Progressive Taxation versus College Subsidies When College Dropout is Endogenous*

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This version is very preliminary.

[Link to the latest version]

Abstract

This paper examines what is the optimal policy against the rising skill premium in a heterogeneous agent macroeconomic model in which agents make endogenous enrollment and dropout decisions. In particular, I consider two types of policies: progressive income tax and college subsidies. I derive the optimal progressive labor income tax and optimal college subsidies separately and compare the social welfare. The optimal college subsidies reduce consumption inequality and improve social welfare than the optimal progressive labor income tax.

1 Introduction

Wage inequality has been rising in the United States. Autor, Katz and Kearney (2008) document that the log real weekly wages by the 90th percentile of earners rise by approximately more than 55% relative to 10th percentile earners. Wage differentials by education and by age rose substantially at the same time. In particular, the skill premium—the ratio of wages of college graduates to high school graduates—has risen from 1.5 in 1980 to 1.9 now. These increases in wage inequality contributes to an increase in consumption inequality (Cutler and Katz (1992)). We need to understand how policy can affect wage inequality. There are two branches of policies.

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First, progressive labor income tax offers social insurance in many countries (Guvenen, Kuruscu and Ozkan (2013) for OECD countries). Second, college subsidies are considered as a policy to increase college enrollment and reduce the skill premium. A large literature (ex. Goldin and Katz (2007) and Katz and Murphy (1992)) argues that the skill premium is determined as a result of “The Race between Technology and Education.”—the skill premium rises because the increase in the supply of college graduates does not catch up with the increase in the demand for skilled labor. In this framework, the speed of the increase in the skill premium can be reduced by increasing college graduates in the economy. The question of this paper is which policy is better in terms of welfare and reducing inequality and what mix of the two policies are optimal.

Although there is a long tradition of optimal tax and college subsidy policy, they abstract from college dropout. College dropout is an important factor when considering progressive income tax and college subsidies. In the United States, more than 50% of college enrollees drop out before earning a bachelor’s degree and this fact draws attention to the policy circle. While the wage premium for college graduates is 90%, the wage premium for college dropouts is 20% and there is a significant difference between college graduates and college dropouts. Progressive income tax decreases the difference in disposable income between college graduates and college dropouts. At the same time, it discourages enrollees from graduating with a bachelor’s degree and college dropout might increase. The decrease in the number of college graduates might increase the skill premium and wage inequality might increase. Second, in the current system, many people have already enrolled but drop out and increasing college subsidies just encourages people who are likely to drop out to enroll. Increasing college subsidies does not lead to an increase in college graduates in the society and the skill premium might not decrease.

In this paper, I examine which is better for reducing inequality and social welfare or what is the best mix, progressive labor income tax or college subsidies when college dropout is endogenous. I build a life-cycle general equilibrium model with endogenous enrollment and dropout decisions and credit constraints based on Matsuda (2018). Agents who are heterogeneous with regard to initial asset and high school ability—ability as of the graduation of high school—make enrollment and dropout decisions. Importantly, agents are assumed to be optimistic with regard to the expectation of college ability before enrollment, which is consistent with the findings of Stinebrickner and Stinebrickner (2012) and Zafar (2011) using a unique longitudinal survey of students. Agents observe college ability only after enrollment and decide to drop out or not. These educational decisions shape the aggregate skill in the economy and the skill premium is determined with imperfect substitution between skilled
<table>
<thead>
<tr>
<th>HGPA Quantile</th>
<th>% graduation</th>
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<tr>
<td>Q1</td>
<td>19%</td>
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<tr>
<td>Q2</td>
<td>31%</td>
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<tr>
<td>Q3</td>
<td>48%</td>
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<tr>
<td>Q4</td>
<td>63%</td>
</tr>
<tr>
<td>total</td>
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Table 1: College graduation Rates for High school GPA Quartiles

Source: NLSY97. I use the sample of only 25 year old people. Family income is defined as the average of parental income at 16 and 17 if both are available. I use the one if only one of the two is available.

and unskilled labor. I parameterize and calibrate the model to match the US economy given the current policy. With the model, I solve for the mix of progressivity of labor income tax and college subsidies maximizing a Utilitarian social welfare function.

The main finding of this paper is the following.

1.1 Related Literature

There is a significant amount of literature on college dropout. One of the early papers of a model with college dropout is Manski (1989), who show that dropout has an option value and college enrollees can experiment on the real value of college going. Arcidiacono, Aucejo, Maurel and Ransom (2015), Athreya and Eberly (2016), Lee, Shin and Lee (2015), and Castex (2017) analyze how introducing college dropout in the models change the previous results. The sequential papers of Stinebrickner and Stinebrickner (2008), Stinebrickner and Stinebrickner (2012), Stinebrickner and Stinebrickner (2014) show that learning academic ability during college is a main driver of college dropout rather than credit constraints. Stange (2012) and Trachter (2015) quantitatively show the importance of the option value of college dropout and that increases welfare. Hendricks and Leukhina (2017) argue that college dropout is predictable before enrollment due to the strong correlation between high school GPA and the dropout rate. Bound and Turner (2007) and Bound, Lovenheim and Turner (2010) argue that college dropout increases due to a lower quality of education. This paper is based on these early literature but introduce subsidies that can vary across college years.

Hanushek, Leung and Yilmaz (2014) analyze the effect of various college aid with exogenous college dropout risk. Although the majority of the literature on college subsidies regard dropout as exogenous, there are some exceptions. Ionescu (2011) shows the effect of default policies of student loan on educational decisions. Garriga and Keightley (2007) show the effect of an increase in subsidies on the dropout decision and labor supply in a general equilibrium framework. The largest difference is that they consider only year-invariant subsidies. Chatterjee and Ionescu (2012) argue it is welfare improving to insure student loan against exogenous financial risk of dropping out with endogenous dropout decisions. These papers do not analyze the optimal progressivity of labor income tax.

The optimal tax and human capital investment subsidies has been analyzed in a dynamic public finance. For example, Stantcheva (2017) derives optimal income tax and human capital subsidies with risky human capital in a Mirrleesian style. In contrast, this paper derives optimal policy in a quantitative macroeconomic model in a Ramsey style. Krueger and Ludwig (2016) analyze the optimal income tax and subsidies simultaneously and show that the less progressive labor income tax and a large amount of subsidies than the current state are optimal for a social utilitarian welfare function.

The rest of this paper is organized as follows. Section 2 outlines the model and defines an equilibrium. Section 3 makes the model quantitative by calibration and estimation. Section 4 presents results and, in section 5, I provide discussion and concluding remarks.

2 Model

The model is based on Matsuda (2018) and has three main building blocks. The first is a model of college attendance featuring endogenous enrollment and graduation decisions. At the first period after high school graduation, individuals make an enrollment decision based on their initial asset and high school ability. College enrollees learn their college abilities and decide to drop out of college. College ability is a key factor in that it determines utility in college and the returns to education, which is consistent with Stinebrickner and Stinebrickner (2014).

The second building block is an overlapping generations life cycle with incomplete markets with inter-generational linkage of ability and wealth. Individuals in the model face uncertainty with regard to college ability and labor productivity over life cycle with no insurance available. Individuals give birth to children with ability which is inter-generationally correlated and make an endogenous transfer to their children. As Abbott et al. (2013) claims, it is important to make transfers endogenous for considering the effect of subsidies.
The third building block is a general equilibrium framework with an aggregate production function featuring imperfect substitution between skilled and unskilled labor. The educational decisions aggregate to the supply of skill in the economy, which determines the skill premium. The skill premium is a key factor of social welfare in terms of inequality. In addition, because the education sector is intensive in skilled labor, the skill premium affects tuition in equilibrium, which has a potential impact on the educational decisions.

Since I focus on a stationary equilibrium in which the cross-sectional allocation within each cohort is invariant and prices are constant, I do not include any time subscript in the description of the economy.

2.1 Demography

The economy is inhabited by a continuum of over-lapping generations individuals. Age is indexed by \( j \in \{1, 2, \ldots, J\} \). Each individual in the economy has one offspring that lives with them before the offspring becomes independent. At the beginning of age 1, individuals become economically independent. I will identify age 1 with the graduation of high school (biological age 18), so that everyone begins their life as an independent individual as a high school graduate.

Figure 1 is the timeline. At the beginning of age 1, individuals make enrollment decisions—whether to enroll in college. Once they do not enroll in college, then they cannot enroll later. Time is discrete and one period in the model corresponds to two years. Consistent with college typically requiring four years in reality, college graduation requires two periods in the model. At the beginning of age 2, a college enrollee will observe his or her college ability and a productivity shock for working outside and then makes a decision about whether to continue in college or become a college dropout. Once an individual finishes their schooling, they will be one of three types: high school graduates \((e = HS)\) for those who do not enroll at age 1, college dropouts \((e = CD)\) for those who do not continue college at the beginning of age 2, and college graduates \((e = CG)\) for those who finish two periods of college. After that, they face a standard life cycle problem with income risk where markets for insurance and credit are incomplete.

Individuals give birth to children at age \( j_f = 7 \) which is biological age \( 30 (j_f = (30 - 18)/2 + 1 = 7) \). At age \( j_b = 16 \) (at biological age 48), their children leave and become independent and individuals retire at age \( j_r = 25 \) (at biological age 66) and the maximum age is \( J = 42 \) (at biological age 100). It is at age \( j_b \) when the child leaves the household with a wealth transfer from parents. There are no
transfers allowed at other ages\(^1\).

Individuals survive with probability \(\varphi_j \in [0, 1]\) between age \(j\) and \(j + 1\). Moreover, I assume \(\varphi_j = 1\) for \(j \in [0, j_r - 1]\). The survival rate between \(j_r\) and \(J - 1\) is taken from US Life Tables 2000.

### 2.2 Preferences

When an individual becomes economically independent at age 1, he or she has preferences that are the sum of three components:

1. The expected discounted sum of instant utility:

   \[
   \mathbb{E}_1 \sum_{j=1}^{J} \bar{\beta}^{j-1} u(c_j, \ell_j)
   \]

   \[
   (1)
   \]

   where

   \[
   u(c, \ell) = \frac{(\rho^\mu \ell^{1-\mu})^{1-\sigma}}{1 - \sigma}
   \]

   and \(c_j\) denotes consumption and \(\ell_j\) is leisure at age \(j\). \(\mathbb{E}_1\) is the expectation operator conditional on the information at the beginning of age 1. The individuals are endowed with one unit of time each period. At age \(j \in [j_f, j_b - 1]\), individuals live with their children and consumption is discounted by \(1 + \zeta\) where \(\zeta\) is an adult equivalence parameter. \(\beta\) is the time discount rate.\(^2\)

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\(^1\) If transfers are allowed at other ages such as age 2, the state variables of parents have to include their children’s state variables and solving the individuals’ problem becomes formidable. Transfers from parents changes the result mainly when credit constraints bind for their children. As you will see later, the credit limit for age 1 is tighter than the limit for age 2 and it is unlikely that the transfers from parents at age 1 changes the outcome.

\(^2\)\(\bar{\beta}\) is the effective time discount rate: \(\bar{\beta}_j = \beta j \left(\prod_{k=1}^{j} \varphi_k\right)\)
2. The expected college utility:

\[ \mathbb{E}_1 d_0(s_0) \lambda_1(\theta, \phi) + \beta \mathbb{E}_1 d_1(s_1) \lambda_2(\theta, \phi) \]  

(3)

where

\[ \lambda_j(\theta, \phi) = \lambda + \lambda^\theta \theta + \lambda^\phi \phi \]  

(4)

and \( d_0(s_0) \) is an indicator function which is one if the individual enrolls and \( d_1(s_1) \) is an indicator function for graduation. Individuals derive this utility only while in college. As in Heckman, Lochner and Todd (2006), the psychic cost of education is an important factor determining college education. I define \( \lambda_j(\theta, \phi) \) not as disutility but as utility without loss of generality. College utility is dependent on two components: ability \( \theta \) and college taste \( \phi \). \( \phi \) is fixed over lifetime while the coefficient \( \lambda_j^\phi \) can vary across periods (different loading). I explain ability and college taste in more detail in the individual problems section.

3. Parental altruism.

\[ \beta J_1 - 1 \nu \mathbb{E}_1 V_0 \]  

(5)

where \( V_0 \) is the value of their children at the beginning of age 1. I will explain the detail of the value function later. Individuals enjoy their children’s lifetime utility with a weight \( \nu \). This is a motive of transfers from parents to children.

### 2.3 Goods Sector

There exists a representative firm producing the final good from capital \( K \) and aggregate labor services \( H \) following a production function:

\[ Y = F(K, H) = K^\alpha H^{1-\alpha} \]  

(6)

where aggregate labor services \( H \) is a function of the inputs of two skill levels of labor: skilled labor \( S \) and unskilled labor \( U \).  

\[ H = (a^S H^S)^{\rho} + (1-a^S)(H^U)^{\rho} \]  

(7)
where $\frac{1}{1-\rho}$ is the elasticity of substitution and $H^s$ is the aggregate labor services of skill $s = S, U$. This representative firm rents capital at prices $r + \delta$ where $r$ is the interest rate and $\delta$ the depreciation rate and hires two skills of labor at wages $w^S$ and $w^U$ respectively. I assume markets for output and inputs are competitive, so that the first order conditions for profit maximization yield:

$$r = \alpha \left( \frac{K}{H} \right)^{\alpha-1} - \delta$$  \hspace{1cm} (8)

$$w^S = (1 - \alpha)a^S \left( \frac{K}{H} \right)^{\alpha} \left( \frac{H}{H^S} \right)^{1-\rho}$$  \hspace{1cm} (9)

$$w^U = (1 - \alpha)(1 - a^S) \left( \frac{K}{H} \right)^{\alpha} \left( \frac{H}{H^U} \right)^{1-\rho}$$  \hspace{1cm} (10)

Note that there are two types of skill in production while there exist three levels of education. In the literature on the skill premium as in Katz and Murphy (1992), high school equivalents are assumed to provide unskilled labor and college equivalents provide skilled labor. In this framework, high school equivalents are measured as the sum of high school dropout workers, high school graduate workers, and a fraction of college dropout. College equivalents are calculated as college-plus workers and a fraction of college dropout. Following this classification, I assume high school graduates provide only unskilled labor, college graduates provide only skilled labor, and college dropouts work for a $\chi$ fraction of total effective labor as skilled and $1 - \chi$ as unskilled. For convenience, I define the wage per efficiency unit for college graduates, college dropouts, and high school graduates as

$$w^{CG} = w^S$$  \hspace{1cm} (11)

$$w^{CD} = \chi w^S + (1 - \chi)w^U$$  \hspace{1cm} (12)

$$w^{HS} = w^U$$  \hspace{1cm} (13)

Effective labor is defined as $\varepsilon^e_j(\theta, \eta)h$ where $\varepsilon^e_j(\theta, \eta)$ is labor productivity and $h$ is hours. $\varepsilon^e_j(\theta, \eta)$ is dependent on education $e$, age $j$, ability $\theta$, and idiosyncratic productivity $\eta$. The stochastic productivity shock $\eta$ is mean-reverting and follows an education-specific Markov chain $\pi^e_j(\eta'|\eta) > 0$ and $\Pi^e_\eta$ denotes the invariant distribution function. Note that the effective labor depends on education $e$, not skilled or unskilled. Since college dropouts devote $\chi$ (resp. $1 - \chi$) fraction of effective labor as skilled (resp. unskilled), their wage per unit is $\chi w^S \varepsilon^{CD}_j(\theta, \eta) + (1 - \chi)w^U \varepsilon^{CD}_j(\theta, \eta) = w^{CD} \varepsilon^{CD}_j(\theta, \eta)$.  

8
I assume that high school ability determines labor productivity of high school graduates while college ability determines labor productivity of college graduates and dropouts.

2.4 College

There is a representative college. To provide a student with one period of education requires $\kappa$ units of skilled labor. An interpretation of this assumption is that college enrollees obtain education from professors who are college-equivalent workers. In this formulation, education does not require any capital or unskilled labor.\(^3\)

The profit of college is

$$p_c E - w^s \kappa E$$

(14)

where $E$ is the measure of college enrollees and $p_c$ denotes tuition. I assume colleges are competitive and there is free entry. This implies, in equilibrium with positive units of students, $p_c = w^s \kappa$. In the United States, colleges receive subsidy from governments, which should make the sticker tuition smaller than the actual education cost for students. I reinterpret this situation as follows: colleges do not receive any subsidy while college enrollees receive subsidies instead. In both cases, enrollees pay $p_c$ less the subsidy for education.

2.5 Financial Markets

I assume that financial markets are incomplete. There is no insurance market against idiosyncratic risks and individuals can self-insure using only trade risk-free assets.

Lenders incur the cost of overseeing borrowers to lend capital to workers and the cost per unit of capital is $\iota > 0$. With non-arbitrage condition, the interest rate to workers is $r^- = r + \iota$. In addition, the borrowing limits for workers of education level $e$ is assumed to be $A^e$ and retired individuals have no access to loans.

I also assume that the cost of overseeing college enrollees is $\iota + \iota^s$. With non-arbitrage condition, the interest rate to enrollees is $r^e = r + \iota + \iota^e = r^- + \iota^e$. I assume these overseeing costs to match the fraction of borrowers in the economy and to imitate the actual student loan system in the United States. The borrowing limit for college enrollees is $A^e_j$ at age $j$.

\(^3\) While this is a strong assumption, this formulation captures an important aspect of tuition. When considering policy changes, it is important to keep track of what happens to tuition. Archibald and Feldman (2011) argue that college tuition reflects wages of college graduates. This implies that policies that affect the wages of skilled labor can also affect tuition which potentially has an effect on enrollment and graduation. While this specification is too simple, it captures the effect of the skill premium on tuition.
2.6 Individual Problems

The lifecycle of individuals is basically composed of education, working, and retirement stages. Although college enrollees can also work in this model, I call the individuals who are not in college “workers”. Likewise, I call the periods when the individuals are not in college “working stage”.

2.6.1 Education Stage

Enrollment

At the beginning of $j = 1$, individuals become independent as high school graduates and their first decision is whether to enroll in college or not. I define $V_0$ to be the value function.

\[
V_0(a, \theta_h, \eta, q, \phi) = \max_{\text{enrolling}} \left[ V_1(a, \theta_h, \eta, q, \phi), V_1(a, HS, \theta_h, \eta) \right] \tag{15}
\]

An individual’s initial state is composed of initial assets $a$, high school ability $\theta_h$, an idiosyncratic transitory productivity $\eta$ from $\Pi^{HS}$, parents’ (family) income level $q$, and unobserved education taste $\phi$.

I assume that there are two types of ability that are distinct but related to each other: high school ability $\theta_h$ and college ability $\theta_c$. I assume that individuals observe high school ability but do not observe college ability before the enrollment decision. Individuals observe their high school ability through high school grade point average (GPA) or test scores before age 0 during high school. College ability is only observed after 1 period of college. Stinebrickner and Stinebrickner (2012) present evidence that enrollees do not have perfect foresight of their college ability before enrollment. However, college ability is correlated with high school abilities and

\[
\theta_c = \theta_h + \epsilon_c \quad \text{where} \quad \epsilon_c \sim N(0, \sigma_c^2) \tag{16}
\]

In addition, I assume that college enrollees are optimistic about their college ability, in order to be consistent with an empirical finding of Stinebrickner and Stinebrickner (2012) that optimism is a key factor of enrollment. They have a longitudinal survey of students, which asks each student his or her expectation of GPA multiple times. They show that the expectation is higher than the actual GPA on average and that students who drop out in early years are the most optimistic and had the largest
downward revision of their expectation. Given \( \theta_h \), enrollees expect that

\[
\theta_c = \theta_h + \mu_c(\theta_h) + \epsilon_c \text{ where } \epsilon_c \sim N(0, \sigma^2_c)
\]

(17)

\( \mu_c(\theta_h) \) is the bias between the actual mean of college ability and the one expected by them. If \( \mu_c(\theta_h) \) is positive, it implies that enrollees are optimistic about their college abilities. Furthermore, the bias can depend on high school ability and I assume \( \mu_c(\theta_h) = \mu_{c0} + \mu_{c1}\theta_h \), which implies that college enrollees with different high school ability can have different bias. I assume that the variance of the residual term is identical to the actual one.

College taste \( \phi \) is observable to the individuals before enrollment but not observable to econometricians. This is necessary because people make different enrollment and dropout decisions within the same category of ability which is observable to econometrician. Within the same category of family income and high school ability, people with high college taste are more likely to enroll than those with low taste for college. Initial wealth \( a \) is endogenously determined as a transfer from their parents as will be shown later. If an individual enrolls, he or she enters the first half of college and the value is \( V_1^c \). If they do not enroll, they start working as high school graduates and its value is \( V_1 \).

**First half of college**

The value of being in the first half of college \( V_1^c \) is

\[
V_1^c(a, \theta_h, \eta, q, \phi) = \max_{c,h,a',y} u(c, 1-h-\tilde{h}) + \mathbb{E}_{\theta_c|\theta_h} \lambda_1(\theta_c, \phi)
\]

\[
\quad + \beta \mathbb{E}_{\theta_c|\theta_h} \mathbb{E}_{\eta'} \max_{\text{continue}} [V_2^c(a', \theta_c, \eta', q, \phi), V_2(\tilde{a}(a'), CD, \theta_c, \eta')]
\]

subject to

\[
c + a' + p_c - s_j(q) = a + y - T(c, a, y)
\]

(18)

\[
y = w^{HS}_{\epsilon_j} \epsilon_j^{HS}(\theta_h, \eta) h, \quad a' \geq -A^c_1, \quad c \geq 0, \quad 0 \leq h \leq 1 - \tilde{h}
\]

(19)

\[
\theta_c = \theta_h + \mu_c(\theta_h) + \epsilon_c, \quad \epsilon_c \sim N(0, \sigma^2_c) \quad \text{(perceived process)}
\]

(20)

Going to college requires a fraction \( \tilde{h} \) of time, tuition \( p_c \) and additive utility \( \lambda_j(\theta, \phi) \) for each enrolling period. \( c \) is consumption, \( y \) is labor earnings and \( a' \) is next period assets. The total tax \( T(c, a, y) \) is dependent on consumption, asset, and earnings. College enrollees receive subsidies \( s_j(q) \). The subsidies are need-based (dependent on family income \( q \)). They can work as high school graduates
during the first half of college.

At the end of the first half of college, college enrollees observe their college ability $\theta_c$ and draw an idiosyncratic productivity as college dropout $\eta'$ from $\Pi^{CD}$. College enrollees choose whether or not to drop out of college given college ability $\theta_c$ and $\eta'$. If the individual drops out, his or her education level becomes college dropout ($e = CD$) and their value is $V_2$. Furthermore, all the student loan is refinanced into a single bond that carries interest rate $r^-$. $\bar{a}(a)$ is the transformation from the asset position during college to the position after college so that the total payment is identical. When making this calculation I assume that fixed payments would have been made for 20 years (10 periods) after dropout\(^4\). If the individual does not drop out, they proceed to the second half of college with value $V_2^\prime$.

**Second half of college**

The Bellman equation for the second half of college is

$$V_2^\prime(a, \theta_c, \eta, q, \phi) = \max_{c,h,a',y} \ u(c, 1 - h - \bar{h}) + \lambda_2(\theta_c, \phi) + \beta \mathbb{E}_{\eta'} V_3(\bar{a}(a'), CG, \theta^c, \eta)$$  \hspace{1cm} (21)

subject to

$$c + a' + p_e - s_j(q) - y + T(c, a, y) = \begin{cases} (1 + r)a & \text{if } a \geq 0 \\ (1 + r^s)a & \text{if } a < 0 \end{cases}$$ \hspace{1cm} (22)

$$y = w^{CD} \xi^j \eta(h), \ a' \geq -A^\prime_2, c \geq 0, \ 0 \leq h \leq 1 - \bar{h}$$ \hspace{1cm} (23)

They can work as college dropout during the second half of college. At the end of period, they complete college and acquire education level $e = CG$ and draw an idiosyncratic productivity as college graduates $\eta'$ from $\Pi^{CG}$. As in the case of dropout, student loan is refinanced into a single bond and the transformation is $\bar{a}(a')$. The value of workers at age $j$ is $V_j$.

**2.6.2 Working Stage**

The Bellman equation for workers is\(^5\)

$$V_j(a, e, \theta, \eta') = \max_{c,h,a',y} \ u \left( \frac{c}{1 + \sum_{j=1}^{\infty} \frac{1}{(1+r)^j}}, 1 - h \right) + \beta \mathbb{E}_{\eta'} [V_{j+1}(a', e, \theta, \eta')]$$  \hspace{1cm} (24)

\(^4\) $\bar{a}(a) = a' \times \frac{r^s}{1 - (1 + r^s)^{-10}} \times \frac{1 - (1 + r^-)^{-10}}{1 - (1 + r^-)^{-10}}$.

\(^5\) After retirement, labor productivity is no longer a state variable. Thus the Bellman equation for the last period of workers is $V_{j_{ret}}(a, e, \theta, \eta) = \max_{c,h,a',y} u(c, 1 - h) + \beta V_{j_{ret}}(a', e, \theta)$.
subject to 
\[ c + a' - y + T(c, a, y) = \begin{cases} (1 + r)a & \text{if } a \geq 0 \\ (1 + r^-)a & \text{if } a < 0 \end{cases} \]  
(25)

\[ y = w^c \varepsilon_j^c(\theta, \eta) h, \quad a' \geq -\Delta c^e, \quad c \geq 0, \quad 0 \leq h \leq 1 \]  
(26)

where \( \mathbb{I}_{\mathcal{J}_f} \) is an indicator function which is one when the individuals live with their children \( j \in [j_f, j_b - 1] \). Ability is \( \theta = \theta_h \) for high school graduates and \( \theta = \theta_c \) for college dropout and college graduates. At each period, idiosyncratic productivity \( \eta \) transitions according to \( \pi^\eta_i \).

### 2.6.3 Transfer

At the age \( j_b \), the individuals’ children become independent and they determine the amount of transfer. Its Bellman equation is

\[ V_j(a, e, \theta, \eta) = \max_{c(\theta_h'), h(\theta_h'), a(\theta_h'), y(\theta_h')} \mathbb{E}_{\theta_h'|e, \theta} \{ u(c(\theta_h'), 1 - h(\theta_h')) + V_{j+1}(a', \theta, \theta_h', e, \eta) \} \]  
(27)

subject to

\[ c(\theta_h') + a'(\theta_h') - y(\theta_h') + T(c(\theta_h'), a(\theta_h'), y(\theta_h')) = \begin{cases} (1 + r)a & \text{if } a \geq 0 \\ (1 + r^-)a & \text{if } a < 0 \end{cases} \]  
(28)

\[ y(\theta_h') = w^c \varepsilon_j^c(\theta, \eta) h(\theta_h'), \quad a' \geq -\Delta c^e, \quad c(\theta_h') \geq 0, \quad 0 \leq h(\theta_h') \leq 1 \]  
(29)

where

\[ \tilde{V}_{j+1}(a, \theta, \theta_h', e, \eta) = \max_{b \in [0, a]} \beta \mathbb{E}_{\eta'|\theta} V_{j+1}(a - b, e, \theta, \eta') + \nu \mathbb{E}_{\eta'|\theta} V_0(b, \theta', \eta'', \tilde{q}(w^c \varepsilon_j^c(\theta, \eta)), \phi) \]  
(30)

for all \( \theta_h' \). Before making any decisions, the individuals observe their children’s high school ability \( \theta_h' \).

The density function for the child’s ability is \( \pi_\theta(\theta_h'|\theta) \). Parents can observe neither their children’s idiosyncratic productivity \( \eta'' \) drawn from \( \Pi^{HS} \) nor college taste \( \phi \) drawn from the normal distribution \( N(0, \sigma_\phi^2) \). Consumption, leisure, savings, and parental transfers can be dependent on \( \theta_h' \). Note that the value of their children depends on family income level \( q \) which reflects the potential labor income of the parental individuals.\(^6\) Family income determines the amount of need-based subsidies.

---

\(^6\)Note that the parental income is not the actual labor income. The parents can control the actual labor income by adjusting their working hours. In this setting, this manipulation of parental income is not allowed and parental income is a function of “potential” income which is labor earnings if they spend 35% working. Thus the family income mapping
2.6.4 Retirement Stage

After retirement at age $j_r$, I assume individuals provide no labor. The Bellman equation is

$$V_j(a, e, \theta) = \max_{c, a'} u(c, 1) + \beta \varphi_{j+1} V_{j+1}(a', e, \theta)$$

(32)

subject to

$$c + a' = (1 + r) \varphi_j^{-1} a + p(e, \theta) - T(c, \varphi_j^{-1} a, 0)$$

(33)

$$a' \geq 0 \quad c \geq 0$$

(34)

The sources of income are asset earnings and retirement benefits $p(e, \theta)$. In the United States, retirement benefits are determined according to the labor earnings before retirement (see Appendix B). To capture this, I assume the retirement benefits are dependent on their ability and education. The asset inflated by $\varphi_j^{-1}$ reflects that assets of expiring individuals are distributed within cohorts (perfect annuity market).

2.7 Government

The government collects tax revenue $T(c, a, y)$ from individuals and spends the revenues on subsidies $G_c$, other government consumption $G_c$ and retirement benefits. Government consumption $G_c$ is assumed to be exogenous and proportional to output, which follows $G_c = gY$. The total college subsidies is

$$G_c = \sum_{j=1,2} \int_{S_j} s_j(q) d\mu^c_j$$

(35)

The tax function is assumed to be

$$T(c, a, y) = \tau_c c + \tau_k ra \mathbb{1}_{a \geq 0} + \tau_y - \frac{Y}{N}$$

(36)

where the proportional consumption tax is $\tau_c$ and the proportional capital income tax $\tau_k$ is levied only on positive net worth. I assume the government refunds a lump-sum transfer $\frac{Y}{N}$ to each individual where $N$ is the population. This reflects the progressivity of income tax observed in the United States.

$$q(w^e e_j^0(\theta, \eta)) = \begin{cases} 1 & \text{if } w^e e_j^0(\theta, \eta) \times 0.35 \in [0, q_1] \\ 2 & \text{if } w^e e_j^0(\theta, \eta) \times 0.35 \in [q_1, q_2] \\ 3 & \text{else} \end{cases}$$

(31)
\( \tau_i \) is the proportional part of labor income tax.

### 2.8 Equilibrium

The model includes \( J \) overlapping generations and is solved numerically to characterize a stationary equilibrium. Stationarity implies that the cross-sectional allocation within each cohort \( j \) is invariant. In equilibrium, individuals maximize expected lifetime utility, firms maximize profits, the government budget is balanced each period, and prices clear all the markets. Let \( s^c_j \in S^c_j \) be the age-specific state vector for college enrollees and \( s_j \equiv S_j \) for workers and retirees and \( s_0 \in S_0 \) for individuals at the beginning of age 1. I also define the age-specific state vector for workers and retirees conditional on education \( e \) as \( s^c_j \in S^c_j \). Computation is described in Appendix A.

**Definition 1** A stationary equilibrium is a list of value functions of workers and college enrollees \( \{V_j(s_j), V_j^e(s^c_j)\} \), decision rules of enrollment \( d_0(s_0) \) and graduation \( d_1(s^c_1) \), decision rules of consumption, asset holdings, labor, output, parental transfers of workers \( \{c_j(s_j), a_j(s_j), h_j(s_j), y_j(s_j), b(s_j)\} \), decision rules of college enrollees \( \{c^e_j(s^c_j), a^e_j(s^c_j), h^e_j(s^c_j), y^e_j(s^c_j)\} \), aggregate enrollees, capital, and labor inputs \( \{E, K, H^S, H^U\} \), prices \( \{r, w^S, w^U, p_e\} \), policy \( \tau_i \), measures \( \mu = \{\mu^e_j(s^c_j), \mu_j(s_j), \mu^c_j(s^c_j)\} \) such that

1. Taking prices and policies as given, value functions \( \{V_j^e(s^c_j), V_j(s_j)\} \) solve the individual Bellman equations and \( d_0(s_0), d_1(s^c_1), \{c_j(s_j), a_j(s_j), h_j(s_j), y_j(s_j), b(s_j)\}, \{c^e_j(s^c_j), a^e_j(s^c_j), h^e_j(s^c_j), y^e_j(s^c_j)\} \) are associated decision rules.

2. Taking prices and policies as given, \( K, H^S, H^CG \) solve the optimization problem of the good sector and \( E \) solves the optimization problem of the education sector.

3. The government budget is balanced.

\[
G_c + G_e + \sum_{j=1}^{J} \int_{S_j} p(e, \theta) d\mu_j = \sum_{j=1, 2} \int_{S^c_j} T(c^e_j(s^c_j), a^e_j(s^c_j), y^e_j(s^c_j)) d\mu^e_j + \sum_{j} \int_{S_j} T(c_j(s_j), a_j(s_j), y_j(s_j)) d\mu_j
\]

where

\[
G_e = gF(K, H)
\]
\[ G_e = \sum_{j=1,2} \int_{s_j} s_j(q) d\mu_j^x \]  

(38)

4. Labor, asset, and education markets clear.

\[ H^S + \kappa E = H^{CG} + \chi H^{CD} \]  

(39)

\[ H^U = H^{HS} + (1 - \chi) H^{CD} \]  

(40)

where

\[ H^{CG} = \sum_{j=3}^{j_r-1} \int_{s_j^{CG}} \epsilon_j^{CG}(\theta, \eta) h_j(s_j) d\mu_j^{CG} \]  

(41)

\[ H^{CD} = \sum_{j=2}^{j_r-1} \int_{s_j^{CD}} \epsilon_j^{CD}(\theta, \eta) h_j(s_j) d\mu_j^{CD} + \int_{s_2^D} \epsilon_2^{CD}(\theta, \eta) h_2^D(s_2^D) d\mu_2^{D} \]  

(42)

\[ H^{HS} = \sum_{j=1}^{j_r-1} \int_{s_j^{HS}} \epsilon_j^{HS}(\theta, \eta) h_j(s_j) d\mu_j^{HS} + \int_{s_1^H} \epsilon_1^{HS}(\theta, \eta) h_1^H(s_1^H) d\mu_1^{H} \]  

(43)

and

\[ K = \sum_{j=1}^{j_r-1} \int_{s_j} a_j^e(s_j) d\mu_j + \sum_{j=1,2} \int_{s_j^x} a_j^e(s_j^x) d\mu_j^x \]  

(44)

\[ E = \sum_{j=1,2} \int_{s_j^x} d\mu_j^x \]  

(45)

5. Measures \( \mu \) are reproduced for each period: \( \mu(S) = Q(S, \mu) \) where \( Q(S, \cdot) \) is a transition function generated by decision rules and exogenous laws of motion, and \( S \) is the generic subset of the Borel-sigma algebra defined over the state space.

3  Calibration

This section describes how the model is parameterized and estimated. There are two sets of parameters: (1) those that are estimated outside of the model or fixed based on literature and (2) the remaining parameters to match key moments given the first set of parameter values.

3.1 Labor Productivity Process

I assume labor productivity

\[ \ln \epsilon^*_j(\theta, \eta) = \ln e^e + \ln \psi^x_j + \epsilon^*_0 + \ln \eta \]  

(46)
where $\psi_j^e$ is the age profile of workers at age $j$ at education level $e$ estimated from PSID (See Appendix C). The coefficients can vary across education levels.

The ability used in the wage process differs across education levels. For high school graduates, $\theta = \theta_h$ which is approximated by $\ln \text{AFQT80}$. $\eta$ is an idiosyncratic productivity shock uncorrelated with $\theta_h$ and I can estimate the coefficient $\theta_h^{HS}$ using $\ln \text{AFQT80}$. For college dropouts and college graduates, $\theta = \theta_c$. College ability is not only a college GPA but also includes other general components represented by quality of college and college majors. Since it is hard to measure the composite, I instrument college ability using high school ability. From $\theta_c = \theta_h + \varepsilon_c$, the log labor productivity is

$$\ln \epsilon^e + \ln \psi_j^e + e_\theta^c \theta_h + \ln \eta = \ln \epsilon^e + \ln \psi_j^e + e_\theta^c \theta_c + (\ln \eta + e_\theta^c \varepsilon_c)$$  \hspace{1cm} (47)$$

Since $\theta_h$ is uncorrelated with $\ln \eta + e_\theta^c \varepsilon_c$, I can estimate the coefficient $e_\theta^c$ using $\ln \text{AFQT80}$ for college dropouts and college graduates in the same way as high school graduates\(^7\). Table 2 shows the coefficient on ability for each education level. As in the literature, the slope of ability is higher as the education level is higher and returns to education vary across ability.

I assume $\pi^e_\eta(\eta' | \eta)$ is a Markov chain with two states $\eta_H$ and $\eta_L$, which is specific to each education level which has exactly the same persistence and conditional variance as the AR(1) process:

$$\ln \eta' = \rho^e \ln \eta + \epsilon_{\eta'}^c, \quad \epsilon_{\eta'}^c \sim N(0, \sigma_{\eta'}^e)$$  \hspace{1cm} (48)$$

In addition, I assume the initial $\eta$ for each worker is drawn from the invariant distribution $\Pi^e$. After filtering out age effects, I include a fixed effect and a measurement error and employ a Minimum Distance Estimator, which use as moments the covariances of the wage residuals at different lags and age groups, separately for each education level. In Appendix C, I discuss sample selections and the detail of the estimation procedures. Table 3 is the estimates of the parameters.

\(^7\)Since students with high $\varepsilon_c$ are self-selected as college graduates or college dropouts, I estimate using the Heckman two step estimators.
<table>
<thead>
<tr>
<th></th>
<th>HS</th>
<th>CD</th>
<th>CG</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho^x)</td>
<td>0.9311</td>
<td>0.9439</td>
<td>0.9352</td>
</tr>
<tr>
<td>(\sigma^2_y)</td>
<td>0.0171</td>
<td>0.0217</td>
<td>0.0273</td>
</tr>
</tbody>
</table>

Table 3: Estimated parameters of wage process

3.2 Intergenerational Ability Transmission

Newborns draw their high school ability \(\theta_h\) from a normal distribution whose mean depends on the ability of their parents.

\[
\theta_h | \theta \sim N(\bar{m} + m_\theta \theta, \sigma^2_y)
\]

The high school ability is formed partly as a result of genetics, which leads to a correlation between parents’ and children’s ability. Daruich (2017) argues that educational investment by their parents before high school is correlated with the parents’ earnings and education levels, which are based on parent’s ability. In addition, as Cunha and Heckman (2007), Cunha (2013), and Daruich (2017) argue, educational investment by parents has a significant effect on ability. It follows that parents’ ability also affects children’s ability.

In order to estimate the conditional mean of inter-generational ability transmission, I regressed children’s ability on parents’ ability in NLSY79 to obtain the parameters 0.46\(^8\). A standard deviation increase in parent’s ability leads to an increase in children’s high school ability by .46 of a standard deviation.

3.3 Subsidies and Loans

I measure the cost of education from the US Department of Education’s Digest of Education Statistics. As in Jones and Yang (2015), the education cost is education and general (E&G) category which excludes dormitories and hospitals. The education cost per student is $17,187 in 2000.

Since the Federal Pell subsidy Program, which is the largest source of subsidies, is need-based and only a small fraction of state subsidies are merit-based (less than 18% according to Abbott et al. (2013)), I assume subsidies are not merit-based in the status-quo.

I adopt Abbott et al. (2013) for the cost of college for enrollees and the subsidy system of the Unites States (see Table 4 for federal and state subsidies). The cost of college for enrollees is set to

\(^8\)For college dropouts and college graduates, \(\theta = \theta_c\) but I use ln AFQT80 as an instrument as in the estimation of labor productivity process.
$6,710. It follows that the government subsidizes the education sector by the difference between the
cost of education above and the cost for enrollees, $17,187 – $6,710 = $10,477. In the model, the
subsidies for enrollees are the sum of this subsidy and the subsidies as in Table 4.

The largest federal loan program in the US is the Federal Family Education Loan Program. Among
federal loans, the Stafford loan program was the most common for the undergraduates so I focus on
Stafford loans. A Stafford loan can be either subsidized or unsubsidized. The difference between these
two is interest payments during college but borrowers have to pay interest after college for either type.
I focus on unsubsidized loan. Students’ interest rate is the prime rate plus 2.3% (= r*, annual). I
assume students face a borrowing limit dependent on age. The annual Stafford loan limits are $2,625
and $3,500 for freshmen and sophomores. The loan limit for the first half is assumed to be $6,125
(= $2,625 + $3,500). The loan limit for the second half is $23,000 which is the aggregate Stafford
loan limit. The borrowing limits for workers are based on self-reported limits on unsecured credit by
education level from 2001 Survey of Consumer Finances.

### 3.4 Share of Skilled Labor from College Dropouts

The share of skilled labor among college dropouts χ is one of the key parameters for the effect of
year-dependent subsidies. If χ = 0 and all the labor provided by college dropouts is unskilled labor,
encouraging enrollees to graduate increases the fraction of skilled labor in the economy and has a large
effect on the skill premium. If χ = 1 and all the labor provided by college dropouts is skilled labor,
it is just a redistribution within skilled labor and year-dependent subsidies will have a minor effect on
the skill premium.

Moreover, it is hard to calibrate χ using the skill premium of college dropouts. The fact that
college dropouts earn more than high school graduates does not determine whether the labor of college
dropouts is unskilled but they are more productive than high school graduates (high ε^{CD}) or because
their labor is skilled whose wage is higher than that of unskilled labor by high school graduates (high
χ). I need to calibrate χ using another source of data and calibrate ε^{CD} to match the skill premium
Figure 2: The share of jobs and workers of each education level

I rely on two separate sources of data. First, Torpey and Watson (2013) present the proportion of jobs in the United States that were in occupations that typically require each education level. They show that 23%, 11%, 39%, 27% of jobs require college graduation, some college, high school graduation, less than high school respectively, which is shown in the top bar of Figure 2. I interpret jobs for some college and more as skilled labor and others as unskilled labor. Then the share of skilled labor is 34% and the share of unskilled labor is 66% in the United States. The second data I use is the shares of workers of each education level at age 25 are 28%, 39%, 24%, 9% for each according to NLSY97, which is shown in the bottom bar of Figure 2. I assume college graduates provide only skilled labor and that high school graduates and high school dropouts provide only unskilled labor. Then the share of jobs of skilled labor left for college dropouts is 6% = (34% − 28%) of all the jobs in the economy and those of unskilled labor for college dropouts is 33% = (39% + 27% − 24% − 9%). This implies that the share of skilled labor by college dropouts is \( \chi = 15\% (= 6\%/33\% + 6\%) \).

### 3.5 Government Policy

The government consumption and investment over GDP in the United States in 2000 is 17.8% from Bureau of Economic Analysis. Since the government expenditure on tertiary education in the United States in 2000 is 0.7% of GDP (OECD), \( g \) is set to 17.8% − 0.7% = 17.1%. The tax on consumption and capital income are \( \tau_c = 0.07 \) and \( \tau_k = 0.27 \) respectively (see McDaniel (2007)).

---

9 They use the May 2013 data of Occupational Employment Statistics survey (employment data) and Employment Projections program (occupational education-level designations) by the U.S. Bureau of Labor Statistics. I assume the jobs for “Bachelor’s degree”, “Master’s degree”, and “Doctoral or professional degree” in their categories require college graduation. I assume the jobs for “Some college, no degree”, “Associate’s degree”, and “Postsecondary nondegree award” require some college.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Coef of relative risk aversion</td>
<td>4</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>Study time</td>
<td>0.25</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Equivalence scale</td>
<td>0.3</td>
</tr>
<tr>
<td>$\chi$</td>
<td>CD share of skilled labor</td>
<td>0.15</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>33.3%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation (annual)</td>
<td>7.55%</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity of substitution in production</td>
<td>1.41</td>
</tr>
<tr>
<td>$\iota^s$</td>
<td>Stafford interest premium (annual)</td>
<td>2.3%</td>
</tr>
<tr>
<td>$A_{t/1}^c$</td>
<td>Borrowing constraint for 1st half (Stafford loan)</td>
<td>$6,125$</td>
</tr>
<tr>
<td>$A_{t/2}^c$</td>
<td>Borrowing constraint for 2nd half (Stafford loan)</td>
<td>$23,000$</td>
</tr>
<tr>
<td>$A_{HS}^c$</td>
<td>Borrowing constraint, HS (SCF)</td>
<td>$20,754$</td>
</tr>
<tr>
<td>$A_{CD}^c$</td>
<td>Borrowing constraint, CD (SCF)</td>
<td>$24,833$</td>
</tr>
<tr>
<td>$A_{CG}^c$</td>
<td>Borrowing constraint, CG (SCF)</td>
<td>$37,832$</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Consumption tax rate</td>
<td>7%</td>
</tr>
<tr>
<td>$\tau_h$</td>
<td>Capital income tax rate</td>
<td>27%</td>
</tr>
<tr>
<td>$g$</td>
<td>Gov cons to GDP ratio</td>
<td>17.1%</td>
</tr>
</tbody>
</table>

Table 5: Parameters determined outside the model.

3.6 The Remaining Parameters

Given the parameter values set outside the model in Table 5, there are 16 remaining parameters: bias of expectation of college ability ($\mu_0^c, \mu_1^c$), college utility ($\lambda_0^0, \lambda_1^0, \lambda_2^0$), the variance of college ability $\sigma_c$, productivity of labor ($a^S, e^{CD}$), education cost $\kappa$, utility parameters ($\mu, \beta, \nu$), lump-sum transfer $d$, overseeing cost $\iota$, and inter-generational ability parameters ($\bar{m}, \sigma_h$).

I choose 27 moments in Table 7 and minimize the average Euclidean percentage deviation of the model from the data\textsuperscript{10}. The enrollment and graduation rates across ability and family income and the skill premiums are the main theme of the paper. Optimism is a key driver of college dropouts and I try to match the difference between the graduation rates students expect and the actual one. According to Stinebrickner and Stinebrickner (2012), on average, students of the college they survey believe that there is an 86% chance of graduating while approximately 60% of students graduate. The percent difference is $43% (= 0.86/0.60 - 1)$. Since the educational decisions are strongly dependent on ability, matching the mean and standard deviation of high school ability are also important.

The third column of Table 12 presents the calibrated values. Note that the calibrated value of $\mu_c$ is positive. This implies that enrollees are optimistic about their college ability on average. Since the standard deviation of college ability is 0.36\textsuperscript{11}, the bias for the mean ability is 49% of the standard

\textsuperscript{10}For the mean of high school ability, I chose 5.03, which is the mean of ln AFQT80 before normalization, for the denominator of the percent deviation. I do not take the percent deviation for the enrollment and graduation rates.

\textsuperscript{11}The square root of the sum of the variance of high school ability and $\sigma_c^2$. $0.36 = \sqrt{0.215^2 + 0.287^2}$
deviation of college ability. In addition, enrollees with lower high school ability are more optimistic than enrollees with higher high school ability. These characteristics are consistent with the bias of college GPA observed in Stinebrickner and Stinebrickner (2012). is negative and agents derive disutility from college. A positive implies that the disutility is smaller for agents with high ability than agents with low ability.

The model fit is presented in Table 7 and Figures 3 and 4. In general, the model fits well considering over-identification of 16 parameters against 27 moments. In the data, ability is correlated with enrollment and graduation more than family income and the model captures this pattern. Although the graduation rates across family income are somewhat flatter than the data, they capture the key pattern. The enrollment and graduation rates are higher for the second quartile than for the third quartile. This might be because there are only three bins for family income and there is a jump of subsidies when people cross over the threshold of family income.

### 3.7 Validation Exercises

**Partial Equilibrium Effect of Year-Invariant subsidies**

The elasticity of enrollment with regard to tuition or subsidies has been extensively examined in the micro empirical literature. I simulate the partial equilibrium response of enrollment to a $1,000 increase
<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment rate of ability quartile</td>
<td>(figure)</td>
<td>(figure)</td>
</tr>
<tr>
<td>Graduation rate of ability quartile</td>
<td>(figure)</td>
<td>(figure)</td>
</tr>
<tr>
<td>Enrollment rate of family income quartile</td>
<td>(figure)</td>
<td>(figure)</td>
</tr>
<tr>
<td>Graduation rate of family income quartile</td>
<td>(figure)</td>
<td>(figure)</td>
</tr>
<tr>
<td>Skill premium for CG(^a)</td>
<td>89.0%</td>
<td>89%</td>
</tr>
<tr>
<td>Skill premium for CD</td>
<td>19.8%</td>
<td>20%</td>
</tr>
<tr>
<td>Expected/Actual graduation rate -1</td>
<td>0.442</td>
<td>0.433</td>
</tr>
<tr>
<td>Educ cost/mean income at 48</td>
<td>0.309</td>
<td>0.33</td>
</tr>
<tr>
<td>Hours of work</td>
<td>33.7%</td>
<td>33.3%</td>
</tr>
<tr>
<td>(K/Y)</td>
<td>1.312</td>
<td>1.325</td>
</tr>
<tr>
<td>Transfer/mean income at 48</td>
<td>67.0%</td>
<td>66%</td>
</tr>
<tr>
<td>log pre-tax/post-tax income</td>
<td>60.7%</td>
<td>61%</td>
</tr>
<tr>
<td>borrowers</td>
<td>6.83%</td>
<td>6.3%</td>
</tr>
<tr>
<td>mean of AFQT</td>
<td>-0.0594</td>
<td>0</td>
</tr>
<tr>
<td>std deviation of AFQT</td>
<td>0.215</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 7: Moments matched.

\(^a\)The skill premiums are from full-time workers in Current Population Survey (CPS) IPUMS (Flood, King, Rodgers, Ruggles and Warren (2018))

in subsidies for all the college years and family income evenly. All the prices and the distribution of initial state are fixed at the status-quo level and additional subsidies are given to only one generation.

The aggregate enrollment rate of the affected generation increases by 0.60 percentage points in the simulation. The micro-empirical literature has estimates of the effect of subsidies on enrollment by Dynarski (2002), Kane (1994), and Cameron and Heckman (2001). While this literature argues that the enrollment rate of groups benefitting from an additional subsidy of $1,000 increases by between 3 to 6 percentage points, Hansen (1983) and Kane (1994) argue that there is less evidence of a rise in college enrollment of the target of the Pell Grant program (See Kane (2006) for the empirical literature). Therefore the simulation is broadly in the range of the literature.

Since this paper studies the effect of subsidies on graduation, it is interesting to know how an increase in enrollment leads to changes in the fractions of college graduates and dropouts separately. An increase in subsidies has two effects. Enrollment can increase due to an increase in subsidies for the first half period. Graduation can increase due to an increase in subsidies for the second half. In the simulation, the fraction of college graduates increases by 0.33 percentage points and that of college dropouts increases by 0.28 percentage points. Not only the people who are induced to enroll by the additional subsidy but also those who would already have enrolled without the additional subsidy have incentive to complete. This is consistent with Dynarski (2008), Castleman and Long (2016), Scott-Clayton (2011), who all find a positive effect of subsidies on graduation.
Figure 3: Model fit: enrollment and graduation rate for each ability quartile.

Sluggish Increase in College Graduates

The sluggish increase in college graduates in the United States between 1980 and 2000 is a crucial factor to explain the increase in the skill premium. In this subsection, I examine how well the model can explain this sluggish increase. The benchmark calibration is targeted to the United States in 2000 and I assume only the productivity of skilled labor $a^S$ and productivity of college dropouts $\epsilon^{CD}$ change in the model between 1980 and 2000. In particular, I set the values of $a^S$ and $\epsilon^{CD}$ to match the college graduate wage premium 45.9% and the college dropout wage premium 12.4% as observed in 1980 in the United States with the other parameter values fixed. I compute the steady state with the new values and call it “1980 steady state.” While I target the change in the skill premium, I use the change
in the share of college graduates and dropouts as non-targeted moments to compare with the data.\footnote{I use the Current Population Survey IPUMS for the wage premiums in 1980 and the change in the shares of college graduates and dropouts between 1980 and 2000. For the shares of college graduates and dropouts, I follow the definition of Castro and Coen-Pirani (2016).}

Table 8 shows the change in the share of college graduates and dropouts between “1980 steady state” and the benchmark calibration targeted to year 2000. As the college graduate premium increases by 43.2 percentage points from 1980 to 2000, the third column shows the share of college graduates increases by 5.44 percentage points. Although this is smaller than the data in the fourth column, the model can explain the sluggish increase in the share of college graduates. Interestingly, the share of college dropouts does not change in the model with the college dropout premium increasing, which is
consistent with the data. The increase in the college graduate wage premium cancels out the effect of the increase in the college dropout wage premium.

4 Results

4.1 Progressive Tax versus College Subsidies

In this chapter, I examine which is more effective in terms of reducing inequality and social welfare, progressive income tax or college subsidies. First, I derive the optimal progressivity of labor income tax given the current college subsidies and find the welfare gain. The optimal policy problem is defined as

$$\max_{d, \tau} W = \sum_j N_j \left( \int V_j(s_j)d\mu_j(s_j) + \int V_s(s_j)d\mu_j(s_j) \right)$$  \hspace{1cm} (50)

subject to the government budget constraint. $N_j$ is the relative population of age $j$ calculated from survival rates $\varphi_j$. Next, I derive the optimal college subsidies given the current progressivity of labor income tax and find the welfare gain. The optimal policy problem is defined as

$$\max_{s, \tau} W = \sum_j N_j \left( \int V_j(s_j)d\mu_j(s_j) + \int V_s(s_j)d\mu_j(s_j) \right)$$  \hspace{1cm} (51)

subject to the government budget constraint.

The result of the optimal progressivity of labor income tax and college subsidies are in Table . The second column of the table shows the optimal tax is more progressive than the current policy. With this policy, the second column of Table 17 show that the enrollment rate decreases by 0.5 percentage points. However, the decrease is only from the decrease in college dropouts in the economy. With progressive income tax, marginal enrollees lose incentive to enroll due to a small difference in disposable income between college dropouts and high school graduates and stop enrolling. While the difference in disposable income between college graduates and dropouts decreases, the effect is larger for the
Table 9: Optimal policy.

<table>
<thead>
<tr>
<th></th>
<th>status quo</th>
<th>optimal tax</th>
<th>optimal subsidies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>0.127</td>
<td>0.167</td>
<td>(0.127)</td>
</tr>
<tr>
<td>$s$ ($\times$ benchmark)</td>
<td>1</td>
<td>(1)</td>
<td>2.62</td>
</tr>
<tr>
<td>$\tau_\ell$</td>
<td>36.6%</td>
<td>42.3%</td>
<td>37.8%</td>
</tr>
</tbody>
</table>

marginal enrollee than the marginal college graduate. The skill premiums decrease a little and it is counterintuitive because the share of skilled workers decrease. However, there is another counter effect: the labor hours by unskilled workers decrease more than that by skilled labor. The income effect on leisure is higher for low-income workers if the lump-sum transfer is the same amount for everyone. The standard deviations of consumption and asset holdings decreases significantly due to progressive labor income tax. The consumption equivalence of the welfare gain by this policy is 0.24%.

The third column of the table shows that more college subsidies are optimal. With this policy, the second column of Table 17 show that the enrollment rate increases by 5.9 percentage points and the share of college graduates increases by 2.9 percentage points. With the increase in college subsidies, more people enroll and graduate and the college wage premium decreases by 18.2 percentage points. The wage of college graduates decreases while the wages of college dropouts and high school graduates increase. The consumption equivalence of the welfare gain by this policy is 0.67%.

To summarize, the welfare gain is higher for the optimal college subsidies than for the optimal progressive labor income tax. The reason is similar to Krueger and Ludwig (2016). First, the level effect is significantly negative for the optimal progressive labor income tax because of the distortion of labor hours. In addition, with the large decrease in the skill premium, inequality and uncertainty decrease.

To compare how college dropout affects the optimal policy, I consider a hypothetical economy in which there is no dropout options for agents. In particular, college enrollees are not allowed to drop out at the beginning of the second period. I re-calibrate college utility parameters $\lambda$, $\lambda^B$, $\lambda^S_1$, and productivity of skilled labor $a^S$ to match the share of college graduates and the skill premium from the data (I assume $\lambda^S_2 = \lambda^S_1$). The parameters are in Table 12. I solve again the same optimal problems in this economy and compare with the previous results.

Table 13 shows the optimal policy in this economy without college dropout. Although the optimal progressivity does not change from the previous case, the optimal college subsidies are much higher. In this economy, enrollment necessarily leads to graduation and college subsidies are more powerful
<table>
<thead>
<tr>
<th></th>
<th>status-quo</th>
<th>optimal tax</th>
<th>optimal subsidies</th>
</tr>
</thead>
<tbody>
<tr>
<td>enrollment rate</td>
<td>73.7%</td>
<td>73.2%</td>
<td>79.6%</td>
</tr>
<tr>
<td>graduation rate</td>
<td>43.2%</td>
<td>43.6%</td>
<td>43.8%</td>
</tr>
<tr>
<td>% of college graduates</td>
<td>31.9%</td>
<td>31.9%</td>
<td>34.8%</td>
</tr>
<tr>
<td>% of college dropouts</td>
<td>41.9%</td>
<td>41.3%</td>
<td>44.7%</td>
</tr>
<tr>
<td>college graduates premium</td>
<td>89.1%</td>
<td>86.4%</td>
<td>70.9%</td>
</tr>
<tr>
<td>college dropouts premium</td>
<td>19.8%</td>
<td>19.9%</td>
<td>17.6%</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.289</td>
<td>0.289</td>
<td>0.289</td>
</tr>
<tr>
<td>$K$</td>
<td>0.379</td>
<td>0.380</td>
<td>0.380</td>
</tr>
<tr>
<td>$C$</td>
<td>0.192</td>
<td>0.192</td>
<td>0.192</td>
</tr>
<tr>
<td>$u_{CG}$</td>
<td>0.380</td>
<td>0.380</td>
<td>0.368</td>
</tr>
<tr>
<td>$u_{CD}$</td>
<td>0.384</td>
<td>0.384</td>
<td>0.393</td>
</tr>
<tr>
<td>$u_{HS}$</td>
<td>0.384</td>
<td>0.384</td>
<td>0.398</td>
</tr>
<tr>
<td>std $c$</td>
<td>0.113</td>
<td>0.103</td>
<td>0.111</td>
</tr>
<tr>
<td>std $a$</td>
<td>0.431</td>
<td>0.387</td>
<td>0.423</td>
</tr>
<tr>
<td>std $h$</td>
<td>0.0852</td>
<td>0.0920</td>
<td>0.0858</td>
</tr>
<tr>
<td>std wage</td>
<td>0.480</td>
<td>0.469</td>
<td>0.470</td>
</tr>
<tr>
<td>CEV</td>
<td>0.24%</td>
<td></td>
<td>0.67%</td>
</tr>
</tbody>
</table>

Table 10: Aggregates.

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Level</th>
<th>Uncertainty</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal tax</td>
<td>-0.45%</td>
<td>-3.22%</td>
<td>+0.86%</td>
<td>+0.36%</td>
</tr>
<tr>
<td>Optimal subsidies</td>
<td>+4.02%</td>
<td>-1.24%</td>
<td>+3.88%</td>
<td>+0.76%</td>
</tr>
</tbody>
</table>

Table 11: Welfare decomposition.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>college utility intercept</td>
<td>-11.75</td>
</tr>
<tr>
<td>$\lambda^0$</td>
<td>college utility slope</td>
<td>35.27</td>
</tr>
<tr>
<td>$\lambda^\phi$</td>
<td>first period college taste</td>
<td>10.64</td>
</tr>
<tr>
<td>$a^S$</td>
<td>productivity of skilled labor</td>
<td>0.509</td>
</tr>
</tbody>
</table>

Table 12: Parameters re-calibrated for the case without college dropout.
<table>
<thead>
<tr>
<th></th>
<th>status-quo</th>
<th>optimal tax</th>
<th>optimal subsidies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>0.127</td>
<td>0.157</td>
<td>(0.127)</td>
</tr>
<tr>
<td>$s$ (× benchmark)</td>
<td>1</td>
<td>(1)</td>
<td>8.68</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>36.8%</td>
<td>41.1%</td>
<td>40.7%</td>
</tr>
</tbody>
</table>

Table 13: Optimal policy without college dropout.

<table>
<thead>
<tr>
<th></th>
<th>status-quo</th>
<th>prev optimal subsidies</th>
<th>optimal subsidies</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of college graduates</td>
<td>34.0%</td>
<td>45.8%</td>
<td>59.3%</td>
</tr>
<tr>
<td>college graduates premium</td>
<td>84%</td>
<td>32.1%</td>
<td>-4%</td>
</tr>
<tr>
<td>CEV</td>
<td>3.87%</td>
<td></td>
<td>5.75%</td>
</tr>
</tbody>
</table>

Table 14: Aggregates without college dropout.

tools to increase college graduates and decrease the skill premium. To see this, the second column of Table 14 show the case without college dropout under the previous optimal college subsidies. With the same subsidies as before, the share of college graduates increases and the skill premium decreases considerably more than the case with college dropout. Accordingly, the welfare gain is also much greater. Including college dropout in the model changes the effect of college subsidies on the share of college graduates, the skill premium, and welfare gain.

With the new optimal policy, college graduates increase by 25.3 percentage points and the skill premium decrease to be -4%. The welfare gain is 6% and it has a large effect. The optimal college subsidies are smaller if we consider college dropout in the model. Still, college subsidies are more powerful than progressive labor income tax.

The optimal mix of progressive income tax and college subsidies is very similar to the mix of the optimal progressive income tax and the optimal college subsidies. The effects and welfare gain are basically the combination of those of the previous optimal policies. I show the results in Appendix.

5 Conclusion

References


A Computation of Equilibria

This section describes the method of computing equilibria. Prices are normalized such that the average annual income of high school graduates at age 48 is $51,933.

1. Starting from an initial vector of aggregate variables $\mathbf{w} = \left( \frac{K}{H}, \frac{H^S}{H}, H, \tau_1, \hat{h} \right)$, compute prices $r, w_S, w_U$ and pension $p(e, \theta)$ required for individual decision problems.

2. Given these variables, solve individuals decision problems. This step consists of sub-steps.

(a) Solve backward the Bellman equations for age $j = J, \ldots, j_b + 1$. The number of grids for assets is 30 and that for high school ability and college ability is 5. The number of grids for college taste is $30^{13}$. I apply the Endogenous grid method.

---

$^{13}$The grids of assets depend on age. The range of the grids for high school ability is $[-.55,.55]$ and that for college ability is $[-1.1,1.1]$. The range of grids for college ability is broader because of the higher variance. That of college taste is $[-2,2]$. 

33
(b) Given an initial guess of the value function of newborns $V^0$, solve backward the individual problems from $j = j_b, \ldots, 0$ for value functions and policy functions. It leads to a new $V_0$.

(c) I implement a Howard-type improvement algorithm: that is, with the decision rules fixed, update $V_0$ until the guess and the value functions converge.

(d) Given the converged $V_0$, resolve decision rules of individuals until convergence.

3. I interpolate linearly assets and ability to 80 and 25.

4. Starting from an initial measure $\mu_0$ and given decision rules, solve forward from $\mu_0$ to $\mu_f$ and update $\mu_0$ until convergence.

5. Given the measures, derive the new aggregate variables $K, H, H^S, \bar{h}$ and $\tau_t$ from the government budget constraint and go back to step 2.

B Pension

The average life time income is

$$\hat{y}(e, \theta) = \frac{\sum_{j=2}^{j-1} \omega^j e_j^e(\theta, 1) \bar{h}}{j_r - 2}$$

(52)

The pension formula is given by

$$p(e, \theta) = \begin{cases} 
  s_1 \hat{y}(e, \theta) & \text{for } \hat{y}(e, \theta) \in [0, b_1) \\
  s_1 b_1 + s_2 (\hat{y}(e, \theta) - b_1) & \text{for } \hat{y}(e, \theta) \in [b_1, b_2) \\
  s_1 b_1 + s_2 (b_2 - b_1) + s_3 (\hat{y}(e, \theta) - b_2) & \text{for } \hat{y}(e, \theta) \in [b_2, b_3) \\
  s_1 b_1 + s_2 (b_2 - b_1) + s_3 (b_3 - b_2) & \text{for } \hat{y}(e, \theta) \in [b_3, \infty) 
\end{cases}$$

(53)

where $s_1 = 0.9$, $s_2 = 0.32$, $s_3 = 0.15$, $b_1 = 0.22\bar{y}$, $b_2 = 1.33\bar{y}$, $b_3 = 1.99\bar{y}$, $\bar{y} = $28,793 annually.

C Labor Productivity Process

I use the Panel Study of Income Dynamics (PSID). I use data for the waves from 1968 to 2014 (from 1997 the PSID has become biannual). I restrict the SRC sample of heads whose age is between 25 and 63, which leads to 11,512 samples. I restrict observations to those with positive hours of labor in the individual (but lower than 10,000 annually). I keep only people who do not report extreme changes of
hourly wages (changes in log earnings larger than 4 or less than -2) or extreme hourly wages (less than $1 or larger than $400). I keep only people with 8 or more year observations, which leads to 3,518 samples. Quadratic age polynomials are separately estimated, by education group with year dummies. High school graduates are people with 12 years highest grade completed. College dropouts are with highest grade completed between 13 and 15. College graduates are with highest graded completed greater than 16. The estimation result is in Table 15. I take the average of the productivity of the corresponding two years for the productivity of \( j \) in the model and normalize the process so that the productivity at the first period after education is unity.

For the law of motion of residuals, I use the same sample and use the residuals of the regression for the age profile. For estimation, I normalize job experience to 0 as age minus 18 for high school graduates, age minus 20 for college dropouts, and age minus 22 for college graduates and apply a Minimum Distance Estimator for different lags and different experience of the residuals for age 25 to 40. I assume there is a measurement error from an identical and independent distribution. I also assume there is a fixed effect and estimate the persistence \( \rho^* \), the variance of the residual \( \sigma^* \), the variance of the fixed effect, and the variance of the measurement error for each education level.

Ability is approximated by the log of AFQT80 raw score. To estimate the coefficient on ability in effective labor, I use NLSY79 of 11,864 people. For the ability regression, I restrict samples aged between 25 and 63, which leads to 11,627 people. Since NLSY79 does not include old people, I rely on PSID to estimate the age effects. After the age effect is filtered out, I regress hourly wages on ability for each education levels (HS, CD, and CG). As in the selection of PSID, I keep only people who do not report extreme changes of hourly wages (changes in log earnings larger than 4 or less than -2) or extreme hourly wages (less than $1 or larger than $400). I keep only people with 8 or more year observations, which leads to 3,851 people. I exclude enrolled students and hours worked per week less than 20. I also control dummies for each year.

To handle the selectivity bias problem, I use Heckman two step estimators. For high school graduates, I assume a linear selection equation of log AFQT80 and year dummies and the whole sample

<table>
<thead>
<tr>
<th></th>
<th>HS</th>
<th>CD</th>
<th>CG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>.0530181</td>
<td>.0684129</td>
<td>.0955783</td>
</tr>
<tr>
<td></td>
<td>(.0030501)</td>
<td>(.0040353)</td>
<td>(.0036997)</td>
</tr>
<tr>
<td>Age^2</td>
<td>-.0005314</td>
<td>-.0006872</td>
<td>-.0009521</td>
</tr>
<tr>
<td></td>
<td>(.0000356)</td>
<td>(.0000474)</td>
<td>(.0000429)</td>
</tr>
</tbody>
</table>

Table 15: Age profile estimates of each education.
is people whose educations are higher than high school graduates. Among the people who graduate high school, people with less ability are self-selected as high school graduates. For college dropouts and college graduates, I assume a linear selection equation of log AFQT80 and year dummies and the whole sample is both college dropouts and graduates. Among the people who enroll in college, people with high ability are self-selected as college graduates and people with low ability as college dropouts.

D Intergenerational ability transmission

To estimate the transmission of ability from parents to children, I rely on the data from NLSY79 to approximate parents’ ability and ”NLSY79 Child & Young Adult” for children. The ”NLSY79 Child & Young Adult” survey started in 1986 and has occurred biennially since then. This survey provides information of test scores of the children of the women in the NLSY79 dataset. The test scores reported include the PIAT Math, the PIAT reading recognition, and the PIAT reading comprehension.

There are 11,521 children born to 4,934 female respondents of NLSY79. To focus on cognitive ability, I use the PIAT Math to approximate high school ability of children. In particular, I use the standardized PIAT Math score, which adjusts different age in which the test is taken and is comparable across age. If there are multiple PIAT Math scores for a child, I use only the latest score. I exclude the children whose PIAT Math scores are missing. This leaves me with 9,232 children born to 4,055 mothers.

I use AFQT scores to measure mothers’ ability. In particular, I only use the respondents whose both AFQT scores and education levels are not missing. I focus on people with high school degrees. This leaves me with 6,193 children born to 2,828 mothers.

D.1 Optimal Mix

Using this model, I derive what is the optimal progressivity of labor income tax and college subsidies. The objective of the optimal policy is Utilitarian for people with weights of the initial distribution of households in the equilibrium under the status-quo as follows. The optimal policy problem is defined as

$$
\max_{d,s,r} W = \sum_j N_j \left( \int V_j(s_j) d\mu_j(s_j) + \int V_j^c(s_j^c) d\mu_j(s_j^c) \right)
$$

(54)

subject to the government budget constraint.
<table>
<thead>
<tr>
<th></th>
<th>status-quo</th>
<th>optimal mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
<td>0.127</td>
<td>0.158</td>
</tr>
<tr>
<td>(s) (× benchmark)</td>
<td>1</td>
<td>2.40</td>
</tr>
<tr>
<td>(\tau_f)</td>
<td>36.6%</td>
<td>42.1%</td>
</tr>
</tbody>
</table>

Table 16: Optimal policy.

<table>
<thead>
<tr>
<th></th>
<th>Status-quo</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>enrollment rate</td>
<td>73.7%</td>
<td>78.9%</td>
</tr>
<tr>
<td>graduation rate</td>
<td>43.2%</td>
<td>44.3%</td>
</tr>
<tr>
<td>% of college graduates</td>
<td>31.9%</td>
<td>34.9%</td>
</tr>
<tr>
<td>% of college dropouts</td>
<td>41.9%</td>
<td>43.9%</td>
</tr>
<tr>
<td>college graduates premium</td>
<td>89.1%</td>
<td>70.0%</td>
</tr>
<tr>
<td>college dropouts premium</td>
<td>19.8%</td>
<td>18.0%</td>
</tr>
<tr>
<td>(Y)</td>
<td>0.289</td>
<td>0.273</td>
</tr>
<tr>
<td>(K)</td>
<td>0.379</td>
<td>0.350</td>
</tr>
<tr>
<td>(C)</td>
<td>0.192</td>
<td>0.183</td>
</tr>
<tr>
<td>(w^{CG})</td>
<td>0.380</td>
<td>0.362</td>
</tr>
<tr>
<td>(w^{CD})</td>
<td>0.384</td>
<td>0.389</td>
</tr>
<tr>
<td>(w^{HS})</td>
<td>0.384</td>
<td>0.394</td>
</tr>
<tr>
<td>std (c)</td>
<td>0.113</td>
<td>0.103</td>
</tr>
<tr>
<td>std (a)</td>
<td>0.431</td>
<td>0.390</td>
</tr>
<tr>
<td>std (h)</td>
<td>0.0852</td>
<td>0.0910</td>
</tr>
<tr>
<td>std wage</td>
<td>0.480</td>
<td>0.462</td>
</tr>
</tbody>
</table>

CEV 0.89%

Table 17: Optimal policy of year-dependent subsidies.

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Level</th>
<th>Uncertainty</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>+3.49%</td>
<td>-3.42%</td>
<td>+4.33%</td>
<td>+0.93%</td>
</tr>
</tbody>
</table>

Table 18: Welfare decomposition.