Discussion: “Narrative Restrictions and Proxies” by Giacomini, Kitagawa & Read

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1 Robustness of narrative proxy approach

2 Inference with a weak proxy

3 Conclusion
Bivariate SVAR model with narrative signals

\[
\begin{pmatrix}
 y_{1t} \\
y_{2t}
\end{pmatrix} = \begin{pmatrix}
 \Theta_{11} & \Theta_{12} \\
 \Theta_{21} & \Theta_{22}
\end{pmatrix}
\begin{pmatrix}
 \varepsilon_{1t} \\
 \varepsilon_{2t}
\end{pmatrix}, \quad t = 1, \ldots, T.
\]

• Shocks \((\varepsilon_{1t}, \varepsilon_{2t})\) are i.i.d. mean zero, variance 1, mutually independent.

• **Narrative signals** about \(\varepsilon_{1t}\) (for simplicity, no info about \(\varepsilon_{2t}\)):

\[
Z_t = \begin{cases}
  1 & \text{if we believe } \varepsilon_{1t} > 0, \\
  0 & \text{if sign unknown}, \\
  -1 & \text{if we believe } \varepsilon_{1t} < 0.
\end{cases}
\]

• Data: \((y_{1t}, y_{2t}, Z_t)\). Shocks are unobserved.

• Identified set for impulse responses \(\Theta_{ij}\) is large if we ignore \(Z_t\). How to model+exploit \(Z_t\)?
Bayesian inference when signals are perfect

- Assume first that signals are perfect:
  \[ \text{sign}(\varepsilon_{1t}) = Z_t \quad \text{whenever } Z_t \neq 0. \]

- This implies restrictions that can substantially sharpen inference about impulse responses: Antolín-Díaz & Rubio-Ramírez (2018); Ludvigson, Ma & Ng (2019); Giacomini, Kitagawa & Read (2021)
  \[ \text{sign} \left( \frac{\Theta_{22} y_{1t} - \Theta_{12} y_{2t}}{\Theta_{11} \Theta_{22} - \Theta_{12} \Theta_{21}} \right) = Z_t \quad \text{whenever } Z_t \neq 0. \]

- Subjective/robust Bayesian inference based on these restrictions imposes strong as’ns:
  - Signals are perfect and arrive randomly (likelihood function appears to impose that the event \( \{Z_t \neq 0\} \) is independent of the shocks).
  - SVAR model assumes shocks are invertible (functions of only current and past data).
Proxy approach

More generally, we could assume that the signal $Z_t$ is a potentially imperfect proxy:

$$Z_t = F(\varepsilon_{1t}, u_t), \quad u_t \perp (\varepsilon_{1t}, \varepsilon_{2t}),$$

where $F: \mathbb{R}^2 \rightarrow \{-1, 0, 1\}$ is unknown, and $u_t$ is unobserved measurement error.

Example: $Z_t = \text{sign}(\varepsilon_{1t} + u_t) \times \mathbb{1}(|\varepsilon_{1t}| \geq 2)$.

Yet, we assume that signals are not too inaccurate overall:

$$\text{Cov}(Z_t, \varepsilon_{1t}) > 0.$$
Proxy approach: Robust estimation

\[ Z_t = F(\varepsilon_{1t}, u_t), \quad \text{Cov}(Z_t, \varepsilon_{1t}) > 0 \]

- Consider estimation of the relative IR \( \theta \equiv \Theta_{21}/\Theta_{11} \). Since

\[ y_{2t} = \theta y_{1t} + (\Theta_{22} - \Theta_{21} \frac{\Theta_{12}}{\Theta_{11}}) \varepsilon_{2t}, \]

we can estimate \( \theta \) by 2SLS regression of \( y_{2t} \) on \( y_{1t} \), with \( Z_t \) as IV.

Romer & Romer (1989); Hamilton (2003); Budnik & Rünstler (2020)

- Appealing robustness properties:

1. IV exclusion restriction holds even under misclassification and non-random signal arrival.

2. Can allow shocks to be non-invertible with 2SLS version of Local Projection (or recursive VAR with \( Z_t \) ordered first). Stock & Watson (2018); Plagborg-Møller & Wolf (2021)
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Proxy approach: Weak identification

- Credible signals are usually only available for a small number $K$ of periods $\implies$ proxy $Z_t$ is mostly zero $\implies$ weak identification.

- G, K & R helpfully point out that the standard weak-IV robust SVAR procedure does not work if we model $K$ as finite asymptotically. Montiel Olea, Stock & Watson (2021)
  - This procedure requires asy. normality of reduced-form sample moments.
  - But $T^{-1/2} \sum_{t=1}^{T} Z_t y_{1t} \overset{p}{\to} 0$, since $Z_t = 0$ for all but finite no. of obs.
  - Can fix the procedure if shocks are Gaussian, but this seems fragile.

- Should we therefore give up on the robustness afforded by the proxy approach?
Proxy inference in small samples

• Fundamentally, the setting with a sparse proxy $Z_t$ is a **small-sample** problem. Can we apply weak-IV procedures that are geared towards small samples?

• **Permutation Anderson-Rubin test:** Under $H_0: \theta = \bar{\theta}$,

  $$y_{2t} - \bar{\theta}y_{1t} = (\Theta_{22} - \Theta_{21} \Theta_{12} \Theta_{11}) \varepsilon_{2t} \implies (y_{2t} - \bar{\theta}y_{1t}) \perp \perp Z_t.$$  

  Fisher permutation test of independence: **Imbens & Rosenbaum (2005)**

  1. Compute $|\widehat{\text{Corr}}(y_{2t} - \bar{\theta}y_{1t}, Z_t)|$.
  2. Compute same statistic over all possible permutations of the IV data points $Z_1, \ldots, Z_T$.
  3. Reject if original statistic exceeds 95th percentile of permutation distribution.

• **Exact size** in finite samples. Does not require shocks to be Gaussian.
Simulated size/power of nominal 5% permutation test

DGP: Same as G, K & R, except perfect signals arrive in first $K$ periods where $\varepsilon_{1t} > 0$ (so uncond’l shock distribution is normal). $T = 500$. 5,000 simulations. 1,000 random permutations.
Outline

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• G, K & R are doing important work on understanding+improving narrative identification.

• Very helpful to point out the small-sample problem that (i) causes a weak proxy issue and (ii) cannot be tackled with the usual inference procedure based on asy. normality.

• My opinion: It would be a shame to give up on the proxy approach, as it is robust to misclassification, non-random signal arrival, and non-invertibility.

• My suggestion: Use small-sample weak-IV robust procedures, such as permutation test.
  • Power may be low, which seems inevitable given nature of restrictions.
  • May want to relax shock independence assumptions (also true for other approaches).