Spatial Equilibrium, Search Frictions and Dynamic Efficiency in the Taxi Industry

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Abstract

This paper analyzes the dynamic spatial equilibrium of taxicabs and shows how common taxi regulations lead to substantial inefficiencies as a result of search frictions and misallocation. To analyze the role of regulation on frictions and efficiency, I pose a dynamic model of spatial search and matching between taxis and passengers. Using a comprehensive dataset of New York City yellow medallion taxis, I use this model to compute the equilibrium spatial distribution of vacant taxis and estimate intraday demand given price and medallion regulations. My estimates show that the weekday New York market achieves about $5.5 million in daily welfare split almost equally by customers and drivers comprising 182 thousand trips, but an additional 48 thousand customers fail to find cabs due to search frictions. Counterfactual analysis shows that implementing simple tariff pricing changes can enhance allocative efficiency and expand the market, offering daily net surplus gains of up to $420 thousand and 76 thousand additional daily taxi-passenger matches, a similar magnitude of gains generated by adopting a perfect static matching technology.

Key Words: dynamic games, spatial equilibrium, search frictions, dynamic pricing, regulation, taxi industry

JEL classification: C73; D83; L90; R12

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1 Introduction

It has been well documented that search frictions lead to less efficient outcomes. One particularly stark reason for the existence of search frictions is that buyers and sellers are spatially distributed across a city or region, so that meeting to trade requires costly transportation of one or both sides of the market. When locations are fixed, say between households and potential employers, search frictions arise from these added costs. In some spatial settings, however, every trade involves a future re-allocation of buyers or sellers. This is a prominent feature of transportation markets, where every trade entails a vehicle’s movement from one place to another. When transportation and search intersect, dynamic search externalities arise as each trip affects the search frictions faced by future buyers and sellers at each destination. In this paper I study the regulated taxicab industry in New York City, where a decentralized search process and a uniform tariff leads to distortions in the intra-daily equilibrium spatial patterns of supply and demand. I ask how much spatial misallocation is induced by search externalities in this setting and to what extent can simple changes to pricing regulations serve to enhance allocative efficiency over time.

The taxicab industry is a critical component of the transportation infrastructure in large urban areas, generating about $23 billion in annual revenues. New York City has long been the largest taxicab market in the United States, accounting for about 25% of national industry revenues in 2013. In New York, as well as many other cities, the taxi market is distinguished from other public transit options by a lack of centralized control; taxi drivers do not service established routes or coordinate search behavior. Instead, drivers search for passengers and, once matched, move them to destinations. Since different types of trips are demanded in different areas of the city, how taxi drivers search for passengers directly impacts the subsequent availability of service across the city. These movements of capacity give rise to equilibrium patterns that can leave some areas with little to no service while in other areas empty taxis will wait in long queues for passengers.

In this paper, I model taxi drivers’ location choices in a dynamic spatial search framework in which vacant drivers choose where to locate themselves given both the time-of-day pattern of trip demand as well as the distribution of rival taxi drivers throughout the day. While the spatial search process under current regulations often generates mis-allocation across locations, I also model frictions within each location to account for a block-by-block search process within small

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1 Models of search conduct and equilibrium have been widely studied. Since the pioneering work of Diamond (1981, 1982b), Mortensen (1982a,b) and Pissarides (1984, 1985), the search and matching literature has focused on understanding the role of search frictions in impeding the efficient clearing of markets. The search and matching literature examines many markets where central or standardized exchange is not possible, including labor markets (e.g., Rogerson, Shimer, and Wright (2005)), marriage markets (e.g., Mortensen (1988)), monetary exchange (e.g., Kiyotaki and Wright (1989, 1993)), and financial markets (e.g., Duffie, Gärleanu, and Pedersen (2002, 2005)).

2 This value is based on my own calculation combining data from the NYC Taxi and Limousine Commission and a national industry report Brennan (2014).
windows of time. I show that spatial frictions are largely attributable to inefficient pricing, as tariff-based prices fail to account for driver opportunity costs and the heterogeneity in consumer surplus that is not internalized by drivers. To empirically analyze this model, I use data from the New York City Taxi and Limousine Commission (TLC), which provides trip details including the time, location, and fare paid for all 27 million taxi rides in New York between August and September of 2012. Using TLC data together with a model of taxi search and matching, I estimate the spatial and intra-daily distribution of supply and demand in equilibrium. Importantly, the data only reveal matches made between taxis and customers as a consequence of search activity, but do not show underlying supply or demand; I therefore can not observe the locations of vacant taxis or the number of customers who want a ride in different areas of the city. Because these objects are necessary to measure search frictions and welfare in the market, I develop an estimation strategy using the dynamic spatial equilibrium model together with a local matching function. The observed distribution of taxi-passenger matches is a sufficient statistic to solve for driver policies and compute the equilibrium distribution of vacant taxis without direct knowledge of demand. I then invert each local matching function to recover the implied distribution of customer demand up to an efficiency parameter. Finally I estimate matching efficiency using moments related to the variance of matches across days of the month.

I use this model to evaluate welfare and search frictions in the New York taxi market. Baseline estimates of welfare indicate that the New York taxi industry generates $3.0 million in consumer surplus and $2.4 million in taxi driver variable profits during each 9-hour day-shift and across 184 thousand taxi-passenger matches. Commensurate with these surpluses, however, is an average of 48 thousand failed customer searches per day and 5,405 vacant drivers at any point in the day. To what extent can a more sophisticated pricing policy mitigate these costs by better allocating available supply to demand? By simulating market equilibrium over nearly one million potential pricing rules, I am able to solve for a dynamically optimal flexible fare structure and show that a flexible tariff that changes with time-of-day can provide up to a 12% increase in consumer welfare, a 4% increase in taxi profits and a 2% improvement in taxi utilization. These results utilize an estimated demand system that incorporates an elasticity of waiting time calibrated from recent work. Alternative policies offering flexible tariffs by location and distance yield slightly smaller benefits to consumers in favor of driver profits and higher utilization rates, but all of the counterfactual policies tested offer unambiguous benefits to the market even after accounting for search and matching frictions. I contrast these results with a counterfactual simulation of ride-sharing technology that offers frictionless within-location matching, and show that optimal pricing policies can produce 33% more trips than the matching technology alone and deliver a similar magnitude of welfare gains.
Related Literature

This paper integrates ideas from the search and matching literature with empirical industry dynamics. The key component is a model of dynamic spatial choices that adapts elements from Lagos (2000). Lagos (2000) studies endogenous search frictions using a stylized environment of taxi search and competition. The model predicts how meeting probabilities adjust to clear the market and how misallocation can occur as an equilibrium outcome. Lagos (2003) uses the Lagos (2000) model to empirically analyze the effect of fares and medallion counts on matching rates and medallion prices in Manhattan. I draw elements from the Lagos search model, but make several changes to reflect the real-world search and matching process. Specifically, I add non-stationary dynamics, a more realistic and flexible spatial structure, stochastic and price-sensitive demand, fuel costs, and heterogeneity of the matching process in different locations. Further, I build a tractable framework for the empirical analysis of dynamic spatial equilibrium by providing tools for the estimation and identification of the model. Examples include specifying flexible location-specific matching functions and accounting for unobservables in drivers’ location choices. I also model a static, localized market clearing process via an aggregate matching function. Hall (1979) introduces the aggregate matching function concept, and notably uses the urn-ball specification adapted in this paper. In recent work Brancaccio, Kalouptsidi, and Papageorgiou (2019) apply a related search model to study endogenous trade costs in the bulk shipping industry.

I integrate the dynamic spatial search framework with a dynamic model in the tradition of Hopenhayn (1992) and Ericson and Pakes (1995), which characterize Markov-perfect equilibria in entry, exit, and investment choices given some uncertainty in the evolution of the states of firms and their competitors. Here, each taxi operates as a firm that is optimizing where to search in a city. The state variable is the distribution of taxi driver locations. To facilitate computation, I make a large-market assumption that both taxi drivers and customers are non-atomic. As with Hopenhayn (1992) this will enable computing deterministic state transitions without integrating over a high-dimensional space of states and future periods. The mass of customers in each location varies from day to day in each location and period. Drivers do not condition on these shocks, assumed to be non-observable to individual drivers, but rather the expectation of consumer demand. The equilibrium is therefore similar to an Oblivious Equilibrium (Weintraub, Benkard, and Van Roy (2008b)) in which policies are formed with respect to limits taken across many days in the market. This notion is also similar to an Experience-Based Equilibrium (Fershtman and Pakes (2012)) in which firms’ information set is restricted and strategies are conditioned on repeated experiences.

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3 Mortensen (1986), Mortensen and Pissarides (1999) and Rogerson, Shimer, and Wright (2005) survey the labor-search literature and the implementation of aggregate matching functions.

4 There is also a literature in empirical industrial organization which studies the allocative distortions induced by search frictions in different industries. This includes work on airline parts (Gavazza (2011) and mortgages (Allen, Clark, and Houde (2014)).
with market outcomes. This is the first empirical analysis of pricing and welfare in a taxi market and the first to study how price regulations impact the spatial allocation of service. A related study is Frechette, Lizzeri, and Salz (2019), which models the dynamic entry game among taxi drivers to ask how customer waiting times and welfare are impacted by medallion regulations and dispatch technology. As with my paper, Frechette, Lizzeri, and Salz (2019) study the effect of regulations on search frictions and welfare. The key difference is that they focus on the labor supply decision rather than the spatial location decision. Though these research questions and approaches differ substantially, they lead to similar predictions when comparing similar counterfactuals.

There is a recent literature on the benefits of dynamic pricing used in ride-hail services (e.g., Hall, Kendrick, and Nosko (2015), Castillo, Knoepfle, and Weyl (2017)). This paper similarly highlights the impact of pricing on efficiency, but with two distinct differences. First, I focus on posted tariffs as opposed to real-time price adjustment. Posted tariffs are a feature of both traditional taxis and ride-hail services that affect the search behavior of taxi drivers. Second, I explicitly model the influence of prices on the dynamic path of supply and demand. I leverage this model to show how prices can be configured to induce efficient allocations of supply and demand while accounting for the flow of reallocated cabs due to passenger trips.

Finally, a diverse set of literature exists to address whether taxi regulation is necessary at all. Among this literature, both the theoretical and empirical findings offer mixed evidence. These studies point to successful regulation’s function to reduce transaction costs (Gallick and Sisk (1987)), prevent localized monopolies (Cairns and Liston-Heyes (1996)), correct for negative externalities (Schrieber (1975)), and establish efficient quantities of vacant cabs (Flath (2006)). Other authors assert that regulations have lead to restricted quantities and higher prices (Winston and Shirley (1998)) and that low sunk and fixed costs in this industry are sufficient to support competition (Häckner and Nyberg (1995)). My paper shows how existing regulatory levers are inefficiently configured, and that a better implementation of posted tariffs leads to more efficient spatial allocations and higher rates of utilization.

This paper is organized as follows. Section 2 details taxi industry characteristics relating to search, regulation, and spatial sorting, as well as a description of the data. Section 3 presents

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5This approach also relates to auction models with many bidders (e.g., Hong and Shum (2010)). It also serves as an empirical exercise in studying non-stationary firm dynamics (e.g., Weintraub, Benkard, Jeziorski, and Van Roy (2008a), Melitz and James (2007)).

6There is an additional body of literature on taxi drivers’ labor supply choices, including Camerer, Babcock, Loewenstein, and Thaler (1997), Farber (2005, 2008), Crawford and Meng (2011), and Thakral and Töll (2017). These studies investigate the labor-leisure tradeoff for drivers. They ask how taxi drivers’ labor supply is determined and to what extent it is driven by daily wage targets and other factors. Buchholz, Shum, and Xu (2017) estimate a dynamic labor supply model of taxi drivers to show that behavior consistent with dynamic optimization may appear as a behavioral bias when viewed in a static setting.
the dynamic model of taxi search and matching. Section 4 discusses the empirical strategy for computing equilibrium and estimating model parameters. Results are presented in section 5, with an analysis of counterfactual policies in section 6. Section 7 concludes.

2 Regulation and Search Frictions in The Taxi Industry

2.1 Market Characteristics

New York City is the largest taxi market in the United States, with 236 million passenger trips in 2014, about 25% of all U.S. service. As with nearly all major urban taxi markets, the New York taxi industry is highly regulated. Two regulations imposed by the New York Taxi and Limousine Commission (TLC) directly impact market function and efficiency. The first is a fixed two-part tariff fare pricing structure, where fares are based on a one-time flag-drop fee and a distance-based fee. Except for separate fares for some airport trips, this fare structure does not depend on location. Except for an evening flat-rate surcharge, fares do not depend on time of day. The second type of regulation is entry restrictions through a limit on the number of legal taxis that may operate. This is implemented by requiring drivers to hold a “medallion” or permit, the supply of which are capped (Schaller (2007)). Medallion cabs may only be hailed from the street and are not authorized to conduct pre-arranged pick-ups, a service exclusively granted to separately licensed livery cars.

In recent years, several ride-hail firms including Uber and Lyft have entered the taxi industry including the New York market. Such firms operate a mobile platform to match customers with cabs, thereby greatly reducing frictions associated with taxi search and availability. Often these platforms implement some form of real-time pricing which will adjust trip prices given current supply and demand conditions to aid in market clearing. The precipitous expansion and popularity of ride-hail is suggestive of the benefits associated with both the reduced search costs and more flexible pricing compared with traditional taxi markets. Another potential reason for this expansion is that taxi regulations are often at odds with this new wave of technology-centered entrants. These firms tend to enjoy much less stringent entry restrictions than the more regulated incumbents, leading to a variety of legal disputes as stakeholders in the traditional taxi business absorb losses. The stakes of this debate are large, and highlight the need for analysis surrounding the effects of these new entrants. This paper aims to understand how regulation and matching technology impact the equilibrium spatial allocations of supply and demand as well as the corresponding impact on market welfare and efficiency.

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7 These licenses are also tradable, and the mere fact that they tend to have positive value, sometimes in excess of one million dollars, implies that this quantity cap is binding and below that of an unrestricted equilibrium.

8 See, e.g., forbes.com/sites/ellenhuet/2015/06/19/could-a-legal-ruling-instantly-wipe-out-uber-not-so-fast/.

9 The spatial availability of taxis is of evident concern to municipal regulators around the country: policies
2.2 Data Overview

In 2009, the New York TLC initiated the Taxi Passenger Enhancement Project, which mandated the use of upgraded metering and information technology in all New York medallion cabs. The technology includes the automated data collection of taxi trip and fare information. I use TLC trip data from all New York City medallion cab rides given from August 1, 2012 to September 30, 2012. An observation consists of information related to a single cab ride. Data include the exact time, date and GPS coordinates of pickup and drop-off, trip distance, and trip time length for approximately 27 million rides. New York cabs typically operate in two separate shifts of 9-12 hours each, with a mandatory shift change between 4-5pm. I focus on the day-shift period of 7am until 4pm, after which I assume all drivers stop working.

Due to New York rules governing pre-arranged trips, the TLC data only record rides originating from street-hails. This provides an ideal setting for analyzing taxi search behavior since all observed rides are obtained through search. Table [1] provides summary facts for this data set. Additional monthly-level statistics are in Appendix A.3.

Most of the time, New York taxis operate in Manhattan. When not providing rides within Manhattan, the most common origins and destinations are to New York’s two city airports, LaGuardia (LGA) and John F. Kennedy (JFK). At the airports taxis form queues and wait in line for next available passengers. Table [2] below provides statistics related to the frequency and revenue share of trips between Manhattan, the two city airports, and elsewhere.

Uber began operating in New York City beginning in 2011, but service remained minimal. In an October 2012 interview, the CEO reported that 160 drivers had participated in providing trips in the city since opening. This represents about 1% of licensed yellow cab drivers, and likely much less in trip volume as these drivers were not necessarily operating consistently throughout the prior year, nor full time.

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11 Using this information together with geocoded coordinates, we might learn for example that cab medallion 1602 (a sample cab medallion, as the TLC data are anonymized) picks up a passenger at the corner of Bowery and Canal at 2:17pm of August 3rd, 2012, and then drives that passenger for 2.9 miles and drops her off at Park Ave and W. 42nd St. at 2:39pm, with a fare of $9.63, flat tax of $0.50, and no time-of-day surcharge or tolls, for a total cost of $10.13. Cab 1602 does not show up again in the data until his next passenger is contacted.

Table 1: Taxi Trip and Fare Summary Statistics

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rate Type</th>
<th>Variable</th>
<th>Obs.</th>
<th>10%ile</th>
<th>Mean</th>
<th>90%ile</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Data</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>27,475,614</td>
<td>4.50</td>
<td>9.51</td>
<td>16.00</td>
<td>5.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>27,475,621</td>
<td>1.50</td>
<td>5.59</td>
<td>12.00</td>
<td>6.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>27,475,621</td>
<td>2.50</td>
<td>2.83</td>
<td>3.50</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>27,475,621</td>
<td>0.82</td>
<td>2.70</td>
<td>6.00</td>
<td>2.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>27,475,621</td>
<td>4.00</td>
<td>12.04</td>
<td>22.52</td>
<td>8.23</td>
</tr>
<tr>
<td>JFK Fares</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>491,689</td>
<td>45</td>
<td>48.32</td>
<td>52</td>
<td>3.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>491,689</td>
<td>3.02</td>
<td>16.25</td>
<td>20.58</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>491,689</td>
<td>22.75</td>
<td>39.49</td>
<td>60.00</td>
<td>17.33</td>
</tr>
<tr>
<td>Weekdays, Day-Shift, Manhattan &amp; Boro.</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>8,164,678</td>
<td>4.50</td>
<td>10.17</td>
<td>17.70</td>
<td>6.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>8,122,794</td>
<td>1.20</td>
<td>4.66</td>
<td>9.60</td>
<td>5.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>8,122,794</td>
<td>2.50</td>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>8,122,794</td>
<td>0.71</td>
<td>2.28</td>
<td>4.67</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>8,122,794</td>
<td>4.00</td>
<td>12.74</td>
<td>23.80</td>
<td>8.49</td>
</tr>
<tr>
<td>JFK Fares</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>171,223</td>
<td>45.00</td>
<td>48.28</td>
<td>52.00</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>171,223</td>
<td>2.00</td>
<td>16.14</td>
<td>20.91</td>
<td>6.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>171,223</td>
<td>26.18</td>
<td>45.65</td>
<td>67.00</td>
<td>19.16</td>
</tr>
</tbody>
</table>

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to individual taxi trips taken in New York City between August 1, 2012 and September 30, 2012 for two fare types. The first is the standard metered fare (TLC rate code 1), in which standard fares apply, representing 98.1% of the data. The second is a trip to or from JFK airport (TLC rate code 2). Total Fare and Distance data are reported for each ride in the dataset. The two main fare components are a distance-based fare and a flag-drop fare. I predict these constituent parts of total fare using the prevailing fare structure on the day of travel and the distance travelled, though they are not separately reported from each other or from waiting costs. Flag fare calculations include the presence of time-of-day surcharges. Any remaining fare is due to a charge for idling time. The first set of statistics relate to the full sample of all New York taxis rides across the two months, and the second set relates to the smaller sample used in this analysis: weekdays, day-shift trips occurring within the space described in Figure 1.

2.3 Discretizing time and space

To analyze time and geography, I aggregate time into five minutes periods.\textsuperscript{13} I also define 39 spatial areas and link these with observed GPS points of origin and destination for each taxi trip.\textsuperscript{14} These locations represent 98% of all taxi ride originations, and are depicted in Figure 1. The average observed travel time between one location to a neighboring location is 2 minutes, 45 seconds, or about one-half of a five-minute period. This suggests that the 5-minute period is reasonably well-suited to this geographic partitioning. For additional details on location selection and construction see Appendix A.2.

I further denote five \textit{regions} as distinct subsets of all 39 locations. Regions are depicted as

\textsuperscript{13}Note in Table 1 that standard trips average 11 minutes, with a 10th percentile of 4 minutes.
\textsuperscript{14}This association is achieved via the point-in-polygon matching procedure outlined in Brophy (2013).
### Table 2: Taxi Trips and Revenues by Area

<table>
<thead>
<tr>
<th>Time</th>
<th>Place</th>
<th>Obs.</th>
<th>Mean Fare</th>
<th>Trip Share</th>
<th>Rev. Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Times</td>
<td>Intra-Manhattan Trips</td>
<td>24,835,103</td>
<td>$9.28</td>
<td>89%</td>
<td>73%</td>
</tr>
<tr>
<td></td>
<td>Airport Trips</td>
<td>1,568,699</td>
<td>$33.77</td>
<td>6%</td>
<td>17%</td>
</tr>
<tr>
<td></td>
<td>Other Trips</td>
<td>1,563,501</td>
<td>$19.83</td>
<td>6%</td>
<td>10%</td>
</tr>
<tr>
<td>Weekdays, Day-shift</td>
<td>Intra-Manhattan Trips</td>
<td>7,813,226</td>
<td>$9.33</td>
<td>91%</td>
<td>76%</td>
</tr>
<tr>
<td></td>
<td>Airport Trips</td>
<td>503,711</td>
<td>$34.80</td>
<td>6%</td>
<td>18%</td>
</tr>
<tr>
<td></td>
<td>Other Trips</td>
<td>270,883</td>
<td>$19.62</td>
<td>3%</td>
<td>6%</td>
</tr>
</tbody>
</table>

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to the locations of taxi trips taken in New York City between August 1, 2012 to September 30, 2012. Intra-Manhattan denotes trips which begin and end within Manhattan, Airport Trips are trips with either an origin or destination at either LaGuardia or JFK airport, and Other Trips captures all other origins and destinations within New York City. Statistics are reported for all times as well as the day-shift period of a weekday, from 6am until 4pm. The latter category is the focus of my analysis.

Each region is characterized by unique mixes of geographical features and transit infrastructure. I will estimate the efficiency of search across each of these five regions. Region I is lower Manhattan, an older part of the city where streets follow irregular patterns, and where numerous bridges, tunnels and ferries connect to nearby boroughs and New Jersey. Region II is midtown Manhattan, with fewer traffic connections away from the island, but denser centers of activity including the major transit hubs Penn Station and Grand Central Station. Region III is uptown Manhattan, where streets follow a regular grid pattern, but at the same time are longer and more spread out. Few bridges, tunnels or stations offer direct connection outside to other boroughs. Region IV is the large area encompassing Brooklyn and Queens. Region V consists of the two airports, John F. Kennedy (JFK) and LaGuardia (LGA).

#### 2.4 Evidence of Frictions

Search frictions occur when drivers cannot locate passengers even though supply and demand coexist at some point in time. Frictions in this market manifest as waiting time experienced by drivers to match with a passenger. The TLC data provide evidence of search frictions for drivers that vary across space and time of day. Using driver ID together with the time of pick-up and drop-off, I compute the waiting time between trips. The mean waiting time for different trips is displayed in Figure 2. Panel I shows the probability that a driver will find a passenger in each five-minute period, as well as the expected waiting time to find a passenger in units of 10-minutes.

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15In a discrete-time sense, this means that after some interval of time some taxi drivers will remain empty despite the presence of demand somewhere else in the market.
Each divided section of Manhattan depicts each of 39 locations indexed by \( i \) in the model. Locations are created by aggregating census-tract boundaries, which broadly follow major thoroughfare divisions. The expected travel time and distance between these locations is computed separately for each origin and destination pair as the average of all observations within each \( ij \) cell. Each shaded section depicts a region \( r \), indicated with Roman numerals I-V. Regions are characterized by similarities in transit infrastructure, road layouts, and zoning.

(i.e., a value of 0.5 equals 5 minutes). There is substantial intra-day variation in search times, with the best times of day for finding passengers around 9am and closer to 4pm, with average wait times around six minutes and five-minute finding rates around 50%. The worst times are in early morning and mid-day, where average wait times are nearly 10 minutes and finding rates fall as low as 25%. Panel 2 divides New York shows the same driver match probabilities and waiting times by the 37 non-airport locations, taken as an average from 7am-4pm across all weekdays of the month. Again there is heterogeneity across space, with relatively higher match probabilities and lower waiting times in lower Manhattan (1-8) and Midtown (9-18), declining match probabilities in upper Manhattan (19-34), and even lower in Brooklyn (35-37).\(^{16}\) In aggregate, drivers spend

\(^{16}\)There is additional evidence that drivers often relocate to find passengers: 61.3% of trips begin in a different neighborhood than the neighborhood where drivers last dropped off a passenger. This suggests that some spatial
about 47% of their time vacant during the sample period of weekdays during the day-shift. This suggests that among 11,500 active drivers, an average of 5,405 are vacant at any one time.

Figure 2: Taxi wait times and match probabilities by time-of-day and location

TLC Data from August 2012, Monday-Friday from 7am until 4pm, within regions indicated on Figure 1. Left Panel: Each series shows taxi drivers’ five-minute probability of finding a customer and mean waiting times, averaged across all drivers and all weekdays of the month. Dotted lines depict 25th and 75th percentiles. Right Panel: Each bar shows driver waiting times and matching probabilities by location of drivers’ last drop-off. Manhattan locations follow a roughly South-to-North trajectory from index 1-34. Brooklyn locations are indexed 35-36. Queens is location 37.

Figure 2 provides a snapshot of the frictions faced by drivers by time-of-day and neighborhood. The data do not reveal the frictions faced by customers; it is impossible to tell how long a customer has been waiting before pick-up, nor is it possible to tell if a customer arrived to search for a taxi and gave up.

3 Model

A city is a network of $L$ nodes called “locations”, connected by a set of routes. A location can be thought of as a spatial area within the city.\textsuperscript{17} Time within a day is divided into discrete intervals with a finite horizon, where $t = \{1,\ldots,T\}$. At time $t = 1$ the work day begins; at $t = T$ it ends. Model agents are vacant taxi drivers who search for customers within a location $i \in \{1,\ldots,L\}$. When taxis find passengers, they drive them from origin location $i$ to a destination location $j \in \{1,\ldots,L\}$. Denote $v^i_t \in \mathbb{R}$ as a measure of vacant taxis and denote $u^i_t \in \mathbb{R}$ as a measure of customers looking

search frictions are present for drivers, as finding a customer requires relocation.
\textsuperscript{17}e.g., a series of blocks bounded by busy thoroughfares, different neighborhoods, etc.
for a taxi in each location at each time. The distance between each location is given by $\delta_{ij}$ and the travel time between each location is given by $\tau_{ij}$.

The model has four basic ingredients. First, there is a demand system that describes, for every neighborhood pair $ij$, how many customers will arrive to the market to search for a taxi as a function of the price of service along that route. Second, there is a payoff vector associated with every route that taxis service. Payoffs include the revenues from each ride as well as a service cost due to fuel expenses. Third, there is a model of period-by-period market clearing. Here I use an aggregate matching function to map supply and demand into match probabilities, which adjust payoffs depending on the relative quantities of taxis and customers.$^{18}$ Finally, these three components are combined in a dynamic model of location choice. In this model vacant drivers make period-by-period location choices accounting for the expected match probabilities and payoffs associated with future locations. These four components are presented in more detail below.

### 3.1 Demand

In each location $i$ at time $t$, the measure of customers that wish to move to a new location, $u^t_i$, is drawn from a Poisson distribution with parameter $\lambda^t_i$. Moreover, $\lambda^t_i$ is a sum of Poisson parameters $\lambda^t_i = \sum_j \lambda^t_{ij}(P_{ij})$, where $\lambda^t_{ij}(P_{ij})$ represents the destination-$j$-specific Poisson arrival of customers in location $i$ at time $t$. The parameters $\lambda^t_{ij}$ are functions of the price of a taxi ride between $i$ and $j$, $P_{ij}$. Denote the probability that a customer in $i$ wants to travel to location $j \in \{1, \ldots, L\}$ at time $t$ by $M^t_{ij}$, so that $\lambda^t_{ij}(P_{ij}) = M^t_{ij} \cdot \lambda^t_i(P)$, where $P$ is a vector of prices between all locations.

I assume that taxi drivers face a constant-elasticity demand curve. Demand depends on the origin and destination of the trip, its price, and the time of day. Price elasticities depend on whether the trip involves an airport (a binary index denoted $a$) and the distance of the trip (indexed by discrete categories $s$). Taxi demand takes the form:

$$\ln(\lambda^t_{ij}(P_{ij})) = \alpha_{0,t,s,a} + \alpha_{1,s,a} \ln(P_{ij}) + \eta_{t,s,a}.$$  

In addition, I assume that customers demand taxi services for one period. After this period, consumers use a different method of transit.$^{19}$

**Waiting Time** Customer waiting time is unobserved, and yet may be an important determinant of demand for taxi rides. To identify the price elasticity of demand in this specific exercise, how-

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18 Note that in a setting of ride-hail, in which prices adjust to neighborhood market conditions, we might instead recast this model as one of localized price formation instead of search frictions.

19 In the empirical analysis to follow, I define one period as five minutes. In the context of New York City, there are plenty of alternative transport options, and this assumption suggests that customers will choose to travel via one of these alternatives upon failing to find a taxi.
however, I provide evidence that waiting time is negligible or of second-order importance. There are two primary reasons for this. First, as the estimation of demand parameters $\lambda_{ij}$ will not require knowledge of waiting time or price elasticities, I compute a measure of waiting time via customer match probabilities.\textsuperscript{20} The September price change, later used to identify price elasticities, leads to an estimated average waiting time change of approximately -3.8 percent, or a reduction of only 11.4 seconds. Second, what little empirical evidence for waiting time elasticities exists suggests that it should be relatively small. \cite{Frechette2019} estimate waiting time elasticity of demand to be about -1.2, while \cite{Buchholz2019} estimate average waiting time elasticity in a large European taxi market to be -0.66. In the latter case, authors estimate a convex relationship between cost of waiting and length of waiting, which suggests that small waiting times are even less impactful to demand than would be suggested by the average elasticity.

### 3.2 Revenue and Costs

Taxis earn revenue from giving rides. At the end of each ride, the taxi driver is paid according to the fare structure. The fare structure is defined as follows: $b$ is the one-time flag-drop fare and $\pi$ is the distance-based fare, with the distance $\delta_{ij}$ between locations $i$ and $j$. The total fare revenue earned by providing a ride from $i$ to $j$ is $b + \pi \delta_{ij}$.

Drivers have two sources of costs. First, there is a fixed daily fee for leasing the taxi and medallion license (or a financing cost for drivers who also own their own medallion). Second there are per-mile fuel costs, which I denote as $c_{ij}$. On any particular day a driver is working, medallion leasing costs for that day are sunk and therefore independent of the driver’s search choices. As the analysis holds fixed the entry decisions of taxis, I ignore these costs in the model and focus on drivers’ optimization while working.

The net revenue of any passenger ride is given by

$$\Pi_{ij} = b + \pi \delta_{ij} - c_{ij}. \quad (2)$$

This profit function sums the total fare revenue earned net of fuel costs in providing a trip from location $i$ to $j$.

\textsuperscript{20}This estimate is premised on the assumption that consumer search takes at most five minutes. Consumers find a match with probability $q_i = m_i / \lambda_i$, where $m_i$ is the observed matches in each $i,t$ cell. A measure of expected waiting time in minutes is then computed as $5 \cdot (1 - q_i)$ assuming any matches are made instantaneously.
3.3 Searching and Matching

At the start of each period, taxis search for passengers. The number of taxis in each location at the start of the period is given by the sum of previously vacant taxis who have chosen location $i$ to search, plus the previously employed taxis who have dropped off a passenger in location $i$. This sum is denoted as $v^t_i$. I make the following assumptions about matching: (1) matches can only occur among cabs and customers within the same location, (2), matches are randomly assigned between taxis and customers, and (3) once a driver finds a customer, a match is made and the driver may not refuse a ride. The expected number of matches made in location $i$ and time $t$ is given by an aggregate matching function $m_i(\lambda^t_i, v^t_i)$. The ex-ante probability that a driver will find a customer is then given by $p^t_i = \frac{m_i(\lambda^t_i, v^t_i)}{v^t_i}$. Figure 3 illustrates the within-period search and matching process.

### 3.3.1 A Model of Neighborhood Search

There are two types of locations, neighborhoods and airports. Neighborhoods comprise most of a city; they are locations in which cabs drive around to search for passengers. Below I detail how matches are formed in neighborhood locations. The next subsection discusses airport locations.

When model locations are specified as spatial areas such as a neighborhood, search within this area will exhibit search frictions even when block-by-block search is nearly frictionless. This echoes the result of Lagos (2000) that search frictions arise endogenously from spatial search. To model

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the search frictions within each location, I use an aggregate matching function given by equation (3):

$$m(\lambda^t_i, v^t_i, \alpha_r) = v^t_i \cdot \left(1 - e^{-\frac{\lambda^t_i}{\alpha_r v^t_i}}\right)$$

Equation (3) is a reduced-form model of intra-location matching. It can flexibly reproduce frictions (i.e. such that $$m(\lambda^t_i, v^t_i, \alpha_r) < \min(\lambda^t_i, v^t_i))$$, the extent of which are controlled by the search efficiency parameter $$\alpha_r > 0$$. All else equal, larger values of alpha generate fewer matches. $$r$$ denotes a region, or a subset of locations as described in section 2.3 $$\alpha_r$$ is region-specific as it reflects the difficulty of search within a region, such as the complexity of the street grid or limitations of visibility. These are physical characteristics of a region which are assumed to be fixed across the day. An illustration of the aggregate matching function and the role of $$\alpha_r$$ is depicted in Figure 4.

Moreover, this equation is specified in terms of expected demand $$\lambda^t_i$$ and not the daily draws $$u^t_i$$. It represents the expected number of matches produced in a location-time with demand parameter $$\lambda^t_i$$ and vacant taxi supply $$v^t_i$$. This is the relevant object from the perspective of taxi drivers’ location optimization problem as outlined below. Hereafter denote $$m_i(\lambda^t_i, v^t_i) = m(\lambda^t_i, v^t_i, \alpha_r)$$ to be the location-specific matching function, with the only difference across locations coming from the efficiency of the region $$r$$ containing location $$i$$. The probability of a match from a taxi driver’s perspective is therefore given by

$$p_i(\lambda^t_i, v^t_i) = \left(1 - e^{-\frac{\lambda^t_i}{\alpha_r v^t_i}}\right).$$

### 3.3.2 Airport Queuing

At airports, taxis pull into one of multiple queues and wait for passengers to match with cabs at the front of the queue.23 I assume there is some measure of congestion in the taxi lane, so that no more than $$\bar{v}^t_i$$ cabs can clear the queue in a period. This condition prevents instantaneous clearing of the taxi queue. Thus the total number of matches made is given by $$\min\{u^t_i, w^t_i\}$$, where $$w^t_i = \min(\bar{v}^t_i, v^t_i)$$ and the total measure of cabs at the airport in each period is $$v^t_i$$. From a taxi driver’s perspective, airports represent match probabilities of one, but at the expense of time spent waiting for the match. The more taxis there are in line, the more periods it will take to match a driver entering the queue.

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22This function is derived from an urn-ball matching problem first formulated in Butters (1977) and Hall (1979). Where the original model characterizes matches from discrete (i.e., integer) inputs, this function to characterizes urn-ball matching with a large number (or continuum) of inputs. See, e.g., Petrongolo and Pissarides (2001).

23In the data, rides involving one of the two major New York airports comprise roughly 6% of all taxi trips, and 16% of revenues.
This figure shows contour plots of the matching function over three values of $\alpha$. Contour levels depict the expected number of matches produced in a given location when the level of taxis is $v$ and the expected arrivals of customers is $\lambda$, for each shown level of $\alpha$.

### 3.4 Dynamic Model of Taxi Drivers’ Locations, Actions and Payoffs

A taxi driver’s behavior is dependent on his own private state $(\ell_{a}^{t}, e_{a}^{t})$ and the market state, $S^{t}$. Specifically a driver $a$’s own location at time $t$ is given by $\ell_{a}^{t} \in \{1, ..., L\}$, and employment status (vacant or employed) is $e_{a}^{t} \in \{0, 1\}$. Let $m_{i,j}^{k}$ denote the set of $ij$ matches occurring with starting time $k$. Then the market state $S^{t}$ at time $t$ is a measure of vacant taxis $v_{i}^{t}$ in each location $i$ and a measure of employed taxis $m_{i,j}^{k}$ that are in-transit between locations.\(^{24}\) Thus the market state at time $t$ is summarized by

$$S^{t} = \{\{v_{i}^{t}\}_{i \in \{1, ..., L\}}, \{m_{i,j}^{k}\}_{i,j \in \{1, ..., L\}^{2}, k \leq t}\}. \tag{5}$$

Denote $S = \{S^{t}\} \forall t$ so that $S$ reflects the entire spatial and intertemporal distribution of vacant and employed taxis. At the beginning of each period, taxi drivers make a conjecture about the current-period state and how it will evolve going forward. Given this conjecture, they assign value $V_{i}^{t}$ to each $i, t$-pair.

I define the drivers’ ex-ante (i.e., before observing any shocks and before any uncertainty in passenger arrivals is resolved) value as

\(^{24}\)At any moment, employed taxis are not directly competing with vacant taxis for passengers. Accounting for the distribution of employed taxis is an important component of the state variable because the eventual arrival of employed taxis and subsequent transition to vacancy is payoff-relevant when deciding how to conduct future search.
\[ V_i^t(S^t) = \mathbb{E}_{S_{i+1}} \left[ p_i(\lambda_i^t, v_i^t) \left( \sum_j M_{ij}^t \cdot (\Pi_{ij} + V_j^{t+\tau_{ij}}(S^{t+\tau_{ij}})) \right) + \right. \\
(1 - p_i(\lambda_i^t, v_i^t)) \cdot \mathbb{E}_{\varepsilon, a} \left[ \max_{j \in A(i)} \left\{ V_j^{t+\tau_{ij}}(S^{t+\tau_{ij}}) - c_{ij} + \varepsilon_{j,a} \right\} \right] \right]. \] (6)

This expression is decomposed as follows. Drivers in \( i \) at time \( t \) expect to contact a passenger with probability \( p_i(\lambda_i^t, v_i^t) \). Drivers’ payoff for providing a trip is equal to the net profit of a trip \( \Pi_{ij} \) plus continuation values \( V_j^{t+\tau_{ij}} \) of being in location \( j \) \( \tau_{ij} \) periods later. Therefore the expected value of a trip is simply the value of a trip to each \( j \) weighted by the probability that a passenger picked-up in \( i \) chooses destination \( j \), which is given by \( M_{ij}^t \).\(^{25}\)

At the end of the period, any cabs which remain vacant can choose to relocate or stay put to begin a search for passengers in the next period. Vacant drivers choose to search next period in the location that maximizes total expected payoff as the sum of continuation values \( V_j^{t+\tau_{ij}}(S) \), fuel costs \( c_{ij} \) and a contemporaneous and an idiosyncratic shock \( \varepsilon_{j,a} \). \( \varepsilon_{j,a} \) is a driver \( a \)-specific i.i.d. shock to the perceived value of search in each alternative location \( j \), which I assume to be drawn from a Type-I extreme value distribution. This shock accounts for unobservable reasons that individual drivers may assign a slightly greater value to one location over another. For example, traffic conditions and a taxi’s direction of travel within a location may make it inconvenient to search anywhere but further along the road in the same direction.\(^{26}\)

The set \( A(i) \) reflects the set of locations available to vacant taxis. I assume that \( A(i) \) is the entire set of locations in the city. Drivers are therefore not limited to search in adjacent locations. If a driver chooses an \( j \in A(i) \) which is not adjacent to \( i \), this means the driver opts out of search along the path from \( i \) to \( j \) but potentially have access a faster route, for example by taking a highway. This implies that \( \tau_{ij} \leq \tau_{ik} + \tau_{kj} \) for all \( k \). However such a decision will involve opportunity costs of not searching sequentially because adjacent location travel times are almost always less than more distant locations.

Time ends at period \( T \). Continuation values beyond \( t = T \) are set to zero: \( V_i^t = 0 \) \( \forall t > T \), \( \forall i \). Employed taxi drivers with arrival times beyond period \( T \) are assumed to finish en-route trips before quitting.

\(^{25}\)Note that \( M_{ij}^t \) has superscript \( t \) because preferences of passengers change throughout the day.
\(^{26}\)The terms \( \varepsilon_{j,a} \) also ensure that vacant taxis leaving one location will mix among several alternative locations rather than moving to the same location, a feature broadly corroborated by data.
At the end of each period, vacant drivers decide where to search for passengers in the next period by choosing the location with the highest present value of search net of transportation costs. Vacant drivers in location \( i \) move to location \( j^* \) by solving the last term in equation 6:

\[
j^* = \arg \max_j \{ V_{jt}^{t+\tau_{ij}}(S^{t+\tau_{ij}}) - c_{ij} + \varepsilon_{ja} \}. \tag{7}
\]

To compute the driver’s strategies, I define the ex-ante choice-specific value function as \( W^t_i(j_{a}, S^t) \), which represents the net present value of payoffs conditional on taking action \( j_a \) while in location \( i \), before \( \varepsilon_{ja} \) is observed:

\[
W^t_i(j_{a}, S^t) = \mathbb{E}_{S^{t+\tau_{ija}}} \left[ V_{j_{a}}^{t+\tau_{ija}}(S^{t+\tau_{ija}}) - c_{ija} \right]. \tag{8}
\]

Defining \( W^t_i \) allows for an expression of taxi drivers’ conditional choice probabilities: the probability that a driver in \( i \) will choose \( j \in A(i) \) conditional on reaching state \( S^t \), but before observing \( \varepsilon_{ja} \), is given by

\[
P^t_i[j_{a}|S^t] = \frac{\exp(W^t_i(j_{a}, S^t)/\sigma_{\varepsilon})}{\sum_{k \in A(i)} \exp(W^t_i(j_{k}, S^t)/\sigma_{\varepsilon})}. \tag{9}
\]

This expression defines aggregate policy functions \( \sigma^t_i = \{ P^t_i[j|S^t]\}_{j \in \{1, \ldots, L\}} \) as the probability of optimal transition from an origin \( i \) to all destinations \( j \) conditional on future-period continuation values.

### 3.5 Intradays timing

There is an exogenous initial distribution of vacant taxis labeled \( S^1 \) which is known to all drivers. This distribution accounts for the early morning position of taxis, all of which are assumed vacant at this time, as they leave from garages and arrive to the search regions of the city. Taxis conduct search in the first period and at the end of the period the newly employed taxis disappear from the stock of vacant cabs and earn revenue. Vacant taxis earn no revenues but face continuation values associated with each possible move. In period \( t = 2 \), the locations of vacant taxis are updated based on movement from both relocating vacant taxis as well as any taxis dropping off passengers matched in the previous period (for trips in which travel time takes one period, or \( \tau_{ij} = 1 \), and the distribution of employed taxis who are still in-transit, \( \tilde{v}^k_{ij} \), is updated). Any vacant taxi then search for passengers, repeating the same process as above. Since the expectation of a Poisson random variable with parameter \( \lambda \) is also equal to \( \lambda \), denote \( \lambda^t_i \) be the expected demand which is observed by taxi drivers. Drivers form policies based on the following sequence of events:

1. Taxis are exogenously distributed according to \( S^1 \).
2. \( m_i(\lambda^1_i, v^1_i) \) taxis become employed with matched customers in each location.

3. The remaining \( \lambda^1_i - m_i(\lambda^1_i, v^1_i | \alpha_i) \) unmatched customers leave the market.

4. The remaining \( v^1_i - m_i(\lambda^1_i, v^1_i | \alpha_i) \) vacant taxis choose a location to search in next period according to policy functions.

5. Previously vacant and some previously employed taxis arrive in new locations, forming distribution \( S^2 \).

6. The process repeats from \( S^2, S^3 \), etc. until reaching \( S^T \).

3.6 Transitions

Policy functions \( \sigma^t \) form a matrix of transition probabilities from origin \( i \) to all destinations \( j \in L \). Note that only vacant taxis transition according to these policies. Employed taxis will transition according to a different matrix of transition probabilities given by \( M^t_i \) denoting the probability that a matched customer in \( i \) will demand transit to any destination \( j \in L \). Together, these two transition processes combine to generate a law of motion for the state variable \( S \).

The transition kernel of employed taxis is given by \( \nu(v^{t+1}_v | v^t_v, M^t, m^t) \) where \( v^t_v \) is the distribution of employed taxis across locations in period \( t \), \( M^t = \{M^t_{ij}\} \) for \( i, j = \{1, ..., L\} \) is the set of transition probabilities of each matched passenger at time \( t \) and \( m^t = \{m_i(\lambda^1_i, v^1_i)\} \) for \( i = \{1, ..., L\} \) is the distribution of matches in each time \( t \). \( \nu \) specifies for all employed taxis in location \( i \) at time \( t \), \( v^t_v, i^t \), their expected distribution across locations in period \( t + 1 \).

Likewise, the transition kernel of vacant taxis is given by \( \mu(v^{t+1}_v | v^t_v, \sigma^t) \). As with \( \nu \), \( \mu \) specifies the expected \( t + 1 \) spatial distribution of period \( t \) vacant taxis, given the transitions generated from policies \( \sigma^t = \{\sigma^i_t\} \) for \( i = \{1, ..., L\} \). The combined set of transitions forms an aggregate transition kernel that defines the law-of-motion, given by \( Q(S^{t+1} | S^t) = \nu(v^{t+1}_v | v^t_v, M^t, m^t) + \mu(v^{t+1}_v | v^t_v, \sigma^t) \).

Explicit formulas for the state transitions are provided in Appendix A.5.

3.7 Equilibrium

Taxi drivers’ policy functions depend on the initial state, beliefs about the distribution and policy functions of their competitors, and an information set which includes the fixed price schedule, demand parameters and the geography of route times and distances. Beliefs over competitors’ policies given the distribution of all vacant cabs allow taxi drivers to infer how the distribution of vacant cabs at time \( t \) will update in future periods. This transition is denoted as \( \tilde{Q}^t_i \).

\footnote{Regarding item 7, “some previously employed taxis arrive”: many hired taxis are in-transit for more than one period. Suppose hired taxis providing service from location \( i \) to \( j \) will take 3 periods to complete the trip. Then only the taxis who were 1 period away at time \( t - 1 \) will arrive in \( j \) in period \( t \).}
Although drivers have knowledge of the Poisson demand parameters $\lambda^t_i$, I assume they do not see actual draws $u^t_i$, which are spread across multiple blocks within a location. Any successful or unsuccessful match is attributed to a long-run probability of matching in each location and period and there is no intra-day updating of beliefs based on whether the driver is matched or not.

Taxis optimize over where to locate when vacant. Since beliefs about the state and transitions at time $t$ summarize all relevant information about distribution of competition, taxis condition only on beliefs over the current-period so that an optimal location choice at time $t$ can be made using time $t-1$ information. This Markovian structure permits a definition of equilibrium as follows:

**Definition** Equilibrium is a sequence of state vectors $\{S^t_i\}$, transition beliefs $\{\tilde{Q}^t_i\}$ and policy functions $\{\sigma^t_i\}$ over each location $i = \{1, ..., L\}$, and an initial state $\{S^0_i\}_{vi}$ such that:

(a) In each location $i \in \{1, ..., L\}$, at the start of each period, matches are made according to equation 3 and are routed to new locations according to transition matrix $M^t$. The aggregate movement generates the employed taxi transition kernel $\nu(v^t+1_e|v^t_e, M^t, m^t)$ where $v^t_e$ is the distribution of employed taxis across locations in period $t$ and $m^t$ is the distribution of matches across locations.

(b) In each location $i \in \{1, ..., L\}$, at the end of each period, vacant taxi drivers (indexed by $a$) follow a policy function $\sigma^t_i,a(S^t, \tilde{Q}^t_i)$ that (a) solves equation 7 and (b) derives expectations under the assumption that the state transition is determined by transition kernel $\tilde{Q}^t_i$. The aggregate movement generates the vacant taxi transition kernel $\mu(v^t+1_v|v^t_v, \tilde{\sigma}^t, S^t)$ where $v^t_v$ is the distribution of vacant taxis in period $t$.

(c) State transitions are defined by the combined movement of vacant taxis and employed taxis, defined by $Q(S^{t+1}|\tilde{S}^t) = \nu(v^t+1_e|v^t_e, M^t, m^t) + \mu(v^t+1_v|v^t_v, \tilde{S}^t)$.

(d) Agent’s expectations are rational, so that transition beliefs are self-fulfilling given optimizing behavior: $\tilde{Q}^t_i = Q^t_i$ for all $i$ and $t$.

**Proposition 3.1.** Period payoff functions, which can be written as $F(i,a,S^t)$, are strictly concave and continuously differentiable in probabilities over actions, $\sigma^t_i$, and exhibit increasing differences in $(\sigma^t_i, v^t_i)$ where $v^t_i$ is the $i$-th element of $S^t$.

**Proof.** See Appendix A.7

Proposition 3.1 provides a set of sufficient conditions for existence and uniqueness of equilibrium, as demonstrated by Theorems 2 and 4, respectively, of Light and Weintraub (2018).

Equilibrium delivers a distribution of vacant taxi drivers such that no one driver can systematically profit from an alternative policy: there is no feasible spatial arbitrage opportunity that
would make search more valuable (ex-ante) in any location other than the optimum one. Vacant taxis are therefore more clustered in locations more profitable customers, but the associated profits are offset by higher search frictions. One implication of this is that equilibrium value functions are nearly identical across space in each time period. Moreover, equilibrium value functions would equate across locations in each period if not for transportation costs in time and fuel, which limit an equilibrium with perfect spatial arbitrage. See additional discussion and illustration in Appendix A.11.

4 Empirical Strategy

4.1 Computing Equilibrium

The equilibrium location choices of vacant taxi drivers and their resulting spatial allocations must be computed in order to estimate model parameters. I assume that taxi drivers have knowledge of the initial state and demand parameters \( \{ \lambda_i^t \} \). Taxi driver form policies that condition on expected demand only and do not observe specific draws of demand or any consequential deviations from the equilibrium state. This assumption is motivated by drivers’ inability to observe supply or demand beyond the particular streets driven on in one period.\(^{28}\)

I solve the model using a two-step procedure where in the first step, the set of equilibrium moments, \( \{ m_i^t \} \) are non-parametrically estimated using trip data as the mean matches in each \( i, t \) cell taken over each day of the month. These moments contain all relevant information needed for the second step, which is to solve the taxi drivers’ spatial equilibrium up to \( \sigma_\varepsilon \). This step is the most involved as it entails solving for non-stationary equilibrium value functions and policy functions given an exogenous initial condition (i.e. the distribution of locations where drivers start their shift) and an exogenous labor supply.

Computing a dynamic equilibrium in large markets with many states is typically challenged by the curse of dimensionality. In this setting the problem is mitigated by assumptions on taxi drivers’ information set, in particular the assumption that deviations from the average distribution of supply and demand is not observed; instead all drivers form strategies based on the expected distribution of both. This assumption gives rise to a limited-information equilibrium similar to that of [Weintraub, Benkard, and Van Roy (2008b)](https://doi.org/10.1257/jep.2008b103) and [Pershtman and Pakes (2012)](https://doi.org/10.1257/jep.2012b103) in which all agents play strategies and form equilibria without full knowledge of the payoff-relevant state variables.

\(^{28}\)An alternative assumption that drivers have full information of demand would be implementable within the same framework. Information assumptions in between these two extremes, such as a drivers’ learning about demand and subsequently updating policies would impose much greater computational burden due to the challenges in computing expectations over a large set of feasible future states.
State transitions are composed of the combined transitions of vacant and employed taxis. Under the continuum model both of these transitions are deterministic given a set of matches and vacancies. Given an exogenous initial allocation of all 11,500 taxis in the market, assumed vacant at time \( t = 1 \), drivers compute a single, deterministic equilibrium path for the state \( \{S^t_i\} \) for \( t = \{1, ..., T\} \). Taxi cabs start the day in garages around the city. When initially arriving to the set of locations in Figure [I], they form an initial distribution of vacant supply. Because such locations are unobserved, I approximate this distribution using the empirical distribution of early morning matches. To compute equilibrium conditional on this initial state, I devise a numerical algorithm that couples backwards induction and value function iteration. The details this algorithm and tests for robustness to the initial condition are in Appendix (A.6). Once the spatial equilibrium is computed, and once \( \sigma_\varepsilon \) is estimated, I then estimate the remaining parameters as discussed below.

4.2 Model Estimation and Identification

This subsection proceeds in four parts. I first discuss parameters that are identified directly from data. Second, I discuss using the first set of parameters together with a computed equilibrium allocation of taxis to estimate the demand and matching efficiency parameters. Third, I discuss estimation of the variance of the driver shocks to location values. Finally, I detail using the recovered demand parameters from before and after a fare change to estimate the price elasticities of demand along different routes.

4.2.1 Objects Identified Directly from Data

Five parameters of the model are identified directly from data. Each is a set of time and location averages. The first four parameters are expected quantities related to time, distance, and transitions. The fifth parameter relates to fuel costs, where average values of the taxi fleet’s fuel economy are the only available data.

1. \( M^t_{ij} \) is the transition probability of employed taxis in each period and location. In each period, I record the probability of transition from each origin to each destination conditional on a taxi matching with a passenger. The mean of these probabilities over each weekday of the month, computed for each origin \( i \), destination \( j \) and hour \( t \), generates expected transition probabilities \( M^t_{ij} \).

2. \( \tau_{ij} \) is the travel time between each origin and destination. As above, I record the average of all travel times between each \( i \) and \( j \), for each hour \( t \), over all weekdays of the month. I set \( \min(\tau_{ii}) = 1 \), so that within-location trips must take at least 1 period.
3. \( \delta_{ij} \) is the distance between each origin and destination. With the trip distance variable in TLC data, I record the mean distance between each \( i \) and \( j \) across all weekdays of the month. Note that \( \delta_{ii} > 0 \) as trips can occur within a location.

4. \( \omega^t_i \) is the congestion measure at each airport \((i = \{38, 39\})\). I treat the mean number of pick-ups in each airport location as direct observations of \( \lambda_{38}^t \) and \( \lambda_{39}^t \). However, there are sequences of large and small queues at each airport and a logistical system to allocate arriving taxis to these queues. I also compute the mean rate of pickups at each airport as the per-taxi expected wait time. With this estimate, taxis’ expected wait times upon arrival at the airport queue each period are computed as the product of all taxis at the airport \( v^t_i \) (having arrived in previous periods) multiplied by this mean wait time.

5. Finally the cost of fuel per mile \( c \), taken as the average fuel price in New York City in 2012, divided by the average fuel economy in the New York taxi fleet, 29 mpg.\(^{29}\) Using \( c \), I compute the cost of traveling between any origin and destination as \( c_{ij} = c \cdot \delta_{ij} \). Note that \( \delta_{ii} > 0 \) implies \( c_{ii} > 0 \).

After I record the distances between each origin and destination, I can derive \( \Pi_{ij} \), the expected profit associated with each possible trip. Recall from equation 2 that \( \Pi_{ij} = b + \pi \delta_{ij} - c_{ij} \), where the regulated fare structure is given by the set \( \{b, \pi\} \). With these parameters, and given data on expected matches, the spatial equilibrium model is configured to identify the equilibrium spatial distribution of vacant taxis.

### 4.2.2 Summary

In the following, I show that the 4,212 Poisson demand parameters \( \{\lambda^t_i\} \), the four matching efficiency parameters for regions I-IV, \( \{\alpha_r\} \), and the parameter \( \sigma_\varepsilon \) can be identified given the available data. A summary of this process is as follows.

1. Given the estimated objects that are derived directly from the data, \( \{\tau_{ij}, \delta_{ij}, \Pi_{ij}\} \), and given some value of \( \sigma_\varepsilon \), all that is necessary to solve for the equilibrium state \( \{v^t_i\} \) are the expected number of matches across locations and times, \( m^t_i \) and the variance parameter \( \sigma_\varepsilon \).

2. I resolve step 1 over a grid of different values of \( \sigma_\varepsilon \), and choose the value that best matches a set of simulated moment conditions to data.

---

\(^{29}\)Data come from the New York City Taxi and Limousine Commission 2012 Fact Book and the U.S. Energy Information Administration. The taxi fleet is approximately 60% hybrid vehicles. Volatility of fuel prices is low in this period: cost-per-mile fluctuates within a range of $0.01 during the sample period.
3. Given the expected number of matches and the corresponding equilibrium \( \{v_t^i\} \), I can identify the ratio \( \frac{\lambda t}{\alpha r} \) by inverting \( m_t^i(v_t^i, \frac{\lambda t}{\alpha r}) \) in its second argument.

The next two subsections describe this process in more detail.

### 4.2.3 Estimation of \( \sigma_\varepsilon \)

Since the profit function is observed directly and carries no parameters to be estimated, the variance of \( \varepsilon \) impacts how much drivers’ spatial search behavior is explained by trip profits versus other unobserved factors. Section 4.1 describes equilibrium computation given the scale parameter \( \sigma_\varepsilon \). To estimate this parameter, I re-solve for the equilibrium state \( \{v_t^i\} \) over a grid of different values of \( \sigma_\varepsilon \). For each grid value, I generate a set of simulated moments related to \( \sigma_\varepsilon \): drivers’ vacant waiting times between trips, total distance traveled with passengers, the probability of the next ride being given from location \( i \) conditional on the last drop-off being in location \( i \), and average matching probabilities within sections I-IV depicted in Figure 1. Each of these moments can then be compared to their data counterparts in a Method of Simulated Moments (MSM) estimator.

A high-\( \sigma_\varepsilon \) equilibrium will lead to drivers that are more spread out spatially (as choice probabilities tend towards a uniform distribution) so that drivers choose to search patterns that are less centralized in Manhattan. This will lead to longer times between trips as more drivers will choose to search farther from their current location despite the fact that, all else equal, nearby search is more profitable. Longer trips are correlated with long between-trip times. Thus, remote regions will be searched more often under a high-\( \sigma_\varepsilon \) equilibrium.

### 4.2.4 Estimation of \( \lambda_t^i \) and \( \alpha_r \)

Let \( m_t^i \) denote the expected number matches in each location \( i \) and time \( t \). Because matches are observed over many days, the \( m_t^i \) are therefore obtained in the data. As discussed above, \( m_t^i \) and \( \sigma_\varepsilon \) together with observed data moments \( \tau_{ij}, \delta_{ij}, \) and \( \Pi_{ij} \) give rise to a unique equilibrium state \( S^* = \{v_t^{*i}\} \) where \( m = \{m_t^i\} \). This notation is explicit that only data on equilibrium matches \( m \) are necessary to recover \( S^* \). I can then identify the set of ratios \( \{\frac{\lambda_t^i}{\alpha_r}\} \) using \( \{v_t^{*i}(m)\} \) by inverting Equation 3.

**Proposition 4.1.** Suppose a vector of expected matches by location and time, \( m \), is observed. Further, suppose \( \{v_t^{*i}(m)\} \) is unique and \( v_t^{*i}(m) \neq 0 \forall i, t \). Then the ratio \( \frac{\lambda_t^i}{\alpha_r} \) is identified.

**Proof.** It is straightforward to show that Equation [3] is strictly increasing in \( \lambda/\alpha \) given \( v > 0 \). Since a strictly increasing function is one-to-one, \( \{m(\cdot, \cdot), v\} \leftrightarrow \lambda/\alpha \) is one-to-one. It therefore
follows that as long as $v^*(m)$ is unique and strictly positive, then (3) can be uniquely inverted for $\frac{\lambda_i}{\alpha_r}$ given $v_{it} = v_{it}^*(m)$ and $m_{it} \in m$.

$$\frac{\lambda_i}{\alpha_r} = -v_{it}^*(m) \cdot \ln \left( 1 - \frac{m_{it}}{v_{it}^*(m)} \right)$$ (10)

Since $m$ is observed and $v^*(m)$ is uniquely determined upon recovering $\sigma_\epsilon$, the right-hand-side of Equation 10 can be computed to recover $\frac{\lambda_i}{\alpha_r}$. The non-zero condition on the state vector is confirmed by the numerically obtained equilibrium.

To separately identify $\lambda_i$ and $\alpha_r$, I leverage an additional moment in the data: the variance of matches in each $i,t$ cell, across days of the sample. Specifically, using the density function of the Poisson distribution, an analytic expression for the variance of matches can be derived that may be used to form an estimator.\(^{30}\)

$$\text{Var}(m_{it}) = (v_{it}^*)^2 e^{-2\frac{\lambda_i}{\alpha_r}} \left( e^{2\frac{\lambda_i}{\alpha_r} v_{it}^*(m)} - 1 \right).$$ (11)

**Proposition 4.2.** Suppose all assumptions of Proposition 4.1 hold, and suppose a vector $\tilde{\sigma}_m^2$ of the variance of matches by time and location across days is observed. Further suppose that the $v_{it}^*$ are constant across days. Then $\{\lambda_i\}$ and $\{\alpha_r\}$ are identified.

**Proof.** $\frac{\lambda_i}{\alpha_r}$ and $v_{it}^*$ are previously obtained as in Proposition 4.1. Denote $\hat{\lambda} = \frac{\lambda_i}{\alpha_r}$, and $\tilde{\sigma}_m^2 = \{\text{Var}(m_{it})\}$, where the variance of matches in each location and time is taken across days. Then inverting equation 11 for $\alpha_r$ gives:

$$\alpha_r = \frac{\hat{\lambda}_{it}}{v_{it}^*(m)} \left( \ln \left( e^{2\hat{\lambda}_{it} \left( \frac{\tilde{\sigma}_m^2}{v_{it}^*(m)} \right)} + 1 \right) \right)^{-1}.$$ (12)

With estimates of $\alpha_r$ and $\frac{\lambda_i}{\alpha_r}$, the demand parameters $\{\lambda_i\}$ may be recovered directly. \(\square\)

Equation 12 shows that $\alpha_r$ is overidentified as each region $r$ is made up of several locations $i$ and times-of-day $t$. While $\alpha$ could be treated as $i$- or $t$-specific, a choice to model frictions on the basis of broader regions will help obtain more credible results for each $\alpha_r$, as there could be error in the measurement and estimation of the right-hand-side parameters and moments. From here, $\alpha_r$ can be estimated via NLLS. Additional details are in Appendix A.10.

\(^{30}\)A derivation of this expression and additional discussion is in Appendix A.9. XXX Equation 10 is derived under the assumption that any variance in $v_{it}^*$ across days is negligible, consistent with the continuum of drivers. This assumption is consistent with the model and empirically motivated by two additional facts. First, the daily variance in $v_{it}^*$ induced by variance in prior matches is low: $E_{i,t}[\text{Var}(v_{it}^*)/\text{Var}(u_{it}^*)] = 0.087$. Second, on average, supply exceeds demand in equilibrium: $E_{i,t}[v_{it}^* / \lambda_i] = 5.87$. Thus the contribution of the variance in taxis to the variance of all matches will be proportionally smaller since the demand side represents the short side of the market.
4.2.5 Estimation of Demand Elasticities

To compute market welfare, I estimate the demand elasticity parameters in equation 1. On September 4, 2012, the distance fee increased by $0.50 per-mile, and the JFK airport flat-fee increased by $7. Using September 2012 data, I re-estimate the model for $\{v^i_t\}$, $\{\lambda^i_t\}$, and $\{\alpha_r\}$.\(^{31}\) In the analysis that follows, I use the price variation from this regulatory change to estimate demand elasticities across different types of trips, where the demanded quantity is the average customer arrivals in each $i$ with destination $j$ at time $t$ given prices $P_{ij}$.

I estimate demand via the following specification.

$$\ln(\lambda^i_t(P_{ij})) = \alpha_{0,s,a} + \alpha_{1,s,a} \ln(P_{ij}) + \delta_{ht} + \gamma_{ri} + \gamma_{rj} + \kappa' \cdot X_{ij} + \eta_{i,j,t}. \tag{13}$$

In a slight abuse of notation I define the index $s = s(i,j)$ as a set of the distance categories associated with a trip $i,j$, such that $s \in \{s_0, s_1, s_2, s_3\} = \{0-2$ mi., $2-4$ mi., $4-6$ mi., $6+$ mi.\}, roughly corresponding to trip-distance quartiles. The index $a$ indicates an airport trip. Price elasticities $\alpha_{1,s,a}$ are different for each distance category among trips without airports, and different for airport trips. I include hourly fixed effects $\delta_{ht}$ and region fixed effects for drop-off region $r_i$ and pickup region $r_j$. $X_{ij}$ is a vector of data summarizing the number of public transit stations at the origin location $i$ and destination location $j$, as well as the indicator for whether the trip origin $i$ contains Broadway Ave, a particularly busy thoroughfare for taxi rides. The above controls allow for sufficiently rich fit of the demand model given the highly heterogeneous spatial patterns of demand, while at the same time limiting any over-fitting such as when a few specific neighborhoods saw an increase in average demand after the price increase. Such an increase would, in the above specification, be instead attributed to $\eta_{i,j,t}$.

Origin and time fixed effects capture the heterogeneity of locations: some have more or less public transit stations, bus stops, or walkability.

In this demand system, all customers of a given type $s,a$ have the same price elasticities. Identifying variation comes from two sources: differences in prices for trips from a given origin to all other destinations (within category $s,a$), and differences in prices before and after the September 2012 fare change.\(^{32}\) To estimate parameters, I estimate an empirical analogue of equation 1 for each $s$ category using OLS. Since prices are fixed within a location and time period, this specification does not suffer from simultaneity bias as would traditional non-instrumented demand models.

\(^{31}\)Since road conditions, traffic patterns, average weather patterns, etc. may change, I allow $\alpha_r$ to change by month. I assume that $\sigma_\epsilon$ is unchanged.

\(^{32}\)For example, a non-airport trip of distance 2.2 miles from location $i$ is compared with another non-airport trip of distance 2.4 miles, also from $i$. These trips have slightly different prices as well as different customer arrivals. In addition, there is a change in prices and arrival rates for all trips between August and September.
Table 3: Results Summary

### Panel A: Parameter Estimates Summary

<table>
<thead>
<tr>
<th>Estimated Parameter</th>
<th>Number of Estimates</th>
<th>Point Estimate</th>
<th>Mean/Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_t^i )</td>
<td>4,212 (108 × 39)</td>
<td>See Figs. (A6)-(A7)</td>
<td>58.85 / 2.39 / 543.94</td>
</tr>
<tr>
<td>( \alpha_r )</td>
<td>4</td>
<td>{0.86,0.86,1.16,1.11}</td>
<td>n.a.</td>
</tr>
<tr>
<td>( \sigma_\varepsilon )</td>
<td>1</td>
<td>3.16</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

### Panel B: Equilibrium Summary

<table>
<thead>
<tr>
<th>Estimated Object</th>
<th>Number of Elements</th>
<th>Computed Value</th>
<th>Mean/Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S = {v_t^i} )</td>
<td>4,212 (108 × 39)</td>
<td>See Figs. (A6)-(A7)</td>
<td>161.4 / 5.29 / 562.7</td>
</tr>
<tr>
<td>( {V_t^i} ) (non-airport i)</td>
<td>3996 (108 × 37)</td>
<td>See Fig. (A8)</td>
<td>$186.6 / $0.45 / $367.5</td>
</tr>
<tr>
<td>( {V_t^i} ) (airport i)</td>
<td>216 (108 × 2)</td>
<td>See Fig. (A8)</td>
<td>$188.6 / $0 / $571.76</td>
</tr>
</tbody>
</table>

This table presents a summary of estimation results and equilibrium solutions using the baseline August 2012 data. Point estimates for matching efficiency parameters \( \alpha_r \) correspond to Table 1 sections I-IV, respectively.

## 5 Empirical Results

This section presents estimation results of the dynamic spatial equilibrium model. Table 3 Panel A shows estimation results for the per-period Poisson parameters for customer arrivals \( \lambda_t^i \), as well as point estimates for additional parameters: the variance of unobservable shocks, \( \sigma_\varepsilon \) and the four region-specific matching efficiency parameters \( \alpha_r \). Panel B summarizes the computed equilibrium objects: the supply of vacant taxis across time and locations, \( \{v_t^i\} \) and the corresponding value functions \( \{V_t^i\} \) at each time and location.\(^{33}\)

### 5.1 Spatial Distributions and Intra-day dynamics

Figure 5 depicts supply and demand for taxi rides across all locations, averaged across all periods of the day. It shows that both taxi supply and passenger demand are most highly concentrated in the central part of Manhattan. For the most part, the number of vacant taxis is sufficient to meet demand in the absence of search frictions. The notable exception is in two central regions with very high demand, where average demand exceeds average supply.

Figure 7 provides an inter-temporal view of results for two busy locations, Greenwich Village/SoHo (Panel 1) and Central Midtown (Panel 2). Both graphs depict the equilibrium supply of vacant taxis, estimated arrival of customers looking for a taxi, the equilibrium number of matches, \(^{33}\)

\(^{33}\)A full set of result figures is provided in Appendix Figure A.11.
Figure 5: Map of Estimated Supply and Demand: Mean Across $t$

This figure shows the supply and demand estimates for August 2012 averaged over time-of-day. The left panel shows the mean estimated per-period customer arrivals and the right panel shows the mean level of vacant taxis.

and the model’s fit against the observed number of matches in the data. Each series is shown from 8a-4p, in 5-minute increments. Panel 1 shows that there are periods of relative oversupply and undersupply (compared to demand) of taxis at different times of day. Panel 2 shows an oversupply of taxis at the same moment there is an undersupply shown in Panel 1. These illustrate evidence of spatial misallocation as an equilibrium outcome: there is mismatch across locations, as across Panels 1 and 2. Within-location matching frictions are captured as the vertical space between the minimum of supply and demand at any point (i.e., $\min\{v_t^i, \lambda_t^i\}$) compared with matches (i.e., $m_t^i$).\(^{34}\)

When taxis and customers match each period vacant capacity is redistributed across space. How much is the spatial distribution of taxi supply driven by customer destination preferences versus taxis’ search behavior? Figure 9 aggregates taxi supply across all 39 locations into five regions (identical to those in Figure 1) and depicts the net flow of matches by region, defined as the sum of drop-offs minus pick-ups in each location, summed across all locations per region. Panel (b) shows the net flow of taxis due to vacant taxis’ location choices by region. It shows, for example, that in the first half of the day, employed taxis are traveling into Midtown and out of most other regions, while at the same time, vacant taxis are proportionally exiting Midtown. Across the day, the choices of vacant taxis almost perfectly offset the movement of employed taxis, as these choices maintain an equilibrium by equating value functions in each location.

\(^{34}\)Results for more locations are available in Appendix A.11.
This figure shows intra-day results for two example locations. Panel 1 shows the location that encompasses Greenwich Village and SoHo between Canal and 14th Street. Panel 2 is Central Midtown from 37th Street to 59th Street. Each figure depicts the equilibrium supply of taxis (red, dashed line) and the estimated arrival of passengers (blue, dot-dash line) from 7am to 4pm, compared with the expected number of matches. Matches are shown in two forms: the purple (dotted) line shows the expected matches in each minute, for each location, where the expectation is taken over days of the month. The yellow (solid) line shows a smoothed-over-time version of the former, with smoothing implemented by fitting a sixth-order polynomial. Each point depicts the over- or under-supply of taxis relative to demand in each 5-minute interval from 8am until 4pm.

5.2 Frictions

These results allow us to impact of search frictions within and across locations. Aggregate excess demand over the course of the day-shift is given by \( \sum_{i,t} \left( \lambda_{t}^{l} - m_{i}(\lambda_{t}^{l}, v_{t}^{*}; \alpha_{r}) \right) \). Total daily demand is 241,187 whereas total matches is 183,839, implying that 57,348 demanded customer trips are unmet each day, or an average of 531 each period. This contrasts with the 5,405 taxis that are vacant at any one time, suggesting the presence of substantial search frictions on both sides of the market given the current set of price regulations.

This can be further decomposed by attributing unmet demand to within-location versus across-location frictions. Within-location frictions are due to the presence of a frictional matching technology. These can be measured via \( \sum_{i,t} \left( \min(\lambda_{t}^{l}, v_{t}^{*}) - m_{i}(\lambda_{t}^{l}, v_{t}^{*}; \alpha_{r}) \right) \), where the first term reflects a frictionless matching technology. These are frictions that may be directly mitigated with better matching, for example through app-based ride-hail platforms. Within-location frictions amount to 48,292 unmet passengers, or 84%. The remaining 9,056 lost trips are due to spatial mismatch between vacant drivers and passengers. Even with better technology, these frictions exist when supply and demand are farther apart and thus not readily matched. The residual spatial mismatch
Figure 9: Equilibrium Flow of Matches and Vacancy Choices by Region

This figure shows the net flow of matches and vacant taxis for August 2012. The top panel shows the net flow of matches, defined as the sum of matches with destinations into each region minus the sum of pick-ups headed out of each region. The bottom panel similarly shows the net movement of vacant taxis into and out of locations within each region. Positive values therefore reflect a net inflow of vacant cabs in each location due to taxis dropping off customers (in Panel I) and previously vacant cabs (in Panel II).

5.3 Demand Elasticities

Table 4 provides estimation results for the demand model in equation (1). As outlined in Section 4.2.5, I separately estimate the model for August 2012 and September 2012 following a change in the regulated tariff. I then leverage the change in prices over this period to identify price elasticities of demand across trips of differing lengths. An observation is an arrival-rate of customers within an origin-location, destination-location, and five-minute period during a weekday from 7a-4p. Table 4 reports price elasticities of passenger arrivals between -1.19 to -2.53.

I compute welfare by first integrating demand over $q \in [0, \lambda_{ij}^t]$ for each combination of origin, destination and hour. From this measure, welfare accrues only to the fraction of customers served, or $m_i^t(\lambda_i^t, v_i^t)/\lambda_i^t$. These calculations are illustrated in Figure 11. The area $A \cup B$ reflects the entire available surplus in this market at price $p_i^t$. The area $B$ is the lost surplus due to frictions and random matching. $A$ is therefore the realized welfare for each sub-market $(i,j,t)$. Note that $A$

Explicit formulas are provided in Appendix A.12
Table 4: Estimation results: Demand Elasticities for Non-airport Origins

<table>
<thead>
<tr>
<th>Description</th>
<th>Trip Type</th>
<th>0-2 mi.</th>
<th>2-4 mi.</th>
<th>4-6 mi.</th>
<th>&gt;6 mi.</th>
<th>Airport Trips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Elasticity</td>
<td></td>
<td>−1.243**</td>
<td>−1.192**</td>
<td>−1.673**</td>
<td>−2.533**</td>
<td>−1.487**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.296)</td>
<td>(0.306)</td>
<td>(0.452)</td>
<td>(0.639)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Pickup Hour FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Pickup Region FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Drop-off Region FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Pickup MTA Station FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Drop-off MTA Station FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>N</td>
<td>99,336</td>
<td>87,012</td>
<td>60,960</td>
<td>48,396</td>
<td>16,848</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.290</td>
<td>0.366</td>
<td>0.503</td>
<td>0.344</td>
<td>0.189</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses, ** p<0.01

Demand data come from model estimates. The dependent variable is $\log(\lambda_{ij}^t)$ and the independent variable of interest is $\log(p_{ij})$. Columns 1-4 report estimates for trips within non-airport locations. Column 5 reports estimates for trips that end at an airport. Standard errors are clustered at the level of origin-location.

is an equilibrium object due to the dependency on $p_{ij}^t$ and $m_{ij}^t$. Since elasticity estimates are local, welfare at high prices involves extrapolation far out of sample. To ensure that welfare valuation is not driven by these observations I implement a choke price of $\bar{p} = $100. Aggregate welfare is then computed as $\sum_{i,j,t} A_{ijt}(p_0)$ where $p_0$ reflect trip prices in each $i,j,t$ sub-market given the observed tariff pricing.

Total estimated welfare for each weekday, day-shift is shown in Table 5. Consumer welfare for New York taxi service is $2.96M per day-shift. Taxi profits in each shift are $2.40M, or $208 per driver. There is substantial heterogeneity by time, place and trip length. For example, mid-day trips generate twice as much consumer surplus and about 33% more profits than morning trips. The welfare from trips in manhattan are vastly more valuable than those in Brooklyn, and shorter trips make up the bulk of welfare and profits. This variation suggests that if different pricing regimes were to affect the spatial allocation of supply and demand, then we should expect them to also substantially impact aggregate welfare.
This figure describes how welfare is calculated in each sub-market \((i,j,t)\) in the presence of search frictions under random matching. The depicted shape of each curve is for illustration purposes; the scale and curvature of demand differs in each sub-market.

Table 5: Estimated Results: Daily, Single-Shift Welfare Measures

<table>
<thead>
<tr>
<th>Grouping Category</th>
<th>Grouping</th>
<th>Cons. Surplus ($, thousand)</th>
<th>Taxi Profits ($, thousand)</th>
<th>Matches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>-</td>
<td>2957.1</td>
<td>2399.5</td>
<td>183,839</td>
</tr>
<tr>
<td></td>
<td>7a-9a</td>
<td>556.2</td>
<td>535.0</td>
<td>29,535</td>
</tr>
<tr>
<td></td>
<td>9a-11a</td>
<td>984.6</td>
<td>725.5</td>
<td>46,249</td>
</tr>
<tr>
<td></td>
<td>11a-1p</td>
<td>922.1</td>
<td>686.3</td>
<td>42,082</td>
</tr>
<tr>
<td></td>
<td>1p-4p</td>
<td>494.1</td>
<td>452.7</td>
<td>64,443</td>
</tr>
<tr>
<td>Time-of-day</td>
<td>Sec. I</td>
<td>741.2</td>
<td>618.8</td>
<td>32,738</td>
</tr>
<tr>
<td></td>
<td>Sec. II</td>
<td>1257.6</td>
<td>908.5</td>
<td>87,563</td>
</tr>
<tr>
<td></td>
<td>Sec. III</td>
<td>921.1</td>
<td>829.1</td>
<td>52,753</td>
</tr>
<tr>
<td></td>
<td>Sec. IV</td>
<td>37.2</td>
<td>43.1</td>
<td>3,614</td>
</tr>
<tr>
<td>Origin Region</td>
<td>0-2 mi.</td>
<td>1698.3</td>
<td>1130.8</td>
<td>114,472</td>
</tr>
<tr>
<td></td>
<td>2-4 mi.</td>
<td>988.5</td>
<td>825.3</td>
<td>43,355</td>
</tr>
<tr>
<td></td>
<td>4-6 mi.</td>
<td>218.8</td>
<td>315.6</td>
<td>11,405</td>
</tr>
<tr>
<td></td>
<td>6+ mi.</td>
<td>51.5</td>
<td>127.7</td>
<td>13,014</td>
</tr>
</tbody>
</table>

This table depicts welfare measures decomposed by category. Consumer welfare is summed across all \(i,j,t\) in each category. Taxi profits derive from total matches multiplied by prices for each origin, destination, and time-of-day. Profits reflect daily, single-shift revenues net of fuel costs. Sections I-V refer to those in Fig. [1]
6 Counterfactual: Pricing for Dynamic Efficiency

With prices set via the same tariff on nearly all trips, there are likely to be static inefficiencies due to mis-pricing in many neighborhoods. In addition, as illustrated above, the prevailing equilibrium in the New York taxi market involves spatial misallocation. A natural conjecture would be that such a coarse pricing policy may also lead to misallocation, a dynamic inefficiency caused by both distorted search incentives (drivers are profit-maximizing, but not social surplus maximizing) and consumer prices which do not internalize the cost of re-allocating future supply. Thus, the static and dynamic inefficiency are intimately linked and from a welfare perspective. It is therefore insufficient to independently optimize prices at each location and time-period.

This section studies a set of simple and easily implementable changes to existing tariffs in the New York market with the purpose of generating better welfare outcomes in a manner consistent with the equilibrium dynamics of supply and demand. The changes introduce flexibility of the current tariffs with respect to location, time-of-day, or distance. Flexibility along these dimensions plays two important roles. First, it offers the efficiency of a price mechanism to better clear markets. Second, flexible prices can lead to an endogenous spatially re-allocation of empty capacity to different regions of the city. Results will show that it is possible to increase both profits and consumer welfare by optimally implementing each type of flexible tariff.

To study flexible tariffs, I create three possible regimes: location-based pricing, time-based pricing and distance-based (non-linear) pricing. In the first regime, I allow prices to vary in segments of 2-3 hours across the day (the four segments are 7a-9a, 9a-11a, 11a-1p, 1p-4p). In the second experiment, I allow prices to vary in each of four areas, depicted as zones I-IV in Figure 1. The third experiment considers non-linear pricing, where prices may vary in three dimensions: the fixed fare, per-mile fare, and a squared-distance fare. For each, I search across a set of multipliers to existing fares associated with each origin, destination and time. For each candidate set of multipliers, I recompute origin-destination-specific demand, profits, transitions, and resolve the dynamic spatial equilibrium among taxis.

To account for the potential impact of changing wait times among consumers, I calibrate waiting time elasticities to be -1.0, close to the estimates of Frechette, Lizzeri, and Salz (2019) and Buchholz, Doval, Kastl, Matejka, and Salz (2019). I compute a measure of waiting time as the mass of unmatched consumers in each period multiplied by the period length. The demand response to counterfactual waiting times is evaluated with respect to differences between the waiting time at baseline prices and the waiting time at any counterfactual price. Details of this procedure are available in Appendix A.12.

---

36For example, if at the fare structure of $2.50 + $2.00/mile, a trip from location i to j costs $10.00, a multiplier of 0.8 on trips from i to j would change this fare to $8.00.
Within each flexible price regime, I numerically solve for an optimal configuration according to three different criteria: maximize total surplus, maximize consumer surplus and maximize the total number of trips. The different criteria represent different planner objectives. Because this paper does not study the extensive margin of taxi drivers’ labor supply decision, I constrain the numerical search for optimal prices to outcomes in which taxi revenues are at least as high as the baseline case.\textsuperscript{37} In addition, as a benchmark I compare results with the equilibrium welfare induced by a frictionless matching technology at baseline prices. In this counterfactual, the matching function is changed to a Leontief function of taxis and consumers. Such technology is representative of modern app-based matching that connects taxis and consumers when they are close to each other.

### 6.1 Results

Table 6 displays the results of the dynamically constrained-efficient prices associated with location-based, time-based and distance-based fares. It shows that total welfare gains up to 7.5% and consumer welfare gains up to 12.3% are possible. Interestingly, different planner objectives within a price regime often coincide: location based prices and non-linear prices both reveal that efficient prices maximize total and consumer surplus. All optimal price regimes improve taxi utilization rates, which highlights the role of flexible prices to allocate spare capacity to more productive areas. In addition, they all give a slightly higher share of welfare to consumers. Total welfare gains are on the order of $190-420 million per day, accounting for just the day-time shift, or around $1.00-1.20 per trip. Matches grow by much more, consistent with lower average prices introducing more marginal consumers to the market.\textsuperscript{38}

The final row of Table 6 contrasts these results with a counterfactual in which the within-location matching technology is frictionless, given by \( m(\lambda^t_i, v^t_i) = \min(\lambda^t_i, v^t_i) \). This model approximates that of a modern ride-share platform, where local supply and demand are guaranteed to find one another, but taxis must still choose locations to search. Consumer welfare benefits to this technology are on a similar scale as the benefits to optimal pricing and somewhat better for taxis. Interestingly, it only generates about 12% more matches than baseline, as supply exceeds demand enough on average to render the friction due to within-location matching relatively small. Thus, matching the two sides more efficiently only produces modest overall gains to trip volumes compared with policies that explicitly target changes to the spatial distribution of supply.\textsuperscript{39}

\textsuperscript{37}This avoids pathological solutions which would set driver profits to zero even though they are assumed to continue working.

\textsuperscript{38}The flexibility results corroborate theoretical insights of Schmalensee (1981) and Varian (1985) which find that a necessary condition for price discrimination to enhance social welfare is that it accompanies an increase in output.

\textsuperscript{39}Note that this counterfactual does not simulate gains from other attributes of ride-sharing services such as the value of less waiting, the certainty of a match, app-based payments, etc.
Table 6: Efficient Pricing and Matching Technology: Counterfactual Results

<table>
<thead>
<tr>
<th>Price Type</th>
<th>Price Multipliers</th>
<th>Total Surplus (,000 USD)</th>
<th>Consumer Surplus (,000 USD)</th>
<th>Consumer Rent Share (percent)</th>
<th>Matches (000)</th>
<th>Taxi Utilization (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline - 8/2012</td>
<td>1.00 1.00 1.00 1.00</td>
<td>5984.2</td>
<td>3071.7</td>
<td>55.0</td>
<td>155.0</td>
<td>46.3</td>
</tr>
<tr>
<td><strong>Location-Based Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>1.10 0.70 0.85 1.50</td>
<td>5807.2 (+4.0 %)</td>
<td>3293.7 (+7.2 %)</td>
<td>56.7</td>
<td>186.5</td>
<td>(+20.3 %) 47.3</td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>1.10 0.70 0.85 1.50</td>
<td>5807.2 (+4.0 %)</td>
<td>3293.7 (+7.2 %)</td>
<td>56.7</td>
<td>186.5</td>
<td>(+20.3 %) 47.3</td>
</tr>
<tr>
<td>Max Matches</td>
<td>1.25 0.65 0.90 1.50</td>
<td>5770.8 (+3.3 %)</td>
<td>3256.0 (+6.0 %)</td>
<td>56.4</td>
<td>187.8</td>
<td>(+21.1 %) 47.2</td>
</tr>
<tr>
<td><strong>Time-Based Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>1.50 0.70 0.50 0.50</td>
<td>6004.6 (+7.5 %)</td>
<td>3413.5 (+11.1 %)</td>
<td>56.8</td>
<td>223.9</td>
<td>(+44.5 %) 48.3</td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>1.35 0.70 0.50 0.50</td>
<td>5971.0 (+6.9 %)</td>
<td>3449.7 (+12.3 %)</td>
<td>57.8</td>
<td>224.7</td>
<td>(+44.9 %) 48.3</td>
</tr>
<tr>
<td>Max Matches</td>
<td>1.50 0.55 0.50 0.50</td>
<td>5940.4 (+6.4 %)</td>
<td>3421.2 (+11.4 %)</td>
<td>57.6</td>
<td>231.0</td>
<td>(+49.0 %) 48.5</td>
</tr>
<tr>
<td><strong>Non-Linear Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>0.95 0.90 -0.30</td>
<td>5833.7 (+4.5 %)</td>
<td>3320.0 (+8.1 %)</td>
<td>56.9</td>
<td>183.2</td>
<td>(+18.2 %) 48.9</td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>0.95 0.90 -0.30</td>
<td>5833.7 (+4.5 %)</td>
<td>3320.0 (+8.1 %)</td>
<td>56.9</td>
<td>183.2</td>
<td>(+18.2 %) 48.9</td>
</tr>
<tr>
<td>Max Matches</td>
<td>1.00 1.30 -0.50</td>
<td>5804.7 (+3.9 %)</td>
<td>3257.2 (+6.0 %)</td>
<td>56.1</td>
<td>190.1</td>
<td>(+22.6 %) 53.6</td>
</tr>
<tr>
<td><strong>Matching Technology</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Prices</td>
<td>1.00 1.00 1.00 1.00</td>
<td>6193.1 (+10.9 %)</td>
<td>3340.6 (+8.8 %)</td>
<td>53.9</td>
<td>173.3</td>
<td>(+11.8 %) 47.4</td>
</tr>
</tbody>
</table>

This table shows, for each weekday day-shift (7a-4p), the estimated change in total welfare (profits plus consumer surplus), consumer surplus, the consumer surplus share of total surplus, total matches, and utilization rates across each counterfactual price policy. Each pricing policy shown is a rule that applies to four policy-specific multipliers on the baseline price $p_{ijt}$ for every route, given by $2.50 + \theta_1 \cdot \text{base fare} + \theta_2 \cdot \text{fare per-mile} + \theta_3 \cdot \text{fare per-mile}^2$. The final row depicts equilibrium outcomes under a simulated matching technology in which the matching function takes the form $m_t^i = \min(\lambda_t^i, v_t^i)$.

6.2 Discussion: Comparing Tariff Changes with Real-time Pricing

In pursuit of enhanced intra-day dynamic efficiency, the above pricing counterfactuals are configured to expected patterns of supply and demand. The optimal policies do not have any direct relation to real-time pricing, however; real-time pricing would re-adjust prices in each period to accommodate unexpected shifts in supply or demand. Note that some degree of real-time pricing, by both location and time, is implemented, for example, in Uber’s “Surge Pricing” or Lyft’s “Prime Time”, whereas implementing better dynamic pricing across hours of the day is not the goal of these pricing rules. Nevertheless, better average pricing of the form indicated in Table 6 may enable the regulator to alleviate some of the need for additional real-time pricing, as supply and demand will be more efficiently allocated across space and time in a way that is consistent with dynamic evolution of supply from one period to the next.
7 Conclusion

Supply and demand in the taxi market are uniquely shaped by space. Regulation influences how taxis and their customers search for one another and how often they find each other. This paper models a dynamic spatial equilibrium in the search and matching process between taxis and passengers, showing how both supply and demand can be recovered using data on intra-daily spatial matches. Using such data from New York yellow taxis, I estimate this model to recover the expected spatial and inter-temporal distribution of taxis and mean customer arrivals. By using variation in prices, I further specify and estimate demand curves for each time-of-day and across 39 locations within New York. Demand elasticities enable welfare measurement and a counterfactual analysis of demand. I recompute the equilibrium taxi supply, spatial matches, search frictions and welfare outcomes under alternative price schedules and a frictionless matching technology.

I show that consumer welfare attained in the New York market is $1.6 million per day-shift on a typical weekday, but a more flexible tariff pricing system could provide up to 12% more welfare and 25% more trips. Implementing an optimal distance-based tariff alone leads to an aggregate welfare gain of at least $153 thousand per-shift by better allocating vacant taxis with respect to current and future customer demand through the end of the day. A more sophisticated tariff might offer different prices by location, time and distance, and these results suggest additional benefits to increasing the complexity and granularity of the price, provided they are optimally configured.\textsuperscript{40} The gains due to optimal dynamic pricing are substantially better than gains due to better inter-location matching, suggesting the importance of effective tariff pricing implementation. Such a mechanism is well within scope of the modern app-based ride-hailing platforms that permit transparent price communication to both drivers and customers.

\textsuperscript{40}In other words, offering both additional dimensions on which to set price as well as more categories within each dimension could further boost efficiency.
Bibliography


A Appendix

A.1 Data Cleaning

Taxi trip and fare data are subject to some errors from usage or technology flaws. A quick analysis of GPS points reveals that some taxi trips appear to originate or conclude in highly unlikely locations (e.g., the state of Maine) or even impossible locations (e.g., the ocean). I first drop any apparently erroneous observations. Next, I drop observations outside of the locations of interest, Manhattan and the two airports. This section describes how data are cleaned and provides some related statistics.

Data Cleaning Routine

1. Begin with merged trip and fare data from August 2012 to September 2012.

2. Drop observations outside of USA boundaries.

3. Drop observations outside of the New York area.

4. Drop duplicates in terms of taxi driver ID and date-time of pickup.

5. Drop observations outside of Manhattan (bounded above by 125th st.) or either airport.

6. Drop observations that cannot be mapped to any of the 39 locations in Figure 1

Table A1 shows the incidence of each cleaning criterion.

Table A1: Data Cleaning Summary

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Criterion Applied</th>
<th>Obs. Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop Errors</td>
<td>1. Initial Data</td>
<td>28,927,944</td>
</tr>
<tr>
<td></td>
<td>2. obs. outside USA</td>
<td>-749,623</td>
</tr>
<tr>
<td></td>
<td>3. obs. outside NYC</td>
<td>-5,298</td>
</tr>
<tr>
<td></td>
<td>4. drop duplicates</td>
<td>-57</td>
</tr>
<tr>
<td></td>
<td>5. keep manhattan + airports</td>
<td>-3,622,803</td>
</tr>
<tr>
<td></td>
<td>6. un-mapped data</td>
<td>-117,249</td>
</tr>
</tbody>
</table>

Final Data Set: 24,432,914 observations

This table summarizes the data cleaning routine for TLC data from 8/1/2012-9/30/2012.
A.2 Map Preparation

The 39 spatial locations shown in Figure 1 are created by uniting census tracts, representing 98% of all taxi ride originations. While there is some arbitrariness involved in their exact specification, the number of locations used is a compromise between tradeoffs; more locations give a richer map of spatial choice behavior, but impose greater requirements on both data and computation. Because of the sparsity of data in the other boroughs, I focus on the set of locations falling within Manhattan below 125th street, three nearby areas within Brooklyn and Queens, and the two New York City airports, Laguardia and J.F.K.

The following graphics show how raw GPS data points are converted to locations. I begin with New York census tracts, 425 of which cover the locations of interest. From these, I examine taxi activity, and group census tracts into areas with clusters of activity. Figure A1 shows the origin of each trip in a 10-percent sample of TLC data. It can be seen that trip origins are most heavily concentrated around major streets, particularly north-south and diagonal thoroughfares in the north, with more scattered origin points in lower Manhattan and Midtown Manhattan. The densest neighborhoods are clearly those in Midtown. I have grouped census tracts to form locations in a way that attempts to minimize the number of location boundaries that overlap clusters of activity, for example the clusters around a busy transit station.

A.3 Summary Statistics by Month

Table A2 decomposes the trip and fare summary statistics by month, before and after the fare change.
This figure shows TLC data for a 10 percent sample of taxi trips taken in August 2012. Each dot on the map is the GPS origin of a trip.

Table A2: Taxi Trip and Fare Summary Statistics by Month

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rate Type</th>
<th>Variable</th>
<th>Obs.</th>
<th>10%ile</th>
<th>Mean</th>
<th>90%ile</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekdays, Day-Shift, Manhattan &amp; Boro. (Aug. 2012)</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>4,299,644</td>
<td>4.50</td>
<td>9.20</td>
<td>15.7</td>
<td>5.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>4,299,645</td>
<td>1.04</td>
<td>4.19</td>
<td>8.96</td>
<td>4.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>4,299,645</td>
<td>2.50</td>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>4,299,645</td>
<td>0.72</td>
<td>2.29</td>
<td>4.68</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>4,299,645</td>
<td>4.00</td>
<td>12.25</td>
<td>22.48</td>
<td>7.95</td>
</tr>
<tr>
<td></td>
<td>JFK Fares</td>
<td>Total Fare ($)</td>
<td>85,531</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>85,531</td>
<td>1.87</td>
<td>15.99</td>
<td>20.88</td>
<td>6.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>85,531</td>
<td>26.00</td>
<td>45.27</td>
<td>65.83</td>
<td>19.27</td>
</tr>
<tr>
<td>Weekdays, Day-Shift, Manhattan &amp; Boro. (Sep. 2012)</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>3,823,147</td>
<td>5.00</td>
<td>11.23</td>
<td>20.00</td>
<td>7.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>3,823,149</td>
<td>1.20</td>
<td>5.17</td>
<td>11.00</td>
<td>5.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>3,823,149</td>
<td>2.50</td>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>3,823,149</td>
<td>0.70</td>
<td>2.27</td>
<td>4.65</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>3,823,149</td>
<td>4.12</td>
<td>13.30</td>
<td>25.0</td>
<td>9.01</td>
</tr>
<tr>
<td></td>
<td>JFK Fares</td>
<td>Total Fare ($)</td>
<td>85,692</td>
<td>52.00</td>
<td>51.56</td>
<td>52.00</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>85,692</td>
<td>3.42</td>
<td>16.29</td>
<td>20.93</td>
<td>5.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>85,692</td>
<td>26.82</td>
<td>46.04</td>
<td>68.42</td>
<td>19.05</td>
</tr>
</tbody>
</table>

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to individual taxi trips taken in New York City in the months of August 2012 and September 2012 for two fare types. The first is the standard metered fare (TLC rate code 1), in which standard fares apply. The second is a trip to or from JFK airport (TLC rate code 2). Total Fare and Distance data are reported for each ride. While not reported directly or separated from waiting costs, I predict distance and flag fares using the prevailing fare structure on the day of travel and the distance travelled.
A.4 Medallion Counts

Figure A2 shows the unique number of medallions observed each day of August and September 2012 in the TLC data during weekdays during the day shift. The mean across all days is 11,911.88. It should be noted however, that about 2% of trips occur outside of the 39 locations defined in this paper during this period. This implies that approximately 11,673 medallions are active within the locations, with some additional diminishment in reality due to breaks, refueling, etc. The second point of this figure is that the medallion counts seem fairly stable between price changes, lending support for the assumption that this overall level remains constant. The drop on September 3rd seems to reflect the extra servicing of metering equipment just prior to the tariff change on September 4th.

![Figure A2: Medallions per day, Aug-Sep 2012](image)

This figure depicts the unique number of medallions observed each day of August and September 2012 in the TLC data during weekdays during the day-shift.

A.5 Details on State Transitions

The combined set of transitions forms an aggregate transition kernel that defines the law-of-motion, given by $Q(S_{t+1} | S_t) = \nu(v^t_{e+1} | v^t_e, M^t, m^t) + \mu(v^t_{v+1} | v^t_v, \sigma^t)$.

Let the following objects be defined:

- $v^t_e$ be the $(L + K) \times 1$ vector of employed cabs at the start of period $t$, where $L$ is the total number of search locations and $K$ is the total number of positions between locations (e.g., if a route takes 4 periods to travel, there is a pickup-location $i$, 2 in-between positions, and a drop-off location $j$). $m^t$ is the $(L + K) \times 1$ vector of matches in period $t$, where the first $L$ entries are the matches in each location and the next $K$ entries are zeros (as no matches occur while cabs are
employed and in-transit. $\mathbf{M}_e^t$ be the $(L+K) \times (L+K)$ vector of one-period transition probabilities of customers from all locations $\{1,...,L\}$ and all in-between positions $\{1,...,K\}$. The number of in-between positions is based on the mean number of periods it takes to travel from any locations $i$ to $j$, rounded to the nearest period (e.g., an average 16-minute trip would be considered 3.2 periods, and then rounded to be 3 periods, with a single in-between position). $\mathbf{m}^{t-\tau_{ji}}$ describes how many drop-offs will occur in period $t$, which is the number of matches made in each pick-up location in $\tau_{ji}$ prior periods, and transition matrix $\mathbf{M}_e^{t-\tau_{ji}}$ re-distributes those earlier matches to locations at time $t$.

Given these objects, we can write the state transitions of employed cabs as follows, reflecting the transitions of new matches and already-employed taxis at time $t$, minus the time $t$ drop-offs:

$$
\mathbf{v}_{e}^{t+1} = ((\mathbf{v}_{e}^t + \mathbf{m}^t) \times \mathbf{M}_e^t) - (\mathbf{m}^{t-\tau_{ji}} \times \mathbf{M}_e^{t-\tau_{ji}}).
$$

(14)

Next, I define the state transitions of vacant taxis. Let $\mathbf{v}_v^t$ be the $(L + K) \times 1$ vector of vacant taxis in all search locations and in-between locations $\{1,...,(L+K)\}$. Note that there may be taxis in the in-between locations. For example, driving vacant to the airport may take more than one period. Let $\mathbf{v}_v^t$ be the $(L + K) \times (L + K)$ vector of one-period transition probabilities of vacant taxis from all locations $\{1,...,L\}$ and all in-between positions $\{1,...,K\}$. Then the state transitions of vacant cabs is given by the vector of vacant cabs at the start of period $t$ minus the period $t$ matches, multiplied by the policy functions in each period:

$$
\mathbf{v}_v^{t+1} = (\mathbf{v}_v^t - \mathbf{m}^t) \times \mathbf{\sigma}^t.
$$

(15)

Summing these two transition formulas defines the state transitions from $t$ to $t + 1$.

### A.6 Taxi Equilibrium Algorithm Details

The algorithm that I implement takes as inputs all model primitives, parameters, and a time zero state, and returns the equilibrium state and policy functions for each location and each time period. Equilibrium states constitute a $L \times T$ matrix (i.e., how many taxis are in each location in each period), and equilibrium policy functions constitute a $L \times L \times T$ matrix (i.e., the probability of vacant taxi transition from any location $i \in \{1,...,L\}$ to any location $j \in \{1,...,L\}$ in each period). Broadly, the algorithm uses backwards iteration to solve for continuation values and forward simulation to generate transition paths. The algorithm moves in an alternating, asymmetric backwards and forwards sequence through the current time step $t \in \{1,...,T\}$, where backwards moves update continuation values and forwards moves update transition paths. The algorithm terminates when all transition paths and continuation values are self-fulling and consistent with equilibrium. Below I provide an outline of the taxi equilibrium algorithm.
Algorithm 1 Taxi Equilibrium Algorithm

1: Load empirical matches \( \{ \tilde{m}_{ij}^t \} \) and \( \{ \tilde{m}_i^t \} = \{ \sum_j \tilde{m}_{ij}^t \} \)
2: Fix parameters \( \sigma, \varepsilon \) (solved outside of algorithm)
3: Set counter \( k = 0 \)
4: Guess \( S_0^T \) and compute \( V^T(S_0^T, \lambda^T) \)
5: for \( t = T - 1 \) to 1 do \( \triangleright \) Backwards Iteration
6: Guess \( S_0^t \) and compute \( V^t(S_0^t, \lambda^t) \)
7: for \( t = 1 \) to \( T - 1 \) do \( \triangleright \) Fwd. Iteration to \( T \) for each step back
8: Derive choice-specific value functions \( W_i^t(j, S^t) \) for all \( t, i, j \).
9: Find policy fcts. \( \sigma_k^t(W_{k+1}^t) \) to determine vacant taxi transitions
10: \( \sigma_0^t \) and \( m_{ij}^t \) imply transition to \( \tilde{S}^{t+1} \)
11: Update next period state \( S_{k+1}^{t+1} \leftarrow \tilde{S}^{t+1} \)
12: Update next period continuation values as \( V^{t+1}(S_{k+1}^{t+1}, \tilde{m}^t) \)
13: \( k \leftarrow k + 1 \)
14: end for
15: end for
16: repeat
17: Iterate on steps 6 to 15
18: until \( |V_k^T - V_{k-1}^T| \leq \varepsilon \) \( \forall t \)

The Taxi Equilibrium Algorithm begins with an initial guess of the market state \( S_0 \) (i.e., the number of all vacant taxis across locations and time-of-day).\(^{41}\) With \( S_0 \) as well as observations of the empirical distributions of taxi-passenger matches, \( \tilde{m} \), I compute value functions \( V_i^t(S_0; \tilde{m}) \) for each \( i \) and \( t \) via backwards induction, beginning at period \( T \) and stepping backwards to period 1, updating continuation values in each step. Next, using the value functions, I compute choice-specific value functions and optimal policies as in equation 9. Next, I use the computed policy functions and, starting at time \( t = 1 \) at \( S_0^1 \), I forward simulate the optimal transition paths and update the initial state for \( t = 2, \ldots, T \), resulting in a new guess of the state, \( S_1 \). With \( S_1 \), I again combine the same observations \( \tilde{m} \) to update value functions \( V_i^t(S_1; \tilde{m}) \). This process repeats until value and policy functions converge.

A.6.1 Initial conditions

Recall that \( S_0^t \) is a state vector of the number of vacant taxis in each location at time \( t \). The initial guess of the state in each period, \( S_0^0 \), is assigned by allocating the exogenous total number of taxis according to the empirical distribution of matches.\(^{42}\) As the algorithm runs, each vector \( S_0^t \) for

\(^{41}\)Note that the algorithm will take as given the parameters \( \gamma \) and \( \sigma, \varepsilon \). These are solved for in a second stage, as an outer loop to the algorithm.

\(^{42}\)Recall the total number of taxis equals 11,500, as discussed in Section 4.1.
Table A3: Alternative Initial Conditions

<table>
<thead>
<tr>
<th>Initial Condition</th>
<th>$\Delta v^t_i$ (mean)</th>
<th>$% \Delta v^t_i$ (mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Uniform</td>
<td>2.10</td>
<td>0.0175</td>
</tr>
<tr>
<td>Edge</td>
<td>4.52</td>
<td>0.0336</td>
</tr>
</tbody>
</table>

This table shows the change in taxis’ spatial equilibrium distribution given changes in initial conditions. Baseline is the initial condition used throughout the paper, as described above. Uniform imposes an initial distribution that is uniform across all locations at 6am. Edge imposes an initial distribution that uniformly puts all vacant taxis across edge locations: all peripheral locations with adjacent access to the outer boroughs and New Jersey.

$t \geq 2$ is updated as $t - 1$ transitions are computed given the $t - 1$ initial state and value functions for $t, t + 1, \ldots, T$. Only one term, $S_0^t$, remains exogenously chosen.

To mitigate any issues related to this remaining first-period exogenous initial state, I define $t = 1$ as 6:00am. In this period, the assumption that all available cabs are actively searching or with customers is less credible. Nevertheless, by starting the equilibrium algorithm at 6:00am, a wide range of initial conditions quickly wash out within 5-6 periods. This is verified by setting alternative initial conditions and comparing equilibrium levels of taxi supply across locations. By the intended starting time for estimation, of 7:00am, then, the spatial distribution of taxis is in equilibrium. Table A3 shows the impact of initial conditions on the equilibrium supply of taxis under increasingly heterogeneous starting points. The baseline case, as described above, is compared with (1) a uniform initial distribution and (2) a distribution in which all initial vacant cabs are distributed at edge locations: those locations adjacent to the boundaries of the map. The latter edge distribution is meant to simulate the case in which taxis start the day by driving from garages where they are stored. The locations of these garages are not available in my data, so this condition serves as an extreme case in which all taxis are stored in outer boroughs.

A.7 Proofs

Proof of Proposition 3.1

Part (i): Period profits are concave in actions. Therefore, best response mappings are single-valued.

Proof. This proof largely follows the argument for uniqueness in Mean Field Games outlined in [Light and Weintraub (2018)](LightAndWeintraub2018). To begin, the authors provide a set of conditions under which best

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43Recall that the data do not allow for distinguishing whether fewer matches in the morning are due to low supply or demand; and thus it is impossible to say how many cabs are actually on the road at any point.

44Boundary locations are all peripheral locations with adjacent access to the outer boroughs and New Jersey. This includes all locations in Manhattan with bridges and those bordering 125th street, all Brooklyn and Queens locations, and each Airport.
response mappings are single-valued. The main step is to demonstrate strict concavity of period payoff functions with respect to probabilities over actions $\sigma_t$. To show this, I first re-write the choice-specific value functions (8). For exposition it will be convenient to re-write current location $i$ and choice-location $k$ as arguments instead of subscripts.

$$W^t(i, k, S) = p(k, S) \cdot \bar{\pi} + (1 - p(k, S)) \cdot (-c(i, k)) + \epsilon(a, k) +$$

$$\left( p(k, S) \cdot V^{t+\tau(k,k')}(k', S) + (1 - p(k, S)) \cdot \arg \max_{\ell \in A(k)} W^{t+\tau(k,\ell)}(k, \ell, S) \right), \quad (16)$$

Where $\bar{\pi}(k) = \sum_j M^t_{kj} \Pi_{kj}$. The above expression explicitly separates period payoff functions from continuation values. To demonstrate concavity of the former, denote period payoffs as follows:

$$F(i, k, S) = p(k, S) \cdot \bar{\pi}(k) + (1 - p(k, S)) \cdot (-c(i, k)) + \epsilon(a, k).$$

Concavity implies that for $\gamma \in (0, 1)$ and $k, k' \in A(i)$, that

$$F(i, \gamma k + (1 - \gamma)k', S) > \gamma F(i, k, S) + (1 - \gamma) F(i, k', S).$$

Since profits $\bar{\pi}(k) > 0$ and fuel costs $-c(i, j) < 0$, and since location choice $k$ only indexes a discrete point of $\bar{\pi}, c(i, k)$, and $\epsilon(a, k)$, it is sufficient to show that $p(k, S)$ is concave. Let $v(k)$ be the $k$-th element of the state $S$ and $\hat{\lambda}(k)$ an exogenous parameter for location $k$. Then

$$p(k, S) = 1 - \exp \left( \frac{\hat{\lambda}(k)}{v(k)} \right).$$

Thus, concavity of $p(k, S)$ follows from the convexity of $-p(\cdot, \cdot)$ via the convexity of $\exp(\cdot)$.

Remaining conditions for single-valued best-response mappings are straight-forward to satisfy; the constraint correspondence $\Gamma(A(i))$ is the convex set of mixed strategies over the set of feasible locations accessible from location $i$, and the transition function is weakly convex as a linear function of $\sigma^t$. Thus by Lemma 1 of [Light and Weintraub 2018], best response mappings are single-valued.

$\Box$

**Part (ii):** Let $\sigma \equiv \sigma(k)$ be the action that assigns probability to choosing location $k$. Then period profits exhibit decreasing differences in $(\sigma, v)$.

**Proof.** Return to the function $F(i, k, S)$ above. Let $v$ denote the $i$-th element $v(i) \in S$. To show that $F$ has decreasing differences in $(\sigma, v)$, let $\sigma' > \sigma$ and $v' > v$, first note that $v$ does not enter $F$ except through $p(k, S)$. Re-writing $F(i, k, S) = p(k, S) \cdot \Theta + g(\sigma, i, k)$ for some $\Theta > 0$ we only
need to show that $p(\cdot, \cdot)$ has decreasing differences on $(\sigma, v)$. Thus for some $k \in \{1, \ldots, L\}$ and some $\hat{\lambda}(k) \in \mathbb{R}$, we need to show that

$$\sigma' \left( 1 - \exp \left( \frac{\hat{\lambda}(k)}{v'(k)} \right) \right) - \sigma \left( 1 - \exp \left( \frac{\hat{\lambda}(k)}{v(k)} \right) \right) > \sigma' \left( 1 - \exp \left( \frac{\hat{\lambda}(k)}{v'(k)} \right) \right) - \sigma \left( 1 - \exp \left( \frac{\hat{\lambda}(k)}{v(k)} \right) \right).$$

This condition is satisfied, as $v'(k) > v(k)$ implies that $- \exp \left( \frac{\hat{\lambda}(k)}{v'(k)} \right) > - \exp \left( \frac{\hat{\lambda}(k)}{v(k)} \right)$.

### A.8 Estimation and Simulated Moments

I identify $\sigma_\varepsilon$ by simulating individual taxi trip data and comparing simulation moments with their empirical counterparts. The moments are as follows: (1) Mean total vacancy times per taxi (2) Mean total distance travelled with passengers, (3) the probability that a driver’s next match is in the same location as his most recent drop-off and (4)-(7) the average probability of matching with a customer in each of sections I-IV, where sections are defined in Figure 1. Note that these moments will depend at least in part on these parameters; $\sigma_\varepsilon$ reflects how much of a taxi driver’s location choice depends on observable features within the model. A high value of $\sigma_\varepsilon$ should lead to behavior that appears random from the perspective of the model, including longer vacancy periods, whereas a low value implies that the model is capturing incentives well, and thus behavior should conform to the model’s valuation of locations. Empirical moments of these probabilities are recorded as the percentile associated with the distribution of waiting times in each location being equal to five minutes, the length of one period.\textsuperscript{45} Table A4 displays each simulation moment compared with its observed value.

### A.9 Relative Variances of $v_t^i$ and $u_t^i$

Equation 11 is derived under the assumption that the variance of $v_t^i$ is negligible as a determinant of $\alpha$. This subsection provides additional support for this assumption by highlighting two sets of equilibrium moments that are independent of $\alpha$. First, across days, the variance in the equilibrium level of taxis in this model is solely driven by variance in the arrival of customers each day, which induces variance in taxi-customer matches as well as variance in the number of vacant taxis left to search. Below I provide evidence that this variance is small relative to that of demand. Second, I will show that generally large levels of supply relative to demand are present in equilibrium,\textsuperscript{45} For example, if half of all taxis in a particular location matched with a passenger within 2.5 minutes, then the probability of matching within a period would equal 50%.

\textsuperscript{49}
Table A4: Data and Simulation Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data Average</th>
<th>Simulation Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Vacant Wait Time (min.)</td>
<td>155.21</td>
<td>184.95</td>
</tr>
<tr>
<td>Total Employed Distance Travelled (mi.)</td>
<td>38.37</td>
<td>35.74</td>
</tr>
<tr>
<td>$\Pr(\text{pickup in } i \mid \text{drop-off in } i)$</td>
<td>.3714</td>
<td>.3809</td>
</tr>
<tr>
<td>Match Probability (Section 1)</td>
<td>.4205</td>
<td>.3809</td>
</tr>
<tr>
<td>Match Probability (Section 2)</td>
<td>.4912</td>
<td>.4543</td>
</tr>
<tr>
<td>Match Probability (Section 3)</td>
<td>.3842</td>
<td>.2214</td>
</tr>
<tr>
<td>Match Probability (Section 4)</td>
<td>.2477</td>
<td>.4735</td>
</tr>
</tbody>
</table>

This table summarizes the data and simulation moments used to estimate remaining model parameters.

implying that any variance in supply will have a comparatively small impact on the variance of matches.

The level of taxis in each location and time in each day is correlated with the levels in all other locations and previous times of that same day, and likewise correlated with the draws of $u_{ti}^t$ in all previous times of that same day. An exact result for taxi variance would require a large simulation and lack an interpretable analytical solution. Instead I will provide evidence that the variance in $v_{ti}^t$ is small relative to that of $u_{ti}^t$ by computing an upper-bound on the variance of incoming taxis in each location attributable to past draws of $u_{ti}^t$. This is done by fixing $\text{Var}(m_{ti}^t) = \text{Var}(u_{ti}^t)$, an overestimate of match variance, and then computing the variance of incoming matches to each location and time, $\text{Var}(v_{ti,\text{matches}})$ plus the variance of incoming searchers, $\text{Var}(v_{ti,\text{search}})$.

Note that $\text{Var}(u_{ij}^t) = (M_{ij}^t)^2 \cdot \text{Var}(u_{ti}^t)$ and $\text{Var}(u_{ti}^t) = \lambda_{ti}^t$. If matches have $\text{Var}(u_{ij}^t)$, then

$$\text{Var}(v_{ti,\text{matches}}) = \sum_j \text{Var}(u_{ij}^{t-\tau_{ij}}) = \sum_j (M_{ij}^{t-\tau_{ij}})^2 \lambda_{ti}^{t-\tau_{ij}},$$

and

$$\text{Var}(v_{ti,\text{search}}) = \sum_j (\sigma_{ij}^t)^2 \cdot \text{Var}(1 - u_{i}^{t-\tau_{ij}}) = \sum_j (\sigma_{ij}^{t-\tau_{ij}})^2 \lambda_{ti}^{t-\tau_{ij}}.$$

Thus,

$$\text{Var}(v_{ti}^t) = \text{Var}(v_{ti,\text{matches}}) + \text{Var}(v_{ti,\text{search}}) - 2 \sum_j (M_{ij}^{t-\tau_{ij}})^2 (\sigma_{ij}^{t-\tau_{ij}})^2$$

is an upper-bound measure of taxi variance attributable to demand. I compute this number and divide by the variance of demand $\hat{\lambda}_{ti}^t$ in all non-airport locations from 7a-4p. Figure A3 Panel 1 shows the mean variances for all locations within each region. The mean taxi variance as a fraction

50
of demand variance is 0.0308, or 3.1%.

Figure A3, Panel 2 shows the equilibrium ratio of supply to demand across regions. While there are many reversals (see detailed estimates in Figure A6), the average ratio of taxis-to-customers across regions is between 3-10 throughout most of the day. Since the more constrained input to the matching function will exert a proportionally greater influence on matches (see, e.g., Figure 4), this further diminishes the role of supply variance on the realized variance of matches each day.

Together these facts suggest that the variance of taxis can be regarded as negligible with respect to their impact on the overall variance of matches used to identify the efficiency parameter $\alpha$.

A.10 Details on Estimating $\alpha_r$

Given parameter values $\sigma_\varepsilon$ and $\gamma$, I use matches date $\{\tilde{m}\}$ and the TEA procedure to numerically solve for the equilibrium state, $S = \{v_{ti}^*(\tilde{m})\}$. Proposition 4.1 shows that, given a level of taxis $v_{ti}$, the matching function can be inverted to solve for $\lambda_{ti}^{\alpha_r}$ in location $i$ (within region $r$) and time $t$. Next, I use an analytic expression of the variance of matches, given by equation 11. This function depends on both $\alpha_r$ as well as the ratio $\lambda_{ti}^{\alpha_r}$. From here I set up the following estimator:

$$\alpha_r = \arg\min_\alpha \sum_{i \in \mathcal{R}_r,t} \left( Var_d(\tilde{m}_{i,d}^{t}) - \left( v_{ti}^* \right)^2 e^{-2 \frac{\lambda_{ti}^{\alpha_r}}{\alpha_r} \frac{1}{v_{ti}^{\alpha_r}}} \left( \frac{\lambda_{ti}^{\alpha_r}}{\alpha_r} \frac{1}{v_{ti}^{\alpha_r}} - 1 \right) \right), \quad (17)$$

where $\mathcal{R}_r$ denotes the set of locations within region $r$, and $\tilde{m}_{i,d}^{t}$ refers to the number of observed taxi-passenger matches that take place in location $i$, time $t$, and day-of-month $d$. The variance is then taken with respect to all observations within the weekdays in a given month, across days of the month.
Figure A4: Equilibrium Vacant Taxis: Weekdays 7a-4p, 9/2012 (Five Region Aggregates)

This figure depicts the equilibrium spatial distribution of taxis and mean arrival of customers across the Five Regions shown in Figure 1. Results across all 39 locations are summed to these five areas. Results are depicted for the weekday taxi drivers’ day shift, from 7a-4p in September 2012.

A.11 Detailed Estimation Results

Figure A4 shows aggregate supply and demand results, summing all 39 locations into the five regions corresponding to Figure 1. The results above demonstrate that while taxi supply maintains some coverage across all locations throughout the day, there are intra-day trends in spatial availability and demand. Spatial mismatch is evident, as the relative proportions of supply and demand are not the same across each region.

Figures A6 and A7 show detailed results of supply and demand in all locations. Note that location numbers 1-34 roughly track from South to North in Manhattan, locations 35-37 track South to North from Brooklyn to Queens, location 38 is LaGuardia airport and location 39 is JFK airport. We see that most locations have a surplus of taxis except for a few areas of very high demand. Lower Manhattan, parts of midtown Manhattan and far North-east Manhattan all demonstrate particularly large constraints in the ratio of vacant taxis to demand. All locations demonstrate some search frictions on both sides of the market, but we see here that the impact is felt more on the taxi side.

Figure A8 shows the evolution of Value functions by time of day. Each series is the value for a single location. The high correlation between each value function reflects the equilibrium result that drivers’ policy functions ensure that there is no spatial arbitrage possible. The remaining differences between each location’s value is due to the transportation cost that prevents perfect cross-location arbitrage. As the day reaches its 4pm end, the value of search in each location
A.12 Welfare Calculation

Consumer welfare is computed by integrating under the estimated CES demand curves in each origin, destination, time pair (i.e., each \(i, j, t\)). The integral can be computed analytically as follows:

\[
\text{Consumer welfare} = \int \text{CES demand curve} \, dt
\]

systematically drops to zero.
Figure A8: Equilibrium Value Functions

This figure depicts the equilibrium value functions for all 39 locations, by time of day, estimated from August 2012 data. Each line depicts a separate location. The highest-valued function is that of LGA airport and the least-valued function is that of JFK airport. All other locations’ values fall in-between.

\[
W_{ijt}(m_{ijt}^t, \hat{\lambda}_{ijt}, p_{ij}, \beta) = \frac{m_{ijt}^t(\hat{\lambda}_{ijt}, v_{ijt}(\hat{\lambda}))}{\lambda_{ij}^t(p_{ij})} \cdot \left( \frac{\alpha_{1,s,a}}{\alpha_{1,s,a} + 1} \cdot e^{\frac{\alpha_0,i,t,s,a}{\alpha_{1,s,a}} \cdot \hat{\lambda}_{ijt}(p_{ij})^{\alpha_{1,s,a}^{-1}} + 1 - \hat{\lambda}_{ijt}(p_{ij}) \cdot p_{ij}} \right),
\]

(18)

where \(\alpha_{0,i,t,s,a}\) and \(\alpha_{1,s,a}\) are the estimated demand parameters, \(\hat{\lambda}_{ijt}\) is the predicted level of demand given price \(p_{ij}\), and \(v_{ijt}(\hat{\lambda})\) is the equilibrium mass of taxis in each location (a function of the distribution of demand across locations and time). In counterfactuals I incorporate waiting time elasticities into this calculation. To do this I augment Equation 14 with waiting times:

\[
\ln(\lambda_{ij}^t(P_{ij})) = \alpha_{0,i,t,s,a} + \alpha_{1,s,a} \ln(P_{ij}) + \alpha_{2,i,t} \Delta w_{i,j,t} + \eta_{i,t,s,a},
\]

(19)

where \(\Delta w_{i,j,t}\) is computed as the percentage change in the consumer waiting time measure between the baseline case and the counterfactual waiting time evaluated as if there were no waiting time elasticity. Thus, to create the variable \(\Delta w_{i,j,t}\) I need to predict demand and compute equilibrium once with no waiting time elasticity, generate the predicted change in waiting times, and then re-compute demand with the calibrated waiting time elasticity \(\alpha_{2,i,t} = -1\) and again re-compute equilibrium. After the second round is complete, I evaluate welfare. This is a computationally intensive process, so I do not conduct additional iterations. However, after many trials adding one
additional iteration the differences in outcomes seem to be very small.

Finally taxi profits are computed as follows:

$$W_{ijt}^{\text{taxi}}(m_{ijt}, \hat{\lambda}_{ijt}, p_{ij}, c_{ij}) = \frac{m_{ijt}(\hat{\lambda}_{ijt}, v_{ij}(\hat{\lambda}))}{\hat{\lambda}_{ijt}(p_{ij})} \left( \frac{\hat{\lambda}_{ijt}(p_{ij}) : (p_{ij} - c_{ij})}{\text{trip revenues at price } p_{ij}} \right),$$

where $c_{ij}$ is the fuel cost for a trip from $i$ to $j$. 

$$55$$