The Reversal Interest Rate*

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Abstract

The "reversal interest rate" is the rate at which accommodative monetary policy "reverses" its intended effect and becomes contractionary for the economy. It occurs when recapitalization gains from duration mismatch are more than offset by decreases in net interest margins, lowering banks' net worth and tightening its capital constraint. The determinants of the reversal interest rates are (i) banks asset holdings with fixed (non-floating) interest payments, (ii) the strength of the constraints that they face, (iii) the degree of interest rate pass-through to deposit rates, and (iv) the initial capitalization of banks. Furthermore, quantitative easing increases the reversal interest rate and hence should only be employed after interest rate cut is exhausted. Over time the reversal interest rate creeps up, since the capital gains effect fades out as long-term bonds holdings mature while the net interest margin effect does not. Finally, we calibrate a New Keynesian model which embeds our banking model.

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1 Introduction

In most New Keynesian models, the transmission of monetary rates into deposit and lending rate is perfect. The economy enters a liquidity trap as policy rates approach zero only because of an exogenously assumed zero lower bound. This assumption has been questioned, especially since a growing group of central banks – the Bank of Japan, the ECB, the Swiss National Bank, the Swedish Riksbank and Danmarks Nationalbank – have set negative interest rates.

This motivates the broader question: what is the effective lower bound on monetary policy? Given that subzero rates are technically feasible, we argue in this paper that the effective lower bond is given by the "reversal interest rate", the rate at which accommodative monetary policy "reverses" its effect and becomes contractionary for output. Below the "reversal interest rate", a decrease in the monetary policy rate depresses rather than stimulates the economy.

Importantly, the reversal interest rate is not (necessarily) zero. Hence, unlike what some commentators suggest, negative interest rates are not fundamentally different. In our model, when the reversal interest rate is positive, say 1 %, then a policy rate cut from 1% to 0.9% is already contractionary. On the other hand, if the reversal interest rate is -1 %, there is room to go negative up to that point.

The exact level of the reversal interest rate depends on macro-prudential policy, especially financial regulation, as well as other parameters of the economic environment and financial sector's balance variables. Restrictive financial regulation in bad times can undermine monetary policy or render it ineffective. Further determinants of the reversal interest rate in our model include banks' equity capitalization, banks' interest rate exposure, banks' liquidity constraints, the market structure of the financial sector, and the elasticity of deposit rates.

Quantitative easing (QE) increases the reversal interest rate, as it takes fixed income out of the balance sheets of the banks. In that sense, QE should only employed after interest rate cuts are exhausted.

In our multi-period partial equilibrium extension, the path of policy rates that is most stimulatory has rates increasing over time. This is because effects on net interest income (NII) are negative and cumulate every period, while capital gains fade out over time, making the path of optimal rates "creeps up". In other words, exceedingly long low interest rate environments can depress lending in this setting.

Our reversal interest rate existence result continues to hold in general equilibrium. We embed our banking model in an otherwise standard New-Keynesian model and show that a reversal rate still obtains quantitatively, whose magnitude depends on the calibration of the model. With sticky prices an interest cut triggers a demand boost. This accommodating force also raises banks' interest income and hence lowers the reversal interest rate, but not enough to prevent its existence.

In our two-period model, we identify three channels to determine how an interest rate cut by the central bank affect banks' profits, equity and credit growth.

First, banks with long-term legacy assets with fixed interest payments benefit from a policy interest rate cut. As the central bank lowers the interest rate, banks can refinance their long-term assets at a cheaper rate. This increases the value of their equity; they are better capitalized, which relaxes their regulatory or economic constraint. Viewed differently, banks' fixed interest rate holdings experience capital gains. Hence, an interest rate cut is essentially a "stealth recapitalization" of the banks, as stressed in Brunnermeier and Sannikov (2016)'s "I Theory of Money".

Second, a lower policy rate negatively affect banks' profits on new business, through lowering banks' net interest margins. In the hypothetical case of a perfectly competitive financial sector without frictions, any monetary policy rate cut is passed through fully to the deposit and loan rate. Lower loan rates then lead to increased credit growth, and the real economy expands. Since profits from margin business are fully competed away – except for an eventual risk premium –, they are always zero and are not affected by rates changes. Hence banks with legacy asset holdings with fixed interest rates unambiguously benefit from an interest rate cut. In the real world, however, financial markets are not perfectly competitive and banks have market power. In our baseline model the banking sector grants risky loans, holds safe assets (in form of safe bonds and reserves) and issues bank deposits as well as equity. Importantly, we assume that banks' have market power over their ability to grant loans and raise deposits. When a central bank cut the interest rate, the yield on safe assets and reserves goes down. As a consequence, the marginal benefit from raising deposits decreases, which leads the banks' to decrease deposit rates, inducing the desired substitution effect on depositors that the central bank seeks. Overall, however, the banking sector is hurt on its deposit business, since the marginal benefit from its investments decreases; the decrease in the deposit rate is only a typical quantity restriction a monopsonist imposes following a decline in the marginal benefit of lending. Furthermore, as yields on safe assets decrease, banks' decrease their lending rates for risky loans in order to substitute their safe assets positions into riskier high-yield ones, another effect which the central bank seeks to induce. This decrease in the lending rate, although optimal, participates in the overall decline of bank profits.

Third, the change in profits induced by lower policy rates can feed back into lending. In our model, as in reality, the risk-taking ability of the banking sector is constrained by its net worth. If the latter is high enough so that the constraint does not bind, or if capital gains are strong enough to actually increase net worth, then an interest cut generates the boom in lending that the central bank seeks to induce. However, if capital gains are too low to compensate the loss in net interest income, net worth decreases to the point where the constraint binds, limiting banks' ability to take on risk. At that point, i.e. at the reversal interest rate, any further interest cuts generate a decline in lending though the net-worth feedback. Moreover, an interesting amplification mechanism emerges. As the negative wealth effect further tightens banks' equity constraint, banks cut back on their credit extension and are forced to increase their safe asset holdings. As safe assets yield lower returns, banks' profits decline even more, forcing banks to substitute out of risky loans into safe assets, which in turn lowers their profit, and so on.

We then uncover the determinants of the reversal interest rate in our baseline model. The reversal interest rate depends on bank assets interest rate exposure, the tightness of financial regulation, as well as the market structure of the banking sector. If banks hold more long-term bonds and mortgages with fixed interest, the "stealth recapitalization" effect due to an interest rate cut is more pronounced, and the reversal interest rate is lower. Stricter capital requirements rise the reversal interest rate. Lower market power, which decreases profits, also generates a higher reversal interest rate. For example, in a negative interest rate environment, innovations that allow depositors to substitute bank accounts for cash more easily hurt the banks' margins and raise the reversal interest rate; if such innovation occurs below the reversal interest rate, it directly feeds back into lower lending. We also find that our baseline model has important implications for the timing and sequencing of Quantitative Easing measures (QE). The optimal sequencing is the following. First, induce banks (possibly through favorable refinancing operations) to hold long-term bonds with a fixed interest rate; second, cut the policy interest rate to generate capital gains for a "stealth recapitalization" of the banking sector; third, conduct QE and lift the long-term assets of banks' balance sheets so that banks realize their capital gains: banks sell their long-term bonds to the central bank in exchange for short-term bonds or reserves at high prices. However, after QE a further interest rate cut is less effective (and might be even counterproductive) since now banks hold mostly short-term reserves. QE undermines the power of future interest rate cuts and increases the “reversal interest rate”.

In the multiple period extension, our analysis shows that the length of the interest rate cut can last longer if banks hold fixed income assets with longer maturity. Holding policy rates low for longer leads to lower net interest margins for a longer period, but also allows banks to refinance their legacy fixed income assets at a cheaper rate (or equivalently enjoy immediate capital gains appreciation). While the latter effect fades out as long-term bond holdings mature, the former persists. In other words, if bank assets are of shorter duration, then a longer interest rate cut might lead to larger NII profit losses than fixed income capital gains. In that sense, the reversal interest rate “creeps up” over time: an exceedingly long period of low rates may end up lower lending from today onwards, amid feedback effects on the banks’ valuations. This result augments the literature on the forward guidance puzzle opened by the work of Del Negro et al. (2012).

Finally, we embed our banking model in a New Keynesian model and calibrate it in order to study quantitatively the importance of General Equilibrium effects. We find that demand effects indeed work towards supporting banks’ profits and hence their ability to grant loans, but that this effect is mitigated: the pass-through of changes in monetary rates is not one-to-one, but impaired because of the decreased mark-downs as the level of nominal rates drops. This mitigates the strength of the inter-temporal substitution channel, hence generating weaker demand effects. We find in our calibration that the rate at which monetary policy rate cuts reverse its effect on consumption – a general equilibrium reversal interest rate – is around -1%. As emphasized in the conclusion, though, alleviating policies such as ECB’s long-term refinancing operations can help support bank profitability and decrease that number, while other policies such as quantitative easing can help lift up aggregate demand – our main focus is on the interest-rate channel of monetary policy in general equilibrium.

The rest of this paper is organized as follows. First, we provide an overview of the related literature. Second, we present a partial-equilibrium, two-period model where we can explicitly prove the existence of a reversal rate under certain assumptions. We also provide comparative statics and implications of our model for Quantitative Easing as well as the deposit rate pass-through in this section. Third, we extend the model to multiple periods while keeping the analysis in partial equilibrium. This allows us to uncover results the above mentioned creeping up effect. Fourth, we introduce our simple banking model inside an otherwise standard New Keynesian model. We calibrate the model and illustrate the implications of our model for impulse responses of aggregate variables to a monetary shock, illustrating in particular how general equilibrium feedbacks affect our mechanisms in a world with and without price stickiness. The last section concludes.

1.1 Literature Review

Our modeling of the microeconomics of banks stands on the shoulders of a large literature which formally started with Klein (1971) and Monti (1972). Santomero (1984) provides a good survey of this early theoretical literature. Empirical work also justify our chosen competitive structure. Sharpe (1997) suggests evidence of switching costs for depositors, and Kim et al. (2003) of relationship costs for banks. We use these costs to justify the imperfect competitive structure of our model – both papers offer micro-foundations for these costs. Maudos and Fernandez de Guevara (2004), Saunders and Schumacher (2000), and Drechsler et al. (2017b) offer evidence that imperfect competition affects the pass-through of rates. In fact, Petersen and Rajan (1995), in an influential paper, suggest that a monopolistically competitive banking structure better reflects reality, arguing that banks need some monopoly power to sustain their businesses.

A separated strand of literature developed the concept "balance sheet" channel of monetary policy, and the associated bank lending channel, emphasizing the importance of the balance sheet structure and net worth of intermediaries for the transmission of monetary policy (Bernanke and Blinder, 1988; Bernanke and Gertler, 1995; Van den Heuvel, 2007). In our model, the structure of the balance sheet of banks, and how the net worth of the latter is determined, are also key determinants of the transmission of monetary policy.

Other papers have documented that banks do not perfectly hedge their exposure to interest rate movements. Abad et al. (2016) show that banks on net buy interest rate protection on the derivatives market, although they do not cover themselves fully, and large banks even increase their exposures due to their intermediation business. Begenau et al. (2015) also document that although banks do participate on the markets for derivatives to hedge their interest rate risk, banks cannot or do not fully hedge their interest rate exposure. Landier et al. (2013a) focus directly on the real lending of banks. They document in a panel study that the income gap – the sensitivity of banks' profits to interest rates – has a causal impact on their lending behavior. Landier et al. (2013b) offer a rationalization for this absence of exposure *in expectation*: banks' valuations going forward increase following a rate hike, hence taking capital loss is an optimal hedging strategy. Drechsler et al. (2017a) document that in both long-term aggregate series and the cross section of banks, *realized* net interest income as well as return on assets varies little with the level of policy rates. As in our model, this happens because capital gains from maturity transmformation are offset by changes in market power. However, these hedging strategies only works in expectation; upon interest rate realizations the mark-to-market valuation of banks will fluctuate, and the recent long-lasting low/negative interest rate environment was unexpectedly long, hence damaging bank valuation significantly. At the micro level, English et al. (2012), Ampudia and Van den Heuvel (2017), Eisenschmidt and Smets (2018), and Claessens et al. (2017) provide evidence that banks' NIIs and equity valuations substantially vary with the level of interest rates, in a decisively non-linear way. For example, Claessens et al. (2017) find that a one percentage point interest rate drop implies an 8 basis points lower net interest margin, with this effect greater (20 basis points) at low rates. This effect carries through bank profitability, and moreover, for each additional year of low rates, margins and profitability fall further. Finally, Altavilla et al. (2017) find that the banks' return on assets in the recent negative interest rate environment in Europe has not fluctuated for too long; as they document, the effect on net interest income was largely negative, but other factors helped lift up bank profitability, in particular because other policies were in place in Europe at the same time.

A related empirical literature has studied the pass-through of monetary policy interest rates to banks' net worth, and the economy. De Bondt (2005), using European data, shows that the immediate pass-through to lending and deposit rates is at most 50 % at a three-month horizon. Bech and Malkhozov (2016) show that the recent drop in reserve rates below zero transmitted through all risk-free short term assets of the economy, but find that the pass-through seemed imperfect for retail deposit rates. Mortgage rates in their data also showed no response, or even increased in certain countries. Drechsler et al. (2017b) focus on the transmission to deposit rates, and show in particular that mark-ups on deposits tend to decrease with the reserve rate. Rognlie (2016)'s work suggest that, although there is no effective zero lower bound on deposit rates, the elasticity of demand changes at zero and sub-zero rates, which affects the pass-through.

Heider et al. (2017) employ a difference-in-difference analysis using syndicated loans data to document that banks with a high deposit base suffered relatively more from the decision of the European Central Bank to implement negative interest rates than low-deposit, wholesale-funded banks. Eggertsson et al. (2017) document a collapse of the pass-through of monetary policy rate when the latter hits sub-zero territory, because of cash competition at the intensive margin.

2 Two-Period Partial Equilibrium Setup

In this section, we analyze a two-period banking model in partial equilibrium. That is, we hold prices and quantities that are not directly determined by banks' decisions fixed.

Each of a continuum of (ex-ante) identical banks are initially endowed with equity funding of E_0. In addition, banks can raise liabilities D in form of bank deposits. On the asset side of the balance sheet, banks have two investment opportunities: loans L to firms and safe assets S. Banks will compete on the market for deposits from households and loans to firms, while they take the return on safe assets as well as their initial equity as given. Figure 1 displays a stylized balance sheet of a bank.

2.1 Timing of events

Let i_0 be the interest rate between time 0 and 1 that banks expected with probability 1. We consider an experiment in which the central bank unexpectedly changes the level of the interest rate to a new level i. This surprise yields capital gains for banks. Having realized these gains, banks then set new interest rates and quantities for loans and deposits with the objective to maximize their period 1 net worth.

2.2 Bank Assets

Safe Assets. Safe assets S are available in perfectly elastic supply. After the surprise policy interest rate cut/hike they yield a nominal return equal to the new policy rate i in period 1. There can be many such safe assets (i.e. bonds, reserves, cash etc.). By no arbitrage, the yields of all these safe assets equal i. Hence, we only model the total quantity S of safe assets yielding i.

Bank loans. Each bank grants loans L to a unit measure of its customer-firms. We assume for simplicity that informational friction lock-in firms in house-bank relations. The loan demand that bank j faces is denoted by $L(i_j^L)$, where i_j^L is the nominal rate on bank loans that bank j offers. In our partial equilibrium analysis we consider L as a function of i_j^L only, since banks control the nominal loan interest rate. In our general equilibrium section, we micro-found firms' loan demand and make them dependent on other prices and quantities in the economy.

2.3 Bank Liabilities

DEPOSITS. Each bank is naturally associated with a continuum of depositor households. Their deposit supply is sticky in the sense that the depositors shop around for better deposit rates, i^D, if the spread between i and the rate i_j^D from their associated bank j is larger than some threshold, the "activation level", $\eta^D(i)$. Hence, banks compete on prices, but only if the spread they charge relative to some baseline rate i is large enough.[1]

Importantly, we assume that the search "activation level" $\eta^D(i)$ is decreasing in the interest rate i. In other words, depositors become more sensitive to spreads when the policy rate is low. This generates pressure on the extensive margin of banks' deposit margins as rates decrease. For example, depositors are more prone to switch the bank if the interest rate is negative, as empirically documented in Hainz et al. (2017). In addition, banks deposit rate choices are also driven by intensive margin considerations. That is, conditional on keeping a customer, the bank might decide to offer an attractive interest rate to ensure that the customer supplies a sufficient amount of deposits instead of simply consuming his income or substituting to alternative savings vehicles, like cash.

Concretely, each depositor household $h \in [0, 1]$ in the continuum associated with bank j has an activation level $\eta^D(i)$. He only considers looking at the rates offered by competing bank j' if the rate offered by his bank, i_j^D, is below $i - \eta^D(i)$. We assume that $\partial\eta^D(i)/\partial i \geq 0$, that is, the activation level is increasing in the level of interest rate, so that spreads are less tolerated at low levels of the policy rate.

Let us denote $\boldsymbol{i}_{-j}^D \equiv \{i_{j'}^D\}_{j' \neq j}$ to be the vector of competitors' deposit rates. Hence, the share of costumers ϑ_j^D that actually stay with bank j is:

$$\vartheta_j^D(i_j^D; \boldsymbol{i}_{-j}^D, i) \equiv \mathbf{1}_{\{i - i_j^D \leq \eta^D(i) \,\vee\, i_j^D > \max_{j' \neq j} i_{j'}^D\}}$$

We can then decompose the residual deposit supply faced by bank j as consisting of an extensive and an intensive margin:

$$D_j(i_j^D; \boldsymbol{i}_{-j}^D, i) = \underbrace{\vartheta_j^D(i_j^D; \boldsymbol{i}_{-j}^D, i)}_{\text{extensive margin}} \times \underbrace{d(i_j^D)}_{\text{intensive margin}}$$

[1] Varying mark-downs at the extensive margin can be modeled in numerous ways. A large literature focuses on on switching costs (see e.g. Klemperer (1995) for a review), which is sometimes applied to banking (as eg. in Sharpe (1997)). Our goal here is to have a realistic yet parsimoniously parametrized model that easily fit in a large-scale New Keynesian model.

In our calibration, the extensive margin will turn out to be the key driver of that residual elasticity.

EQUITY. Let $E_0(i_0)$ be the banks' book equity before the surprise policy rate change. We assume that banks' book equity after the surprise change, $E_0(i)$, is a function of the nominal policy interest rate i. This captures the fact that banks might make unexpected capital gains on the portfolio with which it enters the initial period, after monetary policy changes its stance. Formally, we decompose the equity after the monetary policy shock into $E_0(i) = \bar{e}_0 + e_0(i)$, where $\bar{e}_0$ is the interest-insensitive component of initial equity and $e_0(i)$ the interest rate sensitive part. We assume throughout the paper that banks' interest exposure is such that $\partial E_0(i)/\partial i = \partial e_0(i)/\partial i < 0$. Banks' duration risk will lead to capital gains that are key determinants for the reversal interest rate.[2]

2.4 Financial Frictions

Banks face two forms of financial frictions. First, banks are subject to a capital constraint of the form:

$$\psi^L L + \psi^S S \leq N_1$$

where N_1 denotes the bank's (nominal) net worth (defined below). That is, a weighted average of book assets must be covered by the value of the bank. We set the risk coefficient on safe asset equal to $\psi^S = 0$. Two remarks are in order. First, the capital constraint could be economic reasons or regulatory reasons. For simplicity we refrain from a full micro-foundation. Second, one could replace N_1 in the constraint by book equity E_0, especially if the constraint is motivated by regulatory purposes. Note that E_0 does enter the constraint, indirectly through N_1, since ceteris paribus a larger E_0 leads to higher net worth. Moreover, this assumption is essentially inconsequential in our dynamic setting where net worth is persistent.

[2]As a clarifying example, suppose that the portfolio of a bank at time -1 consisted on the asset side of one-period loans $L_{-1,0}$, one-period safe assets $S_{-1,0}$, two-period (liquid) safe assets $S_{-1,1}$ (with fixed interest rate maturing at $t = 1$), and on the liability side, one-period deposits $D_{-1,0}$. Upon entering period 0, all one-period safe assets and liabilities mature, and the two period assets generate interest income and expense, yielding total earnings $\mathcal{E}_0$. A share δ^{div} of this $\mathcal{E}_0$ is paid out as dividend. Suppose the bank keeps its long-term assets. The equity of the bank then is $E_0(i) = (1 - \delta^{\text{div}})\mathcal{E}_0 + p^S(i)S_{-1,1}$, where i is the interest rate between time 0 and 1 and $p^S(i)$ is the price of the two-period bond after its time 0 coupon is paid. Clearly, if there's a *surprise* interest rate cut (hike) to i at time 0, the maturity mismatch generated by the long-term bond will generate capital gains (losses) as $\partial p^S(i)/\partial i < 0$.

Second, banks face a liquidity constraint of the form:

$$S \geq \psi^D D$$

That is, each bank's safe asset holding must cover a certain fraction of deposits. Such a constraint could be micro-founded by ensuring that banks have sufficient funds to avoid run risk.

2.5 Banks' problem

Finally, let $L + S = D + E_0(i)$ be the balance sheet identity of the bank. Then, we can write its problem as:

$$\begin{aligned}
\max_{i^L, i^D, L, D, S, N_1} \quad & N_1 = (1 + i^L)L + (1 + i)S - (1 + i^D)D \\
L + S &= D + E_0(i) \\
\psi^D D &\leq S \\
\psi^L L &\leq N_1 \\
L &= L_j(i_j^L) \\
D &= D_j(i_j^D; \boldsymbol{i}_{-j}^D, i)
\end{aligned}$$

This problem offers no particular mathematical difficulties, and hence we omit conditions for existence and uniqueness of a symmetric equilibrium. In our micro-foundations of $D(\cdot), L(\cdot)$ in later sections, existence and uniqueness are straightforward to show.

3 Partial Equilibrium Reversal Interest Rate

3.1 The reversal interest rate: Definition

We now explicitly define the "reversal interest rate" as the rate such that a decrease in the nominal policy rate, i, stimulates lending if and only if the current level of the interest rate, i, is above the reversal interest rate i^{RR}.

Definition 1 (Reversal interest rate)**.** *Let i^{RR} define the "reversal interest rate" such that*

1. *$i > i^{RR}$ implies $\frac{dL^*}{di} < 0$;*

2. $i = i^{RR}$ *implies* $\frac{dL^*}{di} = 0$;

3. $i < i^{RR}$ *implies* $\frac{dL^*}{di} > 0$.

In what follows, we first derive the bank's optimal setting rules. We then spell out sufficient conditions under which a reversal interest rate obtains.

3.2 Banks' rate setting rules

Monetary policy affects the marginal investment opportunity of banks. Given that a bank can earn a return of i from holding a safe asset it considers the loan demand function only for loan interest rates above i. Furthermore, i encodes the opportunity cost of granting loans, and banks charges a mark-up above it. Similarly, for deposits it applies a mark-down.

The constraints limit this optimal behavior. When the capital constraint binds, banks charge higher than desired lending rates in order to decrease their leverage. Similarly, when the liquidity constraint binds, banks offer a higher than desired lending and deposit rates in order to bring its liquidity ratio up.

Let ε^{f*} denote the semi-elasticity of the function f with respect to the relevant rate, evaluated at the optimal pricing rules.[3] The next lemma formally encodes these results.

Lemma 1 (Rate setting rules)**.** *The optimal rate on loans is given by*

$$i^{L*} = \underbrace{i}_{\substack{\text{Marginal}\\ \text{opportunity cost}}} + \underbrace{\frac{1}{\varepsilon^{L*}}}_{\text{Mark-up}} + \underbrace{\frac{\psi^L}{1+\psi^L}\lambda^{L*}}_{\text{capital constraint}} . \tag{3.1}$$

The optimal rate on deposits is given by

$$i^{D*} = \underbrace{i}_{\substack{\text{Marginal}\\ \text{benefit}}} - \underbrace{\min\left(\eta^D(i), \frac{1}{\varepsilon^{D*}} - \frac{\psi^D}{1+\psi^D}\lambda^{D*}\right)}_{\text{Mark-down}} \tag{3.2}$$

When constraints are slack the Lagrangian multipliers are simply zero; when they do bind, the Lagrange multipliers are defined by the FOCs and actual rates are given by the

[3] That is, $\varepsilon^{L*} = \frac{\partial \log L(i^L)}{\partial i^L}\Big|_{i^L = i^{L*}}$. Although mathematically these are semi-elasticities, economically they are elasticities, since the units of i^L, i^D are percentage points. An alternative is to work with log gross rates and pure elasticities, in which case the log-mark-up takes the common form $\log\left(\frac{\varepsilon^{L*}}{\varepsilon^{L*}-1}\right)$.

constraints themselves.

3.3 Existence of i^{RR}

We now show how the constraints lead to a reversal of the bank lending channel. Remember that the capital constraint depends on how profitable the bank is. The next lemma shows that profits of banks have two components: net interest income (NII) and capital gains (CG). NII is defined as

$$NII = \underbrace{i^{L*} L^* + i\, S^*}_{\text{interest income}} - \underbrace{i^{D*} D^*}_{\text{interest expenses}}$$

Capital gains, on the other hand, are simply the change in initial equity (retained earnings) created by the surprise change in interest rates:

$$\text{CG} = E_0(i) - E_0(i_0)$$

The following lemma also shows that when there are no capital gains, that is CG $= 0$, then the change in profits following a cut in interest rate is strictly negative. We focus on the case in which the liquidity constraint does not bind.

Lemma 2 (Profit response)**.** *Assume the liquidity constraint does not bind. The change in profits following a change in i is then given by:*

$$\frac{dN_1^*}{di} = \underbrace{(1 + \lambda^{L*})}_{amplification} \Big(\underbrace{\frac{dNII}{di}}_{NII} + \underbrace{(1+i)\frac{dE_0(i)}{di}}_{CG} \Big) \tag{3.3}$$

Moreover, if $\frac{dE_0(i)}{di} = 0$, then $\frac{dN_1^*}{di} > 0$.

This result is intuitive. An interest cut depresses the return on new investments in safe assets. Because safe assets are always held by banks (whether the liquidity constraint binds or not), this decreases net interest income and hence profits. That this is sufficient is a consequence of the envelope theorem – the fact that the first-order conditions described above apply. However, an interest rate cut also leads to capital gains, which boosts profits. Without such gains, profits unambiguously decrease following an interest rate cut.

Note, moreover, that an amplification occurs when the bank's capital constraint binds (implying that $\lambda^{L*} > 0$). From that point onwards the bank is forced to partly re-route its loan investment into safe asset investment. Ceteris paribus, that makes further cuts more harmful to banks' profits.

Returning to our main result, when the capital gains are sufficiently small – that is the change in $E_0(i)$ is small enough – then the NII channel dominates. Hence profits decrease with a decline in i. Moreover, as long as the capital constraint does not bind, $di^{L*}/di > 0$, so that an interest rate cut lowers the loan interest rate leading to more loans. Both forces tightens the constraint. Eventually, the constraint inevitably binds: at that point the policy rate hits the reversal interest rate, because any further decrease in i will decrease profits which through the constraint must decrease L^*, so that dL^*/di flips sign.

Proposition 1. *(Existence of i^{RR}) When capital gains $E_0'(i)$ are uniformly bounded from below and either (1) capital constraints are sufficiently tight or (2) initial equity is sufficiently high, there exists a finite reversal interest rate i^{RR}.*

Furthermore, nothing guarantees the reversal interest rate to be zero, or any particular number.

Corollary 1. *Generically $i^{RR} \neq 0$.*

A NUMERICAL EXAMPLE. Figure 2 displays a numerical example of a reversal rate triggered by a binding capital constraint for a baseline interest rate of $i_0 = 1.5\%$. An interest rate cut lowers banks' net worth (lower left panel), as the decline in net interest income exceeds the increase in capital gains (lower right panel). The capital constraint tightens, until it inevitably binds. At this stage, i falls below the reversal interest rate and a further decrease in the policy rate lowers loan volume (top left panel). Interestingly, the loan interest rate then rises below that interest rate (top right panel).

3.4 Comparative statics results

We now derive comparative statics result. Unsurprisingly, the reversal rate is lower when capital constraints are looser or initial equity is low. This is consistent with Corbae and D'Erasmo (2014), who find in a structurally estimated banking model that an increase in capital requirements leads to a decline in aggregate loan supply and an increase in the loan interest rate.[4,5] Our third result in the next proposition states that, everything else equal, higher capital constraints on leverage makes subsequent interest rate cuts below the reversal rate even more harmful for lending.

Proposition 2 (Capital constraint and Equity)**.** *The reversal interest rate i^{RR} has the following properties:*

1. *The reversal interest rate i^{RR} increases in the risk-weight of the capital constraint ψ^L.*
2. *The reversal interest rate i^{RR} decreases in the interest rate sensitivity of the initial equity $\partial e_0(i)/\partial i$ (keeping $E_0(i_0)$ constant).*
3. *An interest rate cut below the reversal is more detrimental for lending for an economy with a tighter capital constraint. Specifically, consider two economies A, B with the same reversal interest rate $i_A^{RR} = i_B^{RR}$ and that are identical in all expects except that $\bar{e}_{0,A} < \bar{e}_{0,B}$ and $\psi_A^L < \psi_B^L$. Then, for any $i < i_A^{RR} = i_B^{RR}$, $L_A^*(i) > L_B^*(i)$.*

Arguably, one of the most striking features of our reversal result is that it does not rely on stickiness of the deposit rate. Deposit rate stickiness on its own can generate a reversal due to the binding of the liquidity constraint. Intuitively, once the constraint binds, safe assets simply become a burden that banks must hold in order to stay sufficiently liquid. Decreasing the return on holding safe assets then decreases lending. Hence, in order to generate a reversal, we only need assumptions guaranteeing that the liquidity constraint eventually binds as i decreases. A lower bound on deposit rate – at the extensive or intensive margin – does just that: as i goes below it, the liquidity constraint must bind. Proposition **??** shows that an increase in this lower bound, and more generally a decrease in mark-downs, increase the reversal interest rate.

Proposition 3 (Liquidity constraint and deposit rate pass-through)**.** *The reversal interest rate i^{RR} has the following properties:*

1. *The reversal interest rate i^{RR} increases in the tightness of the liquidity constraint ψ^D.*
2. *The reversal interest rate i^{RR} increases with competition at the extensive margin $\eta^D(i)$.*

[4]It is important to keep in mind that we neglect potential risk-taking effects of decreasing interest rates, which might be the basis for a constraint – see for example Di Tella (2013) or Klimenko et al. (2015). In a theory encompassing both channels, a trade-off would emerge between the two; we are only modeling one-side of a trade-off, and hence our results are unambiguous here.

[5]Note that for both Propositions 2 and 3 below the statements hold only weakly; however for some reasonable constellations of parameters they hold strictly.

3. *For $\underline{i}^D$, such that $d(i^D) = 0$ for all $i^D \leq \underline{i}^D$, where $d(\cdot)$ is the intensive margin of the deposit supply from depositors, i^{RR} is increasing in $\underline{i}^D$. In words, the reversal rate is increasing with in the lower bound on deposits.*

3.5 Optimal sequencing of QE

Our model also implies an optimal sequencing of interest rate policy and other monetary operations such as Quantitative Easing (QE). QE changes the bond holdings of the banking sector, and hence their interest rate risk exposure. QE reduces the banks' holdings of long-term bonds and hence after QE banks' interest rate sensitivity of their equity $\partial e(i)/\partial i$ is reduced. This increases the reversal interest rate.

Proposition 4 (QE and capital gains)**.** *Quantitative Easing which lowers the interest rate sensitivity of banks' initial equity $\partial e(i)/\partial i$ while leaving overall level of $E_0(i_0)$ unchanged, lowers potential capital gains from a subsequent interest rate cut and hence increases the reversal interest rate i^{RR}.*

The optimal sequence of stimulating monetary policy is to cut the interest rate all the way towards the reversal interest rate before conducting QE measures.

We emphasize that this is a partial equilibrium results – in general equilibrium, other forces might pull towards an alternative sequencing.[6]

3.6 Additional results on deposit pass-through

Our results so far concern only the bank lending channel. In New Keynesian models, however, monetary policy operates also through the intertemporal substitution channel. We show that our simple banking model has important implications for this channel too. In particular, we show that deposit flight – from the extensive or intensive margin – decreases the pass-through of monetary policy into deposit rates, hence weakening the effects of an interest rate cut.[7] This is true even without the liquidity constraint binding. Specifically, if semi-elasticity of deposit supply is decreasing with the level of the deposit rate, then the pass-through is less than 1, meaning that one basis point decrease in the gross rate results in a less than one basis point decrease in the gross deposit rate. If, in contrast, the elasticity were constant, then

[6]In particular, the sequencing describes above stealthy recapitalizes banks, meaning that another sectors – the government, households, foreigners, or others – could lose and hence distort the economy in undesirable ways too.

[7]Note that the same can be true for the loan rates.

the pass-through would be exactly 1; the last case is easy to guess. Note that the change in the elasticity of *residual* deposit supply comes from either the extensive or the intensive margin.

Proposition 5 (Deposit pass-through)**.** *Suppose that the liquidity constraint does not bind. If the mark-down on deposits is given by the extensive margin parameter $\eta^D(i)$, then:*

$$\frac{di^{D*}}{di} = 1 - \frac{\partial \eta^D(i)}{\partial i}$$

Hence if competition gets tougher as rates decrease, that is $\partial\eta^D(i)/\partial i > 0$, then the pass-through of monetary policy rates into deposit rates is less than one-for-one, i.e. $\frac{di^{D*}}{di} < 1$.

If instead the mark-down is given by the intensive margin $1/\varepsilon^{D*}$, then:

$$\frac{di^{D*}}{di} = \frac{1}{1 + \frac{\partial(1/\varepsilon^D)}{\partial i^D}}$$

4 The "creeping-up" effect

In this section, we extend the model to a three-period setting. This allows us to study how announcements about a *path* of policy rates impact the business of the bank, in particular net interest margins in the future and their feedback on lending today. Our main result is that the optimal length of interest rate cuts should be related to the maturity of the banks' existing assets. The reason is as follows. as in the two-period model, a cut in an interest rate in the future has two effects: fixed-income assets experience capital gains, while net interest income will be depressed. Since the fixed income assets mature over time, the first force slowly fades out, whereas the loss in margins on future business does not. Hence, the interest rates that maximize lending "creep up" over time.

To make that intuition concrete, we consider banks in a similar setting than our two-period model, except that banks enter the period with two assets on their books: a one-period bond (that matures next period) and a two-period bond. Moreover, their equity in the second-period is endogeneous and depends on profits that banks make in the first period. We then ask: what is the path of interest rates that maximizes banks' loan supply? Since fixed-income holdings of the first period are larger than that of the second period, the case for cutting the interest rate is stronger in the first period, where the capital gains are higher, while the effect on net interest income is similar across both periods. In that sense, the optimal path of (reversal) interest rates is increasing, and an exceedingly long-lasting low interest rate environment might hurt lending.

4.1 Three-period model extension

In our three-period model the monetary authority controls a *path* of one-period interest rates $\{i_{0,1}, i_{1,2}\}$: the rate between the first two periods, $i_{0,1}$, and the rate between the last two periods, $i_{1,2}$. Banks enter the period with their past book, consisting of equity $\bar{e}_0$, one-period bonds $B_{0,1}$ and two-period zero-coupon bonds $B_{0,2}$.[8] Bonds are priced competitively at prices $p^B_{0,1} = \frac{1}{1+i_{0,1}}$ and $p^B_{0,2} = \frac{1}{(1+i_{0,1})(1+i_{1,2})}$. Hence the bank's equity entering the period is $E_0(i_{0,1}, i_{1,2}) = \bar{e}_0 + p^B_{0,1} B_{0,1} + p^B_{0,2} B_{0,2}$: when rates and therefore price changes, so does equity entering the new period.

Each period, the bank is able to grant loans and deposits, and invest in safe assets. The demand for loans $L(\cdot)$ and supply of deposits $D(\cdot)$ is the same both periods. We also assume that before the policy experiment, the interest rates are equal in both periods, denoted by i^*. Hence, the two periods are identical in every aspect to repeating our two-period model twice, except that we have one- and two-period bonds and that the equity level in the second period is now endogenously specified. As before, let N_1 be the net worth of the bank after optimization in the first period. We assume that part of the earning are retained. Specifically, we assume a dividend (payout) rate $\upsilon \in (0, 1)$ of the net worth, so that $E_1 = (1 - \upsilon)N_1$. We further assume that υ is such that $E_0(i^*, i^*) = E_1$. In sum, the environment is totally stationary when there are no policy changes, and the bank makes similar decisions in both periods should rates stay at i^*. Finally, we keep the analysis in partial equilibrium, so that there are no general equilibrium feedbacks from the policy changes and banks' endogenous responses.

4.2 Loan-maximizing policies

We define the loan-maximizing policies $i^P_{0,1}, i^P_{1,2}$ as those that maximize the discounted sum of loans:

$$(i^P_{0,1}, i^P_{1,2}) = \underset{i_{0,1}, i_{1,2}}{\arg\max}\ L^*_{0,1}(i_{0,1}, i_{1,2}) + \beta^P L^*_{1,2}(i_{0,1}, i_{1,2}),$$

[8]More generally, these bonds should represent the duration structure of banks' balance sheet as they enter the period, in the spirit of Begenau et al. (2015).

where $L_{0,1}^*, L_{1,2}^*$ are the optimal choices of banks' loan supply given the interest rates, and β^P is a "policy-specific" discount factor (of the social planner). We assume that $\beta^P \leq 1$, that is the policy cares more about present loans than future loans.

Our goal in this section is to characterize the choices $i_{0,1}^P, i_{1,2}^P$.

4.3 "Creeping-up" result

Our main result is that under mild conditions, $i_{0,1}^P < i_{1,2}^P$, that is the interest rate path "creeps up". The key reason for this result is that although the loss in net interest income following an interest rate cut is similar in both periods, the capital gains from cutting the short-term rate are larger than cutting the long-term rates, since assets mature. In other words, a long-lasting low interest rate environment is going to hurt banks' flow profits every period, while generating low capital gains in the later periods since assets have matured by then. As a consequence, it is optimal to cut the short-term rate more deeply than the long-term one.

One condition that we need when capital gains on long-term assets are present, however, is that the policy makers care about loans in the second period. To see this, suppose that $\beta^P = 0$, that is, the policy maker is myopic and only cares about current loan volume. Suppose moreover that $B_{0,2} > 0$, that is, there are capital gains to be made on long-term assets. Then, the policy response will naturally be to cut the long rate as deep as possible, as to maximally boost capital gains on long-term assets. This will drive down long-term loans to very low levels, and bank net worth tanks and with it loan volume in the second period. Hence, to avoid these "myopic" cases, we need that β^P is sufficiently close to one whenever $B_{0,2} > 0$.

Proposition 6. *Assume that $B_{0,1}, B_{0,2}$ are small enough that the loan-maximizing rates $i_{0,1}^P, i_{1,2}^P$ are well defined. Then $i_{0,1}^P < i_{1,2}^P$.*

Note that we have not assumed that a low bank net worth in the long-term feeds back on the ability of the bank to lend in the first period.[9] This would make the case for cutting the long-rate even weaker, as there would be an additional motive to raise the long-term rate further in order to avoid the drop in long-term net worth to feed back on the bank's risk-taking ability.

[9] That is, we could have assumed that $\psi^L L_{0,1} \leq N_1 + \beta N_2$, where β is some discount factor relevant to evaluate how much long-term net worth impact a bank's ability to take on risk.

5 Reversal in a New Keynesian DSGE

We now asks whether our existence result derived in Proposition 1 still hold in a quantitatively realistic general equilibrium setting where changes in the policy rate can stimulate aggregate demand through the inter-temporal substitution channel. The answer is a qualified yes. As in our partial equilibrium model, a fall in net interest rate income of banks not adequately compensated by capital gains decreases banks' profitability and hence threatens their ability to provide risky but productive lending to the economy. However, standard New Keynesian forces operating through the inter-temporal substitution channel of households' optimization problem are still able to generate a boom and even lift bank profitability. Hence, as long as standard New Keynesian forces operate, our partial equilibrium results do not apply for aggregate variables.

However, as policy rates enter low level territories, the pass-through to deposit rates worsens because depositors become more and more aware of the spreads that banks charge, amid concerns of low nominal returns on their deposits. Banks respond by decreasing their mark-downs and hence decrease the pass-through. This in turn weakens the standard New Keynesian forces, very much like an economy entering a liquidity trap. That weakens loan demand and the associated intermediation boom undertaken by banks. Now, problems would not occur if bank lending did not suffer from any further frictions, as the bank lending channel of monetary policy would remain active. However, the combination of both problems generates the conditions for a reversal rate.[10] Moreover, even before the reversal rate is reached, the effectiveness of monetary policy can be severely dampened by the weakening of both the bank lending and inter-temporal substitution channels.

5.1 Environment

Time is discrete and the horizon is infinite. Households choose consumption, savings and labour supply to maximize their lifetime utility over consumption and leisure. Homogeneous intermediate goods producers competitively sell goods to retailers. These retailers differentiate these goods at no cost and sell them to competitive final good producers. Retailers are subject to price frictions, in a New Keynesian fashion. Final good producers then bundle retail goods into final goods usable for consumption and capital. Finally, a monetary authority (government) supplies nominal safe assets to banks elastically at a decided interest

[10] With flexible prices, general equilibrium forces will work to alleviate the negative consequences of a reversal rate, but cannot question its existence.

rate, taxing (or redistributing gains to) households lump-sum to finance such assets.

Households. A unit continuum of identical households with separable preferences over consumption and labor choose consumption C_t, labour supply H_t, and deposits D_t in order to maximize their lifetime utility:

$$\max_{\{C_t, N_t, D_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} - \chi \frac{l_t^{1+\varphi}}{1+\varphi} \right)$$

subject to their budget constraint

$$p_t C_t + D_{t+1} = p_t w_t l_t + (1 + i_{t-1}^D) D_t + p_t \Pi_t^R + p_t \Pi_t^B - p_t T_t,$$

where $X_t = hC_{t-1}$ denotes the external habit, Π_t^R denotes profits of retail good producers, Π_t^B denote dividend payments from the banks, and T_t are government lump-sum transfers.

Final good producers. Final good producers purchase retail goods $j \in [0,1]$ at price $p_t(j)$ and aggregates them into the final good, with production function

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

They then sell these final goods on competitive markets on competitive markets to households for consumption and intermediate firms for capital investment.

Retailers. A unit continuum of retailers indexed by $j \in [0,1]$ each produces its own retail good variety j by costlessly transforming intermediate goods. They face the demand function for their retail variety derived from the problem of the final good producers. Retailers are subject to Rotemberg adjustment costs to price adjustments. The problem of a single retailer j is then summarized by:

$$\begin{aligned} \max_{\{p_{t+s}(j)\}} \mathbb{E}_t \Bigg[\sum_{s=0}^{\infty} \Lambda_{t+s} \Big[& p_{t+s}(j)^{1-\epsilon} p_{t+s}^{\epsilon-1} Y_{t+s} - mc_{t+s} p_{t+s}(j)^{-\epsilon} p_{t+s}^{\epsilon} Y_{t+s} \\ & - \frac{\theta}{2} \left[\frac{p_{t+s}(j)}{p_{t+s-1}(j)} - 1 \right]^2 p_{t+s}(j)^{1-\epsilon} p_{t+s}^{\epsilon-1} Y_{t+s} \Big] \Bigg] \end{aligned}$$

where real marginal costs mc_t are equal to p_t^I, the price of intermediate goods.

Intermediate firms. Labour and capital are combined using a Cobb-Douglas production function to produce intermediate goods. These goods are then sold competitively to retailers. Labor is hired competitively on labor markets. Capital is purchased a period in advance from final goods producers, and depreciates slowly. Every period, a share ξ of incumbent firms exogenously close after having produced, and are replaced by an equivalent share of new firms.[11] These new firms are subject to financial frictions in that they can only obtain funding from banks.[12]

Specifically, the problem of an incumbent firm is to maximize its profits, given that it rents labour at the real wage w_t and acquires capital a period in advance. In so doing it values the opportunity cost of its capital acquisition at the effective rental rate r_t defined by:

$$r_t = \Lambda_{t-1} - 1 + \delta \tag{5.1}$$

where Λ_{t-1} is the real discount factor of households between period $t-1$ and period t, and δ is the depreciation rate of capital. It then produces output and sells it at the given price p_t^I.

Next, to obtain realistic capital demand we introduce costs of investment rate deviations from its steady state value given by the quadratic formulation $\frac{\kappa^{\mathrm{AC}}}{2}(\iota_t - \iota^*)^2 K_{t-1}$, where $\iota_t = \frac{k_t-(1-\delta)K_{t-1}}{K_{t-1}}$, K_{t-1} is the past capital value of incumbent firms, and ι^* is the steady state value of investment for incumbent firms. Note that the usage of the average previous capital to compute investment costs keeps the problem static while keeping the economics similar. This is important since our constrained firms will face the same frictions, and an easy computation of the elasticity of their demand for capital is crucial to keep the model analytically tractable, a feature that would be lost should the adjustment costs take a dynamic form.

We also introduce decreasing returns to scale in the production function, parametrized by ν. This is important for two reasons: (1) it hinders the financially unconstrained production sector from taking over production, and creates a well defined capital demand schedule for financially constrained firms, a necessary object to banks' problems.

[11] Alternatively, one can justify this setting by saying that firms' projects eventually needs to be replaced, and new projects require funding only available through banks because of financial frictions – for example because new projects are created by entrepreneurs without funding and then sold to production firms.

[12] ξ will hence parametrize the reliance of the economy on bank loans. Albeit simplistic, this way of introducing financial frictions is both transparent and tractable.

Hence the problem of an incumbent reads:[13]

$$
\begin{aligned}
&\max_{y_t,\ell_t,k_t,\iota_t} p_t^I y_t - w_t\ell_t - r_t k_t - \frac{\kappa^{\text{AC}}}{2}(\iota_t - \iota^*)^2 K_{t-1} \quad \text{s/t} \\
&\quad y_t = A_t\left(k_t^{\alpha}\ell_t^{1-\alpha}\right)^{\nu} \\
&\quad \iota_t = \frac{k_t - (1-\delta)K_{t-1}}{K_{t-1}}
\end{aligned}
$$

As said, every period a share ξ of incumbents are replaced by an equivalent share of new firms. New firms must obtain a specialized bank loan to finance their first capital purchases, meaning that they cannot directly issue equity to households in order to finance such purchases.[14] For such a firm, hence, the rental rate is:

$$
\tilde{r}_t = \frac{1 + i_{t-1}^L}{1 + \pi_t} - 1 + \delta
$$

Where i_{t-1}^L, π_t are respectively the bank loan rate and inflation rates between $t-1$ and t. Note that since Λ_{t-1} will be given by the deposit rate, and the latter will usually be smaller than the loan rate, we obtain that $\tilde{r}_t > r_t$ in general, meaning that it is more expensive for new firms to obtain capital. Finally, we assume that after their initial investment in the first period, new firms can freely access equity markets and hence directly borrow from households. The production function is identical to that of incumbents, as is the structure of adjustment costs. The problem of a new firm hence reads:

$$
\begin{aligned}
&\max_{\tilde{y}_t,\tilde{\ell}_t,\tilde{k}_t} p_t^I \tilde{y}_t - w_t\tilde{\ell}_t - \tilde{r}_t\tilde{k}_t - \frac{\kappa^{\text{AC}}}{2}(\tilde{\iota}_t - \tilde{\iota}^*)^2 \tilde{K}_{t-1} \quad \text{s/t} \\
&\quad \tilde{y}_t = A_t\left(\tilde{k}_t^{\alpha}\tilde{\ell}_t^{1-\alpha}\right)^{\nu} \\
&\quad \tilde{\iota}_t = \frac{\tilde{k}_t - (1-\delta)\tilde{K}_{t-1}}{\tilde{K}_{t-1}}
\end{aligned}
$$

Banks. There is a continuum of identical banks that are infinitely lived. The problem of each bank is identical to the one described in our two-period model, with one exception: to keep the equilibrium differentiable, we replace our capital and liquidity inequality constraints with smooth leverage costs. Specifically, adjustment costs for capital leverage are given by

[13] Note that k_t is effectively chosen at $t-1$.

[14] Hence, and in accordance with our assumption of perfect market power of banks on their loan size, only one particular bank j is able to give such loans.

$\frac{\kappa^L}{2}\left(\frac{\psi^L L_t}{N_{t+1}}-1\right)_+^2$, where L_t are the loans given between t and $t+1$ and N_{t+1} is the net worth of the banks in $t+1$, before dividend payments. Similarly, liquidity leverage costs are set to $\frac{\kappa^D}{2}\left(\frac{\psi^D D_t}{S_t}-1\right)_+^2$.

We need to describe the capital accumulation process of banks between perios. We assume that banks accumulate equity according to the following formula:

$$E_{t+1} = (1-\upsilon)\frac{1}{1+\pi_{t+1}}N_{t+1}$$

Where υ is a fixed dividend rate, and N_{t+1} is the nominal (in period t terms) net worth of net banks, appropriately deflated. Dividends are rebated directly to households. The fixed dividend assumption makes certain that firms do not grow out of their financial frictions. It can be viewed as a financial friction, since banks will not be able to raise capital when most needed. This however is consistent with the empirical evidence in Gropp et al. (2016): when under-capitalized, banks tend to deleverage by decreasing the size of their balance sheets or rebalance their assets towards less risky ones, instead of increasing their capitalization. One theoretical underpinning for this behaviour is debt overhang (Admati et al., 2017).

Next, we assume that the deposit rate in the model is given by the extensive margin competition between banks given their consumers' preferences (Equation 3.2), that is $i_t^D = i_t - \eta^D(i_t)$ for all t, and assume that $\eta^D(i) = \left(1-\frac{\eta_1}{\eta_1+\exp(-\eta_2 i)}\right)i$. With $\eta_2 > 0$, the pass-through of changes in i into i^D becomes worse as the level of i decreases, consistent with empirical evidence.

Finally, we need to model capital gains. There are none in the steady state, by assumption of perfect foresight. Let $E_{0,SS}$ then be the steady state level of equity of banks, and assume that after an interest rate cut, banks' capital gains are linear in the level of the interest rate. Using the notation of Section 2.3, we set $\bar{e}_0 = E_{0,SS}$ and $e_0(i) = \zeta_0(i - i_{SS})\bar{e}_0$, with $\zeta_0 < 0$ since a decrease in the policy rate yields capital gains, and i the surprise new rate after the interest rate cut.[15]

Government. The government lump-sum taxes (or transfers) an amount T_t to households, which allows it to pay the nominal interest rates i_t on safe assets. The Taylor rule is

[15] Note that in the steady state, banks are indifferent between holding short-term or long-term safe assets. Hence, calibrating ζ_0 can be thought as calibrating the share of safe assets that are long-term – on which capital gains will be made when i moves. For simplicity we model these gains as only being functions of the rate between period 0 and 1 – as if banks only had at most two-period bonds – which will be the rate that moves the most given that our shock lacks persistence.

assumed to take the following common form:

$$1 + i_t = (1 + i^*) \left(\frac{1 + \pi_t}{1 + \pi^*} \right)^{\phi^\pi} e^{\varepsilon_t^m}$$

Where i^* is the steady state policy rate, π^* is the steady state inflation rate and ε_t^m is a monetary policy shock.

5.2 Solution concept

We assume that every agent in the economy has perfect foresight over the future, and solve the deterministic equilibrium after a one-time unexpected monetary policy shock ε_t^m hits the economy. Our computational algorithm solves for the full non-linear system of equations, and hence does not rely on perturbation techniques. This is important since our economy inherently features non-linearities and state dependence.

5.3 Calibration

The calibration of the New Keynesian block of the model is standard and the parameters are summarized in Table 1. The value of these parameters affect the estimate of the Reversal Interest Rate quantitatively; however, they do not affect its existence qualitatively, for reasons discussed below. One noteworthy difference is that we set no persistence of the monetary policy shock – real persistence is given by habit formation and the various frictions of our model. This allows us to focus on the transmission mechanism of a single monetary shock and abstract from forward guidance.[16] Given the absence of persistence, we calibrate the model annually.

We calibrate interest rates as well as the banking part of the model. We choose the steady state nominal rate to be 2.12%, the average value of the annualized yield on 3-months treasury bills for the period 1997-2016. From aggregated banks' balance sheet and income statements obtained from the FDIC, we compute the average deposit rate during the same period to be 1.82%. So far, the U.S. has never implemented negative interest rates, where the non-linearity in the deposit pass-through is strongest. We therefore calibrate the deposit pass-through parameters, η_1 and η_2, jointly to match the spread between the two

[16] Moreover, when shocks are persistent, even in the presence of a lower bound (or a reversal) in the first period shock will not prevent the second-period shock from affecting the economy positively, and so on. Our goal here is not to study the effects of forward guidance, so we focus on one-period-only shocks.

Table 1: Parameters: New Keynesian Block.

γ	CRRA parameter	2
h	Habit Formation	0.6
ϕ	Frisch Elasticity	1
δ	Cap. Deprec.	0.065
α	Capital share	0.33
ν	Scale parameter	0.81
κ^{AC}	Adj. costs parameter	0.05
ϵ	Retail Price Elastic.	6
θ	Rotemberg cost param.	60
ϕ^{π}	Taylor Rule Coeff.	1.2

interest rates given above, as well as the negative spread recently observed in Europe, where the nominal rate stood at -0.31% while the deposit rate was 0.08%.[17] Next, our loan rate spreads in the model comes from two sources: leverage costs coming from capital or liquidity constraints as well as market power held by banks. For simplicity, we assume that the former do not bite in the steady state, so that market power fully explains the spread: using our values for α, ν, the spread obtained yields a loan rate of 6.54%, as in the data.[18,19] We set the risk-weight on loans ψ^L such that its inverse matches the steady state capital leverage ratio of banks, which stands at 4.5. The liquidity ratio target ψ^D is also set to match the inverse steady state liquidity ratio, which is directly calibrated from the data.[20] The capital leverage cost parameter κ^L is chosen to match the elasticity of rates to changes in capital leverage in the MAG report of the Financial Stability Board (Macroeconomic Assessment Group, 2010), as in Alpanda et al. (2014). This report estimates that a one percentage point increase in capital leverage results in a 0.28 percentage point increase in capital funding costs. Absent a good estimate, we calibrate κ^D to the same value. Next, we calibrate the dividend rate υ such that the net interest income to equity matches that of the data in

[17] Our calibration implies that, possibly, $\eta(i) < 0$ and hence $i^D > i$. Banks are still profitable overall, only less so, which is why they are still willing to keep their costumers despite earning negative margins on safe assets.

[18] Specifically, we take a weighted-average of the new rates for aggregate banks' main assets categories deemed non-safe, which mainly consist of mortgages, commercial & industrial loans, as well as personal credit loans. Computing the average of that loan rate for the 1997-2016 time period, we obtain 6.82%.

[19] An alternative would be to make firms more elastic in their choices, reducing banks' mark-ups, for example by assuming monopolistic competition on the loan side of banks' balance sheet. This would rise the reversal rate, as leverage costs would not be second-order in the neighbourhood of the steady state.

[20] The ratio of deposits to safe assets is not identified in the steady state. Creation of nominal safe assets will only work to inflate banks' balance sheets, without changing any real quantities, since safe assets are indirectly held by households through the government and banks' are owned by households.

Table 2: Calibrated parameters

i_{SS}	Steady-state policy rate	2.12%
β	Implied discount factor of households	0.985%
η_1	Deposit pass-through slope	25.3
η_2	Deposit pass-through shape	198.7
ψ^L	Inverse capital leverage	0.2740
ψ^D	Inverse liquidity leverage	0.3440
κ^L	Capital leverage cost elasticity	0.28
κ^D	Liquidity leverage cost elasticity	0.28
ζ_0	Capital gains elasticity	-0.44
ξ	Share of bank-dependent firms	0.41

1996-2017, which equals approximately 25%. Finally, we model adjustment costs linearly, that is $E_0(i) = E_{0,SS} + E_{0,SS}\zeta_0 i$, where $\zeta_0 = -0.44$ matches the empirical evidence on our definition of capital gains.[21] This parametrization implies that banks' equity increases by 0.44% after a one-percent decrease in interest rates.

We next calibrate the share ξ of firm in need for loans to match the aggregate bank debt as a fraction of firms' liabilities and net worth, which is approximately 19%. This yields $\xi = 41\%$, which is implies that bank-dependent firms are relatively smaller firms with equity-markets access in the model (as in the data), because they face higher funding costs.

5.4 Results

We find that a reversal not only in loans but also aggregate capital and consumption arises in our economy. To substantiate this statement, we conduct the following exercise. First, we study the effects of a marginal monetary policy shock in the vicinity of the steady state and report the resulting impulse response function. Then, we generate at first a large monetary policy shock, which brings the economy far away from the steady state, and study the impulse response functions (IRFs) of the system of an additional marginal shock in that new vicinity. Note that if the system were to be log-linearized, the then-obtained impulse responses would

[21] In our model, we only have short-term assets. Capital gains can be thought as long-term assets (and liabilities) being re-priced at new short-term rates after an interest rate change. Since we only consider a one-time change, we only need to know capital gains (long-term assets/liabilities) on impact.

be *identical*.[22] [23]

Figures 3, 4, 5, 6 show different IRFs to marginal monetary policy shocks: (i) from the steady state level of interest rate (2.12%); (ii) after previous monetary shocks have already brought the interest rate down by 1% (that is, around 1.12%); (iii) after the rate is down by 2% below it's steady-state value; and (iv) after it is down by 4%, which is deep into negative territory.

Figure 3 displays the marginal response of consumption and capital to a monetary shock in each of these economies. In the vicinity of the steady state – where interest rates are at normal levels – the response of consumption takes the usual hump-shape form, while capital jumps on impact and then slowly decreases as households consume. However, things change when the shock occurs in a lower-rate environment. After monetary shocks have brought down the current interest level two percent below the its steady-state value, the effectiveness of monetary policy is already severely dampened, as the consumption and capital responses are both about three times as lower. After shocks have brought the policy rate down by 4%, we have crossed the "general equilibrium" reversal rate and the effects of a marginal monetary policy shock are contractionary – not only for bank lending but also consumption.

Figure 4 confirms that the weakening and then reversal of the effects of monetary policy shocks are due to imperfect pass-through to deposit and loan rates. The left panel shows that, as rates get into lower territory, the effect on deposit rates gets lower to then vanishes when we reach negative territory. The right panel shows that the loan responses follows a similar pattern, except that it eventually reverses in that a monetary policy shocks raise lending rates. Figure 5 confirms that this is due to poor bank profitability and raising leverage costs, apparent in the right panel. Note that bank net worth actually increases following a shock around the steady state, due to the boom on quantities that lift banks' net interest income. In contrast, a cut into negative territory deeply depresses the net worth of banks. This is consistent with the evidence documented in Ampudia and Van den Heuvel (2017), who document that banks' stock market valuations' responses to monetary policy shocks changes sign as the level of interest rates decreases.

[22]When the system is close to linear – when the pass-through of policy rates into deposit rates and loan rates is perfect – they are close to identical. We check that this is indeed the case. The only main source of non-linearity far from the steady state are adjustment costs, but they do not explain the differences in magnitude that we observe in the figures.

[23]In practice, we compute three impulse responses: that of a small shock in the vicinity of the steady state, that of a large shock, and finally a small shock in addition to the large one; we then compare the first IRFs with the difference between the last two IRFs. To be clear, these impulse responses include all general equilibrium feedbacks.

Finally, Figure 6 shows what happens to the capital of the two groups of firms separately. Banks loans and hence the capital of constrained firms respond ambiguously to monetary shocks. This reflects the fact that, even near the steady state, the two groups of firms face different elasticities to interest rate changes; and the rates they face have different elasticities with respect to the monetary policy rate. The deposit rate is cut slightly more than the loan rate, due to endogenous mark-up changes by banks (Figure 3). General equilibrium changes then make constrained firms slightly decrease their physical capital holdings. This reverses as the deposit rate pass-through worsens more than the loan rate pass-through; but eventually, the rise in leverage costs and hence loan rates following the shock in negative territory eventually makes bank-dependent firms sharply decrease their investment. Note that despite that, unconstrained firms' investment continue to be stimulated even when the overall capital response is negative: this is because general equilibrium changes in prices incentivize them to partially compensate the lost investment capacity of bank dependent firms. That substitution is imperfect, though, as firms face decreasing returns to scale. Note that the presence of decreasing returns-to-scale means that measured aggregate TFP endogenously decreases in this economy.

5.5 Forward Guidance

In Section 4, we showed that as long as the loan supply is concerned, it might be desirable under some conditions to make the path of interest rates to slope upward. This is because a cut in long rates generate sizeable losses in net interest income, while generating little capital gains. However, in general equilibrium, forward guidance is implausibly powerful in New Keynesian models, a fact labeled as the "forward guidance puzzle" (Del Negro et al., 2012). For this reason, our partial equilibrium result is largely overturned in our current general setting, although we believe that this might change in a model in which forward guidance is more timid, as for example is the case in Angeletos and Lian (2016).

6 Conclusion

We have shown the possible existence of a Reversal Interest Rate, the rate at which standard monetary policy stimulus reverses its intended effect and becomes contractionary. Its existence relies on the net interest income of banks decreasing faster than recapitalization gains from banks' balance sheets. We showed that its level depends on the magnitudes of these capital gains, the overall capitalization of banks, and the strength of the various leverage constrains faced by banks. Moreover, we have shown that occurrence of a reversal also depends on a weakening of a pass-through of monetary rates to deposit rates, which our theory rationalizes through higher depositor' spread awareness in low-rate environments. Without this weakening, our mechanisms might participate only in weakening the overall response to monetary policy shocks, without necessarily overturning it.

For the sake of tractability, we have omitted other channels through which monetary policy can affect banks' as well as the real economy. In particular, we believe that policies such as ECB's Long Term Refinancing Operations could reduce some of the consequences of low interest rates environments on banks' margins, hence alleviating concerns about banks' margins. Moreover, we have omitted the explicit modelling of risk; hence we have remained agnostic on how low rates change non-performing loans and the associated provisions. We believe including these forces and quantifying them in a bank-augmented general equilibrium model is an important area for future research.

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A Proofs

Lemma 1

Proof. The Lagrangian of this problem is

$$\begin{aligned}\mathcal{L} =&(1+i^L)L + (1+i)S - (1+i^D)D - \mu(L+S-D-E_0(i)) + \lambda^D(\psi^D D - S)\\ &- \lambda^L(\psi^L L - (1+i^L)L - (1+i)S + (1+i^D)D) + \zeta_L(i^D - (i-\eta^D(i)))\end{aligned}$$

This formulation uses the fact that it is never optimal for a bank to set i^D below $i - \eta^i(D)$, since it then receives no deposits, whereas it can earn a positive spread above the deposit rate if it sets $i^D = i - \eta^i(D)$.

The first-order conditions with respect to S, i^L and i^D are (ignoring the lower bound constraint on i^D)

$$\mu = (1+\lambda^L)(1+i)$$
$$1 + i^L = \frac{1}{1+\lambda^L}\left(\mu - (1+\lambda^L)\frac{L}{L'} + \lambda^L\psi^L\right)$$
$$1 + i^D = \frac{1}{1+\lambda^L}\left(\mu - (1+\lambda^L)\frac{D}{D'} + \lambda^D\psi^D\right)$$

Define $\frac{D'}{D} = \epsilon^D$, $\frac{L'}{L} = -\epsilon^L$. Then rearrangement of these first-order conditions yields the equations in Lemma 1.

□

Lemma 2

Proof. Ignoring the extensive margin constraint on deposits, we can write a bank's problem as

$$\begin{aligned}N(i) = \max_{i^L, i^D, S}\ & (1+i^L)L + (1+i)S - (1+i^D)D\\ s.t.\ \ & L + S = D + E_0(i),\ \ \psi^L L \leq (1+i^L)L + (1+i)S - (1+i^D)D,\ \ \psi^D D \leq S\end{aligned}$$

The envelope condition of this problem implies

$$\frac{dN}{di} = \mu E_0'(i) + (1+\lambda^L)S$$

where the Lagrange multipliers μ and λ^L are as defined in Lemma 1. This yields

$$\frac{dN}{di} = (1+\lambda^L)(S + (1+i)E_0'(i))$$

Write

$$CG = E_0'(i), \quad NII = i^L L + iS - i^D D$$

Combining these definitions with the expression for $\frac{dN}{di}$, we obtain

$$\frac{dN}{di} = (1+\lambda^L)\left(\frac{dNII}{di} + (1+i)CG\right)$$

as desired $\square$

Proposition 1

Proof. Define $N(i)$ as in the proof of Lemma 2. We outline sufficient conditions for $N(i)$ to be an increasing function:

1. $i^L L(i^L) \leq K$ for some $K \geq 0$.
2. $E_0'(i) > -\epsilon$ for sufficiently small $\epsilon > 0$.
3. Either ψ^L or $\bar{e}_0$ is sufficiently large.

We now show these conditions are sufficient. As shown in the proof of Lemma 2,

$$\frac{dN}{di} = (1+\lambda^L)(S + (1+i)E_0'(i)) \geq (1+\lambda^L)(S - (1+i)\epsilon)$$

In order to show $N(i)$ is increasing, we must uniformly bound S from below. We have

$$\begin{aligned} S &= E + D - L \\ &\geq E - \frac{1}{\psi^L}N \\ &= (1 - \frac{1}{\psi^L})E - \frac{1}{\psi^L}NII \\ &= (1 - \frac{1}{\psi^L})E - \frac{1}{\psi^L}(i^L L + iS) \\ \Rightarrow S &\geq \frac{1}{1+\frac{i}{\psi^L}}\left((1 - \frac{1}{\psi^L})E - \frac{1}{\psi^L}i^L L\right) \end{aligned}$$

By assumption, $i^L L \leq K$. Then

$$S \geq \frac{1}{1+\frac{i}{\psi^L}}\left((1 - \frac{1}{\psi^L})E - \frac{1}{\psi^L}K\right)$$

Examination of the expression on the right-hand side shows that for E or ψ^L sufficiently large, the right-hand side will be larger than $(1+i)\epsilon$, so $\frac{dN}{di} > 0$.

For $i < i_0$, then, we have

$$\begin{aligned}\frac{dN}{di} &\geq \frac{1+\lambda^L}{1+\frac{i}{\psi^L}}\left((1-\frac{1}{\psi^L})E(i) - \frac{1}{\psi^L}K - (1+i)\epsilon\right) \\ &\geq \frac{1}{1+\frac{i_0}{\psi^L}}\left((1-\frac{1}{\psi^L})E(i_0) - \frac{1}{\psi^L}K - (1+i_0)\epsilon\right) \\ &= G(i_0)\end{aligned}$$

Thus the derivative of $N(i)$ is bounded from below for $i < i_0$, and

$$N(i) \leq N(i_0) - G(i_0)(i_0 - i)$$

When the capital constraint does not bind, the quantity of loans made by the bank is given by $L(i^{L*})$, where i^{L*} satisfies the equation

$$i^{L*} = i + \frac{1}{\varepsilon^L(i^{L*})}$$

Note that $L(i^{L*})$ is decreasing in i. For sufficiently low i, then,

$$\psi^L L(i^{L*}) \geq N(i_0) - G(i_0)(i_0 - i) \geq N(i)$$

meaning there exists a largest interest rate $\hat{i}$ such that the capital constraint binds for all $i < \hat{i}$. In this region, $L(i^L) = \frac{1}{\psi^L}N(i)$, so $\frac{dL}{di} = \frac{1}{\psi^L}\frac{dN}{di} < 0$. Therefore $i^{RR} = \hat{i}$, since $\frac{dL}{di} < 0$ for all $i < \hat{i}$ and $\frac{dL}{di} > 0$ for $i > \hat{i}$. □

Main Lemma

Suppose $i_0 > i^{RR}$ when the parameters of the bank's problem are $\theta = (\psi^L, \psi^D, \bar{e}_0, e_0(i))$. Under an alternative set of parameters $\hat{\theta} = (\hat{\psi}^L, \hat{\psi}^D, \hat{\bar{e}}_0, \hat{e}_0(i))$ such that $N(i,\theta) > N(i,\hat{\theta})$ for $i \leq i_0$, the reversal interest rate is lower under parameters θ than under θ' (so long as it is unique under both sets of parameters).

Proof. Define $i^{L*}(i)$ implicitly as the solution to the equation

$$i^L - \frac{L(i^L)}{L'(i^L)} = i$$

Note that $i^{L*}(i)$ is increasing in i, so $L(i^{L*}(i))$ is decreasing in i. Furthermore, i^{L*} does not depend on parameters. The reversal interest rate i^{RR} is the solution to the equation

$$\psi^L L(i^{L*}(i)) = N(i, \theta)$$

Let $i^{RR}(\theta)$ be the reversal interest rate under parameters θ. With parameters θ', for any $i \leq i^{RR}(\theta)$ we have

$$\psi^L L(i^{L*}(i)) \geq N(i, \theta) > N(i, \theta')$$

This is because by the definition of the reversal interest rate, the function $N(i, \theta)$ must be increasing in i in the region $i \leq i^{RR}(\theta)$. Thus it cannot be that $i^{RR}(\theta') \leq i^{RR}(\theta)$. □

Proposition 2

Proof.

1. Clearly, an increase in the capital constraint ψ^L weakly decreases $N(i)$ for all i. Then by the main lemma, the reversal interest rate must increase.

2. Consider a shift in the interest rate sensitivity of equity such that $E_0(i_0)$ remains constant but $\frac{de_0}{d_i}$ is uniformly increased for all i. Then $N(i)$ is uniformly increased for all $i \leq i_0$, since capital gains following an interest rate cut are larger. By the main lemma, the reversal interest rate must decrease.

3. Note that the first-order condition for i^D implies that i^D is the same in both economies for a given level of $i < i^{RR}$, so $D_A(i) = D_B(i)$. Then the equation

$$L_j(i) + S_j(i) = D_j(i) + E_0^j(i)$$

for $j \in \{A, B\}$ implies $L_A(i) + S_A(i) < L_B(i) + S_B(i)$, since $E_0^A(i) < E_0^B(i)$ for all i. Furthermore, note that when $i = i^{RR}$, i^L is the same in both economies, so it must be that $S_A(i) < S_B(i)$. Thus

$$\frac{dN}{di} = (1 + \lambda^L)(S(i) + (1 + i)E_0'(i))$$

must be larger in economy B when evaluated at i^{RR} because $S(i^{RR})$ is larger. Thus

$$\frac{dL_A(i)}{di} = \psi_A^L \frac{dN_A(i)}{di} < \psi_B^L \frac{dN_B(i)}{di} = \frac{dL_B(i)}{di}$$

at i^{RR}, so the balance sheet constraints $L_j(i) + S_j(i) = D(i) + E_0^j(i)$ yield $\frac{dS_A(i)}{di} > \frac{dS_B(i)}{di}$ at $i = i^{RR}$. But clearly, then, we can keep iterating this argument to obtain $S_A(i) < S_B(i)$ for all $i < i^{RR}$, which then implies $L_A(i) > L_B(i)$ for all $i < i^{RR}$ through the equation for $\frac{dN}{di}$ above (using the fact that the constraint is tighter in economy B).

□

Proposition 3

Proof.

1. An increase in the liquidity coefficient ψ_D can only decrease $N(i)$. Therefore, i^{RR} must increase by the main lemma.

2. A larger $\eta^D(i)$ must make $N(i)$ (weakly) decrease, so i^{RR} must weakly increase by the main lemma.

3. A higher lower bound on the deposit rate also decreases $N(i)$, so the main lemma implies that i^{RR} increases.

□

Proposition 4

Proof. Here we consider a perturbation $E_0(i) \to \tilde{E}_0(i)$ such that $\tilde{E}_0(i) \geq E_0(i)$ if and only if $i \geq i_0$. In particular, for $i < i_0$, $\tilde{E}_0(i) < E_0(i)$, so $N(i)$ is shifted uniformly downwards below i_0. Hence, by the main lemma, i^{RR} must increase. □

Proposition 5

Proof. Differentiating the expression for deposit rate yields the results. This expression is

$$i^D = i - \min\{\eta^D(i), \frac{1}{\varepsilon^D} - \frac{\lambda^D}{1+\lambda^L}\psi^D\}$$

and the liquidity constraint is assumed to be slack, so $\lambda^D = 0$. □

Proposition 6

Proof. There are four possible cases: (1) the capital constraint does not bind in either period at an optimum, (2) the capital constraint binds in both periods, (3) the capital constraint binds only in the first period, and (4) the capital constraint binds only in the second period. I consider these cases in turn.

1. If the bank is unconstrained in both periods, $i_t^L = i_{t-1,t} + \frac{1}{\varepsilon_t^L}$ for $t = 1, 2$. Thus it is possible to increase the quantity of loans made in both periods by decreasing both $i_{0,1}$ and $i_{1,2}$ by a small constant $\epsilon > 0$. Therefore, case (1) is never optimal for the central bank.

2. If the capital constraint binds in both periods, it is possible to increase the bank's net worth in both periods by increasing $i_{0,1}$ as long as $B_{0,1}$ and $B_{0,2}$ are sufficiently low

that $\frac{dNII_1}{di_{0,1}} > \frac{d}{di_{0,1}}(\frac{B_{0,1}}{1+i_{0,1}} + \frac{B_{0,2}}{(1+i_{0,1})(1+i_{1,2})})$. This increase in interest rates at $t = 1$ then increases the quantity of loans made at $t = 1$ and $t = 2$, so it is never optimal for the constraint to bind in both periods.

3. When the capital constraint binds only in the first period, the central bank can increase the net worth of the bank (and thus the quantity of loans made) by cutting $i_{1,2}$, since this increases the value of equity $E_0(i_{0,1}, i_{1,2}) = \frac{B_{0,1}}{1+i_{0,1}} + \frac{B_{0,2}}{(1+i_{0,1})(1+i_{1,2})}$ and N_1 is increasing in equity. This cut in $i_{1,2}$ does not change the quantity of loans made at $t = 2$ because the bank's choice of i_2^L is unconstrained by assumption. Hence it cannot be that the capital constraint binds only in the first period.

4. Given the analysis of the three cases above, it must be that the capital constraint binds only in the second period. We now argue that it must be *exactly* binding (in the sense that $\lambda_2^L = 0$) when $B_{0,2}$ is sufficiently low. Suppose $\lambda_2^L > 0$. Then the quantity of loans made in the second period is increasing in the bank's net worth N_2 in period 2, as $\psi^L L_2 = N_2$. A change in the interest rate at $t = 2$ has two effects: it changes the value of bank equity at $t = 1$, which feeds into $t = 2$ net worth, and it directly impacts net interest income at $t = 2$. Formally,

$$N_2(i_{0,1}, i_{1,2}) = \max_{i^L, i^D, S} (1 + i^L)L + (1 + i_{1,2})S - (1 + i^D)D$$
$$s.t.\ L + S = D + E_1(i_{0,1}, i_{1,2}),\ \psi^L L \leq N_2,\ \psi^D D \leq S$$

Then the envelope theorem implies

$$\frac{dN_2}{di_{1,2}} = (1 + \lambda^L)\left(S^* + (1 + i_{1,2})\frac{dE_1}{di_{1,2}}\right)$$

Note that using the formula $E_1 = (1 - \nu)N_1$, we can write

$$\begin{aligned}\frac{dE_1}{di_{1,2}} &= (1 - \nu)\frac{dN_1}{dE_0}\frac{dE_0}{di_{1,2}} \\ &= -(1 - \nu)\frac{dN_1}{dE_0}\frac{B_{0,2}}{(1 + i_{0,1})(1 + i_{1,2})^2}\end{aligned}$$

Recall from the proof of Lemma 1 that $\frac{dN_1}{dE_0} = 1 + i_{0,1}$ when the capital constraint does not bind. Thus

$$\frac{dN_2}{di_{1,2}} = (1 + \lambda^L)\left(S^* - (1 - \nu)\frac{B_{0,2}}{1 + i_{1,2}}\right)$$

so net worth is increasing in $i_{1,2}$ when $B_{0,2}$ is sufficiently small. Therefore, when $B_{0,2}$ is small and the constraint in the second period binds, loans are increasing in $i_{1,2}$. Loans in the first period do not depend on $i_{1,2}$ because the capital constraint is slack at $t = 1$. The capital constraint must then bind *exactly* at $t = 2$.

The proof in part (1) shows that when $\lambda_2^L = 0$, it is strictly suboptimal to raise interest rates in the first period as long as the constraint in the first period is slack as well. Consider setting $i_{0,1} = i_{0,2} = \hat{i}$ such that $\hat{i}$ is the highest interest rate for which the constraint binds in the second period. We now show that under these interest rates, the constraint in the first period will be slack. By assumption, $B_{0,1}$ and $B_{0,2}$ are small enough that $N_1(\hat{i}, \hat{i}) < N_1(i^*, i^*)$. Then

$$E_1(\hat{i}, \hat{i}) = (1 - \nu)N_1(\hat{i}, \hat{i}) < (1 - \nu)N_1(i^*, i^*) = E_1(i^*, i^*) = E_0(i^*, i^*)$$

so equity in the second period is lower than in the first. By the main lemma, the reversal rate must be higher in the second period because net worth is increasing in equity. Hence at $\hat{i}$, the capital constraint in the first period must be slack, so by the argument above it is never optimal to increase $i_{0,1}$ from $\hat{i}$, meaning that at an optimum $i_{0,1} \leq i_{1,2}$.

□

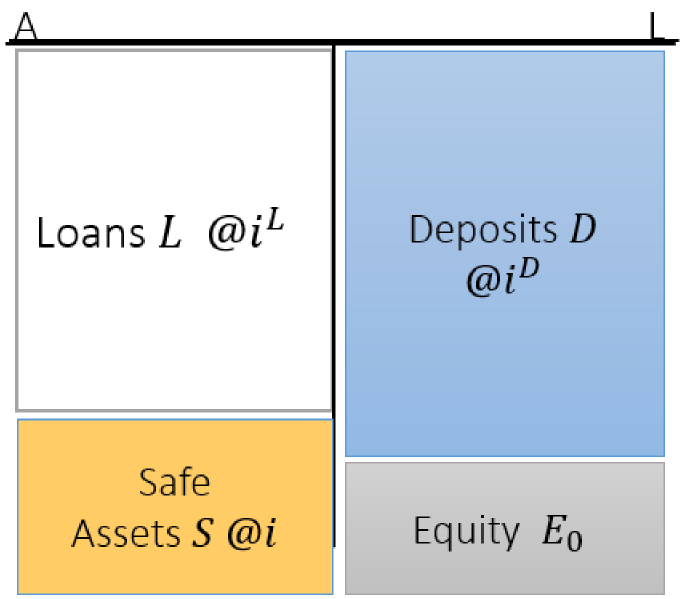

Figure 1: Bank's balance sheet: two-period model.

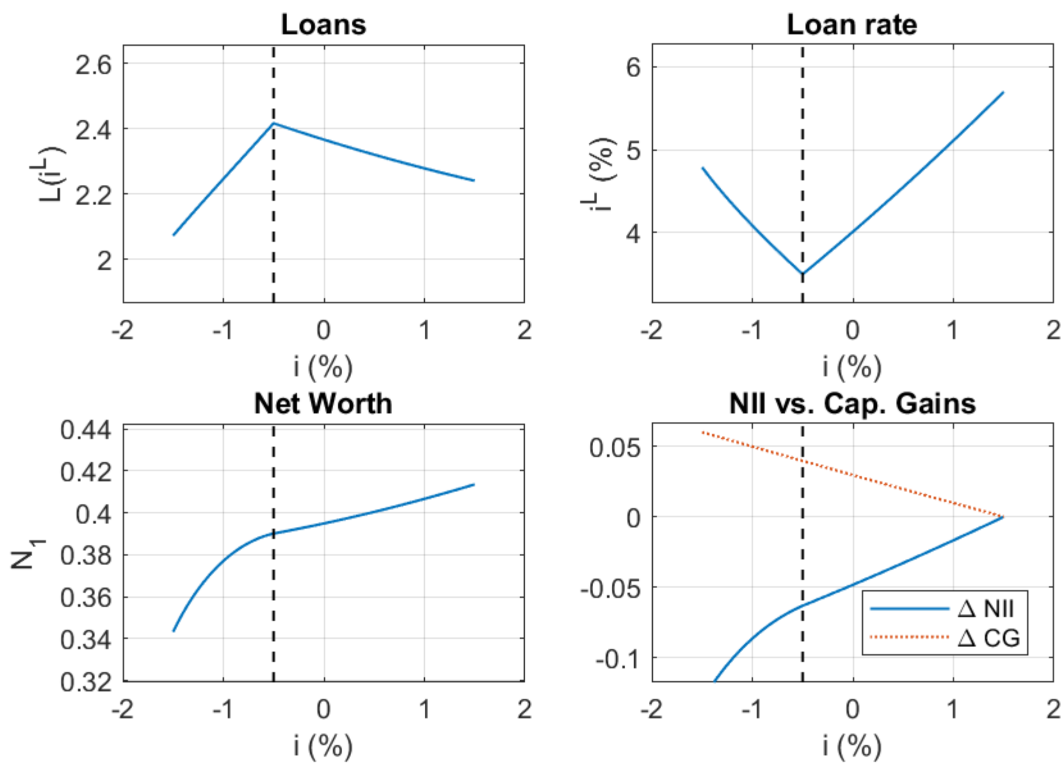

Figure 2: A numerical example of a reversal rate (dashed vertical line) due to a binding capital constraint. The constraint binds since the losses on net interest income (NII) are not sufficiently compensated by capital gains (CG). The ΔCG, ΔNII refer to changes relative to their respective value at a baseline rate of $i_0 = 1.5\%$.

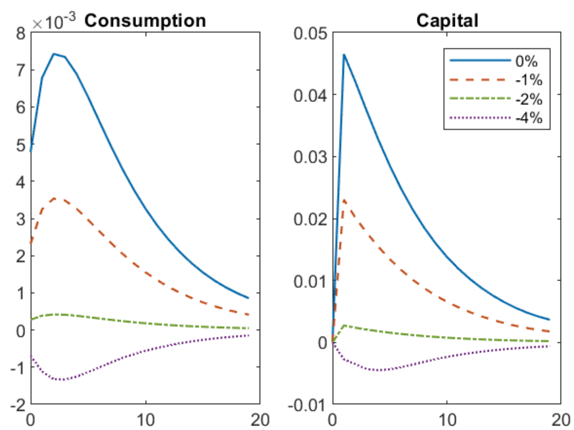

Figure 3: Marginal impulse responses of consumption and capital to a monetary policy shock in four economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state, implying a perfect pass-through. In the second economy (dashed line), the marginal shock occurs after monetary shock depressing the policy rate by -1% have already occured. In the third economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -2% have already occured. In the fourth economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -4% have already occured. The reversal rate has been crossed at this stage, as the effects are recessionary.

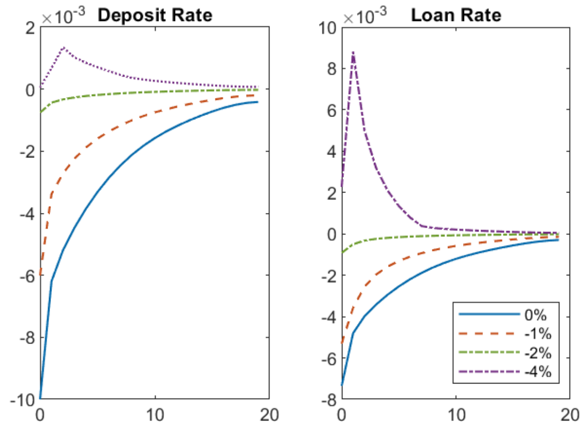

Figure 4: Marginal impulse responses of deposit and loan rates to a monetary policy shock in four economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state, implying a perfect pass-through. In the second economy (dashed line), the marginal shock occurs after monetary shock depressing the policy rate by -1% have already occured. In the third economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -2% have already occured. In the fourth economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -4% have already occured.

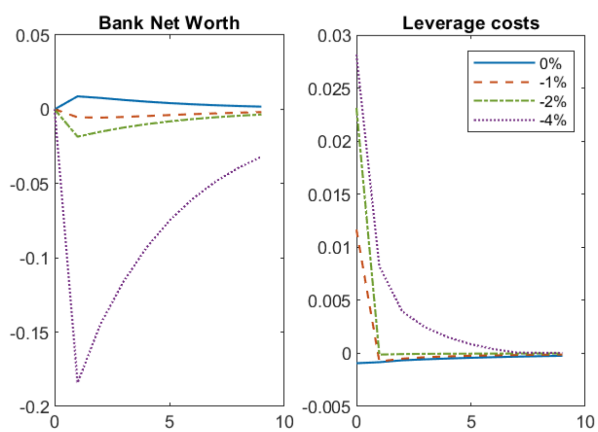

Figure 5: Marginal impulse responses of bank net worth and leverage costs to a monetary policy shock in four economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state, implying a perfect pass-through. In the second economy (dashed line), the marginal shock occurs after monetary shock depressing the policy rate by -1% have already occured. In the third economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -2% have already occured. In the fourth economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -4% have already occured.

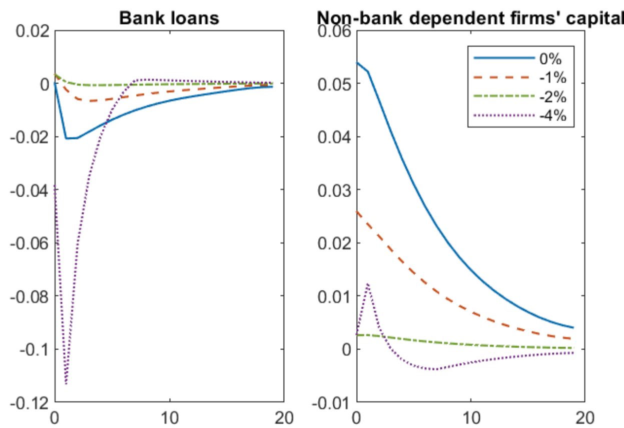

Figure 6: Marginal impulse responses of bank net worth and capital funding outside the banking system to a monetary policy shock in four economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state, implying a perfect pass-through. In the second economy (dashed line), the marginal shock occurs after monetary shock depressing the policy rate by -1% have already occured. In the third economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -2% have already occured. In the fourth economy (dashed-dotted line), the marginal shock occurs after shocks depressing the policy rate by -4% have already occured.