Eco529: Lecture 07
The I Theory of Money 6.0
Markus Brunnermeier & Yuliy Sannikov
“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
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- Banking → “diversifier”
  holds risky assets, issues inside money

- Value of capital declines due to fire-sales
  Liquidity spiral
- Flight to safety
  Value of money rises
  Disinflation spiral
  a la Fisher
- Demand for money rises – less idiosyncratic risk is diversified
- Supply for inside money declines – less creation by intermediaries

- Endogenous money multiplier = f(capitalization of critical sector)

- Paradox of Thrift (in risk terms)

Money and Banking, part 3: Redistributive Monetary...

Watch “Money and Banking”
YouTube Video Channel: “markus.economicus”
https://www.youtube.com/channel/UCV8DkoTKvTvWkI4UsRYlqA/videos?pbjreload=10
"Money and Banking" (in macro-finance)

- Money → store of value/safe asset
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Amplification/endogenous risk dynamics

- Value of capital declines due to fire-sales \textbf{Liquidity spiral}
  - Flight to safety
- Value of money rises \textbf{Disinflation spiral} a la Fisher
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“Money and Banking” (in macro-finance)

- Money: store of value/safe asset
- Banking: “diversifier” holds risky assets, issues inside money

Amplification/endogenous risk dynamics
- Value of capital declines due to fire-sales **Liquidity spiral**
  - Flight to safety
- Value of money rises **Disinflation spiral** a la Fisher
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- Paradox of Thrift (in risk terms)
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- Banking “diversifier” holds risky assets, issues inside money

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- Value of capital declines due to fire-sales Liquidity spiral
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Paradox of Prudence

- Paradox of Thrift (in risk terms)

Monetary Policy (redistributive)
Some literature

- **Roles of money**
  - Unit of account
  - Medium of exchange (Clower, Lagos & Wright)
  - Store of value (Samuelson, Bewley, Aiyagari, Scheinkman & Weiss, Kiyotaki & Moore)

- Models without inside money imply inflation in downturns
  - Less money needed to perform fewer transactions

- **“Money view”** (Friedman & Schwartz)
  - Downturns $\rightarrow$ Bank liabilities decrease

- **“Credit view”**
  - Downturns $\rightarrow$ equity capital $\rightarrow$ bank cuts assets/credit
  - BGG, Kiyotaki & Moore, He & Krishnamurthy, BruSan2014, Drechsler, Jeanne & Korinek, Savov & Schnabl

- **Financial Stability**
  - Diamond & Rajan 2010, Curdia & Woodford 2010, Stein 2012
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Model

- **Agents**

- **Preferences**

- **Firm's production technology**

- **Capital evolution**
  - Reinvestment rate \( \iota_t \), \( \Phi(\iota_t) = \frac{1}{\kappa} \log(\kappa \iota_t + 1) \)
  - \( \frac{ak_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma dZ_t + \tilde{\sigma} d\tilde{Z}_t^i \)

- **Outside equity issued by firms**

- **Money supply**
  \( \frac{dM_t}{M_t} = \mu_t^M dt + \sigma_t^M dZ_t \)

- **Derivatives**

- **Frictions: Incomplete markets**
  - No idiosyncratic risk sharing
  - Limited outside equity issuance (skin in the game constraint)
Without “I” Intermediaries

- Recall from earlier lecture

Diagram:
- Physical Capital
- Firms $i$
- Inside equity
- Households $i$
- Net worth
- Money
Equilibrium – recall from previous lecture

- Collecting the three equations:
  \[ q = 1 + \kappa \lambda \]
  \[ \rho (p + q) = A - \iota \]
  \[ \frac{q \tilde{\sigma}^2}{p + q} = \frac{A - \iota}{q} \]

- Equilibrium solved for \( \mu^M = 0 \)

  \[ p = \frac{\tilde{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q, \]
  \[ q = \frac{1 + \kappa A}{\kappa \sqrt{\rho} \tilde{\sigma} + 1}. \]

Flight-to-safety comparative static
Main insights

- Moneyless equilibrium with $p = 0$, shadow $r^f$ very low
- Money is a bubble with $p > 0$ if $\tilde{\sigma} > \sqrt{\rho}$
- Money takes on insurance role
  - $r^f$ is higher compared to moneyless equilibrium
  - Increases households’ welfare
- Non-stationary equilibria with exploding hyperinflation

- “Tax backing” (even if only tiny $\varepsilon$)
  - Money is not a bubble ($p =$ discounted value of taxes)
  - Eliminates non-stationary equilibria & moneyless equilibrium
    - Off-equilibrium belief alone are sufficient
With Intermediaries: Overview

- Markets are complete w.r.t. aggregate risk
  - \( dZ \)-derivatives can be traded, \( \varsigma = \varsigma \)
  - Incompleteness only w.r.t. idiosyncratic risk
  - Advantages:
    - Clear welfare benchmark
    - Monetary policy does not “complete markets” (no ‘chicken model’)

- Markets are incomplete w.r.t. aggregate & idio risk
  - \( dZ \)-derivatives cannot be traded
  - Advantage:
    - Larger amplification effects
    - Larger pecuniary externalities
Frictions:
- Household cannot diversify idio risk
- Limited risky claims issuance
- Only nominal deposits
With Intermediaries: with $\eta$-Derivative

- As I’s risk share $\chi$ increases, $A(\chi)$ declines due to monitoring cost/moral hazard.
Model

- Agents
- Preferences
  \[ E \left[ \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \]
- Firm’s production technology
  \[ \Phi(\iota_t) = \frac{1}{\kappa} \log(\kappa \iota_t + 1) \]
- Capital evolution
  \[ \frac{d\kappa_t}{\kappa_t} = (\Phi(\iota_t) - \delta) dt + \sigma dZ_t + \tilde{\sigma} d\tilde{Z}_t \]
- Outside equity issued by firms
- Money supply
  \[ \frac{dM_t}{M_t} = \mu^M_t dt + \sigma^M_t dZ_t \]
- Derivatives
- Frictions: Incomplete markets
  - No idiosyncratic risk sharing
  - Limited outside equity issuance (skin in the game constraint)
Model

- $q_t K_t$ value of physical wealth/capital
- $p_t K_t$ value of nominal wealth/money

- $\vartheta_t = \frac{p_t}{q_t + p_t}$ share of (net) wealth due to (outside) money

- Now amplification will be

$$\sigma_t \eta_t = \frac{(1-\vartheta)\chi(1-\overline{\chi})}{1-\chi_t - \eta_t \vartheta'(\eta)} \sigma$$

  **sum of geometric series**

  - Depends on $q(\eta)$ and $p(\eta)$
  - $(1 - \overline{\chi})$ is risk of intermediaries’ stake relative to economy-wide
Digression: Identical risk aversion $\gamma = \gamma$

- Conjecture that $q_t, p_t$ are not affected by $\sigma dZ_t$ aggregate shocks, i.e. $\sigma^q = \sigma^p = 0$

- $q_tK_t$ value of physical capital
  - $dr^K_t = \frac{A_i - \epsilon}{q} dt + \mu^q_t dt + (\Phi(i_t) - \delta) dt + \sigma dZ_t + \tilde{d}d\tilde{Z}_t$

- $p_tK_t$ value of outside money
  - $dr^M_t = (\Phi(i) - \delta) dt + \mu^p_t dt + \sigma dZ_t$

- Wealth risk exposure to aggregate risk is $\sigma dZ_t$ independent of portfolio choice

- Hence $\sigma^\eta = 0$, which confirms our conjecture.

- Remark:
  - If an aggregate risk asset can be traded, then agents do not want to trade it because $\zeta = \zeta = \gamma \sigma$ (absent stochastic investment opportunities)
Consequences of a Shock in 4 Steps

1. **Shock**: destruction of some capital
   - % loss in intermediaries net worth > % loss in assets
   - Leverage shoots up
   - Intermediaries %-loss > Household %-losses, since $\gamma < \gamma$
     - $\eta$-derivative shifts losses to intermediaries

2. Response: shrink balance sheet / delever
   - For given prices no impact

3. Asset side: asset price $q$ shrinks
   - Liquidity spiral
   - Further losses, leverage, further deleveraging

4a. Liability side: money supply declines
   - Value of money $p$ rises

4b. Households' money demand rises
   - HH face more idiosyncratic risk (can't diversify)
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Risk-equivalence & $A(\psi)$-microfoundations

- **Risk-equivalent representation**
  - Express $\chi$-risk exposure by shifting $\psi$-capital shares

- **$A(\psi)$ interpretation**
  - As intermediaries capital share increases $A(\psi)$ declines due to monitoring cost

- Recall in international paper (lecture 04) with 2 goods and CES aggregation
  - Also feasible, but more complicated
  - 2 sectors are needed of which one is bank independent
With Intermediaries: with $Z$-Derivative

- Physical Capital
- Inside equity
- Risky claims
- Firms $i$...

Intermediaries
- Out-Money
- Risky claims
- Money
- Derivative
- Net worth

Households $i$...
- Money
- Net worth
- Derivative
With Intermediaries: with $Z$-Derivative

- As intermediaries’ risk share $\chi$ increases, $A(\chi)$ declines due to monitoring cost/moral hazard.
Risk-equivalent Representation

- Intermediaries hold fraction $\psi_t$ of physical capital

- Households hold fraction $1 - \psi_t$ of physical capital

$A(\psi)$
Equilibrium is a map

Histories of shocks $\{Z_{\tau}, 0 \leq \tau \leq t\}$

prices $q_t, p_t, \psi_t$ allocation

wealth distribution

$$\eta_t = \frac{N_t}{(p_t+q_t)K_t} \in (0,1)$$

intermediaries’ wealth share

• All agents maximize utility
  ▪ Choose: portfolio, consumption

• All markets clear
  ▪ Consumption, capital, money, (outside equity)
Solving MacroModels Step-by-Step

0. Postulate aggregates, price processes & obtain return processes

1. For given SDF processes
   a. Real investment $\iota$, (portfolio $\theta$, & consumption choice of each agent)
   
      ▪ Toolbox 1: Martingale Approach
   b. Asset/Risk Allocation across types/sectors & asset market clearing
      
      ▪ Toolbox 2: “price-taking social planner approach” – Fisher separation theorem

2. Value functions
   a. Value fcn. as fcn. of individual investment opportunities $\omega$
      
      ▪ Special cases
   b. De-scaled value fcn. as function of state variables $\eta$
      
      ▪ Digression: HJB-approach (instead of martingale approach & envelop condition)
   c. Derive $\zeta$-risk premia, $C/N$-ratio from value fcn. envelop condition

3. Evolution of state variable $\eta$
   
   ▪ Toolbox 3: Change in numeraire to total wealth (including SDF)
   ▪ (“Money evaluation equation” $\mu^\theta$)

4. Value function iteration & goods market clearing
   a. PDE of de-scaled value fcn.
   b. Value function iteration by solving PDE
Step-by-Step Approach

0. Postulate aggregate, price/return/SDF processes
\[ dq_t / q_t = \mu^q_t \, dt + \sigma^q_t \, dZ_t, \quad dp_t / p_t = \ldots, \quad d\xi_t / \xi_t = \ldots, \quad d\bar{\xi}_t / \bar{\xi}_t = \ldots \]

1. For given SDF processes \textit{static}
   a. As before \[ \kappa \iota_t = q_t - 1 \]

Recall after using market clearing
\[ \iota_t = \frac{(1-\vartheta_t)A(\psi_t)-\bar{\zeta}}{1-\vartheta_t+\kappa \bar{\zeta}}, \]
where \( \bar{\zeta} \) is the “average” consumption-networth ratio.

This formula is always the same.
Step-by-Step Approach

0. Postulate aggregate, price/return/SDF processes
\[ \frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t, \frac{dp_t}{p_t} = \ldots, \frac{d\xi_t}{\xi_t} = \ldots, \frac{d\_t}{\_t} = \ldots \]

1. For given SDF processes \textit{static}
   a. As before \[ \kappa_t = q_t - 1 \]
   b. Asset/Risk allocation via “Price-taking Planner”
   \[ \max_{\psi_t} A(\psi_t) - \psi_t \zeta_t \phi \tilde{\sigma} - (1 - \psi_t)\_t \tilde{\sigma} \]
   FOC: \[ \frac{A'(\psi_t)}{q} = (\zeta_t \phi - \_t) \tilde{\sigma} \]

2. Value function \textit{backward eqn}
   \[ \tilde{\zeta}_t = \gamma \tilde{\sigma}_t^n = \gamma \frac{(1 - \vartheta_t)\psi_t}{\eta_t} \phi \tilde{\sigma} \]
   Idio-risk premium on portfolio
   \[ \gamma(\tilde{\sigma}_t^N)^2 = \frac{\psi_t^2}{\eta_t^2} (1 - \vartheta_t)^2 \gamma \phi^2 \tilde{\sigma}^2, \quad \gamma \left( \frac{\tilde{\sigma}_t^N}{(1 - \eta_t)^2} \right)^2 = \frac{(1 - \psi_t)^2}{(1 - \eta_t)^2} (1 - \vartheta_t)^2 \gamma \tilde{\sigma}^2 \]
   \[ \zeta_t = \gamma \sigma_t^c = -\sigma_t^v + \sigma_t^n + \sigma_t^q + \sigma_t^p + \gamma \sigma = \_t = \zeta_t = \gamma \sigma_t^c = -\sigma_t^v - \frac{\eta_t \sigma_t^n}{1 - \eta_t} + \sigma_t^q + \sigma_t^p - \gamma \sigma \]
Step-by-Step Approach

2. Value function

\[ \tilde{\zeta}_t = \gamma \tilde{\sigma}_t^n = \gamma \frac{(1-\theta_t)\psi_t}{\eta_t} \phi \tilde{\sigma} \]

Idio-risk premium on portfolio

\[ \gamma (\tilde{\sigma}_t^N)^2 = \frac{\psi_t^2}{\eta_t^2} (1 - \theta_t)^2 \gamma \phi^2 \tilde{\sigma}^2, \quad \gamma \left( \frac{\tilde{\sigma}_t^N}{\eta_t} \right)^2 = \frac{(1-\psi_t)^2}{(1-\eta_t)^2} (1 - \theta_t)^2 \gamma \tilde{\sigma}_t^2 \]

\[ \zeta_t = \gamma \sigma_t^c = -\sigma_t^v + \sigma_t^\eta + \sigma_t^q + \sigma_t^p + \gamma \sigma = \zeta_t = \]

\[ \gamma \sigma_t^c = -\sigma_t^v - \frac{\eta_t \sigma_t^\eta}{1-\eta_t} + \sigma_t^q + \sigma_t^p - \gamma \sigma \]

From Ito’s Lemma \( \sigma_t^v = \frac{\nu'}{\nu} \eta_t \sigma_t^\eta \) and \( \frac{\nu'}{\nu} (1 - \eta_t) \frac{\eta_t \sigma_t^\eta}{1-\eta_t} \)

\[ \frac{C_t}{N_t} = \frac{(\eta_t(q_t+p_t))^{1/\gamma - 1}}{v_t^{1/\gamma}} \quad \frac{C_t}{N_t} = \frac{((1-\eta_t)(q_t+p_t))^{1/\gamma - 1}}{v_t^{1/\gamma}} \]

3. Evolution of \( \eta \)

\( \eta \)-derivative \( \Rightarrow \zeta_t = \zeta_t, \Rightarrow \sigma_t^\eta = \frac{(1-\eta_t)(\gamma-\gamma)\sigma}{(1-\eta_t)(\frac{\nu'}{\nu}-\frac{\nu'}{\nu})+1} \)
Step-by-Step Approach

2. Value function

\[ \gamma (\tilde{\sigma}_t^N)^2 = \frac{\psi_t^2}{\eta_t^2} (1 - \vartheta_t)^2 \gamma \phi^2 \tilde{\sigma}^2, \quad \gamma \left(\tilde{\sigma}_t^N\right)^2 = \frac{(1-\psi_t)^2}{(1-\eta_t)^2} (1 - \vartheta_t)^2 \gamma \tilde{\sigma}^2 \]

\[ \frac{C_t}{N_t} = \frac{(\eta_t(q_t+p_t))^{1/\gamma-1}}{\nu_t^{1/\gamma}} \]

3. Evolution of \( \eta \)

\( \eta \)-derivative \( \Rightarrow \zeta_t = \zeta_t \), \( \Rightarrow \sigma_t^{\eta} = \frac{(1-\eta_t)(\gamma-\gamma)\sigma}{(1-\eta_t)(\frac{v'}{v}-\frac{v'}{v})+1} \)

Recall from earlier lecture (and since \( \zeta_t = \zeta_t \) and \( r^F = r^F \)),

\[ \mu_t^{\eta} = (1 - \eta_t)(\zeta_t - \sigma_t^{\bar{N}}) \left( \sigma_t^{\eta} - \sigma_t^{\bar{N}} \right) \]

\[ + (1 - \eta_t)\tilde{\zeta}_t \tilde{\sigma}_t^{n} - (1 - \eta_t)\tilde{\zeta}_t \tilde{\sigma}_t^{n} - \left( \frac{C_t}{N_t} - \frac{C_t + C_t}{q_t K_t} \right) \]

\[ \mu_t^{\eta} = \sigma_t^{\eta} (\zeta_t - \sigma - \sigma_t^q - \sigma_t^p) - (1 - \eta_t) \left( \frac{C_t}{N_t} - \frac{C_t}{N_t} + \gamma (\tilde{\sigma}_t^N)^2 - \gamma \left(\tilde{\sigma}_t^N\right)^2 \right) \]
Step-by-Step Approach

3. Evolution of $\eta$

The forward eqn for $\eta$

$$\sigma_t^\eta = \frac{(1-\eta_t)(\gamma - \gamma)}{(1-\eta_t)(\frac{v'}{v} - \frac{v''}{v}) + 1} \sigma$$

$$\mu_t^\eta = \sigma_t^\eta (\zeta_t - \sigma - \sigma_t^{q+p}) - (1 - \eta_t) \left( \frac{C_t}{N_t} - \frac{C_t}{N_t} + \gamma (\tilde{\sigma}_t^N)^2 - \gamma (\tilde{\sigma}_t^N)^2 \right)$$

Money evaluation equation

Use same approach as for wealth share $\mu_t^\eta$ for economy wide "money share" $\vartheta_t$

$$\mu_t^\vartheta = +\sigma_t^\vartheta (\zeta_t - \sigma - \sigma_t^{q+p}) +$$

$$+ \frac{C_t + C_t}{(q_t + p_t)K_t} - \mu^M - \eta_t \gamma (\tilde{\sigma}_t^N)^2 - (1 - \eta_t) \gamma (\tilde{\sigma}_t^N)^2$$

4. Value function $v(\eta), \underline{v}(\eta) & \vartheta(\eta)$

- Solve PDE (growth equation)
Numerical Example

- \( \rho = \rho = .05, \gamma = 1.5, \gamma = 4, \delta = .03, \sigma = 0, \gamma \phi^2 \tilde{\sigma}^2 = .1, \gamma \tilde{\sigma}^2 = .4, \)

\[
A(\psi) = \psi (1 - \psi)
\]

Assumes more than simply monitoring costs
Numerical Example

- $\rho = \overline{\rho} = .05, \gamma = 1.5, \gamma = 4, \delta = .03, \sigma = 0, \gamma \phi^2 \tilde{\sigma}^2 = .1, \gamma \tilde{\sigma}^2 = .4,$
  \[ A(\psi) = \psi(1 - \psi) \]

- $\vartheta(\eta)$

Poll 40:
Why does the value increase as $\eta$ goes it very high?
Amplification

- What’s the right benchmark?

- Assume $\frac{C_t}{N_t}$ and $\frac{C_t}{N_t}$ were constant
  - Would be the case with Epstein-Zin preferences when $IES = 1$, risk aversion still differ

\[ \eta^{-1}(q + p)^{-1}v = \text{const.} \Rightarrow \frac{v'}{v} = (1 - \gamma) \left( \frac{1}{\eta} + \frac{q' + p'}{q + p} \right) \]

- Similarly for households
  \[ \Rightarrow \frac{v'}{v} = \left( 1 - \gamma \right) \left( \frac{-1}{1 - \eta} + \frac{q' + p'}{q + p} \right) \]

- Assume also that $q + p$ were constant, then level of risk without amplification (but risk sharing)

\[ \sigma^\eta_t = \frac{(1 - \eta_t)(\gamma - \gamma)}{\gamma \eta_t + \gamma (1 - \eta_t)} \sigma \]
With Intermediaries, but no $\eta$-Derivative
Welfare analysis – I Theory 5.0

- Challenge: Heterogeneous agents with idiosyncratic risks
- Inefficiencies in
  - Production
  - Investment
  - Risk sharing
Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply - Amplification/endogenous risk
    - Liquidity spiral asset side of intermediaries’ balance sheet
    - Disinflationary spiral liability side

- Monetary Policy
- Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Monetary Policy: Ex-post perspective

- **Money view**
  - Friedman-Schwartz
  - Restore money supply
  - Replace missing inside money with outside money
  - Aim: Reduce deflationary spiral
    - ... but banks extent less credit & diversify less idiosyncratic risk away
    - ... as households have to hold more idiosyncratic risk, money demand rises
    - Undershoots inflation target

- **Credit view**
  - Tobin
  - Restore credit
  - Aim: Switch off deflationary spiral & liquidity spiral
Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ... held by intermediaries in equilibrium
    - Value of long-term bond is endogenous
      \[ \frac{dB_t}{B_t} = \mu_t^B dt + \sigma_t^B dZ_t \]

Perpetual bonds:
- pay in money (at unit rate)
- endogenous price \( B_t \) (in money)

Value \( b_t K_t \)

Money

Capital

Value \( p_t K_t \)

Value \( q_t K_t \)
- Adverse shock → value of risky claims drops
- Monetary policy
  - Interest rate cut ⇒ long-term bond price
  - Asset purchase ⇒ asset price
  - ⇒ “stealth recapitalization” - redistributive
  - ⇒ risk premia
- Liquidity & Deflationary Spirals are mitigated
- Adverse shock → value of risky claims drops

- Monetary policy
  - Interest rate cut ⇒ long-term bond price
  - Asset purchase ⇒ asset price
  - ⇒ “stealth recapitalization” - redistributive
  - ⇒ risk premia

- Liquidity & Deflationary Spirals are mitigated
Monetary policy and endogenous risk

- Intermediaries’ risk (borrow to scale up)

\[
\sigma_t^\eta = \frac{x_t \left(1^b \sigma^b - \sigma^K_t\right)}{1 + \left(\frac{x_t \psi_t - \eta}{\eta_t}\right) \frac{\vartheta'(\eta_t)}{\vartheta/\eta_t} - \left(x_t + \vartheta_t \frac{1-\eta_t}{\eta_t}\right) b_t \frac{B'(\eta_t)}{p_t B(\eta)/\eta}}
\]

- MoPo works through \(\frac{B'(\eta_t)}{B(\eta_t)/\eta_t}\)
  - with right monetary policy bond price \(B(\eta)\) rises as \(\eta\) drops “stealth recapitalization”
  - Switch off liquidity and disinflationary spiral

- Example:
  Remove amplification s.t.

\[
\sigma_t^\eta = x_t \left(1^b \sigma^b - \sigma^K_t\right)
\]
Numerical example with monetary policy

- Prices

$q$, without policy
$q$, with policy
$p$, without policy
$p$, with policy

$q$ is more stable
$p$ less disinflation
Numerical example with monetary policy

- Drift and volatility of $\eta$

![Graph showing drift and volatility of $\eta$.](Graph.png)
Observations

- As interest rates are cut in downturns, bonds held by intermediaries appreciate, this
  - protects intermediaries against shocks
  - increases the supply of assets that can be used as storage (weakens disinflation)

- Ex-post stabilization
  - Liquidity spiral
  - Disinflationary spiral

- Ex-ante
  - Higher leverage
  - (shift in steady state)
Monetary policy ... in the limit

- full risk sharing of all aggregate risk

\[ \sigma_t^\eta = \frac{x_t(1^b \sigma^b - \sigma^K)}{1 - (\frac{\chi \psi - \eta}{\eta} - \vartheta'(\eta)) \vartheta/\eta + \left( (1 - \vartheta) \frac{\psi \chi - \eta}{\eta} + \vartheta \frac{1 - \eta}{\eta} \right) \frac{b_t - B'(\eta)}{p_t B(\eta)/\eta}} \]

- \( \eta \) is deterministic and converges over time towards 0
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Monetary policy ... in the limit

- full risk sharing of all aggregate risk

- Aggregate risk sharing makes $q$ deterministic

- Like in benchmark toy model
  - Excessive $k$-investment
  - Too high $q$
    (pecuniary externality)
    - Lower capital return

- Endogenous risk corrects pecuniary externality
MacroPru

- MacroPru complements MoPo
  - Not substitutes

- Good MacroPru enables more aggressive MoPo
  - More redistribution ex-post
  - More risk-transfers/insurance ex-ante
  - Lower $q$
    - reduces cost to repurchase capital after shock
    - Lowers importance of idiosyncratic shocks
MacroPru policy

- Regulator can control
  - Portfolio choice $\psi_s, x_s$

- cannot control
  - investment decision $\iota(q)$
  - consumption decision $c$

of intermediaries and households
MacroPru policy

- Regulator can control
  - Portfolio choice $\psi_s, x_s$
  - investment decision $\iota(q)$
  - consumption decision $c$
- cannot control
  - of intermediaries and households
  - De-facto controls $q$ and $p$ within some range
  - De-factor controls wealth share $\eta$
    - Force agents to hold certain assets and generate capital gains

- In sum, regulator maximizes sum of agents value function
  - Choosing $\psi^b, q, \eta$
Recall

- Unified macro “Money and Banking” model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

- Exogenous risk &
  - Sector specific
  - Idiosyncratic

- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking

- Macro-prudential policies
  - MacroPru + MoPo to achieve superior welfare optimum
Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes
# Redistributive Monetary Policy

## (New) Keynesian Demand Management

- Stimulate aggregate consumption
- Woodford (2003)
  - Price stickiness & ZLB
  - Perfect capital markets
- Both
- Representative Agent
- Cut $i$
- Reduces $r$ due to price stickiness
- Consumption $c$ rises
- Changes bond prices
- Redistributions from low MPC to high MPC consumers

## Theory of Money Risk (Premium) Management

- Alleviate balance sheet constraints
- Tobin (1982)
  - Both
  - Financial frictions
  - Incomplete markets
- Heterogeneous Agents
- Cut $i$
- Changes asset prices
  - Ex-post: Redistributions to balance sheet impaired sector
  - Ex-ante: insurance -> reduces endogenous risk
    - (Hanson-Stein,...)
    - Moral hazard -> role for MacroPru

## Focus on LEVELS

- Focus on levels and RISK DYNAMICS