Online Appendix
“Dollar Invoicing and the Heterogeneity of Exchange Rate Pass-Through”

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PRIORS AND MODEL SELECTION

A1. Hyper-priors

Here we describe the remaining parts of the prior not specified in the main text. We incorporate time fixed effects δ_t by adding $T-1$ dummies in the covariate vector X_t, so the parameter vector θ includes these parameters. We impose the following priors, all mutually independent:

$$\alpha \sim \text{Cauchy}(0,5), \quad \theta_j \sim \text{Cauchy}(0,5),$$

$$\sigma \sim \text{HalfCauchy}(0,1), \quad \tau \sim \text{HalfCauchy}(0,1).$$

Cauchy$(0,a)$ is the centered Cauchy distribution with interquartile range $2a$. HalfCauchy$(0,a)$ is the restriction of the Cauchy$(0,a)$ distribution to the positive real line. Since the units of our outcome variables $Y_{ij,t}$ are log points, the above priors are highly diffuse. As for the MGLR prior, we assume[1]

$$\omega_k \sim \text{HalfCauchy}(0,2), \quad \begin{pmatrix} \mu_{0,k} \\ \mu_{1,k} \end{pmatrix} \mid \omega_k \sim N\left(0, \begin{pmatrix} \omega_k^2 & 0 \\ 0 & \omega_k^2 \end{pmatrix}\right), \quad k = 1, \dots, K,$$

$$\zeta_k(\cdot) \sim GP(0, C(\cdot; A_k)), \quad A_k \sim \text{Exponential}(1), \quad k = 1, \dots, K-1,$$

independently across k. Here $GP(0, C(\cdot; A))$ denotes a Gaussian process with Gaussian radial covariance kernel

$$C(s_1, s_2; A) = \exp\{-A(s_1 - s_2)^2\} + 0.0001 \times \mathbb{1}(s_1 = s_2), \quad s_1, s_2 \in [0,1].$$

The second term on the right-hand side above helps avoid numerical issues in the warm-up phase of the MCMC algorithm, but it is small enough to negligibly affect the final output (the dollar invoicing share S_j is measured as a fraction between 0 and 1).

A2. Bayesian leave-one-out cross-validation

The Bayesian Leave-One-Out (LOO) cross-validation criterion of (Gelfand, Dey and Chang 1992) is given by the cross-sectional sum of leave-one-out predictive densities

$$\begin{aligned} LOO &= \sum_{ij} \log f(Y_{ij} \mid R_{ij}, X_{ij}, Y_{-(ij)}, R_{-(ij)}, X_{-(ij)}) \\ &= \sum_{ij} \log \int f(Y_{ij} \mid R_{ij}, X_{ij}, \vartheta)\, f(\vartheta \mid R_{ij}, X_{ij}, Y_{-(ij)}, R_{-(ij)}, X_{-(ij)})\, d\vartheta. \end{aligned}$$

Here ϑ collects all model parameters. $Y_{ij} = (Y_{ij,1}, \dots, Y_{ij,T})$ collects all observed outcomes for dyad (i,j) across time, and similarly for the covariates R_{ij} and X_{ij}.[2] The notation $Y_{-(ij)}$ means all observed outcomes for dyads other than (i,j), and similarly for $R_{-(ij)}$ and $X_{-(ij)}$. The LOO criterion is large when the model yields good (leave-one-out) out-of-sample fit, given knowledge of the covariates. This is similar in spirit to the well-known non-Bayesian leave-one-out cross-validation criterion. We use a Pareto-smoothed importance sampling estimate of LOO, as developed by (Vehtari, Gelman and Gabry 2017) and implemented in Stan.

[1] Because the mixture component labels are not identified, we additionally impose the normalization $\mu_{0,1} < \mu_{0,2} < \cdots < \mu_{0,K}$. Stan accomplishes this by reparametrizing the vector $(\mu_{0,1}, \dots, \mu_{0,K})'$ into an unconstrained parameter, while adjusting for the Jacobian of the transformation in the posterior density.

[2] Since we have an unbalanced panel, the dimension of Y_{ij}, R_{ij}, X_{ij} actually varies across dyads.

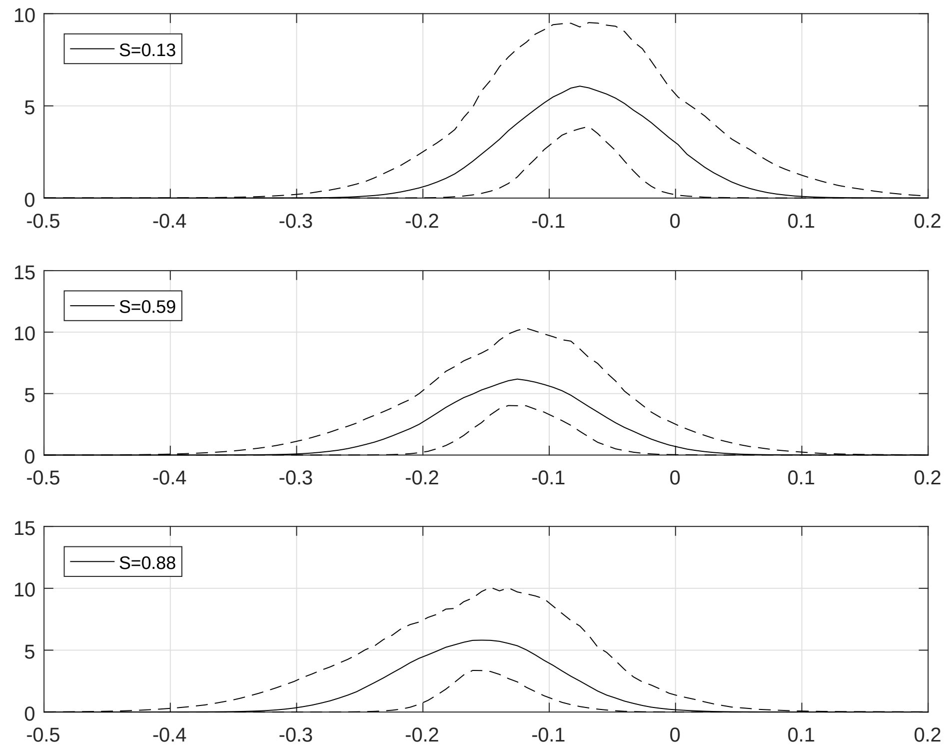

FIGURE B1. DENSITY OF DOLLAR TRADE ELASTICITY GIVEN DOLLAR INVOICING SHARE

Note: Model-implied conditional density $f(\gamma_{ij} \mid S_j)$ plotted at the dollar import invoicing shares S_j of Switzerland (top), Turkey (middle), and Argentina (bottom). Solid lines are posterior medians, dashed lines are 95% pointwise equal-tailed posterior credible intervals.

SUPPLEMENTARY RESULTS

This section provides supplementary results and implementation details for the Bayesian model.

B1. Trade elasticity

Similar to the price pass-through results, we find that the cross-dyad heterogeneity of the elasticity of trade quantities with respect to the dollar exchange rate is related to the dollar invoicing share. However, the results in this subsection generally come attached with higher posterior uncertainty. Section B.B2 provides additional results on parameters not highlighted below.

Our empirical specification again follows (Boz, Gopinath and Plagborg-Møller 2017). We use the log growth rate of bilateral trade volume $\Delta y_{ij,t}$ as the outcome variable, but otherwise follow Equation (1) of the main text. We control for one lag of bilateral and dollar exchange rates, as well as the contemporaneous value and lag of importer log real GDP growth. The sample of dyad-year observations is the same as for the price pass-through results.

We report results for $K = 4$ mixture components. The LOO model selection criterion strongly favors $K = 3, 4, 5$ against either $K \leq 2$ or $K = 6, 7, 8$. $K = 4$ has a slightly higher LOO score than $K = 3, 5$. However, we remark again that the results presented below are little changed across specifications with $K \geq 3$. We report results for $K = 8$ in Section B.B3.

Figure B1 shows that the conditional density of the dollar trade elasticity (expected to be a negative number, as also estimated in (Boz, Gopinath and Plagborg-Møller 2017)) shifts leftward when the importer's country-level dollar invoicing share increases. That is, the higher the dollar invoicing share, the larger in magnitude is the dollar trade elasticity, on average. Notice, however, that the

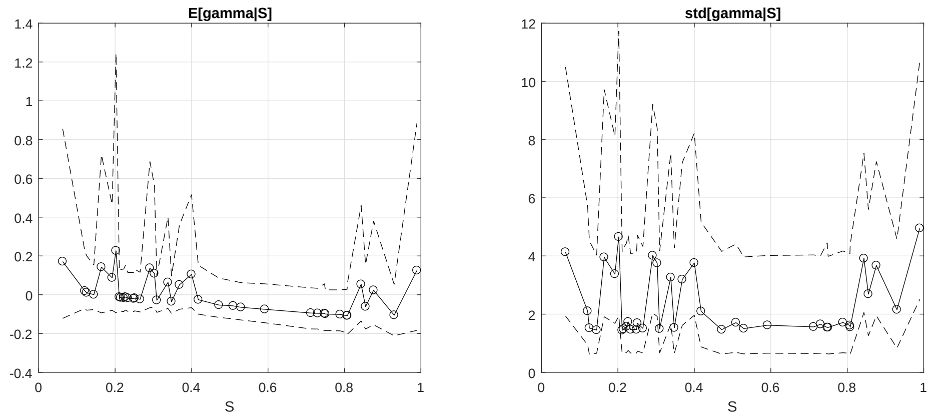

FIGURE B2. CONDITIONAL MEAN AND STANDARD DEVIATION OF DOLLAR TRADE ELASTICITY

Note: Model-implied conditional mean (left) and standard deviation (right) of γ_{ij} given S_j. Solid lines are posterior medians, dashed lines are 95% pointwise equal-tailed posterior credible intervals. Circles indicate observed S_j values.

credible bands are much wider here than for the price pass-through results. This is consistent with the larger standard errors on the interaction terms in the trade elasticity panel regressions in (Boz, Gopinath and Plagborg-Møller 2017). Figure B2 shows the conditional mean and standard deviation. While the posterior medians indicate that the conditional mean function is downward-sloping over most of the range of S_j, the function is estimated with substantial uncertainty.

Figure B3 summarizes the posterior of the sample distribution of γ_{ij}. The median γ_{ij} is in line with the panel regression results in (Boz, Gopinath and Plagborg-Møller 2017) (posterior median of median: -0.11), but the heterogeneity is substantial (posterior median of IQR: 0.09). Again we find a strong (here: negative, as expected) correlation between γ_{ij} and S_j (posterior median of correlation: -0.41), after winsorizing γ_{ij} at 5% in each tail. Thus, trade elasticities with respect to the dollar are highly heterogeneous, but dyads with the largest-in-magnitude dollar elasticities tend to be the dyads with the highest importer dollar invoicing share. The 95% equal-tailed posterior credible interval for the R^2 in a cross-dyad regression of (winsorized) dollar elasticity on the importer's dollar invoicing share is [2.6%, 34.0%].

B2. Additional model parameters

For completeness, we now report posterior summaries of the model parameters that are not of primary interest to us.

First we report results for the price pass-through model with $K = 2$. Figure B4 reports the posterior distribution of the cross-sectionally constant regression coefficients. The results are consistent with the panel regressions in (Boz, Gopinath and Plagborg-Møller 2017). In particular, the lagged exchange rate changes are economically insignificant. The posterior for the parameter $\bar{\gamma}$ (the sum of the dollar and bilateral pass-throughs) is concentrated close to 1, indicating near-complete *total* pass-through within a year. Figure B5 reports the posterior of the mean α and standard deviation τ of the random effects distribution for the dyad-specific effects λ_{ij}, as well as the idiosyncratic standard error σ.

Figures B6 and B7 provide the same posterior summaries for the trade elasticity model with $K = 4$. Again, these results are consistent with the panel regressions from (Boz, Gopinath and Plagborg-Møller 2017).

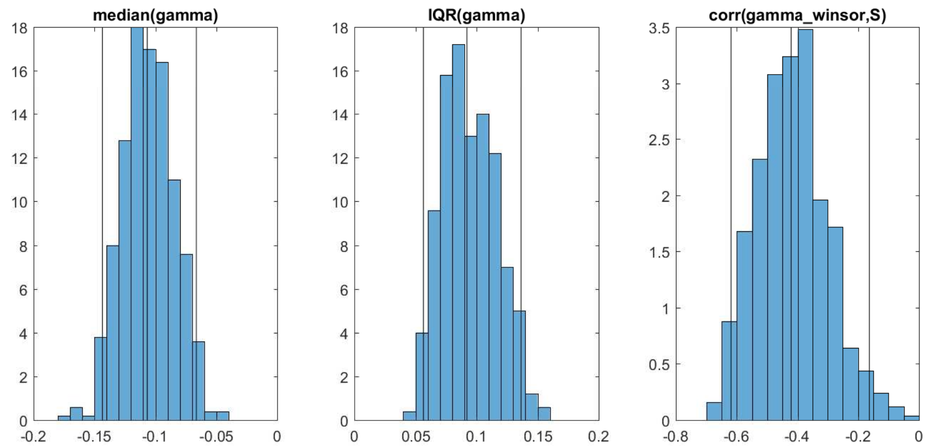

FIGURE B3. SAMPLE DISTRIBUTION OF DOLLAR TRADE ELASTICITY

Note: Histogram of posterior draws of the sample median of γ_{ij} (left), the sample interquartile range of γ_{ij} (middle), and winsorized correlation of γ_{ij} and S_j (right). That is, for each posterior draw, we compute the sample median, IQR, and winsorized correlation across the 1856 dyads in our sample. Vertical lines mark the 2.5, 50, and 97.5 posterior percentiles.

B3. Robustness to number of mixture components

Here we show that the results in the main text are robust to varying the number K of components in the MGLR prior for the cross-sectional distribution of dollar pass-through. Specifically, we here report results for $K = 8$. Figures B8 and B9 are the $K = 8$ analogues of the price pass-through Figures 1 and 3 in the main text, while Figures B10 and B11 are the $K = 8$ analogues of the trade elasticity Figures B1 and B3 (which had $K = 4$). Clearly, the additional mixture components in the $K = 8$ specifications receive very low posterior probability.

B4. MCMC settings and diagnostics

We execute Stan through Matlab R2016b using MatlabStan 2.7.0.0, which in turn calls CmdStan 2.14.0. For each model specification, we run Stan's No U-Turn Sampler for 2,500 iterations after discarding 1,000 warm-up iterations, storing every 5th draw. The MCMC routine is initialized at parameter values drawn uniformly at random (after the parameters have been transformed to unconstrained support). We use Stan's default settings for adaptively tuning the MCMC routine in the warm-up phase. Our results are completely insensitive to the initialization.

The sampler robustly delivers near-independent draws from the posterior distribution in reasonable time. The stored posterior draws of most model parameters exhibit essentially zero serial correlation after a handful of lags. The only parameters that do not exhibit rapid mixing are those MGLR parameters $\mu_{0,k}$, $\mu_{1,k}$, ω_k, A_k that correspond to mixture components k with low posterior probability $\pi_k(\cdot)$ in model specifications with large K, but these parameters negligibly influence the features of the posterior that we care about. Depending on K and the random initial parameter draw, it takes 2–60 hours to run the MCMC routine for each specification on a personal laptop with a 2.30 GHz processor and 8 GB RAM (no parallel computing is involved). In our experience, it is often sufficient to run the algorithm for 2–4 hours to get a sense of the results.

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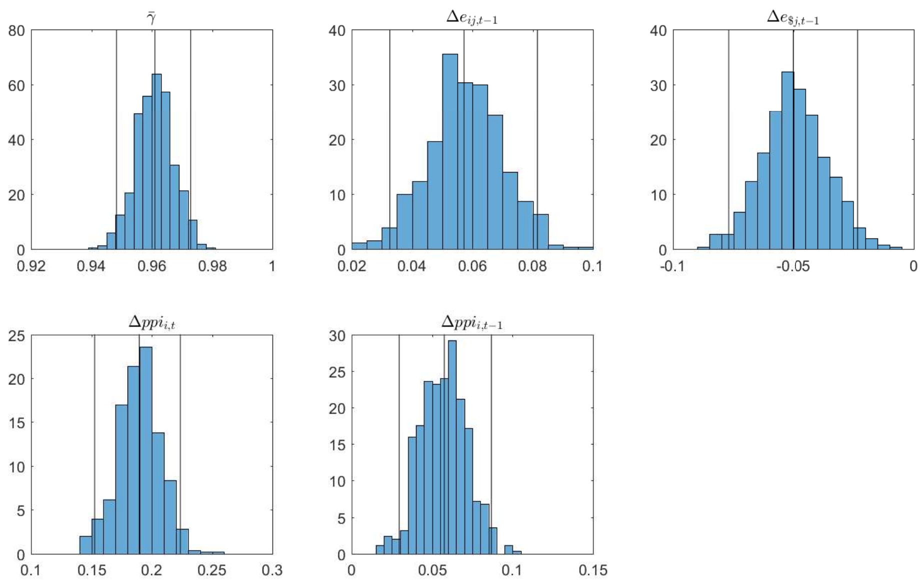

FIGURE B4. POSTERIOR OF CONSTANT REGRESSION COEFFICIENTS, PRICE PASS-THROUGH

Note: Histogram of posterior draws of elements in θ, the regression coefficients that are assumed constant across dyads. The top left display shows the parameter $\bar{\gamma}$ in Equation 1 in the main text. The remaining displays show the coefficients on the indicated exogenous covariates. Vertical lines mark the 2.5, 50, and 97.5 percentiles. For brevity, we do not show the time fixed effects.

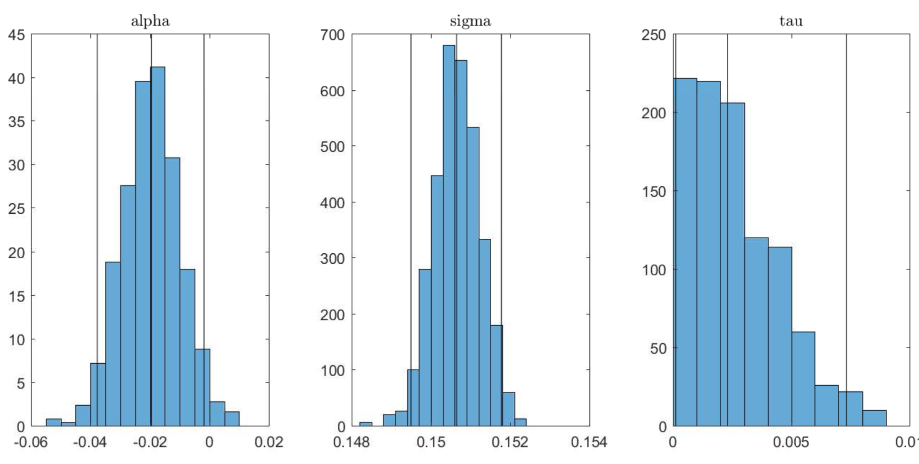

FIGURE B5. POSTERIOR OF OTHER PARAMETERS, PRICE PASS-THROUGH

Note: Histogram of posterior draws of α (left), σ (middle), and τ (right). Vertical lines mark the 2.5, 50, and 97.5 percentiles.

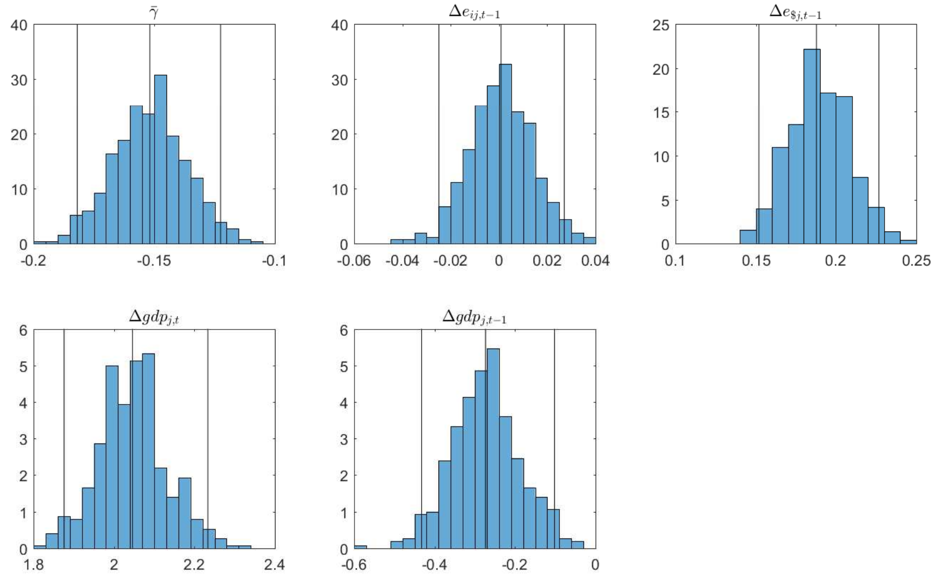

FIGURE B6. POSTERIOR OF CONSTANT REGRESSION COEFFICIENTS, TRADE ELASTICITY

Note: See caption for Figure B4.

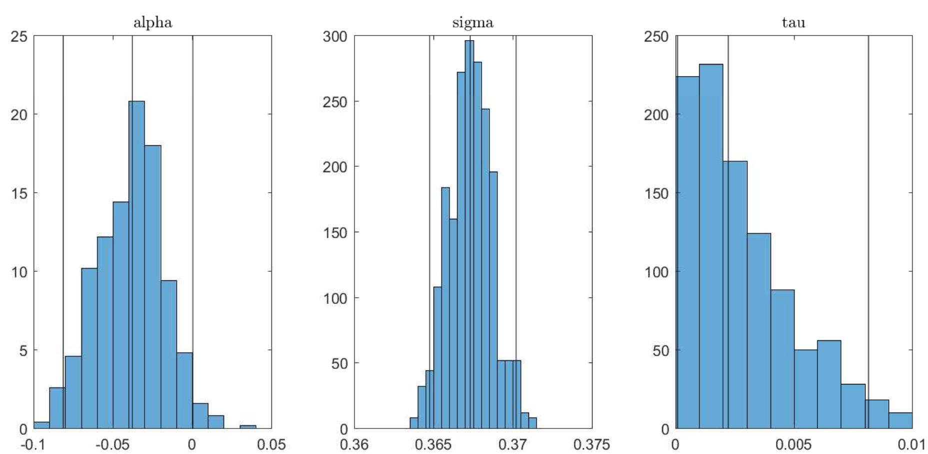

FIGURE B7. POSTERIOR OF OTHER PARAMETERS, TRADE ELASTICITY

Note: See caption for Figure B5.

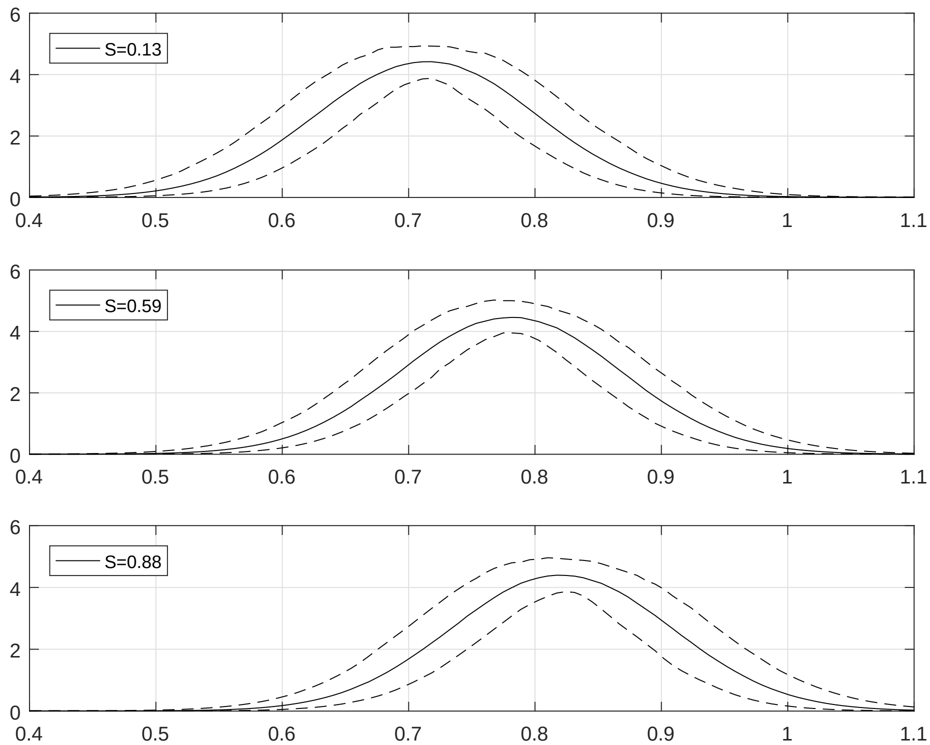

FIGURE B8. DENSITY OF DOLLAR PRICE PASS-THROUGH GIVEN DOLLAR INVOICING SHARE, $K = 8$

Note: Model-implied conditional density $f(\gamma_{ij} \mid S_j)$ plotted at the dollar import invoicing shares S_j of Switzerland (top), Turkey (middle), and Argentina (bottom). Solid lines are posterior medians, dashed lines are 95% pointwise equal-tailed posterior credible intervals.

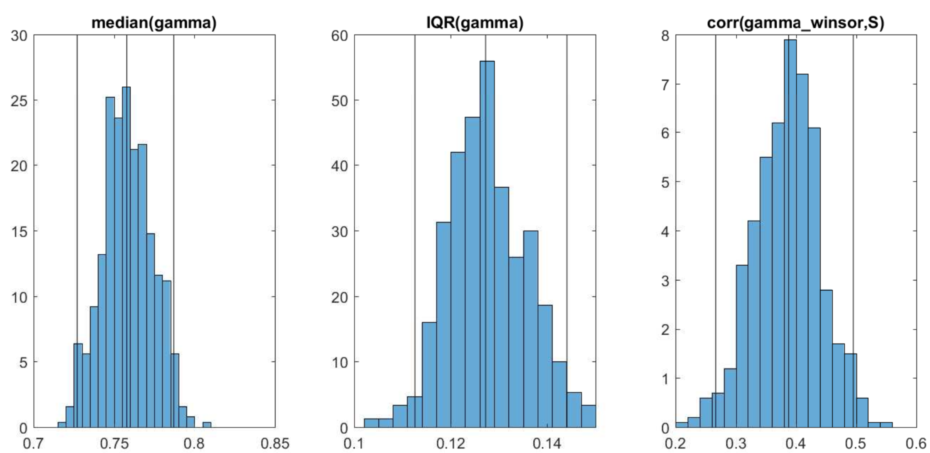

FIGURE B9. SAMPLE DISTRIBUTION OF DOLLAR PRICE PASS-THROUGH, $K = 8$

Note: Histogram of posterior draws of the sample median of γ_{ij} (left), the sample interquartile range of γ_{ij} (middle), and winsorized correlation of γ_{ij} and S_j (right). That is, for each posterior draw, we compute the sample median, IQR, and winsorized correlation across the 1856 dyads in our sample. Vertical lines mark the 2.5, 50, and 97.5 posterior percentiles.

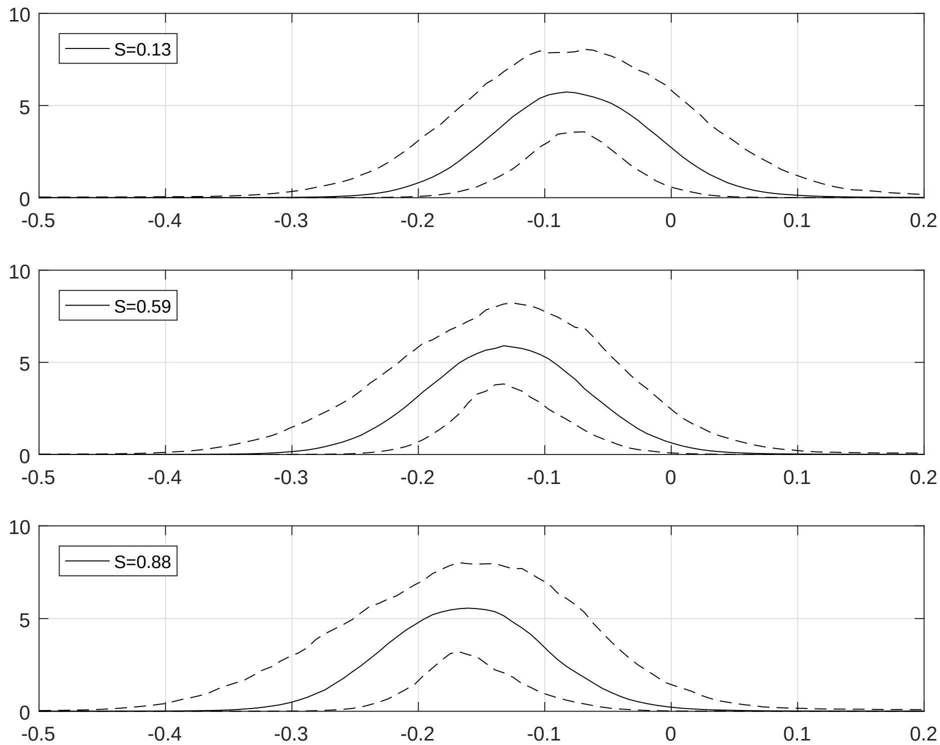

FIGURE B10. DENSITY OF DOLLAR TRADE ELASTICITY GIVEN DOLLAR INVOICING SHARE, $K = 8$

Note: See caption for Figure B1.

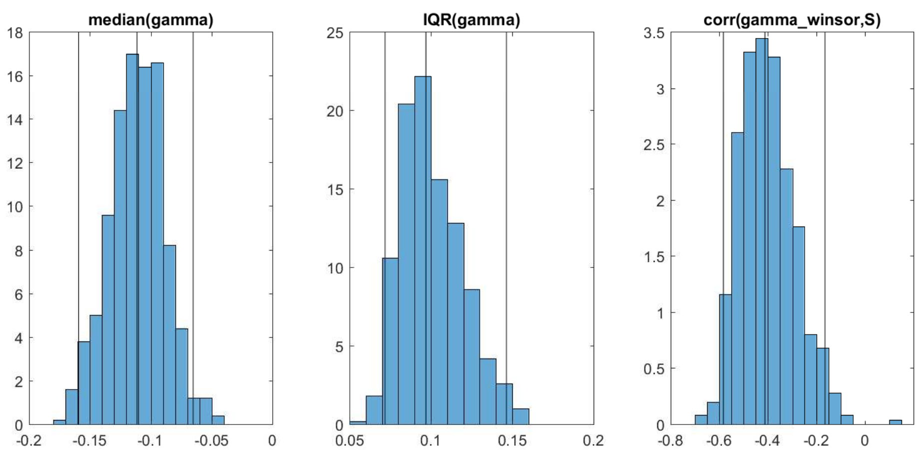

FIGURE B11. SAMPLE DISTRIBUTION OF DOLLAR TRADE ELASTICITY, $K = 8$

Note: See caption for Figure B3.

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