - $\sigma_t^\eta = \frac{x_t - \eta_t}{\eta_t} \sigma$ (recall $\vartheta_t = 0$)

- No amplification since $q^K$ is constant

- Imperfect risk-sharing for $\chi_t \neq \eta_t$
Inflation Risk allows Perfect Risk Sharing

- **Nominal Debt**
  - Note $\sigma_t^{r^K} = \sigma_t^{1-\vartheta} = -\frac{\vartheta_t}{1-\vartheta_t}\sigma_t^{\vartheta}$
  - $\sigma_t^{\eta} = \sigma_t^{\vartheta} - \sigma^B + \frac{\chi_t}{\eta_t}(1-\vartheta_t)\left(-\frac{\vartheta_t}{1-\vartheta_t}\sigma_t^{\vartheta} - \sigma_t^{\vartheta} + \sigma^B\right)$
  - Use $\sigma_t^{\vartheta} = \frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)}\eta_t\sigma_t^{\eta}$ and solve for $\eta_t\sigma_t^{\eta}$ yields
    $$\eta_t\sigma_t^{\eta} = \frac{(\chi_t-\eta_t)\sigma_t^B}{1-\frac{\chi_t-\eta_t}{\eta_t}\left(-\frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)}\right)}$$

- **Intermediaries’ balance sheet**
  - perfectly hedges agg. risk for $\sigma^B = 0$!

- **Proposition**: Aggregate risk is perfectly shared for $\sigma^B = 0$!
  - Via inflation risk
  - Stable inflation (targeting) would ruin risk-sharing
    - Example: Brexit uncertainty. Use inflation reaction to share risks within UK
2. Within Type $\tilde{\eta}$-Risk

- Within intermediary sector
  \[
  \tilde{\sigma}_t^{\tilde{\eta}^i} = (1 - \theta_t^I) \varphi \tilde{\sigma} = \frac{x_t}{\eta_t} (1 - \vartheta_t) \varphi \tilde{\sigma}
  \]

- Within household sector
  \[
  \tilde{\sigma}_t^{\tilde{\eta}^h} = (1 - \theta_t^h) \tilde{\sigma} = \frac{1 - x_t}{1 - \eta_t} (1 - \vartheta_t) \tilde{\sigma}
  \]
Solving for $\chi_t$

- **Recall planner condition:** (equality if $\chi_t < \bar{\chi}$)

$$
\zeta^l_t \sigma^x_t + \tilde{\zeta}^l_t \phi \tilde{\sigma} \leq \zeta^h_t \sigma^x_t + \tilde{\zeta}^h_t \tilde{\sigma}
$$

<table>
<thead>
<tr>
<th>Price of Risks</th>
<th>Real Debt</th>
<th>Nominal Debt with $\sigma^B = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta^l_t = \sigma^\eta_t$</td>
<td>$= \frac{\chi_t - \eta_t}{\eta_t} \sigma$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$\zeta^h_t = -\frac{\eta_t}{1 - \eta_t} \sigma^\eta_t$</td>
<td>$= \frac{\chi_t - \eta_t}{1 - \eta_t} \sigma$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$\tilde{\zeta}^l_t = \frac{\chi_t}{\eta_t} (1 - \vartheta_t) \varphi \tilde{\sigma}$</td>
<td>$= \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma}$</td>
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<td>$\tilde{\zeta}^h_t = \frac{1 - \chi_t}{1 - \eta_t} (1 - \vartheta_t) \tilde{\sigma}$</td>
<td>$= \frac{1 - \chi_t}{1 - \eta_t} \tilde{\sigma}$</td>
<td>$= \frac{1 - \chi_t}{1 - \eta_t} (1 - \vartheta_t) \tilde{\sigma}$</td>
</tr>
</tbody>
</table>
Solving for $\chi_t$

- **Real debt**

\[
\chi_t = \min \left\{ \frac{\eta_t (\sigma^2 + \bar{\sigma}^2)}{\sigma^2 + [(1 - \eta_t)\phi^2 + \eta_t]\bar{\sigma}^2}, \bar{\chi} \right\}
\]

- **Nominal debt**

\[
\chi_t = \min \left\{ \frac{\eta_t}{(1 - \eta_t)\phi^2 + \eta_t}, \bar{\chi} \right\}
\]
<table>
<thead>
<tr>
<th></th>
<th>Real Debt</th>
<th>Nominal Debt with $\sigma^B = 0$</th>
</tr>
</thead>
</table>
| $\chi_t$              | \[
\min \left\{ \frac{\eta_t (\sigma^2 + \tilde{\sigma}^2)}{\sigma^2 + [(1 - \eta_t)\phi^2 + \eta_t]\tilde{\sigma}^2}, \bar{\chi} \right\} \] | \[
\min \left\{ \frac{\eta_t}{(1 - \eta_t)\phi^2 + \eta_t}, \bar{\chi} \right\} \] |
| $\mu_t^\eta$          | \[
\frac{\chi_t - \eta_t \chi_t - 2\chi_t \eta_t + \eta_t^2}{\eta_t (1 - \eta_t)} \sigma^2 + \\
+ (1 - \eta_t) \left( \left( \frac{\chi_t}{\eta_t} \right)^2 \phi^2 - \left( \frac{1 - \chi_t}{1 - \eta_t} \right)^2 \right) \tilde{\sigma}^2 \] | \[
(1 - \eta_t)(1 - \vartheta)^2 \left( \left( \frac{\chi_t}{\eta_t} \right)^2 \phi^2 - \left( \frac{1 - \chi_t}{1 - \eta_t} \right)^2 \right) \tilde{\sigma}^2 \] |
| $\sigma_t^\eta$       | \[
\frac{X_t - \eta_t \chi_t \sigma}{\eta_t} \] | 0 |
| $q_t^K$               | \[
\frac{1 + \phi a}{1 + \phi \rho} \] | \[
(1 - \vartheta_t) \frac{1 + \phi a}{(1 - \vartheta_t) + \phi \rho} \] |
| $q_t^B$               | 0 | \[
\vartheta_t \frac{1 + \phi a}{(1 - \vartheta_t) + \phi \rho} \] |
| $\vartheta_t$         | 0 | \[
\rho - \mu_t^\vartheta + \mu_t^B \]
\[
= (1 - \vartheta_t)^2 \left( \eta_t \frac{\chi_t^2 \phi^2}{\eta_t^2} - (1 - \eta_t) \left( \frac{1 - \chi_t}{1 - \eta_t} \right)^2 \right) \tilde{\sigma}^2 \] |
| $\iota_t$             | \[
\frac{a - \rho}{1 + \phi \rho} \] | \[
\frac{(1 - \vartheta_t) a - \rho}{(1 - \vartheta_t) + \phi \rho} \] |
Example: Nominal Debt/Money with $\bar{\chi} = 1$

- $a = .15, \rho = .03, \sigma = .1, \phi = 2, \delta = .03, \bar{\sigma}^e = .2, \bar{\sigma}^h = .3, \varphi = ., \bar{\chi} = 1$

Blue: real debt model
Red: nominal model
Contrasting Real with Nominal Debt

- **Real** debt model:
  - Changes in $\eta$ are absorbed by risk-free rate moves
  - Aggregate risk
  - $\iota(\eta)$ and $q^K(\eta)$ are constant

- **Nominal** debt/money model:
  - Inflation risk completes markets
  - Perfect aggregate risk sharing
    - Banks balance sheet is perfectly hedged!!!
  - Risk-free rate is high
  - $\iota(\eta)$ and $q^K(\eta)$ are functions of $\eta$
Example: Nominal Debt with Limit on Risk Offloading

- $\rho = .05, \kappa = 2, \tilde{\sigma} = .5, \phi = .4, \bar{\chi} = .8$
Combining Nominal & Real Debt

- Adding real debt to money model does not alter the equilibrium, since
  - Markets are complete w.r.t. to aggregate risk (perfect aggregate risk sharing)
  - Markets are incomplete w.r.t. to idiosyncratic risk only
  - Real debt is a redundant asset

- Note: Result relies on absence of price stickiness

- Both Settings: Real Debt and Money/Nominal Debt converge in the long-run to the “I Theory without I” steady state model of Lecture 05 if $\bar{\chi} = 1$. 
Claim: $\vartheta(\eta)$ and average idiosyncratic risk exposure, $X(\eta)$, is minimized at the stochastic steady state of $\eta$.
- Intuition: at steady state both sectors earn same risk premia + idiosyncratic seems well spread out ... less desire to hold money to self-insure

With $\sigma_t^B = 0 \forall t$
- $\sigma_t^\eta = 0$, (perfect risk sharing with nominal debt)
- $\mu_t^\eta = (\bar{\sigma}_t^I)^2 - \eta_t(\bar{\sigma}_t^I)^2 - (1 - \eta_t)(\bar{\sigma}_t^h)^2 = (1 - \eta_t)(1 - \vartheta_t)^2 \left( \frac{\chi_t^2 \phi^2}{\eta_t^2} - \frac{(1 - \chi_t)^2}{(1 - \eta_t)^2} \right) \bar{\sigma}^2$

Money valuation equation

$$\rho - \mu_t^{\vartheta/B} = (1 - \vartheta_t)^2 \left( \eta_t \frac{\chi_t^2 \phi^2}{\eta_t^2} - (1 - \eta_t) \frac{(1 - \chi_t)^2}{(1 - \eta_t)^2} \right) \bar{\sigma}^2$$

where $\chi_t = \min \left( \frac{\eta_t}{\eta_t + (1 - \eta_t)\phi^2}, \bar{\chi} \right)$
Cashless/Bondless Limit with Jump

- Removing cash/nominal gov. bonds (comparative static)
  - $B > 0$ vs. $B = 0$
    - Price flexibility $\Rightarrow$ Neutrality of money
  - Discontinuity at $\lim_{B \to 0}$

- Remark:
  - Different from Woodford (2003) – medium of exchange role of money
    - CIA becomes relevant for fewer and fewer goods

- Inflation on nominal claims (bond/cash)
  - Change $\mu^B$ and subsidize capital
  - Continuous process
I Theory of Money

- Aim: intermediary sector is not perfectly hedged
- Idiosyncratic risk that HH have to bear is time-varying
- Needed: Intermediaries’ aggregate risk ≠ aggregate risk of economy
- One way to model: 2 technologies \(a\) and \(b\)

<table>
<thead>
<tr>
<th>Technology</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share (Leontieff)</td>
<td>(1 - \bar{\kappa})</td>
<td>(\bar{\kappa})</td>
</tr>
<tr>
<td>Risk</td>
<td>(\frac{dk_t}{k_t} = (\cdot)dt + \sigma^a dZ_t + \bar{\sigma} d\tilde{Z}_t)</td>
<td>(\frac{dk_t}{k_t} = (\cdot)dt + \sigma^b dZ_t + \bar{\sigma} d\tilde{Z}_t)</td>
</tr>
<tr>
<td>Intermediaries</td>
<td>No</td>
<td>Yes, reduce (\bar{\sigma}) to (\varphi \bar{\sigma})</td>
</tr>
<tr>
<td>Excess risk (\sigma_t^a xK^a, \sigma_t^b xK^b)</td>
<td>(-\bar{\kappa}(\sigma^b - \sigma^a) - \frac{\sigma^\theta - \sigma^B}{1 - \vartheta})</td>
<td>((1 - \bar{\kappa})(\sigma^b - \sigma^a) - \frac{\sigma^\theta - \sigma^B}{1 - \vartheta})</td>
</tr>
</tbody>
</table>
I Theory: Balance Sheets

- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits
Overview Slide that Explains the Role of Each Model Ingredient

- $\tilde{\chi}$ -- avoid degenerated distribution (households dying out)
- $\varphi$
  - if $\varphi = 1$ intermediaries would die out,
  - if $\varphi = 0$ don’t earn risk premium (except for aggregate risk)
- $\sigma^b > \sigma^a$ – avoid perfect hedging for intermediaries
  - (except $\sigma^B \neq 0$ – for example risk-free asset is in zero net supply)
    (like AER paper/handbook chapter)
- Fraction $\tilde{\kappa}$ of $K$ has aggregate risk of sigma rest has risk of zero (it’s exogenous) (allocation does not determine total risk in aggregate economy)
  (To keep it clean (taste choice): price-taking planner’s choice is less involved)

- ...

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1b. Price-taking Planner’s Allocation

- Minimize weighted average cost of financing

\[
\min_{\chi_t \leq \bar{\chi}} (1 - \bar{\kappa}) \zeta_t^h \sigma_t^{xK^a} + \left( \zeta_t^l \chi_t + \zeta_t^h (\bar{\kappa} - \chi_t) \right) \sigma_t^{xK^b} + \left( \tilde{\zeta}_t^l \varphi \chi_t + \tilde{\zeta}_t^h (1 - \chi_t) \right) \tilde{\sigma}
\]

- FOC: (equality if \(\chi_t < \bar{\chi}\))

\[
\zeta_t^l \sigma_t^{xK^b} + \tilde{\zeta}_t^l \varphi \tilde{\sigma} \leq \zeta_t^h \sigma_t^{xK^b} + \tilde{\zeta}_t^h \tilde{\sigma}
\]

- \(\sigma_t^{xK^b} = (1 - \bar{\kappa}) \sigma - \frac{\sigma^g - \sigma^B}{1 - \vartheta}\)

- Price of risk with log-utility in total wealth numeraire:

  **Intermediaries**
  \[
  \zeta_t^l = \sigma_t^\eta
  \]

  **Households**
  \[
  \zeta_t^h = -\eta_t \sigma_t^\eta / (1 - \eta_t)
  \]

- Aggregate risk

  \[
  \sigma_t^\eta \left( (1 - \bar{\kappa}) \sigma - \frac{\sigma^g - \sigma^B}{1 - \vartheta} \right) + \left[ (1 - \vartheta) \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma} \right] \varphi \tilde{\sigma} \leq -\eta_t \sigma_t^\eta \left( (1 - \bar{\kappa}) \sigma - \frac{\sigma^g - \sigma^B}{1 - \vartheta} \right) + \left[ (1 - \vartheta) \frac{(1 - \chi_t)}{(1 - \eta_t)} \tilde{\sigma} \right] \tilde{\sigma}
  \]
1c. Money Evaluation + 2. $\eta$-Drift

- As before in money/nominal debt model

- Money evaluation

$$\rho - \mu_t^{\theta/B} = \eta_t \left( (\sigma_t^\eta)^2 + (\tilde{\eta}_t^\eta)^2 \right) + (1 - \eta_t) \left( \left( \frac{\eta_t \sigma_t^\eta}{1 - \eta_t} \right)^2 + (\tilde{\eta}_t^\eta)^2 \right)$$

- $\eta$-drift

$$\mu_t^\eta = (1 - \eta_t) \left( (\sigma_t^\eta)^2 + (\tilde{\eta}_t^\eta)^2 \right) - \left( \frac{\eta_t \sigma_t^\eta}{1 - \eta_t} \right)^2 - (\tilde{\eta}_t^\eta)^2 - \frac{\eta_t \sigma_t^{\theta/B}}{\sigma_t^{\theta/B} - \sigma^B}$$
\( \eta \)-Volatility and Amplification

\[
\sigma_t^\eta = \sigma_t^B + (1 - \theta_t^l)\sigma_t^xK^b
\]
- Where portfolio share \( 1 - \theta_t^l = \frac{\chi_t}{\eta_t}(1 - \vartheta_t) \)

\[
\sigma_t^\eta = \sigma_t^\vartheta - \sigma^B + \frac{\chi_t(1 - \vartheta_t)}{\eta_t} \left( (1 - \bar{\kappa})\sigma - \frac{\sigma_t^\vartheta - \sigma^B}{1 - \vartheta_t} \right)
\]

\[
\Rightarrow \eta_t\sigma_t^\eta = \frac{(1 - \vartheta_t)\chi_t(1 - \bar{\kappa})\sigma + (\chi_t - \eta_t)\sigma^B}{1 - \chi_t(1 - \eta_t)\frac{\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}}
\]
- Note that \( \frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1 - \vartheta_t) \left( \frac{q^K'(\eta_t)\eta_t}{q^K(\eta_t)} + \frac{-q^B'(\eta_t)\eta_t}{q^B(\eta_t)} \right) \)

\[
\eta_t\sigma_t^\eta = (1 - \vartheta_t)\chi_t(1 - \bar{\kappa})\sigma \text{ if } \sigma^B = \sigma^\vartheta
\]

Policy removes endog. amplification

Liquidity Spiral - Disinflationary Spiral

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I Theory: Balance Sheets

- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits
Consequences of a Shock in 4 Steps

1. **Shock:** destruction of some capital
   - % loss in intermediaries net worth > % loss in assets
   - Leverage shoots up
   - Intermediaries %-loss > Household %-losses
     - $\eta$-derivative shifts losses to intermediaries

2. **Response:** shrink balance sheet / delever
   - For given prices no impact

3. **Asset side:** asset price $q^K$ shrinks
   - Liquidity spiral
   - Further losses, leverage $\uparrow$, further deleveraging

4a. **Liability side:** money supply declines
   - Value of money $q^B$ rises
   - Disinflationary spiral

4b. **Households’** money demand rises
   - HH face more idiosyncratic risk (can’t diversify)
Policy

- Fiscal policy
- Monetary policy without fiscal implications
- Macroprudential policy
Fiscal policy

- Includes monetary policy that has fiscal implications
- ...
Monetary Policy

- No fiscal implications, no seigniorage $\tau^{i,\tilde{i}} = 0 \ \forall i, \tilde{i}$
- Any seigniorage is paid out to government debt/money holders in form of interest
- Introducing interest rates on bond/reserves $i_t$.

\[
\begin{align*}
\frac{dr_t^B}{1/P_t} &= i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^K/B_t)}{q_t^K K_t/B_t}
\end{align*}
\]

\[
\begin{align*}
&= \left\{ i_t + \Phi(t_t) - \delta + \mu_t^q - \left[ \mu_t^B + (\sigma_t^q - \sigma_t^B)\sigma_t^B \right] \right\} dt + (\sigma_t^q - \sigma_t^B) dZ_t^\sigma.
\end{align*}
\]

To study monetary policy without fiscal implications, we let $\sigma_t^B = 0$, so

\[
\begin{align*}
\frac{dr_t^B}{1/P_t} &= \left\{ i_t - \mu_t^B + \Phi(t_t) - \delta + \mu_t^q \right\} dt + \sigma_t^q dZ_t^\sigma.
\end{align*}
\]
Monetary Policy: Super-neutrality

- If interest paid on bond holdings is simply financed by issuing new bonds (issuing money), then money is
  - Neutral
  - Super-neutral

\[
\frac{dB_t}{B_t} = i_t dt
\]

- Fisher equation

\[
dr_t^B = i_t dt - d\pi_t
\]
Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ... held by intermediaries in equilibrium

Value of long-term fixed $i$-bond is endogenous

$$d P_t^L / P_t^L = \mu_t^P dt + \sigma_t^P dZ_t$$
Redistributive MoPo: Ex-post perspective

- Adverse shock $\rightarrow$ value of risky claims drops
- Monetary policy
  - Interest rate cut $\Rightarrow$ long-term bond price
  - Asset purchase $\Rightarrow$ asset price
  - $\Rightarrow$ “stealth recapitalization” - redistributive
  - $\Rightarrow$ risk premia
- Liquidity & Deflationary Spirals are mitigated
Introducing long-term bonds

- **Long-term bond**
  - yields fixed coupon interest rate on face value $F(i,m)$
  - Matures at random time with arrival rate $1/m$
  - Nominal price of the bond $P_t^{B(i,m)}$
  - Nominal value of all bonds outstanding of a certain maturity
    \[ B_t^{(m)} = P_t^{B(i,m)} F(i,m) \]
  - Nominal value of all bonds
    \[ B_t = \sum_m B_t^{(m)} \]

- **Special bonds**
  - Reserves: $B_t^{(0)}$ and note $P_t^{B(0)} = 1$
  - Consol bond: $B_t^{(\infty)}$
Debt evolution w/o fiscal implications

\[ dB_t^{(0)} = i_t B_t^{(0)} dt + \sum_{(i,m)} \left[ \left( i + \frac{1}{m} \right) F_t^{(i,m)} dt - \frac{B_t^{(i,m)}}{F_t^{(i,m)}} (dF_t^{i,m} + \frac{1}{m} F_t^{(i,m)} dt) \right] \]

- Money \( B_t^{(0)} \) is different since it pays floating interest rate

- If we have only reserves and consol bond, then

\[ dB_t^{(0)} = i_t B_t^{(0)} dt + iF_t^{(i,\infty)} dt - \frac{B_t^{(i,\infty)}}{F_t^{(i,\infty)}} dF_t^{i,\infty} \]
\[ dM_t = i_t M_t dt + i^L F_t^L dt - P_t^L dF_t^L \]
Define fraction of value of bonds that are not in short-term reserves

\[ \vartheta_t^L = \frac{p_t^L F_t^L}{B_t} \]

Let’s postulate the price of a single long-term consol bond

\[ \frac{dP_t^L}{P_t^L} = \mu_t^L dt + \sigma_t^L dZ_t \]

In the total net worth numeraire the

\[ E_t[dr_t^L - dr_t^M] = \sigma_t^{PL} \sigma_t^{\eta} \] (for now assuming that only intermediaries find it worthwhile to hold consul bonds)

\[ \sigma_t^\eta = \cdots \text{(in net worth numeraire)} \] (5.3)

\[ dr_t^L = dr_t^M + \sigma_t^{PL} \sigma_t^{\eta} dt + \sigma_t^{PL} dZ_t \]
- Return of total bond portfolio (in total net worth numeraire)
- \( dr_t^B = \mu_t^B dt + \sigma_t^B dZ_t \) (since no fiscal implications)
- \( dr_t^B = (1 - \varphi_t^L)dr_t^M + \varphi_t^L dr_t^L \)
- \( dr_t^B = dr_t^M + \varphi_t^L (\sigma_t^{PL} \sigma_t^\eta dt + \sigma_t^{PL} dZ_t) \)

- Return of a single coin (reserve unit/short-term bond)
- \( dr_t^M = (\mu_t^M - \varphi_t^L \sigma_t^{PL} \sigma_t^\eta)dt + (\sigma_t^g - \varphi_t^L \sigma_t^{PL})dZ_t \)
- \( \varphi_t^L \sigma_t^{PL} \) shows importance of long-term bond price variation
  - the \( dZ \)-term is a “risk-transfer”.
  - The dt-term shows that it also affects risk premia
\[ \eta_t \sigma_t^\eta = \sigma_t^\tau + (1 - \theta_t^M, I - \theta_t^L, I) \sigma_t^\chi \kappa^b + \theta_t^L, I (\sigma_t^L - \sigma_t^\tau) \]

- Where portfolio share \( 1 - \theta_t^M, I - \theta_t^L, I = \frac{\chi_t}{\eta_t} (1 - \theta_t) \) and \( \theta_t^L, I = \frac{\theta_t}{\theta_t/\eta_t} \)

\[ \sigma_t^\eta = \sigma_t^\vartheta - \theta_t^L \sigma_t^P_L + \frac{\chi_t(1 - \theta_t)}{\eta_t} \left( (1 - \kappa) \sigma - \frac{\sigma_t^\vartheta}{1 - \theta_t} + \theta_t^L \sigma_t^P_L \right) + \frac{\theta_t^L \theta_t}{\eta_t} \sigma_t^P_L \]

Collect \( \sigma_t^P_L \)-terms

\[ \sigma_t^\eta = \sigma_t^\vartheta + \frac{\chi_t(1 - \theta_t)}{\eta_t} \left( (1 - \kappa) \sigma - \frac{\sigma_t^\vartheta}{1 - \theta_t} \right) + \frac{\chi_t(1 - \theta_t) + \theta_t - \eta_t}{\eta_t} \theta_t^L \sigma_t^P_L \]

Replace \( \sigma_t^\vartheta = \frac{\vartheta' (\eta_t) \eta_t}{\vartheta (\eta_t)} \sigma_t^\eta \) and \( \sigma_t^P_L = \frac{p_L' (\eta_t) \eta_t}{p_L (\eta_t)} \sigma_t^\eta \)

\[ \Rightarrow \eta_t \sigma_t^\eta = \frac{(1 - \theta_t) \chi_t (1 - \kappa) \sigma}{1 - \chi_t - \eta_t \frac{-\vartheta' (\eta_t) \eta_t}{\vartheta (\eta_t)} + \theta_t^L \left( \frac{p_L' (\eta_t) \eta_t}{p_L (\eta_t)} \right) \chi_t (1 - \theta_t) + \theta_t - \eta_t \}

- Note that \( \frac{-\vartheta' (\eta_t) \eta_t}{\vartheta (\eta_t)} = (1 - \theta_t) \left( \frac{q^K' (\eta_t) \eta_t}{q^K (\eta_t)} + \frac{-q^K' (\eta_t) \eta_t}{q^K (\eta_t)} \right) \)... and is the mitigation term due to policy

**Liquidity Spiral**

**Disinflationary Spiral**

Note that money is our benchmark asset (since HH cannot go short L-bond)
Derive $\mu_t^\eta$

- Same steps as before
Monetary Policy: Ex-post perspective

- Money view  
  - Friedman-Schwartz
  - Restore money supply
    - Replace missing inside money with outside money
  - Aim: Reduce deflationary spiral
    - ... but banks extent less credit & diversify less idiosyncratic risk away
    - ... as households have to hold more idiosyncratic risk, money demand rises
  - Undershoots inflation target

- Credit view  
  - Tobin
  - Restore credit
  - Aim: Switch off deflationary spiral & liquidity spiral

- Theory: “Stealth” recapitalization of impaired sector
  - Interest policy and OMO affect asset prices
MoPo Benchmark 1: Removing endogenous Risk

- The policy that removes endogenous risk, $\sigma^M_t = \sigma^\theta_t$
- FOC gives (in closed form)

\[
\chi_t = \min \left( \frac{\eta_t}{\eta_t + (1 - \eta_t)\phi^2 + (1 - \bar{\psi})^2 (\sigma^b)^2 / \bar{\sigma}^2}, \bar{\psi} \right)
\]

- $\eta$-Evolution

\[
\sigma^\eta = (1 - \theta_t) \frac{\chi_t}{\eta_t} (1 - \bar{\psi}) \sigma^b
\]

\[
\eta_t \mu_t^\eta = \eta_t (1 - \eta_t)(1 - \theta_t)^2 \left( \frac{1 - 2\eta_t \chi_t^2}{(1 - \eta_t)^2 \eta_t^2} (1 - \bar{\psi})^2 (\sigma^b)^2 + \frac{\chi_t^2 \phi^2 \bar{\sigma}^2}{\eta_t^2} - \frac{(1 - \chi_t)^2}{\eta_t^2} \phi^2 \bar{\sigma}^2 \right)
\]

Closed form up to $\theta_t$ (which is choice of planner)
Numerical Example

- $\rho = .05$, $\phi = 2$, $\tilde{\sigma} = .5$, $\varphi = .4$, $\bar{\chi} = .8$, $\sigma^a = 0$, $\sigma^b = .1$

policy: $\mu^M = 0$, $\sigma^M = \sigma^g \eta$
Numerical Example

- \( \rho = .05, \phi = 2, \tilde{\sigma} = .5, \varphi = .4, \bar{\chi} = .8, \sigma^a = 0, \sigma^b = .1 \)

\( \vartheta \) falls (because low \( \eta \)-regions with high idiosyncratic risk are visited less frequently)

Policy: \( \mu^M = 0, \quad \sigma^M = \sigma^\vartheta \eta \)

Volatility goes down/amplification removed

With policy, risk is lower, but recovery is faster

No amplification when \( \eta_t = \kappa_t = \bar{\kappa} \)
Optimal Policy

- Next lecture after we have covered welfare analysis
Recall

- Unified macro “Money and Banking” model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

- Exogenous risk &
  - Sector specific
  - Idiosyncratic

- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking

"Paradox of Prudence"
Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes
Thank you!

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