On the Optimal Inflation Rate

by

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Princeton University
Motivation

- What should the (long-run) optimal inflation rate be?

- What role do financial frictions play?
  - Can financial frictions destroy the superneutrality of money?

- Should emerging markets, with less developed financial markets, have a higher inflation rate/target?
## Inflation Target

### Table 4.1. Inflation Targeters

<table>
<thead>
<tr>
<th>Emerging market countries</th>
<th>Inflation Targeting Adoption Date(^1)</th>
<th>Unique Numeric Target = Inflation</th>
<th>Current Inflation Target (percent)</th>
<th>Forecast Process</th>
<th>Publish Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Israel</td>
<td>1997:Q2</td>
<td>Y</td>
<td>1–3</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>1998:Q1</td>
<td>Y</td>
<td>3 (+/-1)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Korea</td>
<td>1998:Q2</td>
<td>Y</td>
<td>2.5–3.5</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Poland</td>
<td>1999:Q1</td>
<td>Y</td>
<td>2.5 (+/-1)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Brazil</td>
<td>1999:Q2</td>
<td>Y</td>
<td>4.5 (+/-2.5)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Chile</td>
<td>1999:Q3</td>
<td>Y</td>
<td>2–4</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Colombia</td>
<td>1999:Q3</td>
<td>Y</td>
<td>5 (+/-0.5)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>South Africa</td>
<td>2000:Q1</td>
<td>Y</td>
<td>3–6</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Thailand</td>
<td>2000:Q2</td>
<td>Y</td>
<td>0–3.5</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Mexico</td>
<td>2001:Q1</td>
<td>Y</td>
<td>3 (+/-1)</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Hungary</td>
<td>2001:Q3</td>
<td>Y</td>
<td>3.5 (+/-1)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Peru</td>
<td>2002:Q1</td>
<td>Y</td>
<td>2.5 (+/-1)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Philippines</td>
<td>2002:Q1</td>
<td>Y</td>
<td>5–6</td>
<td>Y</td>
<td>Y</td>
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<th>Industrial countries</th>
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<td>New Zealand</td>
<td>1990:Q1</td>
<td>Y</td>
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<tr>
<td>Canada</td>
<td>1991:Q1</td>
<td>Y</td>
<td>1–3</td>
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</tr>
<tr>
<td>United Kingdom</td>
<td>1992:Q4</td>
<td>Y</td>
<td>2</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Australia</td>
<td>1993:Q1</td>
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<tr>
<td>Switzerland</td>
<td>2000:Q1</td>
<td>Y</td>
<td>&lt;2</td>
<td>Y</td>
<td>Y</td>
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<td>Iceland</td>
<td>2001:Q1</td>
<td>Y</td>
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Source: IMF, WEO, Sept. 2005
### Literature

- **Money as store of value = bubble**

<table>
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<tr>
<th>Friction</th>
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<td>Risk</td>
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</tr>
<tr>
<td></td>
<td></td>
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<td>With capital</td>
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Risk tied up with individual capital
### Literature

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<th>Angeletos ( q = 1 )</th>
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<tr>
<td></td>
<td></td>
<td>Dynamic inefficiency</td>
<td>capital shock</td>
</tr>
<tr>
<td></td>
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<td>Inefficiency</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r &lt; r^<em>, K &gt; K^</em> )</td>
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- depends on price of capital \( q \)
## Literature

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<td></td>
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<td>Pecuniary externality</td>
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<td></td>
<td></td>
<td>Inefficiency</td>
</tr>
<tr>
<td>f'(k*) = r*</td>
<td></td>
<td>r &gt; r*, K &lt; K*</td>
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<td>r^m = g</td>
<td></td>
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Brunnermeier & Sannikov: Optimal Inflation Rate
Main results

- HH portfolio choice
  - Physical capital: w/ idiosyncratic risk + dividend
  - Money: w/o idiosyncratic risk + no dividend (bubble)
    - Tilted inefficiently towards money

- Money growth ⇒ inflation ⇒ “tax on money”
- ⇒ lowers real interest rate ⇒ tilts portfolio choice
- ⇒ boosts physical investment ⇒ higher economic growth
- ⇒ raises real interest rate (partially undoes inflation tax)

- Pecuniary externality:
  - individual households do not take this GE effect into account.
  - Planner who can print money and distribute seignorage can improve growth + Pareto welfare.

- Derive optimal money growth rate/inflation rate
Model setup

- In each period $j$
  - HH enters with physical capital $k_t$ & nominal money $m_t$
  - Produce output $A k_t \Delta t$
  - Real cash flow shock $z_j = \sigma \varepsilon_j k_j \sqrt{\Delta t}$
  - Transfer from government $\tau w$ (proportional to wealth)
  - Decide
    - Investment rate $\iota$
    - Adjustment cost function $\Phi(\iota) = \frac{1}{\kappa} \log(1 + \kappa \iota)$
  - Portfolio & consumption choice
    - Purchase/sell physical capital $x^k_j = \text{portfolio share}$
    - Consume $c_j$

\[
\max_{\{c_j,k_{j+1},m_{j+1},\iota_j\}} \mathbb{E} \left[ \sum_{j=0}^{\infty} \left( \frac{1}{1 + \rho \Delta t} \right)^j \log c_j \cdot \Delta t \right]
\]
Model setup

- Consumption good is numeraire
- $q$ price of physical capital
  real value of all physical capital $qK_j$
- $p$ real value of all nominal wealth $pK_j$

- $M_j$ aggregate nominal money supply
  - grows at a rate $\mu$
  - Seignorage income is $\frac{\mu \Delta t}{1+\mu \Delta t} pK_j$
- $\varphi_j := \frac{M_j}{pK_j}$ is the price level
Model setup

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- $M_j$ aggregate nominal money supply
  - grows at a rate $\mu$ policy variable of government
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Model setup

- HH’s budget constraint

\[(c_j + \nu_j k_j)\Delta t + qk_{j+1} + \frac{m_{j+1}}{p_j} =
\]

\[Ak_j\Delta t + z_j + q(1 + (\Phi(\nu_j) - \delta)\Delta t)k_j + R_{j-1}m_j \frac{m_j}{p_{j-1}} + \tau w_j\]

- Government’s budget constraint
  - Seignorage income

\[S_j := \frac{M_j - M_{j-1}}{M_j}pK_j = \left(1 - \frac{1}{1 + \mu\Delta t}\right)pK_j = \frac{\mu\Delta t}{1 + \mu\Delta t}pK_j.\]

  - Distribution through transfers \(\tau\)

\[\frac{w_j}{(p + q)K_j}S_j = \frac{p}{p + q} \frac{\mu\Delta t}{1 + \mu\Delta t}w_j =: \tau\]
Optimality conditions

- Optimal investment rate \( \iota^* \)
  \[ q = \frac{1}{\Phi'(\iota^*)} = 1 + \kappa \iota^* \]
  Tobin’s q

- Optimal consumption
  \[ c^* = \frac{\rho}{1+\rho\Delta t} w' \]
  due to log utility
  - Where \( w' = R^k q k + R^m m + \tau w \) wealth just prior to consumption
  - \( R^k = 1 + \left( \frac{A-\iota^* + \Phi(\iota^*) - \delta}{q} \right) \Delta t + \frac{\sigma \varepsilon \sqrt{\Delta t}}{q} \) “capital return”
  - \( R^m = \frac{1 + g \Delta t}{1 + \mu \Delta t} = 1 + \frac{g - \mu}{1 + \mu \Delta t} \Delta t \) “money return”
  - \( R^p(x^k) := x^k R^k + (1 - x^k) R^m + \tau \) “portfolio return”

- Optimal Portfolio
  \[
  \max_{x^k} \frac{1}{1+\rho\Delta t} \alpha_1 E[\log R^p(x^k)]
  \]
  \[
  E[\log R^p(x^k)] = E\left[ \left( R^p(x^k) - 1 \right) - \frac{1}{2} (R^p(x^k) - 1)^2 \right] + o(\Delta t) = 
  \approx \left( \Phi(\iota^*) - \delta - \frac{q}{p+q} \mu + x^k \left( \frac{A - \iota^*}{q} + \mu \right) - \frac{1}{2} \left( x^k \right)^2 \frac{\sigma^2}{q^2} \right) \Delta t
  \]
  \[ x^k = q (A-\iota^*) \frac{\sigma^2}{q^2} + q^2 \mu \frac{\sigma^2}{\sigma^2} \]
Market clearing conditions

- **Goods market**
  - \( AK_j \Delta t = i^* K_j \Delta t + \frac{\rho}{1 + \rho \Delta t} W_j' \Delta t \)
  - \((A - i^*) \Delta t = \rho [\Delta t + (\Phi(i^*) - \delta)(\Delta t)^2](p + q)\)
  - \( A - i^* = \rho (p + q) \) for \( \Delta t \to 0 \)

- **Capital market**
  - \( \frac{x^k W_j}{q} = K_j \Rightarrow q \frac{K_j}{W_j} x^k = \frac{q(A-i^*)}{\sigma^2} + \frac{q^2 \mu}{\sigma^2} \)
  - \( \frac{1}{p+q} = \frac{A-i^*}{\sigma^2} + \frac{q \mu}{\sigma^2} \)

- **Money market**
  - clears by Walras law
Equilibrium

- Collecting the three equations:
  
  \[ q = 1 + \kappa \iota^* \]
  \[ \rho(p + q) = A - \iota^* \]
  \[ \frac{\sigma^2}{q + p} = A - \iota^* + q\mu \]

- Equilibrium solved in terms of \( \hat{\mu} := x^k\mu \) (monotone transformation)
  
  \[ p = \frac{\sigma(1 + \kappa \rho)}{\sqrt{\rho + \hat{\mu}}} - (1 + \kappa A) \]
  \[ q = 1 + \kappa A - \frac{\kappa \rho \sigma}{\sqrt{\rho + \hat{\mu}}} \]
  \[ \iota^* = A - \rho \frac{\sigma}{\sqrt{\rho + \hat{\mu}}} \]

Closed form!
Welfare

- Plug in FOC in value function
- Plug in equilibrium
- All households start symmetrically

- Expected Utility of an individual household

\[
V = V_0 + \frac{1}{\kappa} \log \left( 1 + \kappa A - \frac{\kappa \rho \sigma}{\sqrt{\rho + \hat{\mu}}} \right) - \delta + \rho - \frac{1}{2} (\rho + \hat{\mu}) \frac{\log \left( \frac{\sigma}{\sqrt{\rho + \hat{\mu}}} \right)}{\rho}.
\]

Closed form!
Optimal inflation rate

- Money growth $\mu$ affects (steady state) inflation in two ways
  
  $$\pi = \mu - \left( \Phi(i^*(\mu)) - \delta \right)$$

- Proposition:
  - If $\frac{\sigma}{\sqrt{\rho}} > \frac{2(A\kappa+1)}{1+2\kappa\rho}$, welfare maximizing money growth rate $\mu^* > 0$.
    - Market outcome is not even constrained Pareto efficient
    - Economic growth rate, $g > r^m$, is also higher
  - Growth maximizing $\mu^{g*} \geq \mu^*$, s.t. $p^{g*} = 0$, Tobin (1965)
    $$i^* = A - \rho \frac{\sigma}{\sqrt{\rho} + \mu} \text{ increasing in } \hat{\mu}$$

- Corollary: No super-neutrality of money
  - Nominal money growth rate affects real economy
    - No price/wage rigidity, no monopolistic competition
Proposition: (comparative static)

- $\mu^*$ does not depend on depreciation rate $\delta$, but inflation does.
- $\mu^*$ is strictly increasing in idiosyncratic risk $\sigma$.

“Emerging markets should have higher inflation target.”
Conclusion: our 3 initial questions

- What should the (long-run) optimal inflation rate be?
  - Competitive market outcome is constrained Pareto inefficient.
  - Inflation is Pigouvian & internalizes pecuniary externality!
    - HH take real interest rate as given, but
    - Portfolio choice affects economic growth and real interest rate

- What role do financial frictions play?
  - incomplete markets $\Rightarrow$ no superneutrality of money
    - No price/wage rigidity needed

- Emerging markets, with less developed financial markets, should have higher inflation rate/target
  - Higher idiosyncratic risk $\Rightarrow$ higher pecuniary externality