The "Reversal Interest Rate": An Effective Lower Bound on Monetary Policy*

Markus K. Brunnermeier and Yann Koby†

This version: July 10, 2017

Abstract

The "reversal interest rate" is the rate at which accommodative monetary policy "reverses" its intended effect and becomes contractionary for lending. It occurs when recapitalization gains from the duration mismatch are offset by decreases in net interest margins, lowering banks' net worth and tightening its capital constraint. The determinants of the reversal interest rates are (i) banks asset holdings with fixed (non-floating) interest payments, (ii) the degree of interest rate pass-through to deposit rate, (iii) the capital constraints that they face. Low interest rates beyond the time when fixed interest rate mature do not lead to recapitalization gains while still lowering banks' margins, suggesting a shorter forward guidance policy: the reversal interest rates "creep up". Moreover, interest rate cuts can have heterogeneous effects across regions where monetary policy operates, being possibly expansionary in one region and contractionary in another. Furthermore, quantitative easing increases the reversal interest rate. QE should only be employed after interest rate cut is exhausted.

*We are grateful to comments by Sam Langfield and participants of seminars at the European Central Bank, the Bank of International Settlement, Princeton University, the Federal Reserve Board, Banque de France, the Danish Central Bank, the Finnish Central Bank, the London School of Economics, ETH Zurich, the Swiss Economists Abroad Meetings, the Swiss National Bank, the 2017 AEA Meetings, Humboldt University, and the Philadelphia Macro Workshop.

†Brunnermeier: Department of Economics, Princeton University, markus@princeton.edu, http://scholar.princeton.edu/markus; Koby: Department of Economics, Princeton University, ykoby@princeton.edu

1 Introduction

In most New Keynesian models, the economy enters a liquidity trap as policy rates approach zero, because of the assumed zero lower bound. Yet, a growing group of central banks – the Bank of Japan, the ECB, the Swiss National Bank, the Swedish Riksbank and Danmarks Nationalbank – have set negative interest rates.

This begs the question: what is the effective lower bound on monetary policy? Given that subzero rates are technically feasible, we argue in this paper that the effective lower bond is given by the "reversal interest rate", the rate at which accommodative monetary policy "reverses" its effect and becomes contractionary for lending. Below the "reversal interest rate", a decrease in the monetary policy rate depresses rather than stimulates lending and hence the macro-economy.

Importantly, the reversal interest rate is not (necessarily) zero. Hence, unlike what some commentators suggest, negative interest rates are not fundamentally different. In our model, when the reversal interest rate is positive, say 1 %, then already a policy rate cut from 1 % to 0.9 % is contractionary. On the other hand, if the reversal interest rate is -1 %, there is room to go negative up to that point, provided that financial stability is secured.[1]

The exact level of the reversal interest rate depends on macro-prudential policy, especially financial regulation, as well as other parameters of the economic environment and financial sector's balance variables. Restrictive financial regulation in bad times can undermine monetary policy or render it ineffective. Further determinants of the reversal interest rate in our model include banks' equity capitalization, banks' interest rate exposure, and the market structure of the financial sector. In our multi-period extension, the reversal interest rate varies over time: in fact, as the effects on NIMs is negative, while capital gains fade out over time, the reversal interest rate "creeps up". In other words, exceedingly long low interest rate environments can depress lending. Furthermore, quantitative easing increases the reversal interest rate, as it takes fixed-income out of the balance-sheets of the banks. In that sense, QE should only employed after interest rate cuts are exhausted.

How does an interest rate cut by the central bank affect banks' profit, equity and credit

[1]It is no secret that the US Federal Reserve feels that it cannot lower the interest rate below .25 %, since it would otherwise create a run on money market funds. Nevertheless, discussions over negative rates have reached the U.S. as well.

growth? We identify three channels.

First, banks with long-term legacy assets with fixed interest payments benefit from a policy interest rate cut. As the central bank lowers the interest rate, banks can refinance their long-term assets at a cheaper rate. This increases the value of their equity; they are better capitalized, which relaxes their regulatory or economic constraint. Viewed differently, banks' fixed interest rate holdings experience capital gains. Hence, an interest rate cut is essentially a "stealth recapitalization" of the banks, as stressed in Brunnermeier and Sannikov (2012)'s "I Theory of Money".

Second, a lower policy rate negatively affect banks' profits on new business, through lowering banks' net interest margins. In the hypothetical case of a perfectly competitive financial sector without frictions, any monetary policy rate cut is passed through fully to the deposit and loan rate. Lower loan rates then lead to increased credit growth, and the real economy expands. Since profits from margin business are fully competed away – except for an eventual risk premium –, they are always zero and are not affected by rates changes. Hence banks with legacy asset holdings with fixed interest rates unambiguously benefit from an interest rate cut. In the real world, however, financial markets are not perfectly competitive and banks have market power. Formally, in our baseline model we model the banking sector as having three investment opportunities: risky loans, safe bonds, and reserves. Banks' raise deposits to finance these investments, alongside equity. Importantly, we assume that banks' have market power on their ability to grant loans and raise deposits. When a central bank cut the interest rate, the yield on safe assets and reserves goes down. As a consequence, the marginal benefit from raising deposits decreases, which leads the banks' to decrease deposit rates', inducing the desired substitution effect on depositors that the Central Bank seeks. Overall, however, the banking sector is hurt on its deposit business, since the marginal benefit from its investments decreases; the decrease in the deposit rate is only a typical quantity restriction a monopsonist imposes following a decline in the marginal benefit on lending. Furthermore, as yields on safe assets decrease, banks' decrease their lending rates for risky loans in order to substitute their safe assets positions into riskier high-yield ones, an effect which the Central Bank also seeks to induce. This decrease in the lending rate, although optimal, participate in the overall decline of bank profits'.

Third, the change in profits induced by lower policy rates can feedback into lending. In our model, as in reality, the risk-taking ability of the banking sector is constrained by its net worth. If the latter is high enough so that the constraint does not bind, or if capital gains are strong enough to actually increase net worth, then an interest cut generates the boom in lending that the Central Bank seeks to induce. However, if capital gains are too low to compensate the loss in NIMs, net worth decreases to the point where the constraint binds, limiting banks' ability to take on risk. That point is the reversal interest rate, below which further interest cuts generate a decline in lending though the net-worth feedback. Moreover, an interesting amplification mechanism emerges. As the negative wealth effect further tightens banks' equity constraint, banks cut back on their credit extension and are forced to scale up their safe asset holdings. As these assets have lower yields, their profits decline even more, forcing banks' to substitute out of risky loans into safe assets, which in turn lowers their profit, and so on.

We then uncover the determinants of the reversal interest rate in our baseline model. The reversal interest rate depends on banks' assets' interest rate exposure, the tightness of financial regulation, as well as the market structure of the banking sector. If banks hold more long-term bonds and mortgages with fixed interest, the "stealth recapitalization" effect due to an interest rate cut is more pronounced, and the reversal interest rate is lower. Stricter capital requirements rise the reversal interest rate. Lower market power, which decreases profits, also generates a higher reversal interest rate. For example, in a negative interest rate environment, innovations that allow depositors to substitute bank accounts for cash more easily hurt the margins of the banks' and raise the reversal interest rate; if such innovation occurs below the reversal interest rate, it directly feeds back into lower lending.

We then study three extensions of our baseline model, separately. First, we extend the model to multiple periods, to see how changes in the whole interest rate policy path affects current lending activity. Second, we consider the case of multiple "regions" that are differentially affected by an interest cut, owing to the particularities of their respective banking sectors. Third, we introduce competition among banks and study the differential impacts that the rate cut has on different banks.

In the multiple period extension, our analysis shows that the length of the interest rate cut can last longer if banks hold fixed income assets with longer duration. A lower interest rate allows banks to refinance their fixed come assets up to the point when they mature. A longer anticipated interest rate cut only translates into higher reevaluation gains for assets with higher maturity, which can offset the loss resulting from lower NIM profits for more periods in the future. If banks assets are of shorter duration, then a longer interest rate cut might lead to larger NIM profit losses than fixed income capital gains. In that sense, the reversal interest rate "creeps up" over time: an exceedingly long period of low rates may end up lower lending from today onwards, amid feedback effects on the banks' valuation.

In our heterogenous regions extension, the economics are direct extensions of the comparative statics that were developed for the baseline model. An interest cut might be expansionary in one region, and contractionary in another, owing to differential affect on profits. This might be desirable, as the Central Bank might want to stimulate one region and not the other. Importantly, we show that the inclusion of an interbank market – directly between banks or intermediated via the Central Bank – influence the effects of changes in interest rates, because it changes the exposure of the different regions to interest rate cut, in the absence of interest derivatives that hedge the exposure. Regions whose banks borrow in the interbank markets benefit more from a cut than banks who lend on these markets.

In our extension with competition, we show that competitive behavior and variable mark-ups create forces that amplify banks' heterogeneous exposure to an interest rate cut, at the expense of lending. Intuitively, weaker banks are forced to decrease their risky-lending business following an interest rate cut. This strengthens' the market power of stronger banks: although this raises their profits and hence their ability make risky loans, the lack of competition means that this ability is also used to increase mark-ups, which is detrimental for lending.

Finally, we also believe that our model has important implications for the timing and sequencing of Quantitative Easing measures (QE). The optimal sequencing is the following. First, induce banks (possibly through favorable refinancing operations) to hold long-term bonds with a fixed interest rate; second, cut the policy interest rate to generate capital gains for a "stealth recapitalization" of the banking sector; third, conduct QE and lift the long-term assets of banks' balance sheet so that banks realize their capital gains: banks sell their long-term bonds to the central bank in exchange for short-term bonds or reserves at high prices. However, after QE a further interest rate cut is less effective (and might be even counterproductive) since now banks hold mostly short-term reserves. QE undermines the power of future interest rate cuts and increases the "reversal interest rate".

If banks suffer losses, e.g. because of higher delinquency rates in their mortgages, the (endogenous) "reversal interest rate" rises. If it does so beyond the policy rate, a subsequent interest rate cut is contractionary, and QE has used up the "single bullet". Under such circumstances, it might better to raise interest rates, which improve banks' net interest rate margin. Since banks have passed on large parts of their bond holdings to the central bank, the latter suffers the capital losses on these bonds. Only after raising the interest rate and devaluing the long-term bonds, and only after conducting a "Reverse-QE", which replaces banks' reserve holdings with long-term bonds again, is the reversal interest rate restored at a lower level. In a sense this reloads the gun – for a further round of interest rate cuts.

1.1 Literature Review

Our modeling of the microeconomics of banks stands on the shoulders of a large literature which formally started with Klein (1971) and Monti (1972). Santomero (1984) provides a good survey of this early theoretical literature. Empirical work also justify our chosen competitive structure. Sharpe (1997) suggests evidence of switching costs to depositors, and Kim et al. (2003) of relationship costs for banks. We use these costs to justify the imperfect competitive structure of our model âĂŞ both papers offer micro-foundations for these costs. Maudos and Fernandez de Guevara (2004), Saunders and Schumacher (2000), and Drechsler et al. (2015) offer evidence that imperfect competition affects the pass-through of rates. In fact, Petersen and Rajan (1995), in an influential paper, suggested that a monopolistically competitive banking structure better reflects reality, arguing that banks need some monopoly power to sustain their businesses.

A separated strand of literature developed the concept "balance sheet" channel of monetary policy, and the associated bank lending channel, emphasizing the importance of the balance sheet structure and net worth of intermediaries for the transmission of monetary policy (Ben S. Bernanke, 1988; Bernanke and Gertler, 1995; den Heuvel, 2006). In our model, the structure of the balance sheet of banks, and how the net worth of the latter is determined, are also key determinants of the transmission of monetary policy.

Other papers have documented that banks do not perfectly hedge their exposure to interest rate movements. Abad et al. (2016) show that banks on net buy interest rate protection on the derivatives market, although they do not cover themselves fully, and large banks even increase their exposures for intermediation reasons. Begenau et al. (2015) also document that although banks' do participate on the markets for derivatives to hedge their interest rate risk, banks cannot or do not fully hedge their interest rate exposure. Landier et al. (2013a) focus directly on the real lending of banks. They document in a panel study that the income gap – the sensitivity of banks' profits to interest rates – has a causal impact on their lending behavior. Landier et al. (2013b) and Drechsler et al. (2017) offer rationalization for this absence of exposure: banks' valuation going forward increase following a rate hike, hence taking capital loss is an optimal hedging strategy. Our model also relies banks' valuation going forward, and long-lasting low or negative interest rates can be thought as an un-expected event hindering banks' valuation. Finally, English et al. (2012), Van den Heuvel and Ampudia (2017) and Claessens et al. (2017) provide evidence that banks' NIMs and equity valuations substantially vary with the level of interest rates, in a decisively non-linear way.

An empirical literature has studied the pass-through of monetary policy interest rates to banks net worth, and the economy. De Bondt (2005), using European data, shows that the immediate pass-through to lending and deposit rates is at most 50 % at a three-month horizon. Bech and Malkhozov (2016) show that the recent drop in reserve rates below zero transmitted through all risk-free short term assets of the economy, but find that the pass-through seemed imperfect for retail deposit rates. Mortgage rates in their data also showed no response, or even increased in certain countries. Drechsler et al. (2015) focus on the transmission to deposit rates, and show in particular that mark-ups on deposits tend to decrease with the reserve rate. Rognlie (2016)'s work suggest that, although there is no effective zero lower bound on deposit rates, the elasticity of demand changes at zero and sub-zero rates, which affects the pass-through.

Heider et al. (2017) find in Diff-in-Diff settings using syndicated loans that banks with a high deposit base suffered relatively more from the decision of the ECB to implement negative interest rates, relative to low-deposit, wholesale-funded banks. This fed back on their ability to give lending at attractive rates, as our model would predict.

2 A two-period model

In order to highlight the key mechanism that we have in mind, we start by setting up a two-period economy where a monopoly bank – which can be viewed as the banking sector as a whole – has the simplest structure of assets and liabilities that leads rise to a reversal interest rate. We first spell out the structure of the bank's problem, and derive its optimal pricing rules. We then show that below a certain level of policy rate – the rate paid by the central

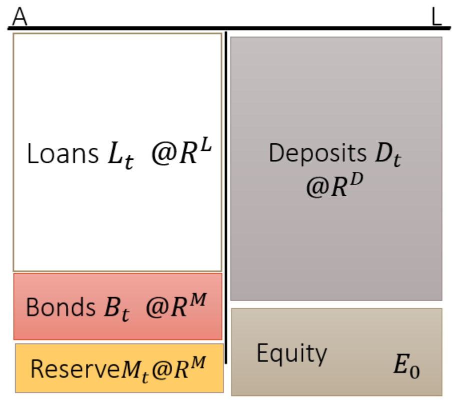

Figure 1: Bank's balance sheet.

bank on excess reserves – further decreases in interest rates decrease total lending. Finally, we identify the key determinants of the reversal interest rate in that simplified setting.

2.1 Setting

A monopoly bank enters the period with its past book, and after the Central Bank unexpectedly changes monetary policy, takes new economic decisions – granting loans and issuing deposits and other debt.

On the asset side of its balance sheet, the bank has essentially three investment opportunities. First, it can grant risky loans L in the economy. Second, the bank can place reserves M at the central bank. Third, it can also purchase safe assets B, e.g. bonds, that yields fixed interest payments.

On the liability side, the bank funds itself by issuing deposits D, and by using its book equity E_0. Its book equity is formed by the past book of the bank: after loans have been repaid and deposits credited, the bank is left with a stock of existing reserves (or cash) holdings M_0 and securities B_0, so that $E_0 = M_0 + p^B B_0$. Figure 1 summarizes the balance sheet of the bank.

Within a period t, the exact timing of events we consider is as follows.

1. The bank enters the period with its "past book", after past loans and deposits have been repaid.
2. The Central Bank unexpectedly may change the policy rate.

3. The bank and other agents choose loan sizes, debt issuances, etc.
4. In the next period, repayments are made and profits are realized.

We now discuss how the quantities and interest rates for each of the balance sheet items are pinned down. We assume for now that inflation is stable and that all rates are also real rates.

Reserves and safe assets

Banks can adjust the amount of reserves M elastically at a rate R^M, paid by the central bank. The bank needs to hold $M \geq \alpha D$ amount of reserves for some $\alpha > 0$, reflecting regulatory requirements or liquidity management; the interest paid on reserves and excess reserves is the same.[2]

Bonds B are also supplied elastically, and generate interest and principal payments R^B after which they mature. These assets can be purchased at a competitive price $p^B = R^B/R^M$.[3]

Loan market and Capital Constraints

The bank – or the banking sector – has a monopoly on the market for loans. The interest it pays to loan seekers is R^L. The total demand for such loans is given by the function $L(R^L)$.

Importantly, we assume that banks are subject to restrictions on the amount of risk they can take. We think of these restrictions as either being directly imposed by regulatory constraints (i.e. Basel regulations[4]) or emerging as the solution to some economic problem – incentives, information friction, or investor risk-aversion.

Concretely, denote γ the weight on risky assets, E_0 the current equity of the bank, Π_1 it's

[2]Note that we do not allow the bank to borrow at the central bank at the rate R^M. Banks have been available to borrow from central banks; however, this type of lending is being offered as emergency lending due to freezes in the inter-banking market, at rates usually entailing a large premium above R^M as well as reputational costs for the borrowing banks. The implications of this assumption are explored when we'll allow interbank lending.

[3]Implicitly, we assume here that the Central Bank is able to affect the yields of all safe assets.

[4]Basel regulations apply constraints to book equity, while we apply it to mark-to-market equity. We hence implicit assume a link between the two, stemming from some decision problem faced by the bank's owners, reflecting incentives to inject equity.

end-of-period profits, and $1/R^E$ the discount factor of the banks' owners. Then

$$\gamma L \leq E_0 + \frac{\Pi_1}{R^E}$$

Deposit market

The bank – or the banking sector – has a monopsony on the deposit market. The interest rate it pays to depositors/bank debt holders is R^D, and the total supply of deposits is given by $D(R^D)$.

Stating the bank's problem

Given the above, the resource constraint of the bank is given by[5]

$$p^B B + M + L = D + E_0$$

When deriving the bank's problem, one faces the difficulty that its profits solve a fixed-point, as profits themselves enter the constraint through leverage. Formally, let $\boldsymbol{\xi}$ stack the parameters that the bank takes as given. First, for any given $\tilde{\Pi}$ one can find:

$$\begin{aligned}\Pi_1(\boldsymbol{\xi}, \tilde{\Pi}) = \max_{M,B,R^L,R^D} \quad & R^L L(R^L) + R^B B + R^M M - R^D D(R^D) - R^E E_0 \\ \text{s.t.} \quad & p^B B + M + L = D + E_0 \\ & \gamma L \leq E_0 + \tilde{\Pi}_1 / R^E \\ & \alpha D \leq M\end{aligned}$$

Then, profits are the maximum such $\tilde{\Pi}$ that solves the fixed-point problem:

$$\Pi_1(\boldsymbol{\xi}) = \max_{\tilde{\Pi}:\ \Pi_1(\boldsymbol{\xi},\tilde{\Pi})=\tilde{\Pi}} \Pi_1(\boldsymbol{\xi}, \tilde{\Pi}))$$

Despite the fixed-point, this problem offers no unknown mathematical difficulties, and hence we omit conditions for existence and uniqueness and simply assume them.

[5] Note that B is recorded in the trading book of the bank.

2.2 Pricing rules and profit margins

In what follows, we restrict the analysis – but not our proofs – to the case where the banking sector holds excess reserves, that is $M \geq \alpha D$ does not bind. This is a good description of the low-interest environment that has followed the 2007-08 financial crisis. In this world, monetary policy works as a floor system: by the elastic supply of reserves, the Central Bank essentially controls the return on safe asset investments.

The effective asset demand that the bank faces is $L(R^L)$ truncated at R^M, given the elasticy supply of safe assets. R^M effectively becomes the opportunity cost of giving out loans, and banks charge a mark-up above it. For deposits, in contrast, R^M is the marginal benefit from raising deposits, and banks apply a mark-down.

Proposition 1 (Interest rates & Pass-through)**.** *Let ε^{*f} denote the elasticity of the function f, evaluated at the optimal pricing rules. The optimal rate on deposits is given by:*

$$R^{*D} = \underbrace{\frac{\varepsilon^{*D}}{\varepsilon^{*D}+1}}_{\textit{Mark-down}} \underbrace{R^M}_{\substack{\textit{Marginal} \\ \textit{benefit}}}$$

While the optimal rate on loans is given by:

$$R^{*L} = \underbrace{\frac{\varepsilon^{*L}}{\varepsilon^{*L}-1}}_{\textit{Mark-up}} \Bigg(\underbrace{R^M}_{\substack{\textit{Marginal} \\ \textit{opportunity cost}}} + \underbrace{\lambda^*}_{\textit{Shadow cost of constraint}} \Bigg)$$

This proposition highlights how the Central Bank affects rates being offered in the economy: essentially, it is because it controls the marginal investment opportunity of the bank – as long as the latter is unconstrained. The pass-through depends on (a) the mark-ups/downs that the bank imposes, and (b) whether the bank is constrained in its behaviour.

A binding leverage constraint decreases the total amount of granted loans, through a higher loan rate.[6] From these pricing rules we immediately get the Net Interest Spreads, $R^{*L} - R^M$ and $R^M - R^{*D}$: holding elasticities constant, these are increasing in R^M. The Net Interest Margins, which are the spreads multiplied by quantities, are the focus of our study: the one for

[6] One could also consider models in which the adjustments takes place at the extensive margin, i.e. in which number of granted loans decreases.

loans, $(R^{*L} - R^M)L(R^{*L})$, is decreasing in R^M, while the one for deposits is increasing in R^M. This is a consequence of the envelope theorem: a decrease in R^M reduces the opportunity cost of granting loans, while it decreases the marginal benefits from deposits. However, overall, the bank is harmed by a decreasing in R^M, simply because the banking sector is a net investor in safe assets – its market power on liabilities is hence consequential in generating that result.

We now derive how profits react to changes in the policy rate R^M. There are essentially two level effects. First, decreasing R^M decrease the returns that banks make on safe asset investments. Hence when banks have a large share of their portfolio invested into safe assets – be it excess returns or bonds – a drop in interest rates decrease this return and hence, in fact, the net interest margin that the bank obtains on its investments. Second, however, decreasing R^M raises the value of the non-floating fixed income legacy asset that the bank held when it entered the period: this is the capital gain channel. Moreover, when it is constrained on its loans, losses from further decreases are even more harmful, as the bank is forced to substitute risky investments for safe assets which have lower returns. The next lemma encodes these results.

Lemma 1. *The derivative of profits with respects to a change in the risk-free rate is:*

$$\frac{d\Pi_1(\boldsymbol{\xi})}{dR^M} = \underbrace{\frac{1}{1-\lambda^*}}_{\text{Leverage amplification}} \left(\underbrace{M^* + p^B B^*}_{\substack{\text{Lower margins} \\ \text{on safe assets}}} - \underbrace{p^B B_0}_{\text{Capital gains}} \right)$$

2.3 The Existence of a reversal interest rate

Does the reversal interest rate always exist? Given our assumptions, a necessary condition for its existence is that B_0 is small enough, or alternatively that M_0 is large enough, holding $E_0 = M_0 + p^B B_0$ constant, so that capital gains do not offset the losses of margins. In particular, existence does not depend on properties of the loan and demand functions, ceteris paribus.[7]

Proposition 2 (The "reversal interest rate")**.** *If B_0 is sufficiently small, then there exists a policy rate R^{RR} at which a reversal occurs in that further decreases to policy rates are contractionary for lending.*

[7]Restricting $M \geq \alpha D$ to not bind isn't necessary for this result.

Formally, there exists R^{RR} such that:

$$R^M < R^{RR} \Leftrightarrow \frac{dL^*(R^M)}{dR^M} > 0$$

Importantly, zero does *not* have to be, generically, the rate at which such reversal occurs – it can be positive as well as negative.

Corollary 1. $R^{RR} - 1 \neq 0$ *generically.*

2.4 Determinants of the reversal interest rate

Having established the conditions for the existence of a reversal interest rate, we now study its determinants in this simple model.

Past book and Quantitative Easing

Our first result suggests an interplay between interest rate policy and other monetary operations such as Quantitative Easing, which change the bond holdings of banks. Before Quantitative Easing, where bank holdings of long term bonds B_0 are high, interest rates cuts are likely to lead to large capital gains and hence rise lending. After QE, however, banks are more exposed to interest rate movements due to the large, positive amount of reserves they hold. This makes them more sensitive to interest cuts – with the risk that a future cut actually hurts lending. In that sense, further rate cuts to be effective needs a "reloading of the gun" by letting bank acquire fixed-income securities whose value intrinsically rise with decreases in risk-free rates.

Proposition 3 (Bonds holdings and returns)**.** *Holding $E_0 = M_0 + p^B B_0$ is held fixed, and replacing B_0 with an equivalent amount of $p^B B_0$ before an interest cut occurs ("QE"), makes R^{RR} increase.*

Capital Constraints

Next we study how the reversal interest rate changes with the structure of capital constraints. Unsurprisingly, R^{RR} is lower when capital constraints are looser or equity buffers are high. Of course, we neglect potential risk-taking effects of decreasing interest rates, which is the basis for a regulatory constraint – see for example Di Tella (2013) or Klimenko et al. (2015). In a theory encompassing both channels, a trade-off would emerge between the two. Moreover, when the leverage costs γ are higher, the negative effects of further drops is more pronounced.

Proposition 4 (Shape of the regulatory constraint)**.** *R^{RR} has the following properties:*

1. *R^{RR} decreases in initial book equity E_0.*
2. *R^{RR} increases if bank investors' are more impatient, that is $1/R^E$ is lower.*
3. *R^{RR} increases in capital requirements γ.*
4. *A higher γ implies that interest rate cuts below R^{RR} depress lending more.*

Note that although increasing equity E_0 reduces the reversal interest rate, this is not necessarily optimal from the point of view of the owners' of the bank.[8]

Elasticities

Our last comparative static involves the properties of the deposit function. It states that if the banks face a more elastic deposit supply function, then R^{RR} increases. This is unsurprising, as it reduces profits and hence the willingness of equity holders to keep investing in the banking business. One can interpret this result as follows: notice from Proposition 1 that ε^{*D} possibly changes as R^M does. One can interpret this change as coming from the possibility of households to save in cash, which offers a nominal rate of zero: although the bank may still charge a mark-down, which would depend on the liquidity services provided by a deposit account, that mark-down would have to be greatly reduced if R^M were to turn negative.[9] In fact, banks who have large institutional investors have been able to pass-through negative rates onto their depositors better than banks whose depositors base consists mostly of small savers.

[8]We have in mind a classic debt overhang problem, through which equity investors are not able to obtain the full return on their investment, which is partially captured by existing debtholders. See Admiti et al. (2016) for a recent discussion of the problem.

[9]R^D would never be larger in our model than R^M, which is what has been observed in some countries who have implemented negative interest rates. One could model this as costs from losing depositors. Moreover, although banks have been reluctant to pass-through negative rates on standard deposit accounts, they have increased the fees associated with the usage of these accounts.

Proposition 5 (Deposit elasticity)**.** *R^{RR} increases in ε^D in the sense that making a deposit supply more elastic ceteris paribus increases the reversal interest rate.*[10]

Hence this result suggests that technologies that allow depositors to substitute their deposits towards cash holdings could raise the reversal interest rate by making the demand supply more elastic at sub-zero rates. It also suggest that banks who rely on large institutional investors are less harmed by a decrease in interest rates than savings' banks that rely heavily on small depositors, as negative rates can be easily passed-on to the former but not the latter, for whom storing cash is less costly.

3 Three-period extension

In this section, we extend the model to a three-period setting. This allows us to study how announcements about a *path* of policy rates impact the business of the bank, in particular net interest margins in the future and their feedback on lending today. Our main result is that the optimal length of interest rate cuts should be related to the maturity of the banks' existing assets. The reason is as follows. A cut in an interest rate in the future has two effects, as in our two-period model: fixed-income assets that pay off in the future gain in value, while NIMs in that period will be depressed. Since the fixed income assets mature over time, the first force slowly fades out, while the loss in margins on future business does not. Hence, the interest rates that maximize lending "creep up" over time.

To make that intuition concrete, consider a bank that enters the period with two assets on its books: a one-period bond (that hence matures next-period) and a two-period bond. Suppose the bank holds more of the former in book value. The Central Bank is considering decreasing the policy rate for two periods. What is the path of interest rates it should choose? As the fixed-income holdings of the first-period are larger than that of the second period, the case for cutting the interest rate is stronger in the first-period, where the capital gains are higher, while the effect on NIMs, discounting apart, is the same. In that sense, the optimal path of (reversal) interest rates is increasing, and an exceedingly long-lasting low interest rate environment might hurt lending.

[10] Mathematically, fix a deposit supply function $D(\cdot)$, and associated reversal interest rate $R^{\mathrm{RR},D}$. Consider another deposit supply function $D'(\cdot)$ such that $D(R^{\mathrm{RR},D}) = D'(R^{\mathrm{RR},D})$, and $\varepsilon^{D'}(R^D) \geq \varepsilon^{D}(R^D)$ for all R^D. Then, $R^{\mathrm{RR},D'} \geq R^{\mathrm{RR},D}$.

3.1 Setting

There are now three periods. The Central Bank controls now a *path* of interest rates $\left\{R^M_{0,1}, R^M_{1,2}\right\}$: the rate between the first two periods, $R^M_{0,1}$, and the rate between the last two periods, $R^M_{1,2}$. A monopoly bank enters the period with its past book, consisting of reserves M_0, one-period bonds $B_{0,1}$ and two-period zero-coupon bonds $B_{0,2}$. Bonds are priced competitively at prices $p^B_{0,1} = R^B/R^M_{0,1}$ and $p^B_{0,2} = (R^B)^2/(R^M_{0,1}R^M_{1,2})$. Hence the bank's equity entering the period is $E_0 = M_0 + p^B_{0,1}B_{0,1} + p^B_{0,2}B_{0,2}$.

Each period, the bank is able to grant loans and deposits, and invest in safe assets. The demand for loans $L(\cdot)$ and supply of deposits $D(\cdot)$ is the same both periods. There are no dividend payments between the two periods: all the earnings are retained and re-injected as equity in the next period.

Denote by Π_2 the profits in the second-period. The problem of maximizing these profits is akin to that of our two-period model, except that equity now also includes the earnings of the bank in the first period. Hence the profits in that period depend the earnings in the first period Π_1, and the risk-free rate in the second-period $R^M_{1,2}$:

$$\Pi_2 = \Pi_2(\Pi_1; R^M_{1,2})$$

We now switch to the first period problem. We assume that the capital constraint is a function of the value of the bank, which hence includes the second period profits too. These profits are discounted by $1/R^E$ as before. Hence, the constraint on the amount of loans L that can be given in the first period is:

$$\gamma L \leq E_0 + \frac{\Pi_1}{R^E} + \frac{\Pi_2}{(R^E)^2}$$

The remainder of the problem of the bank is unchanged compared to our two-period model.

3.2 Analysis

The introduction of a further period generates two changes in the bank's problem. First, profits in the future now influences its ability to lend today. That makes today's problems sensitive to the risk-free rate in the second-period $R^M_{1,2}$. Note that the reverse also holds: low earnings in the first-period might constraint the ability of the bank in the future period too.

Second, the current equity of the bank is now also sensitive to changes in future rates, through the term $p^B_{0,2}B_{0,2}$. Simply, cuts in future rates increases the value of fixed-income assets with long-maturity today already, generating capital gains that relax the capital constraint the bank and hence increase its ability to lend.

More formally, we can derive the changes in the first period's profits as a function of the interest rate today:

$$\frac{d\Pi_1}{dR^M_{0,1}} = \underbrace{\frac{1}{1-\lambda^*(1+\frac{\partial\Pi_2}{\partial\Pi_1})}}_{\text{Leverage amplification}} \Big(\underbrace{M^* + p^B B^*}_{\substack{\text{Lower margins}\\ \text{on safe assets}}} - \underbrace{p^B_{0,1}B_{0,1} + p^B_{0,2}B_{0,2}}_{\text{Capital gains}} \Big)$$

$$\frac{d\Pi_1}{dR^M_{1,2}} = \underbrace{\frac{1}{1-\frac{1}{R^E}\,\lambda^*\,\frac{\partial\Pi_2}{\partial\Pi_1}}}_{\text{Leverage amplification}} \Big(\underbrace{\frac{1}{R^E}\,\lambda^*\,\frac{\partial\Pi_2}{\partial\Pi_1}}_{\text{Lower future margins}} - \underbrace{\frac{p^B_{0,2}}{R^M_{1,2}}B_{0,2}}_{\text{Capital gains}} \Big)$$

From the first expression, we see that the amplification can be larger due to the effect that earnings in the first-period have on future profits, which feeds-back into the capital constraint. The second-expression had no equivalent in the two-period model. It shows that the impact of future rates on profit is the difference between lower future margins and capital gains today, with an amplification term as before.[11]

Given this discussion, we can now discuss the property of the reversal rates. Note that the monetary policy uses two instruments here, $R^M_{0,1}$ and $R^M_{1,2}$. It is important to clarify that we are considering here the maximization of lending in the first period.[12] Clearly, an existence result for a reversal rate $R^{\text{RR}}_{0,1}$ still hold if bonds holdings are low enough, by the same logic as before. More interesting is the fact that the impact of future rates on current profits, and hence lending, are likely to be low. Simply, if the bank has most of its assets maturing in the first period, there are no capital gains following an interest rate cut, while the decrease in Net Interest Margins stays the same. The next proposition encodes this logic by stating the following result: suppose that we re-shuffle the past-book of the bank as to making it more short-term. Then, the reversal rate for short-term rates decreases, while the reversal

[11] The amplification occurs for the same reasons as before: constraining the bank today means that it is forced to hold more safe assets, which makes further cuts more harmful.

[12] Of course, if the bank is constrained both periods, there is not trade-off between lending in the first and second periods, as profits entirely determine banks' lending decisions; however this might not be true when the bank is constrained, as a cut in future periods might benefit lending in the future but not today.

rate increases.

Proposition 6. *Suppose that, maintaining* $E_0 = M_0 + p^B_{0,1} B_{0,1} + p^B_{0,2} B_{0,2}$ *constant, we increase* $B_{0,1}$ *and decrease* $B_{0,2}$*. Then, the first period reversal rate* $R^{RR}_{0,1}$ *decreases while the second period reversal rate* $R^{RR}_{1,2}$ *increases.*

This result easily extends to more than three-periods, with the logic staying the same: if the maturity structure of the banks' is such that their fixed-income holdings decreases with the horizon, then the sequence of reversal rates – which is the sequence of rates that most stimulate lending – increases or "creeps" up with the horizon.

4 Competition, heterogeneous banks and the interbank market

We now go back to our two-period model, but extend our setting along another dimension: we let multiple banks compete with each other. We let market power vary across banks: banks with higher market shares are able to charge higher premia. We show that competitive behavior and variable mark-ups create forces that amplify banks' heterogeneous exposure to an interest rate cut, at the expense of lending. Intuitively, weaker banks are forced to decrease their risky-lending business following an interest rate cut. This strengthens' the market power of stronger banks: although this raises their profits and hence their ability make risky loans, the lack of competition means that this ability is also used to increase mark-ups, which is detrimental for lending. We also study again the implication of an interbank market, and find that "weak banks" are harmed more.

4.1 A simple example

Consider two banks, A and B. Assume that the two banks operate on two segmented markets with similar fundamentals. Banks differ, however, on the composition of their initial capitalization: bank A has more bonds (or outstanding assets) than bank B. As a consequence, following a decrease in the policy rate R^M, bank A enjoys more capital gains than bank B, and its reversal interest rate is lower.

Corollary 2. *Suppose that* $E^A + p^B B^A_0 = E^B + p^B B^B_0$*, but* $B^A_0 \geq B^B_0$*. Then* $R^{RR,A} \leq R^{RR,B}$*.*

Of course, we could have assumed that the two banks differ in any other sense that gives differential reversal rates, as highlighted in our section on the determinants of the reversal rate. For example, savings banks may differ from larger banks because of the composition of their depositors: they face higher elasticity of substitutions on their deposits as rates decrease, since their funding consists mostly of small depositors, while larger banks also borrow from large institutional investors to whom it might be easier to pass on negative rates.

4.1.1 Implications of an interbank market

Suppose that banks have access to a competitive interbank market as above, with an offering rate $R^{\mathcal{I}} = R^M$. Intuitively, this can change the picture because banks with high liquidity needs may start to borrow on the interbank market. Naturally, this reduces their exposure to cuts in interest rates, in the absence of derivatives that would hedge the new exposures created by the interbank market.

Consider again the case where the two banks only differ in that $B_0^A > B_0^B$. Then, if lending demand is sufficiently strong so that bank A is liquidity-constrained on its lending, bank B starts lending to bank A, which may see no reversal as a consequence. Aggregate lending may not decrease, so that an economy-wide reversal interest rate may not exist.

Corollary 3. *The inclusion of an interbank market (weakly) decreases $R^{RR,A}$, with possibly $R^{RR,A} = -\infty$. Aggregate lending may experience no reversal.*

4.2 General Competition

We now extend the model to allow for competition between heterogeneous banks in a fairly arbitrary way. Although this is at the expense of tractability – which motivates us to consider first a monopoly case to highlight our mechanisms – it allows us to analyse new channels that influence the reversal rate.

N banks compete as follows. For risky loans, banks offer slightly differentiated products, so that if $R_i^L, \boldsymbol{R}_{-i}^L$ are the rates offered by bank i and its competitors, the total demand it faces is $L_i(R_i^L; \boldsymbol{R}_{-i}^L)$. We assume that $L_i(\cdot)$ is strictly decreasing in R_i^L and strictly increasing in all other arguments, and assume that the underlying demand system is invertible. Beyond that, we do not make particular assumptions about the structure of competition: instead, we assume that each bank has a conjectural variation function $\boldsymbol{h}_{-i}^L(R_i^L; \boldsymbol{R}_{-i}^L)$ that captures the behaviour of its competitors.[13] Furthermore, we now assume that the capital constraint takes the form now of a smooth cost function, $f\left(\frac{L_i}{E_1};\gamma\right)E_1$, where $E_1 = \Pi_1 + R^E E_0$ is the future value of market equity. A larger γ increases that function. Our previous sharp constraint is hence a limiting case of the analysis undertaken here.

On the deposit side, the market structure is similar: banks face an invertible system of demand supplies $D_i(R_i^D, \boldsymbol{R}_{-i}^D)$, with $D_i(\cdot)$ strictly increasing in R_i^D and strictly decreasing in all other arguments. Competition is captured by a conjectural variation function $\boldsymbol{h}_{-i}^D(R_i^D;\boldsymbol{R}_{-i}^D)$.

The remaining markets are all competitive: safe assets B can be purchased at a price p^B, inter-bank lending and borrowing is done at an interest rate $R^{\mathcal{I}}$, and reserves at the central bank are placed at a rate R^M.

Given this setting, the problem of bank i is now:

$$
\begin{aligned}
\Pi_{1,i}(\boldsymbol{\xi},\tilde{E}_1) = \max_{M_i,\mathcal{I}_i,B_i,R_i^L,\boldsymbol{R}_{-i}^L,R_i^D,\boldsymbol{R}_{-i}^D} \quad & R_i^L L(R_i^L;\boldsymbol{R}_{-i}^L) - f\left(\frac{L_i}{\tilde{E}_1};\gamma\right)\tilde{E}_1 \\
& + R^B B_i + R^M M_i + R^{\mathcal{I}}\mathcal{I}_i - R_i^D D(R_i^D;\boldsymbol{R}_{-i}^D) \\
\text{s.t.} \quad & p^B B_i + L_i + \mathcal{I}_i + M_i = D_i + E_{0,i} \\
& M_i \geq \alpha D_i \\
& \boldsymbol{h}_{-i}^L(R_i^L;\boldsymbol{R}_{-i}^L) = 0 \\
& \boldsymbol{h}_{-i}^D(R_i^D;\boldsymbol{R}_{-i}^D) = 0
\end{aligned}
$$

And the solution to the fixed-point problem is:

$$
\Pi_{1,i}(\boldsymbol{\xi}) = \max_{\tilde{E}:\ \Pi_{1,i}(\boldsymbol{\xi},\tilde{E}_1))=\tilde{E}_1 - R^E E_{0,i}} \Pi_{1,i}(\boldsymbol{\xi},\tilde{E}_1)
$$

And $E_{1,i}(\boldsymbol{\xi}) = \Pi_{1,i} - R^E E_0$.

The next lemma characterizes the new deposit and lending rules.

[13] That is, the loan problem of bank i looks like:

$$
\max_{R_i^L;\boldsymbol{R}_{-i}^L} = R_i^L L_i(R_i^L;\boldsymbol{R}_{-i}^L) - \mathrm{TC}_i(L_i(R_i^L;\boldsymbol{R}_{-i}^L)) \qquad \text{s.t.} \quad \boldsymbol{h}_{-i}^L(R_i^L;\boldsymbol{R}_{-i}^L) = 0
$$

Cournot competition would be captured by $\boldsymbol{h}_{-i}^L(R_i^L;\boldsymbol{R}_{-i}^L) = \boldsymbol{L}_{-i}(\boldsymbol{R}^L) - \bar{L}_{-i}$, while Bertrand competition would be simply $\boldsymbol{h}_{-i}^L(R_i^L;\boldsymbol{R}_{-i}^L) = \boldsymbol{R}_{-i}^L - \bar{R}_{-i}^L$.

Lemma 2 (Interest-rate rules)**.** *The optimal rate on deposits is given by:*

$$R_i^{*D} = \frac{\varepsilon_i^{*D}}{\varepsilon_i^{*D}+1} R^{\mathcal{I}}$$

The optimal rate on loans is given by:

$$R_i^{*L} = \frac{\varepsilon_i^{*L}}{\varepsilon_i^{*L}-1}\Big(R^{\mathcal{I}} + f'\left(L_i(R_i^{*L})/E_{1,i}(\boldsymbol{\xi});\gamma\right)\Big)$$

Where ε_i^{*D} and ε_i^{*L} are now the total elasticities of deposit supply and loan demand faced by bank i.[14]

The elasticities now include the effects of competitors. As before (implicitly), the existence of an interbank lending market implies that $\mathcal{I}_i$ now is the elastic variable that adjusts, and hence guides interest rates charged by banks. We assume that the Central Bank perfectly controls that rate too, so that $R^{\mathcal{I}} = R^M$.

We now compute the *pass-through* of a change in R^M to loan rates charged by banks. This pass-through include the effects on mark-ups in similar way as Amiti, Itskhoki and Konings (2016).

Proposition 7 (Pass-through)**.** *The pass-through of a change in R^M in a change in R_i^L is given by:*

$$\frac{d\log R_i^L}{d\log R^M} = \underbrace{\beta_i\alpha_{ii} + \sum_{j\neq i}\beta_j\alpha_{ij}}_{\textit{Opportunity costs}} + \underbrace{(1-\beta_i)\alpha_{ii}\frac{dL_i^*/E_{1,i}^*}{dR^M}f_i'' + \sum_{j\neq i}(1-\beta_j)\alpha_{ij}\frac{dL_j^*/E_j^*}{dR^M}f_j''}_{\textit{Leverage costs}}$$

[14]That is:

$$\varepsilon_i^{*D} \equiv \frac{\partial\log D_i}{\partial\log R_i^D} + \sum_{j\neq i}\frac{\partial\log D_i}{\partial\log R_j^D}\frac{\partial\log R_j^D}{\partial\log R_i^D}$$
$$\varepsilon_i^{*L} \equiv -\left(\frac{\partial\log L_i}{\partial\log R_i^L} + \sum_{j\neq i}\frac{\partial\log L_i}{\partial\log R_j^L}\frac{\partial\log R_j^L}{\partial\log R_i^L}\right)$$

Where the cross-rates derivatives are given by the implicit functions defined by $\boldsymbol{h}_{-i}^L(R_i^L;\boldsymbol{R}_{-i}^L) = 0$ and $\boldsymbol{h}_{-i}^D(R_i^D;\boldsymbol{R}_{-i}^D) = 0$.

Where the coefficients $\boldsymbol{\alpha}$ are determined by the price-elasticities of mark-ups, and the coefficients $\boldsymbol{\beta}$ are the share of respectively the opportunity cost of lending and the cost of leverage in the marginal cost.[15]

Note that in most models, α_{ij} is positive if $i = j$ and negative otherwise, though note the attenuation effects inside α_{ii}. Hence the first term of the funding cost simply captures the fact that a decreasing in R^M lower the marginal lending opportunity (for a firm long interbank) or the marginal funding cost (for a firm short interbank), while the second term comes from the fact that stronger competition decreases a firm's mark-up and hence forces a stronger rate cut.

Suppose now that $d(L_i^*/E_{1,i}^*)/dR^M \leq 0$, for concreteness, meaning that decreasing the rate leverages the bank. Then, this third (negative) term captures the fact that the bank is forced to set up a higher L_i^* rate to compensate it's higher leverage costs. The fourth term is the opposite: provided $d(L_j^*/E_j^*)/dR^M \leq 0$ too, bank j's deleveraging decreases the competition faced by bank i which responds by raising it's mark-up, hence the positive term.

Of course, the above derivation holds only as a first approximation, but it can be used empirically to evaluate some of the predictions of the model, as in Amiti et al (2016).

4.2.1 A concrete example

Consider again our example with two banks, which now compete in loans (we keep the market for deposits segmented, but it is easy to see how results would extend). Specifically, we assume

[15] That is, denoting $\Gamma_{ij} \equiv \frac{\partial \mu_i^L}{\partial R_j^L}$ we have:

$$\alpha_{ii} = 1 + \Gamma_{ii} + \sum_k \Gamma_{ik}\Gamma_{ki} + \sum_{k,\ell} \Gamma_{ik}\Gamma_{k\ell}\Gamma_{\ell i} + ...$$
$$\alpha_{ij} = \Gamma_{ij} + \sum_k \Gamma_{ik}\Gamma_{kj} + ...$$

And for $\boldsymbol{\beta}$:

$$\beta_i = \frac{R^{\mathcal{I}}}{R^{\mathcal{I}} + f_i'}$$

that banks compete in prices, and that the demand that banks face takes the following form:

$$L_A(R_A^L, R_B^L) = \gamma(R_A^L, R_B^L)\ell(R_A^L)$$
$$L_B(R_A^L, R_B^L) = (1 - \gamma(R_A^L, R_B^L))\ell(R_B^L)$$

This is a discrete-continuous framework, where banks compete to attract consumers at the extensive margin, as captured by the function $\gamma(\cdot)$, and then face the consumers' individual demand function $\ell(\cdot)$, which encodes the intensive margin. Hence the mark-up that banks can charge on loans takes the following form:

$$\mu_j^{*L} = \frac{\varepsilon_j^{*\gamma} + \varepsilon_j^{*\ell}}{\varepsilon_j^{*\gamma} + \varepsilon_j^{*\ell} - 1}$$

For simplicity, let's assume that the intensive margin is fixed, that is $\varepsilon^{*\ell}$ is a constant, while the elasticity at the extensive margin $\varepsilon_j^{*\gamma}$ is increasing in j's rate and decreasing in its competitor's rate, as is usually the case in these frameworks (e.g. the logit case of McFadden (1974)). Then, it is immediate to see that the strong bank's elasticity will be increased by the fact that weak bank is unable to compete. Moreover, as the weak bank gets weaker and weaker, it starts lending to the strong bank, exposing itself even more to further interest rate cuts.

What happens to lending? The existence of an interbank market makes its response ambiguous, as the strong bank is making profits and is able to support lending. However, parts this ability is due to weaker competition, and hence may translate into higher mark-ups instead of higher lending. Total lending, hence, may eventually decrease.

5 Heterogeneous regions

We consider now two regions which each have their own banking sectors, and might have different reversal interest rates owing to differences in their fundamentals. As an immediate consequence of our comparative statics result, a (common) interest rate cut can then have heterogeneous effects across regions: it can be expansionary in a region, while being contractionary in another region. Next, we consider how the inclusion of an interbank market affects the reversal interest rates of the different regions. Our main result is that the existence of an interbank market increases the differential of reversal interest rates between the two regions.

5.1 Using the comparative statics

We note again that given the comparative statics we derived in our first section, it is possible for an interest rate cut to be contractionary in one region while being expansionary in another. Suppose for concreteness that region A has stronger capital constraints than region B, $\gamma_A > \gamma_B$. Then $R_A^{\text{RR}} > R_B^{\text{RR}}$. Hence any interest rate cut while the (common) policy rate lies between the two reversal rates is contractionary in A but expansionary in B.

5.2 Interbank market implications

We now explore the existence of an interbank market between regions. Essentially, the interbank market changes how regions are exposed to interest rate cuts: following a decrease in the policy rate, regions that borrow a lot unambiguously gain, while other regions suffer.

For concreteness, suppose two regions differ in terms of one fundamental: region B has more lending demand than in A, that is $L_B(R^L) = \psi L_A(R^L)$ for some $\psi > 1$. We assume the existence of a competitive interbank market, which can be an explicit market or occur though a central bank. The respective banking sectors in region A and the B can take positions $\mathcal{I}_A, \mathcal{I}_B$ on that market, at a competitive rate $R^{\mathcal{I}}$. Let us assume that the loan disparity is such that for the range of rates R^M of interest the B liquidity constraint $M_B = \alpha D$ binds while in region A it is loose $M_A > \alpha D$ and demand is large enough to accommodate B's demand.

Then, when the interbank market opens, $\mathcal{I}_B < 0$ and $\mathcal{I}_A > 0$, that is the B borrows from the A. The easing of the constraint benefits the B, while the A is left indifferent on impact. Most importantly, the B's exposure to interest rate movement decreases. As a consequence, its reversal interest rate must decreases; potentially, it can become $-\infty$, in the case the B starts to hold short positions, i.e. becomes a net borrower in safe assets.

Corollary 4. *If ψ is large enough, the inclusion of a competitive interbank market makes region B strictly better off. Moreover, the reversal interest rate in region B decreases, possibly to $-\infty$.*

How important is the assumption of a competitive interbank market? Suppose instead that banks bargain in the interbank market: banks in region A know that the B is (liquidity-) leverage constrained, and hence bargains a mark-up such that $R^{\mathcal{I}} > R^M$, yet a rate at which the B finds it still profitable to borrow from. Then, on impact the A is now also better off, and the reversal interest rate lower there; credit growth is smaller in region B, and the reversal interest rate is higher there.

6 Conclusion

The "reversal interest rate" is the rate at which accommodative monetary policy "reverses" its intended effect and becomes contractionary for lending. We have shown that it occurs when recapitalization gains from reevaluation of banks' assets are offset by the future decreases in net interest margins, which lowers banks' net worth and and eventually tightens its capital constraint. Our comparative statics suggest that the reversal rate is higher when (i) banks have fewer asset holdings with fixed (non-floating) interest payments, (ii) when the degree of interest rate pass-through to deposit rates is high, (iii) the capital constraints that they face are tougher. Moreover, cuts in future interest rates beyond the time when fixed interest rate mature do not lead to recapitalization gains but still lowering banks' margins, suggesting a shorter forward guidance policy: the reversal interest rates "creep up". Furthermore, interest rate cuts can have heterogeneous effects across regions where monetary policy operates, being possibly expansionary in one region and contractionary in another. Furthermore, quantitative easing increases the reversal interest rate. QE should only employed after interest rate cut is exhausted.

Does the reversal rate survive in more general settings? The mechanism presented in this paper likely survives in general equilibrium, but it may be only a dampening mechanism, rather than causing an overall reversal rate. Indeed, the rate that depositors face still decreases in our setting, generating the substitution effect that is at the heart of the effects of monetary policy in New Keynesian models. The question then becomes: does that lead banks to capitalize higher profits, and hence possibly increase their lending, undoing the reversal rate? We plan to answer this question in the next iteration of this project, but we foresee the answer to be "it depends". Indeed, it will be important that this increasing demand generate profits that can be captured by the banks, and not other agents in the economy. Moreover, one might suspect that coordination failures could occur, generating multiplicity of equilibria, with one such equilibrium featuring no lending extension and hence mitigated, or even contractionary effects of monetary policy.

References

Abad, J., I. Aldasoro, M. D'errico, L. Fache Rousova, P. Hoffmann, S. Langfield, and T. Roukny (2016). Shedding light on dark markets: first insights from the new eu-wide otc derivatives dataset. *ESRB Occasional Paper Series* (10), 1–32.

Bech, M. and A. Malkhozov (2016). How have central banks implemented negative policy rates? *BIS Quarterly Review*, 31–44.

Begenau, J., M. Piazzesi, and M. Schneider (2015, July). Banks' Risk Exposures. NBER Working Papers 21334, National Bureau of Economic Research.

Ben S. Bernanke, A. S. B. (1988). Credit, money, and aggregate demand. *The American Economic Review 78*(2), 435–439.

Bernanke, B. S. and M. Gertler (1995). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives 9*(4), 27–48.

Brunnermeier, M. K. and Y. Sannikov (2012). The i theory of money. Working Paper, Princeton University.

Claessens, S., N. Coleman, and M. S. Donnelly (2017, February). "Low-For-Long" Interest Rates and Banks' Interest Margins and Profitability : Cross-Country Evidence. International Finance Discussion Papers 1197, Board of Governors of the Federal Reserve System (U.S.).

De Bondt, G. J. (2005). Interest rate pass-through: Empirical results for the euro area. *German Economic Review 6*(1), 37–78.

den Heuvel, S. V. (2006, December). The Bank Capital Channel of Monetary Policy. 2006 Meeting Papers 512, Society for Economic Dynamics.

Di Tella, S. (2013). Uncertainty shocks and balance sheet recessions. Working paper, Stanford University.

Drechsler, I., A. Savov, and P. Schnabl (2015). The deposits channel of monetary policy. Working Paper.

Drechsler, I., A. Savov, and P. Schnabl (2017). Banking on deposits: Maturity transformation without interest rate risk. Working Paper.

English, W. B., S. Van den Heuvel, and E. Zakrajsek (2012). Interest rate risk and bank equity valuations. Working Paper.

Heider, F., F. Saidi, and G. Schepens (2017). Life below zero: Bank lending under negative policy rates. Working Paper.

Kim, M., D. Kliger, and B. Vale (2003). Estimating switching costs: the case of banking. *Journal of Financial Intermediation 12*(1), 25 – 56.

Klein, M. A. (1971). A theory of the banking firm. *Journal of Money, Credit and Banking 3*(2), 205–.

Klimenko, N., S. Pfeil, and J.-C. Rochet (2015). Bank capital and aggregate credit. Working Paper, University of Zrich.

Landier, A., D. Sraer, and D. Thesmar (2013a, February). Banks' Exposure to Interest Rate Risk and The Transmission of Monetary Policy. NBER Working Papers 18857, National Bureau of Economic Research.

Landier, A., D. Sraer, and D. Thesmar (2013b, February). Banks' exposure to interest rate risk and the transmission of monetary policy. Working Paper 18857, National Bureau of Economic Research.

Maudos, J. and J. Fernandez de Guevara (2004, September). Factors explaining the interest margin in the banking sectors of the European Union. *Journal of Banking & Finance 28*(9), 2259–2281.

Monti, M. (1972). Deposit, credit and interest rate determination under alternative bank objective functions. In S. Karl and S. Giorgio P. (Eds.), *Mathematical methods in investment and finance*, pp. 431–454. North-Holland, Amsterdam.

Petersen, M. A. and R. Rajan (1995). The effect of credit market competition on lending relationships. *The Quarterly Journal of Economics 110*(2), 407–443.

Rognlie, M. (2016). What lower bound? monetary policy with negative interest rate. Working Paper.

Santomero, A. M. (1984). Modeling the banking firm: A survey. *Journal of Money, Credit and Banking 16*(4), 576–602.

Saunders, A. and L. Schumacher (2000). The determinants of bank interest rate margins: an international study. *Journal of International Money and Finance 19*(6), 813 – 832.

Sharpe, S. A. (1997). The effect of consumer switching costs on prices: A theory and its application to the bank deposit market. *Review of Industrial Organization 12*(1), 79–94.

Van den Heuvel, S. and M. Ampudia (2017). Monetary policy and bank equity values in a time of low interest rates. Mimeo.