The “Reversal Interest Rate”:
An Effective Lower Bound on Monetary Policy*

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Abstract

The “reversal interest rate” is the rate at which accommodative monetary policy “reverses” its effect and becomes contractionary. The reversal interest rate depends on various factors: (i) banks’ asset holdings with fixed (non-floating) interest payments, (ii) the degree of interest rate pass-through to loan rate and deposit rate, (iii) the amount of bank’s whole sale funding, (iv) their dividend policy. Low interest rates beyond the time when fixed interest rate mature do not lead to recapitalization gains while still potentially lowering banks’ margins, suggesting a shorter forward guidance policy. Moreover, quantitative easing (QE) increases the reversal rate. QE should only employed after interest rate cut is exhausted.

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1 Introduction

In most New Keynesian models, the economy enters a liquidity trap as policy rates approach zero, because of the assumed zero lower bound. Yet recently, the Bank of Japan joined a growing group of central banks – including the ECB, the Swiss National Bank, the Swedish Riksbank and Danmarks Nationalbank – which have set a negative interest rate.

This begs the question: what is the effective lower bound on monetary policy? Given that subzero rates are technically feasible, we argue in this paper that the effective lower bond is given by the “reversal rate”, the rate at which accommodative monetary policy “reverses” its effect and becomes contractionary for lending. Below the “reversal interest rate”, a decrease in the monetary policy rate depresses rather than stimulates lending and hence the macroeconomy.

Importantly, the reversal interest rate is not (necessarily) zero. Hence, unlike what some commentators suggest, negative interest rates are not fundamentally different. In our model, when the reversal interest rate is positive, say 1 %, then already a policy rate cut from 1 % to 0.9 % is contractionary. On the other hand, if the reversal interest rate is -1 %, there is room to go negative up to that point, provided that financial stability is secured.1 The exact level of the reversal interest rate depends on macro-prudential policy, especially financial regulation, as well as other parameters of the economic environment and financial sector’s balance variables. Restrictive financial regulation in bad times can undermine monetary policy or render it ineffective. Further determinants of the reversal interest rate in our model include banks’ equity capitalization, banks’ interest rate exposure, the market structure of the financial sector as well as banks’ dividend policies.

How does an interest rate cut by the central bank affect banks’ profit, equity and credit growth? We focus on the case where (occasionally binding) constraints limit banks’ lending. We identify three channels.

First, banks with long-term legacy assets with fixed interest payments benefit from a policy interest rate cut. As the central bank lowers the interest rate, banks can refinance their long-term assets at a cheaper rate. This increases the value of their equity; they are better capitalized, which relaxes their regulatory or economic constraint. Viewed differently, banks fixed interest rate holdings experience capital gains. Hence, an interest rate cut is

1It is no secret that the US Federal Reserve feels that it cannot lower the interest rate below .25 %, since it would otherwise create a run on money market funds. Yet discussions over negative rates have reached the U.S. as well.
essentially a “stealth recapitalization” of the banks, as stressed in Brunnermeier and Sannikov’s (2012) “I Theory of Money.”

Second, a lower policy interest rate also affects banks’ net interest rate margin (NIM), which depends on the interest rate pass through and the quantity responses. When a central bank cuts the interest rate, it tries to induce a substitution effect: Banks substitute their reserve and safe asset holdings with loans to firms and households. In the hypothetical case of a perfectly competitive financial sector without frictions, any monetary policy rate cut is passed through fully to the deposit and loan rate. Lower loan rates then lead to increased credit growth, and the real economy expands. Since profits from margin business are competed away, they are always zero and are not affected by rates changes. Hence banks with legacy asset holdings with fixed interest rates unambiguously benefit from an interest rate cut. In the real world, however, financial markets are not perfectly competitive and banks have market power. Their NIMs are therefore not competed away, and they charge a mark-up. A policy rate cut can affect the profitability of current and future business in addition to recapitalization effects from their fixed interest legacy holdings. Essentially, decreasing rates hurt the holdings that banks have in reserves and other cash-equivalent assets. Formally, we model banks as having a special relationship to their customers, both on the loan and deposit side. They have more information about their customers than potentially competing banks. This is especially the case for small and medium enterprises which often rely on their “house bank” for repeated financing. Also, households are reluctant to switch their deposits from one bank to another. In our model, we assume that a competing bank has to pay an extra spread to attract a customer away from his house bank. Beyond that, banks are in fierce Bertrand competition, but below the maximum mark-up they can charge, these “switching costs” grant the house bank local monopoly power on the loan market. Similarly, the house bank enjoys monopsony power on the deposit market as long as its mark-up is low enough. An interest rate cut hence affects (a) banks’ net interest rate margins and quantities in this period and (b) in future periods.

Third, in a dynamic setting a change in the policy rate can also alter how future bank profits are discounted in order to obtain the franchise value of the bank. Essentially, when the risk-free rate decreases, equity cushions required to sustain lending are more costly to hold on to if they can’t be re-invested in profitable assets.

Interestingly, the sign of the overall change in banks’ profits following a rate decrease across all three channels depend on a sufficient statistic: the amount of wholesale funding that banks’ need to finance their activities, as measured by the amount of assets earning the
risk-free rate on their balance sheet. As long as the latter is positive, a decrease in interest rates necessarily hurt profits. Hence, if banks’ fixed income holdings are small enough such that reevaluation effects do not fully offset the loss in profit from ongoing net-interest rate margin business, banks decline in value. This, in turn, tightens regulatory constraints and/or economic constraints, up to the point of the reversal interest rate at which lending starts to decrease as the constraint binds and profits decrease. Moreover, an interesting amplification mechanism emerges. As the negative wealth effect further tightens banks’ equity constraint, banks cut back on their credit extension and are forced to scale up their safe asset holdings. As these assets yield a lower interest rate, their profits decline faster. This lowers the value of their equity further, forcing them to substitute out of risky loans into safe assets, which in turn lowers their profit, and so on. The reversal interest rate also depend on banks’ assets’ interest rate exposure, the tightness of financial regulation, as well as the market structure of the banking sector. If banks hold more long-term bonds and mortgages with fixed interest, the “stealth recapitalization” effect due to an interest rate cut is more pronounced, and the reversal interest rate is lower.

The reversal rate rises with banks’ long-dated fixed income asset holdings, with the strictness of capital requirements and competitiveness within the financial sector.

The multi-period extension of our model allows us to study how changes in the whole interest rate policy path affect total lending in the economy - one objective of policy markers. Our analysis shows that the length of the interest rate cut can last longer if banks hold fixed income assets with longer duration. A lower interest rate allows banks to refinance their fixed come assets up to the point when they mature. A longer anticipated interest rate cut only translates into higher reevaluation gains for assets with higher maturity, which can offset the loss resulting from lower NIM profits for more periods in the future. If banks assets are of shorter duration, then a longer interest rate cut might lead to larger NIM profit losses than fixed income capital gains.

Our paper also provides a rationale why central banks should be able to regulate banks’ dividend policy. In order to boost lending, central banks undertake actions to boost banks’ equity. If reevaluation gains simply translates into higher dividend payments, central bank policy is not fully effective. Central banks need a tool to limit dividend payouts in periods in which it is necessary to promote total credit growth. We effectively show how a lending-

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2 Timothy F. Geithner, president of the NY Fed and then head of the Treasury during the crisis, wrote for example in his recent book on the crisis that the Fed Board "considered forcing banks as a group to stop paying dividends in order to conserve capital". (Geithner 2016, p.138).
maximizing policy can improve its effectiveness by imposing a certain dividend structure to banks.

Finally, we also believe that our model has important implications for the timing and sequencing of Quantitative Easing measures (QE). The optimal sequencing is the following. First, induce banks (possibly through favorable refinancing operations) to hold long-term bonds with a fixed interest rate; second, cut the policy interest rate to generate capital gains for a “stealth recapitalization” of the banking sector; third, conduct QE and lift the long-term assets of banks’ balance sheet so that banks realize their capital gains: banks sell their long-term bonds to the central bank in exchange for short-term bonds or reserves at high prices. However, after QE a further interest rate cut is less effective (and might be even counterproductive) since now banks hold mostly short-term reserves. QE undermines the power of future interest rate cuts and increases the “reversal interest rate”.

If banks suffer losses, e.g. because of higher delinquency rates in their mortgages, the (endogenous) “reversal interest rate” rises. If it does so beyond the policy rate, a subsequent interest rate cut is contractionary, and QE has used up the “single bullet”. Under such circumstances, it might better to raise interest rates, which improve banks’ net interest rate margin. Since banks have passed on large parts of their bond holdings to the central bank, the central bank suffers the capital losses on these bonds. Only after raising the interest rate and devaluing the long-term bonds, and only after conducting a “Reverse-QE”, which replaces banks’ reserve holdings with long-term bonds again, is the reversal interest rate restored at a higher level. In a sense this reloads the gun – for a further round of interest rate cuts.

In our last section we formally model the rest of the economy around the banks’, introducing along the way default shocks. This allows us to study how the mechanisms highlighted in the paper show up in impulse-response functions. [This area is still work in progress].

1.1 Related literature.

Our modeling of the pass-through stands on the shoulders of a large literature on the microeconomics of the banks, which formally started with [Klein (1971) and Monti (1972)]. Ho and Saunders (1981) and Prisman et al. (1986) added uncertainty into the analysis. Santomero (1984) provides a good survey of this early theoretical literature.

Recent empirical papers have revived the literature through the lenses of the pass-through of monetary policy interest rates to the economy. De Bondt (2005), using European data, shows that the immediate pass-through to lending and deposit rates is at most 50 % at a
three-month horizon. Bech and Malkhozov (2016) show that the recent drop in reserve rates below zero transmitted through all risk-free short term assets of the economy, but find that the pass-through seemed imperfect for retail deposit rates. Mortgage rates in their data also showed no response, or even increased in certain countries. Drechsler et al. (2015) focus on the transmission to deposit rates, and show in particular that mark-ups on deposits tend to decrease with the reserve rate. Their micro-foundations, as ours, show that there is no effective zero lower bound, as was emphasized as well by Rognlie (2016).

Other papers try to measures banks’ net interest margin from a different angle. Saunders and Schumacher (2000) decompose bank margins into a regulatory component, a market structure component and a risk-premium component and show that all three channels are sizable in the data. Abad et al. (2016) show that banks on net buy interest rate protection on the derivatives market, although large banks do play an intermediary role in that they sell interest rate risk protection to smaller banks buy protection. However, Begenau et al. (2015) document that even if banks do participate on the markets for derivatives to hedge their interest rate risk, banks can’t or don’t fully hedge their interest rate exposure.

Landier et al. (2013) focus directly on the real lending of banks. They document in a panel study that the income gap - the sensitivity of banks profits to interest rates has a causal impact on their lending behavior.

An important aspect of the literature on micro-banking is the competition structure of the banking industry. Petersen and Rajan (1995), in an influential paper, suggested that a monopolistically competitive banking structure better reflects reality, arguing that banks need some monopoly power to sustain their businesses. Sharpe (1997) suggests evidence of switching costs to depositors, and Kim et al. (2003) of relationship costs for banks. We use these costs to justify the imperfect competitive structure of our model both papers offer micro-foundations for these costs. Maudos and Fernandez de Guevara (2004), Saunders and Schumacher (2000), and Drechsler et al. (2015) offer evidence that imperfect competition affects the pass-through of rates.

2 A two-period model

2.1 Structure of the financial sector

We consider first a two-period economy where $I$ symmetric banks owned by investors aim at maximizing their next-period equity. On the asset side, banks hold cash, central bank
reserves and cash-equivalents (like interbank market positions and short-term government bonds), denoted $C$, bonds $B$ inherited from the past period, and loans denoted by $L$. On the liability side, banks raise deposits $D$ and have equity $E_0$ from investors. Hence their balance sheet identity is:

$$C + B + L = D + E_0$$

We now discuss how the interest rates for each of these balance sheet items are pinned down. Also, to abstract from monetary consideration, throughout this paper we assume that inflation is stable and hence any change in nominal rates corresponds to changes in real rates.

**Reserves, legacy assets and equity**

Banks can adjust $C$ elastically at the short-term risk-free rate $r^f$. We assume that $r^f$ is pinned down by the reserve rate offered by the central bank. Next, outstanding bonds $B$ pay-off an interest rate $r^B \geq r^f$ next period, and mature. Finally, the discount factor of investors is assumed to be $r^E \geq r^f$.\(^3\)

**Loan market**

Banks compete on the market for loans. We assume for simplicity that all risky loans are renegotiated every period – this is without loss of generality if old loans contracts are indexed on current economic conditions. The aggregate demand for loans is:

$$L(r^L) = \int_0^1 l^i(r^L)di$$

Where $l^i(r^L)$ are demands for loans from borrowers, firms and households.

Loan costumers take the cheapest loans offered to them, so effectively banks are Bertrand-competing. Each borrower $i$ has a “house bank”. That is, banks have relation-specific partnerships, and each bank has a particular relation with a mass $1/I$ of borrowers. The costs of “stealing” a partnership from other banks $\kappa^L$. They capture informational costs as in [Kim et al. (2003)] or even market power granted from regulation, in the spirit of [Petersen and Rajan (1995)]. We further assume for simplicity that individual loan demands are all

\(^3\)This assumption can reflect the impatience of investors, or risk-premias. We will allow $r^E$ to endogenously vary with $r^f$ in Subsection 3.4.
identical so that effectively $L(r^L) = l(r^L)$. Combined with large enough costs, this effectively creates segmentation of the market, and hence $L(r^L) = L(r^L)/I$.

Banks are subject to quantity restrictions as follows. Similar to [Wong (1997)](#), banks are prevented by regulation from taking an excessively risky position, captured by the following constraint:\footnote{These constraints can also emerge as the solution to some economic problem – incentives or risk-aversion – although the micro-foundation of such constraint is still an active area of research.}

$$\gamma_0 L \leq \left(\frac{E_1}{E_0}\right)^{\gamma_1} E_0$$

Where $\gamma_0 \geq 0$ is the weight assigned to risky positions, $E_1$ is the future equity of banks, $E_0$ current equity, and $\gamma_1 \geq 0$ the relative geometric weight between the two latter, capturing various regulatory possibilities. With $\gamma_1 = 1$, positions must be covered by sufficient amounts of next period’s liquidation value; with $\gamma_1 = 0$, by today’s equity. Note that for simplicity, we don’t include bonds $B$ inside the constraint, although we do think of them as possibly not risk-free.

**Deposit market**

Banks also compete on the deposit market to finance their activities. As per loans, the aggregate supply of deposits is the aggregation of many depositors’ demand, which in turn depends on the deposit rate $r^D$.

$$D(r^D) = \int_0^1 d_i(r^D) di$$

Where $d_i(r^D)$ is supplied from individual depositors, which we assume are all identical for now so that effectively $D(r^D) = d(r^D)$.

\footnote{Ideally, we would want $D(r^D)$ to represent all sources of funding of banks. This is easy to extend, at the cost of tractability and clarity of analysis.} As in [Drechsler et al. (2015)](#), we think of $d(r^D)$ as derived from a depositor maximizing jointly its next-period wealth $W$ and liquidity services $L$, which themselves depend on cash holdings $c$ and deposits $d$. The depositor can invest its
wealth \(W_0\) into cash \(c\) yielding no interest and deposits earning interest \(r^D\). More specifically:

\[
d(r^D) = \arg \max_{c,d} U(W, \mathcal{L}) \tag{2.1}
\]

s.t.

\[
W = (1 + r^D)d + c \tag{2.2}
\]

\[
\mathcal{L} = \mathcal{L}(c, d) \tag{2.3}
\]

\[
W_0 = d + c \tag{2.4}
\]

The properties of the implied deposit function are important, as its semi-elasticity will govern the mark-up banks can charge on deposits. Note that the FOC of the agent’s problem reads:

\[
U_W r^D + U_{\mathcal{L}} d = U_{\mathcal{L}} c
\]

Absent extra-returns on deposits, the agents would just trade-off the marginal liquidity gains of deposits and cash; however the former earns an extra interest \(r^D\). In particular, if \(\mathcal{L}\) is concave in each of its arguments and is supermodular (\(\mathcal{L}_{cd} > 0\)), then an increase in \(r^D\) raises the demand for deposit. Moreover, it is not hard to generate demands that gets more elastic as \(r^D\) decreases, the empirically relevant case, as the next example shows.

**Example 1.** Assume that \(U\) in equation (2.1) of the depositor’s problem is Cobb-Douglas, and \(\mathcal{L}\) is a Dixit-Stiglitz aggregator parameterized by its elasticity of substitution \(\eta\). Consider first the extreme case where \(\eta = \infty\), meaning that cash and deposits are perfect substitutes for liquidity needs. In this scenario, zero is a strict lower bound on deposits: below zero, agents substitute all their savings in cash. At zero, therefore, the demand of agents gets infinitely more elastic. Now consider the other extreme case of \(\eta = 0\), which makes cash and deposits perfect complements. In this scenario, consumers’ deposit supply is nearly always inelastic. Hence the bank enjoys a monopsony power that doesn’t change as the rate that they charge changes. Finally, Figure 7 depict the semi-elasticities of the deposit functions implied by different \(\eta\)’s between our two extreme cases – that is \(\partial \log D(r^D)/\partial r^D\). Not only are these elasticities strictly ordered for given \(r^D\): for high \(\eta\)’s, there is a sharp rise in how elastic deposit supply becomes around zero.

As in the loan market, each depositor also has a “house bank”. Banks have relation-specific partnerships, and each has a particular relation with a mass \(1/I\) of borrowers. As in the rest of the literature (Drechsler et al., 2015; Sharpe, 1997), we assume that there are switching costs \(\kappa^D\) incurred by depositors from changing bank. As a consequence, the
Figure 1: Elasticities of deposit supply functions for varying $\eta$. $U$ is Cobb-Douglas with weight $1/2$, $L$ is a Dixit-Stiglitz aggregator with EOS $\eta$, and $W_0 = 1$.

market for deposits is also segmented, though $\kappa^D$ pins down the maximum mark-up that banks can charge. Since all banks are symmetric, and markets segmented, effectively the demand seen by a particular bank is $D(r_D) = D(r_D)/I$, and all adjustments take place at the intensive margin. Note that when $\kappa^D$ is large enough, banks always charge a monopolistic mark-up on their depositors.

2.2 The reversal interest rate

In this subsection we show how a reversal interest rate can emerge as a result of our assumptions. For clarity of exposition, we assume that the costs $\kappa^D, \kappa^L$ are sufficiently high so that banks have essentially free monopsonies and monopolies over respectively their deposits and their loans; we study in subsection 2.3 how relaxing this assumption affects the reversal interest rate.
Net interest margins

We denote by $E_1$ the next-period equity of the banks. Each bank maximizes $E_1$ by choosing its loan rate $r_L$, and its deposit rate $r_D$, and its amount of cash $C$:

$$E_1(r_f) = \max_{C, r_L, r_D} (1 + r_L)L(r_L) + (1 + r_B)B + (1 + r_f)C - (1 + r_D)D(r_D)$$

s.t. $C + L + B = D + E_0$

Substituting the balance sheet constraint, we then obtain the following expression for the profits:

$$E_1(r_f) = \max_{r_L, r_D} (r_L - r_f)L(r_L) + (r_f - r_D)D(r_D) + (r_B - r_f)B + (1 + r_f)E_0$$

Since risk-free assets holdings $C$, which earn the risk-free rate $r_f$, are elastic, banks’ optimization problems can be separated into two independent problems: the choice of loan rate $r_L$ and that of the deposit rate $r_D$. To characterize the solutions more easily, denote as follows the following semi-elasticities:

$$
\varepsilon^L(r_L) \equiv \left| \frac{\partial \log L}{\partial r_L} \right|, \quad \varepsilon^D(r_D) \equiv \left| \frac{\partial \log D}{\partial r_D} \right|
$$

The following lemma describes the optimal pricing rules, and hence the pass-through of reserve rates to loan and deposit rates in the unconstrained case.

**Lemma 1** (Optimal pricing rules, unconstrained case). Each bank chooses the following pricing rules for loans:

$$r^*_{L} = r_f + \frac{1}{\varepsilon^L(r^*_L)}$$

And for deposits:

$$r^*_{D} = r_f - \frac{1}{\varepsilon^D(r^*_D)}$$

For what follows, denote the resulting optimal mark-ups as $\mu^*_{L}$, $\mu^*_{D}$ – to ease notation we neglect the fact that all optimal variables are functions of $r_f$ and other parameters of the model. Next, denote $L^*$, $D^*$ the associated optimal (aggregate) quantities, which imply the optimal amount of cash holdings $C^* = D^* + E_0 - B - L^*$. Next-period equity can then be
rewritten as:

\[
E_1(r^f) = \mu^*L^* + \mu^*D^* + (r^B - r^f)B + (1 + r^f)E_0
\]

The first two terms capture the *net interest margin* (NIM) of the bank. The third element capture revaluation effects of outstanding bonds, and the fourth element is equity rolled-over.

**Equity response to change in policy interest rate**

Importantly, and irrespective of the shape of the demand for loans and the supply of deposits, we can prove that there exists simple sufficient conditions for the net interest margin as well as next-period equity to diminish after a cut in policy rates.

First consider NIMs. How do they evolve as a function of the reserve rate \(r^f\)? On the one hand, NIMs vary because the optimal mark-ups paid by banks can change when \(r^f\) does. Note in particular that if \(\partial \epsilon^D/\partial r^D \leq 0\), as in Example 1, then the mark-up \(\mu^*D\) on deposits is decreasing in \(r^f\): hence as depositors substitute faster towards cash, this hurts the banks’ ability to make profits out of deposits. On the other hand, NIMs vary with the extent to which loan and deposit quantities vary. Both of these forces can be summarized: the next lemma characterizes how profits from NIMs change with \(r^f\).

**Lemma 2 (Impact on NIMs).** Denote \(\pi^{NIM} = \mu^*L^* + \mu^*D^*\). Then:

\[
\frac{d\pi^{NIM}}{dr^f} = D^* - L^*
\]

Hence if \(D^* > L^*\), decreasing \(r^f\) decreases NIMs.

This is a simple consequence of the Enveloppe theorem. In the case of monopoly pricing the pass-through is imperfect as banks adjust mark-ups in response to changes in the rate \(r^f\). As a consequence, profits from the NIM business are unchanged at the margin, but the risk-free rate changes affect the costs of resources, which is deposits minus loans. When it is positive, a decrease in the risk-free rate necessarily decreases the NIM.

A second channel affecting total profits is that past \(B\)-holdings which pay an interest of \(r^B\) are now refinanced at a lower rate \(r^f\). This channel can also be seen as a revaluation effect as the market value of the bonds with face value \(B\) increases since one can refinance the \(B\)-holdings at a lower rate \(r^f\).
Finally, changing the reserve rate changes the opportunity cost of investing equity: this means next-period’s equity is lower, as today’s equity is rolled-over at a lower rate.

The next proposition characterizes mathematically how next-period equity changes as function of the reserve rate \( r_f \). This takes into account the two channels for profits highlighted above, as well as the fact that changing the reserve rate changes the opportunity cost of investing equity.

**Proposition 1 (Impact on equity).** Suppose that the bank is able to impose monopolistic pricing. Then the total derivative of next-period equity with respect to the risk-free rate \( r_f \) is:

\[
\frac{dE_1(r_f)}{dr_f} = D^* - L^* + E_0 - B = C^*
\]

Hence if \( C^* > 0 \), then \( E_1 \) decreases as the risk-free rate decreases.

Hence, whenever total risk-free assets positions are held by the bank in positive supply — effectively, the monopolistic bank must fund part of its activity on deposits through reserves — decreasing the rate is harmful.

This is a powerful result: it suggests that if one sees banks holding important cash and cash-equivalents relative to their equity — statistics easy to observe — and that the market environment is such that banks are pricing their base of borrowers and consumers as monopolists, then decreasing rates will surely be harmful. Note that this result does not depend on any assumption about the properties of the loan or the deposit functions’ properties.

**The “Reversal Interest Rate”**

Note that the fact that profits drop when rates \( r_f \) decrease does not prevent lending from increasing. The constraint, however, can overturn this result by forcing lending to decrease after an interest rate cut, through decreasing profits. Moreover, we show that the quantity constraint imposed on lending amplifies the negative effect of interest rates cuts on profits by forcing banks to hold even more reserves.

The following proposition guarantees that when capital requirements are high enough — that is \( \gamma_0 \) is high enough — the constraint starts binding, creating a reversal in the lending response. Alternatively, for any given \( \gamma_0 \), we can insure existence as long as profits always
decrease when \( r^f \) goes down, as characterized in the previous section. Importantly, 0 does not have to be, generically, the rate at which such reversal occurs – it can be positive as well as negative.

**Proposition 2** (The “Reversal Interest Rate”). Suppose that \( C^* \geq 0 \) for all \( r^f \). For \( \gamma_0 \) sufficiently large, there exists a policy rate \( r^{RR} \) at which a reversal occurs, that is:

\[ r^f \leq r^{RR} \iff \frac{dL^*(r^f))}{dr^f} \geq 0 \]

Alternatively, fix any \( \gamma_0 > 0 \). If for any \( r^f \), \( C^* \geq \bar{C} \) for some \( \bar{C} > 0 \), then \( r^{RR} \) exists.

**Corollary 1.** \( r^{RR} \neq 0 \) generically.

For simple functional forms, we can compute by hand the reversal rate, as the example below demonstrates.

**Example 2.** Consider the simple case where deposits’ supply is linear and there’s no margins on loans. Specifically, assume that \( \kappa_L = 0 \), \( L(r^L) = \bar{L} \) fixed, \( \kappa_D = \infty \) and \( D(r^D) = \bar{D}(1 + \beta r^D) \). Moreover for simplicity assume \( \gamma_1 = 1 \), \( E_0 = 0 \), and choose bond holdings to be \( B = \bar{D}/2 \). Then the reversal rate exists if and only if:

\[ \gamma_0 \geq \bar{D} \left( \frac{\beta}{4} + r^B \right) \]

In which case it is given by:

\[ r^{RR} = 2 \sqrt{\frac{1}{\beta} \left( \frac{\gamma_0 \bar{L}}{\bar{D}} - r^B \right)^{-1/4}} \]

What happens to the optimal lending rule when \( r^f \) is low enough so that the constraint actually binds? Intuitively, an amplification emerges from the fact that constraining lending forces banks to hold more risk-free assets. A nice closed-form characterization occurs in the case \( \gamma_1 = 1 \), where the sub-optimal loan-rate, denoted \( r^{L_o}(r^f) \), is simply:

\[ r^{L_o}(r^f) = L^{-1} \left( \frac{E_1}{\gamma_0} \right) \]
And hence the total derivative of profits with respects to the policy rate is:

\[
\frac{dE_1(r^f)}{dr^f} = \frac{\gamma_0}{\gamma_0 - \lambda} C^*
\]

Where the lagrangian multiplier \( \lambda \) is defined as \( \lambda = r^{L^0}(r^f) - r^{*L}(r^f) \). This amplification means that profits are even more sensitive to cash holdings when the bank is leverage-constrained. This reflects the fact that banks are forced to substitute their positions towards safe assets with low returns.

### 2.3 Comparative statics

In this subsection, we study in details the comparative statics of the reversal interest rate. Note that in Example 2, we obtained that \( r^{RR} \) increases in regulatory requirements \( \gamma_0 \) but decreases in quantity of deposits \( \bar{D} \), legacy assets’ rate \( r^B \), and \( \beta \), whose higher value indicates a more elastic deposit supply. We explore in more generality how \( r^{RR} \) depends on four factors: the bond holdings and returns of the bank, the shape of the loan demand and the deposit supply, the shape of the regulatory constraint, and the market structure of the banking sector. Throughout, we assume again that \( dE_1/dr^f > 0 \) and that a reversal rate exists – the statements would be meaningless otherwise.

We start with bond holdings and returns. Figure 2 display the reversal interest rate – the rate at which the constraint starts to bind – for various values of \( B \). For \( B \) sufficiently low, the reversal interest rate exists and lending starts to decrease at some point, as the capital gains are not enough to compensate the losses in the traditional business margins of the banks. Moreover, through the inflexion of the profit curves, we observe that these losses get amplified under the reversal interest rate, as predicted by the theory.

**Proposition 3** (Bonds holdings and returns). \( r^{RR} \) is decreasing in \( B \) and \( r^B \).

This simple result suggests an interplay between interest rate policy and other monetary operations such as Quantitative Easing, which change the bond holdings of banks. Before Quantitative Easing, where bank holdings of long term bonds \( B \) are high, interest rates cuts are less likely to harm banks’ margins and hence necessarily rise lending. Quantitative Easing can then be used to allow banks to “realize” their gains. After QE, however, banks are more exposed to interest rate movements due to the large, positive amount of reserves they hold. This makes them more sensitive to interest cuts – with the risk that a future cut actually hurts lending. In that sense, further rate cut to be effective need first banks to ”reload the
Figure 2: Two-period model, various values of $B$.

By acquiring again fixed-income securities whose value intrinsically rise with decreases in risk-free rates.

Next we study how the shape of the loan demand and deposit supply functions matter for the equilibrium. Our next proposition establish a simple result: when loan-seekers and deposit-suppliers have more elastic demand or supply functions, necessarily banks suffer, as they obtain lower mark-ups and mark-downs. Therefore, ceteris paribus, the reversal rate rises.

Proposition 4 (Deposit supply and Loan demand). $r^{RR}$ has the following properties:

1. $r^{RR}$ increases in $\varepsilon^D$ in the sense that making a deposit supply more elastic increases the reversal rate.

   Mathematically, fix a deposit supply function $D(\cdot)$, and associated reversal rate $r^{RR,D}$. Consider another deposit supply function $D'(\cdot)$ such that $D(r^{RR,D}) = D'(r^{RR,D})$, and $\varepsilon^{D'}(r^D) \geq \varepsilon^D(r^D)$ for all $r^D$. Then, $r^{RR,D'} \geq r^{RR,D}$.

2. $r^{RR}$ increases in $\varepsilon^L$ in the same sense.

This result implies that the rate at which elasticities change when interest rates drop is of significant importance. Note that in Example 1 and associated Figure 1, the elasticity of depositors’ supply was increasing as risk-free rates decreased, meaning that consumers substitute toward cash at a faster rate as deposit rates decrease more and more. This reduces the monopsony power of banks, which restrict less its demand for deposits. Hence $D^*$ and $C^*$ decrease more slowly, and a reversal rate occurs even faster. In other words, the faster consumer substitute towards cash or out-of-bank holdings (such as bonds), the faster banks’ profits tank as rates decrease, hence rising the reversal rate.
Next we study how the reversal interest rate changes with the structure of capital constraints. Unsurprisingly, $r^{RR}$ is lower when capital constraints are looser or equity buffers are high.\(^6\) Comparative statics of $r^{RR}$ with respect to $\gamma_1$ are ambiguous and omitted\(^7\); however, and importantly, a large $\gamma_1$ will provoke a larger drop in lending once the reversal rate is crossed.

**Proposition 5** (Shape of the regulatory constraint). $r^{RR}$ has the following properties:

1. $r^{RR}$ increases in capital requirement $\gamma_0$.
2. $r^{RR}$ decreases in equity $E_0$.

Moreover, the drop in lending following a crossing of the reversal rate is more pronounced for $\gamma_1$ larger, ceteris paribus.

Finally, we study how relaxing the assumption of a monopoly – hence letting $\kappa_D, \kappa_L$ be arbitrary – affects the reversal rate. Intuitively, decreasing switching costs decrease the mark-ups that banks can make, and hence their NIMs. By the same logic of Proposition 4, then, the reversal rate must rise.

**Proposition 6** (Market structure). $r^{RR}$ has the following properties:

1. $r^{RR}$ weakly decreases in $\kappa^L$.
2. $r^{RR}$ weakly decreases in $\kappa^D$.

It is important, however, to remember that we have assumed the existence of a reversal rate $r^{RR}$ for this result. To see this, observe that in the case of perfect competition, that is $\kappa^D = \kappa^L = 0$, there is a sense in which a reversal rate is harder to obtain, because profits then only consists of reevaluation gains which rise when $r^f$ rises. If this isn’t compensated by the fact that equity is rolled-over at a lower rate – technically if $B > E_0$ – this precludes the mere existence of a reversal rate. The intuition is that banks’ would not have relied on NIMs in the first place.

\(^6\) Of course, this reflects the fact that we neglect potential risk-taking effects of decreasing interest rates, which could be the basis for a regulatory constraint. See for example [Di Tella (2013)] or [Klimenko et al. (2015)].

\(^7\) Note that with $r^f \geq 0$, $E_1 \geq E_0$ necessarily, and hence raising $\gamma_1$ is always beneficial. But as said in the main body of the text, this is not the interesting static regarding $\gamma_1$. 

17
3 Multi-period extension

In this section, we extend the model to a multi-period setting. This allows us to study dynamic aspects of the impact of policy rates on net interest margins. Indeed, changes in policy rates not only impact NIMs directly as in the previous section: they also impact them indirectly, by constraining or un-constraining banks and hence changing their ability to generate profits. Constraining occurs when a change in rates hurt future NIMs, which impacts the bank’s ability to lever up today; unconstraining occurs when reevaluation gains on the banks’ assets dominate, which allow them to increases their equities and hence leverage. We effectively show in our first proposition that we can decompose the overall effect of monetary policy in four interpretable terms.

Given this decomposition, we can then study the optimal duration of an interest rate policy geared towards maximizing today’s lending. Its duration matters because the optimal length and magnitude of a decrease in interest rates depends on the average duration of the banks’ fixed-income assets as well as the impact the decrease has on its NIMs. When the banks’ assets duration is short, lowering rates further in the future does not impact their value and hence do not lead to any reevaluation gains, while it may hurt future NIMs and hence decrease lending today.

We then study an extension of the multi-period model in which we allow the dividend policy to be endogenous. This allows banks to smooth out shocks to equity, and hence decrease the bite of leverage constraints. However, this force is traded-off with the impatience of the banks’ investor. Therefore, we find that a policy geared towards maximizing lending may need to impose lower dividends – a fact acknowledged by policymakers during the recent financial crisis (T. Geithner 2016, p.138).

3.1 General setting

Time is discrete and indexed by $t = 0, 1, ..., T$ with possibly $T = \infty$. Every period, the timing of events is as follows. First, the Central Bank announces a new path of risk-free rates $\{r_{f_{t+1+s}}\}_{s \geq 1}$. For now, we do not let banks hedge on these announcements – we relax this assumption in Section 4. Next, banks receive interest payments on loans, pay interests on deposits, and sell their bond positions: profits are realized. Then, a fraction $\delta_t$ of (cum-

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8Technically, we’ll always consider the limit as $T \to \infty$. Through backward-induction, this makes it impossible to construct colluding strategies that support mark-ups above switching costs. Hence the latter remain upper-bounds on mark-ups every period.
dividend) equity, $\mathcal{E}_t$ is paid to investors as dividends. We first take the dividend path $\{\delta_s\}_{s \geq 0}$ as exogenous and then endogenize it in Subsection 3.5. Finally, new economic decisions are taken: the amount of loans, bonds, cash and deposits the bank decides to hold.

Hence the cum-dividend value function for an investor with discount rate $r^{E}_{t+1}$ of a bank entering with cum-dividend book equity $E_t$ can be written recursively as:

$$V_t(\mathcal{E}_t) = \delta_t \mathcal{E}_t + \max_{y_t \in \Gamma_t} \mathcal{M}^{E}_{t,t+1} V_{t+1}(\mathcal{E}_{t+1})$$

Where $\mathcal{M}^{E}_{t,t+1} = \frac{1}{1 + r^{E}_{t+1}}$ is the discount factor of investors, and $y_t = (L_t, C_t, B_t, D_t)$ are the decision variables at $t$ given the constraints in $\Gamma_t$, which we describe now. Loans, deposits and cash are determined as in the two-period model, with book equity defined as retained equity: $E_t = (1 - \delta_t) \mathcal{E}_t$. Banks’ now have the possibility to purchase assets $B_t$ at a price $p^B_t$. These assets yield pay-offs $\{x^B_{t+s}\}_{s \geq 1}$, and can be sold every period after the Central Bank’s announcement of new rates. The pricing of these assets is competitive, implying that they sell at the following price:

$$p^B_t = \frac{p^B_{t+1|t} + x^B_{t+1}}{(1 + r^f_{t+1})}$$

Where $p^B_{t+1|t}$ is the next-period price of the bond in period $t$. We assume that every period there’s a supply of these assets.

As a consequence of our modelling choices, the bank’s next-period (cum-dividend) book equity is:

$$\mathcal{E}_{t+1} = (1 + r^L_{t+1}) L_t + (p^B_{t+1|t+1} + x^B_{t+1}) B_t + (1 + r^f_{t+1}) C_t - (1 + r^D_{t+1}) D_t$$

Given that the ex-dividend book equity are $E_t = (1 - \delta_t) \mathcal{E}_t$, the cash-flow constraint reads:

$$L_t + p^B_t B_t + C_t = D_t + E_t$$

Hence solving in the constraint we get:

$$\mathcal{E}_{t+1} = (r^L_{t+1} - r^f_{t+1}) L_t + (r^f_{t+1} - r^D_{t+1}) D_t + (p^B_{t+1|t+1} - p^B_{t+1|t}) B_t + (1 + r^f_{t+1}) E_t$$

Note that should risk-free rates stay unchanged, we would have that $p^B_{t+1|t+1} = p^B_{t+1|t}$ and the third term would drop-out.

We allow as before the loan behavior of the bank to be potentially constrained. Define
\( \tilde{V}_t(\mathcal{E}_t, \cdot) = V_t(\mathcal{E}_t, \cdot) - \mathcal{E}_t \) to be the market value minus the book value, the so-called *franchise value* of the bank. We assume that the constraint takes the form:

\[
\gamma_0 L_t \leq \left( \frac{\tilde{V}_t(\mathcal{E}_t, \cdot)}{E_t} \right)^{\gamma_1} E_t
\]

It is useful to note first that given the form of the constraint, the two relevant state variables are (1) the bank’s profits \( \mathcal{E}_t \) and (2) the value of the bank itself, \( V_t \). Clearly, the latter is increasing in \( \mathcal{E}_t \).

**Lemma 3.** The value of the bank \( V_t \) is strictly increasing in \( \mathcal{E}_t \).

This is intuitive, and useful to note for numerical reasons.\(^9\) Next, denote the optimal profits of the bank on loans and deposits, the net interest margin, to be:

\[
\pi^{\text{NIM}}_{t+1} \equiv \left( r^*_{t+1} - r^f_{t+1} \right) L(t^*_{t+1}) + \left( r^f_{t+1} - r^*_{t+1} \right) D(t^*_{t+1})
\]

The next result characterizes the value function of banks, given optimal decisions for loan rates and deposit rates – that is, taking the so-called policy functions as given.

**Lemma 4.** The franchise value of the bank (after optimization) is:

\[
\tilde{V}_t(\mathcal{E}_t, \cdot) = \sum_{s=0}^{\infty} \mathcal{M}^{E}_{t,t+s} \left( \pi^{\text{NIM}}_{t+s+1} + (1 - \delta_{t+s})(r^f_{t+s+1} - r^E_{t+s+1}) \mathcal{E}^s_{t+s+1} \right)
\]

Where \( \mathcal{M}^{E}_{t,t+s} \equiv \prod_{r=1}^{s} \frac{1}{1 + r^E_{t+r}} \) is the discount factor of equity investors.

The first term corresponds to a standard net present value of the profit stream. The second term represents eventual capital losses due to profits being rolled-over at a rate below the discount rate. This effect would disappear if the bank would always pay dividends.

We now state our main proposition: as in the two-period model, we can decompose the effect of a change in interest rates on banks’ value.

\(^9\)Given a guess for the function \( V_t(\mathcal{E}_t, \cdot) \) and enough stationarity, one can then proceed numerically with standard methods. Indeed, one can rewrite the problem as:

\[
V_t(\mathcal{E}_t, \cdot) = \delta_t \mathcal{E}_t + \max_{x_t \in \mathcal{F}_t} \mathcal{M}^{E}_{t,t+1} V_{t+1}(\mathcal{E}_{t+1}, V^{-1}_{t+1}(\mathcal{E}_{t+1}))
\]
Proposition 7 (Impact on banks’ value). Consider a change in the rate \( r_{t+s} \) announced at time \( t \). The total derivative of \( \tilde{V}_t \) with respect to such change is:

\[
\frac{d\tilde{V}_t}{dr_{t+s}} = M_{t,t+s} \frac{\partial \pi_{t+1}^{NIM}}{\partial \pi_{t+s}^{f}} \sum_{r=1}^{\infty} M_{t,t+r} \frac{\partial \pi_{t+r}^{NIM}}{\partial \pi_{t+s}^{r}} + B_t \sum_{r=1}^{\infty} \frac{\partial M_{t+1,t+s+r}}{\partial r_{t+s}} x_{t+s+r}
\]

There are four effects. First, there’s a direct effect on the net interest margin earned on loans and deposits, which depends on the competitive structure as well as the properties of loan demand and deposit supply. The analysis for this term is not different from the one conducted in Section 2. Second, and unique to the dynamic setting, there’s an indirect effect on all NIMs. This occurs because constraining/unconstraining the bank affects its leverage today which impacts all future NIMs. Constraining occurs when drops in future interest rates decrease the franchise value of the bank beyond reevaluation gains; unconstrained occurs when the latter channel dominates. The third channel is indeed this revaluation effect: a decrease in interest rates makes current bond holdings more valuable. This effect depends on the stream of interest payments of the bond, the amount of bond the bank holds, as well as the change in the risk-free rate decided by the central bank. Note that this affects \( \tilde{V}_t \), the franchise value of the bank, only if the bank is constrained and the extra retained profits help to recapitalize and unconstrain the bank. Finally, a discounting effect is present as in the two-period model, which disappears should there be no retained profits.

Next, note that in the setting under consideration, and contrary to the two-period model, the existence of a path of reversal interest rates is much easier to establish than that of a unique reversal interest rate for a single period. However, even the latter’s existence can be established under the same conditions as the two-period model: as long as the NIM of banks hurts more than the revaluation gains, so that \( dV_t/dr_t^f > 0 \), \( \gamma_1 > 0 \), and \( \gamma_0 \) is sufficiently high so that the bank is constrained, a reversal interest rate exists and displays similar comparative statics. We keep this assumption throughout the section.

3.2 The optimal duration of monetary policy

We can first obtain the following simple, yet general result.
Proposition 8. Suppose that $B_t$ does not yield any pay-offs past some date $\bar{T}$, that is $x_t^B = 0$ for all $t \geq \bar{T}$. Then any decrease in the risk-free rate past $\bar{T}$ has negative effects on the value of the bank. If the bank is constrained, this decreases lending.

When the NIM is positive, the only gains from decreasing rates come the re-evaluation of the banks’ bond positions, as lower rates makes legacy bonds more valuable. However, if only future rates beyond the maturity of bonds are raised, this channel disappears, and decreasing rates is always harmful.

This result only suggests not decreasing rates when it cannot recapitalize constrained banks, as this may decrease lending. More generally, we expect that the longer the maturity of bond holdings of banks, the longer an interest cut should be decided if the objective is to increase lending. The next proposition suggests this result. Consider a sequence of optimal policy rates that were chosen to maximize loans made by a constrained bank. We’re interested in how the duration of this optimal policy rates changes when the duration of assets holdings $B_t$ of banks change, without changing the net present-value of these assets. The results shows that increasing that duration results in a rate increase for rates close to the announcement, and rate decreases after, implying a spread-out in the decrease of the policy rates.

Proposition 9. Denote by $\{r_{t+s}^f\}_{s \geq 0}$ the policy rates that are optimally chosen to maximize lending.

Consider an increase in the duration of the assets $B_t$ held by banks as follows: $\Delta x_{t+s}^B \leq 0$ for all $s \leq S$ and $\Delta x_{t+s}^B \geq 0$ otherwise, while keeping the NPV of these assets constant at the given policy rates.

Then the optimal rates’ duration also increases, in the following sense: there exists $S' > 0$ such that $\Delta r_{t+s}^{f*} \geq 0$ for all $s < S'$ and $\Delta r_{t+s}^{f*} \leq 0$ otherwise.

Sketch of proof. By assumption $\sum_s M_s \Delta x_{t+s}^B = 0$. Hence $B_t \sum_{r=1}^\infty \frac{\partial M_{t+1,t+s+r}^f}{\partial r_{t+s}} x_{t+s+r}^B$ is a hump-shaped function in $s$ with a peak at $\bar{S}$, that attains zero both for $s = 0$ and $s \rightarrow \infty$. Note that the initial equity boost effect is positive and increases the partial effect on the NIMs of an increase in rates; moreover the equity effect slowly fades out, and it’s got to be that at least at $\bar{S}$ the reevaluation channel dominates [showing regularity in this NIM effect is the hardest part, and endogenous dividends characterization may make it easier, in which case this section should appear after. Moreover it might be hard to prove that there can’t be rate increases after $\bar{S}'$, although (1) they should be small and (2) they should be ”controllable” in magnitude with enough regularity conditions]. Use all of that to sign $dV/dr^f$ accordingly,
and use the fact that we started at an equilibrium in which \( dV/dr^f = 0 \) to conclude. [Note that the shape of the dividend policies should not matter for the result, because no matter how the sequence looks like, the ”equity boost” monotonically fades out.]

3.3 Endogenous dividends and the complementarity of monetary & dividend policies

To understand the importance of dividend policy in this framework, observe that gains from bond holdings are realized immediately, as banks clear their bond positions at the time of the announcements. Hence, should these gains be redistributed immediately through dividends – that is \( \delta_t = 1 \) for some \( t \), the franchise value of the bank will necessarily be hurt by lower rates at time \( t \), which necessarily hurts lending if the bank is constrained. Moreover, owing to the impatience of its owners, an ever-unconstrained bank would always choose to pay profits out in dividends, which makes any capital gains on bonds revaluation useless for the banks’ franchise value.

The only reason it may not do so is because of the constraint, since retaining profits can raise its lending abilities and hence its net worth. In other words, with endogenous dividends, a constrained banks will trade-off the loss from rolling-over part of profits at the risk free-rate – instead of paying it out in dividends now – with the unconstraining effect of a larger equity buffer. We record these facts in the next proposition.

Proposition 10 (Complementarity of Lending and Dividend Policies). With endogenous dividends, banks do not maximize today’s lending after a policy rate announcement. Hence, a policy aiming at maximizing lending can improve its effectiveness by imposing a dividend policy.

Obviously, to maximize today’s lending, a bank should choose \( \delta_t^* = 0 \) at the time of the announcement. This does not occur because banks trade-off the impatient of their investors with retained profits: increasing the latter can be beneficial for lending. This is important in the context of the discussions about Central Banks controlling banks’ dividend policies (Citigroup/Fed in the US?) in order to increase lending [Include Geithner’s quote].

It is important to flag out that this result remains even if investors – the banks’ owners – are allowed to form expectations about monetary policy as in Section 4. The only difference is that the central bank may choose not to impose something as extreme as \( \delta_t^* = 0 \), since announcements of decrease in interest rates may make banks’ franchise value to drop in the absence of dividend payments, further reducing their lending abilities.


3.4 Varying investor discount rates

For the sake of clarity, we have so far neglected to analyse cases where the discount rate of investor \( r^E \) depends on \( r^f \). Clearly, one expects this dependence to be positive, as \( r^f \) likely drives many of the investors’ outside options. Hence, as the risk-free rate decreases, we expect investors to be more patient due to a worsening of other investment opportunities.

This dependence has two opposing effects on how the value of banks’ – and hence lending if the bank is constrained – change with the risk-free rate. One effect is potentially positive: as long as banks make positive profits, a decrease in investors’ discount rates make these profits more valuable, and hence helps rising the value of the bank. Of course, this effect flips if profits – which include the opportunity cost of investing equity, the discounting effect in Proposition 7 – were to become negative. However, there is another, second-order effect: a lower discount rate makes future losses in NIMs more costly, since these profits now matter more relative to the immediate capitalization gains. Mathematically, this occurs because the \( \mathcal{M}_{t,t+s}^E \) terms in Proposition 7 become larger.

4 General Equilibrium & Empirical exercises

- Close the economy
- Add Expectations to CB’s shocks
- Add shocks to the NW of banks? (Defaults,...)
- Have a channel by which output decreases (eg inefficiency of investment due to FF)
- Simulate linearly
- Calibrate-Estimate
- IRFs, effect on output
- Negative margins and IRs

5 Extension: heterogeneity

- Banks
- Sources of funding (deposits, wholesale, etc)
6 Policy implications and Conclusion

- to be completed -

A Proofs [NOT UP TO DATE]

TBA

References


