Spatial Equilibrium, Search Frictions and Dynamic Efficiency in the Taxi Industry

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Abstract

This paper analyzes the dynamic spatial equilibrium of taxicabs and shows how common taxi regulations lead to substantial inefficiencies as a result of search frictions and misallocation. To analyze the role of regulation on frictions and efficiency, I pose a dynamic model of spatial search and matching between taxis and passengers. Using a comprehensive dataset of New York City yellow medallion taxis, I use this model to compute the equilibrium spatial distribution of vacant taxis and estimate intraday demand given price and medallion regulations. My estimates show that the weekday New York market achieves about $5.7 million in daily welfare or about $25 per trip, but an additional 53 thousand customers fail to find cabs due to search frictions. Counterfactual analysis shows that implementing simple tariff pricing changes can enhance allocative efficiency and expand the market, offering daily net surplus gains of up to $460 thousand and 65 thousand additional daily taxi-passenger matches, a similar magnitude to the gains generated by adopting a perfect static matching technology.

Key Words: dynamic games, spatial equilibrium, search frictions, dynamic pricing, regulation, taxi industry

JEL classification: C73; D83; L90; R12

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1 Introduction

It has been well documented that search frictions lead to less efficient outcomes. One particularly salient reason for the existence of search frictions is that buyers and sellers are spatially distributed across a city or region, so that meeting to trade requires costly transportation by one or both sides of the market. When locations are fixed, say between households and potential employers, search frictions arise from the added cost of travel associated with meeting. In some spatial settings, however, every trade involves a future re-allocation of buyers or sellers. This is a prominent feature of transportation markets, where every trade entails a vehicle moving from one place to another. When transportation and search intersect, dynamic search externalities arise as each trip affects the search frictions faced by future buyers and sellers at each destination. In this paper I study the regulated taxicab industry in New York City, where a decentralized search process and a uniform tariff leads to distortions in the intra-daily equilibrium spatial patterns of supply and demand. I ask how much spatial misallocation is induced by search externalities in this setting and to what extent simple changes to pricing regulations can enhance allocative efficiency over time.

The taxicab industry is a critical component of the transportation infrastructure in large urban areas, generating about $23 billion in annual revenues. New York City has long been the largest taxicab market in the United States, accounting for about 25% of national industry revenues in 2013. In New York and many other cities, the taxi market is distinguished from other public transit options by a lack of centralized control; taxi drivers do not service established routes or coordinate search behavior. Instead, drivers search for passengers and, once matched, move them to destinations. Since different types of trips are demanded in different areas of the city, how taxi drivers search for passengers directly impacts the subsequent availability of service across the city. These movements of capacity give rise to equilibrium patterns that can leave some areas with little to no service while in other areas empty taxis will wait in long queues for passengers.

In this paper, I model taxi drivers’ location choices in a dynamic spatial search framework in which vacant drivers choose where to locate given both the time-of-day pattern of trip demand as well as the distribution of rival taxi drivers throughout the day. While the spatial search process under current regulations often generates mis-allocation across locations, I also model frictions within each location to account for a block-by-block search process within small windows of time.

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1 Models of search and equilibrium have been widely studied. Since the pioneering work of Diamond (1981), Mortensen (1982a,b) and Pissarides (1984, 1985), the search and matching literature has focused on the role of search frictions in impeding the efficient clearing of markets. The search and matching literature examines many markets where central or standardized exchange is not possible, including labor markets (e.g., Rogerson, Shimer, and Wright (2005)), marriage markets (e.g., Mortensen (1988)), monetary exchange (e.g., Kiyotaki and Wright (1989, 1993)), and financial markets (e.g., Duffie, Gärleanu, and Pedersen (2002, 2005)).

2 This value is based on my own calculation combining data from the NYC Taxi and Limousine Commission and a national industry report Brennan (2014).
I show that spatial frictions are largely attributable to inefficient pricing, as tariff-based prices fail to account for driver opportunity costs and the heterogeneity in consumer surplus that is not internalized by drivers. To empirically analyze this model, I use data from the New York City Taxi and Limousine Commission (TLC), which provides trip details including the time, location, and fare paid for all 27 million taxi rides in New York between August and September of 2012. Using TLC data together with a model of taxi search and matching, I estimate the spatial and intra-daily distribution of supply and demand in equilibrium. Importantly, the data only reveal matches made between taxis and customers as a consequence of search activity, but do not show underlying supply or demand; I therefore cannot observe the locations of vacant taxis or the number of customers who want a ride in different areas of the city. Because these objects are necessary for measuring search frictions and welfare in the market, I develop an estimation strategy using the dynamic spatial equilibrium model together with a local matching function. I show that the observed distribution of taxi-passenger matches is sufficient to solve for drivers’ policy functions and compute the equilibrium distribution of vacant taxis without direct knowledge of demand. I then invert each local matching function to recover the implied distribution of customer demand up to an efficiency parameter. Finally I estimate matching efficiency using moments related to the variance of matches across days of the month.

I use this model to evaluate welfare and search frictions in the New York taxi market. Baseline estimates of welfare indicate that the New York taxi industry generates $2.2 million in consumer surplus and $3.4 million in taxi driver variable profits during each 9-hour day-shift and across 216 thousand taxi-passenger matches, implying a combined surplus of about $25 per trip. Despite these surpluses, however, there are on average 53 thousand failed customer searches per day and 5,756 vacant drivers at any point in the day. To what extent can a more sophisticated pricing policy mitigate these costs by better allocating available supply to demand? By simulating market equilibrium over nearly one million potential pricing rules, I am able to solve for a dynamically optimal flexible fare structure and show that a flexible tariff that changes with time-of-day can provide up to a 21% increase in consumer welfare and a 10% improvement in taxi utilization. These results utilize an estimated demand system that incorporates an elasticity of waiting time calibrated from recent work. Alternative policies offering flexible tariffs by location and distance yield slightly smaller benefits to consumers in favor of driver profits and higher utilization rates, but all of the counterfactual policies tested offer unambiguous benefits to both sides of the market even after accounting for search and matching frictions. I contrast these results with a counterfactual simulation of ride-sharing technology that offers frictionless within-location matching, and show that optimal pricing policies can produce nearly the same number of trips as the matching technology

alone and deliver about 60% of the welfare gains.

Related Literature

This paper integrates ideas from the search and matching literature with empirical industry dynamics. The key component is a model of dynamic spatial choices that adapts elements from Lagos (2000). Lagos (2000) studies endogenous search frictions using a stylized environment of taxi search and competition. His model predicts how meeting probabilities adjust to clear the market and how misallocation can occur as an equilibrium outcome. Lagos (2003) uses the Lagos (2000) model to empirically analyze the effect of taxi fares and medallion counts on matching rates and medallion prices in Manhattan. I draw elements from the Lagos search model, but make several changes to reflect the real-world search and matching process. Specifically, I add non-stationary dynamics, a more realistic and flexible spatial structure, stochastic and price-sensitive demand, fuel costs, and heterogeneity in the matching process across different locations. Further, I build a tractable framework for the empirical analysis of dynamic spatial equilibrium by providing tools for estimating and identifying the model. I also model a static, localized market clearing process via an aggregate matching function. [Hall (1979)] introduces the aggregate matching function concept, using the urn-ball specification adapted in this paper. In recent work [Brancaccio, Kalouptsidi, and Papageorgiou (2019a,b)] study the estimation and identification of matching functions in spatial settings and apply a related search model to study endogenous trade costs in the bulk shipping industry.

I also draw on literature for estimating dynamic models in the tradition of Hopenhayn (1992) and Ericson and Pakes (1995), which characterize Markov-perfect equilibria in entry, exit, and investment choices given some uncertainty in the evolution of the states of firms and their competitors. Here, each taxi operates as a firm that is optimizing where to search in a city. The state variable is the distribution of taxi locations. To facilitate computation, I make a large-market assumption that both taxi drivers and customers are non-atomic. As in Hopenhayn (1992) this allows me to compute deterministic state transitions without integrating over a high-dimensional space of states and future periods. The mass of customers in each location varies from day to day in each location and period. Drivers do not condition on these shocks, which I assume are not observed by individual drivers, but rather the expectation of consumer demand. The equilibrium is therefore similar to an Oblivious Equilibrium (Weintraub, Benkard, and Van Roy (2008b)) in which drivers form their policies with respect to averages taken across many days in the market. This

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4 Mortensen (1986), Mortensen and Pissarides (1999) and Rogerson, Shimer, and Wright (2005) survey the labor-search literature and the implementation of aggregate matching functions.

5 There is also a literature in empirical industrial organization which studies the allocative distortions induced by search frictions in different industries. This includes work on airline parts (Gavazza 2011) and mortgages (Allen, Clark, and Houde 2014).
This notion is also similar to an Experience-Based Equilibrium (Fershtman and Pakes (2012)) in which firms’ information set is restricted and agents condition their strategies on repeated experiences with market outcomes.\footnote{This approach also relates to auction models with many bidders (e.g., Hong and Shum (2010)) and as an empirical exercise in studying non-stationary firm dynamics (e.g., Weintraub, Benkard, Jezioriski, and Van Roy (2008a), Melitz and James (2007)).}

This is the first empirical analysis of pricing and welfare in a taxi market and the first to study how price regulations impact the spatial allocation of service. A related study is Frechette, Lizzeri, and Salz (2019), which models the dynamic entry game among taxi drivers to ask how customer waiting times and welfare are impacted by medallion regulations and dispatch technology. Similar to my paper, Frechette, Lizzeri, and Salz (2019) study the effect of regulations on search frictions and welfare. The key difference is that they focus on the labor supply decision rather than the spatial location decision.\footnote{There is an additional body of literature on taxi drivers’ labor supply choices, including Camerer, Babcock, Loewenstein, and Thaler (1997), Farber (2005, 2008), Crawford and Meng (2011), and Thakral and Tol (2017). These studies investigate the labor-leisure tradeoff for drivers. They ask how taxi drivers’ labor supply is determined and to what extent it is driven by daily wage targets and other factors. Buchholz, Shum, and Xu (2017) estimate a dynamic labor supply model of taxi drivers to show that behavior consistent with dynamic optimization may appear as a behavioral bias in a static setting.}

Though these research questions and approaches differ substantially, they lead to similar predictions when comparing similar counterfactuals.

There is a recent literature on the benefits of dynamic pricing for ride-hail services (e.g., Hall, Kendrick, and Nosko (2015), Castillo, Knoepfle, and Weyl (2017), Castillo (2019)). This paper also highlights the impact of pricing on efficiency, but with two distinct differences. First, I focus on posted tariffs instead of real-time price adjustment. Posted tariffs are a feature of both traditional taxis and ride-hail services that affect the search behavior of taxi drivers. Second, I explicitly model the influence of prices on the dynamic path of supply and demand. I use this model to show how prices can be configured to induce efficient allocations of supply and demand while accounting for the flow of reallocated of cabs due to passenger trips.

Finally, a diverse literature addresses whether taxi regulation is necessary at all. In this literature, both the theoretical and empirical findings offer mixed evidence. These studies point to regulation’s ability to reduce transaction costs (Gallick and Sisk (1987)), prevent localized monopolies (Cairns and Liston-Heyes (1996)), correct for negative externalities (Schrieber (1975)), and establish efficient quantities of vacant cabs (Flath (2006)). Other authors assert that regulations restricted quantities and led to higher prices (Winston and Shirley (1998)) and that low sunk- and fixed-costs in this industry are sufficient to support competition (Häckner and Nyberg (1995)). My paper shows how existing regulatory levers are inefficient due to adverse static and dynamic consequences of mis-pricing, and that a better implementation of posted tariffs leads to more efficient spatial allocations and higher utilization.
In section 2 I detail taxi industry characteristics relating to search, regulation, and spatial sorting, as well as a description of the data. In section 3 I present a dynamic model of taxi search and matching. Section 4 outlines my empirical strategy for computing equilibrium and estimating model parameters. I present my results in section 5 and an analysis of counterfactual policies in section 6. Section 7 concludes.

2 Market Overview and Data

2.1 Regulatory Environment

As with nearly all major urban taxi markets, the New York taxi industry is highly regulated. Two regulations imposed by the New York Taxi and Limousine Commission (TLC) directly impact market function and efficiency. The first is a fixed two-part tariff fare pricing structure, where fares are based on a one-time flag-drop fee and a distance-based fee. Except for separate fares for some airport trips, this fare structure does not depend on location. Except for an evening flat-rate surcharge, fares do not depend on time of day. The second type of regulation is entry restrictions imposed via a limit on the number of legal taxis that can operate. This is implemented by requiring drivers to hold a “medallion” or permit, the supply of which is capped (Schaller (2007)).

Medallion cabs can only be hailed from the street and are not authorized to conduct pre-arranged pick-ups, a service exclusively granted to separately licensed livery cars.

In recent years, several ride-hail firms including Uber and Lyft have entered the taxi industry including the New York market. These firms operate a mobile platform to match customers with cabs, greatly reducing frictions associated with taxi search and availability. The precipitous expansion and popularity of ride-hail suggests there are large benefits associated with both the reduced search costs and more flexible pricing compared with traditional taxi markets. Another potential reason for this expansion is that taxi regulations are often at odds with this new wave of technology-centered entrants. These firms tend to enjoy much less stringent entry restrictions than the more regulated incumbents, leading to a variety of legal disputes as stakeholders in the traditional taxi business absorb losses.

These are high stakes disputes, and they highlight the need for analysis surrounding the effects of these new entrants. My paper aims to understand how regulation and matching technology impact the equilibrium spatial allocations of supply and demand as well as the corresponding impact on market welfare and efficiency.

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8 These licenses are tradable, and the fact that they tend to have positive value, sometimes in excess of one million dollars, implies that this quantity cap is binding and below the quantity that would be supplied in an unrestricted equilibrium.

9 See, e.g., forbes.com/sites/ellenhuett/2015/06/19/could-a-legal-ruling-instantly-wipe-out-uber-not-so-fast/.

10 The spatial availability of taxis is of evident concern to municipal regulators around the country: a number of
2.2 Data

In 2009, the New York TLC initiated the Taxi Passenger Enhancement Project, which mandated the use of upgraded metering and information technology in all New York medallion cabs. The technology includes the automated data collection of taxi trip and fare information. I use TLC trip data from all New York City medallion cab rides given from August 1, 2012 to September 30, 2012. An observation consists of information related to a single cab ride. Data include the exact time, date and GPS coordinates of pickup and drop-off, trip distance, and trip time length for approximately 27 million rides. New York cabs typically operate in two separate shifts of 9-12 hours each, with a mandatory shift change between 4–5pm. I focus on the weekday, day-shift period of 7am until 4pm and I assume all drivers stop working at 4pm.

Due to New York rules governing pre-arranged trips, the TLC data only record rides originating from street-hails. This provides an ideal setting for analyzing taxi search behavior since all observed rides are obtained through search. Table 1 provides summary facts for this data set. I provide additional monthly-level statistics in Appendix A.3.

Most of the time, New York taxis operate in Manhattan. When not providing rides within Manhattan, the most common origins and destinations are New York’s two city airports, LaGuardia (LGA) and John F. Kennedy (JFK). At the airports taxis form queues and wait in line for next available passengers. Table 2 provides statistics related to the frequency and revenue share of trips between Manhattan, the two city airports, and elsewhere.

Uber began operating in New York City in 2011, but service was minimal. In an October 2012 interview, the CEO reported that 160 drivers had provided trips in the city since the company’s entry into New York. This represents about 1% of licensed yellow cab drivers, and likely much less in trip volume as these drivers were not necessarily operating consistently throughout the prior year.

cities have introduced policies to control the spatial dimension of service. For example, in the wake of criticism over the availability of taxis in certain areas, New York City issued licenses for 6,000 additional medallion taxis in 2013 with special restrictions on the spatial areas they may service (See, e.g., cityroom.blogs.nytimes.com/2013/11/14/new-york-today-cabs-of-a-different-color/). Specifically, these green-painted “Boro Taxis” are only permitted to pick up passengers in the boroughs outside of Manhattan. Though the city’s traditional yellow taxis have always been able to operate in these areas, it’s apparent that service was scarce enough relative to demand that city regulators intervened by creating the Boro Taxi service. This intervention highlights the potential discord between regulated prices and the location choices made by taxi drivers.

Using this information together with geocoded coordinates, we might learn for example that cab medallion 1602 (a sample cab medallion, as the TLC data are anonymized) picks up a passenger at the corner of Bowery and Canal at 2:17pm of August 3rd, 2012, and then drives that passenger for 2.9 miles and drops her off at Park Ave and W. 42nd St. at 2:39pm, with a fare of $9.63, flat tax of $0.50, and no time-of-day surcharge or tolls, for a total cost of $10.13. Cab 1602 does not show up again in the data until his next passenger is contacted.

Table 1: Taxi Trip and Fare Summary Statistics

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rate Type</th>
<th>Variable</th>
<th>Obs.</th>
<th>10%ile</th>
<th>Mean</th>
<th>90%ile</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Data</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>27,475,614</td>
<td>5.40</td>
<td>9.51</td>
<td>16.00</td>
<td>5.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>27,475,621</td>
<td>1.50</td>
<td>5.59</td>
<td>12.00</td>
<td>6.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>27,475,621</td>
<td>2.50</td>
<td>2.83</td>
<td>3.50</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>27,475,621</td>
<td>0.82</td>
<td>2.70</td>
<td>6.00</td>
<td>2.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>27,475,621</td>
<td>4.00</td>
<td>12.04</td>
<td>22.52</td>
<td>8.23</td>
</tr>
<tr>
<td>JFK Fares</td>
<td></td>
<td>Total Fare ($)</td>
<td>491,689</td>
<td>45</td>
<td>48.32</td>
<td>52</td>
<td>3.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>491,689</td>
<td>3.02</td>
<td>16.25</td>
<td>20.58</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>491,689</td>
<td>22.75</td>
<td>45.65</td>
<td>70.00</td>
<td>19.16</td>
</tr>
<tr>
<td>Weekdays, Day-Shift, Manhattan &amp; Boro.</td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>8,164,678</td>
<td>4.50</td>
<td>10.17</td>
<td>17.70</td>
<td>6.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>8,122,794</td>
<td>1.20</td>
<td>4.66</td>
<td>9.60</td>
<td>5.33</td>
</tr>
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<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>8,122,794</td>
<td>2.50</td>
<td>2.50</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>8,122,794</td>
<td>0.71</td>
<td>2.28</td>
<td>4.67</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>8,122,794</td>
<td>4.00</td>
<td>12.74</td>
<td>23.80</td>
<td>8.49</td>
</tr>
<tr>
<td>JFK Fares</td>
<td></td>
<td>Total Fare ($)</td>
<td>171,223</td>
<td>45.00</td>
<td>48.28</td>
<td>52.00</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>171,223</td>
<td>2.00</td>
<td>16.14</td>
<td>20.91</td>
<td>6.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>171,223</td>
<td>26.18</td>
<td>45.65</td>
<td>67.00</td>
<td>19.16</td>
</tr>
</tbody>
</table>

Taxi trip and fare data come from the New York Taxi and Limousine Commission (TLC). This table provides statistics related to individual taxi trips taken in New York City between August 1, 2012 and September 30, 2012 for two fare types. The first is the standard metered fare (TLC rate code 1), in which standard fares apply, representing 98.1% of the data. The second is a trip to or from JFK airport (TLC rate code 2). Total Fare and Distance data are reported for each ride in the dataset. The two main fare components are a distance-based fare and a flag-drop fare. I predict these constituent parts of total fare using the prevailing fare structure on the day of travel and the distance travelled, though they are not separately reported either from each other or from waiting costs. Flag fare calculations include time-of-day surcharges. Any remaining fare is due to a charge for idling time. The first set of statistics corresponds to the full sample of all New York taxis rides across the two months, and the second set relates to the smaller sample used in my analysis: weekdays, day-shift trips occurring within the space described in Figure 1.

2.3 Discretizing time and space

To analyze time and geography, I discretize time and space across the weekday, day-shift hours in this market. Time is divided into five minutes periods. I divide space into 39 distinct areas that are linked to observed GPS points of origin and destination for each taxi trip. These locations represent 98% of all taxi ride originations, and I depict them in Figure 1. The average observed travel time from one location to a neighboring location is 2 minutes, 45 seconds, or about one-half of a five-minute period. This suggests that the 5-minute period is reasonably well-suited to this geographic partitioning. For additional details on location selection and construction see Appendix A.2.

I further denote five regions as disjoint subsets of all 39 locations. I depict regions as shaded sections of Figure 1. Each region is characterized by a unique mix of geographical features and
Table 2: Taxi Trips and Revenues by Area

<table>
<thead>
<tr>
<th>Time</th>
<th>Place</th>
<th>Obs.</th>
<th>Mean Fare</th>
<th>Trip Share</th>
<th>Rev. Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Times</td>
<td>Intra-Manhattan Trips</td>
<td>24,835,103</td>
<td>$9.28</td>
<td>89%</td>
<td>73%</td>
</tr>
<tr>
<td></td>
<td>Airport Trips</td>
<td>1,568,699</td>
<td>$33.77</td>
<td>6%</td>
<td>17%</td>
</tr>
<tr>
<td></td>
<td>Other Trips</td>
<td>1,563,501</td>
<td>$19.83</td>
<td>6%</td>
<td>10%</td>
</tr>
<tr>
<td>Weekdays, Day-shift</td>
<td>Intra-Manhattan Trips</td>
<td>7,813,226</td>
<td>$9.33</td>
<td>91%</td>
<td>76%</td>
</tr>
<tr>
<td></td>
<td>Airport Trips</td>
<td>503,711</td>
<td>$34.80</td>
<td>6%</td>
<td>18%</td>
</tr>
<tr>
<td></td>
<td>Other Trips</td>
<td>270,883</td>
<td>$19.62</td>
<td>3%</td>
<td>6%</td>
</tr>
</tbody>
</table>

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to the locations of taxi trips taken in New York City between August 1, 2012 and September 30, 2012. Intra-Manhattan denotes trips that begin and end within Manhattan, Airport Trips are trips with either an origin or destination at either LaGuardia or JFK airport, and Other Trips captures all other origins and destinations within New York City. Statistics are reported for all times as well as for the day-shift period of a weekday, from 6am until 4pm. I focus on weekday day-shifts in my analysis.

transit infrastructure. I will estimate the efficiency of search for each of these five regions. Region I is lower Manhattan, an older part of the city where streets follow irregular patterns, and where numerous bridges, tunnels and ferries connect to nearby boroughs and New Jersey. Region II is midtown Manhattan, with fewer traffic connections away from the island, but denser centers of activity including the major transit hubs Penn Station and Grand Central Station. Region III is uptown Manhattan, where streets follow a regular grid pattern, but are longer and more spread out. Few bridges, tunnels or stations offer direct connections to other boroughs. Region IV is the large area encompassing Brooklyn and Queens. Region V consists of the two airports, John F. Kennedy (JFK) and LaGuardia (LGA).

Table 3 summarizes how cabs move around space. Panel (a) aggregates all passenger trips into Regions and all times of day to display the density of employed-taxi transitions between regions. This matrix represents customer preferences for travel. At the end of a trip, taxis become vacant in these new regions. Panel (b) displays the observed location of a matched taxi, at the start of a ride, conditional on the last observed location of the same taxi, at the previous drop-off location. Thus Panel (b) reveals the transition of vacant cabs, though not accounting for period-by-period choices – only the eventual location of the next pickup. As I will show, an equilibrium estimate of drivers’ period-by-period spatial choices closely mirrors the pattern of Panel (b), but with higher frequency weights put on same-location transitions. The difference occurs because drivers search on average for 2.5 periods before finding a passenger. The ride-to-ride transitions are therefore more dispersed.
Each of the outlined sections of Manhattan is one of the 39 locations indexed by $i$ in my model. I create locations by aggregating census-tract boundaries, which broadly follow major thoroughfare divisions. I compute the expected travel time and distance between these locations separately for each origin and destination pair as the average of all observations within each $ij$ cell. Each shaded section depicts a region $r$, indicated with Roman numerals I–V. Regions are characterized by similarities in transit infrastructure, road layouts, and zoning.

### 2.4 Evidence of Frictions

Search frictions occur when drivers cannot locate passengers even though supply and demand coexist at the same point in time. Frictions in this market manifest as waiting time experienced by drivers looking for a passenger.\(^\text{14}\) The TLC data provide evidence of search frictions for drivers that vary across space and time of day. Using driver ID together with the time of pick-up and drop-off, I compute the waiting time between trips. The mean waiting time for different trips is displayed in Figure 2. Panel (a) shows the probability that a driver will find a passenger in each five-minute period, as well as the expected waiting time to find a passenger in 10-minute units (i.e., a value of 0.5 equals 5 minutes). There is substantial intra-day variation in search times, with the best times of day for finding passengers around 9am to 4pm, with average wait times around six minutes and

\(^{14}\)In a discrete-time sense, this means that after some interval of time some taxis will remain empty despite the presence of demand somewhere else in the market.
Table 3: Observed Pickup and Drop-off Activity by Region

<table>
<thead>
<tr>
<th>Origin</th>
<th>Region I</th>
<th>Region II</th>
<th>Region III</th>
<th>Region IV</th>
<th>Region V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td>0.3800</td>
<td>0.5005</td>
<td>0.0719</td>
<td>0.0215</td>
<td>0.0262</td>
</tr>
<tr>
<td>Region II</td>
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<td>0.6287</td>
<td>0.1681</td>
<td>0.0094</td>
<td>0.0348</td>
</tr>
<tr>
<td>Region III</td>
<td>0.0526</td>
<td>0.3770</td>
<td>0.5469</td>
<td>0.0061</td>
<td>0.0174</td>
</tr>
<tr>
<td>Region IV</td>
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<td>0.2706</td>
<td>0.1067</td>
<td>0.3866</td>
<td>0.0456</td>
</tr>
<tr>
<td>Region V</td>
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<td>0.5417</td>
<td>0.1304</td>
<td>0.1095</td>
<td>0.0584</td>
</tr>
</tbody>
</table>

(a) Passenger Trips

<table>
<thead>
<tr>
<th>Origin</th>
<th>Region I</th>
<th>Region II</th>
<th>Region III</th>
<th>Region IV</th>
<th>Region V</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.1831</td>
<td>0.0286</td>
<td>0.0105</td>
<td>0.0154</td>
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<tr>
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<td>0.7747</td>
<td>0.1409</td>
<td>0.0074</td>
<td>0.0157</td>
</tr>
<tr>
<td>Region III</td>
<td>0.0143</td>
<td>0.1309</td>
<td>0.8340</td>
<td>0.0072</td>
<td>0.0036</td>
</tr>
<tr>
<td>Region IV</td>
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<td>0.1172</td>
<td>0.1488</td>
<td>0.3714</td>
<td>0.1414</td>
</tr>
<tr>
<td>Region V</td>
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<td>0.1305</td>
<td>0.1861</td>
<td>0.0907</td>
<td>0.5306</td>
</tr>
</tbody>
</table>

(b) Vacant Transitions

This table summarizes transitions in the TLC data. Data in the table are aggregated to the regions from Figure 1. Panel (a) depicts the transition density of taxi-passenger matches and Panel (b) depicts the transition of vacant taxis between each drop-off and the same driver’s subsequent pickup.

five-minute finding rates around 50%. The worst times are in early morning and mid-day, when average wait times are nearly 10 minutes and finding rates fall as low as 25%. Panel (b) shows the same driver match probabilities and waiting times by the 37 non-airport locations, taken as an average from 7am-4pm across all weekdays of the month. Again there is heterogeneity across space, with relatively higher match probabilities and lower waiting times in lower Manhattan (1–8) and Midtown (9–18), declining match probabilities in upper Manhattan (19–34), and even lower match probabilities in Brooklyn (35–37). In aggregate, drivers spend about 47% of their time vacant during the sample period of weekdays during the day-shift. This suggests that among 11,500 active drivers, an average of 5,405 are vacant at any time.

Figure 2 provides a snapshot of the frictions faced by drivers by time-of-day and neighborhood. The data do not reveal the frictions faced by customers; it is impossible to tell how long a customer has been waiting before pick-up, and it is similarly not possible to tell if a customer arrived to search for a taxi and gave up.

15 There is additional evidence that drivers often relocate to find passengers: 61.3% of trips begin in a different neighborhood from the neighborhood where drivers last dropped off a passenger. This suggests that there are spatial search frictions for drivers, as finding a customer requires relocation.
Figure 2: Taxi wait times and match probabilities by time-of-day and location

TLC Data from August 2012, Monday–Friday from 7am until 4pm, within regions indicated on Figure 1. Left Panel: Each series shows taxi drivers’ five-minute probability of finding a customer and mean waiting times, averaged across all drivers and all weekdays. Dotted lines depict 25th and 75th percentiles. Right Panel: Each bar shows driver waiting times and matching probabilities by location of drivers’ last drop-off. Manhattan locations follow a roughly South-to-North trajectory from index 1–34. Brooklyn locations are indexed 35–36. Queens is location 37.

3 Model

A city is a network of \( L \) nodes called “locations”, connected by a set of routes. A location can be thought of as a spatial area within the city.\(^{16}\) Time within a day is divided into discrete intervals with a finite horizon, where \( t \in \{1, ..., T\} \). At time \( t = 1 \) the work day begins; at \( t = T \) it ends. Model agents are vacant taxi drivers who search for customers within a location \( i \in \{1, ..., L\} \). When taxis find passengers, they drive them from origin location \( i \) to a destination location \( j \in \{1, ..., L\} \).

Denote \( v^t_i \in \mathbb{R} \) as a measure of vacant taxis and denote \( u^t_i \in \mathbb{R} \) as a measure of customers looking for a taxi in each location at each time. The total number of taxis in the city is given by \( \sum_i v^t_i = \bar{v} \) for all \( t \). The distance between each location is given by \( \delta_{ij} \) and the travel time between each location is given by \( \tau_{ij} \).

My model has four basic ingredients. First, there is a demand system that describes, for every neighborhood pair \( ij \), how many customers will arrive to the market to search for a taxi as a function of the price of service along that route. Second, there is a payoff vector associated with every route that taxis service. Payoffs include the revenues from each ride minus a service cost due to fuel expenses. Third, there is a model of period-by-period market clearing. Here I use an aggregate matching function to map supply and demand into match probabilities, which adjust

\(^{16}\)e.g., a series of blocks bounded by busy thoroughfares, different neighborhoods, etc.
payoffs depending on the relative quantities of taxis and customers. Finally, I combine these components in a dynamic model of location choice. In this model vacant drivers make period-by-period location choices accounting for the expected match probabilities and payoffs associated with future locations. These four ingredients are presented in more detail below.

3.1 Demand

In each location $i$ at time $t$, the measure of customers that wish to move to a new location, $u_{ti}$, is drawn from a Poisson distribution with parameter $\lambda_{ti}$. Moreover, $\lambda_{ti}$ is a sum of Poisson parameters $\lambda_{ij}(P_{ij})$ where $\lambda_{ij}(P_{ij})$ represents the destination-$j$-specific Poisson arrival of customers in location $i$ at time $t$. The parameters $\lambda_{ij}$ are functions of the price of a taxi ride between $i$ and $j$, $P_{ij}$. Denote the probability that a customer in $i$ wants to travel to location $j \in \{1, ..., L\}$ at time $t$ by $M_{ij}$, so that $\lambda_{ij}(P_{ij}) = M_{ij} \cdot \lambda_{i}(P)$, where $P$ is a vector of prices between all locations.

I assume that taxi drivers face a constant-elasticity demand curve. Demand depends on the origin and destination of the trip, its price, and the time of day. Price elasticities depend on whether the trip involves an airport (a binary index denoted by $a$) and the distance of the trip (indexed by discrete categories $s$). Taxi demand takes the form:

$$\ln(\lambda_{ij}(P_{ij})) = \alpha_{0,i,t,s,a} + \alpha_{1,s,a} \ln(P_{ij}) + \eta_{i,t,s,a}. \quad (1)$$

In addition, I assume that customers demand taxi services for one period. After this period, consumers use a different method of transit.

Waiting Time I do not observe customer waiting time, but it may be an important determinant of demand for taxi rides. To identify the price elasticity of demand in this specific exercise, I provide evidence that waiting time is negligible or of second-order importance. There are two primary reasons for this. First, since the estimation of demand parameters $\lambda_{ij}$ does not require knowledge of waiting time or price elasticities, I compute a measure of waiting time via customer match probabilities. The September price change, later used to identify price elasticities, leads to an estimated average waiting time change of approximately 21 seconds. Further, the limited empirical evidence for waiting time elasticities suggests that it should be relatively small. Frechette,
Lizzeri, and Salz (2019) estimate waiting time elasticity of demand to be about -1.2, while Buchholz, Doval, Kastl, Matejka, and Salz (2019) estimate average waiting time elasticity in a large European taxi market to be -0.66. In the European setting, authors estimate a convex relationship between cost of waiting and length of waiting, which suggests that small waiting times have an even smaller impact on demand than would be suggested by the average elasticity. In counterfactuals, however, I study changes to pricing policies that may be large enough to meaningfully impact waiting times. Therefore in all counterfactuals I implement a waiting time elasticity calibrated to -1.0 and allow demand to adjust accordingly.

3.2 Revenue and Costs

Taxis earn revenue from giving rides. At the end of each ride, the taxi driver is paid according to the fare structure. The fare structure is defined as follows: \( b \) is the one-time flag-drop fare and \( \pi \) is the distance-based fare, with the distance \( \delta_{ij} \) between locations \( i \) and \( j \). The total fare revenue earned by providing a ride from \( i \) to \( j \) is \( b + \pi \delta_{ij} \).

Drivers have two sources of costs. First, there is a fixed daily fee for leasing the taxi and medallion license (or a financing cost for drivers who own their own medallion). Second there are per-mile fuel costs, which I denote as \( c_{ij} \). On any particular day a driver is working, medallion leasing costs for that day are sunk and therefore independent of the driver’s search choices. Since my analysis holds fixed the entry decisions of taxis, I ignore these costs in the model and focus on drivers’ optimization while working.

The net revenue of any passenger ride is given by

\[
\Pi_{ij} = b + \pi \delta_{ij} - c_{ij}. \tag{2}
\]

This profit function sums the total fare revenue earned net of fuel costs in providing a trip from location \( i \) to \( j \).

3.3 Searching and Matching

At the start of each period, taxis search for passengers. The number of taxis in each location at the start of the period is given by the sum of previously vacant taxis who have chosen location \( i \) to search, plus the previously employed taxis who have dropped off a passenger in location \( i \). This sum is denoted as \( v^t_i \). I make the following assumptions about matching: (1) matches can only occur among cabs and customers within the same location, (2) matches are randomly assigned between taxis and customers, and (3) once a driver finds a customer, a match is made and the driver cannot refuse a ride.\(^{20}\) The expected number of matches made in location \( i \) and time \( t \) is given by an

passengers arrive with Poisson param. $\lambda_t$

vacant taxis + taxis dropping off passengers = $v_t$

expected matches = $m_i(\lambda_t^i, v_t^i)$

match probability = $p_t^i = \frac{m_i(\cdot, \cdot)}{v_t^i}$

(remaining customers wait 5-minutes and disappear)

vacant taxis

dropping off passengers = $v_t^i$

aggregate matching function $m_i(\lambda_t^i, v_t^i)$. The ex-ante probability that a driver will find a customer is then given by $p_t^i = \frac{m_i(\lambda_t^i, v_t^i)}{v_t^i}$. Figure 3 illustrates the within-period search and matching process.

3.3.1 A Model of Neighborhood Search

There are two types of locations, neighborhoods and airports. Neighborhoods comprise most of a city; they are locations in which cabs drive around to search for passengers. Below I detail how matches are formed in neighborhoods. The next subsection discusses airports.

When model locations are specified as spatial areas such as a neighborhood, search within this area will exhibit search frictions even when block-by-block search is nearly frictionless. This design echoes the setup of Lagos (2000) that allows search frictions to arise endogenously from driver behavior. To model the search frictions within each location, I use an aggregate matching function given by equation 3.

$$m(\lambda_t^i, v_t^i, \alpha_r) = v_t^i \cdot \left(1 - e^{-\frac{\lambda_t^i}{m_t v_t^i}}\right)$$

Equation 3 is a reduced-form model of intra-location matching. It can flexibly reproduce frictions (i.e. such that $m(\lambda_t^i, v_t^i, \alpha_r) < \min(\lambda_t^i, v_t^i)$), the extent of which are controlled by the search

---

21This function is derived from an urn-ball matching problem first formulated in Butters (1977) and Hall (1979). While the original model characterizes matches from discrete (i.e., integer) inputs, my specification characterizes urn-ball matching with a large number (or continuum) of inputs. See, e.g., Petrongolo and Pissarides (2001) and the derivation in Appendix 3.8.
efficiency parameter $\alpha_r > 0$. All else equal, larger values of alpha generate fewer matches. $r$ denotes a region, or a subset of locations as described in section 2.3. $\alpha_r$ is region-specific as it reflects the difficulty of search within a region, such as the complexity of the street grid. These are physical characteristics of a region which are assumed to be fixed across the day. I illustrate the aggregate matching function and the role of $\alpha_r$ in Figure 4.

Moreover, this equation is specified in terms of expected demand $\lambda^t_i$ and not the daily draws $u^t_i$. It represents the expected number of matches produced in a location-time with demand parameter $\lambda^t_i$ and vacant taxi supply $v^t_i$. This is the relevant object from the perspective of taxi drivers’ location optimization problem. From now on I denote $m_i(\lambda^t_i, v^t_i) = m(\lambda^t_i, v^t_i, \alpha_r)$ to be the location-specific matching function, with the only difference across locations coming from the efficiency of the region $r$ containing location $i$. The probability of a match from a taxi driver’s perspective is therefore given by

$$p_i(\lambda^t_i, v^t_i) = \frac{m_i(\lambda^t_i, v^t_i)}{v^t_i} = \left(1 - e^{-\frac{\lambda^t_i}{\alpha_r v^t_i}}\right).$$ (4)

![Figure 4: Matching Efficiency and $\alpha$](image)

This figure shows contour plots of the matching function over three values of $\alpha$. Contour levels depict the expected number of matches produced in a given location when the level of taxis is $v$ and the expected arrivals of customers is $\lambda$, for each level of $\alpha$.

### 3.3.2 Airport Queueing

At airports, taxis pull into one of multiple queues and wait for passengers to match with cabs at the front of the queue.\footnote{In the data, rides involving one of the two major New York airports comprise roughly 6% of all taxi trips, and 16% of revenues.} I assume there is some measure of congestion in the taxi lane, so that no more
than $\bar{v}_i^t$ cabs can clear the queue in each period. This condition prevents instantaneous clearing of the taxi queue. The total number of matches is thus given by \(\min\{u_i^t, w_i^t\}\), where $u_i^t = \min(\bar{v}_i^t, v_i^t)$ and where the total measure of cabs at the airport in each period is $v_i^t$. From a taxi driver’s perspective, airports represent match probabilities of one, but at the expense of time spent waiting for the match. The more taxis there are in line, the more periods it will take to for a new driver entering the queue to find a match.

### 3.4 Dynamic Model of Taxi Drivers’ Locations, Actions and Payoffs

A taxi driver’s behavior depends on his own private state \((\ell_i^t, e_i^t)\) and the market state, $S^t$. Specifically a driver $a$’s own location at time $t$ is given by $\ell_i^t \in \{1, \ldots, L\}$, and his employment status (vacant or employed) is $e_i^t \in \{0, 1\}$. Let $m_{ij}^k$ denote the set of $ij$ matches occurring at starting time $k$. The market state $S^t$ at time $t$ is a measure of vacant taxis $v_i^t$ in each location $i$ and a measure of employed taxis $m_{i,j}^k$ that are in-transit between locations.\(^{23}\) Thus the market state at time $t$ is summarized by

$$S^t = \left\{ \{v_i^t\}_{i \in \{1, \ldots, L\}}, \{m_{i,j}^k\}_{i,j \in \{1, \ldots, L\}^2, k < t}\right\}. \quad (5)$$

Denote $S = \{S^t\} \forall t$ so that $S$ reflects the entire spatial and intertemporal distribution of vacant and employed taxis. At the beginning of each period, taxi drivers make a conjecture about the current-period state and how it will evolve going forward. Given this conjecture, they assign value $V_i^t$ to each $i, t$-pair.

I define the drivers’ ex-ante (i.e., before observing any shocks and before any uncertainty in passenger arrivals is resolved) value as

$$V_i^t(S^t) = \mathbb{E}_{S[S^t]} \left[ p_i(\lambda_i^t, v_i^t) \left( \sum_j M_{ij}^t \cdot (\Pi_{ij} + V_{j}^{t+\tau_{ij}}(S^{t+\tau_{ij}})) \right) + (1 - p_i(\lambda_i^t, v_i^t)) \cdot \mathbb{E}_{\varepsilon_{j,a}} \left[ \max_{j \in \mathcal{A}(i)} \left\{ V_{j}^{t+\tau_{ij}}(S^{t+\tau_{ij}}) - c_{ij} + \varepsilon_{j,a} \right\} \right] \right], \quad (6)$$

This expression has two components. Drivers in location $i$ at time $t$ expect to contact a passenger with probability $p_i(\lambda_i^t, v_i^t)$. Drivers’ payoff for providing a trip is equal to the net profit of a trip $\Pi_{ij}$

\(^{23}\)At any moment, employed taxis are not directly competing with vacant taxis for passengers. Accounting for the distribution of employed taxis is an important component of the state variable because the eventual arrival of employed taxis and subsequent transition to vacancy is payoff-relevant when deciding how to conduct future search.
plus continuation values $V_j^{t+\tau_{ij}}$ of being in location $j$ after $\tau_{ij}$ periods have elapsed. Therefore the expected value of a trip is simply the value of a trip to each location $j$ weighted by the probability that a passenger picked-up in $i$ chooses $j$ as the destination, which is given by $M_{ij}^t$. 24

At the end of the period, any cabs that remain vacant can choose to relocate or stay put to begin a search for passengers in the next period. The set $A(i)$ reflects the set of locations available to vacant taxis and is limited to all adjacent locations in the city, where adjacency is defined as locations that can be reached in one period or less. In addition, all trips to and from airports are included in each choice set.

Vacant drivers choose to search next period in the location that maximizes total expected payoff as the sum of continuation values $V_j^{t+\tau_{ij}}(S)$, fuel costs $c_{ij}$ and a contemporaneous and an idiosyncratic shock $\varepsilon_{ja}^t$. $\varepsilon_{ja}^t$ is a driver $a$-specific i.i.d. shock to the perceived value of search in each alternative location $j$, which I assume to be drawn from a Type-I extreme value distribution. This shock accounts for unobservable reasons that individual drivers may assign a slightly greater value to one location than another. For example, traffic conditions and a taxi’s direction of travel within a location may make it inconvenient to search anywhere but further along the road in the same direction. 25

Vacant drivers in location $i$ move to location $j^*$ by solving the last term in equation 6:

$$j^* = \text{arg max}_j \{V_j^{t+\tau_{ij}}(S^{t+\tau_{ij}}) - c_{ij} + \varepsilon_{ja}\}.$$  

To compute the drivers’ strategies, I define the ex-ante choice-specific value function as $W_i^t(j_a, S^t)$, which represents the net present value of payoffs conditional on taking action $j_a$ while in location $i$, before $\varepsilon_{ja}$ is observed:

$$W_i^t(j_a, S^t) = \mathbb{E}_{S^{t+\tau_{ija}}} \left[V_j^{t+\tau_{ija}}(S^{t+\tau_{ija}}) - c_{ija}\right].$$  

Defining $W_i^t$ allows for an expression of taxi drivers’ conditional choice probabilities: the probability that a driver in $i$ will choose $j \in A(i)$ conditional on reaching state $S^t$, but before observing $\varepsilon_{ja}$, is given by

$$P_i^t[j_a|S^t] = \frac{\exp(W_i^t(j_a, S^t)/\sigma_{\varepsilon})}{\sum_{k \in A(i)} \exp(W_i^t(j_k, S^t)/\sigma_{\varepsilon})}.$$  

This expression defines aggregate policy functions $\sigma_i^t = \{P_i^t[j|S^t]\}_{j \in \{1,\ldots,L\}}$ as the probability of optimal transition from an origin $i$ to all destinations $j$ conditional on future-period continuation

--

24 Note that $M_{ij}^t$ has superscript $t$ because passenger preferences change throughout the day.
25 The terms $\varepsilon_{ja}$ also ensure that vacant taxis leaving one location will mix among several alternative locations rather than moving to the same location, a feature broadly corroborated by data.
values.

Time ends at period $T$. Continuation values beyond $t = T$ are set to zero: $V^t_i = 0 \forall t > T, \forall i$. Employed taxi drivers with arrival times beyond period $T$ are assumed to finish en-route trips before quitting.

### 3.5 Intraday timing

There is an exogenous initial distribution of vacant taxis labeled $S^1$ which is known to all drivers and constant across each weekday. This distribution accounts for the early morning position of taxis, all of which are assumed vacant at this time, as they leave from garages and arrive to the search regions of the city. Vacant taxis conduct search at the start of each period $t$. At the end of a period any newly employed taxis disappear from the stock of vacant cabs and earn revenue. Vacant taxis earn no revenues but face continuation values associated with each possible move. In the next period $t + 1$, the locations of vacant taxis are updated based on movement from both previously vacant taxis who have relocated as well as any taxis dropping off passengers.

As detailed in the equilibrium description, I assume drivers are unaware of the i.i.d. demand shocks in each location and time, and instead condition policies on long-run market averages (which might, for example, be learned through experience). Since the expectation of a Poisson random variable with parameter $\lambda$ is also equal to $\lambda$, $\lambda^t_i$ is the expected demand faced by taxi drivers. Drivers form policies based on forecasting the following sequence of events:

1. Taxis are exogenously distributed each day according to $S^1$.
2. $m_i(\lambda^1_i, v^1_i)$ taxis become employed with matched customers in each location.
3. The remaining $\lambda^t_i - m_i(\lambda^1_i, v^1_i | \alpha_i)$ unmatched customers leave the market.
4. The remaining $v^t_i - m_i(\lambda^1_i, v^1_i | \alpha_i)$ vacant taxis choose a location to search in next period according to policy functions.
5. Previously vacant and some previously employed taxis arrive in new locations, forming distribution $S^2$.\(^{26}\)
6. The process repeats from $S^2$, $S^3$, etc. until reaching $S^T$.

\(^{26}\)Many hired taxis are in-transit for more than one period. Suppose hired taxis providing service from location $i$ to $j$ will take 3 periods to complete the trip. Then only the taxis who were 1 period away at time $t - 1$ will arrive in $j$ in period $t$.  

19
3.6 Transitions

Policy functions $\sigma^t_i$ form a matrix of transition probabilities from origin $i$ to all destinations $j \in L$. Note that only vacant taxis transition according to these policies. Employed taxis will transition according to a different matrix of transition probabilities given by $M^t_i$ denoting the probability that a matched customer in $i$ will demand transit to any destination $j \in L$. Together, these two transition processes generate a law of motion for the state variable $S$.

The transition kernel of employed taxis is given by $\nu(v^{t+1}_e|v^t_e, M^t, m^t)$ where $v^t_e$ is the distribution of employed taxis across locations in period $t$, $M^t = \{M^t_{ij}\}$ for $i, j = \{1, ..., L\}$ is the set of transition probabilities of each matched passenger at time $t$ and $m^t = \{m_i(\lambda^t_{vi}, v^t_i)\}$ for $i = \{1, ..., L\}$ is the distribution of matches. $\nu$ specifies the expected distribution of all employed taxis $v^t_e, i^t$ over locations in period $t + 1$.

Likewise, the transition kernel of vacant taxis is given by $\mu(v^{t+1}_v|v^t_v, \sigma^t)$. As with $\nu$, $\mu$ specifies the expected $t + 1$ spatial distribution of period $t$ vacant taxis, given the transitions generated from policies $\sigma^t = \{\sigma^t_i\}$ for $i = \{1, ..., L\}$. The combined set of transitions forms an aggregate transition kernel that defines the law-of-motion, given by $Q(S^{t+1}|S^t) = \nu(v^{t+1}_e|v^t_e, M^t, m^t) + \mu(v^{t+1}_v|v^t_v, \sigma^t)$. I provide explicit formulas for the state transitions in Appendix A.5.

3.7 Equilibrium

Taxi drivers policies in a given period depend on the beliefs about the distribution of their competitors, policies of competitors when vacant, expected demand across different neighborhoods and the destination preferences of customers across neighborhoods. Beliefs over competitors’ policies given the distribution of all vacant cabs allow taxi drivers to infer how the distribution of vacant cabs at time $t$ will update in future periods. Likewise, driver forecasts about demand and customer destination preferences across neighborhoods allow drivers to infer how the distribution of matched cabs and their movement will affect the future distribution of vacant cabs. The net transition of taxis’ vacant capacity is denoted as $\tilde{Q}^t_i$.

Although drivers have knowledge of the Poisson demand parameters $\lambda^t_{vi}$, I assume they do not see actual draws $u^t_{vi}$, which are spread across multiple blocks within a location. Any successful or unsuccessful match is attributed to a long-run probability of matching in each location and period and there is no intra-day updating of beliefs based on whether the driver is matched or not.

Taxis optimize over where to locate when vacant. Since beliefs about the state and transitions at time $t$ summarize all relevant information about distribution of competition, taxis condition only on beliefs over the current-period so that an optimal location choice at time $t$ can be made using time $t - 1$ information. This Markovian structure permits the following definition of equilibrium:

---

27See Assumption 2 in Section 4.
**Definition** Equilibrium is a sequence of state vectors \( \{ S^t_i \} \), transition beliefs \( \{ \tilde{Q}^t_i \} \) and policy functions \( \{ \sigma^t_i \} \) over each location \( i = \{1, ..., L\} \), and an initial state \( \{ S^0_i \} \) such that:

(a) In each location \( i \in \{1, ..., L\} \), at the start of each period, matches are made according to equation \( \mathbb{3} \) and are routed to new locations according to transition matrix \( M^t \). The aggregate movement generates the employed taxi transition kernel \( \nu(v_{e}^{t+1}|v_{e}^{t}, M^t, m^t) \) where \( v_{e}^t \) is the distribution of employed taxis across locations in period \( t \) and \( m^t \) is the distribution of matches across locations.

(b) In each location \( i \in \{1, ..., L\} \), at the end of each period, vacant taxi drivers (indexed by \( a \)) follow a policy function \( \sigma^t_{i,a}(S^t, \tilde{Q}^t_i) \) that (a) solves equation \( \mathbb{7} \) and (b) derives expectations under the assumption that the state transition is determined by transition kernel \( \tilde{Q}^t_i \). The aggregate movement generates the vacant taxi transition kernel \( \mu(v_{v}^{t+1}|v_{v}^{t}, \tilde{\sigma}^t, S^t) \) where \( v_{v}^t \) is the distribution of vacant taxis in period \( t \).

(c) State transitions are defined by the combined movement of vacant taxis and employed taxis, defined by:

\[
Q(S^{t+1}|\tilde{S}^t) = \nu(v_{e}^{t+1}|v_{e}^{t}, M^t, m^t) \cup \mu(v_{v}^{t+1}|v_{v}^{t}, \tilde{S}^t).
\]

(d) Agents’ expectations are rational, so that transition beliefs are self-fulfilling given optimizing behavior: \( \tilde{Q}^t_i = Q^t_i \) for all \( i \) and \( t \).

**Proposition 3.1.** The equilibrium defined above exists and is unique.

Proof. See Appendix A.6

Equilibrium delivers a distribution of vacant taxi drivers such that no one driver can systematically profit from an alternative policy: there is no feasible spatial arbitrage opportunity that would make search more valuable (ex-ante) in any location other than the optimum one. Vacant taxis are therefore clustered in locations with more profitable customers, but the associated profits are offset by higher search frictions. One implication of this sorting pattern is that equilibrium value functions are nearly identical across space in each time period. Moreover, equilibrium value functions would equate across locations in each period if not for transportation costs in time and fuel, preventing an equilibrium with perfect spatial arbitrage. See additional discussion and illustration in Appendix A.9.

### 4 Empirical Strategy

To estimate model parameters I use each month of the New York TLC data and aggregate trip-specific information into daily quantities related to customer transit preferences, trip times and...
trip distances for each weekday day-shift period. To form moments related to drivers’ beliefs and payoffs I further average these daily quantities into monthly averages. When taxi drivers engage in equilibrium search behavior, they incorporate these monthly averages into forecasts that enter their continuation values. I present an estimation strategy below that is configured to (1) solve for equilibrium and (2) use this equilibrium to recover parameters, both at a monthly-average level of resolution. Below I detail these two main components of estimation.

4.1 Computing Equilibrium

The equilibrium location choices of vacant taxi drivers and their resulting spatial allocations must be computed in order to estimate model parameters. I make the following assumptions about the total supply of taxis and their information set.

**Assumption 1** The total supply of cabs $\bar{v}$ for each weekday day-shift during the period studied is equal to 11,500.

Assumption 1 satisfies a requirement for an exogenous labor supply necessary to compute the equilibrium level of vacant taxis across time and locations.\(^{28}\)

**Assumption 2** Taxi drivers have knowledge of the initial state vector $S^1$ and all model parameters including demand parameters $\{\lambda_t^i\}$. Drivers do not observe the specific draws from the distributions of demand across time and locations.

Assumption 2 not only provides computational tractability, but also provides a behavioral model that is motivated by drivers’ inability to observe supply or demand beyond the particular streets they drive on in one period.

To solve for equilibrium I use a two-step procedure where in the first step the set of equilibrium matches $\{m_{t}^{i}\}$ are non-parametrically estimated using trip data. I estimate matches by computing the mean matches in each $i,t$ cell for each day of the month. These moments contain all relevant information needed for the second step, which solves the taxi drivers’ spatial equilibrium up to $\sigma_{\varepsilon}$. Equation (4) makes this second step explicit: the moments from the first step are the only inputs to the value functions that contain information on demand. This step is the most involved as it entails solving for non-stationary equilibrium value functions and policy functions given an exogenous initial condition (i.e. the distribution of locations where drivers start their shift) and an exogenous labor supply.

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\(^{28}\)The number is data-driven; see Section A.4 for details.
Computing a dynamic equilibrium in large markets with many states is typically confounded by the curse of dimensionality. In this setting the problem is mitigated by Assumption 2. This assumption gives rise to a limited-information equilibrium similar to that of Weintraub, Benkard, and Van Roy (2008b) and Fershtman and Pakes (2012) in which all agents play strategies and form equilibria without full knowledge of the payoff-relevant state variables. This allows me to compute policy functions without keeping track of which drivers are aware of which shocks, and without history dependence (i.e. 2pm in any particular neighborhood is always valued the same). Drivers therefore play against beliefs over average values of the state vector. By playing the same equilibrium policies each day, these beliefs become self-fulfilling in equilibrium.

State transitions are composed of the combined transitions of vacant and employed taxis. Under the continuum model both of these transitions are deterministic given a set of matches and vacancies. Given an exogenous initial allocation of all 11,500 taxis in the market, assumed vacant at time \( t = 1 \), drivers compute a single, deterministic equilibrium path for the state \( \{S_t^i\} \) for \( t = \{1, ..., T\} \). Taxi cabs start the day in garages around the city. When initially arriving to the set of locations in Figure 1, they form an initial distribution of vacant supply. Because these locations are unobserved, I approximate this distribution using the empirical distribution of early morning matches. To compute equilibrium conditional on this initial state, I devise a numerical algorithm that couples backwards induction and value function iteration. See Appendix A.7.1 for details and robustness checks.

4.2 Model Estimation and Identification

This subsection proceeds in four parts. I first discuss parameters that are identified directly from data. Second, I describe how I use the first set of parameters together with a computed equilibrium allocation of taxis to estimate the scale parameter of the driver shocks to location values. Third, I describe how to estimate the demand and matching efficiency parameters given the scale parameter. Finally, I explain how to use the recovered demand parameters from before and after a fare change to estimate the price elasticities of demand along different routes.

4.2.1 Objects Identified Directly from Data

Five parameters of the model are identified directly from data. Each is a set of time and location averages. The first four parameters are expected quantities related to time, distance, and transitions. The fifth parameter relates to fuel costs, where average values for the taxi fleet’s fuel economy are the only available data.

1. \( M_{ij}^t \) is the transition probability of employed taxis in each period and location. In each period, I record the probability of transition from each origin to each destination conditional on a
taxi matching with a passenger. The mean of these probabilities over each weekday of the month, computed for each origin \(i\), destination \(j\) and hour \(t\), generates expected transition probabilities \(M^t_{ij}\).

2. \(\tau_{ij}\) is the travel time between each origin and each destination. As above, I record the average of all travel times between each \(i\) and \(j\), for each hour \(t\), over all weekdays of the month. I set \(\min(\tau_{ii}) = 1\), so that within-location trips must take at least 1 period.

3. \(\delta_{ij}\) is the distance between each origin and each destination. With the trip distance variable in TLC data, I record the mean distance between each \(i\) and \(j\) across all weekdays of the month. Note that \(\delta_{ii} > 0\) since trips can occur within a location.

4. \(\bar{v}^t_i\) is the maximum number of cabs that can collect passengers in one period at each airport \((i = \{38, 39\})\). To compute the number of periods a taxi driver expects to wait in the airport queue, I compute the mean number of pickups per period \(\rho_i\) at each airport as a per-taxi expected wait time and use this to determine the waiting period length as \(\text{round}(\bar{v}^t_i / \rho_i)\). This moment is sufficient to determine how drivers value the airport, upon arrival, as a search location. To estimate demand, I treat the mean number of pick-ups in each airport location as direct observations of \(\lambda^t_{38}\) and \(\lambda^t_{39}\).\(^{29}\)

5. Finally I compute the cost of fuel per mile \(c\) as the average fuel price in New York City in 2012 divided by the average fuel economy in the New York taxi fleet, 29 mpg.\(^{30}\) Using \(c = \$0.124\), I compute the cost of traveling between any origin and destination as \(c_{ij} = c \cdot \delta_{ij}\). Note that \(\delta_{ii} > 0\) implies \(c_{ii} > 0\).

After I record the distances between each origin and each destination, I can derive \(\Pi_{ij}\), the expected profit associated with each possible trip. Recall from equation [2] that \(\Pi_{ij} = b + \pi \delta_{ij} - c_{ij}\), where the regulated fare structure is given by the set \(\{b, \pi\}\). With these parameters, and given data on expected matches by time and space, I use the spatial equilibrium model to solve for the equilibrium distribution of taxis up to a scale parameter. Below I describe how solving for equilibrium and using it to minimize a set of moment conditions allows me to recover the scale parameter and thereby recover an estimate for demand and matching efficiency.

\(^{29}\)Note if airport riders were to wait more than 5 minutes, then due to \(\bar{v}^t_i\) these estimates may be biased. For example if \(\lambda^t_{38} > \bar{v}^t_{38}\), the estimate would be truncated at \(\bar{v}^t_{38}\). However, I keep airport prices fixed in all counterfactuals (and hold the queuing technology fixed), so that model estimates of \(\lambda^t_{38}\) capture a lower bound on demand.

\(^{30}\)Data come from the New York City Taxi and Limousine Commission 2012 Fact Book and the U.S. Energy Information Administration. The taxi fleet is approximately 60% hybrid vehicles. Volatility of fuel prices is low in this period: cost-per-mile fluctuates within a range of \$0.01 during the sample period.
4.2.2 Remaining Parameters: Overview

In the following, I show that the 4,212 Poisson demand parameters \( \lambda_t^i \), the four matching efficiency parameters for regions I-IV, \( \{ \alpha_r \} \), and the parameter \( \sigma_\varepsilon \) can be identified given the available data. Identification has three steps.

1. Given the estimated objects that are derived directly from the data, \( \{ \tau_{ij}, \delta_{ij}, \Pi_{ij} \} \), and given some value of \( \sigma_\varepsilon \), all that is necessary to solve for the equilibrium state \( \{ v_t^i \} \) are the expected number of matches across locations and times, \( m_t^i \).

2. I resolve for equilibrium as described step 1 given different values of \( \sigma_\varepsilon \), and choose the value that best matches a set of simulated moment conditions to data.

3. Given the expected number of matches and the corresponding equilibrium \( \{ v_t^i \} \), I can identify the ratio \( \frac{\lambda_t^i}{\alpha_r} \) by inverting \( m_t^i(v_t^i, \frac{\lambda_t^i}{\alpha_r}) \) in its second argument.

Section 4.1 discusses the computational process used to solve for equilibrium given \( \{ \tau_{ij}, \delta_{ij}, \Pi_{ij} \} \) and \( \sigma_\varepsilon \) with additional detail in appendix A.7.1. The next two subsections describe steps 2 and 3.

4.2.3 Estimating \( \sigma_\varepsilon \)

Since the static profit function is observable, it is possible to test how much drivers depend on observed versus unobserved factors in making search choices. This is done through the scale parameter \( \sigma_\varepsilon \), which impacts how much drivers’ spatial search behavior is explained by trip profits. To estimate this parameter, I search for the value of \( \sigma_\varepsilon \) that generates an equilibrium state vector \( \{ v_t^i(\sigma_\varepsilon) \} \) and corresponding driver-specific moments that are closest to the data. The moments of interest are matching probabilities across different regions (regions I-IV depicted in Figure 1) as well as average number of periods vacant within a shift and the average matched distance travelled per trip. Each of these moments can then be compared to their data counterparts and estimated by Method of Simulated Moments (MSM).

A high-\( \sigma_\varepsilon \) equilibrium will lead to drivers that are more spread out spatially (as choice probabilities tend towards a uniform distribution) so that drivers choose search patterns that are less centralized in Manhattan. This will lead to longer times between trips since more drivers will choose to search farther from their current location despite the fact that, all else equal, nearby search is more profitable. Longer trips are correlated with long between-trip times. Thus, remote regions will be searched more often under a high-\( \sigma_\varepsilon \) equilibrium.
4.2.4 Estimating $\lambda_t^i$ and $\alpha_r$

Before I can estimation $\lambda_t^i$ and $\alpha_r$, I must first estimate the ratio $\frac{\lambda_t^i}{\alpha_r}$. This procedure has three main ingredients: equation 4 defining the matching probabilities, the associated empirical observation of matches, and the equilibrium state vector defining taxis’ allocations.

I denote the state vector as $S^* = \{v_t^i\} = \{v_t^i(m_\sigma)\}$. Let $m_t^i$ denote the expected number matches in each location $i$ and at each time $t$, with estimates $\hat{m}^i_t$ obtained by averaging the data. As discussed above, $m_t^i$ and $\sigma_\epsilon$ together with observed data moments $\tau_{ij}$, $\delta_{ij}$, and $\Pi_{ij}$ give rise to a unique equilibrium state $S^*$. $S^*$ describes drivers’ equilibrium beliefs over the distribution of vacant taxis. According to the definition of equilibrium, these beliefs are correct on average despite the small variation induced by stochastic demand.\(^{31}\) Denote this equilibrium as $\{v_t^i(m_\sigma)\}$ where $m_\sigma = \{\hat{m}^i_t\}$. I can then identify the set of ratios $\{\frac{\lambda_t^i}{\alpha_r}\}$ using $\{v_t^i(m_\sigma)\}$ by inverting Equation 3 at $m_\sigma$.

**Proposition 4.1.** Suppose a vector of expected matches by location and time, $m_\sigma$, is observed. Further, suppose $\{v_t^i(m_\sigma)\}$ is unique and $v_t^i(m_\sigma) \neq 0 \forall i, t$. Then the ratio $\frac{\lambda_t^i}{\alpha_r}$ is identified.

**Proof.** Equation \(^{[3]}\) is strictly increasing in $\lambda_t^i/\alpha_r$ given $v > 0$. Since a strictly increasing function is one-to-one, it follows that as long as $v_t^i(m_\sigma)$ is unique and strictly positive, then by fixing $\{m_t^i = \hat{m}^i_t, v_t^i = v_t^i(\hat{m})\}$ equation \(^{[3]}\) can be uniquely inverted for $\frac{\lambda_t^i}{\alpha_r}$:

$$\frac{\lambda_t^i}{\alpha_r} = -v_t^i(\hat{m}) \ln \left(1 - \frac{\hat{m}^i_t}{v_t^i(\hat{m})}\right).$$

\(^{10}\)

Since $m_\sigma$ is observed and $v_t^i(m_\sigma)$ is uniquely determined upon recovering $\sigma_\epsilon$, I can compute the right-hand-side of Equation \(^{[10]}\) to recover $\frac{\lambda_t^i}{\alpha_r}$. The non-zero condition on the state vector is confirmed by the numerically obtained equilibrium.

To separately identify $\lambda_t^i$ and $\alpha_r$, I leverage an additional moment in the data: the variance of matches in each $i, t$ cell across days in the sample. There is day-to-day variance in matches at any $i, t$ pair because demand is drawn daily from the Poisson($\lambda_t^i$) distribution.\(^{32}\) To illustrate how match variance can held identify $\alpha_r$, suppose that matches were determined by $m(\lambda_t^i, v_t^i) = \frac{\lambda_t^i}{\alpha_r}$. Then for a Poisson parameter $\lambda_t^i$ we have $Var(m) = \frac{1}{\alpha_r^2} Var(\lambda_t^i) = \frac{\lambda_t^i}{\alpha_r} \frac{1}{\alpha_r}$. Thus by comparing the recovered $\frac{\lambda_t^i}{\alpha_r}$ and the observed $Var(m)$ I can identify $\alpha_r$.

Of course the matching model in equation \(^{[3]}\) is more complex than the example above, but the logic is the same: a higher $\alpha_r$ parameter dampens the variance of matches relative to the variance

\(^{31}\)See Appendix A.8 for details regarding this variation.

\(^{32}\)Note that this induces variance directly through the variance in demand as well as indirectly because varying demand leads to some variance in the mass of taxis searching in each neighborhood as a result of varying matches.
of demand. This logic gives rise to the following estimator for the efficiency parameter in each region:

$$\hat{\alpha}_r = \arg\min_{\alpha} \left( \sum_{i,t} \mathbb{I}(i \in r(i)) \left( \text{Var}(m_i(\lambda^t_i, v^t_i|\alpha) - \text{Var}(\hat{m}_i^t) \right) \right),$$

(11)

where $\text{Var}(\hat{m}_i^t)$ denotes the variance of matches observed in the data. Implementing this estimator is challenging because computing $\text{Var}(m_i(\lambda^t_i, v^t_i|\alpha))$ requires simulating equilibrium over a large number of draws in order to obtain a credible estimate of variance and then numerically searching over the vector of $\alpha_r$ to solve the above equation. This process then would have to be repeated many times across bootstrap samples to recover standard errors. To make this estimation practical, I instead make the following assumption:

**Assumption 3** The variance of matches $\text{Var}(m_i(\lambda^t_i, v^t_i|\alpha_r))$ is not significantly impacted by variance in $v^t_i$.

I substantiate Assumption 3 by simulating the market with and without variance in vacant taxis to show that it yields nearly identical variance in matches (see Appendix A.8). This assumption allows for analytically computing the variance of matches and obviates the need for a simulation-based estimator. Specifically, using the density function of the Poisson distribution, I can derive an analytic expression for the variance of matches that I can use to form an estimator:

$$\text{Var}(m_i^t) = (v_i^{t^*})^2 e^{-2\frac{\hat{\lambda}^t_i}{\alpha_r} \frac{1}{v_i^{t^*}}} \left( e^{\frac{\hat{\lambda}^t_i}{v_i^{t^*}}} \frac{1}{v_i^{t^*}} - 1 \right).$$

(12)

**Proposition 4.2.** Suppose Assumption 3 and all assumptions of Proposition 4.1 hold, and suppose a vector $\sigma^2_{m,i,t}$ of the variance of matches by time and location across days is observed. Further suppose that the $v_i^{t^*}$ are constant across days. Then $\{\lambda^t_i\}$ and $\{\alpha_r\}$ are identified.

**Proof.** $\frac{\lambda^t_i}{\alpha_r}$ and $v_i^{t^*}$ are obtained as in Proposition 4.1. Denote $\hat{\lambda} = \frac{\lambda^t_i}{\alpha_r}$, and $\hat{\sigma}^2_{m,i,t} = \{\text{Var}(\hat{m}_i^t)\}$, where the variance of matches in each location and at each time is taken across days. Then inverting equation 20 for $\alpha_r$ gives

$$\alpha_r = \frac{\hat{\lambda}^t_i}{v_i^{t^*}(\hat{\mathbf{m}})} \left( \ln \left( e^{2\hat{\lambda}^t_i \left( \frac{\hat{\sigma}^2_{m,i,t}}{v_i^{t^*}^2(\hat{\mathbf{m}})} \right)} + 1 \right) \right)^{-1}.$$

(13)

With estimates of $\alpha_r$ and $\frac{\lambda^t_i}{\alpha_r}$, I can recover the demand parameters $\{\lambda^t_i\}$ directly. \hfill \Box

Equation 13 shows that $\alpha_r$ is overidentified because each region $r$ is made up of several locations $i$ and times-of-day $t$. While $\alpha$ could be treated as $i$- or $t$-specific, a modeling frictions on the basis of
broader regions obtains more credible results for each \( \alpha_r \), as there could be error in the measurement and estimation of the right-hand-side parameters and moments. From here, I can estimate \( \alpha_r \) via NLLS.\(^{33}\) I provide additional details in Appendix A.8.

### 4.2.5 Estimating Demand Elasticities

To compute market welfare, I estimate the demand elasticity parameters in equation (1). On September 4, 2012, the distance fee increased by \$0.50 per-mile, and the JFK airport flat-fee increased by \$7. Using September 2012, data, I re-estimate the model for \( \{ \nu_i^s \} \), \( \{ \lambda_i^j \} \), and \( \{ \alpha_r \} \).\(^{34}\) In the analysis that follows, I use the price variation from this regulatory change to estimate demand elasticities across different types of trips, where the demanded quantity is the average number of customer arrivals in each \( i \) with destination \( j \) at time \( t \) given prices \( P_{ij} \).\(^{35}\)

I estimate demand via the following specification:

\[
\ln(\lambda_{ij}^t(P_{ij})) = \alpha_{0,s,a} + \alpha_{1,s,a} \ln(P_{ij}) + \delta_{ht} + \gamma_i + \gamma_j + \eta_{i,j,t}. \tag{14}
\]

In a slight abuse of notation I define the index \( s = s(i,j) \) as a set of the distance categories associated with a trip \( i, j \), such that \( s \in \{ s_0, s_1, s_2, s_3 \} = \{ 0–2 \text{ mi.}, 2–4 \text{ mi.}, 4–6 \text{ mi.}, 6+ \text{ mi.} \} \), roughly corresponding to trip-distance quartiles. The index \( a \) indicates an airport trip. Price elasticities \( \alpha_{1,s,a} \) are different for each distance category among trips without airports, and different for airport trips. I include hourly fixed effects \( \delta_{ht} \) and region fixed effects for drop-off location \( j \) and pickup location \( i \).

In this demand system, all customers of a given type \( s, a \) have the same price elasticities. Origin, destination and time fixed effects capture the heterogeneity of locations: some have more public transit stations, bus stops, or walkability. Given these fixed effects, identifying variation comes from the differences in prices before and after the September 2012 fare change. To estimate parameters, I estimate an empirical analogue of equation (1) for each \( s \) category using OLS. Since prices are fixed within a location and time period, this specification does not suffer from simultaneity bias, unlike traditional non-instrumented demand models.

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\(^{33}\)While variation in demand is important to this approach, I note that any unobserved sources of variation, say due to local demand shocks or unobserved variation in vacant taxis’ supply on a given day could raise the variance of matches beyond any variation attributable to demand and search frictions. If such unobserved variation exists, my estimates \( \alpha_r \) would imply lower frictions than the true value. Therefore any estimated welfare gains to improving search frictions can be viewed as a lower bound.

\(^{34}\)Since road conditions, traffic patterns, average weather patterns, etc. may change, I allow \( \alpha_r \) to change by month. I assume that \( \sigma_e \) is unchanged.

\(^{35}\)Based on the discussion in Section 3.1, I compute that average waiting times dropped by 21 seconds between August and September, and in Manhattan the largest drop in location-specific waiting time was 31 seconds. I therefore assume the changes in waiting times are sufficiently small to be negligible for the demand estimation. My counterfactuals, however, will incorporate an elasticity of waiting time.
### Panel A: Parameter Estimates Summary

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Elements</th>
<th>Estimate</th>
<th>Std. Err.</th>
<th>Avg/Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (August 2012)</td>
<td>$\lambda_t^{aug.}$</td>
<td>4,212</td>
<td>See Figures A7, A8</td>
<td>59.2/2.3/339.8</td>
<td></td>
</tr>
<tr>
<td>Demand (September 2012)</td>
<td>$\lambda_t^{sep.}$</td>
<td>4,212</td>
<td></td>
<td></td>
<td>55.3/2.2/446.0</td>
</tr>
<tr>
<td>Efficiency, Region I (Aug.)</td>
<td>$\alpha_1^{aug.}$</td>
<td>1</td>
<td>0.812</td>
<td>(0.016)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region I (Sep.)</td>
<td>$\alpha_1^{sep.}$</td>
<td>1</td>
<td>0.811</td>
<td>(0.009)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region II (Aug.)</td>
<td>$\alpha_2^{aug.}$</td>
<td>1</td>
<td>0.829</td>
<td>(0.012)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region II (Sep.)</td>
<td>$\alpha_2^{sep.}$</td>
<td>1</td>
<td>0.753</td>
<td>(0.007)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region III (Aug.)</td>
<td>$\alpha_3^{aug.}$</td>
<td>1</td>
<td>1.153</td>
<td>(0.013)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region III (Sep.)</td>
<td>$\alpha_3^{sep.}$</td>
<td>1</td>
<td>1.040</td>
<td>(0.006)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region IV (Aug.)</td>
<td>$\alpha_4^{aug.}$</td>
<td>1</td>
<td>0.919</td>
<td>(0.407)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Efficiency, Region IV (Sep.)</td>
<td>$\alpha_4^{sep.}$</td>
<td>1</td>
<td>0.998</td>
<td>(0.017)</td>
<td>n.a.</td>
</tr>
<tr>
<td>Driver Shocks</td>
<td>$\sigma_\varepsilon$</td>
<td>1</td>
<td>3.068</td>
<td>(1.512)</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

### Panel B: Equilibrium Summary

<table>
<thead>
<tr>
<th>Estimated Object</th>
<th>Number of Elements</th>
<th>Computed Value</th>
<th>Avg/Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = {v_t^i}$</td>
<td>4,212 (108 × 39)</td>
<td>See Figures A7, A8</td>
<td>161.4 / 5.29 / 562.7</td>
</tr>
<tr>
<td>${V_t^i}$ (non-airport i)</td>
<td>3996 (108 × 37)</td>
<td>See Figure A9</td>
<td>$186.6 / 0.45 / 367.5</td>
</tr>
<tr>
<td>${V_t^i}$ (airport i)</td>
<td>216 (108 × 2)</td>
<td>See Figure A9</td>
<td>$188.6 / 0 / 571.76</td>
</tr>
</tbody>
</table>

Panel A presents a summary of estimation results from both August and September 2012. Point estimates for matching efficiency parameters $\alpha_r$ correspond to Table 1 sections I–IV, respectively. I compute standard errors by resampling from the set of weekdays in the month, with replacement, to create a new 23-day sample of trips. I then solve and estimate the model for 150 monthly samples. The table reports estimates as the means and standard deviations of the parameters. Panel B shows computed equilibrium objects in the August 2012 sample.

### 5 Empirical Results

This section presents estimation results for the dynamic spatial equilibrium model. Table 4 Panel A shows estimation results for the per-period Poisson parameters for customer arrivals $\{\lambda_t^i\}$, as well as point estimates for additional parameters: the variance of unobservable shocks, $\sigma_\varepsilon$, and the four region-specific matching efficiency parameters $\alpha_r$. Panel B summarizes the computed equilibrium objects: the supply of vacant taxis across time and locations, $\{v_t^i\}$, and the corresponding value functions $\{V_t^i\}$ at each time and location.\[36\]

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\[36\] I provide figures with a full set of results in Appendix Figure A9.
This figure shows the supply and demand estimates for August 2012 averaged over time-of-day. The left panel shows the mean estimated per-period customer arrivals and the right panel shows the mean number of vacant taxis.

5.1 Spatial Distributions and Intra-day dynamics

Figure 5 depicts supply and demand for taxi rides across all locations, averaged across all periods of the day. Both taxi supply and passenger demand are most highly concentrated in the central part of Manhattan. For the most part, the number of vacant taxis is sufficient to meet demand in the absence of search frictions. The notable exception is in two central regions with very high demand, where average demand exceeds average supply.

Figure 7 provides an inter-temporal view of the results for two busy locations, Greenwich Village/SoHo and Central Midtown. Both graphs depict the equilibrium supply of vacant taxis, estimated arrival of customers looking for a taxi, the equilibrium number of matches, and the model’s fit against the observed number of matches in the data. Each series depicts the day-shift in 5-minute increments. Panel (a) shows that there are periods of relative oversupply and undersupply (compared to demand) of taxis at different times of day. Panel (b) shows an oversupply of taxis at the same time where there is an undersupply shown in Panel (a). This simultaneous over-supply and under-supply illustrates evidence of spatial misallocation as an equilibrium outcome: there is mismatch across locations, as across Panels 1 and 2. Within-location matching frictions are captured as the vertical space between the minimum of supply and demand at any point (i.e., \( \min\{v^t_i, \lambda^t_i\} \)) compared with matches (i.e., \( m^t_i \)).

\[ \text{[37] I provide results for more location in Appendix (A.9).} \]
Table 5 shows the five-region average per-period transition matrix of vacant taxis in Panel (a) and of all taxis, vacant and employed, in Panel (b). Comparing these two transitions shows how the spatial distribution of taxi supply is driven by customer destination preferences versus taxis’ search behavior. For example vacant taxis are much less likely to leave from their current location than an average taxi (vacant or not). The movement of vacant taxis also mirrors that shown in Table 3, Panel (b), which depicts the transitions from drop-off location to next pickup location. The fact that Table 5 shows slightly more dispersion (i.e. more weight on the off-diagonal elements) is natural: it reflects that the average time from drop-off to the next pickup is longer than a period (12 minutes 39 seconds, or about 2.5 periods).

The movement of hired and vacant taxis alike shifts the spatial distribution of vacant capacity across space. Figure 7 shows how these two types of spatial flows relate to each other. It aggregates taxi supply across all 39 locations into five regions (identical to those in Figure 1) and depicts the net flow of matches by region, defined as the sum of drop-offs minus pick-ups in each location, summed across all locations in each region. Panel (b) shows the net flow of taxis due to vacant taxis’ location choices by region. It shows, for example, that in the first half of the day employed
Table 5: Equilibrium Capacity Flows

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination Region I</th>
<th>Region II</th>
<th>Region III</th>
<th>Region IV</th>
<th>Region V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td>0.8007</td>
<td>0.1857</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0936</td>
</tr>
<tr>
<td>Region II</td>
<td>0.0643</td>
<td>0.7282</td>
<td>0.1534</td>
<td>0.0000</td>
<td>0.0540</td>
</tr>
<tr>
<td>Region III</td>
<td>0.0068</td>
<td>0.0203</td>
<td>0.9459</td>
<td>0.0022</td>
<td>0.0248</td>
</tr>
<tr>
<td>Region IV</td>
<td>0.2118</td>
<td>0.2452</td>
<td>0.4385</td>
<td>0.0914</td>
<td>0.0131</td>
</tr>
<tr>
<td>Region V</td>
<td>0.1294</td>
<td>0.2123</td>
<td>0.5277</td>
<td>0.0766</td>
<td>0.0415</td>
</tr>
</tbody>
</table>

(a) Vacant Taxis

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination Region I</th>
<th>Region II</th>
<th>Region III</th>
<th>Region IV</th>
<th>Region V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td>0.6372</td>
<td>0.2590</td>
<td>0.0281</td>
<td>0.0085</td>
<td>0.0671</td>
</tr>
<tr>
<td>Region II</td>
<td>0.0990</td>
<td>0.7162</td>
<td>0.1171</td>
<td>0.0041</td>
<td>0.0635</td>
</tr>
<tr>
<td>Region III</td>
<td>0.0126</td>
<td>0.0728</td>
<td>0.8901</td>
<td>0.0024</td>
<td>0.0221</td>
</tr>
<tr>
<td>Region IV</td>
<td>0.2005</td>
<td>0.2519</td>
<td>0.2751</td>
<td>0.2430</td>
<td>0.0295</td>
</tr>
<tr>
<td>Region V</td>
<td>0.1258</td>
<td>0.2839</td>
<td>0.4552</td>
<td>0.0865</td>
<td>0.0487</td>
</tr>
</tbody>
</table>

(b) All Taxis

Data in the table are aggregated across day-shift hours and regions as described in Figure 1. Panel (a) depicts the transition density of vacant taxis in each period and Panel (b) depicts the transition of vacant taxis between each drop-off and the same driver’s subsequent pickup.

Figure 9: Equilibrium Flow of Matches and Vacant Taxi Choices by Region

This figure shows the net flow of matches and vacant taxis for August 2012. The top panel shows the net flow of matches, defined as the sum of matches with destinations into each region minus the sum of pick-ups headed out of each region. The bottom panel similarly shows the net movement of vacant taxis into and out of locations within each region. Positive values therefore reflect a net inflow of vacant cabs in each location due to taxis dropping off customers (in Panel I) and previously vacant cabs (in Panel II).

taxi choices are traveling into Midtown and out of most other regions, while at the same time vacant taxis are proportionally exiting Midtown. Across the day, the choices of vacant taxis almost perfectly offset the movement of employed taxis. This pattern arises from drivers’ time- and location-specific policy functions which maintain a near-equality of equilibrium value functions across locations in each period.
5.2 Frictions

These results allow me to assess the impact of search frictions within and across locations. Aggregate excess demand over the course of the day-shift is given by \( \sum_{i,t} \left( \lambda_{t}^{i} - m_{i}(\lambda_{t}^{i}, v_{t}^{i*}; \alpha_{r}) \right) \). Total daily demand is for 249,552 trips whereas total matches in the data are 196,099, implying that 53,453 demanded customer trips are unmet each day, or an average 494 unmet trips each period. This contrasts with an average of 5,759 taxis that are vacant in each period, suggesting substantial search frictions on both sides of the market.

I can further decompose the equilibrium search frictions by attributing unmet demand to within-location versus across-location frictions. Within-location frictions are due to an imperfect matching technology (in this case, searching on the street). These frictions can be measured via \( \sum_{i,t} \left( \min(\lambda_{t}^{i}, v_{t}^{i*}) - m_{i}(\lambda_{t}^{i}, v_{t}^{i*}; \alpha_{r}) \right) \), where the first term reflects a frictionless matching technology. These are frictions that can be directly mitigated with better matching, for example through app-based ride-hail platforms. Within-location frictions amount to 40,038 unmet passengers, or 75% of the total number of unmatched passengers. The remaining 13,415 unmet trips are due to spatial mismatch between vacant drivers and passengers. Even with better technology, these frictions exist when supply and demand are farther apart and thus not readily matched. The residual spatial mismatch highlights the possibility of distortions in the way trips are priced, as demand and supply distributions are functions of price. Note that this decomposition is not equivalent to a true counterfactual analysis of changes to matching technology. I perform this type of counterfactual analysis in Section 6.

5.3 Demand Elasticities

Table 6 provides estimation results for the demand model in equation 1. As outlined in Section 4.2.5, I separately estimate the model for August 2012 and September 2012 following a change in the regulated tariff. I then leverage the change in prices over this period to identify price elasticities of demand across trips of differing lengths. An observation is an arrival-rate of customers within an origin-location, destination-location, and five-minute period during a weekday from 7a–4p. Table 6 reports price elasticities of passenger arrivals between -1.02 to -2.21. Shorter trips are more price elastic than longer ones. This is not surprising, as short distances tend to be better connected with public transit and more walkable and therefore there are more substitutable options with taxi service.

I compute welfare by first integrating demand over \( q \in [0, \lambda_{ij}^{t}] \) for each combination of origin, destination and hour. From this measure, welfare accrues only to the fraction of customers served, or \( m_{i}^{t}(\lambda_{i}^{t}, v_{i}^{t*}; \alpha_{r}) \). I illustrate these calculations in Figure 11. The area \( A \cup B \) reflects the entire

\[^{38}\]I provide explicit formulas in Appendix A.11.1
Table 6: Estimation results: Demand Elasticities for Non-airport Origins

<table>
<thead>
<tr>
<th>Description</th>
<th>Trip Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-2 mi.</td>
</tr>
<tr>
<td>Price Elasticity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.211**</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
</tr>
<tr>
<td>Pickup Location FE</td>
<td>X</td>
</tr>
<tr>
<td>Drop-off Location FE</td>
<td>X</td>
</tr>
<tr>
<td>Time-of-Day FE</td>
<td>X</td>
</tr>
<tr>
<td>Pickup Region × Hour FE</td>
<td>X</td>
</tr>
<tr>
<td>Time Trend</td>
<td>X</td>
</tr>
<tr>
<td>N</td>
<td>99,336</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.907</td>
</tr>
</tbody>
</table>

Demand data come from model estimates. The dependent variable is \( \log(\lambda_{ij}^t) \) and the independent variable of interest is \( \log(p_{ij}) \). Columns 1–4 report estimates for trips within non-airport locations. Column 5 reports estimates for trips that begin in a non-airport location and end at an airport. Standard errors are clustered at the level of origin-location.

available surplus in this market at price \( p_t^i \). The area \( B \) is the lost surplus due to frictions and random matching. \( A \) is therefore the realized welfare for each sub-market \((i,j,t)\). Note that \( A \) is an equilibrium object due to its dependency on \( p_{tij} \) and \( m_{tij} \). Since elasticity estimates are local, estimating consumer welfare at high prices involves extrapolation far out of sample. To ensure that welfare valuation is not driven by these observations I implement a choke price of \( \bar{p} = $100 \). Aggregate welfare is then computed as \( \sum_{i,j,t} A_{ijt}(p_0) \) where \( p_0 \) reflects trip prices in each \( i,j,t \) sub-market given the observed tariff pricing.

Total estimated welfare for each weekday, day-shift is shown in Table 7. Consumer welfare for New York taxi service is $2.20M per day-shift. Taxi profits in each shift are $3.36M, or $292 per driver.\(^{39}\) There is substantial heterogeneity by time, place and trip length. For example, hourly consumer surplus increases from morning to afternoon. The consumer welfare and profits accrued from trips in Manhattan are vastly higher than those in Brooklyn, and shorter trips make up the bulk of welfare and profits. This variation suggests that if different pricing regimes affect the spatial allocation of supply and demand, then we should expect them to also impact aggregate welfare.

\(^{39}\)Drivers also must pay a leasing fee imposing costs-per-shift of around $100-125 per day, for which detailed data were not available. Including these fees implies daily profits around $180.
Figure 11: Consumer Surplus with Frictions

This figure depicts how welfare is calculated in each sub-market \((i, j, t)\) in the presence of search frictions under random matching. The depicted shape of each curve is for illustration purposes; the scale and curvature of demand differs in each sub-market.

Table 7: Estimated Results: Daily, Single-Shift Welfare Measures

<table>
<thead>
<tr>
<th>Subset Type</th>
<th>Subset</th>
<th>Cons. Surplus ($, thousand)</th>
<th>Taxi Profits ($, thousand)</th>
<th>Matches (thousand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>-</td>
<td>2204.9</td>
<td>3361.9</td>
<td>216.2</td>
</tr>
<tr>
<td>By Time-of-day</td>
<td>7a-9a</td>
<td>395.3</td>
<td>728.9</td>
<td>44.8</td>
</tr>
<tr>
<td></td>
<td>9a-11a</td>
<td>539.0</td>
<td>738.8</td>
<td>48.7</td>
</tr>
<tr>
<td></td>
<td>11a-1p</td>
<td>585.6</td>
<td>737.0</td>
<td>48.3</td>
</tr>
<tr>
<td></td>
<td>1p-4p</td>
<td>927.0</td>
<td>1192.8</td>
<td>77.3</td>
</tr>
<tr>
<td>By Origin Region</td>
<td>Sec. I</td>
<td>477.6</td>
<td>642.8</td>
<td>37.2</td>
</tr>
<tr>
<td></td>
<td>Sec. II</td>
<td>1211.2</td>
<td>1604.7</td>
<td>105.0</td>
</tr>
<tr>
<td></td>
<td>Sec. III</td>
<td>724.2</td>
<td>1053.9</td>
<td>67.0</td>
</tr>
<tr>
<td></td>
<td>Sec. IV</td>
<td>33.8</td>
<td>96.1</td>
<td>4.3</td>
</tr>
<tr>
<td>By Trip-Length</td>
<td>0-2 mi.</td>
<td>1180.2</td>
<td>1526.2</td>
<td>127.6</td>
</tr>
<tr>
<td></td>
<td>2-4 mi.</td>
<td>865.2</td>
<td>1045.1</td>
<td>60.5</td>
</tr>
<tr>
<td></td>
<td>4-6 mi.</td>
<td>310.6</td>
<td>364.7</td>
<td>14.7</td>
</tr>
<tr>
<td></td>
<td>6+ mi.</td>
<td>90.7</td>
<td>461.6</td>
<td>16.2</td>
</tr>
</tbody>
</table>

This table depicts welfare measures decomposed by category. Consumer welfare is summed across all \(i, j, t\) in each category. I compute taxi profits from total matches multiplied by prices for each origin, destination, and time-of-day. Profits reflect daily, single-shift revenues net of fuel costs. Sections I–V refer to those in Fig. 1.
6 Counterfactual: Pricing for Dynamic Efficiency

Aside from certain airport trips, New York TLC regulations on yellow cabs impose the same flag-drop tariff and distance tariff on all trips. Since demand is heterogeneous across neighborhoods, static inefficiencies due to mis-pricing across neighborhoods will naturally exist. In addition the prevailing equilibrium in the New York taxi market involves spatial misallocation between cabs and passengers. Misallocation in this context is a dynamic inefficiency caused by distorted search incentives (drivers are profit-maximizing, but not social-surplus maximizing) and consumer prices that do not internalize the cost of re-allocating future supply. Since both supply-side and demand-side incentives are functions of prices, a coarse pricing policy may also lead to misallocation. Thus, the static and dynamic inefficiency are intimately linked from a welfare perspective. It is therefore insufficient to independently optimize prices at each location and in each time-period.

This section studies a set of simple and easily implemented changes to existing tariffs in the New York market with the purpose of generating better welfare outcomes consistent with the equilibrium dynamics of supply and demand. The changes introduce price flexibility with respect to location, time-of-day, or distance. Flexibility along these dimensions plays two important roles. First, it offers the efficiency of a price mechanism to better clear markets. Second, flexible prices can lead to an endogenous spatial re-allocation of empty capacity to different regions of the city. I show that it is possible to increase both profits and consumer welfare by optimally implementing each type of flexible tariff.

To study flexible tariffs, I create three possible regimes: location-based pricing, time-based pricing and distance-based (non-linear) pricing. In the first regime, I allow prices to vary in segments of 2–3 hours across the day (the four segments are 7a–9a, 9a–11a, 11a–1p, 1p–4p). In the second experiment, I allow prices to vary in each of four areas, depicted as zones I–IV in Figure 1. The third experiment considers non-linear pricing, where prices can vary along three dimensions: the fixed fare, the per-mile fare, and a squared-distance fare. For each, I search across a set of multipliers to existing fares associated with each combination of origin, destination and time. A description of each is outlined in Table 8. For each candidate set of multipliers, I recompute origin-destination-specific demand, profits, and transitions, and I resolve the dynamic spatial equilibrium among taxis.

To account for the potential impact of changing wait times among consumers, I calibrate waiting time elasticities to be -1.0, close to the estimates of Frechette, Lizzeri, and Salz (2019) and Buchholz, Doval, Kastl, Matejka, and Salz (2019). I compute a measure of waiting time as the mass of unmatched consumers in each period multiplied by the period length. Given these two measures I

\[ \text{Waiting time} = \text{Mass of unmatched consumers} \times \text{Period length} \]

For example, if at the fare structure of $2.50 + $2.00/mile, a trip from location $i$ to $j$ costs $10.00, a multiplier of 0.8 on trips from $i$ to $j$ would change this fare to $8.00.
Table 8: Description of Counterfactual Policies and Multipliers

<table>
<thead>
<tr>
<th>Counterfactual Regime</th>
<th>Multiplier Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location-Based Pricing</td>
<td>$\theta_k \cdot q_{ij}^k$ for $i \in \text{Region } k$</td>
<td>$\theta_2 = 0.85 \Rightarrow 15% \text{ discounted baseline price } q_{ij}^k$ for trips starting in Region II</td>
</tr>
<tr>
<td>Time-Based Pricing</td>
<td>$\theta_k \cdot q_{ij}^t$ for $t \in k^{th}$ element of {7-9a, 9-11a, 11a-2p, 2-4p}</td>
<td>$\theta_2 = 0.85 \Rightarrow 15% \text{ discounted baseline price } q_{ij}^t$ for all trips during 9-11a</td>
</tr>
<tr>
<td>Non-Linear Pricing</td>
<td>$\theta_0 \cdot \text{flag fare} + \theta_1 \cdot \text{dist. fare} + \theta_2 \cdot \text{dist.}^2$</td>
<td>$\theta_1 = 0.9, \theta_2 = 0.9, \theta_3 = -0.1 \Rightarrow$ new fare = $1.80 + 2.25/\text{mi.} - 0.1/\text{mi}^2$</td>
</tr>
</tbody>
</table>

This table details the three sets of tariff pricing rules considered in the counterfactual analysis. Prices are denoted $q_{ij}^t$ and refer to the baseline fare price for a trip from $i$ to $j$ at time $t$, where the baseline price is set at August 2012 levels of $2.00$ fixed fare + $2.50$ per-mile.

endogenize the demand response with respect to the percentage change in waiting times, which I compute as the percentage difference between the waiting time at baseline prices and the waiting time at any counterfactual price. I provide details of this procedure in Appendix A.11.1.

Within each flexible price regime, I numerically solve for an optimal configuration according to three different criteria: maximize total surplus, maximize consumer surplus, and maximize the total number of trips. The different criteria represent different planner objectives. Because this paper does not study the extensive margin of taxi drivers’ labor supply decision, I constrain the numerical search for optimal prices to outcomes in which taxi revenues are at least as high as the baseline case. If I assume that the regulatory cap is binding during the weekday, then this constraint ensures that the same number of drivers will be on the road in each counterfactual. If the cap were lifted, additional drivers would weakly increase supply-side profits and benefit consumers with lower waiting times, so my current results would constitute a lower-bound. In addition, as a benchmark I compare each result with the equilibrium welfare induced by a frictionless matching technology at baseline prices. In this counterfactual, the matching function is changed to a Leontief function of taxis and consumers. This technology is representative of modern app-based matching that matches taxis with consumers when they are close to each other.

6.1 Results

Table 9 displays the results of the dynamically constrained-efficient prices associated with location-based, time-based and distance-based fares. Under efficient pricing, prices will fall by around 10–20% in almost all cases. Even then, driver profits improve as utilization rates increase 6–10%.

41 This avoids pathological solutions that would set driver profits to zero even though they are assumed to continue working.
Total welfare gains up to 8.2% and consumer welfare gains up to 20.7% are possible. Interestingly, different planner objectives within a price regime often coincide: given each of the three tariff structures, the optimal prices maximize total and consumer surplus. Nearly all regimes improve taxi utilization rates, which highlights the role of flexible prices in allocating spare capacity to more productive areas. In addition, the optimal prices within each regime give a higher share of welfare to consumers by around 3–5%. Total welfare gains are on the order of $270–460 thousand per day, accounting for just the day-time shift, between $1.03–1.65 per trip. Matches grow substantially, consistent with lower average prices introducing marginal consumers to the market.\textsuperscript{42}

The final row of Table 9 contrasts these results with a counterfactual where prices are set to the baseline tariff but the within-location matching technology is frictionless, given by \( m(\lambda^t_i, v^t_i) = \min(\lambda^t_i, v^t_i) \). This model approximates a modern ride-share platform, where local supply and demand are guaranteed to find one another, but taxis must still choose locations to search. Consumer welfare benefits from this technology are somewhat larger than the benefits to optimal pricing and much better for taxis. The best performing tariff I study reaches about 57\% of the benefits of this technology. Interestingly, it only generates marginally more matches than the optimal tariff; in contrast to the matching technology counterfactual, lower average prices under the new tariffs induce additional customers to enter the market. Unsurprisingly, utilization rates are highest under the frictionless matching technology.\textsuperscript{43}

Comparing the matching technology counterfactual with the optimal tariff regimes illustrates the relative gains from improving within-location frictions compared with across-location frictions. Perfect matching within a location generates predictably large gains and provides a rationale for the growth and popularity of modern ride-hail services. However, allocating supply and demand to the right place and time is also highly beneficial, particularly to consumers, offering welfare gains and improvements to utilization that approach the benefits of the frictionless technology.\textsuperscript{44}

\subsection*{6.2 Discussion: Comparing Tariff Changes with Real-time Pricing}

The pricing counterfactuals in Table 9 are configured to average patterns of supply and demand. Implementing this type of price regime would not require any special technology; the regulator could simply post new tariffs that differ by time or location, both of which have been applied in various cities, such as a zone-based pricing system that used to exist among regulated taxis in Washington,\textsuperscript{45}

\textsuperscript{42}The flexibility results corroborate theoretical insights of Schmalensee (1981) and Varian (1985) which find that a necessary condition for price discrimination to enhance social welfare is that it accompanies an increase in output.

\textsuperscript{43}Note that this counterfactual does not simulate gains from other attributes of ride-sharing services such as the value of less waiting, the certainty of a match, app-based payments, etc.

\textsuperscript{44}The benefits need not be exclusive to traditional taxis, however; ride-hail firms could also implement similarly flexible baseline fares to allocate supply and demand across broader regions, potentially improving capacity utilization and market welfare beyond what I study here.
### Table 9: Efficient Pricing and Matching Technology: Counterfactual Results

<table>
<thead>
<tr>
<th>Price Type</th>
<th>Efficiency-optimized Multipliers</th>
<th>Total Surplus (,000 USD)</th>
<th>Consumer Surplus (,000 USD)</th>
<th>Consumer Rent Share (percent)</th>
<th>Matches (,000)</th>
<th>Taxi Utilization (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline - 8/2012</strong></td>
<td>1.00 1.00 1.00 1.00</td>
<td>5966.8</td>
<td>2264.9</td>
<td>39.6</td>
<td>216.2</td>
<td>43.4</td>
</tr>
<tr>
<td><strong>Location-Based Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>0.89 0.82 0.70 1.19</td>
<td>6024.0 (+8.2 %)</td>
<td>2661.9 (+20.7 %)</td>
<td>44.2</td>
<td>276.6 (+28.0 %)</td>
<td>53.7</td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>0.89 0.82 0.70 1.19</td>
<td>6024.0 (+8.2 %)</td>
<td>2661.9 (+20.7 %)</td>
<td>44.2</td>
<td>276.6 (+28.0 %)</td>
<td>53.7</td>
</tr>
<tr>
<td>Max Matches</td>
<td>0.89 0.82 0.70 1.19</td>
<td>6024.0 (+8.2 %)</td>
<td>2661.9 (+20.7 %)</td>
<td>44.2</td>
<td>276.6 (+28.0 %)</td>
<td>53.7</td>
</tr>
<tr>
<td><strong>Time-Based Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>0.84 0.83 0.90 0.90</td>
<td>5834.0 (+4.8 %)</td>
<td>2471.9 (+12.1 %)</td>
<td>42.4</td>
<td>252.2 (+16.7 %)</td>
<td>49.6</td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>0.84 0.83 0.90 0.90</td>
<td>5834.0 (+4.8 %)</td>
<td>2471.9 (+12.1 %)</td>
<td>42.4</td>
<td>252.2 (+16.7 %)</td>
<td>49.6</td>
</tr>
<tr>
<td>Max Matches</td>
<td>0.72 0.90 0.90 0.90</td>
<td>5830.3 (+4.7 %)</td>
<td>2467.6 (+11.9 %)</td>
<td>42.3</td>
<td>255.4 (+18.1 %)</td>
<td>50.2</td>
</tr>
<tr>
<td><strong>Non-Linear Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>0.90 0.65 -0.10</td>
<td>6004.0 (+7.9 %)</td>
<td>2629.1 (+19.8 %)</td>
<td>44.0</td>
<td>267.2 (+23.6 %)</td>
<td>53.4</td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>0.90 0.65 -0.10</td>
<td>6004.0 (+7.9 %)</td>
<td>2629.1 (+19.8 %)</td>
<td>44.0</td>
<td>267.2 (+23.6 %)</td>
<td>53.4</td>
</tr>
<tr>
<td>Max Matches</td>
<td>0.70 0.99 0.10</td>
<td>5975.4 (+7.4 %)</td>
<td>2614.0 (+18.6 %)</td>
<td>43.7</td>
<td>281.0 (+30.0 %)</td>
<td>52.5</td>
</tr>
<tr>
<td><strong>Matching Technology</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Prices</td>
<td>1.00 1.00 1.00 1.00</td>
<td>7393.1 (+32.8 %)</td>
<td>3005.5 (+36.3 %)</td>
<td>40.7</td>
<td>281.9 (+30.4 %)</td>
<td>56.2</td>
</tr>
</tbody>
</table>

This table shows, for each weekday day-shift period, the estimated change in total welfare (profits plus consumer surplus), consumer surplus, consumers’ share of total surplus, total matches, and utilization rates across each counterfactual price policy. Each pricing policy shown is a rule that applies to four policy-specific multipliers on the baseline price $p_{ijt}$ for every route, given by $2.50 + $2.00/mile. In location-based pricing, the multipliers $\theta_i(t)$ apply to $p_{ijt}$ where $k(i)$ ∈ {1, 2, 3, 4} indexes the region of location $i$ according to [1]. In time-based pricing, the multipliers $\theta_k(t)$ apply to $p_{ijt}$ where $k(t)$ ∈ {1, 2, 3, 4} indexes the time ranges of 7a–9a, 10a–11a, 12p–1p, 2p–4p. In non-linear pricing, the multipliers $\theta_k(i,j)$ apply to $p_{ijt}$ where $k(i,j)$ ∈ {1, 2, 3} are coefficients that change existing tariffs according to $\theta_1 \cdot \text{base fare} + \theta_2 \cdot \text{fare per-mile} + \theta_3 \cdot \text{miles}^2$. The final row depicts equilibrium outcomes under a simulated matching technology in which the matching function takes the form $m^*_t = \min(\lambda^*_t, v^*_t)$. This last counterfactual is computed at baseline prices.

D.C. In contrast, a common feature of ride-hail platforms is to use real-time pricing across different neighborhoods or times (e.g., Uber’s “Surge Pricing”). Real-time pricing re-adjusts prices in each period to accommodate unexpected shifts in supply or demand. However, conventional real-time pricing is not forward-looking. If it were forward-looking then the approach taken in the above counterfactuals would apply. Nevertheless, better average pricing of the form indicated in Table 9 may enable the regulator to alleviate some of the need for additional real-time pricing, as supply and demand will be more efficiently allocated across space and time in a way that is consistent with dynamic evolution of supply from one period to the next.

## 7 Conclusion

Supply and demand in the taxi market are uniquely shaped by space. Regulation influences how taxis and their customers search for one another and how often they find each other. This paper...
models a dynamic spatial equilibrium in the search and matching process between taxis and passengers, showing how both supply and demand can be recovered from data on intra-daily spatial matches. Using data from New York yellow taxis, I estimate this model to recover the expected spatial and inter-temporal distribution of taxis and mean customer arrivals. By using variation in prices, I estimate demand across 5-minute intervals of the day-shift and across 39 spatial regions within New York. Estimating demand in this way allows me to measure the magnitude and sources of search frictions across space and time-of-day. I show that total welfare attained in the New York market is $5.7 million per day-shift on a typical weekday with about 40% of this surplus accruing to consumers.

The estimated model also allows me to recompute the equilibrium taxi supply, spatial matches, search frictions and welfare outcomes under alternative price schedules and a frictionless matching technology. I show that by optimally configuring tariff prices to vary according to spatial regions, vacant taxi capacity can be endogenously and dynamically relocated where it is valued the most, providing up to 8.2% more welfare and 28% more trips. A more sophisticated tariff could offer different prices across a combination of location, time and distance. My counterfactual estimates suggest that additional gains might be possible in this regime, although they would come at the expense of simplicity. The gains due to optimal dynamic pricing are comparable to the gains due to better inter-location matching, suggesting the importance of implementing effective tariff pricing even with the rise of modern ride-hail platforms.
Bibliography


A Appendix

A.1 Data Cleaning

Taxi trip and fare data are subject to some errors from usage or technology flaws. I first drop any apparently erroneous observations (e.g. well outside of the New York Area). Next, I drop observations outside of the locations of interest, Manhattan and the two airports. This section describes how data are cleaned and provides some related statistics.

Data Cleaning Routine

1. Begin with merged trip and fare data from August 2012 to September 2012.
2. Drop observations outside of USA boundaries.
3. Drop observations outside of the New York area.
4. Drop duplicates in terms of taxi driver ID and date-time of pickup.
5. Drop observations outside of Manhattan (bounded above by 125th st.) or either airport.
6. Drop observations that cannot be mapped to any of the 39 locations in Figure 1

Table A1 shows the incidence of each cleaning criterion.

Table A1: Data Cleaning Summary

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Criterion Applied</th>
<th>Obs. Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop Errors</td>
<td>1. Initial data</td>
<td>28,927,944</td>
</tr>
<tr>
<td></td>
<td>2. Obs. outside USA</td>
<td>-749,623</td>
</tr>
<tr>
<td></td>
<td>3. Obs. outside NYC</td>
<td>-5,298</td>
</tr>
<tr>
<td></td>
<td>4. Drop duplicates</td>
<td>-57</td>
</tr>
<tr>
<td></td>
<td>5. Keep manhattan + airports</td>
<td>-3,622,803</td>
</tr>
<tr>
<td></td>
<td>6. Un-mapped data</td>
<td>-117,249</td>
</tr>
</tbody>
</table>

Final Data Set: 24,432,914 observations

This table summarizes the data cleaning routine for TLC data from 8/1/2012-9/30/2012.

A.2 Map Preparation

I generate the 39 spatial locations shown in Figure 1 by uniting census tracts, representing 98% of all taxi ride originations. While there is some arbitrariness involved in their exact specification, the number of locations used is a compromise between tradeoffs; more locations give a richer map of
spatial choice behavior, but impose greater requirements on both data and computation. Because of the sparsity of data in the other boroughs, I focus on the set of locations falling within Manhattan below 125th street, three nearby areas within Brooklyn and Queens, and the two New York City airports, Laguardia and J.F.K.

The following graphics show how I convert raw GPS data points into locations.\textsuperscript{45} I begin with New York census tracts, 425 of which cover the locations of interest. From these, I examine taxi activity, and group census tracts into areas with clusters of activity. Figure A1 shows the origin of each trip in a 10-percent sample of TLC data. It can be seen that trip origins are most heavily concentrated around major streets, particularly north-south and diagonal thoroughfares in the north, with more scattered origin points in lower Manhattan and Midtown Manhattan. The densest neighborhoods are clearly those in Midtown. I have grouped census tracts to form locations in a way that attempts to minimize the number of location boundaries that overlap clusters of activity, for example the clusters around a busy transit station.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figureA1.png}
\caption{Mapping GPS points to Locations}
\end{figure}

This figure shows TLC data for a 10 percent sample of taxi trips taken in August 2012. Each dot on the map is the GPS origin of a trip.

\textsuperscript{45}This association is achieved via the point-in-polygon matching procedure outlined in Brophy (2013).
A.3 Summary Statistics by Month

Table A2 decomposes the trip and fare summary statistics by month, before and after the fare change.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rate Type</th>
<th>Variable</th>
<th>Obs.</th>
<th>10%ile</th>
<th>Mean</th>
<th>90%ile</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>4,299,644</td>
<td>4.50</td>
<td>9.20</td>
<td>15.7</td>
<td>5.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>4,299,645</td>
<td>1.04</td>
<td>4.19</td>
<td>8.96</td>
<td>4.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>4,299,645</td>
<td>2.50</td>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>4,299,645</td>
<td>0.72</td>
<td>2.29</td>
<td>4.68</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>4,299,645</td>
<td>4.00</td>
<td>12.25</td>
<td>22.48</td>
<td>7.95</td>
</tr>
<tr>
<td>JFK Fares</td>
<td></td>
<td>Total Fare ($)</td>
<td>85,531</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>85,531</td>
<td>1.87</td>
<td>15.99</td>
<td>20.88</td>
<td>6.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>85,531</td>
<td>26.00</td>
<td>45.27</td>
<td>65.83</td>
<td>19.27</td>
</tr>
<tr>
<td></td>
<td>Standard Fares</td>
<td>Total Fare ($)</td>
<td>3,823,147</td>
<td>5.00</td>
<td>11.23</td>
<td>20.00</td>
<td>7.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dist. Fare ($)</td>
<td>3,823,149</td>
<td>1.20</td>
<td>5.17</td>
<td>11.00</td>
<td>5.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flag Fare ($)</td>
<td>3,823,149</td>
<td>2.50</td>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>3,823,149</td>
<td>0.70</td>
<td>2.27</td>
<td>4.65</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>3,823,149</td>
<td>4.12</td>
<td>13.30</td>
<td>25.0</td>
<td>9.01</td>
</tr>
<tr>
<td>JFK Fares</td>
<td></td>
<td>Total Fare ($)</td>
<td>85,692</td>
<td>52.00</td>
<td>51.56</td>
<td>52.00</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance (mi.)</td>
<td>85,692</td>
<td>3.42</td>
<td>16.29</td>
<td>20.93</td>
<td>5.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trip Time (min.)</td>
<td>85,692</td>
<td>26.82</td>
<td>46.04</td>
<td>68.42</td>
<td>19.05</td>
</tr>
</tbody>
</table>

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to individual taxi trips taken in New York City in the months of August 2012 and September 2012 for two fare types. The first is the standard metered fare (TLC rate code 1), in which standard fares apply. The second is a trip to or from JFK airport (TLC rate code 2). Total Fare and Distance data are reported for each ride. While not reported directly or separated from waiting costs, I predict distance and flag fares using the prevailing fare structure on the day of travel and the distance travelled.

A.4 Medallion Counts

Figure A2 shows the unique number of medallions observed each day of August and September 2012 in the TLC data during weekdays during the day shift. The mean across all days is 11,911.88. It should be noted however, that about 2% of trips occur outside of the 39 locations defined in this paper during this period. This implies that approximately 11,673 medallions are active within the locations, with some additional diminishment in reality due to breaks, refueling, etc. The second point of this figure is that the medallion counts seem fairly stable between price changes, lending support for the assumption that this overall level remains constant. The drop on September 3rd seems to reflect the extra servicing of metering equipment just prior to the tariff change on September 4th.
This figure depicts the unique number of medallions observed each day of August and September 2012 in the TLC data during weekdays during the day-shift.

Figure A3 illustrates the time-of-day appearance of medallions by showing the number of active taxis by hour beginning at 6am. To compute this, I first calculate the hours in which a taxi is on a shift by using the shift definitions common in the literature, which specify that a driver is on a shift when he shows up in the data (i.e., a ride is given by the driver) and remains on shift until a gap of five or more hours between observed rides occurs. Next, I count how many taxis are working a shift in each hour and plot this count by hour. Note that levels shown will be smaller than the total medallion count of 13,237. One reason is that medallions will not appear immediately when taxis enter the shift because it takes time to find passengers. Thus, there may be significant downward bias in the mornings.

A.5 Details on State Transitions

The combined set of transitions forms an aggregate transition kernel that defines the law-of-motion, given by $Q(S^{t+1}|S^t) = \nu(v_e^{t+1}|v_e^t, M^t, m^t) + \mu(v_v^{t+1}|v_v^t, \sigma^t)$. Here I use addition instead of union notation to signify that the transitions of both employed and vacant cabs lead to new stocks of vacant cabs.

Let the following objects be defined:

$v_e^t$ be the $(L + K) \times 1$ vector of employed cabs at the start of period $t$, where $L$ is the total number of search locations and $K$ is the total number of positions between locations (e.g., if a route takes 4 periods to travel, there is a pickup-location $i$, 2 in-between positions, and a drop-off location $j$). $m^t$ is the $(L + K) \times 1$ vector of matches in period $t$, where the first $L$ entries are
This figure is derived from August 2012 TLC trip data. It shows the number of unique medallions present in the data by hour-of-day, where presence is determined by finding the first- and last-appearance times within the day-shift, and counting each taxi as active in the hours between (inclusive of end-points). It approximates the total number of taxis working during the day shift, by hour, averaged across each day in the data set. Note that earlier hours will be systematically downward biased because drivers who begin a shift are not observed until finding their first passenger.

The matches in each location and the next $K$ entries are zeros (as no matches occur while cabs are employed and in-transit. $M_e^t$ be the $(L + K) \times (L + K)$ vector of one-period transition probabilities of customers from all locations and all in-between positions. The number of in-between positions is based on the mean number of periods it takes to travel from any locations $i$ to $j$, rounded to the nearest period (e.g., an average 16-minute trip would be considered 3.2 periods, and then rounded to be 3 periods, with a single in-between position). $m^{t-\tau_{ji}}$ describes how many drop-offs will occur in period $t$, which is the number of matches made in each pick-up location in $\tau_{ji}$ prior periods, and transition matrix $M_e^{t-\tau_{ji}}$ re-distributes those earlier matches to locations at time $t$.

Given these objects, we can write the state transitions of employed cabs as follows, reflecting the transitions of new matches and already-employed taxis at time $t$, minus the time $t$ drop-offs:

$$v_{e+1}^t = ((v_e^t + m^t) \times M_e^t) - (m^{t-\tau_{ji}} \times M^{t-\tau_{ji}}).$$  \hspace{1cm} (15)

Next, I define the state transitions of vacant taxis. Let $v_v^t$ be the $(L + K) \times 1$ vector of vacant taxis in all search locations and in-between locations. Note that there may be taxis in the in-between locations. For example, driving vacant to the airport may take more than one period. Let $v_v^t$ be the $(L + K) \times (L + K)$ vector of one-period transition probabilities of vacant taxis from all locations and all in-between positions. Then the state transitions of vacant cabs is given by the vector of vacant cabs at the start of period $t$ minus the period $t$
matches, multiplied by the policy functions in each period:

\[ v^{t+1}_v = (v^t_v - m^t) \times \sigma^t. \]  

(16)

Summing these two transition formulas defines the state transitions from \( t \) to \( t + 1 \).

A.6 Proofs

Proof of Proposition 3.1

Part (i): Equilibrium Existence

Proof. In the last period in which actions may be taken, \( t = T-1 \), policy functions are simply determined by the payoffs attributable to search in the final period. The sum of policies among agents in a particular location are summarized by the following

\[ \sigma^{T-1}_{ij} \equiv P^{T-1}_{i}[j \mid \mathcal{S}^{T-1}] = \frac{\exp \left( p_j(\lambda^T_{ij}, v^T_{ij}) \cdot \sum_l \Pi_{jl}/\sigma - c_{ij} \right)}{\sum_k \exp \left( p_k(\lambda^T_{ik}, v^T_{ik}) \cdot \sum_l \Pi_{kl}/\sigma - c_{ik} \right)}. \]

(17)

Further note that the final period state \( v^T_{ij} \) can be described by \( v^T_{ij} = \sum_i (v^{T-1}_i - m^{T-1}_i) \times \sigma^{T-1}_{ij} + \left( \sum_{i, \tau} m^{T-\tau}_{ij} M^{T-\tau}_{ij} \right) \) from the definition of the transition kernel (see Section A.5). Let \( \sigma^{T-1} = \sigma^{T-1}_{ij} \) for all \( i \in \{1, \ldots, L\} \) and all \( j \in \{1, \ldots, L\} \). In prior periods, policies are similarly defined except for the inclusion of continuation values, which are bounded below by zero and above by the maximum fare revenue attainable in the remaining periods up to \( T \). For each period let \( F(\sigma^t) \) define the above mapping between the vector \( \sigma^t \in \mathbb{S}^{L \times L} \) to itself, where \( \mathbb{S}^n \) denotes the \( n \)-dimensional unit ball. Direct application of Brouwer’s Fixed-Point theorem implies the existence of a fixed point in \( F \), which by repeated application for \( t = 1, \ldots, T \) implies that the equilibrium \( \hat{\sigma}^t \) for \( t = 1, \ldots, T \) exists.

Part (ii): Equilibrium Uniqueness

Proof. To show that the fixed point defining equilibrium is unique, suppose there are two such equilibria given by \( \sigma^1_{T-1} \) and \( \sigma^2_{T-1} \). Without loss of generality this implies that there exists some \( i, j \) for which \( \sigma^1_{T-1,ij} > \sigma^2_{T-1,ij} \). Thus the ratio defined in equation (17) is greater for equilibrium 1. Since all objects indexed by \( i \) in this ratio are taken to be exogenous and fixed (e.g., \( c_{ij} \)), this further implies that \( \sigma^1_{T-1,ij} > \sigma^2_{T-1,ij} \) for all \( i \). In turn equation (16) defining vacant state transitions implies that \( v^T_j \) is higher under equilibrium 1 than in equilibrium 2. Since \( p_j(\lambda^T_j, v^T_j) \) is decreasing in \( v^T_j \), and since \( \sum v^T_j = \bar{v} \) (i.e. the total number of taxis in the city is exogenous and fixed across equilibria), the following must hold:
\[
\sum_k \exp \left( p_k(\lambda_{1,k}^T, v_{1,k}^T) \cdot \sum_l \Pi_{kl}/\sigma_e - c_{ik} \right) < \sum_k \exp \left( p_k(\lambda_{2,k}^T, v_{2,k}^T) \cdot \sum_l \Pi_{kl}/\sigma_e - c_{ik} \right).
\]

However, since for all \(i\), \(\sum_k \sigma_{ik} = 1\) by construction, for some \(l \neq j\) we have \(\sigma_{1,l}^T < \sigma_{2,l}^T\). By the same logic as above, equation 17 then implies that \(p_l(1) > p_l(2)\). Thus, it must be that

\[
\sum_k \exp \left( p_k(\lambda_{1,k}^T, v_{1,k}^T) \cdot \sum_l \Pi_{kl}/\sigma_e - c_{ik} \right) > \sum_k \exp \left( p_k(\lambda_{2,k}^T, v_{2,k}^T) \cdot \sum_l \Pi_{kl}/\sigma_e - c_{ik} \right).
\]

This implies a contradiction, so there cannot exist two equilibria to this system. This proves that policies \(\sigma^T(S^T-1)\) and therefore also values \(V(S^T)\) are unique.

To show that uniqueness of values and policies in period \(t + 1\) implies uniqueness in period \(t\), I first show that two conditions hold.

**Condition 1:** Flow payoffs defined by equation 6 are bounded, continuous, and strictly concave with respect to probabilities over actions \(\sigma^T_i\).

To show this, I first re-write the choice-specific value functions (8). For exposition it will be convenient to re-write current location \(i\) and choice-location \(k\) as arguments instead of subscripts.

\[
W^t(i, k, S) = p(k, S) \cdot \bar{\pi} + (1 - p(k, S)) \cdot (-c(i, k)) + \epsilon(a, k) + \left( p(k, S) \cdot V^{t+\tau(k,k')}(k', S) + (1 - p(k, S)) \cdot \arg \max_{\ell \in A(k)} W^{t+\tau(k,\ell)}(k, \ell, S) \right), \quad (18)
\]

Where \(\bar{\pi}(k) = \sum_j M_{kj} \Pi_{kj}\). The above expression explicitly separates period payoff functions from continuation values. To demonstrate concavity of the former, denote period payoffs as follows:

\[
F(i, k, S) = p(k, S) \cdot \bar{\pi}(k) + (1 - p(k, S)) \cdot (-c(i, k)) + \epsilon(a, k).
\]

Concavity implies that for \(\gamma \in (0, 1)\) and \(k, k' \in A(i)\), that

\[
F(i, \gamma k + (1 - \gamma)k', S) > \gamma F(i, k, S) + (1 - \gamma)F(i, k', S).
\]

Since profits \(\bar{\pi}(k) > 0\) and fuel costs \(-c(i, j) < 0\), and since location choice \(k\) only indexes a discrete point of \(\bar{\pi}, c(i, k), \) and \(\epsilon(a, k)\), it is sufficient to show that \(p(k, S)\) is concave. Let \(v(k)\) be the \(k\)-th element of the state \(S\) and \(\lambda(k)\) an exogenous parameter for location \(k\). Then
\[ p(k, S) = 1 - \exp \left( \frac{\lambda(k)}{\nu(k)} \right). \]

Thus, concavity of \( p(k, S) \) follows from the convexity of \(-p(\cdot, \cdot)\) via the convexity of \( \exp(\cdot) \).

**Condition 2:** Feasibility constraints \( \Gamma(S) \) on vacant state transitions are nonempty, compact and convex.

\( \Gamma(S^t) \) represents the constraint set of possible actions to be taken by drivers. Constraints are defined to be the set of locations \( A(i) \) adjacent to drivers in location \( i \). Non-emptiness follows from the adjacency of each location with itself. Compactness follows from the finite measure of agents choosing over a finite set of adjacent locations. To show that \( \Gamma(S) \) is convex, let \( s_1 \in \Gamma(S_1) \) and \( s_2 \in \Gamma(S_2) \). \( s_j \) is a feasible allocation of vacant taxis starting from \( S_j \). Given that vacant taxis are non-atomic, it follows that for any \( a > 0 \), \( a \cdot s_j \in \Gamma(a \cdot S_j) \). Further, \( (s_j + s_k) \in \Gamma(S_j + S_k) \) as one can label drivers belonging to either set and independently assign feasible allocations according to \( \Gamma(\cdot) \). Therefore it follows that \((a \cdot s_j + (1 - a) \cdot s_k) \in \Gamma(a \cdot S_j + (1 - a) \cdot S_k) \).

Together these conditions imply that the value functions defined by equation 6 are strictly concave (c.f., Theorem 4.8 in Stokey and Lucas (1989)), and therefore since best response functions defined by equation 9 are single-valued, there is a unique solution to the driver’s problem in period \( t - 1 \). We may now implement backwards induction (c.f., Rust (2016)): substitute the recovered \( V_T^i(S_T^T) \) into the set of equations defining optimal policies at \( T-2 \) and repeat the above exercise to recover unique \( V_{T-1}^i(S_{T-1}^T) \) and iterate until period 1, in which \( S^1 \) is exogenous and known to drivers.

**A.7 Computational Details**

**A.7.1 Taxi Equilibrium Algorithm**

The algorithm that I implement takes as inputs all model primitives, parameters, and an initial state. It returns the equilibrium state and policy functions for each location and each time period. Equilibrium states constitute a \( L \times T \) matrix (i.e., how many taxis are in each location in each period), and equilibrium policy functions constitute a \( L \times L \times T \) matrix (i.e., the probability of vacant taxi transition from any location \( i \in \{1, ..., L\} \) to any location \( j \in \{1, ..., L\} \) in each period). The algorithm uses backwards iteration to solve for continuation values and forward iteration to generate transition paths. Each component repeats, updating the state and policy vectors in each iteration. The process terminates when all policies and continuation values converge to a fixed point. The algorithm is described below.

The Taxi Equilibrium Algorithm begins with an initial guess of the state vector \( S_0 = \{S^t\} \) for all \( t \). With \( S_0 \) as well as observations of the empirical distributions of taxi-passenger matches, \( \tilde{m} \),
**Algorithm 1** Taxi Equilibrium Algorithm

1: Input empirical matches \( \tilde{m}_{ij}^t \) and \( \tilde{m} = \{ \sum_j \tilde{m}_{ij}^t \} \)
2: Fix parameter value \( \sigma_\varepsilon \).
3: Set counter \( k = 0 \)
4: Input initial and future states guess \( S_t^0 \) for \( t = 1, ..., T \).
5: repeat
6: \( \text{for } t = T \text{ to } 1 \) do \( \triangleright \) Backwards Iteration
7: \quad Compute \( V_i^t(S_k^t; \tilde{m}^t) \) for all \( i \)
8: \end for
9: \( \text{for } t = 1 \text{ to } T - 1 \) do \( \triangleright \) Fwd. Iteration to \( T \) for each step back
10: \quad Derive choice-specific value functions \( W_i^t(j; S^t) \) for all \( t, i, j \) per equation 8
11: \quad Find policy fcts. \( \sigma_k^t(W_{k+1}^t) \) per equation 7
12: \quad Given \( S_k^t \), compute transition to \( S_{k+1}^{t+1} \) per equations 15 and 16
13: \end for
14: Update next period state \( S_{k+1}^{t+1} \leftarrow \tilde{S}^{t+1} \)
15: Update next period continuation values as \( V^{t+1}(S_{k+1}^{t+1}; \tilde{m}^t) \)
16: \( k \leftarrow k + 1 \)
17: until \( |V_k^t - V_{k-1}^t| \leq \epsilon \) \( \forall t \)

I can compute value functions \( V_i^t(S_0; \tilde{m}) \) for each \( i \) and \( t \) via backwards induction, beginning at period \( T \).\(^{46}\) Next, using the value functions, I compute choice-specific value functions and optimal policies as in equation 9. Next, I use the computed policy functions and, starting at time \( t = 1 \) at \( S_0^1 \), I forward simulate the optimal transition paths and update the initial state for \( t = 2, ..., T \), resulting in a new guess of the state, \( S_1 \). With \( S_1 \), I again combine the same observations \( \tilde{m} \) to update value functions \( V_i^t(S_1; \tilde{m}) \). This process repeats until value and policy functions converge.

**A.7.2 Initial conditions**

Recall that \( S_0^t \) is a state vector of the number of vacant taxis in each location at time \( t \). The initial guess of the state in each period, \( S_0^t \), is assigned by allocating the exogenous total number of taxis according to the empirical distribution of matches.\(^{47}\) As the algorithm runs, each vector \( S_0^t \) for \( t \geq 2 \) is updated as \( t - 1 \) transitions are computed given the \( t - 1 \) initial state and value functions for \( t, t + 1, ..., T \). Only one term, \( S_0^1 \) remains exogenously chosen.

To mitigate any issues related to this remaining first-period exogenous initial state, I define \( t = 1 \) as 6:00am. In this period, the assumption that all available cabs are actively searching or

\(^{46}\) Note this process requires integrating over future period states and epsilons. State transitions are deterministic and thus the first integral is trivial. To integrating over driver-specific shocks \( \varepsilon_{j,a} \) I leverage the fact that for discrete-choice logit models there is a closed form expression for the conditional expectation of unobservables given by \( E[\varepsilon_j | S] = \gamma_0 - \sum_j \log(\sigma(\varepsilon_j | S) \cdot \sigma(\varepsilon_j | S)) \) where \( \gamma_0 \) is Euler’s constant. See e.g., Arcidiacono and Ellickson (2011).

\(^{47}\) Recall the total number of taxis equals 11,500, as discussed in Section 4.1.
Nevertheless, by starting the equilibrium algorithm at 6:00am, a wide range of initial conditions quickly wash out within the first hour. This is verified by setting alternative initial conditions and comparing equilibrium levels of taxi supply across locations. As results are reported starting at 7:00am, the spatial distribution of taxis reaches an equilibrium mostly unaffected by the initial state assumption. Table A3 shows the impact of more extreme initial condition assumptions on the equilibrium supply of taxis under increasingly heterogeneous starting points. The baseline case, as described above, is compared with (1) a uniform initial distribution and (2) a distribution in which all initial vacant cabs are distributed at edge locations: those locations adjacent to the boundaries of the map. The latter edge distribution is meant to simulate the case in which taxis start the day by driving from garages where they are stored. The locations of these garages are not available in my data, so this condition serves as a check on any misspecification due to unobserved initial conditions, where all taxis are stored in outer boroughs. In both cases, the equilibrium supply of taxis is very close to the baseline, with average percentage differences across all i, t pairs no worse than 2.5% and average level differences no worse than 3.4 vacant taxis.

Table A3: Alternative Initial Conditions

<table>
<thead>
<tr>
<th>Initial Condition</th>
<th>( \Delta v^t_i ) (mean)</th>
<th>( % \Delta v^t_i ) (mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Uniform</td>
<td>3.379</td>
<td>0.025</td>
</tr>
<tr>
<td>Edge</td>
<td>0.463</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

This table shows the change in taxis’ spatial equilibrium distribution given changes in initial conditions. Baseline is the initial condition used throughout the paper, as described above. Uniform imposes an initial distribution that is uniform across all locations at 6am. Edge imposes an initial distribution that uniformly puts all vacant taxis across edge locations: all peripheral locations with adjacent access to the outer boroughs and New Jersey.

A.8 Details on Estimating \( \alpha_r \)

The variance of matches The matching function given in \ref{eq:matching} describes the number of average number of matches produced in an area with \( v^t_i \) taxis and demand parameter \( \lambda^t_i \). Estimating \( \alpha \) will require computing the variance in matches produced by the daily draws \( u^t_i \) from each i, t-

---

\footnote{Recall that the data do not allow for distinguishing whether fewer matches in the morning are due to low supply or demand; and thus it is impossible to say how many cabs are actually on the road at any point.}

\footnote{Boundary locations are all peripheral locations with adjacent access to the outer boroughs and New Jersey. This includes all locations in Manhattan with bridges and those bordering 125th street, all Brooklyn and Queens locations, and each Airport.}
specific Poisson distribution. To compute this, I first determine the day-specific matching function $m^d_i(u^i_t, v^i_t)$ that would give rise to the equation \(3\). This function is given by

$$m_i(u^i_t, v^i_t; \alpha_r) = v^i_t \left(1 - \left(1 - \frac{1}{\alpha_r v^i_t}\right)^{u^i_t}\right). \quad (19)$$

Let $\rho = \left(1 - \frac{1}{\alpha_r v^i_t}\right)$. From here we can integrate equation \(19\) over the distribution of $u^i_t$.

$$E[m^i_t|v^i_t, \lambda^i_t] = v^i_t \sum_{k=0}^{\infty} \left(1 - \left(1 - \frac{1}{\alpha_r v^i_t}\right)^k\right) f_{\lambda^i_t}(k)$$

$$= v^i_t - v^i_t \sum_{k=0}^{\infty} \rho^k \lambda^i_t^k e^{-\lambda^i_t}$$

$$= v^i_t - v^i_t \sum_{k=0}^{\infty} \rho^k \lambda^i_t^k e^{-\rho \lambda^i_t}$$

$$= v^i_t - v^i_t \cdot e^{-\lambda^i_t(1-\rho)}$$

$$= v^i_t(1 - e^{-\frac{\lambda^i_t}{\alpha_r v^i_t}}).$$

The second to last equation follows as integrating the probability mass function of the Poisson distribution over its entire support is equal to one. I now repeat the exercise to compute the variance of matches implied by the stochastic process governing $\{u^i_t\}$ as $E[m^2_i|v^i_t, \lambda^i_t] - E[m^i_t|v^i_t, \lambda^i_t]^2$. First, following directly from equation \(19\) we have

$$E[m^2_i|v^i_t, \lambda^i_t] = (v^i_t)^2(1 - e^{-\frac{\lambda^i_t}{\alpha_r v^i_t}})^2$$

$$= (v^i_t)^2(1 - 2e^{-\frac{\lambda^i_t}{\alpha_r v^i_t}} + e^{-\frac{2\lambda^i_t}{\alpha_r v^i_t}}),$$

and second,
\[
E[m_i^2 | v_t^i, \lambda_t^i] = v^2 \sum_{k=0}^{\infty} \left(1 - \left(1 - \frac{1}{\alpha_r v_t^i}\right)^k\right)^2 f_{\lambda_t^i}(k) \\
= v^2 \sum_{k=0}^{\infty} \left(1 - \rho^k\right)^2 f_{\lambda_t^i}(k) \\
= v^2 \sum_{k=0}^{\infty} \left(1 - 2\rho^k + \rho^{2k}\right) f_{\lambda_t^i}(k) \\
= v^2 \left(1 - 2 \sum_{k=0}^{\infty} \rho^{k} f_{\lambda_t^i} + \sum_{k=0}^{\infty} \rho^{2k} f_{\lambda_t^i}(k)\right) \\
= v^2 \left(1 - 2 e^{-\frac{\lambda_t^i}{\alpha_r v_t^i}} + \frac{e^{-\lambda_t^i}}{e^{-\rho^2 \lambda_t^i}} \sum_{k=0}^{\infty} \left(\rho^{2k} \lambda_t^i e^{-\rho^2 \lambda_t^i}\right)\right) \\
= v^2 \left(1 - 2 e^{-\frac{\lambda_t^i}{\alpha_r v_t^i}} + e^{-\lambda_t^i(1-\rho^2)}\right) \\
= v^2 \left(1 - 2 e^{-\frac{\lambda_t^i}{\alpha_r v_t^i}} + e^{-2\frac{\lambda_t^i}{\alpha_r v_t^i} + \lambda_t^i(1-\rho^2)}\right).
\]

Putting these terms together gives,

\[
\text{Var}[m_i | v_t^i, \lambda_t^i] = E[m_i^2 | v_t^i, \lambda_t^i] - E[m_i | v_t^i, \lambda_t^i]^2 \\
= (v_t^i)^2 \left(e^{-2\frac{\lambda_t^i}{\alpha_r v_t^i}} + \frac{\lambda_t^i}{\alpha_r v_t^i} - e^{-2\frac{\lambda_t^i}{\alpha_r v_t^i}}\right) \\
= (v_t^i)^2 e^{-2\frac{\lambda_t^i}{\alpha_r v_t^i}} \left(\frac{\lambda_t^i}{\alpha_r v_t^i} - e^{-\frac{\lambda_t^i}{\alpha_r v_t^i}} - 1\right). 
\]

An estimator for \(\alpha\) Given parameter value \(\sigma_\varepsilon\), I use matches date \(\{\tilde{m}\}\) and the TEA procedure to numerically solve for the equilibrium state, \(S = \{v_t^i(\tilde{m})\}\). Proposition 4.1 shows that, given a level of taxis \(v_t^i\), the matching function can be inverted to solve for \(\lambda_t^i\) in location \(i\) (within region \(r\)) and time \(t\). Next, I use an analytic expression of the variance of matches, given by equation 20. This function depends on both \(\alpha_r\) as well as the ratio \(\frac{\lambda_t^i}{\alpha_r v_t^i}\). From here I set up the following estimator:

\[
\alpha_r = \arg \min_{\alpha} \sum_{i \in R, t} \left(\text{Var}_d(\tilde{m}_{i,d}^t) - (v_t^i)^2 e^{-2\frac{\lambda_t^i}{\alpha_r v_t^i}} \left(\frac{\lambda_t^i}{e^{\frac{\lambda_t^i}{\alpha_r v_t^i}}} - 1\right)\right),
\]

56
where $\mathcal{R}_r$ denotes the set of locations within region $r$, and $\tilde{m}_{i,t,d}$ refers to the number of observed taxi-passenger matches that take place in location $i$, time $t$, and day-of-month $d$. The variance is then taken with respect to all observations within the weekdays in a given month, across days of the month.

**Accounting for the variance of $v_i^t$**  
Equation [20] is derived under Assumption 3 that the variance of $v_i^t$ in each region $r$ is negligible as a determinant of $\alpha_r$. To motivate this assumption, Table A4 compares the outcome of two simulation exercises. First is a simulation over one month of the market, in which demand is drawn in each $i,t$ from the corresponding $\lambda_i^t$ parameters. This process generates some variance in the distribution of taxis. From this simulation the variance of matches across days of the month are computed. Second, a second simulation is computed that is identical except imposes that the level of taxis is fixed at the equilibrium forecast generated by the model. In essence this shuts down the possibility that any variance in matches is attributable to variance in taxis. Finally, the table reports the theoretical variance in matches via equation [20]. This comparison shows that the variances are for the most part not statistically distinguishable from one another.

<table>
<thead>
<tr>
<th>Table A4: Simulated Variance Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Simulated $Var(m_i^t)$</td>
</tr>
<tr>
<td>(Fixed $v_i^t$)</td>
</tr>
<tr>
<td>Simulated $Var(m_i^t)$</td>
</tr>
<tr>
<td>(Fixed $v_i^t$)</td>
</tr>
<tr>
<td>Theoretical $Var(m_i^t)$</td>
</tr>
<tr>
<td>(Fixed $v_i^t$)</td>
</tr>
</tbody>
</table>

This table shows the variance of matches in New York Regions I-IV, generated under two sets of simulations of one month of data each. 95% Confidence intervals are reported. Confidence intervals are based on standard deviations of variance estimates produced by replicating each simulation 150 times and computing standard errors for each variance measure.

The result that the variance of taxis has little apparent bearing on the variance of matches is not surprising, as taxis most often outnumber customers across the city. Even with some degree of search frictions, this discrepancy implies that variation in taxis will have less impact than variation in demand. Figure A4 Panel 2 shows the equilibrium ratio of supply to demand across regions. While there are many reversals (see detailed estimates in Figure A7), the average ratio of taxis-to-customers across regions is between 3-10 throughout most of the day.
Together these results suggest that the variance of taxis can be regarded as negligible with respect to their impact on the overall variance of matches used to identify the efficiency parameters \( \alpha_r \).

### A.9 Detailed Estimation Results

Figure A5 shows aggregate supply and demand results, summing all 39 locations into the five regions corresponding to Figure 1. The results above demonstrate that while taxi supply maintains some coverage across all locations throughout the day, there are intra-day trends in spatial availability and demand. Spatial mismatch is evident, as the relative proportions of supply and demand are not the same across each region.

Figures A7 and A8 show detailed results of supply and demand in all locations. Note that location numbers 1-34 roughly track from South to North in Manhattan, locations 35-37 track South to North from Brooklyn to Queens, location 38 is LaGuardia airport and location 39 is JFK airport. We see that most locations have a surplus of taxis except for a few areas of very high demand. Lower Manhattan, parts of midtown Manhattan and far North-east Manhattan all demonstrate particularly large constraints in the ratio of vacant taxis to demand. All locations demonstrate some search frictions on both sides of the market, but we see here that the impact is felt more on the taxi side.

Figure A9 shows the evolution of Value functions by time of day. Each series is the value for a single location. The high correlation between each value function reflects the equilibrium result that drivers’ policy functions ensure that there is no spatial arbitrage possible. The remaining differences between each location’s value is due to the transportation cost that prevents perfect cross-location arbitrage. As the day reaches its 4pm end, the value of search in each location systematically drops to zero.
Figure A5: Equilibrium Vacant Taxis: Weekdays 7a-4p, 9/2012 (Five Region Aggregates)

This figure depicts the equilibrium spatial distribution of taxis and mean arrival of customers across the Five Regions shown in Figure 1. Results across all 39 locations are summed to these five areas. Results are depicted for the weekday taxi drivers’ day shift, from 7a-4p in September 2012.

A.10 Model Fit

The primary moments used in this study are the spatial and inter-temporal patterns of taxi-passerger matches. Here I describe and analyze the model fit.

Since taxis’ dynamic spatial equilibrium is computed via the inversion described in section X, this part of the model perfectly fits the taxi-passerger matches data by construction. In other words, given estimated parameters, the model would generate a set of equilibrium matches across periods and location that perfectly line up with the data.

For the model to generate counterfactuals and welfare estimates, I rely on the estimation of demand elasticities. The ability of the model to fit data thus rests on the fit associated with the demand model as well as the subsequent computation of equilibrium. Figure A10 shows the predicted and actual spatial heterogeneity of demand and matches respectively. Each comparison involves an aggregation over 164,268 (39 x 39 x 108) predictions of \( \lambda_t \) at observed prices. Demand fits quite well across time and space. Lower Manhattan demand is under-predicted in the morning while Uptown demand is over-predicted, but the ordering across regions and broad time trajectories lines up well.

Computing equilibrium matches requires demand predictions as inputs. The model uses all demand estimates to compute equilibrium supply in each location and time, which further generates 4,212 (38 x 108) predictions on matches. Since taxi supply is increasing in demand for any location and time, any bias in demand will be exacerbated in computing matches for two reasons. First,
demand is most often the short side of the market, so that an additional unit of demand leads to almost one additional match. Second, in equilibrium taxi drivers’ search in a location increases with demand in that location, which implies that matches will increase with demand from both a direct and indirect effect. Nevertheless, the model is able to fairly-well reproduce a complex system of spatial and inter-temporal patterns of matches. It tends to under-predict Midtown matches in the morning rush hour and yet over-predicts all Manhattan matches in the early morning. End of day matches generated by the model closely match those in the data.
This figure depicts the equilibrium value functions for all 39 locations, by time of day, estimated from August 2012 data. Each line depicts a separate location. The highest-valued function is that of LGA airport and the least-valued function is that of JFK airport. All other locations’ values fall in-between. Values do not include any medallion leasing fees.

A.11 Details on Counterfactual Results

A.11.1 Welfare Calculation

Consumer welfare is computed by integrating under the the estimated CES demand curves in each origin, destination, time pair (i.e., each \(i, j, t\)). The integral can be computed analytically as follows:

\[
W_{ijt}(m_{ij}^t, \hat{\lambda}_{ij}^t, p_{ij}, \beta) = \frac{m_{ij}^t(\hat{\lambda}_{ij}^t, v_{i}^t(\hat{\lambda}))}{\hat{\lambda}_{ij}^t(p_{ij})} \cdot \frac{\alpha_{1,s,a}}{\alpha_{1,s,a} + 1} \cdot e^{-\frac{\alpha_{0,i,t,s,a}}{\alpha_{1,s,a}}} \cdot \hat{\lambda}_{ijt}(p_{ij})^{\alpha_{1,s,a} - 1} - \hat{\lambda}_{ijt}(p_{ij}) \cdot p_{ij},
\]

where \(\alpha_{0,i,t,s,a}\) and \(\alpha_{1,s,a}\) are the estimated demand parameters, \(\hat{\lambda}_{ijt}\) is the predicted level of demand given price \(p_{ij}\), and \(v_{i}^t(\hat{\lambda})\) is the equilibrium mass of taxis in each location (a function of the distribution of demand across locations and time). In counterfactuals I incorporate waiting
This figure demonstrates model fit across demand and matches. The left panel shows the fit of the demand model. The right panel shows the fit of the equilibrium model to the total number of matches. Each series displayed aggregates origin-destination-specific series into five origin-based regions as shown on Figure 1.

time elasticities into this calculation. To do this I augment Equation (24) with waiting times:

\[
ln(\lambda_{ij}(P_{ij})) = \alpha_{0,i,t,s,a} + \alpha_{1,s,a}ln(P_{ij}) + \alpha_{2,i,t}\Delta w_{i,j,t} + \eta_{i,t,s,a},
\]

where \(\Delta w_{i,j,t}\) is computed as the percentage change in the consumer waiting time measure between the baseline case and the counterfactual waiting time evaluated as if there were no waiting time elasticity. Thus, to create the variable \(\Delta w_{i,j,t}\) I need to predict demand and compute equilibrium once with no waiting time elasticity, generate the predicted change in waiting times, and then re-compute demand with the calibrated waiting time elasticity \(\alpha_{2,i,t} = -1\) and again re-compute equilibrium. After the second round is complete, I evaluate welfare. This is a computationally
intensive process, so I do not conduct additional iterations. However, after many trials adding one additional iteration the differences in outcomes seem to be very small.

Finally taxi profits are computed as follows:

\[
W_{ijt}^{\text{taxi}}(m_{ij}, \hat{\lambda}_{ij}, p_{ij}, c_{ij}) = \frac{m_{ij}^t(\hat{\lambda}_{ij}, v_{st}^t(\hat{\lambda}))}{\hat{\lambda}_{ij}^t(p_{ij})} \left( \hat{\lambda}_{ij}(p_{ij}) \cdot (p_{ij} - c_{ij}) \right),
\]

(26)

where \(c_{ij}\) is the fuel cost for a trip from \(i\) to \(j\).

A.11.2 Robustness to Omitting the Early Morning

Figures A2 and A3 suggest that, despite a consistent daily participation rate of drivers, many fewer matches occur in the period between 7a-9a. One concern is the total supply of vacant taxis is unobservably low during this early period, as drivers’ start times are not observed, only the time of the first passenger match. By assuming supply to be fixed at 11,500 through the day, the model may attribute low match rates in the morning to low demand instead of low supply. Figure ?? demonstrates that counterfactual results are robust to omitting the first two hours of the day. It shows that optimal pricing patterns are very similar to those discussed in Section 6 and displayed in Table 9; all optimized pricing policies lower average prices, increase matches by around 20-30% and increase utilization rates by about 7-10%.
Table A5: Efficient Pricing and Matching Technology: Counterfactual Results (Starting at 10am)

<table>
<thead>
<tr>
<th>Price Type</th>
<th>Price Multipliers</th>
<th>Total Surplus (,000 USD)</th>
<th>Consumer Surplus (,000 USD)</th>
<th>Consumer Rent Share (percent)</th>
<th>Matches (,000)</th>
<th>Taxi Utilization (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline - 8/2012</td>
<td>1.00</td>
<td>4094.5</td>
<td>1783.3</td>
<td>43.6</td>
<td>216.2</td>
<td>43.4</td>
</tr>
<tr>
<td><strong>Location-Based Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>0.70</td>
<td>2110.3 (+18.3 %)</td>
<td>47.7</td>
<td>272.4 (+26.0 %)</td>
<td>53.0</td>
<td></td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>0.70</td>
<td>2110.3 (+18.3 %)</td>
<td>47.7</td>
<td>272.4 (+26.0 %)</td>
<td>53.0</td>
<td></td>
</tr>
<tr>
<td>Max Matches</td>
<td>0.90</td>
<td>2084.8 (+16.9 %)</td>
<td>47.4</td>
<td>276.3 (+27.8 %)</td>
<td>53.6</td>
<td></td>
</tr>
<tr>
<td><strong>Time-Based Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>.</td>
<td>2024.0 (+13.5 %)</td>
<td>46.7</td>
<td>255.4 (+18.1 %)</td>
<td>54.7</td>
<td></td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>.</td>
<td>2024.0 (+13.5 %)</td>
<td>46.7</td>
<td>255.4 (+18.1 %)</td>
<td>54.7</td>
<td></td>
</tr>
<tr>
<td>Max Matches</td>
<td>.</td>
<td>2003.3 (+12.3 %)</td>
<td>46.4</td>
<td>281.6 (+30.3 %)</td>
<td>55.7</td>
<td></td>
</tr>
<tr>
<td><strong>Non-Linear Pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Total Surplus</td>
<td>0.70</td>
<td>2102.4 (+18.0 %)</td>
<td>47.6</td>
<td>280.9 (+30.0 %)</td>
<td>56.1</td>
<td></td>
</tr>
<tr>
<td>Max Cons. Surplus</td>
<td>0.70</td>
<td>2102.4 (+18.0 %)</td>
<td>47.6</td>
<td>280.9 (+30.0 %)</td>
<td>56.1</td>
<td></td>
</tr>
<tr>
<td>Max Matches</td>
<td>0.70</td>
<td>2062.0 (+15.7 %)</td>
<td>47.2</td>
<td>287.5 (+33.0 %)</td>
<td>55.2</td>
<td></td>
</tr>
<tr>
<td><strong>Matching Technology</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Prices</td>
<td>1.00</td>
<td>5242.1 (+28.0 %)</td>
<td>42.6</td>
<td>278.7 (+28.9 %)</td>
<td>57.7</td>
<td></td>
</tr>
</tbody>
</table>

This table shows, for each weekday period from 10a-4p, the estimated change in total welfare (profits plus consumer surplus), consumer surplus, the consumer surplus share of total surplus, total matches, and utilization rates across each counterfactual price policy. Each pricing policy shown is a rule that applies to four policy-specific multipliers on the baseline price \( p_{ijt} \) for every route, given by $2.50 + $2.00/mile. In location-based pricing, the multipliers \( \theta_k(i) \) apply to \( p_{ijt} \) where \( k(i) \in \{1, 2, 3, 4\} \) indexes the region of location \( i \) according to Figure 1. In time-based pricing, the multipliers \( \theta_k(t) \) apply to \( p_{ijt} \) where \( k(t) \in \{1, 2, 3, 4\} \) respectively indexes the time ranges of 7a-9a (omitted here by construction), 10a-11a, 12p-1p, 2p-4p. In non-linear pricing, the multipliers \( \theta_k(i, j) \) apply to \( p_{ijt} \) where \( k(i, j) \in \{1, 2, 3\} \) are coefficients which change existing tariffs according to \( \theta_1 \cdot \text{base fare} + \theta_2 \cdot \text{fare per-mile} + \theta_3 \cdot \text{fare per-mile}^2 \). The final row depicts equilibrium outcomes under a simulated matching technology in which the matching function takes the form \( m^1_i = \min(\lambda^t_i, v^t_i) \). This last counterfactual is computed at baseline prices.