The "Reversal Interest Rate": An Effective Lower Bound on Monetary Policy*

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Abstract

The "reversal interest rate" is the rate at which accommodative monetary policy "reverses" its intended effect and becomes contractionary for lending. It occurs when recapitalization gain from the duration mismatch are offset by decreases in net interest margins, lowering banks' net worth and tightening its capital constraint. The determinants of the reversal interest rates are (i) banks asset holdings with fixed (non-floating) interest payments, (ii) the degree of interest rate pass-through to loan rate and deposit rate, (iii) the capital constraints that they face. Low interest rates beyond the time when fixed interest rate mature do not lead to recapitalization gains while still lowering banks' margins, suggesting a shorter forward guidance policy: the reversal interest rates "creep up". Moreover, interest rate cuts can have heterogeneous effects across regions where monetary policy operates, being possibly expansionary in one region and contractionary in another. Furthermore, quantitative easing increases the reversal interest rate. QE should only employed after interest rate cut is exhausted.

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1 Introduction

In most New Keynesian models, the economy enters a liquidity trap as policy rates approach zero, because of the assumed zero lower bound. Yet, a growing group of central banks – the Bank of Japan, the ECB, the Swiss National Bank, the Swedish Riksbank and Danmarks Nationalbank – have set negative interest rates.

This begs the question: what is the effective lower bound on monetary policy? Given that subzero rates are technically feasible, we argue in this paper that the effective lower bond is given by the "reversal interest rate", the rate at which accommodative monetary policy "reverses" its effect and becomes contractionary for lending. Below the "reversal interest rate", a decrease in the monetary policy rate depresses rather than stimulates lending and hence the macro-economy.

Importantly, the reversal interest rate is not (necessarily) zero. Hence, unlike what some commentators suggest, negative interest rates are not fundamentally different. In our model, when the reversal interest rate is positive, say 1 %, then already a policy rate cut from 1 % to 0.9 % is contractionary. On the other hand, if the reversal interest rate is -1 %, there is room to go negative up to that point, provided that financial stability is secured.[1]

The exact level of the reversal interest rate depends on macro-prudential policy, especially financial regulation, as well as other parameters of the economic environment and financial sector's balance variables. Restrictive financial regulation in bad times can undermine monetary policy or render it ineffective. Further determinants of the reversal interest rate in our model include banks' equity capitalization, banks' interest rate exposure, and the market structure of the financial sector. In our multi-period extension, the reversal interest rate varies over time: in fact, as the effects on NIMs is negative, while capital gains fade out over time, the reversal interest rate "creeps up". In other words, exceedingly long low interest rate environments can depress lending. Furthermore, quantitative easing increases the reversal interest rate, as it takes fixed-income out of the balance-sheets of the banks. In that sense, QE should only employed after interest rate cuts are exhausted.

How does an interest rate cut by the central bank affect banks' profit, equity and credit

[1] It is no secret that the US Federal Reserve feels that it cannot lower the interest rate below .25 %, since it would otherwise create a run on money market funds. Nevertheless, discussions over negative rates have reached the U.S. as well.

growth? We identify three channels.

First, banks with long-term legacy assets with fixed interest payments benefit from a policy interest rate cut. As the central bank lowers the interest rate, banks can refinance their long-term assets at a cheaper rate. This increases the value of their equity; they are better capitalized, which relaxes their regulatory or economic constraint. Viewed differently, banks' fixed interest rate holdings experience capital gains. Hence, an interest rate cut is essentially a "stealth recapitalization" of the banks, as stressed in Brunnermeier and Sannikov (2012)'s "I Theory of Money".

Second, a lower policy rate negatively affect banks' profits on new business, through lowering banks' net interest margins. In the hypothetical case of a perfectly competitive financial sector without frictions, any monetary policy rate cut is passed through fully to the deposit and loan rate. Lower loan rates then lead to increased credit growth, and the real economy expands. Since profits from margin business are fully competed away – except for an eventual risk premium –, they are always zero and are not affected by rates changes. Hence banks with legacy asset holdings with fixed interest rates unambiguously benefit from an interest rate cut. In the real world, however, financial markets are not perfectly competitive and banks have market power. Formally, in our baseline model we model the banking sector as having three investment opportunities: risky loans, safe bonds, and reserves. Banks' raise deposits to finance these investments, alongside equity. Importantly, we assume that banks' have market power on their ability to grant loans and raise deposits. When a central bank cut the interest rate, the yield on safe assets and reserves goes down. As a consequence, the marginal benefit from raising deposits decreases, which leads the banks' to decrease deposit rates', inducing the desired substitution effect on depositors that the Central Bank seeks. Overall, however, the banking sector is hurt on its deposit business, since the marginal benefit from its investments decreases; the decrease in the deposit rate is only a typical quantity restriction a monopsonist imposes following a decline in the marginal benefit on lending. Furthermore, as yields on safe assets decrease, banks' decrease their lending rates for risky loans in order to substitute their safe assets positions into riskier high-yield ones, an effect which the Central Bank also seeks to induce. This decrease in the lending rate, although optimal, participate in the overall decline of bank profits'.

Third, the change in profits induced by lower policy rates can feedback into lending. In our model, as in reality, the risk-taking ability of the banking sector is constrained by its net worth. If the latter is high enough so that the constraint does not bind, or if capital gains are strong enough to actually increase net worth, then an interest cut generates the boom in lending that the Central Bank seeks to induce. However, if capital gains are too low to compensate the loss in NIMs, net worth decreases to the point where the constraint binds, limiting banks' ability to take on risk. That point is the reversal interest rate, below which further interest cuts generate a decline in lending though the net-worth feedback. Moreover, an interesting amplification mechanism emerges. As the negative wealth effect further tightens banks' equity constraint, banks cut back on their credit extension and are forced to scale up their safe asset holdings. As these assets have lower yields, their profits decline even more, forcing banks' to substitute out of risky loans into safe assets, which in turn lowers their profit, and so on.

We then uncover the determinants of the reversal interest rate in our baseline model. The reversal interest rate depends on banks' assets' interest rate exposure, the tightness of financial regulation, as well as the market structure of the banking sector. If banks hold more long-term bonds and mortgages with fixed interest, the "stealth recapitalization" effect due to an interest rate cut is more pronounced, and the reversal interest rate is lower. Stricter capital requirements rise the reversal interest rate. Lower market power, which decreases profits, also generates a higher reversal interest rate. For example, in a negative interest rate environment, innovations that allow depositors to substitute bank accounts for cash more easily hurt the margins of the banks' and raise the reversal interest rate; if such innovation occurs below the reversal interest rate, it directly feeds back into lower lending.

We then study three extensions of our baseline model, separately. First, we extend the model to multiple periods, to see how changes in the whole interest rate policy path affects current lending activity. Second, we consider the case of multiple "regions" that are differentially affected by an interest cut, owing to the particularities of their respective banking sectors. Third, we introduce competition among banks and study the differential impacts that the rate cut has on different banks.

In the multiple period extension, our analysis shows that the length of the interest rate cut can last longer if banks hold fixed income assets with longer duration. A lower interest rate allows banks to refinance their fixed come assets up to the point when they mature. A longer anticipated interest rate cut only translates into higher reevaluation gains for assets with higher maturity, which can offset the loss resulting from lower NIM profits for more periods in the future. If banks assets are of shorter duration, then a longer interest rate cut might lead to larger NIM profit losses than fixed income capital gains. In that sense, the reversal interest rate "creeps up" over time: an exceedingly long period of low rates may end up lower lending from today onwards, amid feedback effects on the banks' valuation.

In our heterogenous regions extension, the economics are direct extensions of the comparative statics that were developed for the baseline model. An interest cut might be expansionary in one region, and contractionary in another, owing to differential affect on profits. This might be desirable, as the Central Bank might want to stimulate one region and not the other. Importantly, we show that the inclusion of an interbank market – directly between banks or intermediated via the Central Bank – influence the effects of changes in interest rates, because it changes the exposure of the different regions to interest rate cut, in the absence of interest derivatives that hedge the exposure. Regions whose banks borrow in the interbank markets benefit more from a cut than banks who lend on these markets.

In our extension with competition, we show that competitive behavior and variable markups create forces that amplify banks' heterogeneous exposure to an interest rate cut, at the expense of lending. Intuitively, weaker banks are forced to decrease their risky-lending business following an interest rate cut. This strengthens' the market power of stronger banks: although this raises their profits and hence their ability make risky loans, the lack of competition means that this ability is also used to increase mark-ups, which is detrimental for lending.

Finally, we also believe that our model has important implications for the timing and sequencing of Quantitative Easing measures (QE). The optimal sequencing is the following. First, induce banks (possibly through favorable refinancing operations) to hold long-term bonds with a fixed interest rate; second, cut the policy interest rate to generate capital gains for a "stealth recapitalization" of the banking sector; third, conduct QE and lift the long-term assets of banks' balance sheet so that banks realize their capital gains: banks sell their long-term bonds to the central bank in exchange for short-term bonds or reserves at high prices. However, after QE a further interest rate cut is less effective (and might be even counterproductive) since now banks hold mostly short-term reserves. QE undermines the power of future interest rate cuts and increases the "reversal interest rate".

If banks suffer losses, e.g. because of higher delinquency rates in their mortgages, the (endogenous) "reversal interest rate" rises. If it does so beyond the policy rate, a subsequent interest rate cut is contractionary, and QE has used up the "single bullet". Under such circumstances, it might better to raise interest rates, which improve banks' net interest rate margin. Since banks have passed on large parts of their bond holdings to the central bank, the latter suffers the capital losses on these bonds. Only after raising the interest rate and devaluing the long-term bonds, and only after conducting a "Reverse-QE", which replaces banks' reserve holdings with long-term bonds again, is the reversal interest rate restored at a lower level. In a sense this reloads the gun – for a further round of interest rate cuts.

1.1 Literature Review

Our modeling of the pass-through stands on the shoulders of a large literature on the microeconomics of the banks, which formally started with Klein (1971) and Monti (1972). Ho and Saunders (1981) and Prisman et al. (1986) added uncertainty into the analysis. Santomero (1984) provides a good survey of this early theoretical literature.

Recent empirical papers have revived the literature through the lenses of the pass-through of monetary policy interest rates to the economy. De Bondt (2005), using European data, shows that the immediate pass-through to lending and deposit rates is at most 50 % at a three-month horizon. Bech and Malkhozov (2016) show that the recent drop in reserve rates below zero transmitted through all risk-free short term assets of the economy, but find that the pass-through seemed imperfect for retail deposit rates. Mortgage rates in their data also showed no response, or even increased in certain countries. Drechsler et al. (2015) focus on the transmission to deposit rates, and show in particular that mark-ups on deposits tend to decrease with the reserve rate. Rognlie (2016) show that there is no effective zero lower bound on deposits.

Other papers try to measures banks' net interest margin from a different angle. Saunders and Schumacher (2000) decompose bank margins into a regulatory component, a market structure component and a risk-premium component and show that all three channels are sizable in the data. Abad et al. (2016) show that banks on net buy interest rate protection on the derivatives market, although large banks do play an intermediary role in that they sell interest rate risk protection to smaller banks buy protection. However, Begenau et al. (2015) document that even if banks' do participate on the markets for derivatives to hedge their interest rate risk, banks cannot or do not fully hedge their interest rate exposure.

Landier et al. (2013) focus directly on the real lending of banks. They document in a panel study that the income gap – the sensitivity of banks' profits to interest rates – has a causal impact on their lending behavior.

An important aspect of the literature on micro-banking is the competition structure of the banking industry. Petersen and Rajan (1995), in an influential paper, suggested that a monopolistically competitive banking structure better reflects reality, arguing that banks need some monopoly power to sustain their businesses. Sharpe (1997) suggests evidence of switching costs to depositors, and Kim et al. (2003) of relationship costs for banks. We use these costs to justify the imperfect competitive structure of our model âĂŞ both papers offer micro-foundations for these costs. Maudos and Fernandez de Guevara (2004), Saunders and Schumacher (2000), and Drechsler et al. (2015) offer evidence that imperfect competition affects the pass-through of rates.

2 A two-period model

In order to highlight the key mechanism that we have in mind, we start by setting up a two-period economy where a monopoly bank – which can be viewed as the banking sector as a whole – has the simplest structure of assets and liabilities that leads rise to a reversal interest rate. We first spell out the structure of the bank's problem, and derive its optimal decision rules. We'll show that below a certain level of policy rate – the rate paid by the central bank on excess reserves – further decreases in interest rates decrease total lending. We then spell out the key determinants of the reversal interest rate in that simplified setting.

2.1 Setting

A monopoly bank enters the period with its past book, and after the Central Bank unexpectedly changes monetary policy, takes new economic decisions – buying assets and raising liabilities.

The exact timing of events we consider is as follows.

1. The bank enters the period with its "past book", after past loans and deposits have been repaid.
2. A new policy rate is decided by the Central Bank.
3. New economic decisions are taken by the bank and other agents in the economy.

4. In the next period, repayments are made and profits are realized.

On the asset side of its balance sheet, the bank has essentially three investment opportunities. First, it can grant risky loans L in the economy. Second, the bank can place reserves M at the central bank that are enumerated at a floating policy rate. Third, it can also purchase safe assets B, e.g. bonds, that yields fixed interest payments.

On the liability side, the bank funds itself by issuing deposits D, and by using its book equity E_0. Its book equity is formed by the past book of the bank: after loans have been repaid and deposits credited, the bank is left with a stock of existing reserves (or cash) holdings M_0 and securities B_0, so that $E_0 = M_0 + p^B B_0$.

We now discuss how the quantities and interest rates for each of the balance sheet items are pinned down. We assume for now that inflation is stable and that all rates are also real rates.

Reserves and safe assets

Banks can adjust the amount of reserves M elastically at a rate R^M, paid by the central bank. The bank needs to hold $M \geq \alpha D$ amount of reserves for some $\alpha > 0$, reflecting regulatory requirements or liquidity management; however the interest paid on reserves and excess reserves is the same.[2]

Bonds B are supplied elastically, and generate interest and principal payments R^B after which they mature. These assets can be purchased at a competitive price $p^B = R^B/R^M$.[3]

Deposit market

The bank – or the banking sector – has a monopsony on the deposit market and uses it as a source of funding. The interest rate it pays to depositors/bank debt holders is R^D, and the total supply of deposits is given by $D(R^D)$. We assume standard conditions on $D(\cdot)$ that

[2] Note that we do not allow the bank to borrow at the central bank at the rate R^M. Banks have been available to borrow from central banks; however, this type of lending is being offered as emergency lending due to freezes in the inter-banking market, at rates usually entailing a large premium above R^M as well as reputational costs for the borrowing banks. The implications of this assumption are explored when we'll allow interbank lending.

[3] Implicitly, we're assuming here that the Central Bank is able to affect the yields of all safe assets.

ensures a solution to the standard monopsony problem with constant marginal costs to have a solution[4].

Loan market and Capital Constraints

The bank – or the banking sector – also has a monopoly on the market for loans. The interest it pays to loan seekers is R^L and includes default risk. The total demand for such loans is given by the function $L(R^L)$, which again is continuously differentiable, positive, decreasing in R^L and not too convex.

Importantly, we assume that banks are subject to restrictions on the amount of risk they can take. We think of these restrictions as either being directly imposed by regulatory constraints (i.e. Basel regulations[5]) or emerging as the solution to some economic problem – incentives, information friction, or investor risk-aversion.

These restrictions are as follows, with E the value of the bank (derived below):

$$\gamma L \leq E$$

Hence γ captures the strength of the capital constraint

Stating the bank's problem

Given the above, the cash-flow constraint of the bank is given by[6]

$$p^B B + M + L = D + E_0$$

The (future-value) equity of the bank is given by the balance sheet identity. One difficulty is that it solves a fixed-point, as equity enters the problem itself through leverage costs. We can write it formally as follows. Let $\boldsymbol{\xi}$ stack the parameters that the bank takes as exogenous.

[4]That is, we assume that $D(\cdot)$ is continuously differentiable, positive, increasing in R^D and satisfies $-2D' - (c - R^D)D'' \leq 0$ to be negative for any c in consideration. Concavity is sufficient for this condition, but not necessary.

[5]Basel regulations apply constraints to book equity, while we apply it to mark-to-market equity. We think that these two objects can be linked by an investor's problem of how much book equity to keep in the bank.

[6]Note that B is recorded in the trading book of the bank.

First, for every given $\tilde{E}$ one can find:

$$
\begin{aligned}
E(\boldsymbol{\xi}, \tilde{E}) = \max_{M,B,R^L,R^D} \quad & R^L L(R^L) + R^B B + R^M M - R^D D(R^D) \\
\text{s.t.} \quad & p^B B + M + L = D + E_0 \\
& \gamma L \leq \tilde{E} \\
& M \geq \alpha D(R^D)
\end{aligned}
$$

Then, next-period equity is the maximum such $\tilde{E}$ that solves the fixed-point problem:

$$
E(\boldsymbol{\xi}) = \max_{\tilde{E}:\ E(\boldsymbol{\xi}, \tilde{E}))=\tilde{E}} E(\boldsymbol{\xi}, \tilde{E}))
$$

Despite the fixed-point, this problem offers no unknown mathematical difficulties , and hence we omit conditions for existence and uniqueness and simply assume them.

2.2 Pricing rules and profit margins

In what follows, we restrict the analysis – but not our proofs – to the case where the banking sector holds excess reserves, that is $M \geq \alpha D$ does not bind, which we believe is a good description of the low-interest environment that has followed the 2007-08 financial crisis. In this world, monetary policy works as a floor system: by the elastic provision of reserves, the Central Bank essentially controls the return on safe asset investments, while open market operations – the purchase of assets B – is inconsequential in a two-period model.

The effective asset demand that the bank faces is $L(R^L)$ truncated at R^M, given that the safe asset provision is elastic. R^M effectively becomes the opportunity cost of giving out loans, and banks charge a mark-up above it. For deposits, in contrast, R^M is the marginal benefit from raising deposits, and banks charge a mark-down.

Proposition 1 (Interest rates & Pass-through). *Let ε^{*f} denote the elasticity of the function f, evaluated at the optimal pricing rules. The optimal rate on deposits is given by:*

$$
R^{*D} = \underbrace{\frac{\varepsilon^{*D}}{\varepsilon^{*D} + 1}}_{\textit{Mark-down}} \underbrace{R^M}_{\substack{\textit{Marginal} \\ \textit{benefit}}}
$$

While the optimal rate on loans is given by:

$$R^{*L} = \underbrace{\frac{\varepsilon^{*L}}{\varepsilon^{*L}-1}}_{\textit{Mark-up}} \Bigg(\underbrace{R^M}_{\substack{\textit{Marginal} \\ \textit{opportunity cost}}} + \underbrace{\lambda^*}_{\textit{Shadow cost of constraint}} \Bigg)$$

This proposition highlights how the Central Bank affects rates being offered in the economy: essentially, it is because it controls the marginal investment opportunity of the bank – as long as the latter is unconstrained. The pass-through depends on (a) the mark-ups/downs that the bank imposes, and (b) whether the bank is constrained in its behaviour.

A binding leverage constraint decreases the total amount of granted loans, through a higher loan rate.[7] From these pricing rules we immediately get the Net Interest Spreads, $R^{*L} - R^M$ and $R^M - R^{*D}$: holding elasticities constant, these are increasing in R^M. The Net Interest Margins, which are the spreads multiplied by quantities, are in a sense more interesting: the one on loans, $(R^{*L}-R^M)L(R^{*L})$, is decreasing in R^M, while the one on deposits is increasing in R^M. This is a consequence of the envelope theorem: a decrease in R^M reduces the opportunity cost of granting loans, while it decreases the marginal benefits from deposits. However, overall, the bank is harmed by a decreasing in R^M, simply because the banking sector is a net investor in safe assets – its market power on liabilities is hence consequential in generating that result.

We now derive how profits react to changes in the policy rate R^M. There are essentially two level effects. First, decreasing R^M decrease the returns that banks make on safe asset investments. Hence when banks have a large share of their portfolio invested into safe assets – be it excess returns or bonds – a drop in interest rates decrease this return and hence, in fact, the net interest margin that the bank obtains on its investments. Second, however, decreasing R^M raises the value of the legacy asset that the bank held when it entered the period: this is the capital gain channel. Moreover, when it is constrained on its loans, losses from further decreases are even more harmful, as the bank is forced to substitute risky investments for safe assets which have lower returns. The next lemma encodes these results.

[7]One could also consider models in which the adjustments takes place at the extensive margin, i.e. in which number of granted loans decreases.

Lemma 1. *The derivative of profits with respects to a change in the risk-free rate is:*

$$\frac{dE(\boldsymbol{\xi})}{dR^M} = \underbrace{\frac{1}{1-\lambda^*}}_{\textit{Leverage amplification}} \Big(\underbrace{M^* + p^B B^* -}_{\substack{\textit{Lower margins} \\ \textit{on safe assets}}} \underbrace{p^B B_0}_{\textit{Capital gains}} \Big)$$

2.3 The Existence of a reversal interest rate

Does the reversal interest rate always exist? Given our assumptions, the only thing we have to guarantee here is that B_0 is small enough, or alternatively that M_0 is large enough, holding $E_0 = M_0 + p^B B_0$ constant, so that capital gains do not offset the losses of margins. In particular, existence does not depend on properties of the loan and demand functions, ceteris paribus.[8]

Proposition 2 (The "reversal interest rate")**.** *If B_0 is sufficiently small (or M_0 sufficiently large, holding $E_0 = M_0 + p^B B_0$ constant), then there exists a policy rate R^{RR} at which a reversal occurs in that further decreases to policy rates are contractionary for lending.*

Formally, there exists R^{RR} such that:

$$R^M < R^{RR} \Leftrightarrow \frac{dL^*(R^M)}{dR^M} > 0$$

Importantly, 0 does *not* have to be, generically, the rate at which such reversal occurs – it can be positive as well as negative.

Corollary 1. $R^{RR} - 1 \neq 0$ *generically.*

2.4 Determinants of the reversal interest rate

Having established the conditions for the existence of a reversal interest rate, we now study its determinants in this simple model.

[8] Restricting $M \geq \alpha D$ to not bind isn't necessary for this result.

Past book and Quantitative Easing

Our first result suggests an interplay between interest rate policy and other monetary operations such as Quantitative Easing, which change the bond holdings of banks. Before Quantitative Easing, where bank holdings of long term bonds B_0 are high, interest rates cuts are likely to lead to large capital gains and hence rise lending. After QE, however, banks are more exposed to interest rate movements due to the large, positive amount of reserves they hold. This makes them more sensitive to interest cuts – with the risk that a future cut actually hurts lending. In that sense, further rate cuts to be effective needs a "reloading of the gun" by letting bank acquire fixed-income securities whose value intrinsically rise with decreases in risk-free rates.

Proposition 3 (Bonds holdings and returns)**.** *If $E_0 = M_0 + p^B B_0$ is held fixed, but B_0 is replaced with an equivalent amount of $p^B B_0$ before an interest cut occurs ("QE"), then R^{RR} increases.*

Capital Constraints

Next we study how the reversal interest rate changes with the structure of capital constraints. Unsurprisingly, R^{RR} is lower when capital constraints are looser or equity buffers are high. Of course, we neglect potential risk-taking effects of decreasing interest rates, which is the basis for a regulatory constraint – see for example Di Tella (2013) or Klimenko et al. (2015). In a theory encompassing both channels, a trade-off would emerge between the two. Moreover, when the leverage costs γ are higher, the negative effects of further drops is more pronounced.

Proposition 4 (Shape of the regulatory constraint)**.** *R^{RR} has the following properties:*

1. *R^{RR} decreases in initial book equity E_0, ceteris paribus.*
2. *R^{RR} increases in capital requirements γ.*
3. *A higher γ implies that interest rate cuts below R^{RR} depress lending more, ceteris paribus.*

Note that although increasing equity E_0 reduces the reversal interest rate, this is not necessarily optimal from the point of view of the owners' of the bank.[9]

[9] We have in mind a classic debt overhang problem, through which equity investors are not able to obtain the full return on their investment, which is partially captured by existing debtholders. See Admiti et al. (2016) for a recent discussion of the problem.

Elasticities

Our last comparative static involves the properties of the deposit function. It states that if the banks face a more elastic deposit supply function, then R^{RR} increases. This is unsurprising, as it reduces profits and hence the willingness of equity holders to keep investing in the banking business. One can interpret this result as follows: notice from Proposition 1 that ε^{*D} possibly changes as R^M does. One can interpret this change as coming from the possibility of households to save in cash, which offers a nominal rate of zero: although the bank may still charge a mark-down, which would depend on the liquidity services provided by a deposit account, that mark-down would have to be greatly reduced if R^M were to turn negative.[10] In fact, banks who have large institutional investors have been able to pass-through negative rates onto their depositors better than banks whose depositors base consists mostly of small savers.

Proposition 5 (Deposit elasticity). *R^{RR} increases in ε^D in the sense that making a deposit supply more elastic ceteris paribus increases the reversal interest rate.*[11]

Hence this result suggests that technologies that allow depositors to substitute their deposits towards cash holdings could raise the reversal interest rate by making the demand supply more elastic at sub-zero rates. It also suggest that banks who rely on large institutional investors are less harmed by a decrease in interest rates than savings' banks that rely heavily on small depositors, as negative rates can be easily passed-on to the former but not the latter, for whom storing cash is less costly.

3 Multi-period extension

In this section, we extend the model to a multi-period setting. This allows us to study how announcements about a *path* of policy rates impact the business of the bank, in particular net

[10] R^D would never be larger in our model than R^M, which is what has been observed in some countries who have implemented negative interest rates. One could model this as costs from losing depositors. Moreover, although banks have been reluntant to pass-through negative rates on standard deposit accounts, they have increased the fees associated with the usage of these accounts.

[11] Mathematically, fix a deposit supply function $D(\cdot)$, and associated reversal interest rate $R^{\mathrm{RR},D}$. Consider another deposit supply function $D'(\cdot)$ such that $D(R^{\mathrm{RR},D}) = D'(R^{\mathrm{RR},D})$, and $\varepsilon^{D'}(R^D) \geq \varepsilon^{D}(R^D)$ for all R^D. Then, $R^{\mathrm{RR},D'} \geq R^{\mathrm{RR},D}$.

interest margins in the future and their feedback on lending today. Our main result is that the optimal length of interest rate cuts should be related to the duration of the banks' existing assets. The reason is as follows. A cut in an interest rate in the future has two effects, as in our two-period model: fixed-income assets that pay-off in that future gain in value, while NIMs in that period will be depressed, decreasing the current value of the bank. Since the pay-offs of fixed-income assets are decreasing over time, the first force slowly fades out, while the loss in margins on future business does not. Hence, the reversal interest rate – which is the point at which present lending is maximized – "creeps up" over time. Moreover, as profits in the next period is also equity in the future period, future lending might also decrease.

A simple example

To make that intuition concrete, consider a bank that enters the period with two assets on its books: a one-period bond (that hence expires next-period) and a two-period bond. The bank holds more of the former in book value. The Central Bank is considering decreasing the policy rate for three periods. What is the path of interest rates it should choose? Clearly, cutting the rate in the third-period leads to no capital gains in the present, and will hurt the future NIMs of the bank at that period. If the bank currently operates at its constraint, that will generate lower lending *today* already – which, through declining profits and the amplification channel we highlighted, may hurt lending in the future too. What about the first and second period? As the holdings of the first-period are larger than that of the second period, the case for cutting the interest rate is stronger in the first-period than the second-period (the effect on NIMs, discounting apart, is the same). Hence, in that sense, the optimal path of interest rates is increasing, and an exceedingly low interest rate environment might hurt lending.

3.1 General setting

Time is discrete and indexed by $t = 0, 1, ..., T$ with possibly $T = \infty$. Every period, the timing of events is as follows. First, the Central Bank announces a new path of risk-free rates $\left\{R^M_{t+1+s}\right\}_{s\geq 1}$. Next, banks receive interest payments on loans, pay interests on deposits, collect bond payments and select new bond positions: profits are realized. Then, a fraction $\bar{\delta}$ of (cum-dividend) equity, $\mathcal{E}_t$ is paid to investors as dividends. Finally, new economic decisions are taken: the amount of loans, cash and deposits the bank decides to hold.

Hence the cum-dividend value function for the investor of a bank entering with cum-dividend book equity $\mathcal{E}_t$ can be written recursively as:

$$V_t(\mathcal{E}_t) = \bar{\delta}\mathcal{E}_t + \max_{\mathbf{y}_t \in \Gamma_t} \mathcal{M}^E_{t,t+1} V_{t+1}(\mathcal{E}_{t+1})$$

Where $\mathcal{M}^E_{t,t+1} = \frac{1}{R^E_{t+1}}$ is the discount factor of investors, and $\mathbf{y}_t = (L_t, M_t, B_t, D_t)$ are the decision variables at t given the constraints in Γ_t, which we describe now. Deposits and cash are determined as in the two-period model. We allow as before the loan behavior of the bank to be potentially constrained. We assume that the constraint takes the form:

$$\gamma L_t \leq V_t(\mathcal{E}_t, \cdot)$$

Where book equity is retained equity: $E_t = (1 - \bar{\delta})\mathcal{E}_t$.

The banking sector can purchase bonds B_t at a price p^B_t. These assets yield pay-offs $\left\{R^B_s\right\}_{s \geq 0}$, and can be sold every period after the Central Bank's announcement of new rates. As before, we assume that the pricing of these assets is competitive, implying that they sell at the following price:

$$p^B_t = \frac{p^B_{t+1|t} + R^B_{t+1}}{R^M_{t+1}}$$

Where $p^B_{t+1|t}$ is the next-period price of the bond in period t. Note that given this pricing, and the absence of expectation about rates, banks are essentially indifferent about the amount of bonds they hold, which we hold fix over time for simplicity.

As a consequence of our modelling choices, the bank's next-period (cum-dividend) book equity is:

$$\mathcal{E}_{t+1} = R^L_{t+1} L_t + (p^B_{t+1|t+1} + R^B_{t+1}) B_t + R^M_{t+1} M_t - R^D_{t+1} D_t$$

Given that the ex-dividend book equity is $E_t = (1 - \bar{\delta})\mathcal{E}_t$, the cash-flow constraint reads:

$$L_t + p^B_t B_t + M_t = D_t + E_t$$

Note that E_t already includes the eventual capital gains that the bank would have done in the previous period. Solving in the constraint we get:

$$\mathcal{E}_{t+1} = (R^L_{t+1} - R^M_{t+1}) L_t + (R^M_{t+1} - R^D_{t+1}) D_t + (p^B_{t+1|t+1} - p^B_{t+1|t}) B_t + R^M_{t+1} E_t$$

Note that should risk-free rates stay unchanged, we would have that $p^B_{t+1|t+1} = p^B_{t+1|t}$ and the third term would drop-out.

It is useful to note first that the two relevant state variables are (1) the bank's profits $\mathcal{E}_t$ and (2) the value of the bank itself, V_t. Clearly, the latter is increasing in $\mathcal{E}_t$, hence the latter is enough to keep track of.[12]

Lemma 2. *The value of the bank V_t is strictly increasing in $\mathcal{E}_t$.*

Next, denote the optimal profits of the bank on loans and deposits, the net interest margin, to be:

$$\pi^{\text{NIM}}_{t+1} \equiv \left(R^{*L}_{t+1} - R^M_{t+1}\right) L(R^{*L}_{t+1}) + \left(R^M_{t+1} - R^{*D}_{t+1}\right) D(R^{*D}_{t+1})$$

The next result characterizes the value function of banks, *given* optimal decisions for loan rates and deposit rates – that is, taking the so-called policy functions as given.

Lemma 3. *The value of the bank (after optimization) is:*

$$V_t(\mathcal{E}_t, \cdot) = \mathcal{E}_t + \sum_{s=1}^{\infty} \mathcal{M}^E_{t,t+s} \left(\pi^{NIM}_{t+s} + (1-\bar{\delta})(R^M_{t+s} - R^E_{t+s})\mathcal{E}^*_{t+s-1}\right)$$

Where $\mathcal{M}^E_{t,t+s} \equiv \prod_{r=1}^{s} \frac{1}{R^E_{t+r}}$ is the discount factor of equity investors.

The first term corresponds to a standard net present value of the profit stream. The second term represents eventual capital losses due to profits being rolled-over at a rate below the discount rate. This effect would disappear if the bank would always pay dividends.

We now state our main proposition: as in the two-period model, we can decompose the effect of a change in interest rates on banks' value.

Proposition 6 (Impact on banks' value)**.** *Consider a change in the rate R^M_{t+s} announced at*

[12] This is important: given a guess for the function $V_t(\mathcal{E}_t, \cdot)$ and enough stationarity, one can then proceed numerically with standard methods. Indeed, one can rewrite the problem as:

$$V_t(\mathcal{E}_t, \cdot) = \bar{\delta}\mathcal{E}_t + \max_{\mathbf{x}_t \in \Gamma_t} \mathcal{M}^E_{t,t+1} V_{t+1}(\mathcal{E}_{t+1}, V^{-1}_{t+1}(\mathcal{E}_{t+1}))$$

time t. The total derivative of V_t with respect to such change is:

$$\frac{dV_t}{dR^M_{t+s}} = \underbrace{\mathcal{M}^E_{t,t+s}\frac{d\pi^{NIM}_{t+s}}{dR^M_{t+s}}}_{\text{Direct effect on NIM}} + \underbrace{\sum_{r=1}^{\infty}\mathcal{M}^E_{t,t+r}\frac{\partial\pi^{NIM}_{t+r}}{\partial\mathcal{E}^*_{t+r}}\frac{d\mathcal{E}^*_{t+r}}{dR^M_{t+s}}}_{\text{Indirect effect on NIMs}} + \underbrace{B_t\frac{dp^B_{t+1|t+1}}{dR^M_{t+s}}}_{\text{Revaluation effect}}$$
$$+ \underbrace{\frac{d}{dR^M_{t+s}}\sum_{r=0}^{\infty}\mathcal{M}^E_{t,t+r}(1-\bar{\delta})(R^M_{t+r+1}-R^E_{t+r+1})\mathcal{E}^*_{t+r}}_{\text{Discounting effect}}$$

There are four effects. First, there's a direct effect on the net interest margin earned on loans and deposits, which depends on the competitive structure as well as the properties of loan demand and deposit supply. The analysis for this term is not different from the one conducted in Section 2. Second, and unique to the dynamic setting, there's an indirect effect on all NIMs. This occurs because constraining/unconstraining the bank affects its leverage today which impacts all future NIMs. Constraining occurs when drops in future interest rates decrease the franchise value of the bank beyond reevaluation gains; unconstrained occurs when the latter channel dominates. The third channel is indeed this revaluation effect: a decrease in interest rates makes current bond holdings more valuable. This effect depends on the stream of interest payments of the bond, the amount of bond the bank holds, as well as the change in the risk-free rate decided by the central bank.[13] Note that this affects V_t, the value of the bank, only if the bank is constrained and the extra retained profits help to recapitalize and unconstrain the bank. Finally, a discounting effect is present as in the two-period model, which disappears should there be no retained profits.

Next, note that in the setting under consideration, and contrary to the two-period model, the existence of a *path* of reversal interest rates is much easier to establish than that of a unique reversal interest rate for a single period. However, even the latter's existence can be established under the same conditions as the two-period model: as long as the NIM of banks hurts more than the revaluation gains, so that $dV_t/dR^M_t > 0$, and γ is sufficiently high so that the bank is constrained, a reversal interest rate exists and displays similar comparative

[13] In particular, the reevaluation effect can be re-written as:

$$B_t\frac{dp^B_{t+1|t+1}}{dR^M_{t+s}} = B_t\sum_{r=1}^{\infty}\frac{d\mathcal{M}^f_{t+1,t+s+r}}{dR^M_{t+s}}(R^B)_{t+s+r}$$

statics. We keep this assumption throughout the section.

3.2 The optimal duration of monetary policy

We can first obtain the following simple, yet general result.

Proposition 7. *Suppose that B_t does not yield any pay-offs past some date $\bar{T}$, that is $R_t^B = 0$ for all $t \geq \bar{T}$. Then any decrease in the risk-free rate past $\bar{T}$ has negative effects on the value of the bank. If the bank is constrained, this decreases lending.*

When the NIM is positive, the only gains from decreasing rates come the re-evaluation of the banks' bond positions, as lower rates makes legacy bonds more valuable. However, if only future rates beyond the maturity of bonds are raised, this channel disappears, and decreasing rates is always harmful.

We can generalize a bit that insight. Consider choosing R_{t+s}^M for all $s \geq 1$ so as to maximize lending at t. As highlighted in our two-period model, the trade-off is that of capital gains vs. losses of margins: the reversal interest rate R_{t+s}^{RR} is the optimal such point (note that there are now one such rate per future time period). Hence, logically, a decrease in the pay-offs R_{t+s}^B at $t+s$ decreases the potential capital gains from a cut, while keeping the losses on margins constant. As B captures the "whole structure" of fixed-income payments the banking sector gets, one could consider R_{t+s}^B being decreasing in s: in that case, the reversal interest rate is increasing in s. We encode this in the next proposition.

Proposition 8. *Ceteris paribus, the reversal interest rate at t+s is decreasing in R_{t+s}^B: $\frac{\partial R_{t+s}^{RR}}{\partial R_{t+s}^B} \leq 0$.*
Hence, if $\frac{\partial R_{t+s}^B}{\partial s}$ is sufficiently negative, then $\frac{\partial R_{t+s}^{RR}}{\partial s} \geq 0$, that is the optimal sequence of lending-maximizing rates is increasing.

Hence we expect that the longer the maturity of bond holdings of banks, the longer an interest cut should be decided if the objective is to increase lending. In a sense, the beneficial effects of low interest rate environment fade out over time. For example, consider two banks. Bank A holds a two-period bond, and bank B holds a one-period bond as well as a three-period bond with the same present-value as well as the same payments in the first period. Then Proposition 8 says that the optimal rates are lower in the short-run and higher in the long-run in bank B's environment. The intuition is that the effect of the low-interest rate environment fades out over time as *current* bond holdings of banks will eventually expire. However, the negative effect of a low-interest rate environment stays the same: hence, the optimal rates increase again.

3.3 A note on dividends

So far we have voluntarily prevented the banks to adjust their dividend policy. However, because they move cash-flows over-time, these decisions are important for the banks, and hence monetary policy. To understand the importance of dividend policy in this framework, observe that gains from bond holdings are realized immediately, as banks clear their bond positions at the time of the announcements. Hence, should these gains be redistributed immediately through dividends – that is $\bar{\delta} = 1$, the franchise value of the bank will necessarily be hurt by lower rates at time t, which necessarily hurts lending if the bank is constrained.

Moreover, if its owners are impatient enough, an ever-unconstrained bank would always choose to pay profits out in dividends, which makes any capital gains on bonds revaluation useless for the banks' franchise value. The only reason it may not do so is because of the constraint, since retaining profits can raise its lending abilities and hence its net worth. In other words, with endogenous dividends, a constrained banks will trade-off the loss from rolling-over part of profits at the risk free-rate – instead of paying it out in dividends now – with the unconstraining effect of a larger equity buffer. Hence, in this framework dividend control during periods where banks are capital constrained might be useful to guarantee that capital gains are not paid away.[14]

4 Heterogeneity and the reversal interest rate

We now go back to our two-period model, but extend our setting along other dimensions.

First, we consider two regions which each have their own banking sectors, and might have different reversal interest rates owing to differences in their fundamentals. A (common) interest rate cut can have heterogeneous effects across regions: it can be expansionary in a region, while being contractionary in another region. Then, we consider how the inclusion of

[14]Timothy F. Geithner, president of the NY Fed and then head of the Treasury during the Great Recession, wrote for example in his recent book on the crisis that the Fed Board "considered forcing banks as a group to stop paying dividends in order to conserve capital". (Geithner 2016, p.138).

an interbank market affects the reversal interest rates of the different regions. Our main result is that the existence of an interbank market increases the differential of reversal interest rates between the two regions.

Second, we go back to studying a single region but this time let multiple banks compete with each other. We let market power vary across banks: banks with higher market shares are able to charge higher premia. We show that competitive behavior and variable mark-ups create forces that amplify banks' heterogeneous exposure to an interest rate cut, at the expense of lending. Intuitively, weaker banks are forced to decrease their risky-lending business following an interest rate cut. This strengthens' the market power of stronger banks: although this raises their profits and hence their ability make risky loans, the lack of competition means that this ability is also used to increase mark-ups, which is detrimental for lending. We also study again the implication of an interbank market, and find that "weak banks" are harmed more.

4.1 Heterogeneous regions

2 regions, A and B, posses a similar banking system as the one described in the previous section. Regions differ in terms of one fundamental: region B has more lending demand than in A, that is $L_B(R^L) = \mu L_A(R^L)$ for some $\mu > 1$. On every other aspect, the two regions are identical. We immediately get that monetary holdings are higher in region B than in region A, and hence the opposite is true for their respective respective rates.

Proposition 9. *Safe assets holdings are higher in region A than in B, $M_A^* > M_B^*$. The reversal interest rate is higher in region A than in region B; the reversal interest rate of a "pooled" economy with a unique banking sector would be in between the two.*

Simply, given the lack of risky loan demand banks in region A are forced to invest in safe assets. As a consequence, the A is more exposed to decreases in rates. Note that we could have generated the results using other differentials: capital gains, for example, could have been more beneficial in one region than in the other.

Let us now assume now the existence of a competitive interbank market, which can be an explicit market or occur though a central bank. The respective banking sectors in region A and the B can take positions $\mathcal{I}^A, \mathcal{I}^B$ on that market, at a competitive rate $R^{\mathcal{I}}$. Let us assume, for interest, that the loan disparity is such that for the range of rates R^M of interest the B

liquidity constraint $M_B = \alpha D$ binds while in region A it is loose $M_A > \alpha D$ and demand is large enough to accommodate the B's demand.

Then, when the interbank market opens, $\mathcal{I}^B < 0$ and $\mathcal{I}^A > 0$, that is the B borrows from the A. The easing of the constraint benefits the B, while the A is left indifferent on impact. Most importantly, the B's exposure to interest rate movement decreases. As a consequence, its reversal interest rate must decreases; potentially, it can become $-\infty$, in the case the B starts to hold short positions, i.e. becomes a net borrower in safe assets. Moreover, it is important to know whether the A scraps sells bonds as opposed to reserves in order to lend to the B. In the former case, for example due to QE, the reversal interest rate rises in region A, due to lessened capital gains.

Proposition 10. *Suppose that L_A is sufficiently small, and L_B is sufficiently large, ceteris paribus. The inclusion of a competitive interbank market makes the B strictly better off, but leaves welfare in region A unchanged on impact. Moreover, the reversal interest rate in region B decreases, possibly to $-\infty$. If the A sold bonds to then finance the B on the interbank market, then the reversal interest rate in region A increases.*

How important is the assumption of a competitive interbank market? Suppose instead that banks bargain in the interbank market: banks in region A know that the B is (liquidity-) leverage constrained, and hence bargains a mark-up such that $R^{\mathcal{I}} > R^M$, yet a rate at which the B finds it still profitable to borrow from. Then, on impact the A is now also better off, and the reversal interest rate lower there; credit growth is smaller in region B, and the reversal interest rate is higher there.

4.2 Heterogeneous banks

4.2.1 No competition

We now switch to a setting with two banks, A and B, operating in the same region. We first consider the case where the two banks operate on two segmented markets within the region with similar fundamentals. Banks differ, however, on the composition of their initial capitalization: bank A has more bonds (or outstanding assets) than bank B. As a consequence, following a decrease in the policy rate R^M, bank A enjoys more capital gains than bank$B$, and its reversal interest rate is lower.

Proposition 11. *Suppose that $E^A + p^B B_0^A = E^B + p^B B_0^B$, but $B_0^A > B_0^B$. Then $R^{RR,A} < R^{RR,B}$.*

Moreover, suppose that banks have access to a competitive interbank market as above, with an offering rate $R^{\mathcal{I}} = R^M$. Then, if lending demand is sufficiently strong so that bank A is liquidity-constrained on its lending, bank B starts lending to bank A, which may see no reversal as a consequence. Aggregate lending may not decrease, so that an economy-wide reversal interest rate may not exist.

Proposition 12. *If lending demand is large enough, then the inclusion of an interbank market decreases $R^{RR,A}$, with possibly $R^{RR,A} = -\infty$. Aggregate lending may experience no reversal.*

4.2.2 Competition

We now extend the model to allow for competition between heterogenous banks in a fairly arbitrary way. This allows us to discuss the effect of competitions.

N banks compete as follows. For risky loans, banks offer slightly differentiated products, so that if $R_i^L, \boldsymbol{R}_{-i}^L$ are the rates offered by bank i and its competitors, the total demand it faces is $L_i(R_i^L; \boldsymbol{R}_{-i}^L)$. We assume that $L_i(\cdot)$ is strictly decreasing in R_i^L and strictly increasing in all other arguments, and assume that the underlying demand system is invertible. Beyond that, we do not make particular assumptions about the structure of competition: instead, we assume that each bank has a conjectural variation function $\boldsymbol{h}_{-i}^L(R_i^L; \boldsymbol{R}_{-i}^L)$ that captures the behaviour of its competitors.[15] Furthermore, we now assume that the capital constraint takes the form now of a "smooth cost" function, $f\left(\frac{L_i}{E}; \gamma\right) E$, with a larger γ increasing that function. Our previous sharp constraint is hence a limiting case of the analysis here.

On the deposit side, the market structure is similar: banks face an invertible system of demand supplies $D_i(R_i^D, \boldsymbol{R}_{-i}^D)$, with $D_i(\cdot)$ strictly increasing in R_i^D and strictly decreasing in all other arguments. Competition is captured by a conjectural variation function $\boldsymbol{h}_{-i}^D(R_i^D; \boldsymbol{R}_{-i}^D)$.

[15] That is, the loan problem of bank i looks like:

$$\max_{R_i^L; \boldsymbol{R}_{-i}^L} = R_i^L L_i(R_i^L; \boldsymbol{R}_{-i}^L) - \mathrm{TC}_i(L_i(R_i^L; \boldsymbol{R}_{-i}^L)) \qquad \text{s.t.} \quad \boldsymbol{h}_{-i}^L(R_i^L; \boldsymbol{R}_{-i}^L) = 0$$

Cournot competition would be captured by $\boldsymbol{h}_{-i}^L(R_i^L; \boldsymbol{R}_{-i}^L) = \boldsymbol{L}_{-i}(\boldsymbol{R}^L) - \bar{L}_{-i}$, while Bertrand competition would be simply $\boldsymbol{h}_{-i}^L(R_i^L; \boldsymbol{R}_{-i}^L) = \boldsymbol{R}_{-i}^L - \bar{R}_{-i}^L$.

The remaining markets are all competitive: safe assets B can be purchased at a price p^B, inter-bank lending and borrowing is done at an interest rate $R^{\mathcal{I}}$, and reserves at the central bank are placed at a rate R^M.

Given this setting, the problem of bank i is now:

$$
\begin{aligned}
E_i(\boldsymbol{\xi}, \tilde{E}) = \max_{M_i, \mathcal{I}_i, B_i, R_i^L, \boldsymbol{R}_{-i}^L, R_i^D, \boldsymbol{R}_{-i}^D} \quad & R_i^L L(R_i^L; \boldsymbol{R}_{-i}^L) \\
& + R^B B_i + R^M M_i + R^{\mathcal{I}} \mathcal{I}_i - R_i^D D(R_i^D; \boldsymbol{R}_{-i}^D) \\
\text{s.t.} \quad & p^B B_i + L_i + \mathcal{I}_i + M_i = D_i + E_{0,i} \\
& M_i \geq \alpha D_i \\
& \boldsymbol{h}_{-i}^L(R_i^L; \boldsymbol{R}_{-i}^L) = 0 \\
& \boldsymbol{h}_{-i}^D(R_i^D; \boldsymbol{R}_{-i}^D) = 0
\end{aligned}
$$

And the solution to the fixed-point problem is:

$$
E_i(\boldsymbol{\xi}) = \max_{\tilde{E}:\ E_i(\boldsymbol{\xi}, \tilde{E})) = \tilde{E}} E_i(\boldsymbol{\xi}, \tilde{E}))
$$

The next lemma characterizes the new deposit and lending rules.

Lemma 4 (Interest-rate rules)**.** *The optimal rate on deposits is given by:*

$$
R_i^{*D} = \frac{\varepsilon_i^{*D}}{\varepsilon_i^{*D} + 1} R^{\mathcal{I}}
$$

The optimal rate on loans is given by:

$$
R_i^{*L} = \frac{\varepsilon_i^{*L}}{\varepsilon_i^{*L} - 1} \Big(R^{\mathcal{I}} + f' \left(L_i(R_i^{*L}) / E_i(\boldsymbol{\xi}); \gamma \right) \Big)
$$

Where ε_i^{*D} and ε_i^{*L} are now the total elasticities of deposit supply and loan demand faced by *bank i.*[16]

The elasticities now include the effects of competitors. As before (implicitly), the existence of an interbank lending market implies that $\mathcal{I}_i$ now is the elastic variable that adjusts, and hence guides interest rates charged by banks. We assume that the Central Bank perfectly controls that rate too, so that $R^{\mathcal{I}} = R^M$.

We now compute the *pass-through* of a change in R^M to loan rates charged by banks. This pass-through include the effects on mark-ups in similar way as Amiti, Itskhoki and Konings (2016).

Proposition 13 (Pass-through)**.** *The pass-through of a change in R^M in a change in R_i^L is given by:*

$$\frac{d\log R_i^L}{d\log R^M} = \underbrace{\beta_i\alpha_{ii} + \sum_{j\neq i}\beta_j\alpha_{ij}}_{\textit{Opportunity costs}} + \underbrace{(1-\beta_i)\alpha_{ii}\frac{dL_i^*/E_i^*}{dR^M}f_i'' + \sum_{j\neq i}(1-\beta_j)\alpha_{ij}\frac{dL_j^*/E_j^*}{dR^M}f_j''}_{\textit{Leverage costs}}$$

Where the coefficients $\boldsymbol{\alpha}$ are determined by the price-elasticities of mark-ups, and the coefficients $\boldsymbol{\beta}$ are the share of respectively the opportunity cost of lending and the cost of leverage in the marginal cost.[17]

Note that in most models, α_{ij} is positive if $i = j$ and negative otherwise, though note the attenuation effects inside α_{ii}. Hence the first term of the funding cost simply captures the fact

[16] That is:

$$\varepsilon_i^{*D} \equiv \frac{\partial\log D_i}{\partial\log R_i^D} + \sum_{j\neq i}\frac{\partial\log D_i}{\partial\log R_j^D}\frac{\partial\log R_j^D}{\partial\log R_i^D}$$
$$\varepsilon_i^{*L} \equiv -\left(\frac{\partial\log L_i}{\partial\log R_i^L} + \sum_{j\neq i}\frac{\partial\log L_i}{\partial\log R_j^L}\frac{\partial\log R_j^L}{\partial\log R_i^L}\right)$$

Where the cross-rates derivatives are given by the implicit functions defined by $\boldsymbol{h}_{-i}^L(R_i^L;\boldsymbol{R}_{-i}^L) = 0$ and $\boldsymbol{h}_{-i}^D(R_i^D;\boldsymbol{R}_{-i}^D) = 0$.

[17] That is, denoting $\Gamma_{ij} \equiv \frac{\partial\mu_i^L}{\partial R_j^L}$ we have:

$$\alpha_{ii} = 1 + \Gamma_{ii} + \sum_k\Gamma_{ik}\Gamma_{ki} + \sum_{k,\ell}\Gamma_{ik}\Gamma_{k\ell}\Gamma_{\ell i} + ...$$
$$\alpha_{ij} = \Gamma_{ij} + \sum_k\Gamma_{ik}\Gamma_{kj} + ...$$

that a decreasing in R^M lower the marginal lending opportunity (for a firm long interbank) or the marginal funding cost (for a firm short interbank), while the second term comes from the fact that stronger competition decreases a firm's mark-up and hence forces a stronger rate cut.

Suppose now that $d(L_i^*/E_i^*)/dR^M \leq 0$, for concreteness, meaning that decreasing the rate leverages the bank. Then, this third (negative) term captures the fact that the bank is forced to set up a higher L_i^* rate to compensate it's higher leverage costs. The fourth term is the opposite: provided $d(L_j^*/E_j^*)/dR^M \leq 0$ too, bank j's deleveraging decreases the competition faced by bank i which responds by raising it's mark-up, hence the positive term.

[TBC]

References

Abad, J., I. Aldasoro, M. D'errico, L. Fache Rousova, P. Hoffmann, S. Langfield, and T. Roukny (2016). Shedding light on dark markets: first insights from the new eu-wide otc derivatives dataset. *ESRB Occasional Paper Series* (10), 1–32.

Bech, M. and A. Malkhozov (2016). How have central banks implemented negative policy rates? *BIS Quarterly Review*, 31–44.

Begenau, J., M. Piazzesi, and M. Schneider (2015, July). Banks' Risk Exposures. NBER Working Papers 21334, National Bureau of Economic Research.

Brunnermeier, M. K. and Y. Sannikov (2012). The i theory of money. Working Paper, Princeton University.

De Bondt, G. J. (2005). Interest rate pass-through: Empirical results for the euro area. *German Economic Review 6*(1), 37–78.

Di Tella, S. (2013). Uncertainty shocks and balance sheet recessions. Working paper, Stanford University.

Drechsler, I., A. Savov, and P. Schnabl (2015). The deposits channel of monetary policy. Working Paper.

And for $\boldsymbol{\beta}$:

$$\beta_i = \frac{R^{\mathcal{I}}}{R^{\mathcal{I}} + f_i'}$$

Ho, T. and A. Saunders (1981). The determinants of bank interest margins: Theory and empirical evidence. *The Journal of Financial and Quantitative Analysis 16*(4), 581–600.

Kim, M., D. Kliger, and B. Vale (2003). Estimating switching costs: the case of banking. *Journal of Financial Intermediation 12*(1), 25 – 56.

Klein, M. A. (1971). A theory of the banking firm. *Journal of Money, Credit and Banking 3*(2), 205–.

Klimenko, N., S. Pfeil, and J.-C. Rochet (2015). Bank capital and aggregate credit. Working Paper, University of Zrich.

Landier, A., D. Sraer, and D. Thesmar (2013, February). Banks' Exposure to Interest Rate Risk and The Transmission of Monetary Policy. NBER Working Papers 18857, National Bureau of Economic Research.

Maudos, J. and J. Fernandez de Guevara (2004, September). Factors explaining the interest margin in the banking sectors of the European Union. *Journal of Banking & Finance 28*(9), 2259–2281.

Monti, M. (1972). Deposit, credit and interest rate determination under alternative bank objective functions. In S. Karl and S. Giorgio P. (Eds.), *Mathematical methods in investment and finance*, pp. 431–454. North-Holland, Amsterdam.

Petersen, M. A. and R. Rajan (1995). The effect of credit market competition on lending relationships. *The Quarterly Journal of Economics 110*(2), 407–443.

Prisman, E. Z., M. B. Slovin, and M. E. Sushka (1986). A general model of the banking firm under conditions of monopoly, uncertainty, and recourse. *Journal of Monetary Economics 17*(2), 293 – 304.

Rognlie, M. (2016). What lower bound? monetary policy with negative interest rate. Working Paper.

Santomero, A. M. (1984). Modeling the banking firm: A survey. *Journal of Money, Credit and Banking 16*(4), 576–602.

Saunders, A. and L. Schumacher (2000). The determinants of bank interest rate margins: an international study. *Journal of International Money and Finance 19*(6), 813 – 832.

Sharpe, S. A. (1997). The effect of consumer switching costs on prices: A theory and its application to the bank deposit market. *Review of Industrial Organization 12*(1), 79–94.