COLLECTIVE HOLD-UP

MATIAS IARYCZOWER AND SANTIAGO OLIVEROS

ABSTRACT. We consider a dynamic process of coalition formation in which a principal bargains sequentially with a group of agents. This problem is at the core of a variety of applications in economics and politics, including a lobbyist seeking to pass a bill, an entrepreneur setting up a start-up, or a firm seeking the approval of corrupt bureaucrats. We show that when the principal’s willingness to pay is high, reallocating bargaining power from the principal to the agents generates delay and reduces agents’ welfare. This occurs in spite of the lack of informational asymmetries or discriminatory offers. When this collective action problem is severe enough, agents prefer to give up considerable bargaining power in favor of the principal.

keywords: bargaining, contracting externalities, political economy, vote buying, delay.
1. Introduction

In this paper, we study how bargaining power affects the efficiency of collective decisions in dynamic processes of coalition formation with externalities. We focus on problems in which a principal bargains bilaterally with members of a group to obtain their agreement. Consider for example the President seeking to influence legislators of his own party to pass a policy proposal, a firm negotiating with buyers the adoption of a new technology with network externalities, an entrepreneur seeking to form a start-up, or a raider attempting to takeover a target firm.

A salient feature of these problems is that the principal typically bargains with agents sequentially. As a result, the offers the principal makes to, or receives from, an agent, will generally depend on how advanced the negotiation process is. This consideration becomes important when agents are strategically farsighted, because the principal’s ability to successfully negotiate with each agent depends on their expectations about the nature of future trades.

In this context, we study how the allocation of bargaining power between principal and agents affects whether good proposals are undertaken and bad ones rejected, whether good proposals are adopted with or without delay, and how rents are distributed between principal and agents. Would legislators be better off if each of them has a stronger bargaining position against a lobbyist or the executive? Would this lead to a less efficient policy-making process? Would markets with network externalities in which buyers have a stronger bargaining position delay innovation to better technologies?

Conventional wisdom suggests that in the absence of asymmetric information, increasing agents’ bargaining power relative to the principal would improve agents’ welfare and have no impact on the efficiency of collective decisions. We show, however, that both of these assertions are generally false. In particular, we show that when the principal’s willingness to pay is high, redistributing bargaining power from the principal to the agents first induces and then increases delay, and reduces agents’ welfare. As a result, agents are better off conceding substantial bargaining power to the principal.

In our model, a principal negotiates with a group of $n$ agents bilaterally and sequentially.\footnote{This contrasts with decentralized processes of coalition formation in the absence of a principal, as in Gul (1989), Baron and Ferejohn (1989), Banks and Duggan (2000) or Gomes (2005).} There is no deadline for reaching an agreement, and no asymmetric information. In each meeting, the principal bargains with an agent over the terms by which the agent would
commit his support to the principal. If an agreement is reached, the agent commits his support to the principal and exits negotiations. Otherwise, the agent remains uncommitted. The principal needs to obtain the agreement of $q < n$ agents to implement a reform, action, or policy change which affects the payoffs of all agents.\footnote{This can be taken as a particularly simple way to micro-found the source of potential externalities, but also appears quite literally in some applications, such as the approval of a bill requires a majority of senators (or some supermajority of a party), takeovers requires a majority of shares, or technology adoption with increasing returns to scale requires some $q$ buyers to be on board.} When this happens, the principal obtains a payoff $v > 0$, agents who committed their support to the principal obtain $z > 0$, and agents who remained uncommitted obtain $w \in \mathbb{R}$, where $w > 0$ ($w < 0$) implies that there are positive (negative) externalities on uncommitted agents, and $w = 0$ implies that there are no externalities on uncommitted agents. All players have a discount factor $\delta \in (0, 1)$.

To consider arbitrary allocations of bargaining power between the principal and each agent while maintaining the structure of the game fixed, we assume that in a bilateral meeting the principal makes an offer with probability $\phi \in [0, 1]$, and the agent makes an offer with probability $1 - \phi$.\footnote{This formulation is formally equivalent to nesting an infinite horizon bilateral bargaining in our game, where one of the sides decides whether to enter in negotiations or not, and in any period of the negotiation phase after a proposal is rejected the principal (agent) makes offers with probability $\phi$ (respectively, $1 - \phi$).} We then solve for Markov perfect equilibria of the game for each allocation of bargaining power $\phi \in [0, 1]$, where strategies depend on the number of agents the principal still needs in order to win. A well known feature in models of this kind is that if the principal can use discriminatory contracts, one can generate equilibria in which the principal obtains a large profit by exploiting coordination failures among agents (this point is made in Genicot and Ray (2006); see Segal and Whinston (2000), Cai (2000), Chowdhury and Sengupta (2012)). To rule this out, we focus on symmetric equilibria of the game.

We prove existence and uniqueness of equilibrium outcomes, and provide a complete characterization for the case in which the principal’s willingness to pay is large. We show that, irrespective of the direction of the externalities on uncommitted agents, if the principal has enough bargaining power in bilateral negotiations, the equilibrium is efficient, and agents’ welfare increases with their bargaining power in bilateral negotiations. When agents have enough bargaining power, instead, the equilibrium is inefficient, and agents’ welfare decreases with their bargaining power in bilateral negotiations.

The inefficiency we identify is due to what we call a \textit{collective hold-up} problem. When agents have the upper hand over the principal in bilateral negotiations, the principal anticipates that agents trading late in the process will extract a large fraction of the surplus, and is not willing to pay much to agents trading early on. This first part of the mechanism we identify is akin
to the standard hold-up problem, in which trading partners negotiate to divide their trade surplus after making relationship specific investments, in this case obtaining the support of other agents (Williamson (1979), Klein, Crawford, and Alchian (1978), Grossman and Hart (1986)).

The key point here, however, is that by refusing to trade, agents can meet the principal later. This gives agents who meet early with the principal an incentive to hold-out, which more than compensates for the loss induced by delaying completion when the principal’s willingness to pay is high. Holding out indefinitely cannot be an equilibrium, however, for this would destroy the incentives to hold out in the first place. But consistency is restored if the agent negotiating with the principal expects the right amount of delay in the event of not reaching an agreement with the principal. This is because delay in a given state of the bargaining process affects the value of holding out, but not the continuation value after agreement. In this way, the collective action problem among agents complements the principal’s hold-up problem and creates delay and inefficiency.

Equilibrium outcomes for large $v$ have three interesting features. First, the number of transactions with positive expected delay is increasing in agents’ bargaining power in bilateral meetings, so that redistributing bargaining power from the principal to the agents expands the set of transactions which fail with positive probability. Second, we show that both the set of transactions for which there is delay with positive probability and the expected length of delay are increasing in the number of agents $q$ whose approval is required for completion. This conforms to the folk result in political economy that more stringent majority rules are costly because they induce more delay. Third, we show that for a given allocation of bargaining power $\phi$ inducing delay, delay is front-loaded, in the sense that it occurs in the first $k$ transactions. In these first $k$ transactions, the expected delay for each deal increases as the process moves forward. But once the principal obtains the support of $k$ agents, the remaining transactions occur without delay.

For any given allocation of bargaining power for which there is delay in more than one transaction, the expected delay is increasing in the principal’s willingness to pay. In fact, in the limit as $v$ goes to infinity, the expected time for completion goes to infinity, and agents’ payoffs go to zero. Thus, when agents anticipate that the collective hold-up problem would

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4The hold-up problem led to a large literature in modern contract and organization theory, exploring institutional remedies against hold-up (see the articles cited in Che and Sákovics (2004)). In many situations, however, investments must be sunk before agents meet (e.g. Acemoglu and Shimer (1999), Cole, Mailath, and Postlewaite (2001)), or contracting is limited, as is often the case in political economy. We discuss the implications of other contracting arrangements in our model in Section 5.2.
be severe, they prefer to grant considerable bargaining power to the principal, to the point of reducing significantly the number of transactions with delay.

It is important to clarify that delay can arise with positive externalities, no externalities, or even negative externalities on uncommitted agents. All else equal, a larger negative externality on uncommitted agents lowers the value of the lottery induced by holding-out. But when the principal’s willingness to pay is sufficiently large and agents have sufficient bargaining power, holding out can still be attractive, and the main logic for delay is unaltered.

In the second part of the paper we consider the case in which an agent who committed his support to the principal obtains a non-positive payoff, as in corporate takeovers ($z = 0$) or vote buying with audience costs ($z < 0$). We show that when there are positive externalities on uncommitted agents, in any equilibrium in which the project is completed with positive probability there can only be delay when the principal is trying to obtain the support of the very first agent. With this exception, equilibrium either has no delay, or is such that there are no transactions in the initial state. Moreover, we show that when agents have enough bargaining power, the equilibrium is inefficient, and there are no transactions even when agreement would be optimal.

The rest of the paper is organized as follows. In section 2 we place our paper in the context of the literature, and in section 3 we describe the model. In section 4 we present the result in a simplified setting. We fix $z = w > 0$ and analyze the two extreme cases in which either the principal or the agents have full bargaining power in bilateral meetings. In section 5 we present the main result in the general model. We conclude in section 6. All proofs are in the Appendix.

2. Related Literature

Our paper builds on the literature on contracting with externalities (see Grossman and Hart (1980), Rasmusen, Ramseyer, and Wiley Jr (1991), Rasmusen and Ramseyer (1994), Segal (1999, 2003), Segal and Whinston (2000), Genicot and Ray (2006), Iaryczower and Oliveros (2017)). With the exception of Iaryczower and Oliveros (2017), which considers multiple principals, these papers explore problems in which a single principal contracts with a group of agents in the presence of externalities among agents (e.g., corporate takeovers, exclusive contracts, public goods, lobbying).

A standard assumption in the literature is that the principal has full bargaining power. As a result, we know very little about how the allocation of bargaining power affects equilibrium
outcomes in this context. The one exception we are aware of is Galasso (2008), who considers a problem in which there are negative externalities across agents and trade is inefficient, but the principal benefits from trading. In this context, Galasso shows that when agents are sufficiently patient, the principal prefers to enter a finite horizon bargaining game in which she is the last mover, to a one-shot game in which she makes a TIOLI offer to agents. This happens because the repeated game allows the principal to profit from breaking coordination among agents.\footnote{With negative externalities, each agent wants to avoid being the last agent left to receive an offer from the principal, and as a result has an incentive to trade early at more favorable terms for the principal.}

Our contribution is to consider arbitrary allocations of bargaining power between the principal and the agents while maintaining the structure of the game fixed. We show that when the principal’s willingness to pay is high, giving more power to the agents induces delay and reduces agents’ welfare.

The emergence of delay in this context is noteworthy. Although delay with incomplete information is well studied, the possibility of inefficient delay in bargaining models with complete information is rare. Chatterjee, Dutta, Ray, and Sengupta (1993), Ray and Vohra (1999), Banks and Duggan (2006) and Gomes (2005) provide examples featuring delay in general bargaining models, and Iaryczower and Oliveros (2016) show existence of an equilibrium with delay in a model of decentralized legislative bargaining, where one agent emerges endogenously as an intermediary.\footnote{In a general version of the Baron Ferejohn model, Banks and Duggan (2006) show that a stationary equilibrium with delay can only exist if the status quo is in the core, which is generally empty in multidimensional policy spaces, or when transfers are possible.}

Jehiel and Moldovanu (1995a,b) obtain delay with complete information in a model in which a seller tries to sell a single object to one of several potential buyers, and non-buyers suffer an externality that is dependent on the identity of the actual buyer.\footnote{In Jehiel and Moldovanu (1995b), non-buyers suffer a negative externality and there is a finite deadline. Jehiel and Moldovanu (1995a) extend the model to allow for positive externalities and an infinite horizon. They show that with negative externalities delay can also arise without deadlines, but that with positive externalities delay can only arise in equilibria of the finite horizon model.}

Closer to our paper, Cai (2000) considers a model in which a principal bargains with \( n \) agents sequentially, and has to obtain unanimous support from all agents, i.e., \( q = n \). The principal meets the agents in a pre-specified order, and the bargaining protocol in each bilateral meeting is a single round of alternating offers. Cai shows that when players are sufficiently patient, there is a multiplicity of subgame perfect NE, including equilibria with and without delay. These two equilibria remain even after imposing the refinement that offers cannot depend on previously rejected offers. Differently than in our paper, delay here appears...
as a result of discriminating offers (Segal and Whinston (2000), Genicot and Ray (2006)) which can be constructed using the predetermined order of meetings. We explicitly rule this out by focusing on symmetric MPE, and show that delay appears (uniquely) when agents have enough bargaining power in bilateral meetings, even in the absence of discriminating contracts.

Other explanations for delay with complete information, less directly related to this paper, have been proposed. Fershtman and Seidmann (1993) show that if a player that rejects an offer is subsequently committed not to accept any poorer proposal, deadlines can lead to delay in bilateral bargaining (with large discount factor). Ma and Manove (1993) show that deadlines can also lead to delay if we assume that (i) a player is permitted to postpone the implementation of his move without losing his turn and (ii) after each offer is made, a random length of time elapses before the other player can respond. Merlo and Wilson (1995) show that efficient delay can emerge when the size of the surplus to be divided evolves stochastically over time. Yildiz (2004) and Ali (2006) show delay in bargaining with heterogeneous priors, and Acharya and Ortner (2013) show that delay can arise in bargaining over multiple issues with partial agreements.

3. The Model

There is a principal and a group of $n$ agents who interact in an infinite horizon, $t = 1, 2, \ldots$. We say the principal wins if and when she obtains the support of $q < n$ agents. In each period $t$ before the principal wins, any one of the $k(t)$ agents who remain uncommitted at time $t$ meets the principal with probability $1/k(t) > 0$. In a meeting between the principal and an agent, principal and agent bargain over the terms of a deal by which $i$ would support the principal. With probability $\phi \in [0, 1]$ the principal makes an offer $p \in \mathbb{R}$ to the agent, and with probability $1 - \phi$ the agent makes an offer $b \in \mathbb{R}$ to the principal. In both cases, the offer is a transfer from the principal to the agent (which can be positive or negative). If the recipient of the offer accepts it, $i$ commits his support for the principal and the transfer takes place; if the offer is rejected, $i$ remains uncommitted. Upon completion, the principal gets a payoff $v \in \mathbb{R}_+$, committed agents get $z \in \mathbb{R}_+$, and uncommitted agents get $w \in \mathbb{R}$. In any period before completion, all players get a payoff of zero, not including any transfer they have received or paid. Principal and agents have a discount factor $\delta \in (0, 1)$.

The solution concept is symmetric Markov perfect equilibria (MPE). The restriction to symmetric MPE rules out discriminatory contracts, in the spirit of Genicot and Ray (2006). In particular, the strategies of principal and agents only condition on the number of agents
$m \leq q$ the principal still needs to obtain for completion. We let the state space be $M \equiv \{1, \ldots, q\}$. Offers when the principal and agents propose in state $m$ are $p(m)$ and $b(m)$, respectively. We let $w(m)$ and $w_{\text{out}}(m)$ denote the continuation values of an uncommitted and a committed agent in state $m \in M$, and $v(m)$ denote the principal’s continuation value in state $m \in M$.

Although quite simple, the model has a number of applications. To fix ideas, we sketch some of these here.

**Corruption.** We consider a simple model of bribes to corrupt bureaucrats, in the spirit of Olken and Barron (2009). Olken and Barron observe bribes paid by truck drivers to police, soldiers, and weigh station attendants in Indonesia. They model checkpoints as a chain of vertical monopolies, where the sequence of meetings is exogenously given, and the agreement of each checkpoint is needed for completion. In our model, instead, a firm needs to get the approval of $q$ out of $n$ bureaucrats, and does not have to get these approvals in a given sequence. This is as in McMillan and Zoido (2004), who document the details of corruption in Peru in the 1990s under President Alberto Fujimori, observing the bribes Fujimori’s secret police chief Vladimiro Montesinos paid to judges, politicians and the news media. An interesting fact that emerges from both of these papers is that there is substantial bargaining for bribes. Olken and Barron (2009) show that prices are in part set through ex post bargaining rather than being fully determined ex ante, while Montesinos’ videotapes show him haggling with the bribe takers (McMillan and Zoido (2004)). We assume that if the project is greenlighted, the firm gets an expected payoff $v > 0$, and the bureaucrat who supports the project obtains $z > 0$ (possibly due to more benefits down the line), while $w \geq 0$ or $w \leq 0$ depending on whether the project benefits or hurts the population at large.

**New Technologies with Increasing Returns to Scale.** Consider exclusive deals contracts for the introduction of a new product with network externalities (Katz and Shapiro (1992), Segal and Whinston (2000)). Suppose there are $n$ buyers and an incumbent producing with an old technology, in a market that can accommodate at most one supplier due to increasing returns to scale or network externalities. Under the incumbent supplier, buyers obtain a per period payoff which we normalize to zero. A challenger $P$ can supply the market with a new technology, but entry is profitable only if it can serve at least $q$ buyers. In each period, the challenger negotiates with a potential buyer an exclusive deal contract, which

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As this paper documents, to form a winning coalition in the Congress, Montesinos had to bribe only some of the opposition politicians. Similarly, “In the Supreme Court, decisions are made by majority vote, so three of the five Supreme Court judges were enough.” (McMillan and Zoido (2004))
can include some advantage in service or tailored design. If $q$ buyers sign exclusive deals, the challenger enters and the incumbent drops out. In this case the challenger firm gets a payoff $v > 0$, buyers who didn’t sign get $w > 0$ and buyers who signed agreements get $z \geq w$.

**Start-Ups.** A firm needs to hire $q$ specialized workers to produce a new product. Upon starting production, the firm obtains an expected payoff of $v > 0$, while each of the workers gets profit participation leading to an expected value $z > 0$. To sign the workers to the company, the firm negotiates with each worker a sign-up bonus. Workers that are not hired by the firm do not benefit (or suffer) from the company’s activities, so $w = 0$.

**Organization of a Protest.** A national union wants to set up a large protest against the government. To do this, the union leadership can offer concessions to convince each of the local or industry chapters to mobilize their members against the government. The protest is successful if the national leadership secures the agreement of $q$ of the local bosses. If organized, the union gets $v > 0$, the local chapters participating get $z > 0$ and the chapters not participating get $w > z$ (a similar logic holds with the organization of a coup, with the rebel elite bargaining with the commander of each regiment).

In Section 5.1 we consider the case $z \leq 0$. This allows us to extend our analysis to other applications, including corporate takeovers ($z = 0$) or vote buying with audience costs ($z < 0$).

### 4. Benchmarks

To convey the key insights of the paper in the simplest way possible, we begin by analyzing the two extreme cases in which either the principal or the agents have full bargaining power in bilateral meetings ($\phi = 1$ and $\phi = 0$ respectively), with $z = w > 0$.

#### 4.1. Principal has Full Bargaining Power.

We begin with the special case analyzed in Iaryczower and Oliveros (2017), where the principal has full bargaining power.\(^9\) Consider an arbitrary state $m \in M$. Note that the agent meeting the principal will accept an offer $p(m)$ only if $\delta w_{\text{out}}(m-1) + p(m) \geq \delta w(m)$, and will accept the offer with probability one if this inequality holds strictly. Thus, whenever the principal makes an offer to agent $i$ in state $m$, she offers

\[
p(m) = -\delta [w_{\text{out}}(m-1) - w(m)].
\]

\(^9\)As it will become clear later, when the principal has all the bargaining power, the dynamics of the game are dichotomous and the game can be solved in isolated steps.
The principal is willing to make the offer $p(m)$ in state $m$ if $p(m) \leq \delta[v(m-1) - v(m)]$, or, substituting, if and only if the bilateral surplus of moving forward is nonnegative; i.e.,

$$s(m) \equiv [v(m-1) - v(m)] + [w_{\text{out}}(m-1) - w(m)] \geq 0$$

Equilibrium strategies can differ in the probability of trade in each state. Let $\gamma_m \in [0, 1]$ and $\alpha_m \in [0, 1]$ denote, respectively, the probability that the principal makes an offer $p(m)$ and that the agent accepts the offer $p(m)$ in state $m$, and let $\lambda_m \equiv \gamma_m \alpha_m$ denote the probability of trade in state $m$. We argue that the strategy profile in which there is trade in every state, or $\lambda_m = 1$ for all $m \leq q$, is an equilibrium (we call this a full trading equilibrium, of FTE for short). Moreover, this equilibrium is unique.

**Proposition 4.1 (Iaryczower and Oliveros (2017)).** The game with $\phi = 1$ and $z = w$ has a unique MPE, a full trading equilibrium. In this equilibrium, the payoff of an agent is given by

$$w^P(m) = \left[ \prod_{k=1}^{m} r(k) \right] \delta^m w \quad \text{with} \quad r(k) \equiv \frac{(n-q+k-1)}{(n-q+k-\delta)}$$

The intuition for why the FTE exists is as follows. First, fix the proposed equilibrium. Since $v > 0$ and $w > 0$, when the principal needs to collect the support of only one additional agent ($m = 1$), principal and agent can create a positive surplus by moving forward. Thus, given full information, there is a price at which this transaction occurs. And since the principal makes the proposal, the solution gives the principal a positive rent. Now consider the situation in which there are $m$ agents remaining. Since in equilibrium there is trade whenever the principal needs to secure the support of $t < m$ additional agents, in state $m$ the principal and the selected agent can also realize a positive surplus if they move forward. Therefore there is again a price at which trade can occur, which gives the principal a positive rent.

To check more formally whether a particular strategy profile is a MPE we need to check consistency with (2).\footnote{In particular, we must have $s(m) \geq 0$ when $\lambda_m = 1$, $s(m) \leq 0$ when $\lambda_m = 0$, and $s(m) = 0$ when $\lambda_m \in (0, 1)$. To see this last point, suppose that in equilibrium $s(m) > 0$. Then it must be that $\alpha_m = 1$, for otherwise the principal could obtain a discrete gain in payoffs by increasing her offer slightly, as any such offer would be accepted. And then it must be that $\gamma_m = 1$. It follows that if $\lambda_m \in (0, 1)$ then $s(m) = 0$.} In a FTE, in particular, we need to verify that $s(m) \geq 0$ for all $m \leq q$ when $w(\cdot), w_{\text{out}}(\cdot)$ and $v(\cdot)$ are computed for a FTE.

Consider the value of an uncommitted agent $i$ in state $m$ in a FTE, $w(m)$. With probability $\beta(m) \equiv 1/(n-q+m)$, $i$ meets the principal, who offers him a transfer satisfying (1),
leading to a continuation payoff $\delta w(m)$. With probability $1 - \beta(m)$, some agent $j \neq i$ meets the principal and ultimately agrees to support her, leaving $i$ with a continuation value $\delta w(m - 1)$. Solving the difference equation with initial condition $w(0) = w$ gives (3). If instead the agent is already committed in state $m$, $w_{\text{out}}(m) = \delta w_{\text{out}}(m - 1)$, and thus

$$w_{\text{out}}^P(m) = \delta^m w.$$  

Using (3) and (4) we can compute the equilibrium transfer as a function of primitives, and then solve for the principal’s value function, which obeys the recursive representation $v(m) = \delta v(m - 1) - p(m)$. Using $v^P(\cdot), w^P(\cdot)$ and $w_{\text{out}}^P(\cdot)$ in the surplus condition (2), we have that $s(m) \geq 0$ for all $m \leq q$.

Uniqueness can be established via an induction argument. Consider any MPE and suppose that there is trade whenever the principal needs to secure the support of $t < m$ additional agents. If in equilibrium there were no trade with positive probability, principal and agent would obtain a lower combined payoff in state $m$ than in the FTE. Hence the gain from moving forward would be higher than in the FTE, and thus positive. It follows that the principal will make an offer, which the agent will accept.

Proposition 4.1 implies that in equilibrium the principal cannot extract all surplus from the agents. The reason for this is similar to the logic behind under-provision of a public good. Note that since the agents benefit from implementing the alternative to the status quo and they cannot obtain a higher terminal payoff by remaining uncommitted ($z = w$), the principal actually charges them to move on. By rejecting the offer, however, an agent can rely on others to pay the bill. This generates an outside option that gives each agent some bargaining power over the principal. Since the cost of deferring implementation of the proposal decreases with $\delta$, the value of the outside option is increasing in $\delta$, and so is agents’ equilibrium payoff. In fact, as $\delta$ approaches 1, $r(m) \to 1$ and $w^P(m) \to w$.$^{11}$

### 4.2. Agents have Full Bargaining Power.

We now consider the case in which agents have full bargaining power in bilateral negotiations with the principal; i.e., $\phi = 0$. We show that this shift in bargaining power dramatically changes the nature of equilibrium outcomes, inducing inefficient delay, due to the fact that agents trading later with the principal can

$^{11}$An indirect consequence of the assumption that $w = z > 0$ is that when $\phi = 1$, transfers from principals to agents are negative, a feature that can be unappealing in some applications. In the general version of the model where we allow payoffs to depend on whether each agent supported the principal or remained uncommitted, most relevant applications involve positive transfers from the principal to agents under reasonable assumptions on parameters, even when the principal has full bargaining power.
extract a disproportionate fraction of the surplus generated by the completion of the project. We call this a collective hold-up problem.

The nature of the collective hold-up problem is particularly transparent when agents have full bargaining power. Note that as in the previous case, the offer \( b(m) \) that an agent makes to the principal in state \( m \) is uniquely pinned down: since the principal accepts an offer \( b(m) \) in state \( m \in M \) iff \( \delta v(m - 1) - b(m) \geq \delta v(m) \), whenever the agent makes an offer to the principal he offers

\[
(5) \quad b(m) = \delta[v(m - 1) - v(m)].
\]

By (5), the continuation value of the principal in state \( m \) is \( \delta v(m) \) independently of whether the agent makes an offer or not. It follows that \( v(m) = \delta v(m) \), which implies that \( v(m) = 0 \) for all \( m \geq 1 \). Because the critical agent trading in state \( m = 1 \) extracts all the principal’s surplus from completion of the project with his offer \( b(1) = \delta v \), the principal is not willing to pay in previous states to move the process forward. Substituting in (5), equilibrium transfers from the principal to the agents are

\[
(6) \quad b(m) = \begin{cases} \delta v & \text{if } m = 1 \\ 0 & \text{for } m \geq 2. \end{cases}
\]

The fact that agents meeting the principal in earlier states \( m > 1 \) cannot extract additional rents from the principal gives them an incentive to hold out their support, which increases with \( v \). This individual behavior, of course, is not consistent with equilibrium. If the agent meeting the principal in state \( m \) never agrees to give his support to the principal the process does not move forward, and the critical state \( m = 1 \) is never reached in the first place. As we show below, consistency is restored with delay in reaching agreements.

Let \( \hat{\lambda}_m \) denote the probability of trade in state \( m \) when the agents have full bargaining power.\(^{12}\) In equilibrium \( s(m) \geq 0 \) when \( \hat{\lambda}_m = 1 \), \( s(m) \leq 0 \) when \( \hat{\lambda}_m = 0 \), and \( s(m) = 0 \) when \( \hat{\lambda}_m \in (0, 1) \), where \( s(m) \) is given by (2) as before. Note that since \( v(m) = 0 \) for all \( m \in M \), for any state \( m > 1 \), \( s(m) \geq 0 \) if and only if \( w_{out}(m - 1) \geq w(m) \). In the absence of side payments, the probability of trade depends on the relative value for an agent of moving the process along supporting the principal for free, \( w_{out}(m - 1) \), versus holding out support with the goal of extracting the rent \( \delta v \) in late trading, \( w(m) \).

\(^{12}\) We let \( \gamma_m \in [0, 1] \) denote the probability that an agent makes the offer \( b(m) \) in state \( m \), \( \hat{\alpha}_m \in [0, 1] \) the probability that the principal accepts the offer \( b(m) \) in state \( m \), and let \( \hat{\lambda}_m \equiv \gamma_m \hat{\alpha}_m \) denote the probability of trade in state \( m \).
The values \( w_{\text{out}}(m) \) and \( w(m) \) for a committed and uncommitted agent depend on expectations of the probability of trade in states \( m' \leq m \). Since once committed, agents do not engage in further negotiations, the payoff of a committed agent \( i \) in state \( m \) is

\[
w_{\text{out}}(m) = \hat{\lambda}_m \delta w_{\text{out}}(m - 1) + (1 - \hat{\lambda}_m) \delta w_{\text{out}}(m),
\]

and solving recursively,

\[
(7) \quad w_{\text{out}}(m) = \left[ \prod_{j=1}^{m} \frac{\delta \hat{\lambda}_j}{1 - \delta (1 - \hat{\lambda}_j)} \right] w,
\]

On the other hand, the payoff of an uncommitted agent in state \( m > 1 \) is

\[
w(m) = \hat{\lambda}_m [\beta(m) \delta w_{\text{out}}(m - 1) + (1 - \beta(m)) \delta w(m - 1)] + (1 - \hat{\lambda}_m) \delta w(m),
\]

where as before \( \beta(m) \equiv 1/(n - q + m) \) denotes the probability that an agent \( i \) meets the principal in state \( m \in M \). Whether agent \( i \) or another agent \( j \neq i \) negotiates with the principal, an unsuccessful meeting implies that the system stays put at \( m \), which gives \( i \) a discounted continuation payoff \( \delta w(m) \). A successful meeting, instead, moves the process to state \( m - 1 \) with the agent being committed to the principal with probability \( \beta(m) \). Solving recursively,

\[
(8) \quad w(m) = \left[ \prod_{j=1}^{m} \frac{\delta \hat{\lambda}_j}{1 - \delta (1 - \hat{\lambda}_j)} \right] (w + \beta(m)v).
\]

Given \( v(1) = 0 \), the surplus in the critical state \( m = 1 \) is given by \( s(1) = v + w - w(1) \). Now, note that \( w(1) \) is maximized at \( \hat{\lambda}_1 = 1 \), where it attains the value \( w(1) = \delta (w + \beta(1)v) < v + w \). Thus \( s(1) > 0 \) for any \( \hat{\lambda}_1 \in [0, 1] \), and in equilibrium there is no delay in \( m = 1 \). Using (7) and (8), the condition for trade with positive probability at \( m > 1 \) that \( w_{\text{out}}(m - 1) \geq w(m) \) boils down to

\[
(9) \quad w \geq \frac{\delta \hat{\lambda}_m}{1 - \delta (1 - \hat{\lambda}_m)} (w + \beta(m)v)
\]

For delay to occur with positive probability at \( m \), we need (9) to hold with equality. Now, note that since \( w(m) \) is increasing in the probability of trade in state \( m \), \( \hat{\lambda}_m \), while \( w_{\text{out}}(m - 1) \) is independent of \( \hat{\lambda}_m \), the right hand side is a continuous increasing function \( f(\cdot; m) \) of \( \hat{\lambda}_m \) such that \( f(0; m) = 0 \) and \( f(1; m) = \delta (w + \beta(m)v) \).

Since (9) is satisfied with \( \hat{\lambda}_m = 0 \), this implies that in equilibrium there is always trade with positive probability in all states \( m > 1 \). On the other hand, there exists a (unique) solution
\( \hat{\lambda}_m \in (0, 1) \) satisfying (9) with equality if and only if

\[
(10) \quad w < \delta (w + \beta(m)v) \iff m < \frac{\delta}{(1 - \delta)} \frac{v}{w} - (n - q) \equiv \overline{m}
\]

It follows immediately from this that there exists a unique cutpoint \( \overline{m} \in \{2, \ldots, q + 1\} \) such that, in equilibrium, there is delay in each state \( m : 2 \leq m \leq \overline{m} \), and trade with probability one in any \( m > \overline{m} \).

From eq. (10), the set of states in which there is delay is weakly increasing in \( v/w \), which captures the relative value of holding out, and for any \( m \in M \) there is a \( v/w \) large enough such that \( m < \overline{m} \). The ratio \( v/w \) also increases the probability of delay in states below the cutpoint. In fact, note that when there is delay in state \( m \), the probability of trade is given by \( \hat{\lambda}_m \in (0, 1) \) solving \( w_{\text{out}}(m - 1) = w(m) \), or

\[
(11) \quad \hat{\lambda}_m = \left(1 - \frac{\delta}{\delta}ight) \frac{w}{v} \frac{1}{\beta(m)}
\]

Note that from (11), the probability of trade is increasing in \( m \). Therefore, we expect transactions to occur at a faster pace initially, with the process of negotiations slowing down as it goes along. Similarly, note that both the set of states in which there is delay and the probability of delay in states below the threshold are increasing in the size of the coalition required to win. This corresponds well to the intuition that more stringent supermajority requirements are costly because they induce delay.

**Remark 1.** The expected time for completion increases with the stringency of the \( q \)-rule. □

The previous discussion fully characterizes equilibria of the game in which agents have all the bargaining power up to the precise determination of the threshold \( \overline{m} \). This threshold, in turn, is pinned down uniquely for given parameters by (10). We are interested in particular in equilibrium for large \( v \), where the collective hold-up problem is severe. The next proposition summarizes our discussion focusing on this case.

**Proposition 4.2.** Consider the game with \( \phi = 0 \) and \( z = w > 0 \). Suppose \( v \geq \frac{(1 - \delta)}{\delta} nw \equiv \overline{v} \). Then there is an (essentially) unique equilibrium with trading at \( m = 1 \) and delay in all
$m : 2 \leq m \leq q$, given by (11).\textsuperscript{13} Agents’ payoffs are

$$w^A(q) = \left(\prod_{j=2}^{q-1} \frac{w}{w + \beta(j)v}\right) \delta w, \quad \text{and} \quad \lim_{v \to \infty} w^A(q) = 0$$

As Proposition 4.2 shows, when the collective hold-up problem is severe, there is delay in all but the critical state $m = 1$. Moreover, delay is increasing in $v$. This poses a tradeoff for agents’ welfare: a larger $v$ increases the total surplus from transacting, but also leads to larger delay. As the proposition shows, in equilibrium the larger delay more than compensates for the increase in total surplus and leads to a loss of welfare for the agents. Proposition 4.2 thus has the direct implication that when the principal’s valuation for winning is sufficiently large, if able to choose, agents would prefer the situation in which the principal has full bargaining power to that in which the agents have full bargaining power.

**Corollary 4.3.** For $v$ sufficiently large, agents are better off when the principal has full bargaining power than when agents have full bargaining power: $w^P(q) > w^A(q)$.

We should point out that the result above (Corollary 4.3) does not hold under unanimity, which is the classic railroad-farmers example considered by Coase (see Cai (2000), Olken and Barron (2009), Chowdhury and Sengupta (2012)). With unanimity, all the analysis of the game in which agents have full bargaining power remains unchanged, and results are essentially unaltered. While the analysis of the game in which the principal has full bargaining power also remains unchanged, agents’ equilibrium payoffs in this case are zero, $w_{un}^P(q) = 0$. To see this, note that with $q = n$, $\beta(1) = 1$, so in the critical state the agent cannot free ride on others. Thus $w(1) = \delta w(1)$, which implies $w(1) = 0$. But then, recursively, $w(m) = 0$ for all $m \in M$. Since $w_{un}^A(n)$ approaches 0 as $v \to \infty$ but doesn’t attain zero, with unanimity, $w_{un}^P(n) < w_{un}^A(n)$.

### 5. Main Result

In the previous section we studied how outcomes and welfare change in the two polar cases in which either the principal or the agents have full bargaining power in bilateral meetings. These extreme allocations of bargaining power are useful as a benchmark, but often unrealistic. Moreover, the extreme allocations of bargaining power are special in the sense that one

\textsuperscript{13}We say that the equilibrium is essentially unique because any pair $(\hat{\gamma}_m, \hat{\alpha}_m)$ such that $\hat{\lambda}_m = \hat{\gamma}_m \hat{\alpha}_m$ satisfies (11) is an equilibrium. What matters for equilibrium is the expectation of delay, and not whether this occurs because of a lower probability that the agent makes a proposal, $\hat{\gamma}_m$, or a lower probability that the principal accepts this offer, $\hat{\alpha}_m$. 
side extracts the entire surplus generated by the transaction, while in general principal and
agent both benefit from the exchange. This implies that in general the system of difference
equations characterizing equilibrium payoffs can not be decoupled as we have done in the
previous sections.

To tackle this problem more generally, we introduce counteroffers in a way that allows us to
vary smoothly the power of agents: we assume that with probability $\phi \in (0, 1)$ the principal
makes an offer to the selected agent, and with probability $1 - \phi$ the agent makes an offer to
the principal. Note that this is formally equivalent to nesting an infinite horizon bilateral
bargaining in our game, where one of the sides decides whether to enter in negotiations or not,
and in any period of the negotiation phase after a proposal is rejected the principal (agent)
makes offers with probability $\phi$ (respectively, $1 - \phi$). In fact, in any meeting between the
principal and an agent in a state $m \in M$, both games give the principal a value $\phi s(m)$, and the
agent a value $(1 - \phi)s(m)$, conditional on trading. Accordingly, we let $\mu_m \equiv \phi \lambda_m + (1 - \phi)\hat{\lambda}_m$
denote the ex ante probability of trade in state $m$. We also allow the payoffs of committed
and uncommitted agents upon completion to be different. In particular, we allow arbitrary
$z \in \mathbb{R}_+$ and $w \in \mathbb{R}$, as in the applications we described.\footnote{In Section 5.1 we consider the case of $z \leq 0$. We show that in this case there is a breakdown of negotiations.}

In this general case, the values of principal and uncommitted agents can be written recur-
sively, letting $s^\pm(m) = \max\{s(m), 0\}$, as follows (see Appendix A.1 for details).

\begin{align}
  v(m) &= \left(\frac{\delta}{1 - \delta}\right) \phi s^+(m), \\
  w(m) &= \left[\frac{\delta \beta(m)}{1 - \delta \beta(m)}\right] (1 - \phi)s^+(m) + \left[1 + \left(\frac{1 - \delta}{1 - \beta(m)}\right) \frac{1}{\delta \mu_m}\right]^{-1} w(m - 1),
\end{align}

Equation (12) says that the value of the principal in state $m$ is proportional to the surplus
in state $m$ whenever this is positive, and zero otherwise.\footnote{The expression eliminates the dependency on the probability of trade $\lambda_m$ using the fact that if $s(m) > 0$
then $\lambda_m = 1$, if $s(m) < 0$ then $\lambda_m = 0$, and that $s(m) = 0$ when $\lambda_m \in (0, 1)$.} A key implication of (12) is that
the principal’s equilibrium payoff in state $m$ is proportional to the surplus $s(m)$ by a factor
that increases with the principal’s nominal bargaining power $\phi$. Because delay can only
occur in equilibrium if $s(m) = 0$, this means that if there is delay in state $m$ in equilibrium,
then $v(m) = 0$. The agent’s equilibrium payoff in state $m$, on the other hand, has two
components. The first is proportional to the surplus $s(m)$ whenever this is positive, by a
factor that increases with the agents’ bargaining power $1 - \phi$. This term comes for the events
in which the agent is negotiating with the principal. But differently to the principal’s value,
the agent’s value \( w(m) \) is positive even when \( s(m) = 0 \), with the second component being a positive fraction (increasing in the probability of trade in state \( m, \mu_m \)) of the state \( m - 1 \) value \( w(m-1) \). This term comes from the fact that the agent receives some value even when he does not meet the principal in any state.

The value of a committed agent, instead, is not directly related to the surplus in each state, and only depends on the probability that the process moves forward or not: if there is a transaction (with probability \( \mu_m \)) the committed agent gets a continuation payoff \( \delta w_{\text{out}}(m-1) \), and otherwise gets \( \delta w_{\text{out}}(m) \). Solving recursively, we obtain

\[
(14) \quad w_{\text{out}}(m) = \prod_{k=1}^{m} \left( \frac{\delta \mu_k}{1 - \delta(1 - \mu_k)} \right) z
\]

We can now prove our first result of this section. We show that equilibrium exists, and is unique up to the probability of trade \( \mu \). Moreover, we show that in equilibrium trade never collapses, in the sense that in all non-terminal states \( m \in M \), the principal transacts with an agent with positive probability; i.e., \( \mu_m > 0 \) for all \( m \leq q \). We also characterize the probability of trade in each state \( m \leq q \) as a function of continuation values \( w_{\text{out}}(m-1), v(m-1) \) and \( w(m-1) \). For any \( m \geq 1 \), let \( \Gamma(m) \equiv w(m)/(v(m) + w_{\text{out}}(m)) \). Then

**Proposition 5.1.** There exists an essentially unique equilibrium, characterized by trade probabilities

\[
(15) \quad \mu_m = \min \left\{ 1, \left( \frac{1 - \delta}{\delta} \right) \left( \frac{1}{1 - \beta(m)} \right) \left( \frac{1}{\Gamma(m-1)} \right) \right\} > 0 \quad \forall m \in M.
\]

To see the logic for this result, note first that with \( v, z > 0 \), a critical meeting \( (m = 1) \) must have trade with positive probability (Lemma A.1), and thus \( v(1) + w_{\text{out}}(1) > 0 \). Now suppose that for all \( k < m \) there are transactions with positive probability, and take the implied continuation values \( w_{\text{out}}(m-1), v(m-1) \) and \( w(m-1) \) as given. Note that since in all states \( k < m \) there is trade with positive probability, the values of a committed agent and of the principal in state \( m - 1 \) are positive \( (w_{\text{out}}(m-1) > 0 \text{ and } v(m-1) > 0) \). Thus inaction at \( m \) is not an equilibrium, for then \( v(m) = w(m) = 0 \text{ and } s(m) = v(m-1) + w_{\text{out}}(m-1) > 0 \), giving principal and agent an incentive to trade. We then show that the “one-shot” game in state \( m \) has a unique SPE, which has no delay if the (lagged) value of an uncommitted agent is low relative to the (lagged) joint value of a committed agent and the principal, and otherwise has delay with positive probability. The result then follows by induction.

Having established existence and uniqueness of equilibrium outcomes, we turn to our main goal of studying how the allocation of bargaining power among principal and agents affects
delay and agents’ welfare. Our main result, Theorem 5.5, provides a complete characteriza-
tion of equilibria for large $v$, when the collective hold-up problem is severe.\footnote{Here and in the rest of the paper, we write the statement “for $v$ large, [A] is true” to mean that for fixed parameters other than $v$, there exists a $\overline{v} > 0$ such that if $v \geq \overline{v}$, [A] is true.} This establishes that redistributing bargaining power from the principal to the agents creates delay and re-
duces agents’ welfare.

Theorem 5.5 builds on four basic results. The first restricts the possible nature of delay across different states of the negotiation process. In principle, delay could be frontloaded (occur at the beginning of the bargaining process), backloaded, or occur in some interior subset of states. In addition, the set of states with delay could potentially be unconnected, with regions of delay followed by states in which trade is efficient. In our first result we show that for large $v$, delay is front-loaded: if in equilibrium there is delay in a state $m' < q$, then there is delay in all states $m > m'$.\footnote{At first sight, the result that delay is front-loaded seems to contradict earlier results in the benchmark case with $\phi = 0$, where we showed that if delay does not occur in all non-critical states, it must happen at the end of the negotiation process (i.e., is backloaded). However, it should be clear that this result relies on $v$ sufficiently small. For large $v$, the unique equilibrium of the $\phi = 0$ case entails delay in all non-critical states, and is thus fully consistent with the result in Lemma 5.2. See footnote 20 for a discussion of equilibria with low $v$ in the general case.}

**Lemma 5.2.** For large $v$, $\mu_{m'} \in (0, 1)$ for $m' < q \Rightarrow \mu_m \in (0, 1)$ $\forall m > m'$.

The proof of Lemma 5.2 involves three steps:

1. First, in Lemma A.2, we characterize agents’ payoffs in each state $m$ as a function of primitives, for any given probability of trade in each state in the continuation, $(\mu_1, \ldots, \mu_{m-1})$.

Differently to the polar cases we analyzed before, the fact that here both principal and agents make proposals with positive probability means that principal and agents can mutually extract rents from one another. This implies that the system of difference equations characterizing equilibrium payoffs can not be decoupled as in Section 4.1, where we could solve for agents’ values independently, use these values to express transfers as a function of primitives, and then solve for the principal’s equilibrium payoffs. To tackle this difficulty, in the proof we use a transformation to express the system of difference equations as a second order difference equation, which we then solve.
(2) Using this result, in Lemma A.3 we provide a necessary and sufficient condition for full trade in any state $m$ for an arbitrary probability of trade of the $m - 1$ subgame. In particular, we obtain an expression $T(m)$, and show that $s(m) \geq (\leq)0$ given $\mu_m \in [0, 1]$ if and only if $T(m) \leq (\geq)0$.

(3) In the proof of Lemma 5.2 we then show that for large $v$, $T(m) \leq 0 \Rightarrow T(m-1) < 0$. This implies that if the surplus is nonnegative in state $m$, it must be positive in state $m-1$, which gives the front-loaded result.

Our second result further restricts equilibrium outcomes by ruling out “cycles of trade” in the spirit of Jehiel and Moldovanu (1995a), and other non-monotonicities in the probability of trade across states of the negotiation process.\textsuperscript{18} We show that in any equilibrium in which there is delay in states $\{m, \ldots, \overline{m}\}$ (as we know is the case for large $v$ by our previous lemma), for any state $m \geq m + 1$, the probability of trade is monotonic. In particular, we show that the probability of trade grows with $m$ at a rate equal to $\beta(m)$, independent of the allocation of bargaining power $\phi$, the discount factor $\delta$, or the valuations $v, w$ and $z$.

Lemma 5.3. Suppose in equilibrium $\mu_m \in (0, 1)$ for all $m \in \{m, \ldots, \overline{m}\}$. Then
\[
\frac{\mu_{m+1} - \mu_m}{\mu_m} = \beta(m) \quad \forall m \in \{m + 1, \ldots, \overline{m} - 1\},
\]
and thus in particular $\mu_{m+1} > \mu_m$ for all $m \in \{m + 1, \ldots, \overline{m} - 1\}$.

Our first two results show that if there is delay in equilibrium, delay is front-loaded, and the probability of trade grows as we move further into the negotiation process until reaching the last state with delay, after which trade occurs with probability one in each state. In our third result, we obtain a necessary and sufficient condition for existence of a full trading equilibrium (no delay), and show that when the hold-up problem has bite, the unique equilibrium of any $m$-subgame is efficient if the principal has sufficient bargaining power, but exhibits delay if the agents have sufficient bargaining power.

To do this, we first use Lemma A.2 (with $\mu_j = 1$ for all $j \in M$) to obtain an expression for FTE payoffs $w^\dagger(m), v^\dagger(m)$ in terms of primitives of the model, and then proceed as in Lemma A.3. For convenience, we define $\bar{\theta}_km \equiv \prod_{j=k}^{m} \left( \frac{\delta \phi}{1 - \delta + \delta \phi (1 - \beta(j))} \right)$.

\textsuperscript{18}In the context of negotiations between the seller of a good and several potential buyers, Jehiel and Moldovanu show that when the seller is sufficiently patient and externalities between buyers are negative, equilibrium outcomes of the class they identify (SPNE in pure strategies with bounded recall) have the property that long periods of waiting alternate with short periods of activity. (When externalities are positive, there is no delay in equilibrium within this class).
Lemma 5.4 (No Delay). (i) There exists a FTE in the subgame starting in state $m'$, with agents’ equilibrium payoffs

$$
\frac{w^\dagger(m)}{\beta(m)} \equiv \overline{\theta}_{1m}(n-q)w + \sum_{k=1}^{m} \frac{1-\delta}{\delta} \frac{1-\phi}{\phi} \theta_{km} \delta^k (v+kz+(n-q)w)
$$

and

$$
v^\dagger(m) = \left( \frac{\delta \phi}{1-\delta(1-\phi)} \right)^m v - \left( \sum_{r=1}^{m} \left( \frac{\delta \phi}{1-\delta(1-\phi)} \right)^r \right) (w^\dagger(m) - \delta^{m-1}z)
$$

iff

$$
T^\dagger(m) \equiv \frac{w^\dagger(m)}{\beta(m)} - \delta^m (v + mz + (n-q)w) \leq 0 \quad \forall m \leq m'.
$$

Moreover, for large $v$ the following is true: for any $m \leq q$, (ii) there exists $\overline{\phi}(m) \in (0,1)$ such that if $\phi > \overline{\phi}(m)$, the unique MPE of the $m$-subgame is a FTE, and (iii) there exists $\underline{\phi}(m) \in (0,1)$ such that if $\phi < \underline{\phi}(m)$, the unique MPE of the $m$-subgame entails delay.

Part (ii) of the proposition generalizes the result of Proposition 4.1, and shows that as long as the principal has enough bargaining power, the unique equilibrium for any $z \in \mathbb{R}_+$ and $w \in \mathbb{R}$ is a FTE. When instead the agents have enough bargaining power, the equilibrium involves delay. In fact, part (iii) of the proposition says that for any $m \in M \setminus \{1\}$ there is a sufficiently low $\phi$ such that the unique MPE of the $m$-subgame entails delay. This implies that if the agents have enough bargaining power, in equilibrium there is delay in all but the critical state, as in Proposition 4.2 of the benchmark model.
Lemma 5.4 leaves open the possibility that the set of states with delay varies non-monotonically with the principal’s bargaining power for intermediate values of φ. In the proof of Theorem 5.5 we show that this is not the case. The proof builds on the fact that the agents’ FTE payoff $w^\dagger(m)$ is decreasing in the principal’s bargaining power φ whenever the equilibrium of the $m$-subgame is a FTE.\(^{19}\) This implies, combined with other results, that the set of states with delay is decreasing in the principal’s bargaining power φ.

The result that $w^\dagger(m)$ is decreasing in φ in a FTE is intuitive but not completely obvious, because changes in φ lead to a redistribution of rents across agents trading in different states. On the one hand, reducing φ increases the ability of agents trading with the principal in late states to extract from the principal, which acts to increase the value of agents trading in earlier states. On the other hand, this simultaneously lowers the principal’s willingness to pay, thus reducing the ability of agents negotiating with the principal in earlier states to extract from the principal. As we show below, however, in a FTE the direct effect dominates.

Note that from (12), $v(m) = \left(\frac{\delta}{1 - \delta}\right) s^+(m)$. Since in a FTE the surplus $s(m)$ does not change with φ, it follows that $v^\dagger(m)$ is increasing in φ. Now, total welfare in state $m$ is

$$J(m) \equiv v(m) + (n - q + m)w(m) + (q - m)w_{out}(m)$$

Since in a FTE both $J(m)$ and $w_{out}(m)$ are constant in φ, it follows that

$$v^\dagger(m; \phi) - v^\dagger(m; \phi') = -(n - q + m)[w^\dagger(m; \phi) - w^\dagger(m; \phi')]$$

and thus $w^\dagger(m)$ is decreasing in φ whenever the equilibrium of the $m$-subgame is a FTE.

We can now establish the main result of the paper:

**Theorem 5.5 (Characterization for large $v$).** For any $\phi \in [0, 1]$ there exists a unique cutpoint $\overline{m}(\phi) \in M$ such that, in equilibrium, there is delay in each state $m \in M$ s.t. $m > \overline{m}(\phi)$, and full trading in any $m \leq \overline{m}(\phi)$. The cutpoint $\overline{m}(\cdot)$ is weakly increasing in φ and has range $M$. Moreover, for any $m > \overline{m}(\phi) + 1$,

$$\mu_m = \left(\frac{n + m - q}{n + \overline{m} + 1 - q}\right) \left(\frac{1 - \delta}{\delta}\right) \left(\frac{\delta\overline{m} z}{(1 - \beta(\overline{m} + 1))(w^\dagger(\overline{m}) - \delta\overline{m} z) + \beta(\overline{m} + 1)(v^\dagger(\overline{m}))}\right)$$

is decreasing in $v$ and goes to zero as $v \to +\infty$.

\(^{19}\)It follows directly from 16 that the FTE value $w^\dagger(m)$ is increasing in the terminal payoffs of the principal $v$, as well as committed and uncommitted agents, $(z, w)$. 
Theorem 5.5 unifies our previous results and provides a complete characterization of equilibria when the collective hold-up problem is severe. We are now in a position to answer the questions we posed in the introduction.

How does the allocation of bargaining power between principal and agents affect the efficiency of collective decisions? The theorem shows that redistributing bargaining power from the principal to the agents creates delay and reduces agents’ welfare. In particular, the number of transactions with positive expected delay is decreasing in $\phi$ and increasing in $q$, so that giving more power to the agents or raising the number of agents needed for success increases the number of states in which transactions fail with positive probability.

How do the characteristics of the collective decision affect this inefficiency? Theorem 5.5 confirms the intuitions regarding the effect of changes in the agents’ preferences: a higher value for belonging to the coalition ($z$ large) reduces delay, as it increases the incentive to trade, while a large positive externality on uncommitted agents (large $w$) has the opposite effect. More importantly, the theorem shows the effect of changes on the principal’s willingness to pay on delay. For any given $\phi$ for which there is delay in more than one state, expected delay grows continuously with $v$ and in the limit with $v \to \infty$, the expected time for completion goes to infinity.

How does this delay appear in the negotiation process? For a given allocation of bargaining power $\phi$ inducing delay, the expected delay for each transaction increases as we move further along the process in the first $q - m - 1$ transactions, possibly decreasing in the last transaction with delay. But once the principal obtains the support of $q - m$ agents, the remaining transactions occur without delay.

How does agents’ bargaining power affect their welfare? Counterintuitively, for large $v$ agents’ welfare is maximized when they relinquish significant bargaining power to the principal. Using (13) we can express the equilibrium payoff of an uncommitted agent as

$$w(q) = \left[ \prod_{k=m+1}^{q} \left( 1 + \frac{1 - \delta}{1 - \beta(k)} \frac{1}{\delta \mu_k} \right)^{-1} \right] w^\dagger(m) \leq w^\dagger(m)$$

20 While Theorem 5.5 focuses on the case in which the collective hold-up problem is severe, most of our results apply generically, for all values of $v$. In particular, there is still trade with positive probability in all states, the equilibrium exists and is still essentially unique, the characterization of values is unchanged, as is the condition for no delay, and the growth of the probability of trade in contiguous states. The result that holds for large $v$ but does not hold in general is Lemma 5.2. In fact, we have constructed examples in which, for low $v$, there is delay in an intermediate set of states.
Note that for if there are two states with delay, the probability of trade vanishes as \( v \to \infty \), and thus \( w(q) \to 0 \). Thus, for large enough \( v \), any \( \phi \) such that \( \overline{m}(\phi) \leq q - 2 \) leads to lower equilibrium payoffs for agents than giving complete bargaining power to the principal, \( \phi = 1 \). In turn, since \( w^\dagger(q) \) is decreasing in \( \phi \) when a FTE exists, agents prefer the smallest \( \phi \) such that \( \overline{m}(\phi) \leq q - 2 \) leads to lower equilibrium payoffs for agents than giving complete bargaining power to the principal, \( \phi = 1 \).

Remark 2 (Vanishing Frictions I). In Theorem 5.5, we characterized equilibrium outcomes for fixed \( \delta < 1 \), and sufficiently large \( v \). From the expression for the trading probability \( \mu_m \) in the theorem one might be tempted to conclude that as \( \delta \to 1 \), the probability of trade goes to zero, so negotiations slow down almost to a halt. This would be incorrect. In fact, making the dependence of each \( \overline{m}(\phi) \) on \( \delta \) explicit, as long as \( z \geq w \), \( \overline{m}_\delta(\phi) \to q \) as \( \delta \to 1 \). Thus for any given \( \phi \in (0, 1] \) and \( v > 0 \) there is a \( \overline{\delta} > 0 \) such that if \( \delta \geq \overline{\delta} \), the unique equilibrium is a FTE. Note that from (16), for any \( m \in M \),

\[
\lim_{\delta \to 1} w^\dagger(m) = \beta(m) \left[ \prod_{j=1}^{m} \left( \frac{1}{1 - \beta(j)} \right) \right] (n - q)w = w,
\]

and

\[
\lim_{\delta \to 1} v^\dagger(m) = v - m(w^\dagger(m) - z) = v + m(z - w).
\]

Thus, from proposition 5.4, the condition for existence of a FTE boils down to

\[
v \geq -m(z - w) \quad \forall m \in M.
\]

Consider the critical state \( m = 1 \). Note that \( p(1) = -[z - w(1)] = -(z - w) \), so that when the principal can make an offer, she keeps \( v \) and can extract the differential \( z - w > 0 \). But even when the agent proposes, he gets \( b(1) = v - v(1) = -(z - w) = p(1) \). Thus, the critical agent cannot extract \( \delta v \) from the principal, and there are no incentives to hold out, and no collective hold-up problem.

The result is due to simple economics. When both principal and agents do not discount the future, both principal and agents are willing to wait to get a better deal, but the principal is a monopolist, while the agent faces competition from other agents. This means that the critical agent cannot extract any surplus from the principal. Because agents are willing to wait, all agents are guaranteed \( w \). But the principal, being the short side of the market, gets the differential \( z - w \) entirely. And once this happens in \( m = 1 \), then by the same logic

\[
b(m) = p(m) = -(z - w)
\]

for all \( m \in M \), and thus \( w^\dagger(m) = w \) and \( v^\dagger(m) = v + m(z - w) \), independently of \( \phi \), provided that \( \phi > 0 \). \( \square \)
Remark 3 (Vanishing Frictions II). In our discrete time model, increasing the discount factor $\delta$ can be interpreted as both reducing the time between offers and increasing the value agents give to distant rewards. To disentangle the effect of time preferences from that of the frequency of offers, we consider a continuous time version of the model, where offers can be made in $\Delta > 0$ time intervals. In this model, reducing $\Delta$ increases the frequency of the offers, but leaves time preferences unaltered. For simplicity, we consider a simple example where three out of four agents are required for completion.\footnote{We assume that agents have all the bargaining power and make $w = z > 0$. (The proof of the result stated below is in the appendix.)}

Example 5.6. Suppose $n = 4$, $q = 3$, $\phi = 0$, and $w = z > 0$. Suppose the game takes place in continuous time, but transactions occur in intervals of size $\Delta > 0$, and let the discount factor across trading opportunities be $\tilde{\delta} \equiv e^{-r\Delta}$. For large $v$, there is a unique equilibrium, with trading probabilities,

$$
\mu(1) = 1 \quad \mu(2) = \frac{3}{2} \left( \frac{r}{1-r\Delta} \right) \frac{w}{w+v} \quad \mu(3) = \left( \frac{r}{1-r\Delta} \right) \frac{2w}{w+v}
$$

The example shares the features of the discrete model: the unique equilibrium for large $v$ has delay in all non-critical states, and delay is increasing in $v/w$. The example also clarifies the distinct effect of time preferences and frequency of offers. As in the discrete model, reducing the discount rate $r$ (increasing $\delta$) makes holding out more attractive and increases equilibrium delay in $m = 2$ and $m = 3$. Holding $r$ constant, reducing the time between offers also leads to increased delay. Suppose the frequency of trade increases and the probability of trade remains unchanged. Since in a given period of time more offers will be made and accepted, declining to trade with the principal becomes less costly. To restore incentives, agents must anticipate higher delay.

5.1. Breakdown of Negotiations. Up to this point, we maintained the assumption that in the event the principal obtains the support of $q$ agents, an agent who committed his support to the principal obtains a positive payoff $z > 0$. In some applications, however, it is reasonable to assume that $z = 0$ (e.g., corporate takeovers) or even $z < 0$ (e.g., vote buying with audience costs). Here we consider the case $z \leq 0$.

\footnote{The model in continuous time allows to separate the effect of time (value of free riding) and frequency of offers (opportunity to extract), but comes at significant increasing technical costs. This is because changes in states lead to discrete changes in value functions, and a combination of techniques from both frameworks is needed to properly define the player’s strategies and generate existence and uniqueness results in a more general set up.}
Consider for example a dynamic version of corporate takeovers model of Grossman and Hart (1980) (GH). GH analyze a problem in which a company (the raider) acquires shares of a target company to control its board of directors. It is assumed that the raider can improve the value of the company. To capture this feature, we assume that under the raider’s control, the value of a share is \( w > 0 \), and we normalize the value of a share under the incumbent management to zero. We distinguish the payoff that a shareholder obtains when the raider wins if the shareholder does not sell to the raider \((w > 0)\) from the payoff he obtains if he does sell to the raider \((z = 0)\).\(^{22}\)

We show that whenever there are positive externalities on uncommitted agents \((w > 0)\), the condition \(z > 0\) is necessary for robust delay. In particular, we show that when contracting with the principal leads to a negative payoff for the agent when the principal wins, in equilibrium there can only be delay in the initial state \(m = q\), a result which holds for a “small” (but not measure zero) set of parameter values. With this exception, equilibrium is either a FTE or is such that there are no transactions in the initial state and thus \(w(q) = 0\).

The result follows from Lemma 5.7 below. In it we establish two results. First we show that if \(z \leq 0 < w\), there cannot be cycles of trade with probability one and trade failure with positive probability; in fact, if in equilibrium there is trade with probability one in a state \(m'\), then this also has to be the case in all states \(m < m'\). This means that if there is delay, delay is front-loaded. The second part of the proposition establishes that there cannot be delay in two contiguous states \(m\) and \(m + 1\). Together, the two results imply that with the exception of possibly mixing in the initial state, the equilibrium is either a FTE, involves no transactions in any state, or has a FTE in a \(m'\)-subgame off the equilibrium path for some \(m' < q\), with no trade for \(m > m'\), which implies that the process of transactions never starts.

**Lemma 5.7.** Suppose \(z \leq 0 < w\). Then (i) \(s(m - 1) \leq 0 \Rightarrow s(m) \leq 0\). Moreover, (ii) if \(s(m') \leq 0\) for some \(m' < q\), then \(\mu_m = 0\) for all \(m > m'\) and \(w(q) = v(q) = 0\).

Why no delay in contiguous states? Suppose there is delay in \(m'\) in equilibrium. Since \(s(m' + 1) \leq 0\), either trade collapses in \(m' + 1\) or again there is delay. If there is delay in both \(m'\) and \(m' + 1\), \(v(m') = v(m' + 1) = 0\), so \(s(m' + 1) = 0\) if and only if \(w(m' + 1) = w_{out}(m')\). But \(w(m' + 1) \geq 0\), as it is the value obtained from being uncommitted and depends on \(w > 0\), while \(w_{out}(m') = \prod_1^{m'} \frac{\delta \mu_k}{1 - \delta (1 - \mu_k)} \) \(z < 0\), so this is impossible. With no possible

\(^{22}\)As in GH and Segal (2003), we assume that shareholders are homogeneous. Unlike GH, we suppose that shareholders are fully aware of the effect of their action on the outcome of the raid attempt.
payments from the principal, all incentives to trade have to come from diminishing the value of holding out through delay. But delay can only lower the (positive) value of not trading, and thus by itself is insufficient to induce agents to trade when \( z \leq 0 \).

The next proposition builds on Lemma 5.7 to provide a characterization of equilibria with \( z \leq 0 \). To do this, we first show that if there exists a FTE, this is the unique MPE. We then provide a necessary and sufficient condition for existence of a FTE. This condition follows as a corollary of previous results. First, an examination of the proof of Lemma 5.4 shows that these results do not require the assumption that \( z > 0 \), and thus also hold for \( z \leq 0 \). Thus, agents’ FTE payoffs are still given by \( w^t(\cdot) \) as defined by (16), and there exists a FTE in the \( m' \)-subgame if and only if \( T^t(m) \leq 0 \) for all \( m \leq m' \). Moreover, we know from Lemma 5.7 that when \( z \leq 0 \), \( s(m) > 0 \Rightarrow s(m - 1) > 0 \). As a result, a necessary and sufficient condition for existence of a FTE when \( z \leq 0 \) is that \( T^t(q) \leq 0 \).

Proposition 5.8. Suppose \( z \leq 0 < w \). The (unique) equilibrium, (i) is a FTE iff \( T^t(q) \leq 0 \), and (ii) has breakdown of negotiations iff \( T^t(q - 1) > 0 \). Otherwise, in equilibrium there is delay in the initial state \( q \), and trade with probability one for all \( m < q \).

Note that adopting the project without delay is efficient for members of the coalition if \( v + qz \geq 0 \), and is efficient for the group as a whole if \( v + qz + (n - q)w \geq 0 \). Thus, for large enough \( v \), it is efficient to adopt the project even if \( w, z < 0 \). In fact, we know from part (ii) of Lemma 5.4 that for large \( v \), if the principal has enough bargaining power the unique MPE of the \( m \)-subgame is a FTE. So here the coalition should form, and it does form in equilibrium when the principal has enough bargaining power. On the other hand, part (iii) of the same lemma shows that when the agents have enough bargaining power there is no full trading equilibrium for large \( v \), even when this would be efficient.

The main point of the GH paper is that externalities across shareholders can prevent takeovers that add value to the company. The idea is that since shareholders that do not sell can capture the increase in value brought by the raider, no shareholder will tender his shares at a price that would allow the raider to profit from the takeover. GH work with a static model, and assume that shareholders ignore the impact of their actions on the outcome of the bid. In our version of the GH model – where the principal buys shares one at a time and shareholders are fully forward looking and strategic – efficient takeovers are not prevented by externalities when \( \delta < 1 \) as long as the raider has enough nominal bargaining power.\(^{23}\)

\(^{23}\text{Holmstrom and Nalebuff (1992) show that when shareholdings are divisible the free-riding problem does not prevent the takeover process in the GH model. In our model with } \phi = 1, \text{ the raider’s profit goes to zero as } \delta \to 1. \text{ Thus, with fixed costs, efficient raids would be prevented in the limit. This result is similar} \)
But when agents do have enough bargaining power, efficient takeovers can fail to occur due to the collective hold-up problem: with \( z \leq 0 \) the collective hold-up problem still exists, but leads not to delay but to breakdown of negotiations.

5.2. Robustness: Contingent Offers. In the model, we assumed that the transfers between principal and agent are a quid pro quo contingent on the behavior of the agent transacting with the principal, but not contingent on the completion of the project. This assumption is by far the most prevalent in the literature, and fits many applications well. In some other cases, however, the transfers between principal and agents only occur if and when the principal attains the prize.

An interesting example where this occurs is corporate restructuring in bankruptcy proceedings. In these cases, the firm or government in distress often negotiates new terms with creditors bilaterally and sequentially, as in our model.\(^{24}\) But the debt shaving that each creditor agrees to is only realized upon completion of the entire restructuring package.

A natural question is whether our results hold in this modified setting. The first thought could be that they won’t. The logic would go as follows: since agents contracting early can commit the principal’s rents and leave nothing up for other agents to grab later on in the bargaining process, incentives to hold out disappear. We show, however, that while contingent contracts allow other equilibria, collective hold up can still occur. In fact, when agents have all the bargaining power, the unique equilibrium outcomes in the “cash transfer” model are still an equilibrium when transfers are contingent on completion of the project. We state this result formally for ease of reference (the proof is included in Appendix B).

Remark 5.9. Consider a variant of the model in which transfers are contingent on completion of the project, and suppose \( \phi = 0 \) and \( z = w > 0 \). For \( v \) large, there is an equilibrium with trading at \( m = 1 \) and delay in all \( m : 2 \leq m \leq q \), given by trading probabilities \((11)\).

A fundamental difference in the contingent transfer model is that by affecting the amount of standing promises, agents can affect the equilibrium play of agents contracting later.\(^{25}\) As to that of Harrington and Prokop (1993), who consider a dynamic version of GH in which the raider can re-approach the shareholders who have not sold (taking all offers at the posted price in each period).

\(^{24}\)Consider for instance the city of Detroit’s bankruptcy restructuring. On December 2014, Detroit exited bankruptcy protection, 18 months after the city filed for Chapter 9 bankruptcy. The city negotiated a settlement with Bank of America and UBS in December 13’, with several bond insurers in January of 14’, pension plans in May 14’, reached a deal with three Michigan counties over regional water and sewer services in September, and bond insurers Syncora and FGIC in September and October 14’.

\(^{25}\)This implies, in particular, that the payoff-relevant state has to be extended to include both the number of agents required for completion and the amount of standing promises.
a result, it is not generally optimal for an agent to propose a transfer that extracts all the principal’s willingness to pay as it is the case in the main model. Instead, the optimal transfer in a non-critical state is the one that maximizes the continuation value of the committed agent subject to the constraints that the principal accepts the offer and the agent is better off than remaining uncommitted. However, in a critical state, the amount of the transfer does not affect future behavior, and as a result incentives for the principal and agent are the same as in the cash transfer model (corrected by using the principal’s value net of outstanding promises). Because of this, the agent contracting in a critical state will extract any surplus the principal arrives to that state with in equilibrium, and as a result, the principal will not be willing to reward agents contracting in earlier states. It follows that the decision of whether to transact or not with the principal is determined by the same tradeoffs than in the cash transfers model, so if an agent anticipates the same delay as in the equilibrium of the main model, he will likewise be indifferent between trading and not trading with the principal.

While the equilibrium outcomes in the “cash transfer” model are still an equilibrium when transfers are contingent on completion of the project, the contingent payment model allows other equilibria. The reason is that agents can use the principal as a vessel to extract payments from agents contracting later. This requires the principal to transitory carry a positive or negative balance even when she will not ultimately benefit from monetary flows. Analyzing all equilibria of the contingent transfer model is both interesting and important to understand sequential contracting in applications such as bankruptcy restructuring, but it is also fundamentally different than the analysis in this paper. We leave this for future research.

6. Conclusion

In this paper, we consider a dynamic process of coalition formation in which a principal bargains sequentially with a group of agents. We study how institutional changes affecting the allocation of bargaining power between principal and agents affect the distribution of rents and the efficiency of collective decisions. This question has only been addressed in models of bilateral bargaining or fully decentralized bargaining, in the absence of a principal. However, there are many applications that require the type of multilateral negotiations we model in this paper.

We give a complete characterization of equilibrium outcomes when the principal’s willingness to pay is high, and uncover new tradeoffs absent in bilateral bargaining models. We show that
redistributing bargaining power from the principal to the agents generates delay and reduces agents’ welfare, even in the absence of informational asymmetries or discriminatory offers, and even with negative externalities on uncommitted agents. Concentrating bargaining power on the principal, instead, leads to efficient collective decision-making and, for any non-unanimous decision rule, does not lead to complete rent extraction by the principal.

Our results have implications for a number of diverse applications in economics and politics, including lobbying, exclusive deals, start-ups, endorsements and corruption. While the model abstracts away from some of the details pertinent to each application, the results shed light on a common idea behind these apparently diverse problems: bargaining institutions that decentralize power to agents can be detrimental to agents’ welfare by making the coalition formation process inefficient.

The source of the inefficiency has two parts. The first is a form of the traditional hold-up problem: when agents have significant bargaining power relative to the principal, the principal anticipates that agents trading late in the process will extract a large fraction of the surplus, and as a result is not willing to pay much to agents trading early on. This is similar to Blanchard and Kremer (1997) and Olken and Barron (2009), where sequential bargaining under unanimity leads to increasing prices. The point we make here, however, is that given this hold-up problem, competition among agents leads to delay if and only if agents have too much bargaining power. This is what we call a collective hold-up problem.

The collective hold-up problem emerges in our model in the absence of discriminatory contracts or asymmetric equilibria, and do not require a particular form of externalities on uncommitted agents (non-traders). While we do not allow the principal to bargain with multiple agents simultaneously, we can show that this is not crucial for our results. In fact it is sufficient to assume that the principal cannot contract with $q$ agents at once.

Other extensions of the model are more challenging, and are left for future work. First, as we discussed in Section 5.2, we believe it is both interesting and important to study sequential contracting in applications such as bankruptcy restructuring, where contingent transfers are paramount. Second, our model does not allow for more general payoff structures in which payoffs depend on the size of the coalition that supports the principal, and can accrue before the coalition is formed. For example, in industries in which new technologies have a component of learning by doing, earlier sales affect later payoffs. Here the incentives to hold out compete with the benefits of joining early. This presents an interesting problem, where the principal may optimally front payments and sell at a loss. In that sense, collective hold-up may manifest itself in delayed learning.
A.1. Values. Consider the value of the principal in state $m$, $v(m)$. With probability $\phi \lambda_m$, the principal has agenda setting power and makes an offer that is accepted by the agent, getting a payoff $\delta v(m - 1) - p(m)$. With probability $1 - \phi \lambda_m$ either there is no transaction in $m$ or there is a transaction following a proposal by the agent, and the principal obtains a discounted continuation value $\delta v(m)$. Thus

$$v(m) = \phi \lambda_m (\delta v(m - 1) - p(m)) + (1 - \phi \lambda_m) \delta v(m).$$

Using (1), and subtracing $\phi \lambda_m \delta v(m)$ on both sides, we have

$$v(m) = \left(\frac{\delta}{1 - \delta}\right) \phi s^+(m)$$

where $s^+(m) = \max\{s(m), 0\}$. Equation (12.A) says that the value of the principal in state $m$ is proportional to the surplus in state $m$ whenever this is positive, and zero otherwise. The expression eliminates the dependency on the probability of trade $\lambda_m$ using the fact that if $s(m) > 0$ then $\lambda_m = 1$, if $s(m) < 0$ then $\lambda_m = 0$, and that $s(m) = 0$ when $\lambda_m \in (0, 1)$.

Consider instead the value of an uncommitted agent $i$ in state $m$, $w(m)$, recalling that $\beta(m) \equiv 1/(n + m - q)$ denotes the probability that agent $i$ meets the principal. With probability $\beta(m)(1 - \phi)\hat{\lambda}_m$, agent $i$ meets the principal, has agenda setting power, and makes an offer $b(m)$ (which is accepted), leading to a payoff $\delta w_{\text{out}}(m - 1) + b(m)$. With probability $(1 - \beta(m)) \mu_m$ another agent $j \neq i$ meets the principal, and the meeting results in a transaction, leading to a payoff $\delta w(m - 1)$ for player $i$. In all other cases ($i$ meets the principal but either the principal has agenda setting power or the transactions falls through, or some other agent $j \neq i$ meets the principal but the transaction falls through), agent $i$ gets a continuation payoff $\delta w(m)$:

$$w(m) = \beta(m)(1 - \phi)\hat{\lambda}_m [\delta w_{\text{out}}(m - 1) + b(m)] + (1 - \beta(m)) \mu_m \delta w(m - 1)$$

$$+ \left[\beta(m)\phi + (1 - \phi)(1 - \hat{\lambda}_m) + (1 - \beta(m)) (1 - \mu_m)\right] \delta w(m)$$

Using (5) for the transfer $b(m)$ and simplifying, we have that for all $m \geq 2$,

$$w(m) = \left[\frac{\delta \beta(m)}{1 - \delta \beta(m)}\right] (1 - \phi) s^+(m) + \left[1 + \left(\frac{1 - \delta}{1 - \beta(m)}\right) \frac{1}{\delta \mu_m}\right]^{-1} w(m - 1).$$ (13.A)

---

26As before, we have used the fact that if $s(m) > 0$ then $\hat{\lambda}_m = \mu_m = 1$, if $s(m) < 0$ then $\hat{\lambda}_m = \mu_m = 0$, and that $s(m) = 0$ when $\mu_m \in (0, 1)$. 

Using (13.A), we can express the current value for an uncommitted agent as a function of the final payoff \( w \) and the sequence of surpluses \( [s_k] \) for \( k \leq m \):

\[
(18) \quad w(m) = (1 - \phi) \sum_{k=1}^{m} \left( \frac{\beta(k)}{1 - \beta(k)} \right) e_{km} s^+(k) + e_{1m} w \quad \forall m \geq 1,
\]

where we have defined

\[
e_{km} = \left[ \prod_{j=k}^{m} \left( 1 + \left( \frac{1 - \delta}{1 - \beta(j)} \right) \frac{1}{\delta \mu_j} \right) \right]^{-1}
\]

A.2. Proofs.

**Lemma A.1 (Equilibrium Trade in state \( m = 1 \)).** The equilibrium probability of trade in state \( m = 1 \) is uniquely determined by the following conditions:

1. If \( v + z \leq 0 \), \( \mu_1 = 0 \) (no trade at \( m = 1 \)),
2. If \( 0 < v + z < \delta \left( \frac{n - q}{n - q + (1 - \delta)} \right) w \), \( \mu_1 \in (0, 1) \) (probabilistic trade at \( m = 1 \)),
3. If \( v + z \geq \delta \left( \frac{n - q}{n - q + (1 - \delta)} \right) w \), \( \mu_1 = 1 \) (trade w.p. 1 at \( m = 1 \)).

**Proof of Lemma A.1.** Fix a MPE \( \sigma \). Since the principal only makes an offer if \( s(m) \geq 0 \), (12) implies \( v(m) \geq 0 \) for all \( m \), and in particular \( v(1) \geq 0 \). Similarly, since the agent only makes an offer if \( s(m) < 0 \) then \( \hat{\lambda}_m = 0 \). Therefore (18) implies \( w(m) \geq 0 \), and in particular \( w(1) \geq 0 \). Since \( s(1) = v + z - v(1) - w(1) \), \( w(1), v(1) \geq 0 \) imply \( s(1) \leq v + z \).

It follows that if \( v + z < 0 \) then \( s(1) < 0 \) and there is no trade in equilibrium at \( m = 1 \). Now suppose \( v + z = 0 \). Then \( s(1) = -[v(1) + w(1)] \). If \( \mu_1 > 0 \), then \( v(1), w(1) > 0 \), and thus \( s(1) < 0 \), which implies \( \mu_1 = 0 \), a contradiction. Thus \( \mu_1 = 0 \) and \( v(1) = w(1) = 0 \). It follows that if \( v + z \leq 0 \), in equilibrium there is no trade in state \( m = 1 \).

Now suppose \( v + z > 0 \). If \( \mu_1 = 0 \) (no trade), then \( v(1) = w(1) = 0 \) and \( s(1) > 0 \), which implies \( \lambda_1 > 0 \), a contradiction. Suppose \( \mu_1 = 1 \). Then (12) gives \( v(1) = \frac{\delta}{1 - \phi} s(1) \) and (18) gives

\[
w(1) = \frac{\delta(1 - \phi) \beta(1)}{(1 - \delta \beta(1))} s(1) + \frac{\delta(1 - \beta(1))}{(1 - \delta \beta(1))} w
\]

Substituting,

\[
s(1) \left[ 1 + \frac{\delta \phi}{1 - \delta} + \frac{\delta(1 - \phi)}{(1 - \delta \beta(1)) \beta(1)} \right] = v + z - \frac{\delta(1 - \beta(1))}{(1 - \delta \beta(1))} w
\]
Thus \( s(1) \geq 0 \), consistent with equilibrium, iff
\[
v + z \geq \frac{\delta(1 - \beta(1))}{(1 - \delta \beta(1))} w
\]
If instead
\[
0 < v + z \leq \frac{\delta(1 - \beta(1))}{(1 - \delta \beta(1))} w = \delta \left( \frac{n - q}{n - q + (1 - \delta)} \right) w
\]
we have \( \mu_1 \in (0, 1) \). Note that with \( s(1) = 0 \), (12) implies \( v(1) = 0 \), and (18) implies that
\[
w(1) = \left( \frac{\delta \mu_1}{\left( \frac{1 - \delta}{1 - \delta \beta(1)} \right) + \delta \mu_1} \right) w
\]
Substituting in (2), the equilibrium probability of trade is given by
\[
\mu_1 = \left( \frac{1 - \delta}{\delta} \right) \frac{1}{1 - \beta(1)} \left( \frac{v + z}{w - (v + z)} \right)
\]
Note that the RHS of (21) \( \in (0, 1) \) iff (19) holds.

**Proof of Proposition 5.1.** Fix an equilibrium in the subgame starting in state \( m - 1 \). This produces continuation values \( \tilde{v}(m - 1) \), \( \tilde{w}(m - 1) \) and \( \tilde{w}_{\text{out}}(m - 1) \). Given these continuation values, let \( v(m; \mu_m) \) and \( w(m; \mu_m) \) denote the values of the principal and uncommitted agent in state \( m \) when transaction probability \( \mu_m \), and let \( s(m; \mu_m) \) denote the surplus in state \( m \) when transaction probability \( \mu_m \).

From (12) and (13), \( v(m; 0) = w(m; 0) = 0 \). Thus \( s(m; 0) = [\tilde{v}(m - 1) - v(m; 0)] + [\tilde{w}_{\text{out}}(m - 1) - w(m; 0)] = \tilde{v}(m - 1) + \tilde{w}_{\text{out}}(m - 1) \). It follows that if \( \tilde{v}(m - 1) + \tilde{w}_{\text{out}}(m - 1) \geq 0 \), inaction at \( m \) is not an equilibrium. But note that \( \tilde{v}(m - 1) \geq 0 \), and by (14), if \( z > 0 \) and \( \mu_k > 0 \) for all \( k < m \), then \( w_{\text{out}}(m) = \left[ \prod_{k=1}^{m} \left( \frac{\mu_k}{1 - \delta(1 - \mu_k)} \right) \right] z > 0 \). Thus \( \mu_m = 0 \) is not part of an equilibrium if \( \mu_k > 0 \) for all \( k < m \).

Suppose \( \mu_m = 1 \). Using the expression for the principal’s value (12) and the expression for the uncommitted agent’s value (13) in the definition of the surplus (2), we have
\[
s(m) \left[ 1 + \frac{\delta}{1 - \delta} + \frac{\left( \frac{\delta}{1 - \delta} \beta(m)(1 - \phi) \right)}{1 + \left( \frac{\delta}{1 - \delta} \right) (1 - \beta(m))} \right] = \tilde{w}_{\text{out}}(m - 1) + \tilde{v}(m - 1) - \frac{1}{1 + \left( \frac{1 - \delta}{\delta} \right) \left( \frac{1}{1 - \beta(m)} \right)} \tilde{w}(m - 1)
\]
Equilibrium requires that \( s(m) > 0 \). From the previous expression, \( s(m) > 0 \) iff
\[
1 + \left( \frac{1 - \delta}{\delta} \right) \left( \frac{1}{1 - \beta(m)} \right) > \frac{\tilde{w}(m - 1)}{\tilde{w}_{\text{out}}(m - 1) + \tilde{v}(m - 1)}.
\]
Next, suppose \( \mu_m \in (0, 1) \). Equilibrium then requires \( s(m) = 0 \), which in turn implies \( v(m) = 0 \) and then \( w(m) = \tilde{v}(m-1) + \tilde{w}_{\text{out}}(m-1) \). Also with \( s(m) = 0 \), (13) gives

\[
  w(m) = \left( \frac{\delta \mu_m}{1-\beta(m)} + \delta \mu_m \right) \tilde{w}(m-1)
\]

Substituting in \( w(m) = \tilde{v}(m-1) + \tilde{w}_{\text{out}}(m-1) \), and then solving for \( \mu_m \) gives

\[
  \mu_m = \left( \frac{1-\delta}{\delta} \right) \left( \frac{1}{1-\beta(m)} \right) \left( \frac{\tilde{v}(m-1) + \tilde{w}_{\text{out}}(m-1)}{\tilde{w}(m-1) - (\tilde{v}(m-1) + \tilde{w}_{\text{out}}(m-1))} \right),
\]

which is the statement in the proposition. This is less than one iff (22) doesn’t hold.

We have shown that if \( \mu_k > 0 \) for all \( k < m \), equilibrium play in state \( m \) is uniquely determined, and is either \( \mu_m = 1 \) if (22) holds or \( \mu_m \in (0, 1) \) given in (23) if (22) doesn’t hold. Finally note that by Lemma A.1, if \( v, z > 0 \) then \( \mu_1 > 0 \). An induction argument then completes the proof. \( \square \)

**Lemma A.2.** Let \( \theta_{km} \equiv \prod_{j=k}^{m} \left( \frac{\delta \mu_j}{1-\delta + \delta \mu_j \phi(1-\beta(j))} \right) \). In a MPE with trade probabilities \( \mu \), the agents’ equilibrium payoff in each state \( m \in M \) is given by

\[
  w(m) = \theta_{1m}(n-q)w + \sum_{k=1}^{m} \theta_{km} \left( \frac{1-\phi}{\phi} \right) \left( \frac{1-\delta}{\delta} \right) \frac{1}{\mu_k} \left( \prod_{j=1}^{k} \frac{\delta \mu_j}{1-\delta(1-\mu_j)} \right) (v + kz + (n-q)w)
\]

**Proof of Lemma A.2.** The value functions of the principal and agents satisfy

\[
  v(m) = \mu_m \frac{\delta}{1-\delta} \phi s(m)
\]

and

\[
  w(m) = \frac{\delta \beta(m)(1-\phi)\mu_m}{1-\delta + \delta(1-\beta(m))\mu_m} s(m) + \frac{\delta(1-\beta(m))\mu_m}{1-\delta + \delta(1-\beta(m))\mu_m} w(m-1)
\]

Substituting (25) in the surplus condition (2) and using that \( \frac{1-\beta(m)}{\beta(m-1)} = \frac{1}{\beta(m-1)} \) we have the system of difference equations:

\[
  (1-\phi)s(m) = \frac{1-\delta}{\delta \mu_m + 1-\beta(m)} \frac{w(m)}{\beta(m)} - \frac{w(m-1)}{\beta(m-1)}
\]

\[
  \frac{1-\delta + \delta \phi \mu_m}{1-\delta} s(m) = \mu_{m-1} \frac{\delta}{1-\delta} \phi s(m-1) + w_{\text{out}}(m-1) - w(m)
\]

Solving the first equation for \( s(m) \) and substituting in the second equation, we transform the system of first order difference equations into a second order difference equation. Letting
\( \alpha_m \equiv \frac{\delta \mu_m}{1 - \delta(1 - \mu_m)} \), and defining

\[
H(m) \equiv \frac{\phi}{1 - \phi} \frac{\delta}{1 - \delta} \left[ \left( 1 - \frac{\delta}{\delta \phi} + \mu_m(1 - \beta(m)) \right) \frac{w(m)}{\beta(m)} - \mu_m \frac{w(m - 1)}{\beta(m - 1)} \right],
\]

we can write this recursion as

\[
H(m) = \alpha_m H(m - 1) + \alpha_m w_{\text{out}}(m - 1) \quad \text{for} \quad m : 3 \leq m \leq m'
\]

Solving recursively, and using that \( w_{\text{out}}(m - 1) = \alpha_m w_{\text{out}}(m - 1) \) we have

\[
H(m) = \left( \prod_{j=3}^{m} \alpha_j \right) H(2) + (m - 2) w_{\text{out}}(m)
\]

Therefore, letting \( \tau_m = \frac{1 - \delta}{1 - \delta + \delta \mu_m(1 - \beta(m))} \) for convenience,

\[
\frac{w(m)}{\beta(m)} = \frac{1 - \delta(1 - \mu_m)}{1 - \delta} \phi \tau_m \alpha_m \frac{w(m - 1)}{\beta(m - 1)} + \tau_m (1 - \phi) \left[ \left( \prod_{j=3}^{m} \alpha_j \right) H(2) + (m - 2) w_{\text{out}}(m) \right]
\]

The boundary conditions follow by (27) for \( m = 1, 2 \) and (28) for \( H(2) \), which give

\[
H(2) = \alpha_2 \alpha_1 \left( v + 2z + \frac{w}{\beta(0)} \right)
\]
\[
\frac{w(2)}{\beta(2)} = \tau_2 \left( \frac{\alpha_2}{\tau_1} \frac{1 - \mu_2 \phi}{1 - \delta} \right) \frac{w(1)}{\beta(1)} - \alpha_2 \tau_2 \mu_1 \frac{\delta}{1 - \delta} \frac{w}{\beta(0)} + \alpha_2 \tau_2 (1 - \phi) w_{\text{out}}(1)
\]
\[
\frac{w(1)}{\beta(1)} = \tau_1 \phi \left[ \frac{\delta}{1 - \delta} \frac{w}{\beta(0)} + \alpha_1 \frac{1 - \phi}{\phi} \left( v + z + \frac{w}{\beta(0)} \right) \right]
\]

Using these initial conditions together with \( w_{\text{out}}(m) = \left( \prod_{j=1}^{m} \alpha_j \right) z \), we obtain a simple recursive representation of the value functions

\[
\frac{w(m)}{\beta(m)} = \frac{1 - \delta(1 - \mu_m)}{1 - \delta} \phi \tau_m \alpha_m \frac{w(m - 1)}{\beta(m - 1)} + \tau_m (1 - \phi) \left( \prod_{j=1}^{m} \alpha_j \right) (v + mz + (n - q)w)
\]
Solving recursively, we obtain

\[
\frac{w(m)}{\beta(m)} = \left( \prod_{j=1}^{m} \alpha_j \right) \left[ \prod_{j=1}^{m} \left( \frac{1 - \delta(1 - \mu_j)}{1 - \delta} \phi \tau_j \right) \right] (n - q)w
\]

\[
+ (1 - \phi) \left( \prod_{j=1}^{m} \alpha_j \right) \sum_{k=1}^{m-1} \left[ \prod_{j=k+1}^{m} \left( \frac{1 - \delta(1 - \mu_j)}{1 - \delta} \phi \tau_j \right) \right] \tau_k (v + kz + (n - q)w)
\]

\[
+ \tau_m (1 - \phi) \left( \prod_{j=1}^{m} \alpha_j \right) (v + mz + (n - q)w),
\]

which is equivalent to (24).

\[\Box\]

**Lemma A.3.** Consider any \( m \leq q \). For any equilibrium \( \mu_1, \ldots, \mu_{m-1} \) of the \( m - 1 \) subgame, \( s(m) \geq (\leq)0 \) given \( \mu_m \in [0, 1] \) if and only if

\[
T(m) \equiv \frac{w(m)}{\beta(m)} - \left( \prod_{j=1}^{m} \frac{\delta \mu_j}{1 - \delta(1 - \mu_j)} \right) (v + mz + (n - q)w) \leq (\geq)0
\]

**Proof of Lemma A.3.** Using (30) we get that the surplus condition (27) is equivalent to

\[
(27b) \quad \left( \frac{\delta}{1 - \delta} \right) \phi_{\mu_m} s(m) = \left( \prod_{j=1}^{m} \frac{\delta \mu_j}{1 - \delta(1 - \mu_j)} \right) (v + mz + (n - q)w) - \frac{w(m)}{\beta(m)}
\]

Therefore \( s(m) > (\leq)0 \) if and only if

\[
\left( \prod_{j=1}^{m} \frac{\delta \mu_j}{1 - \delta(1 - \mu_j)} \right) (v + mz + (n - q)w) > (\leq) \frac{w(m)}{\beta(m)}
\]

\[\Box\]

**Proof of Lemma 5.2.** In Lemma A.3 we showed that for any \( m \leq q \), and for any equilibrium \( \mu_1, \ldots, \mu_{m-1} \) of the \( m - 1 \) subgame, \( s(m) \geq (\leq)0 \) given \( \mu_m \in [0, 1] \) if and only if \( T(m) \leq (\geq)0 \). We now show that for large \( v \), \( T(m) \leq 0 \Rightarrow T(m - 1) < 0 \). Note that

\[
T(m - 1) = \left( \frac{1 - \delta + \delta \mu_m \phi(1 - \beta(m))}{\phi[1 - \delta(1 - \mu_m)]} \right) T(m) + z - \left( \frac{\delta \beta(m) \mu_m}{1 - \delta(1 - \mu_m)} \right) (v + mz + (n - q)w)
\]

so if \( T(m) \leq 0 \), we have

\[
T(m - 1) \leq z - \delta \beta(m) (v + mz + (n - q)w),
\]

where we have used the fact that \( T(m) \leq 0 \) implies \( \mu_m = 1 \). Since the RHS is decreasing in \( v \) and goes to \(-\infty\) as \( v \to \infty \), for sufficiently large \( v \), then \( T(m) \leq 0 \Rightarrow T(m - 1) < 0 \). \[\Box\]
Proof of Lemma 5.3. Here we prove that if in equilibrium $\mu_m \in (0, 1)$ for all $m \in J \equiv \{m_\ell, \ldots, m_u\}$, then
\[
\frac{\mu_{m+1} - \mu_m}{\mu_m} = \beta(m) \quad \forall m \in \{m_\ell + 1, \ldots, m_u - 1\},
\]
and moreover
\[
\mu_m = \left(\frac{n + m - q}{n + m_\ell - q}\right) \left(\frac{1 - \delta}{\delta}\right) \left(\frac{1}{\mu_m} (w(m)/w_{out}(m_\ell) - 1)\right) \quad \forall m \in \{m_\ell + 1, \ldots, m_u\}.
\]
(This second result will be useful in the proof of Theorem 5.5).

Suppose in equilibrium $\mu_m \in (0, 1)$ for all $m \in J \equiv \{m_\ell, \ldots, m_u\}$. Then $s(m) = v(m) = 0$ for all $m \in M$. Since $s(m) = 0$ for all $m \in J$, by (13),
\[
w(m) = \left(\frac{\delta \mu_m}{\frac{1 - \delta}{1 - \beta(m)} + \delta \mu_m}\right) w(m - 1) \quad \forall m \in J
\]
Note that for all $m \in \{m_\ell + 1, \ldots, m_u\}$, $v(m) = v(m - 1) = 0$, and then $s(m) = 0$ implies $w(m) = w_{out}(m - 1)$. Then
\[
\frac{w(m)}{w(m - 1)} = \frac{w_{out}(m - 1)}{w_{out}(m - 2)} \quad \forall m \in \{m_\ell + 2, \ldots, m_u\}.
\]
Using (32) and (14), this is
\[
\left(\frac{\delta \mu_m}{\frac{1 - \delta}{1 - \beta(m)} + \delta \mu_m}\right) = \left(\frac{\delta \mu_{m-1}}{1 - \delta + \delta \mu_{m-1}}\right) \quad \forall m \in \{m_\ell + 2, \ldots, m_u\},
\]
which implies that
\[
\mu_m = \left(\frac{1}{1 - \beta(m)}\right) \mu_{m-1} \quad \forall m \in \{m_\ell + 2, \ldots, m_u\},
\]
This gives the first result using the definition of $\beta(m)$. This result directly implies
\[
\mu_m = \left[\prod_{k=m_\ell+2}^m \left(\frac{1}{1 - \beta(k)}\right)\right] \mu_{m_\ell+1} \quad \forall m \in \{m_\ell + 2, \ldots, m_u\}.
\]
Now, by (15), and noting that $v(m_\ell) = 0$,
\[
\mu_{m_\ell+1} = \left(\frac{1 - \delta}{\delta}\right) \left(\frac{1}{1 - \beta(m_\ell + 1)}\right) \left(\frac{w_{out}(m_\ell)}{w(m_\ell) - w_{out}(m_\ell)}\right).
\]
Substituting gives

\[
\mu_m = \left[ \prod_{k=m\ell+1}^{m} \left( \frac{1}{1 - \beta(k)} \right) \left( \frac{1 - \delta}{\delta} \right) \frac{1}{w(m\ell)/w_{\text{out}}(m\ell) - 1} \right] \quad \forall m \in \{m\ell + 1, \ldots, m_u\}
\]

Noting that \(1 - \beta(k) = \frac{n+m-q-1}{n+m-q}\), and simplifying, gives the result in the lemma. \(\square\)

Proof of Lemma 5.4. Part (i) of the Lemma follows immediately from Lemma A.3, specializing for the case of a FTE. Part (ii) follows from Lemmas A.4 and A.5. \(\square\)

Lemma A.4. Consider \(m \leq q\), and suppose \(v \geq m(w - z)\). Then there is a \(\phi(m) \in (0, 1)\) such that if \(\phi > \phi(m)\), the unique MPE of the \(m\)-subgame is a FTE.

Proof of Lemma A.4. From expression (16),

\[
\lim_{\phi \to 1} \frac{w^\dagger(m)}{\beta(m)} = \left( \prod_{j=1}^{m} \frac{\delta}{1 - \delta \beta(j)} \right) (n - q)w
\]

So in the limit \(T^\dagger(m) \leq 0\) iff

\[
\left( \prod_{j=1}^{m} \frac{\delta}{1 - \delta \beta(j)} \right) (n - q)w \leq \delta^m (v + mz + (n - q)w)
\]

or iff

\[
\left( \prod_{j=1}^{m} \frac{n + j - q}{n + j - q - \delta} \right) \leq \left( \frac{v + mz + (n - q)w}{(n - q)w} \right)
\]

Expanding the product, the LHS is smaller than \(\frac{n+m-q}{n+1-\delta-q} < \frac{n+m-q}{n-q}\), so it is sufficient that \(v \geq m(w - z)\). Thus, for large \(\phi\), a sufficient condition for a FTE in the \(m\)-subgame is \(v \geq q(w - z)\). \(\square\)

Lemma A.5. For any \(m \leq q\), there exists \(\phi(m) \in (0, 1)\) and \(\bar{v}(m) > 0\) such that if \(\phi < \phi(m)\) and \(v > \bar{v}(m)\), the unique MPE of the \(m\)-subgame entails delay.

Proof of Lemma A.5. From expression (16),

\[
\frac{w^\dagger(m)}{\beta(m)} = \sum_{j=1}^{m} \left( \prod_{k=j}^{m} \frac{\delta \phi}{1 - \delta + \delta \phi(1 - \beta(k))} \right) \frac{1 - \delta}{\phi} \frac{(1 - \phi)}{\delta} \delta^j (v + jz + (n - q)w)
\]

\[
+ \left( \prod_{j=1}^{m} \frac{\delta \phi}{1 - \delta + \delta \phi(1 - \beta(j))} \right) (n - q)w
\]
Note that for large \( v \) all terms are positive, except possibly the last one. Dropping the first \( m - 2 \) terms of the summation, and denoting the last term \( C \) for convenience, we have

\[
\frac{w^t(m)}{\beta(m)} > \left( \frac{\delta \phi}{1 - \delta + \delta \phi (1 - \beta(m - 1))} \right) \left( \frac{(1 - \delta)(1 - \phi)}{1 - \delta + \delta \phi (1 - \beta(m))} \right) \delta^{m-1} (v + (m - 1)z + (n - q)w) + \left( \frac{(1 - \delta)(1 - \phi)}{1 - \delta + \delta \phi (1 - \beta(m))} \right) \delta^m (v + mz + (n - q)w) + C
\]

So \( T^t(m) \equiv \frac{w^t(m)}{\beta(m)} - \delta^m (v + mz + (n - q)w) > 0 \) iff

\[
\left( \frac{(1 - \delta)(1 - \phi)}{1 - \delta + \delta \phi (1 - \beta(m - 1))} \right) \left( \frac{1}{1 - \delta + \delta \phi (1 - \beta(m))} \right) (v + (m - 1)z + (n - q)w) + \tilde{C} \geq (v + mz + (n - q)w),
\]

where \( \tilde{C} \) is a term, possibly negative, that does not depend on \( v \).

Taking derivatives of both sides with respect to \( v \), the LHS increases faster than the RHS iff

\[
\phi \leq \frac{(1 - \delta)\beta(m)}{(1 - \delta) + \delta (1 - \beta(m)) (1 - \phi)} \equiv \phi(m)
\]

It follows that if \( \phi < \phi(m) \), then \( T^t(m) > 0 \). □

**Proof of Theorem 5.5.** By Lemma A.5 for any \( m \leq q \), there exists \( \phi(m) > 0 \) and \( \overline{v}(m) > 0 \) such that if \( \phi < \phi(m) \) and \( v > \overline{v}(m) \), then \( T^t(m|\phi) > 0 \). From Lemma A.4, assuming \( v \geq m(w - z) \), we have that for any \( m \leq q \) there is a \( \phi(m) < 1 \) such that if \( \phi > \phi(m) \), \( T^t(m|\phi) \leq 0 \), and the unique MPE of the \( m \)-subgame is a FTE. Since \( T^t(m|\phi) \) is continuous in \( \phi \), for any \( m \) there is a \( c_m \in (0, 1) \) such that \( T^t(m|c_m) = 0 \) (for \( v \) large, fixed). By Lemma 5.2, for large \( v \) in equilibrium \( T(m'|c_m) > 0 \) for all \( m' > m \). It follows that in the unique MPE for \( \phi = c_m \), we have \( \mu_k = 1 \) for all \( k \leq m \) and (provided \( m < q \), \( \mu_k \in (0, 1) \) for \( k > m \).

We have shown before that \( T(m'|c_m) > 0 = T^t(m|m) \) for all \( m' > m \), and that if \( T^t(m|\phi) \leq 0 \) then \( T^t(m'|\phi) \) is decreasing in \( \phi \) for all \( m' \leq m \) (also for \( v \) large). This implies that \( c_{m+1} > c_m \) for all \( m \leq q - 1 \), and that for any \( \phi \in (c_m, c_{m+1}) \), \( T^t(m + 1|\phi) > 0 \) and \( T^t(m|\phi) \leq 0 \). It follows that the equilibrium characterization above for \( \phi = c_m \) applies unchanged to all \( \phi \in [c_m, c_{m+1}] \).

Now take \( \phi \in [0, 1] \) given, and let \( \overline{m} \in M \) denote the cutpoint such that, in equilibrium, there is delay in each state \( m \in M \) s.t. \( m > \overline{m} \), and full trading in any \( m \leq \overline{m} \). In the proof
of Lemma 5.3 we show that if in equilibrium \( \mu_m \in (0, 1) \) for all \( m \in J \equiv \{m_\ell, \ldots, m_u\} \), then

\[
\mu_m = \left( \frac{n + m - q}{n + m_\ell - q} \right) \left( \frac{1 - \delta}{\delta} \right) \left( \frac{1}{w(m_\ell)/w_{out}(m_\ell) - 1} \right) \quad \forall m \in \{m_\ell + 1, \ldots, m_u\}
\]

It follows that here (with \( m_\ell = \overline{m} + 1 \) and \( m_u = q \)), we have

\[(35) \quad \mu_m = \left( \frac{n + m - q}{n + \overline{m} + 1 - q} \right) \left( \frac{1 - \delta}{\delta} \right) \left( \frac{1}{w(\overline{m} + 1)/w_{out}(\overline{m} + 1) - 1} \right) \quad \forall m > \overline{m} + 1
\]

Note that the probability of trade in each state where there is delay is decreasing in the ratio \( w(\overline{m} + 1)/w_{out}(\overline{m} + 1) \). We now argue that this ratio is increasing in \( v \), and that \( \mu_m \to 0 \) as \( v \to \infty \). Note that by (14),

\[
\frac{w_{out}(\overline{m} + 1)}{w_{out}(\overline{m})} = \left( \frac{\delta \mu_{\overline{m} + 1}}{1 - \delta (1 - \mu_{\overline{m} + 1})} \right),
\]

and by Proposition 5.1,

\[
\delta \mu_{\overline{m} + 1} = (1 - \delta) \left( \frac{1}{1 - \beta(\overline{m} + 1)} \right) \left( \frac{v(\overline{m}) + w_{out}(\overline{m})}{w(\overline{m}) - (v(\overline{m}) + w_{out}(\overline{m}))} \right).
\]

Substituting,

\[
\frac{w_{out}(\overline{m} + 1)}{w_{out}(\overline{m})} = \left( \frac{v(\overline{m}) + w_{out}(\overline{m})}{\beta(\overline{m} + 1)(v(\overline{m}) + w_{out}(\overline{m})) + (1 - \beta(\overline{m} + 1))w(\overline{m})} \right).
\]

Now, since \( \mu_{\overline{m} + 1} \in (0, 1) \), then \( v(\overline{m}) + w_{out}(\overline{m}) = w(\overline{m} + 1) \). Substituting, and noting that the equilibrium of the \( \overline{m} \) subgame is a FTE,

\[
\frac{w_{out}(\overline{m} + 1)}{w(\overline{m} + 1)} = \frac{w_{out}(\overline{m})}{(1 - \beta(\overline{m} + 1))w^*(\overline{m}) + \beta(\overline{m} + 1)[(v^*(\overline{m}) + w_{out}^*(\overline{m}))]}.
\]

It follows that for \( m > \overline{m} + 1 \),

\[
\mu_m = \left( \frac{n + m - q}{n + \overline{m} + 1 - q} \right) \left( \frac{1 - \delta}{\delta} \right) \left( \frac{w_{out}(\overline{m})}{(1 - \beta(\overline{m} + 1))(w^*(\overline{m}) - w_{out}^*(\overline{m})) + \beta(\overline{m} + 1)(v^*(\overline{m}))} \right)
\]

Now, \( w_{out}^*(\overline{m}) = \delta \overline{m} z \) is independent of \( v \), while both \( v^*(\overline{m}) \) and \( w^*(\overline{m}) \) are increasing in \( v \), and unbounded. Thus for \( m > \overline{m} + 1 \), \( \mu_m \) is decreasing in \( v \) and goes to zero as \( v \to +\infty \). This completes the proof. \( \square \)
Proof of Proposition 5.7. Using (12), (18), (14) in (2) we obtain, for all \( m \geq 2 \)
\[
(36) 
\left( 1 + \phi \left( \frac{\delta}{1 - \delta} \right) + (1 - \phi) \frac{\delta \beta(m)}{1 - \delta \beta(m)} \right) s(m) \\
= \phi \left( \frac{\delta}{1 - \delta} \right) s(m - 1) + \left[ \prod_{k=1}^{m-1} \left( \frac{\delta \mu_k}{1 - \delta (1 - \mu_k)} \right) \right] z - \pi(1) w \\
- (1 - \phi) \sum_{k=1}^{m-1} \left( \frac{\beta(k)}{1 - \beta(k)} \right) \pi(k) s(k)
\]
Since \( w > 0, z \leq 0 \), and \( \pi(k) s(k) \geq 0 \), (36) implies
\[
(37) 
\left( 1 + \phi \left( \frac{\delta}{1 - \delta} \right) + (1 - \phi) \frac{\delta \beta(m)}{1 - \delta \beta(m)} \right) s(m) \leq \phi \left( \frac{\delta}{1 - \delta} \right) s(m - 1)
\]
It follows that in any equilibrium, \( s(m - 1) \leq 0 \Rightarrow s(m) \leq 0 \). So suppose \( s(m') < 0 \) for some \( m' < q \). Then \( \mu_{m'} = 0 \), and thus \( w(m) = v(m) = 0 \) for all \( m \geq m' \) with no transactions in equilibrium for \( m \geq m' \). Suppose instead \( s(m') = 0 \) for some \( m' < q \). If \( \mu_{m'} = 0 \), the same conclusion holds, so suppose in equilibrium \( \mu_{m'} \in (0, 1) \). Because \( s(m' + 1) \leq 0 \), in equilibrium either \( \mu_{m'+1} = 0 \) or \( s(m' + 1) = 0 \) and \( \mu_{m'+1} \in (0, 1) \). If \( \mu_{m'+1} \in (0, 1) \), then \( v(m') = v(m'+1) = 0 \), and then \( s(m'+1) = 0 \) implies \( w(m'+1) = w_{out}(m') \). But \( w(m'+1) \geq 0 \), while \( w_{out}(m') = \left[ \prod_{k=1}^{m'} \left( \frac{\delta \mu_k}{1 - \delta (1 - \mu_k)} \right) \right] z < 0 \) by (14), which is a contradiction. It follows that if \( s(m') \leq 0 \) for some \( m' < q \), then \( \mu_m = 0 \) for all \( m > m' \) and \( w(q) = v(q) = 0 \).

Corollary A.6. Suppose \( z \leq 0 \). If \( v + z < \delta \left( \frac{n-q}{n-q+1-\delta} \right) w \), then \( s(m) = w(m) = w_{out}(m) = v(m) = 0 \) for all \( m \geq 2 \).

Proof of Corollary A.6. In Lemma A.1 we showed that a necessary condition for trade with probability one at \( m = 1 \) is that \( v + z \geq \delta \left( \frac{n-q}{n-q+1-\delta} \right) w \). Thus, when this condition is violated, \( \mu_1 < 1 \). The result then follows from Proposition 5.7.

Proof of Proposition 5.8. First we show that \( T^+(q) \leq 0 \) is a necessary and sufficient condition for existence of a FTE. This follows as a corollary of previous results. First, an examination of the proof of parts (i) and (ii) of Lemma 5.4 shows that these results do not require the assumption that \( z > 0 \), and thus also hold for \( z \leq 0 \). Thus, agents’ FTE payoffs are still given by \( w^+(\cdot) \) as defined by (16), and there exists a FTE in the \( m' \)-subgame if and only
if $T^\dagger(m) \leq 0$ for all $m \leq m'$. Moreover, we know from Lemma 5.7 that when $z \leq 0$, $s(m) > 0 \Rightarrow s(m-1) > 0$. As a result, a necessary and sufficient condition for existence of a FTE when $z \leq 0$ is that at the FTE profile, $s(q) > 0$, or $T^\dagger(q) \leq 0$.

Second, we show that if there exists a FTE, this is the unique MPE. Fix an equilibrium in the subgame starting in state $m-1$. This produces continuation values $\tilde{v}(m-1)$ and $\tilde{w}_{out}(m-1)$. Given these continuation values, let $v(m; \mu_m)$ and $w(m; \mu_m)$ denote the values of the principal and uncommitted agent in state $m$ when transaction probability $\mu_m$, and let $s(m; \mu_m)$ denote the surplus in state $m$ when transaction probability $\mu_m$. From (12) and (13), if $\tilde{v}(m-1) > 0$ and $\tilde{w}_{out}(m-1) > 0$, then $v(m; \mu_m)$ and $w(m; \mu_m)$ are both increasing in $\mu_m$, and therefore $s(m; \mu_m) = [\tilde{v}(m-1) - v(m; \mu_m)] + [\tilde{w}_{out}(m-1) - w(m; \mu_m)]$ is decreasing in $\mu_m$. It follows that if $s(m; 1) > 0$, then $s(m; \mu_m) > 0$ for any $\mu_m \in (0, 1)$, and as a result, any such $\mu_m \in (0, 1)$ would not be consistent with equilibrium.

We finish the proof of this step with an induction argument. First, note that if the conditions for existence of a FTE are met, then by Lemma A.1 $\mu_1 = 1$ (the unique MPE of the subgame starting at $m = 1$ is a FTE). Second, we argue that if the unique MPE of the subgame starting in state $m-1$ is a FTE, then $\mu_m = 1$. The two conditions establish the result. To prove the induction step, note that if the unique MPE of the subgame starting in state $m-1$ is a FTE, then existence of a FTE in $m \leq q$ (guaranteed by Lemma 5.7 given the existence of a FTE) implies that $s(m; 1) > 0$. Then our previous argument implies that $s(m; \mu_m) > 0$ for any $\mu_m \in (0, 1)$, and as a result we must have $\mu_m = 1$.

Finally, from Proposition 5.7 we know that if $T^\dagger(q) > 0$ there are two possibilities: either trade stops at some $m < q$ and then $w(q) = v(q) = 0$, or there is a FTE in the $(q-1)$-subgame and delay in the initial state $q$. The first case holds if $T^\dagger(q-1) > 0$, and the latter in the intermediate case in which $T^\dagger(q-1) \leq 0 < T^\dagger(q)$. This concludes the proof. \qed

Proof of example 5.6. It remains true that $v(m) = 0$ for any $m = 1, 2, 3$, and that trade occurs with certainty at $m = 1$. This implies that

$$w(1) = \frac{1}{2} \int_0^\Delta e^{-rs}(w + v) + \frac{1}{2} \int_0^\Delta e^{-rs}w = e^{-r\Delta} \frac{2w + v}{2r} \approx (1 - r\Delta) \frac{2w + v}{2r}$$

where we use the linearization for small $\Delta$, $e^{-r\Delta} = 1 - r\Delta$. Similarly

$$w_{out}(1) = (1 - r\Delta) \frac{w}{2r}$$
Let $\mu(m)$ be the instantaneous probability that an agent makes an offer to the principal, and when the offer is made the principal accepts the agent’s offer. The transaction takes place before the beginning of the next period, so we have

$$w(2) = (1 - r\Delta) \left[ (1 - \mu(2)\Delta)w(2) + \mu(2)\Delta \left( \frac{1}{3}w_{out}(1) + \frac{2}{3}w(1) \right) \right]$$

and

$$w_{out}(2) = (1 - r\Delta) \left[ (1 - \mu(2)\Delta)w_{out}(2) + \mu(2)\Delta w_{out}(1) \right]$$

We are going to show first that $\mu(2) \in (0, 1)$ when $v$ is large. First assume that $\mu(2) = 0$, so $w(2) = 0$ but since $w_{out}(1) > 0$ we have that $s(2) = w_{out}(1) - w(2) > 0$; a contradiction. If $\mu(2) = 1$, then

$$w_{out}(1) - w(2) = \frac{(1 - r\Delta)w}{6r} + \frac{(1 - r\Delta)}{r + (1 - r\Delta)} \left( 1 - \frac{v}{2w} \frac{1 - r\Delta}{6r + (1 - r\Delta)} \right)$$

and implies that $s(2) < 0$ for sufficiently large $v$. It follows that in equilibrium we must have that $w(2) = w_{out}(1)$ so the agent is willing to randomize between making an offer that is accepted and remaining uncommitted in state $m = 2$, which gives the trading probabilities

$$\mu(2) = \frac{r}{1 - r\Delta} \frac{w}{w + \frac{3}{2}v}$$

and therefore

$$w(2) = (1 - r\Delta) \frac{w}{2r} \quad \text{and} \quad w_{out}(2) = (1 - r\Delta) \frac{3w}{5w + 2v} \frac{w}{2r}$$

In state $m = 3$ we have

$$w(3) = (1 - r\Delta) \left[ (1 - \mu(3)\Delta)w(3) + \mu(3)\Delta \left( \frac{1}{4}w_{out}(2) + \frac{3}{4}w(2) \right) \right]$$

and

$$w_{out}(3) = (1 - r\Delta) \left[ (1 - \mu(3)\Delta)w_{out}(3) + \mu(3)\Delta w_{out}(2) \right]$$
and

\[
    w_{\text{out}}(3) = (1 - r\Delta) [ (1 - \mu(3)\Delta) w_{\text{out}}(3) + \mu(3)\Delta w_{\text{out}}(2) ]
    = \frac{(1 - r\Delta)^2 \mu(3)}{r + (1 - r\Delta) \mu(3)} \frac{3w}{w} \frac{w}{w}
\]

Using the same arguments as before, we have that if \( \mu(3) = 0 \), then \( s(3) = w_{\text{out}}(2) - w(3) > 0 \) and there is a profitable deviation to make an offer with probability 1 in state \( m = 3 \), and for sufficiently large \( v \) if \( \mu(3) = 1 \), then \( s(3) < 0 \). It follows that for large \( v \), the equilibrium involves \( s(3) = 0 \) which gives the trading probability

\[
    \mu(3) = \frac{r}{(1 - r\Delta)} \frac{2w}{w + v}
\]

and therefore

\[
    w(3) = (1 - r\Delta) \frac{3w}{2r} \frac{w}{5w + 2v} \quad \text{and} \quad w_{\text{out}}(3) = (1 - r\Delta) \frac{w}{3w + v} \frac{3w}{5w + 2v} \frac{w}{r}
\]

yielding collective hold up even in the continuous time case too. \( \square \)
Appendix B. Contingent Transfers (Not for Publication)

Suppose now that transfers are contingent on winning. In this case the state is not just the number of agents remaining in order to win, but also the total accumulated promises before each move, call it $B$. So the state is a pair $(m, B) \in \mathbb{N} \times \mathbb{R}$. We let $w_{\text{out}}(m, B; b)$ denote the eq. payoff in state $(m, B)$ of an agent who committed for the principal with offer $b$. Thus, if an agent committed for the principal in state $(m', B')$ with offer $b(m', B')$, his value in state $(m, B)$ for $m < m'$ is $w_{\text{out}}(m, B; b(m', B'))$.

Note that since offers in each state can affect the probability of trade in subsequent states, it is not necessarily optimal for the agent to propose the largest offer the principal is willing to accept. This makes solving for equilibrium transfers more involved.

Note first that the principal accepts an offer $b$ in state $(m, B)$ only if

$$\delta v(m - 1, B + b) \geq \delta v(m, B).$$

Let $A(m, B)$ denote the set of all proposals $b$ that the principal accepts in state $(m, B)$. Note that if an offer $b$ is accepted in state $(m, B)$, then the agent gets a payoff $\delta w_{\text{out}}(m - 1, B + b; b)$, and if the offer is rejected, or if the agent does not make an offer, the agent gets $\delta w(m, B)$. Let $A(m, B) \equiv \{b \in A(m, B) : w_{\text{out}}(m - 1, B + b; b) \geq w(m, B)\}$. If $A(m, B) = \emptyset$, the agent makes no offer, or equivalently, offers $b = \infty$. If $A(m, B) \neq \emptyset$, the agent makes an offer $b(m, B)$, given by the $b$ which solves

$$\max_{b \in A(m, B)} \delta w_{\text{out}}(m - 1, B + b; b)$$

Critical states are special, however, because conditional on the transaction being successful, new promises don’t affect future play, as before. Consider a critical state $(1, B)$ for some $B > 0$, and suppose the principal accepts if indifferent (this actually has to be the case in equilibrium if $\overline{A}(1, B)$ is not a singleton, for the usual reason in bargaining models). Since $w_{\text{out}}(0, B + b; b) = w + b$, and $v(0, B + b) = v - B - b$, the principal accepts iff $b \leq v - B - v(1, B)$, and the agent is better off making an offer $b$ iff $b \geq w(1, B) - w$. Thus $\overline{A}(1, B) = [w(1, B) - w, v - B - v(1, B)]$. This is nonempty iff $s_1(B) = v - B - v(1, B) + w - w(1, B) \geq 0$. Since $w_{\text{out}}(0, B + b; b) = w + b$, the optimal offer when $s_1(B) \geq 0$ is the largest $b$ in $\overline{A}(1, B)$. i.e.,

$$b(m, B) = v - B - v(1, B).$$
Note that if $\mu_1(B) = 0$, then $A(1, B) = [-w, v - B]$, so to the extent that $w + v - B \geq 0$, no trade in $(1, B)$ is not an equilibrium. Conversely, if $w + v - B < 0$, the equilibrium has no trade in $(1, B)$.

Next, note that

$$v(m, B) = \mu_m(B)\delta v(m - 1, B + b(m, B)) + (1 - \mu_m(B))\delta v(m, B).$$

Then

$$v(m, B) = \left(\frac{\delta \mu_m(B)}{1 - \delta(1 - \mu_m(B))}\right) v(m - 1, B + b(m, B)).$$

so

$$v(1, B) = \left(\frac{\delta \mu_1(B)}{1 - \delta(1 - \mu_1(B))}\right) [v - B - b(1, B)].$$

Suppose for now that $w + v - B \geq 0$. Then $b(1, B) = v - B - v(1, B)$, and

$$v(1, B) = \left(\frac{\delta \mu_1(B)}{1 - \delta(1 - \mu_1(B))}\right) v(1, B) \Rightarrow v(1, B) = 0.$$

Then

$$b(1, B) = v - B$$

If instead $w + v - B < 0$, so $\mu_1(B) = 0$, then also $v(1, B) = 0$.

Note moreover that by (38), $v(1, B) = 0$ for all $B$ implies

$$v(m, B) = 0 \quad \text{for all } B \text{ and } m \geq 1$$

This implies, in particular, that in any state $(m, B)$ with $m > 1$ the principal is indifferent between accepting or rejecting any offer.

Now consider the welfare of an uncommitted agent in state $(m, B)$. Suppose in state $(m, B)$ there is trade with probability $\mu_m(B)$. Then

$$w(m, B) = \beta(m) \left[ \mu_m(B)\delta w_{out}(m - 1, B + b(m, B); b(m, B)) + (1 - \mu_m(B))\delta w(m, B) \right]$$

$$+ (1 - \beta(m)) \left[ \mu_m(B)\delta w(m - 1, B + b(m, B)) + (1 - \mu_m(B))\delta w(m, B) \right].$$

or

$$w(m, B) = \frac{\mu_m(B)\delta}{1 - \delta(1 - \mu_m(B))} \left[ \beta(m)w_{out}(m - 1, B + b(m, B); b(m, B)) + (1 - \beta(m))w(m - 1, B + b(m, B)) \right].$$

We showed before that if $w + v - B < 0$, there is no trade in $(1, B)$. Suppose instead $B \leq w + v$.

Since $w_{out}(0, B + b(1, B); b(1, B)) = w + b(1, B) = w + v - B - v(1, B)$, $w(0, B + b(m, B)) = w$,
and \( v(1, B) = 0 \), this is
\[
w(1, B) = \frac{\mu_1(B)\delta}{1 - \delta(1 - \mu_1(B))} \left[ w + \beta(1)(v - B) \right].
\]

Note that since \( v(1, B) = 0 \),
\[
s_1(B) = w + v - B - v(1, B) - w(1, B) = w + v - B - \frac{\mu_1(B)\delta}{1 - \delta + \delta \mu_1(B)} \left[ w + \beta(1)(v - B) \right]
\]
In equilibrium, \( \mu_1(B) \geq 0 \) iff \( s_1(B) \geq 0 \) and \( \mu_1(B) = 1 \) if \( s_1(B) > 0 \), and similarly \( \mu_1(B) \leq 1 \) iff \( s_1(B) = v - B - v(1, B) + w - w(1, B) \leq 0 \), and \( \mu_1(B) = 0 \) if \( s_1(B) < 0 \), so that \( \mu_1(B) \in (0, 1) \) only if \( s_1(B) = 0 \). For an equilibrium with \( \mu_1(B) = 1 \), we need
\[
(1 - \delta)w + (1 - \delta \beta(1))(v - B) > 0 \iff B < v + \left( \frac{1 - \delta}{1 - \delta \beta(1)} \right) w
\]
Otherwise, i.e., if \( v + w \left( \frac{1 - \delta}{1 - \delta \beta(1)} \right) \leq B \leq v + w \) (recall we have assumed that \( B \leq w + v \)), in equilibrium \( \mu_1(B) \in (0, 1) \), given by
\[
\mu_1(B) = \frac{(1 - \delta)[w + v - B]}{\delta(1 - \beta(1))(B - v)} \in (0, 1).
\]
Recall that we have shown that \( v(m, B) = 0 \) for all \( B \) and \( m \geq 1 \). Then for any \( m > 1 \), \( A(m, B) \) can be an arbitrary subset of the reals, \( A(m, B) \subset \mathbb{R} \), and \( \overline{A}(m, B) = \{ b \in A(m, B) : w_{out}(m - 1, B + b; b) \geq w(m, B) \} \).

Consider an equilibrium in which the principal only accepts offers \( b \leq 0 \) (i.e., \( A(m, B) = (-\infty, 0] \) in any state \( (m, B) \) such that \( m > 1 \). Since it is never optimal in this case for the agent to propose \( b < 0 \) (doesn’t affect the probability of trade and leads to a lower payoff), all equilibrium transactions in such states have \( b(m, B) = 0 \). Note that in this case for large \( v \) the unique equilibrium in the critical state \((1, 0)\) has trade with probability one.

Suppose in all other states \((m, 0)\), the probability of trade is the same as in the benchmark model with cash transfers. Since the value functions for committed and uncommitted agents in each state \((m, 0)\) are exactly as in the benchmark model with cash transfers, agents are indifferent between trading and not trading at zero transfers. Since any positive offer is rejected, requesting a positive offer in any such state is not a profitable deviation. It follows that the equilibrium of the benchmark model with cash transfers is still an equilibrium with conditional payments. There are of course other equilibria, including no trade.
References


