THE BALANCE OF POWER AND THE RISK OF WAR IN CRISIS 
BARGAINING

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ABSTRACT. Understanding how shocks to the technology of warfighting affect the probability a conflict devolves to war is central to many aspects of security studies. In the standard crisis bargaining model, changes to war payoffs have no effect on the equilibrium probability that bargaining ends in war. This neutrality result implies that shifts in military capabilities, as well as tools of statecraft such as third-party intervention, alliances, and arming, have no effect on war onset when war is the result of bargaining failure. The empirical record of war onset seems to contradict this observation. So can we both accept the existing empirical findings on war onset and maintain the theory that war is the result of bargaining breakdown? We show that a series of individually innocuous assumptions combine in the standard crisis bargaining model to produce this result. So while it is true that changes to the war payoffs make one player more aggressive and the other less aggressive, these effects need not balance out. We show that the exact balancing and ensuing neutrality result relies on a form of symmetry in how external shocks influence the tradeoffs between the payoffs each side derives from fighting and reaching a peaceful settlement.
1. Introduction

The crisis bargaining model has become a fixture in theories of conflict (Fearon 1995). One variant of the model is a standard ultimatum bargaining game with one-sided incomplete information. This model is often used to illustrate how uncertainty about payoffs is a sufficient friction to reach bargaining failure and inefficient war fighting. This variant with asymmetric information has received the majority of attention in the literature. The model now represents a popular baseline and variants of it are used to study a host of other important topics ranging from the effects of the balance of power on conflict (Powell 1996, 1999; Reed 2003), to the role of mediation (Kydd 2003, Rauchhaus 2006) and domestic politics (Filson and Werner 2004, Ramsay 2004, Tarar 2006), to the efficacy of third-party intervention (Kydd and Straus 2013, Cetinyan 2002, Werner 2000) as well as alliances and deterrence (Yuen 2009).

The standard crisis bargaining model has substantial appeal. It is both relatively tractable and has many comparative statics that are consistent with stylized facts generally accepted as important from the field of conflict studies. The standard model also produces some surprising results, probably the most unexpected being what is called “neutrality”—i.e., that changes in the war payoffs are offset by equilibrium bargaining behavior in such a way as to result in no change in the equilibrium probability of conflict (Fearon 1993, Cetinyan 2002, Kydd 2010, Kydd and Straus 2013).

The mechanism behind the neutrality result is compelling. If one country is made stronger, it will behave more aggressively while its opponent becomes more conciliatory in the bargaining process. While increases in the one country’s war payoffs lead
to more aggressive demands, and all else equal a higher chance of bargaining failure, all else is not equal. In the face of this shift in the probability of victory in the favor of one actor, the weaker now is willing to make more concessions to the opponent and this reduces the risk of war. In the standard model these offsetting incentives lead both countries to take actions that in equilibrium neither increase nor decrease the probability of war.

However, as we show, neutrality is an unintended consequence of individually reasonable simplifying assumptions in the original specification of the model. To demonstrate this we show why the neutrality result obtains, how it relates to assumptions about players utilities, and then provide a generalization of the original formulation that is better suited for studying the effects of changes to the technology of war-fighting or the distribution of power in the crisis bargaining framework with asymmetric information. The analysis illustrates how a key comparison of the way that war-payoffs change influences whether the risk of war increases or decreases. Our point is not that the modeling choices in Fearon or other studies using this model are wrong. We contend that every model is stylized in particular ways. Rather we hope that this note will reveal how particular features of this widely used model impact its applicability to questions about the effect of changes in the probability of victory on the onset of war. This note offers guidance into how the choices made about war and settlement payoffs can influence the model’s ability to provide leverage on certain types of questions.

We are not the first to discuss the source of the neutrality result. Previous scholars have incompletely attributed the result to the assumption of risk neutrality (Fearon 1993, Kydd 2010), but as we show it is not risk neutrality but rather the assumption
that changes in one player’s payoffs exactly offset changes in the other player’s payoffs at equilibrium that drives the result. Although certain assumptions that relax risk neutrality will also introduce the relevant asymmetry, we caution scholars against equating risk neutrality with the neutrality result.

We think this is an important question because many issues in international relations can be framed in terms of understanding how changes to the war payoffs of countries influence their behavior in crisis bargaining. Such changes may result from exogenous shifts in the distribution of power (Powell 1999) or intervention by third-party defenders (Werner 2000, Yuen 2009). Related to the models of third-party intervention in interstate war are models of third-party intervention in intrastate conflict (Cetinyan 2002, Kydd and Straus 2013). Other factors that may influence war payoffs and crisis bargaining include international organizations (Chapman and Wolford 2010), domestic politics (Filson and Werner 2004, Tarar 2006), and previous actions by the disputants in a dynamic setting such as in arms races (Kydd 2000).

Typically modelers incorporate shifts in war payoffs by either consider changes in the distribution of capabilities or opponents’ costs of war. For example, Powell (1999) models a country’s decline in power as a decrease in its capabilities relative to its opponent, or the probability it will win in the war lottery. In Yuen (2009), third-party intervention also affects disputants’ relative capabilities. Werner (2000) models third-party intervention as altering both the distribution of capabilities as well as war costs. In Chapman and Wolford (2010), support from an external international organization might affect a country’s costs of war. In Filson and Werner (2004), becoming more democratic increases war costs. An exception to shifts affecting either capabilities or war costs is Tarar (2006), in which a leader choosing to go to
war might be rewarded by a domestic audience by retaining office. In this setup, the leader’s war payoff increases by the amount of the private value of public office.

In our analysis below we provide a general intuition for how changes in war payoffs influence the aggressiveness of states in bargaining and affect the probability of war. We show that exact neutrality is a fragile condition, though the cushioning effect of bargaining dynamics is robust. If shocks change the payoffs to war fighting relative to the marginal value of settlements in ways that do not exactly balance out, then the neutrality result will fail. We further show that increases and decreases in the probability of war result from predictable asymmetries in changes in war payoffs. Establishing general conditions for violating neutrality may create opportunities for further theorizing about how policies create asymmetries in payoffs that affect war onset. We provide several applications of the result to published models studying the relationship between balance of power and war onset.

2. Crisis Bargaining and the Neutrality Result

Crisis bargaining considers situations where players, negotiating over an issue or territory, have an outside option in the form of war. In a setting with two countries, say A and B, we may conceive of the negotiations as relating to the division of some disputed territory and possible solutions are characterized by the countries’ relative shares. The share to player A is denoted $x$ and the share to player B is $1 - x$. It is standard to assume that the payoffs from such a split are linear in the shares and thus given an agreed split of $(x, 1 - x)$ the player preferences can be represented by an expected utility function $EU_A(x) = x$ and $EU_B(1 - x) = 1 - x$. If an agreement

\footnote{Fearon (1995) appears to be more general as the model is formulated to allow players to have non-linear utility over the shares. But all of the analysis of the relevant model (claim 2 in Fearon’s}
is not reached, then the players obtain payoffs from a war. With normalized payoffs for owning the entire “pie” the standard formulation is

\[ EU_A(war) = p - c_A, \]

and

\[ EU_B(war) = 1 - p - c_B. \]

The term \( p \) then is interpreted as the probability that \( A \) wins the full prize while \( B \) wins it with probability \( 1 - p \). The costs, \( c_A \) and \( c_B \), capture losses from war fighting. Natural interpretations of war costs in the literature include destruction of property, loss of life, or time costs associated with delay in resolving the issue.

The bargaining protocol is then the ultimatum game. Player \( A \) makes an proposal, \((x, 1 - x)\). Player \( B \) then decides to accept, resulting in the payoffs \( x \) and \( 1 - x \), or reject, resulting in the war payoffs. With no uncertainty the unique subgame perfect equilibrium involves \( A \) offering exactly \( 1 - x = 1 - p - c_B \) so that the dispute is peacefully resolved with the share \( x = p + c_B \) going to \( A \).

However, a rationalist explanation for war is provided if \( A \) is assumed to face uncertainty about the term \( c_B \). For simplicity, we follow the most common formulation and assume that while \( B \) knows her war cost, \( A \) treats it as a random variable drawn from a distribution \( H(\cdot) \) with density \( h(\cdot) \) on the interval \([0, 1]\). The characterization of a Perfect Bayesian Equilibrium in which there is a risk of war is mechanical and well known. \( B \) will accept an offer in which \( 1 - x \geq 1 - p - c_B \). This means that from the perspective of \( A \) deciding what offer to make, the probability of war is a

appendix) takes the utility over shares to be linear. The literature seems to have followed this simplification.
function of her offer, $\pi(x)$, and is given by $H(x - p)$. Accordingly, $A$ selects $x$ to maximize\footnote{Note that the above derivation parallels that given in Fearon’s proof of claim 2.}

$$H(x - p)(p - c_A) + (1 - H(x - p))x.$$ 

The first order condition is

$$h(x - p)(p - c_A - x) - H(x - p) + 1 = 0.$$ 

Throughout we follow the literature and assume that the parameters support an equilibrium in which A’s offer involves risk. This is the assumption that is central to using uncertainty as a rationalist explanation for war.\footnote{See Fey, Meirowitz and Ramsay (2013), for necessary and sufficient conditions.} In this case, let $x^*$ denote the solution to the first order condition. In this context the neutrality result speaks to a comparative static on how changes in $p$ influence the equilibrium probability of war, $\pi(x^*)$. Expanding our notation to capture the dependencies, we may write $x^*(p)$ to denote the solution to the first order condition and $\pi^*(p) = \pi(x^*(p))$ as the equilibrium probability of war.

**Proposition 1.** (neutrality): In the canonical model the probability of war is constant in $p$, i.e., $\frac{d\pi^*}{dp} = 0$.

**Proof.** The proof is an application of the implicit function theorem and the chain rule. The equilibrium affect of $p$ on $x^*$ is found by way of the implicit function theorem, $\frac{dx^*(p)}{dp} = 1$. Recalling that $\pi(x^*(p)) = H(x^*(p) - p)$, the chain rule yields

$$\frac{d\pi^*}{dp} = h(x^*(p) - p)(\frac{dx^*(p)}{dp} - 1).$$
Substituting \( \frac{dx^*(p)}{dp} = 1 \) into the above yields the conclusion, \( \frac{dx^*}{dp} = 0 \)

3. Rethinking payoffs

It is not uncommon for authors to investigate changes to war onset and bargaining outcomes stemming from external shocks to the probability of victory, \( p \) in the standard expected utility set up. But clearly Proposition 1 says we should not bother because there is no effect. To gain some clarity regarding the effects of policy levers that change the payoff to war consider a more general form of the war payoff.

Let the war payoffs to \( A \) be denoted by a function

\[
EU_A(war) = p(a),
\]

and let the war payoffs to \( B \) be denoted

\[
EU_B(war) = w(a) - c_B.
\]

For convenience we include the private cost, \( c_A \), in the term \( p(a) \). Because private information about \( B \)'s cost is the friction that drives inefficiency (risk of war), we keep this term separate for country \( B \).

The parameter \( a \) is included to allow for comparative statics. Here we can think of \( a \) as an input to the war payoffs, like a level of arms, a state of military technology, the number of allies, etc. To facilitate comparative statics analysis we assume that the functions \( p(\cdot) \) and \( w(\cdot) \) are twice differentiable. The assumption that \( p'(a) \geq 0 \) and \( w'(a) \leq 0 \) allows us to interpret an increase in \( a \) is a change that favors player \( A \)'s war payoff.
With these changes we may repeat the analysis above with very minor changes. Country B will accept any offer $x$ satisfying the inequality

$$c_B \geq w(a) + x - 1.$$ 

Thus the probability of war given an offer $x$ is

$$\pi(x; a) = H(w(a) + x - 1).$$

Given this rule, A’s offer maximizes

$$H(w(a) + x - 1)p(a) + (1 - H(w(a) + x - 1))x.$$ 

The first order condition is now

$$h(w(a) + x - 1)(p(a) - x) - H(w(a) + x - 1) + 1 = 0.$$ 

The effect of a change in $a$ on the equilibrium offer $x^*(a)$ is found by using the chain rule and the implicit function theorem,

$$\frac{dx^*(a)}{da} = \frac{-h(w(a) + x - 1)[p'(a) - w'(a)] - h'(w(a) + x - 1)[p(a) - x]w'(a)}{-h(w(a) + x - 1)[2] + h'(w(a) + x - 1)[p(a) - x]}.$$ 

Note that in the canonical model $p(a) = a$ and $w(a) = 1 - a$, and the above simplifies to 1. Using the chain rule again we can express the change in the equilibrium probability of war as

$$\frac{d\pi^*(a)}{da} = h(w(a) + x^*(a) - 1)[w'(a) + \frac{dx^*(a)}{da}].$$
Recall that \( w'(a) \) is negative and so whether this term is positive, zero, or negative depends on whether the magnitude of \( \frac{dx^*(a)}{da} \) is larger, the same, or smaller than the magnitude of \( w'(a) \). To make this comparison it is convenient to factor \(-w'(a)\) out of the expression for \( \frac{dx^*(a)}{da} \). Doing this we obtain

\[
-w'(a) \frac{-h(w(a) + x - 1)[1 - \frac{p'(a)}{w'(a)}] + h'(w(a) + x - 1)[p(a) - x]}{-h(w(a) + x - 1)[2] + h'(w(a) + x - 1)[p(a) - x]}
\]

The fraction is greater than or equal to one if \( \frac{-p'(a)}{w'(a)} \) is greater than one and it is less than or equal to one if the reverse is true. Accordingly, substituting this fraction into the probability of war, the sign of \( \frac{dx^*(a)}{da} \) coincides with the sign of \( p'(a) + w'(a) \). Thus if changes in \( a \) produce larger changes in \( p(a) \) than they do in \( w(a) \), the increase in \( a \) will increase the equilibrium probability of war. We have thus established the following result.

**Proposition 2.** (generalized war payoff model) If changes in the term, \( a \), have a larger effect on the war payoff to \( A \) than they do on the war payoff to \( B \), then an increase in \( a \) will increase the equilibrium probability of war. If the effect on \( B \)'s payoff is larger, then an increase in \( a \) will decrease the equilibrium probability of war.

Fearon motivated his study with a model in which the players’ payoffs over shares were arbitrarily increasing functions. So \( A \) preferred larger values of \( x \), and \( B \) preferred lower values of \( x \). In this model it is assumed that these functions are linear and so settlement payoffs are \( x \) and \( 1 - x \) respectively. One might, however, believe that the players obtain diminishing returns from the settlement. One plausible specification characterizes the split \((x, 1 - x)\) as yielding payoffs of \( x^{\frac{1}{2}} \) and \( (1 - x)^{\frac{1}{2}} \) respectively. With this specification and the assumption that war payoffs are just
\[ p - c_A \text{ and } 1 - p - c_B \text{ as in the canonical specification, if } c_B \text{ is uniform on the unit interval then the equilibrium probability of war for a parameterization which induces war with non-degenerate probability is given by the expression} \]

\[
1 - p - \frac{1}{4}2^{\frac{1}{2}}[c_A^2 - c_A(c_A^2 + 8)^{\frac{1}{2}} + 4]^{\frac{1}{2}},
\]

which is linearly decreasing in \( p \), and thus as \( A \) is made stronger the likelihood of war decreases.\(^4\)

To extend proposition 2 to capture this flexibility, we assume that \( A \) obtains payoff \( s_A(x) \) and \( B \) obtains payoff \( s_B(1 - x) \) from a settlement of \( (x, 1 - x) \) with both \( s_A(\cdot) \) and \( s_B(\cdot) \) being increasing twice differentiable functions that have a non-vanishing second derivative on the interval. Define \( \gamma(x) = \frac{s'_A(x)}{s'_B(1-x)} \) and \( C^*(a) = w(a) - s_B(1 - x^*(a)) \). In this notation, we obtain the following characterization.

**Proposition 3.** (generalized war and settlement payoff model) For a fixed model with parameter \( a \) and equilibrium offer \( x^*(a) \), the sign of \( \pi'(a) \) is the same as the sign of

\[
(1 - H[C^*(a)])\gamma'(x) - h[C^*(a)]s'_B(1 - x)(\gamma(x) + \frac{p'(a)}{w(a)})
\]

\[
\frac{h'[C^*(a)](p(a) - s_A(x))s'_B(1 - x) - h[C^*(a)](2\gamma(x)s'_B(1 - x)) + (1 - H[C^*(a)])\gamma'(x)}{h'[C^*(a)](p(a) - s_A(x))s'_B(1 - x) - h[C^*(a)](2\gamma(x)s'_B(1 - x)) + (1 - H[C^*(a)])\gamma'(x)}.
\]

The proof to this result follows the same steps as before, so we leave the details of the analysis in the appendix. Looking at Proposition 3, the result implies that

\(^4\)In this specification this equilibrium offer solves a second order polynomial. The solution is thus found in closed form, but is not worth reproducing here. Once the appropriate root is substituted into the probability of war function and simplified the above expression obtains.
neutrality requires the numerator of the expression is 0. Formally we then need

\[(1 - H[C^*(a)])\gamma'(x) = h[C^*(a)]s_B'(1 - x)(\gamma(x) + \frac{p'(a)}{w'(a)}).\]

This expression can be written as

\[\frac{(1 - H[C^*(a)])}{h[C^*(a)]} = \frac{s_B'(1 - x)(\gamma(x) + \frac{p'(a)}{w'(a)})}{\gamma'(x)}.\]

The numerator of the right hand side simplifies to

\[s_A'(x) + s_B'(1 - x)(\frac{p'(a)}{w'(a)}),\]

which leads to the following corollary.

**Corollary 1.** In the more general model, neutrality occurs at an equilibrium \(x^*(a)\)

 iff

\[\frac{s_A'(x^*(a))}{s_B'(1 - x^*(a))} + \frac{p'(a)}{w'(a)} = 0.\]

So when the ratio of the rate of change of utilities for the settlement at equilibrium

are the negative of the ratio of the rates of change of the war payoff, neutrality holds.

For example, in the standard model, \(\frac{s_A'(x^*(a))}{s_B'(1 - x^*(a))} = 1\) because the utilities were simply

the size of the share, and when the probabilities of winning are \(p\) and \(1 - p\), then

\[\frac{p'(a)}{w'(a)} = -1\]

and we have neutrality.

### 4. Application to Balance of Power

A longstanding question for theoretical and empirical scholars of security has been

whether a balance of power results in a greater stability (Claude 1962, Morgenthau

2006, Mearsheimer 1990) or more conflict (Blainey 1988, Organski 1968, Organski and
Kugler 1980). Powell (1996) develops a formal model to help answer this question. In Powell’s treatment changes to the balance of power influence the probability that the proposer makes an offer with risk, but the balance of power does not have a direct affect on the probability of conflict conditional on an offer with risk being made. In the Powell model \( u_i(t) = t, \ w(a) = p \) and, and \( p(a) = 1 - p - c_2 \). If we differentiate the relevant functions and evaluate them as required then Corollary (1)’s neutrality condition is satisfied. In fact, looking in Powell’s appendix (1996, p. 266), substituting his solution for the optimal offer into the probability of war function, the probability of war is \( (1 - c_2)/2 \)–and not a function of \( p \)–whenever \( x^* \) is great than the status quo.

Similarly, in the original form of the bargaining game in Fearon (1995) each player has a utility \( u(x) \) that was increasing and concave on the unit interval and normalized so that \( u(1) = 1 \) and \( u(0) = 0 \). Thus \( p(a) = p - c_A, \ w(a) = 1 - p \) and \( \frac{p'(a)}{w'(a)} = -1 \). But even when both players have the same concave \( u(x) \), as long as \( x^* \neq 1/2, \ u'(x) \neq u'(1 - x) \) and by Corollary 1 neutrality does not hold. This is easily seen in our example above where the utilities for a settlement are \( \sqrt{x} \). This leads to an equilibrium probability of war described in Equation 1, which is decreasing in \( p \).

In another example, if war was modeled as in Stam (1996), the payoff to war for countries \( A \) and \( B \) might be

\[
\begin{align*}
p(a) &= \pi(a) + q(a)\delta V_A - c_A \\
w(a) &= q(a)\delta V_B + (1 - \pi(a) - q(a)).
\end{align*}
\]

where \( \pi(a) \) is the probability the war ends this period with a victory for \( A \), \( q(a) \) is the probability of a stalemate where the game will move to the next period– discounted
and valued at $V_i$—and $(1 - \pi(a) - q(a))$ is the probability that country $B$ wins this period. With standard linear payoffs to settlement, by Corollary 1, there can only be neutrality in this model if $q'(a)$ is zero at the equilibrium offer or $\delta V_A = \delta V_B - 1$.

These examples show that many of the models we regularly study will not satisfy the neutrality condition. The fact that the result of the comparative statics exercise depends on how one changes the war payoffs should not be taken as an indictment of the game-theoretic models, but rather a confirmation of the relevance of formal modeling. The fact that the results depend on some nuances of how states value war fighting points to a subtlety which is otherwise unknown to the literature on the consequence of balance of power. Wittman (1979) foresaw an indeterminacy when thinking about balance of power and war risk when he wrote:

The analysis in this article suggests that this debate [balance of power] is irrelevant. There is no relationship between the probability of winning and the probability of war and therefore we will find no consistent empirical relationship between the two.... If one side is more likely to win at war, its peaceful demands increase; but at the same time the other side’s peaceful demands decrease. Thus we do not know whether an overlap is more or less likely. Furthermore, if one side’s increase in subjective probability of winning is equal to the other side’s decrease in subjective probability of winning, there will be no change in the probability of war (p. 751).

But Wittman was writing prior to the application of bargaining models with asymmetric information in international relations and thus he does not actually conduct equilibrium analysis on a model which can sustain the existence of war. Both Fearon
and Powell worked with models in which the risk of war emerged endogenously but they specify payoffs which lack the flexibility of Wittman’s formulation. Although Wittman’s conjecture that offsetting changes in war payoffs will have no affect on the probability of war is correct, his overall conclusion is nevertheless too strong. Our analysis illustrates exactly what additional information is needed to understand the relationship between balance and stability and the two specifications that we applied the result to illustrate how one can incorporate richer models of war-fighting into the canonical bargaining setting.

5. Conclusion

Although the crisis bargaining game developed in Fearon (1995) has been quite influential, it has an important limitation when used as a benchmark model for the analysis of changes to war payoffs. We show that the neutrality result is not a limitation of the underlying framework but rather a consequence of the interaction of some individually reasonable simplifying assumptions that add up to this result. The basic structure and logic of the Fearon model is quite amenable to natural extensions that render the model more appropriate when the theory is focused on changing the military balance. In other words, close cousins of the standard model can be well suited to answering questions regarding the underlying effect of changes in the probability of victory. Specifically, the equilibrium probability of war depends on only whether the affect on A’s war payoff is larger or smaller on the affect of B’s war payoff. When the effect on the privileged disputant dominates, then the risk of war increases. Of course the choice of the appropriate variant of this model for a given question is where the art lies.
A final point is in order. It is of conceptual value to recognize that the neutrality result is not an artifact of risk aversion as others have speculated (Fearon 1993, Kydd 2010). In fact in a model with only two possible outcomes, the idea of risk aversion is not particular relevant. Perhaps a more fitting description of what previous scholars have meant is linearity of utility over the shares of the prize. Under this interpretation too we find that previous assessments of what drives the neutrality result miss the mark. Our analysis maintains linearity of utility over the shares and relaxes the requirement that shocks result in offsetting changes to the payoffs from fighting in a model with two possible war outcomes. Although we conclude that departures from risk neutrality or linearity are not essential to violations of the neutrality result a second example with three possible outcomes does illustrate some role for risk aversion (or acceptance) to fuel violations of the neutrality result. Taken together these theoretical findings indicate that the choice of how to model warfighting is important, likely merits more thought in subsequent theoretical work, but likely this work can be accommodated within the confines of canonical bargaining models.
Proposition 3

Proof. Player A selects $x$ to maximize

$$H[w(a) - s_B(1-x)]p(a) + (1 - H[w(a) - s_B(1-x)])s_A(x).$$

The first order condition for an interior $x$ is

$$h[w(a) - s_B(1-x)]p(a) - s_A(x)] + (1 - H[w(a) - s_B(1-x)]) \gamma(x) = 0.$$

For ease of exposition, we suppress the arguments when the meaning is clear, writing $H[w(a) - s_B(1-x)]$ as $H[\cdot]$, with similar choices for $h$ and the derivative of the density $h'$. Applying the implicit function theorem to the first order condition we obtain,

$$\frac{dx^*(a)}{da} = -w'(a)[h[\cdot](p(a) - s_A(x)) + h[\cdot](\frac{p'(a)}{w'(a)} - \gamma(x))]$$

Applying the chain rule and fact that $\pi^*(a) = H[w(a) - s_B(1-x^*(a))]$, we obtain

$$\frac{d\pi^*(a)}{da} = h[\cdot](w'(a) + s_B'(1-x) \frac{dx^*(a)}{da}).$$

Since $h[\cdot]$ is strictly positive, the sign of the above derivative coincides with the sign of $w'(a) + s_B'(1-x) \frac{dx^*(a)}{da}$. Substituting we see that this expression reduces to

$$1 - s_B'(1-x)[h[\cdot](p(a) - s_A(x)) h'[\cdot](2\gamma(x)s_B'(1-x)) + (1 - H[\cdot]) \gamma'(x)].$$

Finding a common denominator, we can simplify this to

$$\frac{(1 - H[\cdot]) \gamma'(x) - h[\cdot]s_B'(1-x)(\gamma(x) + \frac{p'(a)}{w'(a)})}{h'[\cdot](p(a) - s_A(x)) s_B'(1-x) - h[\cdot](2\gamma(x)s_B'(1-x)) + (1 - H[\cdot]) \gamma'(x)}.$$
References


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