Comparative Valuation Dynamics in Models with Financing Frictions

Overview

Today’s Lecture:
Lars Peter Hansen (University of Chicago)

Based on joint work with:
Paymon Khorrami (University of Chicago)
Fabrice Tourre (Copenhagen Business School)
March 18, 2019
TODAY’S AGENDA

1. Overview of the project
2. Today’s outline
   1. Continuous-time recursive utility (Duffie-Epstein-Zin)
   2. Baseline model with complete markets
   3. Elasticity of intertemporal substitution
   4. Introduction to shock elasticities as a diagnostic tool
• **Research Goal**: Compare/contrast implications of DSGE models with financial frictions through study of their non-linear transmission mechanisms
• **Research Goal**: Compare/contrast implications of DSGE models with financial frictions through study of their non-linear transmission mechanisms

• **Models of Focus**
  - Continuous time with Brownian shocks
  - Financial intermediaries
  - Heterogeneous productivity, market access and preferences
• **Research Goal**: Compare/contrast implications of DSGE models with financial frictions through study of their non-linear transmission mechanisms

• **Models of Focus**
  • Continuous time with Brownian shocks
  • Financial intermediaries
  • Heterogeneous productivity, market access and preferences

• **Comparisons**
  • Macroeconomic quantity implications
  • Asset pricing implications
  • Welfare consequences and policy ramifications
**Research Goal:** Compare/contrast implications of DSGE models with financial frictions through study of their non-linear transmission mechanisms

**Models of Focus**
- Continuous time with Brownian shocks
- Financial intermediaries
- Heterogeneous productivity, market access and preferences

**Comparisons**
- Macroeconomic quantity implications
- Asset pricing implications
- Welfare consequences and policy ramifications

**Approach:** Nesting model
1. Discrete-time Kreps-Porteus recursive preference specification

\[ U_t = \left[ [1 - \exp(-\delta \epsilon)] (C_t)^{1-\rho} + \exp(-\delta \epsilon) \mathbb{R}(U_{t+\epsilon} | \mathcal{F}_t)^{1-\rho} \right]^{\frac{1}{1-\rho}} \]

\[ \mathbb{R}(U_{t+\epsilon} | \mathcal{F}_t) = \left( \mathbb{E} \left[ (U_{t+\epsilon})^{1-\gamma} | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}} \]

2. \( \mathbb{R}(U_{t+\epsilon} | \mathcal{F}_t) \): certainty equivalent with parameter \( \gamma \)

3. Recursion governed by 3 key parameters
   - \( \delta \) – subjective discount rate
   - \( 1/\rho \) – IES
   - \( \gamma \) – relative risk aversion

4. Special case: \( \rho = \gamma \)

\[ U_t = \left( \mathbb{E} \left[ \delta \int_0^\infty \exp(-\delta s) (C_{t+s})^{1-\gamma} ds | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}} \]
Recursive utility risk adjustment

\[ R(U_{t+\epsilon} | \mathcal{F}_t) = \left( \mathbb{E} \left[ (U_{t+\epsilon})^{1-\gamma} | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}} \]

1. Construct logarithmic counterpart

\[ r(\log U_{t+\epsilon}) = \frac{1}{1-\gamma} \log \mathbb{E} \left( \exp \left[ (1 - \gamma) \log U_{t+\epsilon} \right] | \mathcal{F}_t \right) \]

2. Posit \( dU_t = U_t \mu_{u,t} dt + U_t \sigma_{u,t} \cdot dB_t \)

3. Ito’s Lemma: \( d \log U_t = \mu_{u,t} dt - \frac{1}{2} \sigma_{u,t}^2 dt + \sigma_{u,t} \cdot dB_t \).

4. Derivative: \( \frac{d}{d\epsilon} r(\log U_{t+\epsilon}) \big|_{\epsilon=0} = \mu_{u,t} - \frac{\gamma}{2} \sigma_{u,t}^2 \)

Includes an adjustment for the local variance.
Recall

\[ U_t = \left[ [1 - \exp(-\delta \epsilon)] \left( \frac{C_t}{U_t} \right)^{1-\rho} + \exp(-\delta \epsilon) \mathbb{E}(U_{t+\epsilon} | \mathcal{F}_t)^{1-\rho} \right]^{\frac{1}{1-\rho}} \]

1. Logarithmic counterpart

\[
\log U_t = \frac{1}{1-\rho} \log \left[ [1 - \exp(-\delta \epsilon)] \left( \frac{C_t}{U_t} \right)^{1-\rho} + \exp(-\delta \epsilon) \exp \left[ (1 - \rho) \mathbb{E}(\log U_{t+\epsilon} | \mathcal{F}_t) \right] \right]
\]

2. Subtract \( \log U_t \) from both sides and differentiate with respect to \( \epsilon \):

\[
0 = \frac{\delta}{1-\rho} \left[ \left( \frac{C_t}{U_t} \right)^{1-\rho} - 1 \right] + \mu_{u,t} - \frac{\gamma}{2} |\sigma_{u,t}|^2
\]

3. \( \rho = 1 \) limit

\[
0 = \delta (\log C_t - \log U_t) + \mu_{u,t} - \frac{\gamma}{2} |\sigma_{u,t}|^2
\]
**Stochastic Discount Factor Over an Interval**

1. Use homogeneity of the utility recursion and compute three marginal utilities: associated with the two CES recursions:

   \[ MC_t = [1 - \exp(-\delta \epsilon)] (C_t)^{-\rho} (U_t)^{\rho} \]
   \[ MR_t = \exp(-\delta \epsilon) (R_t)^{-\rho} (U_t)^{\rho} \]
   \[ MU_{t,t+\epsilon} = (U_{t+\epsilon})^{-\gamma} (R_t)^{\gamma} \]

   where \( R_t \) is the date \( t \) risk adjusted continuation value.

2. Form the stochastic discount ratio as

   \[
   \frac{S_{t+\epsilon}}{S_t} = \frac{MR_t MU_{t,t+\epsilon} MC_{t+\epsilon}}{MC_t} \\
   = \exp(-\epsilon \delta) \left( \frac{C_{t+\epsilon}}{C_t} \right)^{-\rho} \left[ \frac{U_{t+\epsilon}}{R(U_{t+\epsilon} | \mathcal{F}_t)} \right]^{\rho - \gamma}
   \]
1. Discrete-time SDF over an interval $\epsilon$:

$$
\frac{S_{t+\epsilon}}{S_t} = \exp(-\epsilon \delta) \left( \frac{C_{t+\epsilon}}{C_t} \right)^{-\rho} \left[ \frac{U_{t+\epsilon}}{\mathbb{E}(U_{t+\epsilon} \mid \mathcal{F}_t)} \right]^\rho\gamma
$$

2. Depict continuous-time evolution of SDF as:

$$
dS_t = S_t \mu_s, t \, dt + S_t \sigma_s, t \cdot dB_t
$$

3. Depict a valuation or cumulative return process:

$$
dA_t = A_t \mu_a, t \, dt + A_t \sigma_a, t \cdot dB_t.
$$

where $AS$ is a positive martingale. Martingale restriction:

$$
\mu_a, t + \mu_s, t + \sigma_a, t \cdot \sigma_s, t = 0.
$$

4. Risk free rate $r_t = -\mu_s, t$; risk price vector $\pi_t = -\sigma_s, t$. 
Recall: $\frac{S_{t+\epsilon}}{S_t} = \exp(-\epsilon \delta) \left( \frac{C_{t+\epsilon}}{C_t} \right)^{-\rho} \left[ \frac{U_{t+\epsilon}}{\mathbb{E}(U_{t+\epsilon} | \mathcal{F}_t)} \right]^{\rho-\gamma}$

1. Consumption

$$d \log C_t = \mu_{c,t} dt - \frac{1}{2} |\sigma_{c,t}|^2 dt + \sigma_{c,t} \cdot dB_t$$

2. Continuation value

$$d \log U_t = \mu_{u,t} dt - \frac{1}{2} |\sigma_{u,t}|^2 dt + \sigma_{u,t} \cdot dB_t$$

3. Local coefficients (prices):

$$\sigma_{s,t} = -\rho \sigma_{c,t} + (\rho - \gamma) \sigma_{u,t}$$

$$\mu_{s,t} - \frac{1}{2} |\sigma_{s,t}|^2 = -\delta - \rho \mu_{c,t} + \frac{\rho}{2} |\sigma_{c,t}|^2 dt + \frac{(\rho - \gamma)(\gamma - 1)}{2} |\sigma_{u,t}|^2$$
1. The long-run risk processes \((Z, V)\):

\[
dZ_t = -\lambda_z Z_t dt + \sqrt{V_t \sigma_z} \cdot dB_t
\]

\[
dV_t = -\lambda_v (V_t - 1) dt + \sqrt{V_t \sigma_v} \cdot dB_t
\]

2. The A-K production technology with adjustment costs

\[
\frac{dK_t}{K_t} = \left[ \Phi \left( \frac{I_t}{K_t} \right) + Z_t - \alpha_k \right] dt + \sqrt{V_t \sigma_k} \cdot dB_t
\]

3. \(\Phi\) is the concave and increasing installation cost function

4. The economy’s resource constraint:

\[
C_t + I_t = aK_t
\]
1. State vector $X \equiv (Z, V)$

2. Capital evolution in logarithms:

$$d \log K_t = \left[ \Phi \left( \frac{I_t}{K_t} \right) + Z_t - \alpha_k - \frac{V_t|\sigma_k|^2}{2} \right] dt + \sqrt{V_t\sigma_k} \cdot dB_t$$

2. Homogeneity properties of the model lead to: $\log U_t = \log K_t + \xi(X_t)$.

3. HJB equation for planner problem:

$$0 = \max_{c+i=a} \left\{ \frac{\delta}{1 - \rho} \left( c^{1-\rho} \exp ((\rho - 1)\xi) - 1 \right) + \Phi(i) + Z - \alpha_k - \frac{1}{2} V|\sigma_k|^2 
+ \mu_x \cdot \partial_x \xi + \frac{1}{2} \text{tr} \left( \sigma'_x \partial_{xx} \xi \sigma_x \right) + \frac{1 - \gamma}{2} |\sqrt{V} \sigma_k + \sigma'_x \partial_x \xi|^2 \right\}$$

where $c$ (consumption-to-capital ratio) and $i$ (investment-to-capital ratio).
1. Marginal utility of consumption satisfies

\[ MC_t = \delta C_t^{-\rho} U_t^\rho \]

2. Use (i) Euler’s theorem and (ii) total wealth = value of capital stock to obtain

\[ Q_t K_t = \frac{U_t}{MC_t} = \frac{1}{\delta} \left( \frac{C_t}{K_t} \right)^\rho \left( \frac{U_t}{K_t} \right)^{1-\rho} K_t \]

where \( Q_t \) is the price of capital.

3. Return on wealth is exposed to direct shocks to the capital stock and also to shocks to its value \( Q_t \).
HJB equation for planner problem when $\rho = 1$

$$
O = \max_{c+i=a} \left\{ \delta (\log c - \xi) + \Phi(i) + z - \alpha_k - \frac{1}{2} v|\sigma_k|^2 
+ \mu_x \cdot \partial_x \xi + \frac{1}{2} \text{tr} (\sigma_x' \partial_{xx} \xi \sigma_x) + \frac{1 - \gamma}{2} |\sqrt{v} \sigma_k + \sigma_x' \partial_x \xi|^2 \right\}
$$

where $c$ (consumption-to-capital ratio) and $i$ (investment-to-capital ratio).

1. $i$ and $c$ are constant independent of the Markov state.
2. Affine value function: $\xi(x) = \beta_0 + \beta_1 \cdot x$
   2.1 The dependence on growth state variable $\beta_{1z}z$ satisfies:
   $$
   \beta_{1z}z = \left( \frac{1}{\delta + \lambda_z} \right) z = E \left[ \int_0^\infty \exp(-\delta \tau) Z_{t+\tau} \, d\tau \mid Z_t = z \right]
   $$
   2.2 Coefficient on the volatility state variable $\beta_{1v}$ satisfies a quadratic equation
Sensitivity to changes $\rho$

Construct an expansion around $\rho = 1$.

1. The sign of $\rho$ changes the quantity dynamics

2. Responses when $\rho < 1$
   a. $c^*$ is decreasing in growth $z$
   b. $c^*$ is increasing in volatility $v$

and conversely when $\rho > 1$. 
Two channels:

1. **Stochastic growth** modeled as a process \( G = \{G_t\} \) where \( G_t \) captures growth between dates zero and \( t \).

2. **Stochastic discounting** modeled as a process \( S = \{S_t\} \) where \( S_t \) assigns risk-adjusted prices to cash flows at date \( t \).

Date zero prices of a payoff \( G_t \) are

\[
\text{Price} = \mathbb{E}(S_t G_t | \mathcal{F}_o)
\]

where \( \mathcal{F}_o \) captures current period information.

**Stochastic discounting** reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.
Ragnar Frisch (1933): 

There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution, and by so doing introduces into the system the energy necessary to maintain the swings.

Irving Fisher (1930): 

The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.
Transition dynamics and valuation through altering cash flow exposure to shocks.

1. Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
2. Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
3. Construct pricing counterpart to impulse response functions.
1. Proportional risk premium over horizon $t$:

$$\log \mathbb{E} \left( \frac{G_t}{G_0} \right) - \log \mathbb{E} \left( \frac{S_t G_t}{S_0 G_0} \mid \mathcal{F}_o \right) + \log \mathbb{E} \left( \frac{S_t}{S_0} \mid \mathcal{F}_o \right)$$

where the first term is the **expected cash flow growth**, the second is the **value**, and the third term is the negative of the **expected risk-less return** all in logarithms.

2. Counterparts to impulse response functions pertinent to valuation:
   2.1 **shock-exposure elasticities**
   2.2 **shock-price elasticities**

These are the ingredients to risk premia, and they have a **term structure** induced by the changes in the investment horizons.

1. Construct shock elasticities as counterparts to impulse response functions

2. Use (exponential) martingale $D^{(\tau)}$, perturbing an underlying positive (multiplicative) process $M$ over the time horizon $[0, \tau)$, where:

$$\log M_t = \int_0^t \mu_m(X_s) \, ds + \int_0^t \sigma_m(X_s) \cdot dB_s$$

$$\log D^{(\tau)}_t = - \int_0^{t \wedge \tau} \frac{|\sigma_d(X_s)|^2}{2} \, ds + \int_0^{t \wedge \tau} \sigma_d(X_s) \cdot dB_s$$

where $\mathbb{E} (|\sigma_d(X_t)|^2) = 1$.

3. Shock elasticity (for example, if $\sigma_d = (1, 0)$, then $D^{(\tau)}$ perturbs in the direction of the first shock):

$$\epsilon_m(x, t) \doteq \lim_{\tau \downarrow 0} \frac{1}{\tau} \log \mathbb{E} \left[ \frac{M_t}{M_0} D^{(\tau)}_t | X_0 = x \right]$$
Shock elasticity

\[ \epsilon_m(x, t) \doteq \lim_{\tau \downarrow 0} \frac{1}{\tau} \log \mathbb{E} \left[ \frac{M_t}{M_0} D_t^{(\tau)} | X_0 = x \right] \]

Apply to a cash-flow \( G \), stochastic discount factor \( S \) and product \( SG \)

1. shock exposure elasticity \( \epsilon_g(x, t) \);
2. shock cost elasticity \( \epsilon_{sg}(x, t) \);
3. shock price elasticity \( \epsilon_g(x, t) - \epsilon_{sg}(x, t) \)
Recall

$$
\epsilon_m(x, t) = \lim_{\tau \downarrow 0} \frac{1}{\tau} \log \mathbb{E} \left[ \frac{M_t}{M_0} D^{(\tau)}(\tau) \middle| X_0 = x \right]
$$

where $D^{(\tau)}$ is an exponential martingale perturbation over the time interval $[0, \tau)$.

Two interpretations:

1. Change in **probability measure** - local impulse response
2. Change in **cash flow exposure** - local risk return

Depend on current state and horizon.
1. What shocks investors do care about as measured by expected return compensation?
2. How do these compensations vary across states and over horizons?
3. How do the shadow compensation differ across agent type?
1. $\sigma_k$ only depends on the first shock while $\sigma_z$ depends on both shocks. The numbers were chosen to better track the observed consumption dynamics.

2. The computations used a first-order small noise, large risk aversion parameterization. In this approximation $\gamma$ only alters the implied deterministic steady states and not the impulse responses to shocks.
**Shock Elasticity: Shock 1**

\[ \text{Blue: } \rho = 0.5 \quad \text{Green: } \rho = 1 \quad \text{Red: } \rho = 2 \]
**Shock Elasticity: Shock 1**

- **Blue:** $\rho = 0.5$
- **Green:** $\rho = 1$
- **Red:** $\rho = 2$

### Stochastic Growth, Shock 1

- **Consumption**
  - $\rho = 0.5$
  - $\rho = 1$
  - $\rho = 2$
- **Capital**
  - $\rho = 0.5$
  - $\rho = 1$
  - $\rho = 2$
- **Consumption to Capital**
  - $\rho = 0.5$
  - $\rho = 1$
  - $\rho = 2$
- **Investment to Capital**
  - $\rho = 0.5$
  - $\rho = 1$
  - $\rho = 2$
- **Price Elasticity**
  - $\gamma = 1.5$
  - $\gamma = 2.0$
  - $\gamma = 3.0$
  - $\gamma = 4.0$
  - $\gamma = 5.0$
  - $\gamma = 6.0$
  - $\gamma = 7.0$
  - $\gamma = 8.0$
  - $\gamma = 9.0$
  - $\gamma = 10.0$
Shock Elasticity: Shock 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 1**

Blue: \( \rho = 0.5 \)  \quad \text{Green: } \rho = 1 \quad \text{Red: } \rho = 2
Shock Elasticity: Shock 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Shock Elasticity: Shock 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Shock Elasticity: Shock 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
SHOCK ELASTICITY: SHOCK 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 1**

- **Blue**: $\rho = 0.5$
- **Green**: $\rho = 1$
- **Red**: $\rho = 2$

![Stochastic Growth, Shock 1](image)
Shock Elasticity: Shock 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 1**

Blue: $\rho = 0.5$ \quad Green: $\rho = 1$ \quad Red: $\rho = 2$

Stochastic Growth, Shock 1

- Consumption
- Capital
- Consumption to Capital
- Investment to Capital
- Price Elasticity

$\gamma = 0.0 \quad \gamma = 1.0 \quad \gamma = 2.0 \quad \gamma = 3.0 \quad \gamma = 4.0 \quad \gamma = 5.0 \quad \gamma = 6.0 \quad \gamma = 7.0 \quad \gamma = 8.0 \quad \gamma = 9.0 \quad \gamma = 10.0$
**Shock Elasticity: Shock 1**

Blue: $\rho = 0.5$  \quad Green: $\rho = 1$  \quad Red: $\rho = 2$

Stochastic Growth, Shock 1

- Consumption
- Capital
- Consumption to Capital
- Investment to Capital
- Price Elasticity

$\gamma = 6.5$  
$\gamma = 1.0, \gamma = 2.0, \gamma = 3.0, \gamma = 4.0, \gamma = 5.0, \gamma = 6.0, \gamma = 7.0, \gamma = 8.0, \gamma = 9.0, \gamma = 10.0$
Shock Elasticity: Shock 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 1**

**Blue:** \( \rho = 0.5 \)  \hspace{1cm} **Green:** \( \rho = 1 \)  \hspace{1cm} **Red:** \( \rho = 2 \)

![Stochastic Growth, Shock 1](image-url)
**Shock Elasticity: Shock 1**

**Blue:** $\rho = 0.5$  \hspace{1cm}  **Green:** $\rho = 1$  \hspace{1cm}  **Red:** $\rho = 2$

![Graphs showing the comparison of consumption, capital, consumption to capital, and investment to capital with different values of $\rho$](image)
**Shock Elasticity: Shock 1**

Blue: $\rho = 0.5$  \hspace{1cm} Green: $\rho = 1$  \hspace{1cm} Red: $\rho = 2$

![Graphs showing stochastic growth with different shock values](image-url)
**Shock Elasticity: Shock 1**

**Blue:** $\rho = 0.5$  
**Green:** $\rho = 1$  
**Red:** $\rho = 2$

**Stochastic Growth, Shock 1**

- **Consumption**
- **Capital**
- **Consumption to Capital**
- **Investment to Capital**
- **Price Elasticity**

$\rho = \text{zero.osf}$  
$\rho = \text{five.osf}$  
$\rho = \text{one.osf}$  
$\rho = \text{two.osf}$
SHOCK ELASTICITY: SHOCK 1

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 1**

Blue: $\rho = 0.5$  \hspace{1cm} Green: $\rho = 1$  \hspace{1cm} Red: $\rho = 2$

**Stochastic Growth, Shock 1**

- **Consumption**
  - Blue: $\rho = 0.5$ (Lowest curve)
  - Green: $\rho = 1$ (Middle curve)
  - Red: $\rho = 2$ (Highest curve)

- **Capital**
  - Blue: $\rho = 0.5$ (Lowest curve)
  - Green: $\rho = 1$ (Middle curve)
  - Red: $\rho = 2$ (Highest curve)

- **Consumption to Capital**
  - Blue: $\rho = 0.5$ (Lowest curve)
  - Green: $\rho = 1$ (Middle curve)
  - Red: $\rho = 2$ (Highest curve)

- **Investment to Capital**
  - Blue: $\rho = 0.5$ (Lowest curve)
  - Green: $\rho = 1$ (Middle curve)
  - Red: $\rho = 2$ (Highest curve)

- **Price Elasticity**
  - Blue: $\rho = 0.5$ (Lowest curve)
  - Green: $\rho = 1$ (Middle curve)
  - Red: $\rho = 2$ (Highest curve)
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 2**

Blue: $\rho = 0.5$  \hspace{1cm} Green: $\rho = 1$  \hspace{1cm} Red: $\rho = 2$
SHOCK ELASTICITY: SHOCK 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
# Shock Elasticity: Shock 2

Blue: $\rho = 0.5$  
Green: $\rho = 1$  
Red: $\rho = 2$

## Stochastic Growth, Shock 2

<table>
<thead>
<tr>
<th>Graph</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Consumption" /></td>
<td>Consumption</td>
</tr>
<tr>
<td><img src="image2" alt="Capital" /></td>
<td>Capital</td>
</tr>
<tr>
<td><img src="image3" alt="Consumption to Capital" /></td>
<td>Consumption to Capital</td>
</tr>
<tr>
<td><img src="image4" alt="Investment to Capital" /></td>
<td>Investment to Capital</td>
</tr>
<tr>
<td><img src="image5" alt="Price Elasticity" /></td>
<td>Price Elasticity</td>
</tr>
</tbody>
</table>

### Parameters

- $\gamma = 1.0, 2.0, 3.0, 4.0, 5.0$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$    Green: $\rho = 1$    Red: $\rho = 2$
**Shock Elasticity: Shock 2**

Blue: $\rho = 0.5$   Green: $\rho = 1$   Red: $\rho = 2$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$ \hspace{1cm} Green: $\rho = 1$ \hspace{1cm} Red: $\rho = 2$

Stochastic Growth, Shock 2

- Consumption
- Capital
- Consumption to Capital
- Investment to Capital
- Price Elasticity

$\gamma = 4.5$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$  \hspace{1cm} Green: $\rho = 1$  \hspace{1cm} Red: $\rho = 2$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$   Green: $\rho = 1$   Red: $\rho = 2$
**Shock Elasticity: Shock 2**

- **Blue**: \( \rho = 0.5 \)
- **Green**: \( \rho = 1 \)
- **Red**: \( \rho = 2 \)
SHOCK ELASTICITY: SHOCK 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
Blue: $\rho = 0.5$  \hspace{1cm} Green: $\rho = 1$  \hspace{1cm} Red: $\rho = 2$
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$

Stochastic Growth, Shock 2

- Consumption
- Capital
- Consumption to Capital
- Investment to Capital
- Price Elasticity
SHOCK ELASTICITY: SHOCK 2

Blue: $\rho = 0.5$  Green: $\rho = 1$  Red: $\rho = 2$
**Shock Elasticity: Shock 2**

Blue: $\rho = 0.5$  \hspace{1cm} Green: $\rho = 1$  \hspace{1cm} Red: $\rho = 2$