Optimal Expectations

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Optimal Expectations
Brunnermeier & Parker
Framework
Discussion
Literature
Applications
Portfolio Choice
General
Equilibrium
Consumption & Savings
Conclusion

- rational expectations
  - Lucas rationality

- Bayesian rationality
- Non-Bayesian rational view
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Brunnermeier & Parker

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biases: confirmation, optimism, overconfidence ....

rational expectations
Lucas rationality

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biases: confirmation, optimism, overconfidence ....

rational expectations

Lucas rationality

common priors non-common priors

Harsanyi rationality

Bayesian rationality

Non-Bayesian

behavioral view

rational view
bias: confirmation, optimism, overconfidence....

rational expectations
- Lucas rationality
  - common priors
  - non-common priors

Harsanyi rationality

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Critic: no disagreement
no-trade theorem
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Rational expectations

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everything goes

no structure
Overview: Three Main Elements

1. **Felicity at** $t$: $\hat{E}_t [U(c_1, \ldots, c_T)]$
   - Agents care about utility flow today and
   - expected utility flows in the future
   $\Rightarrow$ happier if more optimistic

2. **No split personality**
   - Distorted beliefs distort actions
   $\Rightarrow$ better outcomes if more rational

3. **Optimal beliefs balance these forces**
   - Beliefs maximize well-being $\frac{1}{T} E \left[ \sum_{t=1}^{T} \hat{E}_t [U(c_1, \ldots, c_T)] \right]$
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1. Optimal Expectations Framework
2. Discussion
3. Related Literature
4. Applications
   - Portfolio Choice
   - General Equilibrium
   - Consumption & Savings
5. Conclusion
The General Framework

**Actions:** At each $t$ agent chooses $c_t$ to maximize felicity $t$ given subjective beliefs $\hat{\pi}(s_t|s_{t-1})$, and resource constraints.

**Felicity at** $t$: $\hat{E}_t[U(c_1, \ldots, c_T)]$

with time-separable exponential discounting equals

$$
\sum_{\tau=1}^{t-1} \beta^\tau u(c_\tau) + \beta^t u(c_t) + \hat{E}_t \left[ \sum_{\tau=t+1}^{T} \beta^\tau u(c_\tau) \right]
$$

- 'memory' utility
- 'expected' utility

Note: $\beta$s for past consumption could be replaced with $\delta$. 
Utility Flow, Felicity and Well-being

Felicity at $t = 1$

$$u(c_1) + \sum_{\tau = 1}^{T-t} \beta^\tau u(c_{1+\tau})$$
Utility Flow, Felicity and Well-being

\[ u(c_1) + \sum_{\tau=1}^{T-t} \beta^{\tau} u(c_{1+\tau}) \]

- **Felicity at \( t = 1 \)**
- **Felicity at \( t = 2 \)**
- **Felicity at \( t = 3 \)**

Well-being
Beliefs: \( \text{At } t = 0 \text{ optimal beliefs are } \hat{\pi}^{OE}(s_t|s_{t-1}) \)

that maximize

**Well-being:** \( \mathcal{W} = \frac{1}{T} E \left[ \sum_{t=1}^{T} \hat{E}_t [U(\cdot)] \right] \)

subject to:

- agent behavior given these beliefs
- \( \hat{\pi}^{OE}(s_t|s_{t-1}) \) are probabilities
- \( \hat{\pi}^{OE}(s_t|s_{t-1}) = 0 \) if \( \pi(s_t|s_{t-1}) = 0 \)
## Two-period Example with Consumption at $t = 2$

<table>
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**Actions maximize felicity:** $\beta \hat{E}[u(c_2)]$

**Beliefs maximize well-being:**

$$\mathcal{W} = \frac{1}{2} \beta \hat{E}[u(c_2)] + \frac{1}{2} \beta E[u(c_2)]$$
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Beliefs maximize well-being: $\mathcal{W} = \frac{1}{2} \beta \hat{E}[u(c_2)] + \frac{1}{2} \beta E[u(c_2)]$
1. Subjective probabilities are chosen once and forever
   - Bayes’ Rule (LIE) holds,
   - Can be interpreted as choice of priors
2. If beliefs are objective, wellbeing = felicity
   - Only incentive to distort beliefs is anticipatory utility gain
3. Rational expectations are optimal only if
   - anticipatory utility does enter felicities or
   - anticipatory utility does not enter well-being $W$.
4. Different memory discounting in felicity
   - Paper’s results hold qualitatively for any memory discounting
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Frictionless Extreme

Why optimal expectations?

- It is optimal: “as if” interpretation
- Parents/Upbringing affects (prior) beliefs
- Neuroscientific “story”:
  
  prefrontal cortex exerts effort to reduce overoptimism
  
  (subconscious process)

Payoff: biases are endogenous

- biases are small when distort behavior a lot
- large when provide the most expected future utility
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Related Literature

1. Adam Smith (1776)
   “That the chance of gain is naturally overvalued, ...”
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2. Anticipatory utility (‘Pleasure of Expectation’):
   - Bentham, Hume, Böhm-Barwerk, Marshall, Loewenstein,
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3. Models of belief distortions:
   - cognitive dissonance (Akerlof-Dickens),
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Applications

• Portfolio choice
  ⇒ preference for skewed returns

• General equilibrium
  ⇒ endogenous heterogeneous prior beliefs
  ⇒ equity premium puzzle versus long shot phenomena

• Consumption-savings problem with stochastic income
  ⇒ optimism and overconfidence in future income
  ⇒ consumption profiles concave due to “news”
  ⇒ choose incomplete consumption insurance

• Optimal timing of a single task
  ⇒ procrastination, planning fallacy, context effect
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Portfolio Choice

- Setup

1. **Two period problem:**
   - invest in period 1, consume in period 2

2. **Two assets:**
   - a risk-free asset, return $R$; a risky asset, return $R + Z$

3. **Uncertainty:**
   - $S > 2$ states, $\pi_s > 0$ for $s = 1$ to $S$,
   - $Z_s < Z_{s+1}$, $Z_1 < 0 < Z_S$

4. $c \geq 0$ in all states
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Portfolio Choice

Stage 2: Agent

\[ \max_{\alpha, \beta} \beta \sum_{s=1}^{S} \hat{\pi}_s u \left( R + \alpha Z_s \right) \]

FOC:
\[ 0 = \sum_{s=1}^{S} \hat{\pi}_s u' \left( R + \alpha Z_s \right) Z_s \quad \Rightarrow \alpha^*(\hat{\pi}) \]

Stage 1: Choose \( \hat{\pi}_s \) to maximize well-being

\[ \frac{1}{2} \beta \sum_{s=1}^{S} \hat{\pi}_s u \left( R + \alpha^* Z_s \right) + \frac{1}{2} \beta \sum_{s=1}^{S} \pi_s u \left( R + \alpha^* Z_s \right) \]

\( \underline{\text{felicity at } t = 1} \)

\( \underline{\text{‘average’ utility at } t = 2} \)

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\[ \frac{\beta}{2} (u_s - u_s') = \frac{\beta}{2} \sum_{s=1}^{S} \pi_s u' \left( R + \alpha^* Z_s \right) Z_s \frac{d\alpha^*}{d\hat{\pi}_s'} \]

\( \underline{\text{benefits of anticipation}} \)

\( \underline{\text{costs of changed behavior}} \)
Portfolio Choice

Stage 2: Agent \( \max_{\alpha \beta} \sum_{s=1}^{S} \hat{\pi}_s u (R + \alpha Z_s) \)

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\]

- \( \frac{1}{2} \beta \sum_{s=1}^{S} \hat{\pi}_s u (R + \alpha^* Z_s) \): felicity at \( t = 1 \)
- \( \frac{1}{2} \beta \sum_{s=1}^{S} \pi_s u (R + \alpha^* Z_s) \): ‘average’ utility at \( t = 2 \)

\[
\text{FOC: } \frac{\beta}{2} (u_s - u_{s'}) = \frac{\beta}{2} \sum_{s=1}^{S} \pi_s u' (R + \alpha^* Z_s) Z_s \frac{d\alpha^*}{d\hat{\pi}_s'}
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- \( \frac{\beta}{2} (u_s - u_{s'}) \): benefits of anticipation
- \( \frac{\beta}{2} \sum_{s=1}^{S} \pi_s u' (R + \alpha^* Z_s) Z_s \frac{d\alpha^*}{d\hat{\pi}_s'} \): costs of changed behavior
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\]

- benefits of anticipation
- costs of changed behavior
**Proposition** Excess risk taking due to optimism

(i) Agents are optimistic about states with high portfolio payout

\[
\text{if } \alpha^* > 0, \sum_{s=1}^{S} (\hat{\pi}_s - \pi_s) u'(R + \alpha^* Z_s) Z_s > 0;
\]
\[
\text{if } \alpha^* < 0, \sum_{s=1}^{S} (\hat{\pi}_s - \pi_s) u'(R + \alpha^* Z_s) Z_s < 0.
\]

(ii) Agents go even more long (short) than agent with RE or in the opposite direction

\[
\text{if } E[Z] > 0, \text{ then } \alpha^* > \alpha^{RE} > 0 \text{ or } \alpha^* < 0;
\]
\[
\text{if } E[Z] < 0, \text{ then } \alpha^* < \alpha^{RE} < 0 \text{ or } \alpha^* > 0;
\]
Preference for Skewed Returns

- **Empirical Phenomena:**
  - Horse race long shots: Golec and Tamarkin (1998)
  - Lottery demand: Garrett and Sobel (1999)
  - Security design? Swedish lottery bonds, PS-Lotteriesparen

- **Setup**
  - 2 states with payoffs: $Z_1 < 0 < Z_2$,
  - hold variance and mean fixed and $E[Z] < 0$
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 increase skewness
Proposition Skewness

An agent with an unbounded utility function holds some of the asset even though its mean payoff is negative if the payoff is sufficiently skewed.

• Remark:
  • Agent goes long for large $\pi_1$ even though $E[Z] < 0$, since
    • there is not much room to short and distort beliefs
    • shorting becomes very risky
General Equilibrium

- **Empirical Phenomena:**
  - betting & gambling
  - high trading volume (stock and FX market)
  - home bias
  ➜ *endogenous* heterogenous prior beliefs?
  - negatively skewed: equity premium puzzle
  - positively skewed: IPO underperformance, long-shots

- **Setup:**
  The portfolio choice problem with
  - A continuum of agents with identical endowments
  - A fixed supply of ‘bonds’ with normalization $R = 1$
  - The risky asset in zero net supply: $1 + Z_s = \frac{1+\varepsilon_s}{P_e}$
Empirical Phenomena:
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Proposition Heterogeneous Priors

For $S > 2$ agents split into two groups with different beliefs

(i) Optimists with $\hat{E}^i [Z^{OE}] > 0$ and $\alpha^{OE,i} > 0 = \alpha^{RE}$

(ii) Pessimists with $\hat{E}^j [Z^{OE}] < 0$ and $\alpha^{OE,j} < 0$

both groups trade against each other and $\{\hat{\pi}^i\} \neq \{\pi\} \neq \{\hat{\pi}^j\}$.

• Example
  • $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ with $\gamma = 3$,
  • $\pi_1 = 0.25$, $\pi_2 = 0.75$,
  • $\varepsilon_1 = -0.6$, $\varepsilon_2 = 0.2$ so $P^{RE} = 1$. 
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- Example
  - \( u(c) = \frac{1}{1-\gamma} c^{1-\gamma} \) with \( \gamma = 3 \),
  - \( \pi_1 = 0.25, \pi_2 = 0.75, \)
  - \( \varepsilon_1 = -0.6, \varepsilon_2 = 0.2 \) so \( P^{RE} = 1 \).
Figure: Wellbeing as a function of subjective beliefs, $\hat{\pi}_2$
In this example, as we vary the economic environment, beliefs change . . .

\[ P^{OE} > P^{RE} = 1 \]  if payoff is positively skewed (long-shots, IPO)

\[ P^{OE} < P^{RE} = 1 \]  if payoff is negatively skewed (stock market).

**Conjecture**

For multi-asset case with positive net supply:

- Heterogeneity in beliefs is less pronounced.
- Agents invest in different skewed assets (forgo diversification benefits to hold skewed assets.)

*Complicates Aggregation:*

Representative agent has different preference structure from individual (possibly identical) investors.
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**Complicates Aggregation:**
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Consumption & Savings

- **Empirical Phenomena:**
  - households *expect* upward sloping consumption profile (Barsky et al. (1997))
  - *actual* average consumption growth is non-positive and profiles are concave (Gourinchas & Parker (2002))

- **Setup:**
  - Finite-lived agent, quadratic utility $u(c_t) = ac_t - \frac{1}{2} bc_t^2$
  - one risk-free asset, $R\beta = 1$
  - i.i.d. income:
    - *Objective prob.* $y_t$ independent over time $\Pi(y_t \mid y_{t-1})$
    - $d\Pi(y_t) > 0$ for all $y \in [\underline{y}, \bar{y}]$
    - *Subjective prob.* $\hat{\Pi}(y_t \mid y_{t-1}) \geq 0$ for all $y \in [\underline{y}, \bar{y}]$
**Optimal Consumption**

Euler equation:

\[ c_t \left( A_t, y_t \right) = \hat{E} \left[ c_{t+1} \left( A_{t+1}, y_{t+1} \right) \mid y_t \right] \]

Consumption rule:

\[ c_t^* \left( y_t \right) = \frac{1 - R^{-1}}{1 - R^{-(T-t)}} \left( A_t + y_t + \sum_{\tau=1}^{T-t} R^{-\tau} \hat{E} \left[ y_{t+\tau} \mid y_t \right] \right) \]

Note: \( c_t^* \) depends only on \( \hat{E} \left[ y_{t+\tau} \mid y_t \right] \) (not higher moments)
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Consumption rule:

\[ c_t^* \left( y_t \right) = \frac{1 - R^{-1}}{1 - R^{-(T-t)}} \left( A_t + y_t + \sum_{\tau=1}^{T-t} R^{-\tau} \hat{E} \left[ y_{t+\tau} \mid y_t \right] \right) \]

Note: \( c_t^* \) depends only on \( \hat{E} \left[ y_{t+\tau} \mid y_t \right] \) (not higher moments)
So $\Rightarrow$ Variance only lowers anticipatory utility, but does not affect $c$

$\Rightarrow$ OE exhibit no uncertainty for quadratic utility.

Therefore

$$\hat{E} \left[ u (c_{t+\tau}^*) | y_t \right] = u \left( \hat{E} \left[ c_{t+\tau}^* | y_t \right] \right)$$

Note: agents who expect risk have the same behavior and lower felicity
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Certainty + Euler equation ⇒ wellbeing simplifies to

\[ \frac{1}{T} \sum_{t=1}^{T} \psi_t E \left[ u \left( c_t^* (y_t) \right) \right] \]

and FOC implies an actual consumption path of

\[ c_t^* (y_t) = \frac{a}{b} - \frac{\psi_{t+\tau}}{\psi_t} R^\tau \left( \frac{a}{b} - E \left[ c_{t+\tau}^* (y_{t+\tau}) \mid y_t \right] \right) \]

where \( \psi_t = \beta^{t-1} \left( 1 + \sum_{\tau=1}^{T-t} (\beta^\tau + (\beta \delta)^\tau) \right) \)
average consumption path for agent with rational expectations

Figure: Consumption Path
Figure: Consumption Path

average consumption path

overconsumption (overoptimism)

consumption at $t = 1$
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consumption at $t = 1$
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expected consumption path for agent with optimal expectations at $t = 1$
Reduce consumption since income in $t=2$ was lower than expected. Average consumption path for an agent with optimal expectations is shown. The expected consumption path at $t=2$ is also indicated.
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Figure: Consumption Path
Figure: Consumption Path

Initial over-consumption (overoptimism)
Figure: Consumption Path

- Initial over-consumption (overoptimism)
- Average consumption path
- $c^{RE}(t)$
- $c^{OE}(t)$
Proposition Undersaving

For all $t < T$

(i) $\hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} \mid y_t \right] > E \left[ \hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} \mid y_t \right] \right]$

(ii) $c^*_t (y_{t}) > E \left[ c^*_{t+1} (y_{t+1}) \mid y_t \right]$

(iii) $\hat{E} \left[ c^*_{t+1} (y_{t+1}) \mid y_t \right] > E \left[ c^*_{t+1} (y_{t+1}) \mid y_t \right]$

(iv) as $T \to \infty$, $c^*_t (y_{t}) \to c^{RE}_t (y_{t})$

- Model predictions
  - optimism and overconfidence
  - consumption profile hump-shaped
  - agent surprised by declining consumption on average
  - “overconsumption” declines with costs (length of life)
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• Rational expectations are sub-optimal:
  • Agents with rational beliefs makes the ex post best decisions
  • but agents that care about the future can be happier with some optimism
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• Optimal expectations is a structural model of non-rational beliefs
  • beliefs are most distorted when decision errors are small
  • beliefs are most distorted when “dream” benefits are largest
  • excess risk taking due to optimism, preference for skewness
  • endogenous heterogenous beliefs; agreeing to disagree
  • overconfidence, optimism, and lack of consumption insurance
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