Abstract
We generalize the Diamond–Mirrlees production efficiency theorem, that there should be no taxes on sectors producing pure intermediate goods, to an environment with political economy constraints. In our economy, allocations and taxation are decided by self-interested politicians without the power to commit to future policies. The Diamond–Mirrlees production efficiency result holds even when political economy constraints introduce distortions on labor supply and capital accumulation. (JEL: H11, H21, E61, P16)

1. Introduction

There is now a large literature investigating the various constraints that political economy interactions—in particular, the self-interested objectives of politicians and conflict among groups—place on policies (see, for example, the excellent overview in Persson and Tabellini 2000). This literature shows that political economy constraints often lead to policy distortions and studies how public policy differs under different political institutions. The theory of public finance has largely developed without taking these political economy constraints into consideration and has derived a number of important normative conclusions about the structure of taxation. An interesting current research area is to integrate the insights of the political economy literature to determine which of these normative conclusions also have positive content.

In this article, we take a step in this direction by studying one of the most celebrated results in theoretical public finance, Diamond and Mirrlees’s (1971, 1976) productive efficiency theorem. In the standard (normative) framework of public
finance analysis, Diamond and Mirrlees show that optimal tax systems should not involve taxation of (pure) intermediate goods, even if the menu of taxes includes only distortionary instruments. The intuition for this result is simple: Taxation of intermediate goods will cause productive inefficiency by distorting the allocation of factors of production between intermediate and final goods. By reducing intermediate goods taxation and increasing the taxation of consumption or income, the total amount of surplus, the “economic pie,” can be increased.

To investigate whether the Diamond and Mirrlees’s result on intermediate goods taxation extends to an environment incorporating political economy distortions, we construct a simple infinite-horizon economy building on our previous work (Acemoglu, Golosov, and Tsyvinski 2007a, 2007b). The political economy dimension of the model is simple: At each date, fiscal and redistribution decisions are delegated to a politician (or a set of politicians). Politicians are self-interested and can use the available tax instruments to extract resources for their own benefit (for example, for their own consumption). Citizens control politicians as in the standard Barro (1973) and Ferejohn (1986) model and can vote the politician out of office if they are dissatisfied with his performance. The production side of the economy is an extension of the neoclassical growth model considered in Acemoglu, Golosov, and Tsyvinski: Households supply labor, but in addition to the final good used for production and savings, there is an intermediate good sector. The intermediate good sector uses capital and labor, and the final good sector uses capital, labor, and the intermediate good. We investigate the subgame perfect equilibria (SPE) of this dynamic game between politicians and citizens, focusing on the best SPE—the SPE that maximizes citizens’ initial expected utility.

Our main result is that the best SPE always satisfies the Diamond–Mirrlees productive efficiency condition and involves no taxation of intermediate goods. This is true despite the fact that political economy does introduce other distortions, and both labor supply and the level of the capital stock may be lower in the best SPE of our dynamic game than in an “efficient” allocation. We establish this result first by focusing on an economy in which the politician has access to an unlimited set of tax instruments. We then generalize this result to the case in which the politician can only use linear taxes.

The intuition for our main result in this paper is similar to the intuition for the classic Diamond–Mirrlees result.¹ Political economy considerations—the presence of a self-interested politician in charge of policies—necessitate rents for the politician. Moreover, these rents typically introduce distortions on

¹ It is also similar to the intuition for the result in Persson and Tabellini (2000) that in Barro–Ferejohn type environments with full information, the level of public good provision is undistorted.
labor supply and capital accumulation. These distortions ensure that the economy achieves the optimal path of output, balancing the benefits to citizens from higher output with the costs, which involve the greater level of rents that need to be paid to the politician when equilibrium output is greater. Nevertheless, in the best SPE, given the path of output balancing these factors, taxes should be raised and rents to the politician should be delivered in the most efficient way. From this viewpoint, distortions in the intermediate goods sector are pure waste. Therefore, any given path of output can be achieved without distorting intermediate good production and thus without using intermediate good taxes.\(^2\)

To put our results in this article in context, it is useful to compare them to our previous results in Acemoglu, Golosov, and Tsyvinski (2007a, 2007b, 2007c), where we also analyzed dynamic economies with self-interested politicians. The focus of these reports is on whether political economy distortions disappear or remain in the long run. The three articles consider different environments but reach parallel results: If the effective discount factor of politicians is equal to or greater than those of citizens, political economy distortions are present in the short run but disappear asymptotically. In contrast, if politicians are more shortsighted than the citizens, political economy distortions remain even in the long run. Here our results are stronger: We show that there is no intermediate good taxation at any point. The other distortions mimic those in our earlier reports and may remain or disappear in the long run. Therefore, the main contribution of the current article is to isolate a major result in the standard theory of public finance and show that it applies even in environments with political economy distortions (provided that we focus on best SPE).

2. Model

2.1. Environment

We consider an infinite horizon economy in discrete time with a unique final good. There is a continuum of identical households (individuals), with total mass normalized to 1. The utility of a typical individual, denoted by \(h\), at time \(t = 0\) is given by

\[
\sum_{t=0}^{\infty} \beta^t u(c_t^h, l_t^h),
\]  

(1)

2. However, we also show that if, for some reason, intermediate goods directly affect the political economy constraints (for example, because politicians can steal from the intermediate goods sector more easily) then this result would no longer hold.
where \( c^h_t \geq 0 \) is the consumption and \( l^h_t \in [0, \bar{L}] \) is the labor supply of individual \( h \) at time \( t \). \( \beta \in (0, 1) \) denotes the common discount factor. The utility function \( u \) is assumed to be twice continuously differentiable, strictly increasing in \( c \), strictly decreasing in \( l \), and jointly strictly concave in \( c \) and \( l \). In addition, to avoid corner solutions, we assume that \( u \) satisfies the following standard Inada conditions.

\[
\forall l \in \mathbb{R}_+: \lim_{c \to 0} \frac{\partial u(c, l)}{\partial c} = \infty, \quad \lim_{c \to \infty} \frac{\partial u(c, l)}{\partial c} = 0.
\]

\[
\forall c \in \mathbb{R}_+: \lim_{l \to 0} \frac{\partial u(c, l)}{\partial l} = 0, \quad \lim_{l \to \infty} \frac{\partial u(c, l)}{\partial l} = -\infty.
\]

The unique final good is produced according to the production function

\[
Y_t = F(Q_t, K^f_t, L^f_t), \quad (2)
\]

where \( Q_t \) is the input of the intermediate good at time \( t \), \( L^f_t \) is labor allocated to the final good sector at time \( t \), and \( K^f_t \) is capital allocated to the production of the final good at time \( t \).

The production function for the intermediate good is given by

\[
Q_t = Q(K^i_t, L^i_t), \quad (3)
\]

Once again we make the standard assumptions on these production functions; \( F \) and \( Q \) are both twice continuously differentiable, strictly increasing, and jointly concave in all of their arguments. Moreover, again to avoid corner solutions we impose the following Inada conditions.

\[
\forall J \in \{Q, K, L\}: \lim_{J \to 0} \frac{\partial F(Q, K, L)}{\partial J} = \infty, \quad \lim_{J \to \infty} \frac{\partial F(Q, K, L)}{\partial J} = 0.
\]

\[
\forall J \in \{K, L\}: \lim_{J \to 0} \frac{\partial Q(K, L)}{\partial J} = \infty, \quad \lim_{J \to \infty} \frac{\partial Q(K, L)}{\partial J} = 0.
\]

The market clearing conditions for capital and labor at time \( t \) are given by

\[
K^f_t + K^i_t \leq K_t, \quad (4)
\]

\[
L^f_t + L^i_t \leq L_t.
\]

We also assume that at each stage, the society needs to spend an amount \( G \) of final goods for government revenues.

To start with, we do not restrict the set of available tax instruments. This implies that a social planner or the politician in power can directly choose the allocation of resources (the amount of labor supply and consumption for each individual). The only constraint on this choice will be feasibility constraints and
a participation constraint for the citizens, which ensures that citizens are willing to take part in the economy. To simplify this constraint, we assume that there is anonymity; thus an individual can opt out of the economy for one period and then participate in the future (so that the participation constraint can be written in terms of static allocations). More specifically, the feasibility constraints are $c_t \geq 0$ and $l_t \in [0, \bar{L}]$. The participation constraint takes the form $u(c^h_t, l^h_t) \geq u(0, 0)$ for each $h$ and $t$, because the individual can always achieve zero consumption and zero labor supply by opting out.\(^3\) For a consumption labor supply pair $(c^h_t, l^h_t)$ that satisfies the feasibility constraints and the participation constraint at time $t$, we write 

\[ (c^h_t, l^h_t) \in \Lambda. \] (5)

**2.2. The Efficient Allocation without Political Economy**

As a benchmark, let us first consider the allocation that would maximize the $t = 0$ utility of a representative individual without political economy constraints. Because individual utility is strictly concave, this best allocation will involve equal allocation of consumption and labor across individuals and can be represented as the solution to the following program:

\[
\max \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l^f_t + l^i_t) \right\},
\] (6)

subject to the resource constraint

\[
c_t + K^f_{t+1} + K^i_{t+1} + G \leq F(Q(K^i_t, l^i_t), K^f_t, l^f_t),
\] (7)

and to the participation constraint (5) for all $t$. Here $l^f_t$ and $l^i_t$ denote the amount of labor supply to the final and the intermediate goods sectors by a typical individual and thus $l^f_t + l^i_t$ is the total labor supply of the individual (thus superscripts now denote sectors not individuals). Given the differentiability and the Inada conditions, a solution to this program will satisfy a simple set of first-order conditions. Moreover, given the strict concavity of the objective function and the convexity of the constraint set, these first-order conditions are sufficient to characterize the unique solution. The following proposition then follows immediately from the inspection of these first-order conditions.

\(^3\) Note that this participation constraint only needs to be satisfied “along the equilibrium path.” When we consider the dynamic political economy game, the politician can deviate and induce an allocation that does not satisfy this constraint.
PROPOSITION 1 (Efficient Allocation I). *The efficient allocation involves no capital or labor supply distortions—that is, for all \( t \),*

\[
\frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial l} \frac{\partial u(c_t, l_t)}{\partial c} = -\frac{\partial u(c_t, l_t)}{\partial l}, \quad (8)
\]

\[
\frac{\partial u(c_t, l_t)}{\partial c} = \beta \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial K} \frac{\partial Q}{\partial Q} \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Q} \frac{\partial K}{\partial K} \quad (9)
\]

—*and also no distortions in the intermediate good sector—that is, for all \( t \),*

\[
\frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial L} = \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial L} \frac{\partial Q}{\partial Q} \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Q} \frac{\partial K}{\partial K} \quad (10)
\]

\[
\frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Q} = \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Q} \frac{\partial Q}{\partial Q} \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Q} \frac{\partial K}{\partial K} \quad (11)
\]

Because there are unrestricted tax instruments, the revenue necessary for financing the government spending, \( G \), will be raised without inducing any distortions. In particular, there will be no distortions in labor supply and investment, (8) and (9), and also no distortions in the intermediate good sector, (10) and (11). The latter simply means that the marginal products of both factors in the final good sector must be equal to the value of marginal products in the intermediate good sector. This consists of their contribution to production of intermediate goods multiplied by the “shadow price” or “value” of the intermediate in final good production, \( \partial F/\partial Q \). This second part of the proposition is therefore a special case of the general Diamond–Mirrlees production efficiency theorem.

For future reference, we also note the stronger version of the Diamond-Mirrlees result (see, for example, Mirrlees 1986). If instead of having access to unlimited tax instruments, the government only has access to linear taxes, for example, consumption tax, \( \tau^c_t \), labor income tax, \( \tau^l_t \), capital income tax, \( \tau^k_t \), and tax on intermediate good production, \( \tau^I_t \).

PROPOSITION 2 (Efficient Allocation II). *In the economy with linear taxes, the efficient allocation may involve capital or labor supply distortions (i.e., \( \tau^l_t > 0 \) and \( \tau^k_t > 0 \) for some \( t \)), but always features \( \tau^I_t = 0 \).*

The proof of this proposition follows from writing a modified program, where the tax authority chooses the linear taxes and individuals make optimal labor supply, consumption, and saving decisions. Alternatively, the whole program can be written in terms of allocations subject to “implementability” constraints (e.g., Chari and Kehoe 1999). Because we will return to this formulation in Section 3, we do not explicitly provide it here.
2.3. Political Economy

We now turn to our main model, in which taxes are not set by a benevolent fictitious social planner, but by a politician. In particular, we assume that the power to set taxes and transfers (and thus decide the allocation of resources) is entrusted to a politician. This assumption captures the notion that society needs to concentrate the monopoly of violence and the power to tax in a single body for purposes of national defense, provision of public goods, and enforcement of law and order. Citizens control politicians via elections. There is a large number of potential (and identical) politicians, denoted by the set $\mathcal{I}$. The politician’s utility at time $t$ is given by

$$E_t \left[ \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \right],$$

where $x_t$ is the ruler’s consumption at time $t$ and $v: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a twice continuously differentiable, strictly increasing, and concave utility function, with $v(0) = 0$. The politician’s discount factor, $\delta \in (0, 1)$, is potentially different from that of the citizens. Our main results apply regardless of how $\beta$ compares to $\delta$. Nevertheless, the reader may want to focus on the case where $\delta < \beta$, because with this configuration, political economy distortions will be more severe and will not disappear in the long run. This will highlight more clearly the distinction between distortions on labor supply and capital accumulation and distortions in the intermediate goods sector. If citizens decide to replace the politician at any point in time one of the other politicians comes to government and is endowed with the same power to determine the allocation of resources. Moreover, again to simplify the analysis, we assume that a politician does not have access to financial markets and cannot smooth consumption.

Because part of final good production now must be spent on rents for the politician, $x_t$, the resource constraints becomes

$$c_t + K_{t+1}^f + K_{t+1}^i + G + x_t \leq F \left( Q(K_t^i, L_t^i), K_t^f, L_t^f \right),$$

(12)

where $L_t^f \equiv \int_0^1 l_{t,h}^f dh$ and $L_t^i \equiv \int_0^1 l_{t,h}^i dh$ are aggregate supplies of labor to the final and the intermediate goods sectors.

Because the politician in power has access to an unrestricted set of tax instruments, he can effectively choose allocations subject to participation by citizens. The dynamic game between the politician and the citizens is therefore as follows.

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4. The configuration with $\delta < \beta$ may arise because the preferences of the politicians are truly different from those of the citizens, or more realistically, because there is an exogenous probability that the politician will be replaced regardless of his performance on the job and this will shorten his planning horizon. In particular, if the politician has the same discount factor as the citizens but faces an exogenous probability $q$ of replacement, then $\delta \equiv \beta(1 - q)$. 
At each time \( t \), the economy starts with a politician \( i_t \in \mathcal{I} \) in power and a stock of capital inherited from the previous period, \( K_t \in \mathbb{R}_+ \). This capital stock will be allocated between the two sectors, \( K^f_t \in \mathbb{R}_+ \) and \( K^i_t \in \mathbb{R}_+ \) during time \( t \), but for notational purposes, it is simpler to think of this allocation as having taken place just before \( t \), so that we take \( K^f_t \) and \( K^i_t \) as the state variables.

1. Each individual \( h \) makes labor supply decisions, \( l^{f,h}_t \) and \( l^{i,h}_t \). Intermediate output \( Q_t = Q(K^i_t, L^i_t) \) and final output \( Y_t = F(Q(K^i_t, L^i_t), K^f_t, L^f_t) \) are produced.

2. The politician chooses the amount of rents \( x_t \in \mathbb{R}_+ \), a consumption function \( c = [c^h_t]_{h=0}^1 \), which assigns a level of consumption for each level of (current) labor supply, and next period’s capital stocks \( K^f_{t+1} \in \mathbb{R}_+ \) and \( K^i_{t+1} \in \mathbb{R}_+ \) subject to the feasibility constraint (12) and participation constraint of citizens.

3. Elections are held and citizens jointly decide whether to keep the politician or replace him with a new one, \( \rho_t \in \{0, 1\} \), where \( \rho_t = 1 \) denotes replacement.

Note that even though individuals make their economic decisions independently, they make their political decisions—the replacement decision—jointly. This is natural, because when it comes to the political decision, there is complete agreement among the citizens. Joint political decisions can be achieved by a variety of procedures, including various voting schemes or the choice on a random as the decision-maker for the replacement decision. For simplicity, we focus on the latter possibility.

Throughout, we will focus on the SPE of this game and in particular on the best SPE, which maximizes average utility of citizens at time \( t = 0 \).

### 2.4. The Best SPE

The described setup implies that the politician in power can always tax (appropriate as rents) the entire output of the economy and consume it himself. Not surprisingly, if a politician were to take such an action (which is not in the interest of the citizens), in the best SPE, he would be replaced (this is established formally in Lemma 1). Because \( v(0) = 0 \) and the politician does not have access to instruments to smooth consumption, after deviation his utility level will be equal to zero. Consequently, we can represent the politician’s sustainability constraint, which will ensure that he does not wish to deviate, as

\[
\sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \geq v(Y_t),
\]

for all \( t \), where \( Y_t = F(Q(K^i_t, L^i_t), K^f_t, L^f_t) \).
Lemma 1. A best SPE is a solution to the following program:

\[
\max_{\{c_t, l^f_t, l^i_t, K^f_t, K^i_t, x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l^f_t + l^i_t),
\]

subject to \(c_t, l^f_t, l^i_t, K^f_{t+1}, K^i_{t+1}, x_t \geq 0\), the resource constraint (12), the sustainability constraint of the politician, (13), and the participation constraint of citizens, (5), for all \(t\) (with \(L^f_t = l^f_t\) and \(L^i_t = l^i_t\)). Moreover, any solution to this program is a best SPE.

Proof. First, a best SPE must involve symmetric treatment of all citizens given the strict concavity of (1). This explains the maximand in (14). Moreover, any SPE must satisfy (5) and (12), and the nonnegativity of consumption, labor, and capital levels. Next, we need to show that it needs to satisfy (13). Suppose that (13) is not satisfied at some time \(t\). Then the current politician can grab all the output and even with the worst punishment, which is replacement, he will achieve utility \(v(Y_t)\). Because (13) is not satisfied, this is a profitable deviation. But this cannot be an allocation that maximizes the utility of the citizens starting from period zero, because setting \(l^f_t = l^i_t = 0\) would increase citizens’ utility. To complete the proof, we need to show that the replacement strategy supporting this allocation is sequentially rational for the citizens. Let the consumption of the politician resulting from this program be denoted by \(\{\hat{x}_t\}_{t=0}^{\infty}\). Then we need to show that at time \(t\), citizens do not replace a politician that chooses consumption \(x_s = \hat{x}_s\) for all \(s \leq t\), and replace a politician who chooses a higher level of consumption than this. Consider the following strategy profile for politicians: If citizens have replaced a previous politician that has chosen \(x_s = \hat{x}_s\) for all \(s \leq t\), then set \(x'_{t+s} = Y_{t+s}\) for all \(s \geq 1\). If citizens have replaced a politician that has chosen consumption \(x_s > \hat{x}_s\) for some \(s \leq t\), then the politician in power at time \(t+1\) chooses an allocation that maximizes (14) starting with the current capital stock \(K_{t+1}\). Given this strategy profile by politicians, it is sequentially rational for citizens not to replace politicians who have not deviated and to replace those who have. This establishes that the best SPE must satisfy (5), (12), and (13) for all \(t\).

To prove the second part of the lemma, consider an allocation \(\{c_t, l^f_t, l^i_t, K^f_t, K^i_t, x_t\}_{t=0}^{\infty}\) that is a solution to this program. Clearly, no other allocation can give higher utility to the citizens without violating (12) or (13)—and any allocation that violates the first one is not feasible and any allocation that violates the second will not be a SPE. Then choose the same strategy profile for politicians and citizens as above, and this makes the allocation \(\{c_t, l^f_t, l^i_t, K^f_t, K^i_t, x_t\}_{t=0}^{\infty}\) a best SPE. \(\square\)
PROPOSITION 3. In a best SPE, the Diamond–Mirrlees production efficiency theorem holds and (10) and (11) are satisfied.

Proof. Let us represent the maximization in (14) by setting up a recursive Lagrangian as in Marcet and Marimon (1998). Following the same steps as in Acemoglu, Golosov, and Tsyvinski (2007b), this Lagrangian takes the form

\[
\max_{\{c_t, l^f_t, l^i_t, K^f_t, K^i_t, x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t u(c_t, l^f_t + l^i_t) + \sum_{t=0}^{\infty} \delta_t \left\{ \mu_t v(x_t) - (\mu_t - \mu_{t-1})vF((Q(K^i_t, l^i_t), K^f_t, l^f_t)) \right\} + \sum_{t=0}^{\infty} \beta_t \zeta_t \left\{ F(Q(K^i_t, l^i_t), K^f_t, l^f_t) - c_t - K^f_{t+1} - K^i_{t+1} - G - x_t \right\},
\]

where \(\mu_t = \mu_{t-1} + \psi_t\) is the cumulative multiplier with \(\mu_{-1} = 0\), \(\zeta_t\) is the Lagrange multiplier on (12), and \(\psi_t \geq 0\) is the Lagrange multiplier on (13). When this constraint is binding, \(\psi_t > 0\).

In view of assumptions on utility functions and on production structure, the first-order conditions are necessary for a constrained efficient allocation. To simplify notation let us suppose that the participation constraint (5) is slack for all \(t\). Then first-order conditions with respect to \(c_t, l^f_t, l^i_t, K^f_{t+1}\) and \(K^i_{t+1}\) can be written as

\[
\frac{\partial u(c_t, l^f_t + l^i_t)}{\partial c} = \zeta_t, \\
- \frac{\partial u(c_t, l^f_t + l^i_t)}{\partial l^f} = \{\delta/\beta\}^t (\mu_t - \mu_{t-1})v'(Y_t) + \zeta_t \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Y}, \\
- \frac{\partial u(c_t, l^f_t + l^i_t)}{\partial l^i} = \{\delta/\beta\}^t (\mu_t - \mu_{t-1})v'(Y_t) + \zeta_t \frac{\partial F(Y_t)}{\partial Q} \frac{\partial Q(K^i_t, l^i_t)}{\partial K},
\]

\[
\zeta_t = \{\delta/\beta\}^t \delta (\mu_{t+1} - \mu_t)v'(Y_{t+1}) + \beta \zeta_{t+1} \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial K}, \\
\zeta_t = \{\delta/\beta\}^t \delta (\mu_{t+1} - \mu_t)v'(Y_{t+1}) + \beta \zeta_{t+1} \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial Q} \frac{\partial Q(K^i_t, l^i_t)}{\partial K},
\]

for all \(t\), where \(Y_t = F(Q_t, K^f_t, l^f_t)\). Combining the second and the third first-order conditions, we obtain (10) and using the fourth and the fifth, we obtain (11).
If (5) is not slack, then there will be an additional multiplier associated with this constraint, say $\chi_t$, but it is straightforward to verify that it will cancel in the comparison of the second and third (and of the fourth and the fifth) equations, thus the same result applies.

The intuition behind this proposition parallels the intuition of the original Diamond–Mirrlees result. Once the level of the output of the final good is chosen appropriately, the society wishes to achieve the desired level of output as efficiently as possible; this implies that the marginal product of factors used in the final goods and intermediate goods sectors have to be equalized. The key to the result in the proposition is that only the output of the final good but not the output of the intermediate good appears in the sustainability constraint of the politician. Once the level of rents to politician and the amount of the final good is determined, there is no reason to distort factors of production. This parallels the intuition for the Diamond–Mirrlees result given in the Introduction.

Proposition 3 does not characterize the entire best SPE allocation. To do this, we need to determine the consumption and labor supply levels and the dynamics of the capital stock. This step of the analysis is similar to Acemoglu, Golosov, and Tsyvinski (2007a). We will therefore simply state the main result focusing on the case where $\delta < \beta$ and refer the reader to Acemoglu, Golosov, and Tsyvinski (2007a) for a proof.\footnote{Acemoglu, Golosov, and Tsyvinski (2007a) also show that when $\beta = \delta$ or when $\beta \geq \delta$, then political economy distortions exist in the short run, but under some regularity assumptions (in particular ensuring that sufficient utility can be given to the politician without violating the sustainability constraint), the distortions on labor supply and capital accumulation disappear as $t \to \infty$. Given our focus here, these results are less central for the current article.}

**Proposition 4.** Suppose that $\delta < \beta$. If a steady state allocation exists, then the best SPE involves downward labor supply and capital accumulation distortions, that is,

\[ \frac{\partial F(Q_t, K^f_t, l^f_t)}{\partial L} \frac{\partial u(c_t, l_t)}{\partial c} > -\frac{\partial u(c_t, l_t)}{\partial l}, \]  

(16)

\[ \frac{\partial u(c_t, l_t)}{\partial c} < \beta \frac{\partial F(Q_t, K^f_{t+1}, l^f_{t+1})}{\partial K} \frac{\partial u(c_{t+1}, l_{t+1})}{\partial c}, \]  

(17)

as $t \to \infty$. If a steady state does not exist, then (16) and (17) hold infinitely often as $t \to \infty$.

We refer to (16) and (17) as “downward distortions” because it can be verified easily that they imply a lower level of labor supply and lower level of capital accumulation than in the unconstrained allocation. Intuitively, these distortions
result from the political economy constraints, because the opportunity cost of production, by supplying labor and delaying consumption are higher than in the environment without political economy; an increase in output makes deviation by the politician more desirable (by raising \( v[Y] \)), and thus necessitates an increase in the payments to the politician. This increase in opportunity cost makes it desirable for the citizens to reduce the level of labor supply and savings.

The main role of Proposition 4 for us is in the contrast it provides to Proposition 3. The latter showed that there are never any intermediate good distortions in the best SPE. This is not because political economy has no effect on the allocation of resources. Proposition 4 shows that political economy leads to lower labor supply and capital accumulation. In fact these distortions could be quite substantial. Nevertheless, the best SPE always involves no distortions in the intermediate good sector. This is the sense in which the Diamond–Mirrlees’s production efficiency theorem generalizes to our environment with political economy.

The intuition described herein also shows the limits of our generalized production efficiency result. Consider the following environment. Suppose that the politician in power can expropriate only a maximum fraction \( \eta < 1 \) of the final good output (i.e., \( x_t \leq \eta F[Q_t, K^f_t, l^f_t] \)), but he can also directly expropriate a fraction \( \varphi \) of the intermediate sector output \( Q_t \). In this case, there may be distortions in the intermediate good for the same reason as there are labor supply and capital accumulation distortions in Proposition 4; now the level of production of intermediate goods directly enters the sustainability constraint, (13), so an increase in \( Q_t \) may necessitate higher rents for the politician, increasing the opportunity cost of producing intermediate goods. The starkest example of the case where our proposition will not hold is if \( \eta = 0 \) and \( \varphi = 1 \).

3. Intermediate Goods Taxation under Linear Taxes

We now briefly discuss how our results generalize to an environment more in line with canonical Ramsey model of taxation, where the government only has access to linear taxes. This environment is investigated in detail in Acemoglu, Golosov, and Tsyvinski (2007b). Here our purpose is to illustrate the implications for intermediate goods taxes. To save space, we focus on the case in which there is no capital and no way of transferring resources across periods and also refer the reader to Acemoglu, Golosov, and Tsyvinski for details. This implies that the production functions for the final and intermediate goods are given by

\[
Y_t = F(Q_t, L^f_t),
\]

\[
Q_t = Q(L^f_t).
\]

The available tax instruments are a linear consumption tax, \( \tau^c_t \), a linear labor income tax, \( \tau^l_t \), and a linear tax on intermediate good production, \( \tau^I_t \).
Given taxes, each citizen will simply maximize his utility by choosing labor supply and the allocation of his labor between the two sectors, that is,

$$\max_{c_t, l^f_t, l^i_t} u(c_t, l^f_t + l^i_t),$$

with consumption given by the budget constraint 

$$(1 + \tau^f_t) c_t \leq (1 - \tau^f_t) w_t (l^f_t + l^i_t),$$

where $w_t$ is the wage rate at time $t$ and the price of the final goods is taken as the numéraire and normalized to 1. Cost minimization in the final good sector is equivalent to

$$\max_{c_t, l^f_t, Q_t} c_t - w_t l^f_t - q_t (1 + \tau^f_t) Q_t,$$

subject to (19), where $\tau^f_t$ is the linear tax on the intermediate good and $q_t$ is the price of the intermediate good. Finally, the problem of the intermediate goods sector is simply

$$\max_{Q_t, l^i_t} q_t Q_t - w_t l^i_t,$$

subject to (3).\footnote{Alternatively, the intermediate goods tax could have been imposed on intermediate good producers, with identical results.}

We follow the primal approach as in Chari and Kehoe (1999) to characterize the Ramsey problem. This involves adding an implementability constraint to the maximization problem in (14). The implementability constraint summarizes the restrictions placed by individual optimization and market clearing on feasible allocations. Here, this constraint is

$$\frac{\partial u(c_t, l^f_t + l^i_t)}{\partial c} c_t + \frac{\partial u(c_t, l^f_t + l^i_t)}{\partial l} (l^f_t + l^i_t) = 0 \quad \forall t.$$

Let us denote the Lagrange multiplier corresponding to this constraint by $\nu_t$ and form the recursive Lagrangian as in the proof of Proposition 3. The first order conditions with respect to $l^f_t$ and $l^i_t$ are identical to those in the proof of Proposition (3) except the addition of the following term to both

$$\nu_t \left[ \frac{\partial^2 u(c_t, l^f_t + l^i_t)}{\partial c \partial l} c_t + \frac{\partial u(c_t, l^f_t + l^i_t)}{\partial l} (l^f_t + l^i_t) + \frac{\partial^2 u(c_t, l^f_t + l^i_t)}{\partial l^2} (l^f_t + l^i_t) \right].$$

This term therefore cancels from both expressions and leads to the following result.
PROPOSITION 5. Suppose that the government only has access to linear taxes and is controlled by self-interested politicians as described. Then, the equilibrium involves no distortions in the intermediate good sector, that is,

\[
\frac{\partial F(Q_t, l^f_t)}{\partial L} = \frac{\partial F(Q_t, l^f_t)}{\partial Q} \frac{\partial Q(l^f_t)}{\partial L} \quad \text{for all } t.
\]

A number of more general results along these lines are derived in Acemoglu, Golosov, and Tsyvinski (2007b), and a proof of this proposition follows as a corollary of the results presented there.

References