Princeton Initiative: Bubbles

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Systemic risk – a broad definition

- Systemic risk build-up during (credit) bubble ... and materializes in a crisis
  - “Volatility Paradox” → contemp. measures inappropriate

- Spillovers/contagion – externalities
  - Direct contractual: domino effect (interconnectedness)
  - Indirect: price effect (fire-sale externalities)
    - credit crunch, liquidity spirals

- Adverse GE response

![Diagram showing the process of Systemic risk]

- Shock to capital → Loss of net worth → Precaution + tighter margins → Volatility price → Fire sales → amplification, persistence
Bubbles and Crashes

Dilip Abreu and Markus Brunnermeier

Princeton University
- Internet bubble - 1990's

- Why do bubbles persist?
- Do professional traders ride the bubble or attack the bubble (go short)?
- What happened in March 2000?

- Loss of ca. 60 % from high of $ 5,132
- Loss of ca. 85 % from high of Euro 8,583
Credit bubble 2004-2006

ABX 7-1 Prices

- AAA
- AA
- A
- BBB
- BBB-
US House price index – Case-Shiller

Housing Price Index (Jan 2000=100, ratio scale)

Year
Do (rational) professionals ride the bubble?

- **South Sea Bubble (1710-1720)**
  - Issac Newton
    - 04/20/1720 sold shares at £7,000 profiting £3,500
    - Re-entered the market later – ending up losing £20,000
    - “I can calculate the motions of the heavenly bodies, but not the madness of people”

  - Druckenmiller of Soros’ Quantum Fund didn’t think that he party would end so quickly.
    - “We thought it was the eighth inning, and it was the ninth."
  - Julian Robertson of Tiger Fund refused internet stocks.

- **Housing bubble (2007)**
  - Chuck Prince “Dance as long as the music is playing”
**Stylized facts**

- Initial innovation justifies some price increase
- Momentum leads to price overshooting
  - Extrapolative expectations
- Many market participants seem to be aware that the “price is too high” but keep on holding the asset
  - “Play as long as the music is playing”
- Resell-option is crucial for speculative bubbles
- Minsky moment – triggered by “trivial news”
- Credit bubbles lead to extra amplification effects in downturn (since they can impair financial sector)
  - subprime borrowing was only 4% of US mortgage market
Minsky moment – Wile E. Coyote Effect
Overview of Bubble Literature

- **Rational bubbles**
  - Difference equation
  \[ b_t = E_t^Q \left[ \frac{1}{1+r} b_{t+1} \right] \]
  - No zero-sum argument

- **OLG and incompleteness frictions** (morning lecture)
  - Samuelson, Triole, ... Bewley, ... Noise trader risk (DSSW)

- **Informational frictions**
  - Synchronization Risk (Abreu & Brunnermeier 2003)

- **Delegated investment friction**

- **Heterogeneous beliefs bubbles**
  - Harrison & Kreps 1978, Scheinkman & Xiong, Hong & Stein
On Market Efficiency

- Keynes (1936)  \[\Rightarrow\] bubble can emerge
  - “It might have been supposed that *competition between expert professionals*, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.”

- Friedman (1953), Fama (1965)
  Efficient Market Hypothesis  \[\Rightarrow\] no bubble emerges
  - “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.”
Limits to Arbitrage

- **Fundamental risk (Campell & Kyle 1993)**
  - Risk that fundamental overturns mispricing

- **Noise trader risk (DSSW)**
  - Risk that irrational traders drive price even further from fundamentals

- **Synchronization risk**
  - One trader alone cannot correct mispricing (can sustain a trade only for a limited time)
  - Risk that other rational traders do not act against mispricing (in sufficiently close time)
  - Relatively unimportant news can serve as synchronization device and trigger a large price correction
(When) will behavioral traders be overwhelmed by rational arbitrageurs?

Collective selling pressure of arbitrageurs more than suffices to burst the bubble.

Rational arbitrageurs understand that an eventual collapse is inevitable. But when?

Delicate, difficult, dangerous TIMING GAME!
Elements of the Timing Game

- **Coordination**: at least $\kappa > 0$ arbs have to be ‘out of the market’
- **Competition**: only first $\kappa < 1$ arbs receive pre-crash price.
- **Profitable ride**: ride bubble as long as possible.
- **Sequential Awareness**

A Synchronization Problem arises!

- Absent of sequential awareness competitive element dominates $\Rightarrow$ and bubble burst immediately.
- With sequential awareness incentive to TIME THE MARKET $\Rightarrow$ “delayed arbitrage” $\Rightarrow$ persistence of bubble
Overview

- Introduction
- Model setup
- Preliminary analysis
- Persistence of bubbles
- Public events
- Price cascades and rebounds
- Empirical evidence & Hedge funds
- Common action of $\kappa$ arbs
- Sequential awareness
  - Random $t_0$ with $F(t_0) = 1 - e^{-\lambda t_0}$

$\tilde{\beta} p_t$

$p_t = e^{gt}$

$(1 - \beta(\cdot))p_t$

maximum life-span of the bubble $\bar{\tau}$

- paradigm shift
  - internet 90's
  - railways
  - etc.
Payoff structure

- **Focus:** “when does bubble burst”
- $t_0$ is only random variables, all other variables are CK
- **Cash payoff (difference)**
  - Sell one share at $t - \Delta$ instead of at $t$
    \[ p_{t-\Delta} e^{r\Delta} - p_t \]
    where $p_t = \begin{cases} 
    e^{gt} & \text{prior to crash} \\
    (1 - \beta(t - t_0))e^{gt} & \text{after the crash}
    \end{cases}$
- **Price at the time of bursting (tie breaking rule)**
  - Pre crash price for first random orders up to $\kappa$
Payoff structure, Trading

- Small transaction costs $ce^{rt}$
- Risk-neutrality but max/min stock position
  - Max long position
  - Max short position
  - Due to capital constraints, margin requirements etc.

Definition 1: trading equilibrium

- Perfect Bayesian Nash Equilibrium
- Belief restriction: trader who attacks at time $t$ believes that all traders who became aware of the bubble prior to her also attack at $t$. 
Sell out condition for $\Delta \rightarrow 0$ periods

- Sell out at $t$ if
  $$\Delta h(t|t_i)E_t[\beta p_t] \cdot \geq (1 - \Delta h(t|t_i)(g - r)p_t\Delta$$

- RHS $\rightarrow (g - r)$ as $t \rightarrow \infty$

- Bursting date: $T^*(t_0) = \min\{T(t_0 + \eta\kappa), t_0 + \bar{t}\}$
Sequential awareness

Distribution of $t_0$

Distribution of $t_0 + \bar{\tau}$
(bursting if nobody attacks)

trader $t_i$

since $t_i \leq t_0 + \eta$

since $t_i \geq t_0$

$t_0$

$t_0 + \bar{\tau}$
Sequential awareness

Distribution of $t_0$

Distribution of $t_0 + \bar{\tau}$
(bursting if nobody attacks)

 trader $t_i$

$t_i - \eta$
since $t_i \leq t_0 + \eta$
$t_i$
since $t_i \geq t_0$

 trader $t_j$

$t_j - \eta$
$t_j$

$t_0$

$t_0 + \bar{\tau}$
Sequential awareness

Distribution of $t_0$

- Distribution of $t_0 + \bar{\tau}$
  (bursting if nobody attacks)

$t_i$, $t_j$, $t_k$, $t_0$, $t_k$, $t_0 + \bar{\tau}$

$t_i - \eta$ since $t_i \leq t_0 + \eta$
$t_i$ since $t_i \geq t_0$
$t_j - \eta$
$t_j$
$t_0$
$t_k$
$t_0 + \bar{\tau}$
Conjecture 1: Immediate attack

\[ \Rightarrow \] Bubble bursts at \( t_0 + \eta \kappa \)

when \( \kappa \) traders are aware of the bubble
**Conjecture 1: Immediate attack**

- Bubble bursts at $t_0 + \eta \kappa$ when $\kappa$ traders are aware of the bubble.

- If $t_0 < t_i - \eta \kappa$, the bubble would have burst already.
Conjecture 1: Immediate attack

\[ \Rightarrow \text{Bubble bursts at } t_0 + \eta \kappa \]
when \( \kappa \) traders are aware of the bubble
**Conjecture 1: Immediate attack**

⇒ Bubble bursts at $t_0 + \eta \kappa$

when $\kappa$ traders are aware of the bubble
**Conjecture 1: Immediate attack**

⇒ Bubble bursts at $t_0 + \eta\kappa$
Conjecture 1: Immediate attack

$\Rightarrow$ Bubble bursts at $t_0 + \eta \kappa$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

Hazard rate of the bubble

$$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa - t)}}$$

Distribution of $t_0$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

Recall the sell out condition:

$$h(t|t_i) \geq \frac{g - r}{\beta^*}$$

Distribution of $t_0$

Hazard rate of the bubble:

$$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa - t)}}$$
**Conjecture 1: Immediate attack**

- **Bubble bursts at** $t_0 + \eta \kappa$

  - hazard rate of the bubble
    $$h = \frac{\lambda}{1 - e^{-\lambda(t_i+\eta\kappa-t)}}$$

  - bubble appreciation / bubble size

  - lower bound: $(g - r) \bar{\beta} > \frac{\lambda}{1-e^{\lambda\eta\kappa}}$

Recall the sell out condition:

$$h(t|t_i) \geq \frac{g - r}{\beta^*}$$

- optimal time to attack $t_i + \tau_i$

  - "delayed attack is optimal"
Endogenous Crash for large enough $\tilde{\tau}$ (i.e., $\bar{\beta}$)

- Proposition 3: Suppose $\frac{\lambda}{1-e^{-\lambda \eta \kappa}} > \frac{g-r}{\bar{\beta}}$
  - Unique trading equilibrium
  - Traders begin attacking after a delay of $\tau^*$ periods
  - Bubble bursts due to endogenous selling pressure at a size of $p_t$ times

\[
\beta^* = \frac{1 - e^{-\lambda \eta \kappa}}{\lambda} (g - r)
\]
Bubble bursts at $t_0 + \eta \kappa + \tau^*$

$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa + \tau^*- t)}}$

Lower bound: $\frac{g - r}{\beta} > \frac{\lambda}{1 - e^{\lambda \eta \kappa}}$

bubble appreciation
bubble size

$t_i - \eta$
$t_i - \eta \kappa$
$t_i$

$t_i - \eta + \eta \kappa + \tau^*$
$t_i + \tau^*$
$t_i + \eta \kappa + \tau^*$

conjectured attack
optimal
Exogenous crash for low $\bar{\tau}$ (i.e. low $\bar{\beta}$)

- Proposition 2: Suppose

$$\frac{\lambda}{1 - e^{-\lambda \eta \kappa}} \leq \frac{g - r}{\bar{\beta}}.$$  

- Unique trading equilibrium
- Traders begin attacking after a delay of $\tau^1 < \bar{\tau}$ periods.
- Bubble does not burst due to endogenous selling pressure prior to $t_0 + \bar{\tau}$. 
Lack of common knowledge 

⇒ standard backwards induction can’t be applied

\[ t_0 + \eta\kappa + \tau^* \]

\[ \ldots \]

\[ t_0 + \bar{\tau} \]

\( \kappa \) traders know of the bubble

\[ \ldots \]

\[ t_0 + 3\eta \]

\[ t_0 + 2\eta \]

\[ t_0 + \eta \]

\[ t_0 \]

everybody knows of the bubble

everybody knows of the bubble

everybody knows that everybody knows of the bubble

everybody knows that everybody knows that everybody knows of the bubble

(same reasoning applies for \( \kappa \) traders)
Role of synchronizing events

- News may have an impact disproportionate to any intrinsic informational (fundamental) content
  - News can serve as a synchronization device
- Fads & fashion in information
  - Which news should traders coordinate on?
- When “synchronized attach” fails, then the bubble is temporarily strengthened
Setting with synchronizing events

- Focus on news with no info content (sunspots)
- Synchronizing events occur with Poisson arrival rate
  - Note that pre-emption argument does not apply since event occurs with zero probability
- Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
  - Only traders who became aware of the bubble more than $\tau_e$ periods ago observe (look out for) this synchronizing event.
Proposition 5: In ‘responsive equilibrium’
Sell out  a) always at the time of the public event $t_e$, 
b) after $t_i + \tau^{**}$ (where $\tau^{**} < \tau^*$)
except after a failed attack at , re-enter the market for $t \in (t_e, t_e - \tau_e + \tau^{**})$.

Intuition for re-entering the market
- For $t_e < t_0 + \eta \kappa + \tau_e$ attack fails, agents learn $t_0 > t_e - \tau_e - \eta \kappa$
- Without public event, they would have learnt this only at $t_e + \tau_e - \tau^{**}$
- Density that bubble burst for endogenous reasons is zero
Conclusion of Bubbles and Crashes

- Bubbles
  - Dispersion of opinion among arbs causes a synchronization problem which makes coordinated price correction difficult.
  - Arbitrageurs time the market and ride the bubble ⇒ Bubbles persist

- Crashes
  - Can be triggered by unanticipated news without any fundamental content, since
  - It might serve as synchronization device.

- Rebound
  - Can occur after a failed attack which temporarily strengthens the bubble
Bubbles with Trading Cost – simplified example

- Two risk-neutral agents: A and B.
  - An asset with fixed supply, 1 unit equally divided bw A & B.
  - Heterogeneous beliefs; short-sales prohibited.

\[
\begin{align*}
\pi_u^A & = 0.8, \pi_u^B = 0.5 \\
\pi_d^A & = 0.2, \pi_d^B = 0.5 \\
\pi_0^A & = 0.5, \pi_0^B = 0.5
\end{align*}
\]

- Time steps:
  - \( t=0 \)
  - \( t=1 \)
  - \( t=2 \)

- Prices:
  - \( 100 \)
  - \( 50 \)
  - \( 0 \)
Two risk-neutral agents: A and B.
- An asset with fixed supply, 1 unit equally divided bw A & B.
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Two risk-neutral agents: A and B.

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**Bubbles with Trading Cost – simplified example**

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![Diagram](image)
Bubbles with Trading Cost—simplified example

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\[
\begin{align*}
E_u^A[R] &= 90, \\
E_u^B[R] &= 75, \\
p_u &= 90
\end{align*}
\]

\[
\begin{align*}
E_d^A[R] &= 10, \\
E_d^B[R] &= 25, \\
p_d &= 25
\end{align*}
\]

\[
\begin{align*}
\pi_u^A &= 0.8, \pi_u^B = 0.5, \\
\pi_d^A &= 0.2, \pi_d^B = 0.5
\end{align*}
\]

\[
\begin{align*}
p_0 &= 57.5 \\
E_0^A[R] &= 50, \\
E_0^B[R] &= 50
\end{align*}
\]
Welfare criterions heterogeneous beliefs

- Given a social welfare function $W$, allocation $x \succeq_W x'$ if
  - $E^A [W (u^A(x), u^B(x))] \geq E^A [W (u^A(x'), u^B(x'))]$ AND
  - $E^B [W (u^A(x), u^B(x))] \geq E^B [W (u^A(x'), u^B(x'))]$

- Back to Bubble example
  - Assume linear and symmetric social welfare function:
    $$W(u_A, u_B) = u(c_A) + u(c_B) = c_A + c_B.$$ 
  - At the status quo:
    $$E_0^j [W(u_A, u_B)] = E_0^j [\tilde{R}] = 50, \forall j \in \{A, B\}.$$ 
  - Suppose that trading costs $k$ per share.
    - $k < 15$ so that trading occurs.
  - In the equilibrium:
    $$E_0^j [W(u_A, u_B)] = E_0^j [\tilde{R}] - \frac{k}{2} = 50 - \frac{k}{2}, \forall j \in \{A, B\}.$$
Overview of Bubble Literature - again

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  - Difference equation: \( b_t = E_t^Q \left[ \frac{1}{1+r} b_{t+1} \right] \)
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