Optimal Fiscal and Monetary Policy with Collateral Constraints

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Abstract

Standard models prescribe the use of state-contingent inflation to absorb government spending shocks and smooth tax distortions. However, inflation can reduce the net worth of banks by revaluing their nominal assets and liabilities. We augment standard models with collateral constraints to account for this effect of inflation and study optimal fiscal and monetary policy. The government should balance the shock-absorbing benefits of state-contingent inflation against the costs of tightening collateral constraints; thus, perfect tax smoothing would no longer be optimal. In the calibration to postwar U.S. data, the optimal policy features a much smaller role of inflation in buffering higher government spending in the absence of any sticky-price friction. We also argue that introducing price stickiness as an additional cost of inflation has limited effects on the role of inflation, as long as government debt has sufficiently long maturity.

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1 Introduction

How should a country respond to government spending shocks? Without defaulting on the government debt, a country can either increase distortionary taxation or use inflation to reduce the real value of government debt denominated in domestic currency. Since the onset of the 2008 financial crisis, deficits and public debt are approaching historical highs in major economies, and many economists have argued that the inflation target should be raised.1

Indeed, standard models in the optimal policy literature prescribe the use of state-contingent inflation to smooth tax distortions (Lucas and Stokey, 1983; Chari et al., 1991). In periods of higher fiscal expenditure, generating inflation allows the government to decrease the real value of its outstanding nominal claims. In this way, the government is able to attenuate the increase in taxes required to maintain the present-value budget balance. In these standard models, inflation is a lump-sum tax on government debt holders and is therefore costless from the ex post point of view.2

This paper considers an important cost of inflation on commercial banks. Higher inflation reduces commercial bank net worth through the revaluation of nominal fixed-income claims. First, commercial banks hold government debt on their balance sheets; monetizing government debt thus directly causes losses for banks.3 Second, due to the maturity mismatch between bank assets and liabilities, a persistent increase in the inflation rate (e.g., a higher inflation target) causes bank asset values to drop faster than liability values.4 Commercial banks play an important role in financial intermediation; because of financial frictions, losses borne by banks hamper credit supply and dampen real economic activity.

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1For example, see Blanchard et al. (2010), Krugman (2013), and Rogoff (2008). Besides reducing government debt, higher inflation is also advocated to address wage rigidity, reduce household debt, and reduce the real interest rate when the nominal interest reaches its zero lower bound.

2Price stickiness can make state-contingent inflation very costly in models where the government issues short-term debt (Schmitt-Grohé and Uribe, 2004; Siu, 2004). However, when the government issues long-term debt, large changes in the value of the debt can be produced by changes in the nominal interest rate, with much smaller and smoother changes in inflation. Optimal policy therefore still features a sizable contribution of inflation to buffering fiscal shocks (Leeper and Zhou, 2013).

3For instance, the holdings of Japanese government bonds by Japan’s banks equate to 900% of their Tier 1 capital (Jenkins and Nakamoto, 2012). When the Bank of Japan started qualitative and quantitative monetary easing in 2013 with the goal of reaching the 2% inflation target, fears arose that banks would bear large losses if the inflation rate were raised.

4In Cao (2014), we find a sizable loss of bank net worth in a moderate inflation episode. See the end of this section for a brief summary.
How does the cost of inflation to commercial banks affect the design of fiscal and monetary policy? To address this question, we adopt the model of Angeletos et al. (2013) and extend it to account for this negative consequence of inflation. We then use the model to study optimal fiscal and monetary policy.

We first consider a simple flexible-price benchmark model to focus exclusively on the cost of inflation to banks. In the benchmark model, the economy is populated with a large number of bankers who provide funds to firms, which are subject to idiosyncratic productivity shocks. For a high-productivity firm to acquire more productive resources, its banker needs to raise external funds through collateralized borrowing. Bankers hold nominal government debt and physical capital, both of which serve as collateral. The government finances fiscal expenditures and interest payments by imposing distortionary labor taxes. When the government generates inflation to reduce the real value of debt, bankers’ collateral constraints are tightened, which impedes resource reallocation across heterogeneous firms and distorts investment decisions. In this sense, state-contingent inflation is no longer a lump-sum tax even ex post. When the government optimizes its policies, it should balance the cost of inflation with the cost of distortionary taxes.

We then use the model to study the response of optimal fiscal and monetary policy to fiscal shocks. We first consider a simplified version of the model where agents’ preferences are assumed to be quasi-linear. Perfect tax smoothing, which emerges from an otherwise identical model without financial frictions, no longer holds in our model. Instead, the optimal response to an expenditure shock features a combination of a higher tax rate and a higher inflation rate. For government expenditure processes calibrated to postwar U.S. data, the response of inflation to an expenditure shock is significantly reduced in our model. Following a 10% increase in government spending, the cumulative inflation rate is 7% in our model and 15% in the frictionless model.

We then relax the assumption of quasi-linear preference and further explore the quantitative properties of the model. We perform a decomposition analysis to study the contribution of inflation and taxes to the financing of higher government expenditures. In the frictionless model, inflation finances almost all increases in government expenditure; in our model, inflation only finances 56% of the increase in government expenditures. Our model also reduces the volatility of inflation by half relative to the frictionless model. To the extent that inflation volatility in the frictionless model is extreme and at odds with the data (Chari et al., 1991), our model provides a rationale for small
inflation volatility in the optimal policy design.

A natural question for our analysis is whether it is relevant in a more realistic environment with nominal rigidities. For instance, if nominal rigidities already greatly reduce the incentive to engineer state-contingent inflation (Schmitt-Grohé and Uribe, 2004), then the introduction of our collateral channel would have no important implications for inflation.

We then extend the benchmark model to incorporate nominal rigidities in the form of price adjustment costs. We find that price stickiness has limited additional effects as long as government debt has long maturity. When government debt has an average maturity of 10 years, the role played by inflation in fiscal financing in the sticky-price model is very similar to that in the benchmark flexible-price model. Real allocations are also very similar in both economies. This is possible because long-term debt allows large changes in the real return of debt through small but smooth inflation. Naturally, inflation becomes very persistent in this model, and inflation in future periods plays a much larger role than inflation in the current period in the financing of higher fiscal spending.

Related literature. This paper relates to the literature on optimal fiscal and monetary policy using a Ramsey approach. Our model builds on Angeletos et al. (2013), who consider optimal fiscal policy when real government debt serves as collateral and focus on the determination of long-run debt level. We extend Angeletos et al. (2013) to introduce nominal government debt and argue that collateral constraints are also important in the optimal design of monetary policy. In particular, the behavior of inflation in our model differs substantially from the frictionless benchmark in Lucas and Stokey (1983) and Chari et al. (1991), where government can use inflation to revalue real returns on nominal debt without cost.

By incorporating price stickiness into the model, our work also connects to the literature on optimal fiscal and monetary policy in sticky-price models. Sims (2013), Leeper and Zhou (2013) and Faraglia et al. (2013) argue that the maturity of government debt matters for the contribution of inflation to fiscal financing. We confirm that their findings carry through to models with financial frictions.

This paper also contributes to the recent literature that examines the redistribution effect of inflation by revaluing nominal contracts in general equilibrium models (Gomes et al., 2014; Garriga et al., 2013; Meh et al., 2010). This literature shares the common notion that nominal contracts create a link between inflation and the real economy and serve as an important source
of monetary non-neutrality, even with fully flexible prices. While the previous studies focus on
nominal household (mortgage) debt and corporate debt, our work highlights the importance of
nominal positions of the banking sector.

At a conceptual level, this paper also relates to a growing literature on the link between sovereign
default and bank fragility (Gennaioli et al., 2014; Sosa-Padilla, 2012; Bolton and Jeanne, 2011). As
inflation can be viewed as a partial default on government liabilities, our model shares with this
literature the idea that the repudiation of government debt tightens financial constraints on the
banking sector. However, our model differs from this literature in two respects. First, the literature
suits emerging economies, which borrow in foreign currencies, and members of the eurozone, which
do not have control over their own monetary policy. Our study applies to advanced economies
such as the U.S. and Japan, which issue debt in their own currencies and have control over their
own monetary policy. Second, the literature usually assumes a lack of commitment on the part of
the government. We instead study optimal policy under full government commitment and focus
exclusively on the frictions in the financial market.

Empirical Relevance. How large is the effect of inflation on the real value of the assets, liabilities,
and net worth of U.S. commercial banks? In Cao (2014), we quantify this effect using bank-
level data from the Bank Reports of Conditions and Income (call reports) filed quarterly by U.S.
commercial banks. We first document that the average maturity of nominal assets is longer than
nominal liabilities by about five years. We then consider a scenario of a 1% unanticipated and
permanent increase in the inflation rate and study its effect on bank balance sheets, in the spirit of
Doepke and Schneider (2006).\footnote{In this experiment, we assume that the only real effect of inflation were to revalue nominal contracts and hold real interest rates constant.} We find an average 15% loss of Tier 1 capital for U.S. commercial
banks in this scenario. The amount of loss is similar for banks that do not hold interest rate
derivatives and therefore do not hedge interest rate risk.\footnote{Recent empirical studies (Begenau et al., 2013; Landier et al., 2013) find that holdings of interest rate derivatives at best partially hedge banks’ exposure to interest rate risk and inflation risk. In particular, Begenau et al. (2013) show that net derivative positions tended to amplify, not offset, balance sheet exposure to interest rate risk for the four largest U.S. banks from 1997 to 2004.} Therefore, even a moderate inflation
episode reduces bank net worth substantially through the revaluation of bank nominal assets and
liabilities.\footnote{Our results are comparable to that in Bank of Japan (2013), who performs a similar analysis to Japanese commercial banks. They find that a 1% parallel shift in the yield curve causes an average 20% loss of Tier 1 capital for Japanese commercial banks in 2012.}
Roadmap. The rest of the paper is organized as follows. We describe the benchmark model in section 2 and describe the Ramsey optimal policy problem in section 3. We study the optimal policy problem under the assumption of quasi-linear utility in section 4 and then relax this assumption in section 5. In section 6, we extend the model to incorporate price stickiness and the long maturity of government debt. We conclude in section 7.

2 The Model

2.1 Environment

The economy consists of a continuum of identical households. Within each household reside equal masses of bankers $i \in [0, 1]$ and workers $j \in [0, 1]$. Each worker supplies labor in a competitive labor market and earns a wage income. Each banker channels funds to a firm that produces final goods used in consumption and investment. Members in each household share consumption perfectly.

Preference and Technology. Preferences over stochastic processes for the household consumption $\{c_t\}_t$ and labor supply $\{h_{j,t}\}_t$ of each worker $i$ are ordered by

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\rho} - 1}{1 - \rho} - \frac{\chi \int_0^1 h_{j,t}^{1+\epsilon} \, dj}{1 + \epsilon} \right).
$$

We ignore financial frictions between a banker and his firm; thus, each banker $i$ effectively owns the firm.\(^8\) We use $i$ to index the firm owned by banker $i$. Firm $i$ uses $k_{i,t}$ units of physical capital and $n_{i,t}$ units of labor to produce output $y_{i,t}$:

$$
y_{i,t} = z_{i,t} F(k_{i,t}, n_{i,t}),
$$

where $z_{i,t}$ is an idiosyncratic productivity shock and $F$ is a production function that has decreasing returns to scale, with $F(k, n) = k^\alpha n^\theta$ and $\alpha + \theta < 1$. We assume that $z_{i,t}$ is independent and

\(^8\)As each banker is the owner of a firm, in the text below we use banker $i$ and firm $i$ interchangeably, with a slight abuse of notation. We abstract from the frictions between bankers and firms by assuming the ownership of a firm by a banker, similar to Gertler and Kiyotaki (2010). This assumption allows us to focus on the bankers’ balance sheets and how bankers’ borrowing capacity is limited by their net worth.
identically distributed across both bankers $i$ and time $t$ and can take two values:

$$z_{i,t} = \begin{cases} 
  z^H & \text{with probability } \sigma \\
  z^L & \text{with probability } 1 - \sigma .
\end{cases}$$

Physical capital depreciates at rate $\delta$. Aggregate capital stock $a_t$ is the sum of the stock of undepreciated capital and current investment $i_t$:

$$a_t = (1 - \delta) a_{t-1} + i_t.$$

**Aggregate uncertainty.** In this model, the only source of aggregate uncertainty is a stochastic government expenditure $g_t$. Aggregate history up until time $t$ is denoted by $g^t = (g_0, ..., g_t)$, and the time-0 probability of $g^t$ is denoted by $\Pr(g^t)$. To save on notation, we use $X_t$ to denote a random variable that is a function of the history $g^t$.

Aggregate output $y_t$ is divided between household consumption $c_t$, investment expenditures, and government expenditures:

$$c_t + a_t + g_t = (1 - \delta) a_{t-1} + y_t. \quad (2)$$

**Capital market and collateral constraint.** The sequence of activities within each time period $t$ is illustrated in Figure 1. At the beginning of period $t$, workers and bankers separate, and they cannot meet each other until the end of the period. We assume that before the separation, each household shares all the assets accumulated during the previous period among all the bankers in the household. Therefore, each banker holds an equal share of the household’s assets. A household’s assets consist of physical capital $a_{t-1}$ and government-issued one-period nominal bonds $B_{t-1}$. The bonds issued in period $t-1$ pays a non-contingent gross nominal interest rate $R_{t-1}^B$.

After the separation of bankers and workers, the idiosyncratic productivity shocks and the aggregate government expenditure shock are realized. High-productivity bankers want to scale up their production and therefore need to finance more labor and capital. As household members are spatially separated at this point, they cannot reshuffle the resources among themselves. To acquire more physical capital, high-productivity bankers can buy it from other bankers in a competitive
capital market. A buyer of capital does not pay for the capital until production is finished; therefore, at this stage, he issues private IOUs to the seller.

However, after employment and production take place, buyers could “run away” and repudiate their IOUs. In this case, sellers could confiscate only some fraction of buyers’ assets. Ex ante, this lack of commitment limits the amount of IOUs buyers can issue. At the end of the period, workers and bankers come back to the household and make consumption and saving decisions together.

![Timeline of activities within period t](image)

In terms of notations, let $q_t$ be the price of capital that clears the time-$t$ capital market and $k_{i,t}$ be the amount of capital used in production by banker $i$. If $k_{i,t} > a_{t-1}$, banker $i$ purchases $k_{i,t} - a_{t-1}$ units of capital and issues IOUs to the seller in order to pay $q_t (k_{i,t} - a_{t-1})$ units of consumption goods.

We now describe the limited commitment problem in the capital market in detail. In the case where a buyer of capital $i$ repudiates the IOUs after production, a seller could confiscate $\xi$ fraction of capital installed in buyer $i$’s firm $k_{i,t}$, and the total real payoff from buyer $i$’s government debt holding $R_{t-1}^B B_{t-1}/P_t$. Therefore, a buyer faces the incentive constraint that the total value of

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9We assume that employment and production happen simultaneously and thus workers can seize bankers’ outputs if bankers refuse to pay wages. As a result, there is no friction in the labor market.

10As will be clear in the capital demand condition (6) in the next subsection, the price of capital $q_t$ equals the gross return of capital for the low-type firm:

$$q_t = 1 - \delta + z^L F_L \left( k^L_t, n^L_t \right).$$

This is because in equilibrium, the bankers who own low-type firms are unconstrained, and are indifferent between selling capital and using it in production.

11A banker cannot pledge the wage incomes of workers in the same household as collateral due to the spatial separation of household members during the period. A banker cannot credibly pledge his own future income either.
IOUs he issues cannot exceed the total value of the confiscable assets, that is,

\[ q_t (k_{i,t} - a_t^{-1}) \leq \xi k_{i,t} + \frac{R^B_{t-1} B_{t-1}}{P_t}. \]

As a buyer’s default happens after production when physical capital can be converted to consumption goods one to one, the real price of capital at this point is 1.

Rearranging this inequality constraint yields

\[ k_{i,t} \leq \frac{1}{q_t - \xi} \times \left( q_t a_t^{-1} + \frac{R^B_{t-1} B_{t-1}}{P_t} \right). \]  

(3)

The left-hand side, which is the total amount of capital that can be used by banker \( i \) in production, is limited by his total net worth (physical capital and government bonds). \( \frac{1}{q_t - \xi} \) is the (within-period) leverage ratio. It has a natural interpretation: for each unit of capital banker \( i \) uses in production, he could credibly pledge \( \xi \) fraction, and therefore he needs to secure the remaining \( q_t - \xi \) fraction using his own net worth.

The bankers’ balance sheets in this model are a simplification of the real-world bank balance sheets. In this model, part of the bankers’ assets (the government bonds) are in nominal terms. Bankers’ liabilities are only within-period, the value of which is not affected by state-contingent inflation. Therefore, they have a shorter maturity than the government bonds that mature in one period. In this sense, this model broadly captures the mismatch of maturity observed in the data.

As a result, this model also captures the negative effect of inflation on bank balance sheets. Other things being equal, when the government engineers inflation and reduces real debt value \( R^B_{t-1} B_{t-1}/P_t \), it reduces the net worth of bankers and tightens their collateral constraints. In this sense, state-contingent inflation is no longer a lump-sum tax on bond holders, even ex post.

Remark. We have assumed that bankers in the same household reshuffle assets among themselves at the end of each period. This assumption allows us to study heterogeneity and capital reallocation while maintaining the tractability of the aggregate economy. Absent this assumption, we would need to keep track of the distribution of assets across bankers. This will greatly increase

\[ \text{This assumption that human capital is inalienable has been followed in much of the literature on financial frictions since Hart and Moore (1994).} \]
the computational burden, especially because we are interested in the optimal policy response to an aggregate shock.

2.2 Households’ decision problem

Before idiosyncratic productivity shocks are realized in each period, all bankers and their firms are ex ante the same. Therefore, the production decisions of a firm only depend on its current productivity shock $z_{i,t}$. I denote variables regarding production decisions by superscript $s$, where $s = L$ if $z_{i,t} = z^L$ and $s = H$ if $z_{i,t} = z^H$.

In each period, a household’s income consists of labor income, profits of bankers’ firms, and savings income. Each worker in a household earns an after-tax wage income $(1 - \tau_t) w_t h_t$,\(^{12}\) where $w_t$ denotes the real wage; each banker earns a profit from his firm:

$$v_s^t = z^s F(k_s^t, n_s^t) - w_t n_s^t - [q_t - (1 - \delta)] k_s^t.$$  \hspace{1cm} (4)

A household’s end-of-period budget constraint is:

$$c_t + a_t + \frac{B_t}{P_t} = [\sigma v_H^t + (1 - \sigma) v_L^t] + (1 - \tau_t) w_t h_t + \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}. \hspace{1cm} (5)$$

A household’s decision problem is to choose $\{k_s^t, n_s^t, h_t, c_t, a_t, B_t\}_{t=0}^\infty$ to maximize utility (1), subject to the end-of-period budget constraint (5) and the collateral constraint (3).

**Firms’ production decision.** The labor and capital demand conditions of a type $s$ firm are

$$z^s F_n(k_s^t, n_s^t) = w_t$$

and

$$z^s F_k(k_s^t, n_s^t) = q_t - (1 - \delta) + \mu_s^t,$$  \hspace{1cm} (6)

where $\mu_s^t U_{c,t}$ is the multiplier on the collateral constraint. In equilibrium, high- and low-productivity bankers carry the same amount of capital and bonds from previous period $a_{t-1}$ and $\frac{R_{t-1}^B B_{t-1}}{P_t}$.

Therefore, the collateral constraint binds at most for the high-productivity bankers, and $\mu_L^t = 0$.

\(^{12}\)As workers within each households are identical, they supply the same amount of labor, i.e., $h_{j,t} = h_t$. 

The beginning-of-period price capital $q_t$ equals the gross rate of return of the low-productivity bankers.

When the constraint strictly binds for the high type and $\mu^H_t > 0$, the marginal product of capital is greater than the cost of capital $q_t - (1 - \delta)$. From the point of view of the aggregate economy, inefficiency occurs because the marginal product of capital does not equalize between the high and low types.\(^\text{13}\)

**Households’ saving decision.** The Euler equations for capital and bond holdings are

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} q_{t+1} \left( 1 + \frac{\sigma \mu^H_{t+1}}{q_{t+1} - \xi} \right) \quad (7)$$

and

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} \frac{R^B_t}{P_{t+1}} \left( 1 + \frac{\sigma \mu^H_{t+1}}{q_{t+1} - \xi} \right). \quad (8)$$

The Euler equations have natural interpretations. When a household invests in one unit of capital at time $t$, in $t + 1$ the low-type bankers in this household receive a return of $q_{t+1}$. The $\sigma$-fraction of high-type bankers in the household can lever up by $\frac{1}{q_t - \xi}$ to acquire more capital for higher production. For each unit of additional capital, they receive an additional return $\mu^H_{t+1}$. Similar logic applies to the Euler equation of the government bonds.

The existence of the collateral constraint also introduces a trade-off for the government on the inter-temporal margin. If the collateral constraint strictly binds with positive probability in the following period, the associated Lagrange multiplier introduces a wedge between the rates of return of capital and government bonds and the inter-temporal marginal rate of substitution. This distorts the household’s investment decision. On the other hand, government bonds (as well as capital) are priced at a premium relative to an asset that is an equally good form of saving but cannot serve as collateral.\(^\text{14}\) A lower interest rate on government debt allows the government to reduce taxes.

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\(^{13}\) As there is no friction in the labor market, the marginal product of labor equalizes across the two types of firms.\(^{14}\) This is consistent with the observations that government bonds pay a lower return due to liquidity attributes (Krishnamurthy and Vissing-Jorgensen, 2012) and that the “natural rate of interest” declines as credit gets tighter (Eggertsson and Krugman, 2012).
2.3 Government policy

The government consists of fiscal and monetary authorities. The fiscal authority imposes proportional taxes on labor income with a tax rate $\tau_t$ and issues new debt with a nominal amount of $B_t$. The monetary authority decides upon the nominal interest rate $R^B_t$. The following consolidated government budget constraint must hold:

$$\tau_t w_t h_t + \frac{B_t}{P_t} = \frac{R^B_{t-1} B_{t-1}}{P_t} + g_t.$$  \hspace{1cm} (9)

2.4 Competitive equilibrium

We now define the competitive equilibrium, taking government policies as given.

**Definition 1.** Given initial conditions $a_{-1}$ and $R^B_{-1} B_{-1}$, a competitive equilibrium is a set of allocation $\{k^H_t, n^H_t, h_t, c_t, a_t, B_t\}_{t=0}^{\infty}$, prices $\{q_t, w_t, P_t\}_{t=0}^{\infty}$, and fiscal and monetary policies $\{\tau_t, R^B_t\}_{t=0}^{\infty}$ satisfying the (consolidated) government budget constraint (9), such that

(i) Given $\{q_t, w_t\}_{t=0}^{\infty}$, bankers choose capital and labor demand $\{k^H_t, n^H_t\}_{t=0}^{\infty}$.

(ii) Given $\{w_t, \tau_t\}_{t=0}^{\infty}$, workers choose labor supply $\{h_t\}_{t=0}^{\infty}$.

(iii) Given $\{q_t, R_t, P_t\}_{t=0}^{\infty}$, households choose savings $\{a_t, B_t\}_{t=0}^{\infty}$.

(iv) Labor, capital, and bond markets clear.

2.5 Aggregation

Next, we characterize the aggregate economy before moving on to the optimal policy problem. To aggregate over the production decisions of the two types of firms, the key is to compute the capital allocation between the two types of firms. Once we know allocations of capital, allocations of labor can be traced down by the equilibrium conditions in the labor market.

Let $x_t \equiv \frac{k^H_t}{a_{t-1}}$ denote the capital used by the high-productivity firms as a fraction of the aggregate capital stock. Then the fraction of capital used by the low-productivity firms is $\frac{k^L_t}{a_{t-1}} = \frac{1-\sigma x_t}{1-\sigma}$. Based on the fact that wage rate equalizes between the two types of firms, the allocation of
labor between the two types is
\[
\frac{n_t^H}{n_t^L} = \left(\frac{z^H}{z^L}\right)^{\frac{1}{1-\theta}} \left(\frac{x_t - \sigma x_t}{1 - \sigma x_t}\right)^{\frac{\alpha}{1-\theta}}.
\]

Let \(y_t = \sigma y_t^H + (1 - \sigma)y_t^L\) be the aggregate output. In appendix A we show that
\[
y_t = \Gamma(x_t) a_{t-1}^\alpha h_t^\theta,
\] (10)

where
\[
\Gamma(x) = \left[\sigma z^H \frac{1}{1-\theta} x \frac{\alpha}{1-\theta} + (1 - \sigma) z^L \frac{1}{1-\theta} \left(\frac{1 - \sigma x}{1 - \sigma}\right)^{\frac{\alpha}{1-\theta}}\right]^{1-\theta}.
\]

\(\Gamma(x)\) is the endogenous aggregate total-factor productivity (TFP) that is affected by the capital allocation between high and low types (measured by \(x\)). Since the production technology has decreasing returns to scale, there exists an efficient level \(x^*\) in the absence of financial friction, that is, \(x^* = \arg\max_x \Gamma(x)\).\(^{15}\) When the collateral constraint binds, \(x\) falls below \(x^*\), and the aggregate TFP falls below the efficient level \(\Gamma(x^*)\).

### 3 Ramsey Optimal policy

The optimal fiscal and monetary policy is the process \(\{\pi_t, R_t^B\}_{t=0}^\infty\) associated with the competitive equilibrium that yields the highest social welfare:
\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( c_t^{1-\rho} - \frac{h_t^{1+\epsilon}}{1 - \rho - \chi} \right).
\]

We will follow the primal approach in the Ramsey policy literature, which involves substituting for prices and policy instruments so that the Ramsey planner directly chooses real allocations.

The wage rate and labor tax rate can be backed out using the labor demand and supply conditions. As there is no friction in the labor allocation across firms, the wage rate equals the

\(^{15}\)\(x^*\) only depends on exogenous parameters:
\[
x^* = \frac{z^H \frac{1}{1-\theta}}{\sigma z^H \frac{1}{1-\theta} + (1 - \sigma) z^L \frac{1}{1-\theta}}.
\]
marginal product of labor using the aggregate production function, that is, 

$$ w_t = \Gamma(x_t)F_h(a_{t-1}, h_t). \tag{11} $$

The labor tax rate is the wedge between the marginal rate of substitution and the marginal product of labor:

$$ \tau_t = 1 - \frac{\Gamma(x_t)F_h(a_{t-1}, h_t)}{U_{h,t}/U_{c,t}}. \tag{12} $$

The price of capital $q_t$ is determined by the marginal product of capital (MPK) of the low-productivity firms, because these firms are not financially constrained. It follows from equation (6) that

$$ q_t = 1 - \delta + \alpha \Gamma(x_t)a_{t-1}^{\alpha-1} h_t^\theta \left[ \frac{L}{\Gamma(x_t)} \frac{1-\sigma(x_t)}{\Gamma(x_t)^{-1}} \right]^{\frac{1}{\alpha+\theta-1}} \equiv q(a_{t-1}, h_t, x_t). \tag{13} $$

The low-productivity firms’ MPK can be decomposed into two terms: the aggregate MPK and the deviation from the aggregate MPK due to capital misallocation. If $x_t = x^*$, then the deviation term is equal to 1, and the low-productivity firms’ MPK equals the aggregate MPK. If $x_t < x^*$, capital allocation is suboptimal and too much capital remains in the low-productivity firms. Therefore, these firms’ MPK is below the aggregate MPK.$^{16}$

The multiplier for the collateral constraint of the high-productivity firms is also determined by equation (6):

$$ \mu_H^H = \frac{1}{\sigma} \Gamma'(x_t) a_{t-1}^{\alpha-1} h_t^\theta \equiv \mu^H(a_{t-1}, h_t, x_t). \tag{14} $$

It is strictly positive if and only if capital allocation is suboptimal, that is, $x_t < x^*$.

At last we substitute for the real return on government debt. We denote the real government debt by $b_t = \frac{B_t}{P_t}$, and the real holding-period return on debt by $r_t^b = \frac{R_t^B P_t^{\eta-1}}{P_t}$. In Appendix B, we show that $r_t^b$ can be substituted for using the household budget constraint (5) and the Euler equation of debt (8), and we arrive at the flow implementability constraint commonly used in the

$^{16}$In our numerical analysis, $q_t$ is always greater than 1. However, it is theoretically possible that $q_t$ falls below 1. This happens when the collateral constraint becomes very tight and the low-type firms are sufficiently unproductive. In this situation, the marginal product of capital of the low-type firms becomes very small and drives $q_t$ below 1.
literature (e.g., Canzoneri et al., 2013).\(^\text{17}\)

\[
\beta \mathbb{E}_{t-1} [U_{c,t}c_t + U_{h,t}h_t - U_{c,t}(1 - \alpha - \theta)y_t] + \beta \mathbb{E}_{t-1} U_{c,t} (a_t + b_t) = U_{c,t-1}(a_{t-1} + b_{t-1}). \tag{15}
\]

Similarly, \(r^b_t\) can be substituted for from the collateral constraint by combining it with the government budget constraint:\(^\text{18}\)

\[
x_t a_{t-1} (q_t - \xi) \leq q_t a_{t-1} + \left( \theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t \right). \tag{16}
\]

We now establish the equivalence between the primal approach and the original Ramsey problem. The proof is in Appendix B.

**Lemma 1.** Allocations \(\{a_t, h_t, x_t, c_t, b_t\}_{t=0}^{\infty}\) satisfying the social resource constraint (2), the flow implementability constraint (15), the Euler equation (7), the collateral constraint (16), \(\mu^H(.) \geq 0\), and the household complementary slackness condition are the same as those in the competitive equilibrium, where price functions \(q(.)\) and \(\mu^H(.)\) are defined in equations (13) and (14).

**Remark.** In this model, only the real debt \(b_t\) and the state-contingent real return on debt \(r^b_t\) matter for real allocations.\(^\text{19}\) As a result, the Ramsey problem only determines the state-contingent return on debt \(r^b_t\). As the government can only adjust the real return through state-contingent inflation, the Ramsey problem determines the state-contingent component of inflation.\(^\text{20}\) On the other hand, the expected gross inflation rate \(\mathbb{E}_{t-1} \pi_t\) is not determined in the Ramsey problem. Without loss of generality, we assume zero expected inflation, that is, \(\mathbb{E}_{t-1} \pi_t = 1\).\(^\text{21}\)

\(^{17}\)One can obtain the present-value implementability condition by iterating the flow-implementability condition over time.

\(^{18}\)See Appendix B.

\(^{19}\)In our model, the government issues nominal debt to the household and uses inflation to adjust the ex post real return on debt. It is equivalent to a model where the government issues real debt paying state-contingent returns. It is also equivalent to a model where the government issues Arrow securities to the household.

\(^{20}\)Denote the gross inflation rate by \(\pi_t\), then the state-contingent component of inflation is given by

\[
\frac{\pi_t}{\mathbb{E}_{t-1} \pi_t} = \frac{\mathbb{E}_{t-1} r^b_t}{r^b_t}.
\]

\(^{21}\)The result of zero inflation can emerge from a sticky price version of our model (see section 6). In the literature, expected inflation can be determined either by incorporating price stickiness, which drives the expected inflation rate to 0, or by introducing non-interest-bearing government liability (money stock) that leads to the Friedman rule (e.g., Chari et al., 1991). Both features are absent in our flexible-price model.
3.1 Recursive representation

For computational purposes, it is convenient to express the optimal policy problem recursively. As a matter of notation, we use variables with a prime to denote the next-period variables and variables with a minus subscript to denote the last-period variables. For example, \( g \) and \( g_- \) are the amounts of government spending in the current and previous periods, respectively. We use \( E(X|g_-) \) to denote the conditional expectation of variable \( X \) in the state \( g_- \).

With this notation in hand, we can describe the recursive representation for the Ramsey problem. Due to time inconsistency, the time-0 Bellman equation differs from that in the subsequent periods. In the main part of this paper, we focus on the time \( t \geq 1 \) continuation problem, where the government fully commits to its policy decisions made in the previous period. We discuss the time-0 problem in Appendix F. The Ramsey problem with full commitment focuses exclusively on the financial friction and provides a clean benchmark.

The Bellman equation involves four state variables: the value of the capital stock \( a \) inherited from the previous period, the real value of government debt issued in the previous period \( b \), the marginal utility of consumption in the previous period \( \lambda \equiv U_c(-) \), and the state of the government expenditure in the previous period \( g_- \). The Bellman equation is

\[
V(a, b, \lambda, g_-) = \max_{x'(g_-), h(g_-), c(g_-), \lambda'(g_-)} E \left[ \frac{c(g)^{1-\rho}}{1-\rho} - \chi \frac{h(g)^{1+\epsilon}}{1+\epsilon} + \beta V(a'(g_-), b'(g_-), \lambda'(g_-), g_-) | g_- \right],
\]

(17)

where the maximization is subject to

\[
E \left[ \beta U_c(g) b'(g) + U_h(g) h(g) + U_c(g)(1 - \alpha - \theta) \Gamma(x(g)) a^\alpha h(g)^\theta | g_- \right] = \lambda(a + b),
\]

(18)

\[
E \left[ U_c(g) q(g) \left( 1 + \frac{\sigma \mu H(g)}{q(g) - \xi} \right) | g_- \right] = \lambda,
\]

(19)

\[
c(g) + g + a'(g) = (1 - \delta) a + \Gamma(x(g)) a^\alpha h(g)^\theta,
\]

(20)

\[
x(g) a | q(g) - \xi | \leq q(g) a + \left[ \theta y(g) + \frac{U_h(g)}{U_c(g)} h(g) + b'(g) - g \right],
\]

(21)
\( \mu^H(g) \geq 0 \), the household complementary slackness condition\(^{22} \), and

\[ \lambda'(g) = U_c(g). \]

Equations (18) to (21) are the implementability condition, the Euler equation of capital, the social resource constraint, and the collateral constraint, respectively. With a slight abuse of notation, we use \( X(g) \) to denote variable \( X \) in state \( g \). We use \( q(g) \) and \( \mu^H(g) \) as short notations for \( q(a(g), h(g), x(g)) \), and \( \mu^H(a(g), h(g), x(g)) \). Compared to an otherwise identical problem without financial frictions, the Ramsey planner in this problem faces two more constraints: the collateral constraint (21) and the household complementary slackness condition (22).

In standard Ramsey policy without financial frictions, state-contingent inflation is a lump-sum tax on bond holders from the ex post point of view, and nominal government debt is no more than a shock absorber. When the government receives a high expenditure shock, it engineers inflation to reduce real debt and smooth tax distortions. If the expenditure shock is persistent, the government also issues less debt to save on debt-servicing costs. However, these policies are distortionary in the presence of the collateral constraint. First, inflation reduces bankers’ net worth, tightens their constraints, and leads to capital misallocation and TFP loss (equation 10). Second, when the government issues less debt, it reduces the amount of collateral in future periods and distorts investment decisions (equation 8). The government balances these distortions caused by state-contingent inflation and debt provision with distortions from labor taxes.

4 The Quasi-Linear Case

In this section, we assume that the household’s utility function is linear in consumption \((\rho = 1)\):

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( c_t - \chi h_t^{1+\epsilon} \right). \]

We adopt this utility function for two reasons. First, the computation of the Ramsey problem simplifies drastically when preferences are quasi-linear. The simplification allows us to adopt a

\(^{22}\)The complementary slackness condition is

\[ \mu^H(g) \left[ q(g) a + \left( \theta y(g) + \frac{U_h(g)}{U_c(g)} h(g) + b'(g) - g \right) - x(g) a [q(g) - \xi] \right] = 0. \]
global solution method in solving the model. In addition, the quasi-linear preference facilitates
the comparison between this model and an otherwise identical model without financial frictions,
because optimal policy in the frictionless model features perfect tax smoothing under this quasi-
linear specification (see below).\textsuperscript{23}

When preferences are quasi-linear, two state variables, real government debt \( b \) and government
expenditure in the previous period \( g \), are now sufficient to describe the state of the economy.
Intuitively, the marginal utility \( \lambda \) is now fixed and equal to one; it can therefore be dropped as a
state variable. Second, by rearranging terms in the objective function and redefining the Bellman
equation, the state variable \( a \) (outstanding capital stock in the current period) can be viewed as a
control variable at the end of the previous period (Farhi, 2010). To see this, use the social resource
constraint to substitute for \( c_t \) in the objective function:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ y_t + (1 - \delta)a_{t-1} - g_t - a_t - \chi h_t^{1+\epsilon} \frac{1}{1+\epsilon} \right] = \frac{1}{\beta} a_{-1} + E_0 \sum_{t=0}^{\infty} \beta^t \left[ y_t + (1 - \delta - \frac{1}{\beta})a_{t-1} - g_t - a_t - \chi h_t^{1+\epsilon} \frac{1}{1+\epsilon} \right].
\]

We define a new value function \( \hat{V}(b, g) \) where the per-period utility is given by the terms inside
the brackets. The value function \( \hat{V}(b, g) \) satisfies the following Bellman equation:

\[
\hat{V}(b, g) = \max_{y'(g), a, h(g), x(g)} \mathbb{E} \left[ y(g) + (1 - \delta - \frac{1}{\beta})a - g - \chi h(g)^{1+\epsilon} \frac{1}{1+\epsilon} + \beta \hat{V}(b'(g), g) | g \right], \tag{24}
\]

where the maximization is subject to

\[
\mathbb{E} \left[ \beta b'(g) + (\beta(1 - \delta) - 1)a + \beta(\alpha + \theta)y(g) - \beta g - \beta \chi h(g)^{1+\epsilon} | g \right] = b, \tag{25}
\]

\[
\beta \mathbb{E} \left[ q(g) \left( 1 + \frac{\sigma \mu^H(g)}{q(g) - \xi} \right) | g \right] = 1, \tag{26}
\]

\[
\theta y(g) - \chi x(g) a \left[ q(g) - \xi \right] \leq q(g) a + \left[ \theta y(g) + U_h(g) h(g) + b'(g) - g \right], \tag{27}
\]

\[
\mu^H(g) \geq 0 \quad \text{and the household complementary slackness condition.} \tag{28}
\]

\textsuperscript{23}I allow for negative consumption. Consumption and investment are determined through interest rates in general
equilibrium. For the size of shock we consider in the numerical simulation, negative consumption does not emerge.
Equations (25) to (27) are the implementability condition, the Euler equation of capital, and the collateral constraint, respectively. The relationship between the new value function $\hat{V}(b, g)$ and the old one $V(a, b, 1, g)$ is that $\hat{V}(b, g) = \max_a V(a, b, 1, g) - \frac{1}{\beta}a$.

The deterministic model. Angeletos et al. (2013) show that the deterministic version of this model features a stable steady state, where the collateral constraint strictly binds.\(^{24}\) This finding sharply contrasts with standard models in which the long-run debt level equals the initial debt level (Barro, 1979). The existence of the collateral constraint introduces a mean-reverting behavior of government debt. On the one hand, increasing government debt relaxes the collateral constraint and improves resource allocation. On the other hand, increasing government debt reduces the collateral value that bankers assign to the government debt and increases the interest rate and servicing cost of debt. These tradeoffs eventually determine the long-run level of government debt and the tightness of the collateral constraint.

In this paper, we are interested in shocks to government expenditures and the response of state-contingent inflation. We first consider an otherwise identical model without financial frictions. This is a useful benchmark to compare with our model in order to identify how financial frictions and collateral constraints shape the optimal fiscal and monetary policy.

4.1 Optimal response to fiscal shocks: without financial frictions

In an otherwise identical model but without financial frictions, we prove the following result (see Appendix D).

**Proposition 1.** (Ramsey policy without financial frictions.) In the absence of the collateral constraint, the Ramsey problem features a constant tax rate and productions across dates and states,

$$a(g^t) = a^*, \quad \text{for } t \geq 0 \text{ and } \forall g^t,$$

$$h(g^t) = h^*, \quad y(g^t) = y^*, \quad \tau(g^t) = \tau^*, \quad \text{for } t \geq 1 \text{ and } \forall g^t,$$

where $a^*$, $h^*$, $y^*$ and $\tau^*$ are constants independent of history $g^t$.\(^{25}\)

\(^{24}\)See Appendix C for a discussion of key policy functions.

\(^{25}\)The initial period allocations $h(g^0)$ and $y(g^0)$ and the tax rate $\tau(g^0)$ differ from $h^*$, $y^*$ and $\tau^*$ for two reasons. First, the government wants to confiscate the entire stock of outstanding debt by infinite price level. Second, the initial level of capital $a_{-1}$ may differ from $a^*$. 

18
Standard Ramsey policy without financial frictions typically finds that the optimal labor tax rate is roughly constant (Chari et al., 1991). Due to the quasi-linear preference, the optimal labor tax rate is exactly constant in our model. As the government spending fluctuates, inflation rate \( \pi \) and the real return on government debt \( r^b \) fluctuate to satisfy the government budget constraint. In particular, when a high government expenditure shock is realized, the government optimally generates higher inflation and reduces the real debt \( r^b b \) by exactly the sum of expected increases in current and future expenditures. In this way, the government maintains a constant labor tax rate \( \tau^* \) regardless of the realization of government expenditure shock \( g \). Intuitively, higher inflation and lower real return on debt resemble a lump-sum tax on households' wealth ex post, but proportional labor tax is distortionary and the efficiency loss is convex. Therefore, the Ramsey planner wants to use state-contingent returns to absorb shocks while making the labor tax rate relatively smooth.\(^{27}\)

### 4.2 Optimal response to fiscal shocks: with financial frictions

We now return to the case of interest, the Ramsey problem in the presence of financial frictions. We use standard policy function iteration to solve the Ramsey problem described by equations (24) to (28).

Table 1 summarizes the parameters used in the numerical exercise. The model is computed at annual frequency, and the discount factor \( \beta \) is set to be 0.96. We set \( \epsilon = 1 \), implying a Frisch elasticity of labor supply of 1. This number, in line with the recommendation of Chetty et al. (2011), is appropriate given that our model does not distinguish between intensive and extensive margins of employment.

Regarding the production technology, the overall returns to scale \( \alpha + \theta \) are set to 0.85 and the share of labor \( \theta \) is set to two thirds of 0.85 (Midrigan and Xu, 2014; Basu and Fernald, 1997). We focus on a symmetric productivity shock process by setting \( \sigma = 0.5 \) and normalize the low realization of productivity \( z^L \) to 1. We choose the high realization \( z^H \) such that the standard deviation of the logarithm idiosyncratic productivity shock is 0.3. This value is in line with the

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\(^{26}\)Our results generalize those of Chari et al. (1991) by incorporating physical capital into the model. On the other hand, it can be shown that in their model, optimal labor tax is constant whenever utility is separable in consumption and leisure, but this is not true in our model.

\(^{27}\)Proposition 1 also shows that after the initial period, capital \( a(g^t) \), labor \( h(g^t) \), and output \( y(g^t) \) are independent of history and state. This is because government consumption shock is the only aggregate shock in this model. Capital, labor, and output will fluctuate if, for example, an aggregate productivity shock is introduced.
estimated size of TFP innovations using U.S. manufacturing firms (Asker et al., 2013). The implied value of \( z^H \) is 1.822.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household discount factor ( \beta )</td>
<td>0.960</td>
<td>exogenous</td>
</tr>
<tr>
<td>Disutility of labor ( \chi )</td>
<td>3.400</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>Inverse Frisch elasticity ( \epsilon )</td>
<td>1.000</td>
<td>in line with Chetty et al. (2011)</td>
</tr>
<tr>
<td>Production Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share of output ( \alpha )</td>
<td>0.283</td>
<td>one third of overall span of control 0.850</td>
</tr>
<tr>
<td>Labor share of output ( \theta )</td>
<td>0.566</td>
<td>two thirds of overall span of control 0.850</td>
</tr>
<tr>
<td>Depreciation rate of capital ( \delta )</td>
<td>0.100</td>
<td>exogenous</td>
</tr>
<tr>
<td>Probability of ( z^H ) ( \sigma )</td>
<td>0.500</td>
<td>exogenous</td>
</tr>
<tr>
<td>High idiosyncratic productivity ( z^H )</td>
<td>1.822</td>
<td>standard deviation of ( \log(z_{i,t}) ) is 0.3</td>
</tr>
<tr>
<td>Low idiosyncratic productivity ( z^L )</td>
<td>1.000</td>
<td>normalized</td>
</tr>
<tr>
<td>Financial Friction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pledgeable share of capital ( \xi )</td>
<td>0.330</td>
<td>steady-state government debt to GDP ratio 61%</td>
</tr>
<tr>
<td>Government Expenditure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS government consumption ( \bar{g}/\bar{y} )</td>
<td>0.151</td>
<td>estimated</td>
</tr>
<tr>
<td>High government consumption ( \bar{g}^H )</td>
<td>1.05( \bar{g} )</td>
<td>estimated</td>
</tr>
<tr>
<td>Low government consumption ( \bar{g}^L )</td>
<td>0.95( \bar{g} )</td>
<td>estimated</td>
</tr>
</tbody>
</table>

The parameter \( \xi \) dictates the severity of financial frictions. We calibrate \( \xi \) such that the debt-to-GDP ratio after a long series of low government spending shocks \( g^L \) converges to 61%, which is the value for the U.S. before the 2008 crises (2007 Q3). We choose to target the debt-to-GDP ratio, because the behavior of state-contingent inflation is very sensitive to it (Siu, 2004).

In standard frictionless models, the government wants to use inflation to reduce real debt by exactly the sum of expected increases in the current and future expenditure. As the debt base increases, the government is able to generate the same change in real claims with smaller variations in the price level. As a result, the inflation volatility required to achieve cross-state tax smoothing becomes smaller.

---

28 As shown in Asker et al. (2013), the firm-level productivity shock is very persistent, with an autocorrelation coefficient of 0.8. Therefore, the standard deviation of the shock is around \( 0.3/\sqrt{1-0.8^2} = 0.5 \). In our model, idiosyncratic productivity shock is i.i.d.. We perform a conservative calibration by calibrating the size of the shock to the size of productivity innovations rather than the productivity process in the data.

29 In standard frictionless models, the government wants to use inflation to reduce real debt by exactly the sum of expected increases in the current and future expenditure. As the debt base increases, the government is able to generate the same change in real claims with smaller variations in the price level. As a result, the inflation volatility required to achieve cross-state tax smoothing becomes smaller.
an otherwise identical asset with no collateral value, which is $1/\beta - \bar{r}b$ in the steady state. Under our choice of $\xi$, the value of the steady state premium is 1.26%. This value is broadly in line with the estimates in the literature, which vary depending on the sample period and the exact type of assets used in the estimation. For instance, Krishnamurthy and Vissing-Jorgensen (2012) estimate that the average liquidity premium from 1926–2008 is 0.46%; Krishnamurthy (2002) documents a liquidity premium of 1.44% in February 2001.

Regarding the bank balance sheets, our parameter choice implies that government debt is 23.3% of total bank assets, and the within-period leverage ratio is 1.45. Compared with U.S. commercial bank data documented in Cao (2014), we overstate the share of government debt as a percentage of bank assets (10%) and understate the leverage ratio of U.S. banks (14). Later in the paper, we will vary the value of $\xi$ and test the robustness of the numerical results.

We adopt an assumption that government expenditure shock follows a two-state Markov process. We use this process to illustrate the transition of the aggregate economy from the low state $g^L$ to the high state $g^H$. Since $g$ does not generate any utility gain for the agents, we calibrate its process to the government consumption data of the U.S. The U.S. data in the sample period 1949 Q1-2007 show that annual government consumption averaged about 15.1% of GDP, with a standard deviation of 1.75% and an autocorrelation of 0.60. The distribution is also very symmetric. Therefore, we set $g^L = 0.194$ and $g^H = 0.213$. We set the transition matrix to

$$
\begin{bmatrix}
0.933 & 0.067 \\
0.067 & 0.933
\end{bmatrix}
$$

The high state $g^H$ is about 10% higher than $g^L$. The transition probability implies that both states have an average duration of 15 years.

Figure 2 shows the dynamics of the model economy as it transitions from the low state $g^L$ to the high state $g^H$. We compare our economy (the blue line) with an otherwise identical economy in the absence of financial frictions (the black dashed line).

We start the economy with financial frictions at a level of government bond $b$ to which the economy converges after a long sequence of $g^L$. Then in year 11, government expenditure switches from $g^L$ to $g^H$ and lasts for 10 years. In the frictionless economy, the level of debt in the stochastic
steady state (after sufficiently long $g^L$ shocks) is indeterminate and depends on the initial level of debt. We therefore set the debt-to-GDP ratio in the frictionless economy to the value in the economy with frictions (61%). That is, before the government spending shock switches from $g^L$ to $g^H$, the two economies have the same debt-to-GDP ratio.

![Graphs showing various economic variables over time]

Figure 2: Stochastic stimulations. High government expenditure shock $g^H$ occurs in year 11 and lasts for 10 years. The debt-to-GDP ratio, the inflation rate, and the interest rates are measured in percentage points; the tax rate is measured in the deviations from its value in year 10 and other variables are measured in percentage deviations from their values in year 10 (before the $g^H$ shock occurs).

Consistent with Proposition 1, the frictionless economy features perfect tax smoothing. As the economy switches from $g^L$ to $g^H$, the government generates 15% state-contingent inflation in one year.\(^\text{30}\) The size of state-contingent inflation is sufficient to reduce the real debt $r^b b$ by exactly the

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\(^{30}\)In this model, we assume that government debt is one-period (year) short-term debt. Therefore, devaluation of debt can only be achieved by inflation in the current period when existing debt matures. If we allow for long-term government debt, then inflation can be spread out into the future. For instance, if the outstanding government debt has a five-year maturity, then roughly speaking, the same debt devaluation could be produced by 3% inflation in the next five years. In this sense, the number we obtain is the cumulative inflation.
sum of expected increases in the current and future government expenditure. Therefore, the labor
tax rate and all the real allocations remain constant. As long as the $g^H$ state continues, real debt
and the debt-to-GDP ratio remain small, and the government responds to the high expenditure
$g^H$ by generating a small amount of inflation. When the economy switches back to the $g^L$ state,
the government generates deflation and maintains a constant tax rate and real allocations. Indeed,
government debt and state-contingent inflation are purely shock absorbers in this economy.

When financial frictions exist, perfect tax smoothing through the monetizing of outstanding
debt is no longer optimal, and the size of the state-contingent inflation is significantly dampened.
As the economy switches from $g^L$ to $g^H$, the optimal policy features a 7% state-contingent inflation,
compared with 15% in the frictionless economy. When financial frictions exist, government
debt provides collateral value to the economy, and it is no longer purely a fiscal cushion. Consequently,
the government faces a tradeoff between the misallocation cost of inflation and the cost of
distortionary labor taxes.

The inter-temporal decisions are also affected by government policies. As government expendi-
ture shocks are persistent, after the initial monetization of debt, the government also issues less
real debt in the subsequent periods as $g^H$ continues, causing a dearth of collateral in the economy.
Bankers are now willing to pay a higher price to hold government debt, leading to a lower real
interest rate paid on debt.\textsuperscript{31} Lower interest rates help the government to reduce the servicing cost
of debt and labor taxes. On the other hand, the dearth of collateral makes capital investment less
appealing. Consequently, capital investment declines sharply.\textsuperscript{32}

Our analysis so far shows the size of inflation starting from a particular level of government
bond $b$ (after the economy converges after a long sequence of $g^L$). In Figure 3, we show state-
contingent inflation when $g^L$ switches to $g^H$, $\pi(b, g^L, g^H)$ as a function of outstanding government
debt. We plot the policy function on the ergodic distribution of government debt in the economy
with financial frictions and transform the horizontal axis to debt-to-GDP ratio.\textsuperscript{33}

\textsuperscript{31}Due to the assumption that the expected inflation rate is zero, the nominal interest rate equals the real interest
rate in our model. As shown in the fifth panel, the size of the interest rate decline is not large enough to hit the zero
lower bound.

\textsuperscript{32}Of course, one reason for the large decline in capital investment is our assumption of quasi-linear preferences. However, we will show that this mechanism is still important after we relax this assumption.

\textsuperscript{33}In the frictionless economy, the ergodic distribution of debt depends on the initial debt level. Therefore, to facilitate a comparison, we show the policy function for the same debt-to-GDP ratio as the economy with financial frictions.
Two observations can be made. First, the size of state-contingent inflation in the economy with financial frictions is always significantly smaller compared with that in the friction economy. Second, in both economies, the optimal size of inflation decreases as the debt level increases. In the frictionless economy, a larger debt base allows the government to generate the same change in real claims with smaller variations in the price level. This mechanism also operates in the economy with financial frictions, but it is accompanied by the effect of the collateral constraint. With a larger amount of real government debt, the collateral constraint is more relaxed and the government is more willing to engineer inflation. As a result, although still downward sloping, the policy function becomes flatter relative to the frictionless economy.

![Figure 3: Policy function of state-contingent inflation \( \pi(b, g^L, g^H) \).](image)

### 4.3 Sensitivity analysis by varying \( \xi \)

We vary the tightness of the collateral constraint \( \xi \) to test the sensitivity of the quantitative results. Higher \( \xi \) means that the bankers can credibly pledge a larger fraction of their capital, relaxing the collateral constraint. Therefore, the government issues a smaller amount of debt in the low state of government expenditure \( g^L \), as in the upper left panel of Figure 4. At the same time, government bonds constitute a smaller fraction of total bank assets (lower left panel).

When the economy switches from \( g^L \) to \( g^H \), the optimal size of state-contingent inflation is always significantly smaller in the economy with collateral constraint (upper right panel). As the
debt base decreases with higher $\xi$, the size of the optimal state-contingent inflation increases in both economies. As we discussed before, with a smaller debt base, the same value of real debt adjustment can only be achieved through larger inflation.

The intra-temporal leverage of bank balance sheets remains small as $\xi$ varies. Leverage equals $1/(q - \xi)$, and capital price $q$ fluctuates around 1. Therefore, the range of leverage is limited.\(^{34}\)

![Figure 4: Comparative statics of $\xi$. The debt-to-GDP ratio, bank bond-to-asset ratio, and the bank leverage are values in the stochastic steady state after a long history of $g_L$. The state-contingent inflation is the size of inflation as the economy switches to $g^H$ after a long history of $g^L$.](image)

5 General Utility Functions

We now relax the quasi-linear preference assumption in the previous section and further explore the quantitative property of the model. As the problem now involves two more state variables that add to the computational burden, we now adopt a local solution method. In particular, we approximate the model economy around the non-stochastic steady state where the collateral constraint strictly binds. When solving the model, we assume that the collateral constraint always binds, and later verify that it is the case for the size of shock we consider.\(^{35}\)

\(^{34}\)In this model, as $\xi$ approaches $\bar{\xi} = x^* - \beta x^*$, the collateral constraint no longer binds (in the steady state).

\(^{35}\)In Appendix E, we evaluate the accuracy of linearized solutions by showing that 1) the linearized solution and the global solution for the quasi-linear model are very similar; 2) the linearized and quadratic solutions for the model with the general utility function are very similar.
We calibrate the model to quarterly frequency. For most of the parameters, we take the calibration from the previous session. We set the relative risk aversion $\rho$ to 2. We estimate an AR(1) process for government consumption. The standard deviation is 1.53% and the autocorrelation is 0.89.

**Fiscal financing decomposition.** To show the contribution of taxes and inflation to the financing of the increases in government spending, we do the following decomposition. Using the first-order-approximation of the inter-temporal government budget constraint, we decompose the increase in government spending $g_t$ into the increase in tax revenue and state-contingent inflation and the decrease in the real interest rate. A higher real interest rate is bad news for fiscal financing, as future primary surpluses are now discounted at a higher rate:

$$
\sum_{s=t}^{\infty} \frac{1}{(\bar{r}_b)^{s-t+1}} \bar{g}_s = \sum_{s=t}^{\infty} \frac{1}{(\bar{r}_b)^{s-t+1}} \bar{T}_s + \frac{1}{\bar{\pi}_t} \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}_b)^{s-t+1}} \bar{\pi}_s - \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}_b)^{s-t+1}} \bar{r}_b.
$$

(29)

where we use $\bar{X}$, $\tilde{X}$, and $\hat{X}$ to denote the steady state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable $X$, respectively. See Appendix G.1 for derivations.

**Table 2: Decomposition of fiscal financing (as a fraction of total increase in $g$)**

<table>
<thead>
<tr>
<th></th>
<th>Frictionless model</th>
<th>Friction model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax revenue</td>
<td>-2.25%</td>
<td>51.87%</td>
</tr>
<tr>
<td>State-contingent inflation</td>
<td>114.68%</td>
<td>55.65%</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-12.43%</td>
<td>-7.52%</td>
</tr>
</tbody>
</table>

We feed in a one standard-deviation government expenditure shock. We then compute the fractions of government spending innovation financed by tax, state-contingent inflation, and real interest rate along the path of the shock (Table 2). In the frictionless economy, inflation finances more than 100% of the present value of the increase in government expenditure.\(^{36}\) In the model with financial friction and liquidity value of debt, inflation only finances 56% of higher government expenditures. This is because of the negative contribution of the real interest rate. After a negative government spending shock hits, consumption drops and grows back to the steady state. Therefore, the real interest rate is higher along this path.

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\(^{36}\)This is because of the negative contribution of the real interest rate. After a negative government spending shock hits, consumption drops and grows back to the steady state. Therefore, the real interest rate is higher along this path.
expenditure, while tax revenue accounts for 52%. The negative contribution of the real interest rate is smaller because the lower value of real debt causes a decline in the real interest rate.

Volatility of inflation and tax rate. In standard models, optimal policy displays large inflation volatility because state-contingent inflation is costless (Chari et al., 1991). We now show that introducing the collateral constraint significantly reduces the volatility of inflation. In this sense, we provide a new explanation in addition to price stickiness as to why volatile inflation is undesirable.

Quarterly standard deviations of inflation and the labor tax rate are shown in Table 3. Without financial friction, volatility of the labor tax rate is near zero, while that of the inflation rate is near 1% per quarter. In contrast, in the model with financial frictions, the standard deviation of inflation is dampened by half, and the labor tax rate becomes much more volatile.

Table 3: Standard deviation of tax rate and inflation (quarterly)

<table>
<thead>
<tr>
<th></th>
<th>Frictionless model</th>
<th>Friction model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.96%</td>
<td>0.47%</td>
</tr>
<tr>
<td>Tax rate</td>
<td>0.02%</td>
<td>0.35%</td>
</tr>
</tbody>
</table>

Robustness as $\xi$ varies. As a robustness check, we vary the tightness of the collateral constraint (Table 4). The first three rows show the contributions of tax revenues, state-contingent inflation, and the real interest rate to the financing of higher government expenditure. The results remain stable as the collateral constraint is tightened or relaxed. In particular, inflation consistently finances around 50% of the present value of higher government expenditures. The contribution of inflation also remains consistently smaller than in the frictionless model.

Table 4: Sensitivity analysis by varying $\xi$

<table>
<thead>
<tr>
<th></th>
<th>$\xi = 0.33$ Baseline</th>
<th>$\xi = 0.23$ Tighter constraint</th>
<th>$\xi = 0.43$ Looser constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax revenue</td>
<td>Frictionless -2.25%</td>
<td>Friction -7.52%</td>
<td>Frictionless -2.13%</td>
</tr>
<tr>
<td></td>
<td>Friction 51.87%</td>
<td>Friction 42.65%</td>
<td>Friction 55.80%</td>
</tr>
<tr>
<td>State-contingent inflation</td>
<td>114.68%</td>
<td>122.73%</td>
<td>106.56%</td>
</tr>
<tr>
<td></td>
<td>55.65%</td>
<td>59.64%</td>
<td>49.50%</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-12.43%</td>
<td>-20.39%</td>
<td>-4.43%</td>
</tr>
<tr>
<td></td>
<td>-7.52%</td>
<td>-2.29%</td>
<td>-5.30%</td>
</tr>
<tr>
<td>SS debt-to-GDP</td>
<td>60.37%</td>
<td>99.10%</td>
<td>21.49%</td>
</tr>
<tr>
<td>Volatility of inflation</td>
<td>0.96%</td>
<td>0.63%</td>
<td>2.51%</td>
</tr>
<tr>
<td></td>
<td>0.47%</td>
<td>0.31%</td>
<td>1.18%</td>
</tr>
</tbody>
</table>

Note: The first three rows show the results of fiscal financing decomposition.
Changing the values of $\xi$ has a large effect on the debt provision of the government in the long-run steady state. For example, when bankers face tighter collateral constraint (smaller $\xi$), the government finds it optimal to issue more debt and provide more collateral to the economy. As a result, the steady state debt-to-GDP ratio becomes larger. For the same reason as the quasi-linear case in the previous section, a larger debt base leads to smaller inflation volatility in both economies. The reverse happens when the collateral constraint is relaxed (larger $\xi$). But importantly, the inflation volatility in our model remains much smaller than in the frictionless model.

To sum up, introducing collateral constraints alone into the standard model significantly reduces the role of inflation in the optimal fiscal and monetary policy, without any sticky-price friction. Inflation now finances only half of higher government expenditures, and it becomes much less volatile compared with the standard model.

6 Introducing Price Stickiness and Long-term Government Debt

We now investigate whether our result in the benchmark flexible-price model is robust in an environment with nominal rigidities, where it is very costly for the government to engineer state-contingent inflation, even in the absence of the collateral channel (Schmitt-Grohé and Uribe, 2004; Siu, 2004).

We extend the benchmark model to incorporate both price stickiness and long-term government debt. In a model calibrated to the degree of price stickiness observed in U.S. data, the quantitative results depend crucially on the average maturity of government debt. When government debt has short maturity, adjustment in the real debt value can only be achieved through large fluctuations in the price level, which has a substantial cost. Consequently, the contribution of inflation to fiscal financing in the optimal policy is significantly smaller than in the benchmark flexible-price model. However, when government debt has long maturity, large changes in the value of the debt can be produced by changes in the nominal interest rate, with much smaller and smoother changes in inflation. As a result, the contribution of inflation to fiscal financing in the optimal policy becomes more similar to that in the benchmark flexible-price model, and the role of the collateral channel remains quantitatively stable.
6.1 The model

*Price stickiness and retail firms.* In order to incorporate price stickiness into the model, we introduce a continuum of retail firms. Retail firms are monopolistic competitors. They buy goods from competitive firms owned by bankers, differentiate these goods costlessly, and resell them to households. The monopoly power of retail firms allows them to set sticky prices above marginal costs; otherwise, they play no role. We assume that profits from retail activity are rebated lump-sum to households.\(^{37}\)

The final goods used in household consumption and investment are aggregated from the differentiated goods using constant elasticity of substitution (CES) technology. The household optimally chooses their demand or each type of good \(j\).

\[
y_{j,t} = y_t \left(\frac{P_{j,t}}{P_t}\right)^{-\nu},
\]

where \(\nu\) measures the elasticity of substitution across goods sold by retail firms, and \(\frac{\nu}{\nu - 1}\) is the static markup. \(P_t\) denotes the aggregate nominal price level, and \(P_{j,t}\) denotes the nominal price of type-\(j\) good.

We introduce price stickiness through a Rotemberg-style price adjustment costs; to adjust nominal price \(P_{j,t}\), retail firm \(j\) pays \(\frac{\psi}{2}(P_{j,t} - 1)^2\) units of final goods. Retail firm \(j\) sets price \(\{P_{j,s}\}_{s \geq t}\) to maximize the expected discounted sum of real profits that it rebates to the household, discounted by the household’s real stochastic discount factor \(\Lambda_{t,s}\) \((s \geq t)\),

\[
\max_{P_{j,s}} \mathbb{E}_t \sum_{s \geq t} \Lambda_{t,s} \left[ \frac{P_{j,s}}{P_s} y_{j,s} - m_{s} y_{j,s} - \frac{\psi}{2} (\frac{P_{j,s}}{P_{j,s-1}} - 1)^2 \right],
\]

subject to the demand function for good \(j\) in equation (30). \(m_t\) is the real price (in the units of final goods) to purchase goods from bankers’ firms. In other words, \(m_t\) is the real marginal cost to produce differentiated goods \(j\).

We focus on a symmetric equilibrium where each retail firm \(j\) sets the same price \(P_{j,t}\) and

\(^{37}\)The separation of competitive and flexible-price firms held by bankers from sticky-price retail firms follows the approach of Bernanke et al. (1999). Directly introducing price stickiness to the firms held by bankers will destroy the tractability of the model because these firms receive idiosyncratic productivity shocks. High-productivity firms would set a lower price and vice versa. Therefore, we would need to keep track of the history and the cross-section distribution of prices.
$P_{j,t} = P_t$ for all $j$. The optimality condition of the retail firms takes the form of the New Keynesian (NK) Phillips curve:\footnote{Log-linearizing equation (31), we get the more familiar-looking NK Phillips curve:}

$$[\nu m_t - (\nu - 1)] y_t - \psi [(\pi_t - 1)\pi_t - \beta E_t \Lambda_{t,t+1} (\pi_{t+1} - 1)\pi_{t+1}] = 0. \quad (31)$$

If $\psi = 0$, that is, in the case without price stickiness (31) reduces to

$$1 = \frac{\nu}{\nu - 1} m_t.$$ 

Intuitively, the real price of good $j$, which is 1 because $P_{j,t} = P_t$ in the symmetric equilibrium, equals the product of the static markup $\frac{\nu}{\nu - 1}$ and the real marginal cost $m_t$. The presence of nominal price rigidities alters this optimality condition.

The social resource constraint now takes into account the real adjustment cost from changing prices:

$$c_t + g_t + a_t + \frac{\psi}{2} \left(\frac{\pi_t}{\pi_{t-1}} - 1\right)^2 = y_t + (1 - \delta) a_{t-1}.$$ 

**Long-term nominal government debt.** We model long-term nominal government debt as a security paying an infinite stream of nominal coupons, which decreases at a constant rate $\eta$. In particular, a bond issued in period $t$ promises to pay one dollar in period $t + 1$ and $(1 - \eta)^{s-1}$ dollars in period $t + s$, with $s \geq 2$. The exogenous parameter $\eta$ dictates the average maturity of government debts. This way of modeling takes the maturity of government debts as given and abstract away from the maturity composition of the government debt portfolio. It allows us to study long-duration bonds without increasing the dimensionality of the state space, and it is commonly adopted in the literature (e.g., Arellano and Ramanarayanan, 2012; Hatchondo and Padilla, 2013).

As in previous sections, we denote the units of nominal government debt by $B_t$. We use $Q_t^B$ to denote the nominal price of debt in period $t$. The household budget constraint in real terms is
given by

\[ c_t + a_t + \frac{Q_t^B B_t}{P_t} = (\sigma v_t^H + (1 - \sigma)\nu_t^L) + \nu_t^R + (1 - \tau_t) w_t h_t + \frac{1 + (1 - \eta)Q_t^B}{P_t} B_{t-1} + g_t a_{t-1}. \]

This constraint differs from the one in the baseline model (equation 5) in terms of the retailers’ profits \( \nu_t \) and the long maturity of government debt \( \eta \). The household maximizes its utility subject to budget constraint (32) and the collateral constraint (3).

Government policies need to satisfy the budget constraint (in real terms).

\[ \tau_t w_t h_t + \frac{Q_t^B B_t}{P_t} = \frac{1 + (1 - \eta)Q_t^B}{P_t} B_{t-1} + g_t \]

The role of the maturity of government debt can be seen from this constraint. In the constraint, the amount of real government debt is \( \frac{Q_t^B B_t}{P_t} \) and the real holding-period return on government debt is \( \frac{1 + (1 - \eta)Q_t^B}{Q_{t-1}^B \pi_t} \). If \( \eta = 1 \) and government debt is one-period debt, the only way to adjust real return ex post is through inflation in the current period \( \pi_t \). Large fluctuations in prices can have a substantial cost in the presence of nominal rigidities. However, if government debt has long maturity, that is, \( \eta < 1 \), adjustment in the real return ex post can be engineered through changes in bond price \( Q_t^B \) (or nominal interest rate), which depends on inflation in future periods. In other words, changes in real debt return can be produced by small and smooth inflation, which is less costly than large fluctuations in inflation. As a result, long-term debt helps the Ramsey government to achieve the desired adjustment in the ex post real return at less cost.

6.2 Numerical analysis

To better understand optimal policy response to government expenditure shocks, we perform a simple numerical exercise by calibrating the model to the U.S. data. Parameters that also appear in the baseline model take the same values in Table 1. The two parameters new to this model are the elasticity of substitution \( \nu \) and the degree of price stickiness \( \psi \). We set these parameters to values estimated from U.S. data in Christiano et al. (2005). In particular, we calibrate \( \nu \) to a 20% markup of retail firms. With respect to nominal rigidities, we set \( \psi \) to a value that would replicate, in a linearized setup, the slope of the price Phillips curve derived using Calvo stickiness with an
average duration of prices of three quarters.\textsuperscript{39}

Similar to the benchmark model, in the steady state of the Ramsey problem, the collateral constraint is strictly binding. We solve the stochastic model by locally approximating the model around the the non-stochastic steady state.\textsuperscript{40}

\textit{Fiscal financing decomposition.} To investigate how the Ramsey optimal policy responds to fiscal shocks, we perform a similar decomposition exercise as in the previous section, using the linear approximation of the inter-temporal government budget constraint. Current inflation, future inflation, taxes, and the real interest rate each finance some fraction of the present value of higher government expenditure $g_t$, as in the following equation:

\begin{equation}
\sum_{s=t}^{\infty} \frac{1}{(\bar{p}_b)^{s-t+1}} \tilde{g}_s = \sum_{s=t}^{\infty} \frac{1}{(\bar{p}_b)^{s-t+1}} \tilde{T}_s + \frac{1}{\bar{\pi}_t} \tilde{\pi}_t + \sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{p}_b)^{s-t+1} \bar{\pi}_s} - \sum_{s=t+1}^{\infty} \frac{1-(1-\eta)^{s-t}}{(\bar{p}_b)^{s-t+1}} \tilde{r}_s.
\end{equation}

See Appendix G.2 for the derivations. Longer maturity of debt (smaller coupon declining rate $\eta$) affects the decomposition in two ways. First, other things being equal, the contribution of inflation in future periods becomes larger because inflation in future periods reduces the real value coupon payments in those periods. Second, the contribution of the real interest rate becomes smaller, as the government only needs to roll over a smaller fraction of debt in each period.

Numerical results of the financing decomposition are reported in Table 5. The first two columns compare the flexible-price economy ($\psi = 0$) and the economy with price stickiness and one-period (three-month) government debt ($\psi > 0$ and $\eta = 1$). In the second economy, the government can only increase the inflation rate in the current period to reduce the real return on debt, but it is costly to do so due to the presence of the price adjustment cost. Therefore, the contribution of current inflation reduces to 14.71%, in contrast with 65.16% in the flexible-price economy.\textsuperscript{41}

\textsuperscript{39}The slope of the Phillips curve in a quarterly Calvo price-setting model is $\frac{(1-\kappa)(1-\beta \kappa)}{\kappa}$, where $\kappa$ is the probability of not being able to re-optimize price (Galí, 2009). $\kappa = 0.667$ is consistent with the average duration of the wage contract being three quarters. The slope in the Rotemberg model in this paper is $\frac{(\nu-1)\bar{y}}{\psi}$, where $\bar{y}$ is the steady-state value of output. We set $\frac{(\nu-1)\bar{y}}{\psi} = \frac{(1-\kappa)(1-\beta \kappa)}{\kappa}$.

\textsuperscript{40}The Ramsey steady state features zero price inflation, that is, $\bar{\pi} = 1$. Intuitively, the only gain from the non-zero inflation rate is to produce state contingency in debt returns in response to fiscal shocks. This gain from inflation does not exist in the non-stochastic steady state in the absence of any fiscal shocks. On the other hand, any deviation from zero inflation leads to a positive adjustment cost in real resources. Thus, we conclude that $\bar{\pi} = 1$.

\textsuperscript{41}The results in column 1 differ from those in the previous section due to the presence of monopolistic competition, that is, $\nu < \infty$. 

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addition, the negative contribution of the real interest rate becomes more severe (-45.72%) in the sticky price model. As the government cannot monetize debt adequately, there exists too much collateral in the economy, causing a higher interest rate. The small contributions of inflation and the real interest rate imply a large contribution of taxes.

Inflation plays a much larger role when government debt has long maturity. Column 3 in Table 5 shows the fiscal decomposition when the average duration of government debt is 5 years, which is consistent with the U.S. data. The contribution of current and future inflation sum up to 33.31%, in contrast with 14.71% in the short-term debt economy. Moreover, long-term government debt allows inflation in future periods to devalue coupon payments in those periods. Hence, future inflation plays a much more important role relative to inflation in the current period (27.69% vs 5.62%). In addition, the negative contribution of the real interest rate becomes smaller, as government only rolls over a small fraction of debt compared with the economy with only three-month debt. Column 4 presents the result when government debt has a longer duration of 10 years. The total contribution of inflation is even larger (38.17%).

Table 5: Decomposition of fiscal financing

<table>
<thead>
<tr>
<th></th>
<th>Flexible price 3-month debt</th>
<th>Sticky price 3-month debt</th>
<th>Sticky price 5-year debt</th>
<th>Sticky price 10-year debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax revenue</td>
<td>44.90%</td>
<td>131.01%</td>
<td>97.3%</td>
<td>84.21%</td>
</tr>
<tr>
<td>Current inflation</td>
<td>65.16%</td>
<td>14.71%</td>
<td>5.62%</td>
<td>3.03%</td>
</tr>
<tr>
<td>Future inflation</td>
<td>0.00%</td>
<td>0.00%</td>
<td>27.69%</td>
<td>35.14%</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-10.06%</td>
<td>-45.72%</td>
<td>-30.62%</td>
<td>-22.37%</td>
</tr>
</tbody>
</table>

Note: All four economies are associated with collateral constraints. They only differ in the degree of price stickiness and the maturity of government debt.

Figure 5 further illustrates the intuition by showing the optimal policy responses to a one standard-deviation fiscal shock of 1.53% with the first-order autocorrelation coefficient of 0.89. When the government issues short-term debt, the inflation rate rises by 0.12% in the same quarter as the fiscal shock occurs. This amount of inflation is much smaller than the 0.56% inflation

\[ D = \frac{1 + \bar{r}^b}{\eta + \bar{r}^b}. \]

42 We use the concept of Macaulay duration, which in the steady state is given by

43 Average duration of 10 years is consistent with the U.K. data.
that emerges in the flexible-price economy. When the government can issue long-term debt, the optimal response of inflation is much smaller (0.03%) but much more persistent. The cumulative inflation in the 10 years after the occurrence of the fiscal shock is 0.37%. Inflation leads to a large decline in the nominal bond price and facilitates the reduction of real debt return in the period when the shock occurs. As a result, the increase in tax rate is greatly dampened. In terms of real allocations, labor and consumption in the economy with long-term debt becomes much more similar to that in the flexible-price economy than to that in the economy with short-term debt.

![Graphs showing optimal policy responses to a fiscal shock](image)

Figure 5: Optimal policy responses to one standard-deviation fiscal shock of 1.53% with the first-order autocorrelation coefficient of 0.89.

### 6.3 Application: war financing

As an application, we use the model to study how the U.S. government should optimally finance the wars in Afghanistan and Iraq. The total appropriations for these wars in 2001-2013 amount to $1.54 trillion (Crawford, 2014). As shown in the left panel of Figure 6, the budgetary costs of wars amount to 7% of total government consumption at its peak.

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44Inflation in the flexible-price economy is not plotted for the consideration of presentation, since it has a much larger magnitude than in the other two economies.
Figure 6: Increases in government consumption caused by wars in Afghanistan and Iraq

We simulate the model with government consumption shocks that match war costs in the data. To do this, we first assume that the government consumption process follows the AR(1) process we estimated in the previous sections. We then calculate the series of shocks that makes the government consumption process match the data in 2001-2013. We assume that after 2013, there are no more shocks, and the government consumption declines at the rate in the AR(1) process. The right panel in Figure 6 shows the series of shocks to the government consumption process.

We calibrate the model to the features of the U.S. before the wars. The debt-to-GDP ratio in 2000 is 54.6%, and the average duration of government debt is 5.8 years. We assume that the economy is in the steady state before the series of war shocks arrive.

Figure 7 displays the prescriptions of three models for war financing. The black dashed line represents the model without financial friction and price stickiness. The government solely relies on inflation to adjust the real debt value, and the tax rate remains relatively constant. The average annual inflation rate in 25 years is around 1.6%. The blue line shows the economy with the existence of the financial friction and collateral constraint. Financial friction dampens the increase in the inflation rate; the average increase in the annual inflation rate is now 1.0%. At the same time, the government raises labor tax rate by an average 0.5 percentage point (from 35 percentage points in the steady state). Finally, the red line represents the economy with both financial friction and prices

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45Strictly speaking, in this model where prices are perfectly flexible and government debt has long maturity, the optimal path of inflation is indeterminate. This is because the government is indifferent between using current and future inflation to adjust the real value of debt. In the simulation, we determine the optimal path of inflation by setting a tiny degree of price-stickiness.
stickiness, which is the most realistic and preferred calibration. Price stickiness further reduces the use of inflation to 0.4% on average, and the rise in the labor tax rate is now 1 percentage point.

In this paper, we have argued that considering the bank net worth effect of inflation substantially changes the optimal fiscal and monetary policy prescriptions. Inflation should play a much smaller role in the financing of higher government spending, compared with standard models where financial frictions and bank balance sheets are not considered.

There are several possibilities regarding extensions of this work. First, we have considered optimal policy when the government can credibly commit to future policies. We conjecture that in a model where the government cannot make a credible commitment, the bank balance sheet effect will make much more of a difference in the optimal policies. In the absence of financial frictions, a government cannot sustain any debt in equilibrium, as it always wants to confiscate the entire stock of government debt by engineering an infinite price level ex post. Yet infinite inflation is not optimal when inflation tightens bankers’ financial constraints. In this sense, financial frictions can be viewed as a commitment device for the government.

7 Conclusion

In this paper, we have argued that considering the bank net worth effect of inflation substantially changes the optimal fiscal and monetary policy prescriptions. Inflation should play a much smaller role in the financing of higher government spending, compared with standard models where financial frictions and bank balance sheets are not considered.

There are several possibilities regarding extensions of this work. First, we have considered optimal policy when the government can credibly commit to future policies. We conjecture that in a model where the government cannot make a credible commitment, the bank balance sheet effect will make much more of a difference in the optimal policies. In the absence of financial frictions, a government cannot sustain any debt in equilibrium, as it always wants to confiscate the entire stock of government debt by engineering an infinite price level ex post. Yet infinite inflation is not optimal when inflation tightens bankers’ financial constraints. In this sense, financial frictions can be viewed as a commitment device for the government.
Second, we have adopted a representative household structure to maintain the tractability of our analysis and to facilitate comparison with standard models. Embedding richer heterogeneity and more realistic dynamics can shed light on the distribution effect of fiscal and monetary policies. Larger bankers and bankers with larger maturity gap between nominal assets and liabilities are hit harder by an increase in inflation and have a greater impact on the aggregate economy. Besides, we have abstracted away from nominal assets other than government debt. A richer quantitative study should account for other large categories of nominal assets, such as mortgage loans.

Third, our model also has implications in terms of the costs of joining a currency union. The main argument against joining a currency union, from the perspective of a member country, is the loss of freedom to tailor monetary policy to local needs. Our model suggests that a country finds it undesirable to resort too much to monetary policy in the first place. Therefore, the cost of joining a monetary union is smaller. It is interesting to quantify this cost by extending our model into an open-economy setting.
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Appendix

A Equilibrium characterization and aggregation

We let $x_t = \frac{k_t^H}{a_{t-1}}$ be the amount of capital used by the high-productivity firm as a fraction of the aggregate capital stock. Then

$$\frac{k_t^L}{a_{t-1}} = \frac{1 - \sigma x_t}{1 - \sigma}.$$ 

There is no friction in the labor market and therefore marginal product of labor is equalized between high and low-productivity firms, i.e.,

$$\theta z^H (k_t^H)^{\alpha} (n_t^H)^{\theta - 1} = \theta z^L (k_t^L)^{\alpha} (n_t^L)^{\theta - 1}.$$ 

Therefore the fraction of labor used by the two types of firms are the following:

$$\frac{n_t^H}{n_t^H} = \left(\frac{z^H}{x_t}\right)^{\alpha} \frac{1 - \sigma x_t}{1 - \sigma} + (1 - \sigma) \left(\frac{z^L}{1 - \sigma x_t}\right)^{\alpha} \Gamma(x_t)^{1 - \theta} = \Gamma(x_t)^{1 - \theta},$$

$$\frac{n_t^L}{n_t^L} = \left(\frac{z^L}{x_t}\right)^{\alpha} \frac{1 - \sigma x_t}{1 - \sigma} + (1 - \sigma) \left(\frac{z^L}{1 - \sigma x_t}\right)^{\alpha} \Gamma(x_t)^{1 - \theta} = \Gamma(x_t)^{1 - \theta}.$$ 

Given the allocations of capital $k_t^H/a_{t-1}$, $k_t^L/a_{t-1}$ and the allocations of labor $n_t^H/h_t$ and $n_t^L/h_t$, the aggregate production function can be written as

$$y_t = \sigma z^H (k_t^H)^{\alpha} (n_t^H)^{\theta} + (1 - \sigma) z^L (k_t^L)^{\alpha} (n_t^L)^{\theta} = \Gamma(x_t) a_{t-1}^h h_t^\theta,$$

where $\Gamma(x_t) = \left[\sigma (z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}} + (1 - \sigma) (z^L)^{\frac{1}{1-\theta}} \left(\frac{1 - \sigma x_t}{1 - \sigma}\right)^{\frac{\alpha}{1-\theta}}\right]^{1-\theta}.$$

B Proof of Lemma 1

Proof of the “only if”

To prove the “only if” part, we need to show that the set of competitive equilibrium conditions imply the set of constraints in Lemma 1. We proceed by showing that competitive equilibrium
conditions imply the implementability constraint (15) and the collateral constraint (16) in Lemma 1. Other constraints can be derived straightforwardly.

**Implementability condition.** Plug the expressions for wage rate and tax rate (11) and (12) into the household budget constraint (5), we have

\[
c_t + \frac{B_t}{P_t} + a_t = (\sigma v_t^H + (1 - \sigma) v_t^L) - \frac{U_{h,t}}{U_{c,t}} h_t + \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}.
\]

Using the definition of firm’s profit (4) we obtain

\[
\sigma v_t^H + (1 - \sigma) v_t^L = y_t - w_t h_t - [q_t - (1 - \delta)] a_{t-1}.
\]

The labor and capital demand conditions (6) imply that

\[
w_t h_t = \theta y_t,
\]

and

\[
[q_t - (1 - \delta)] a_{t-1} = \alpha y_t - \sigma \mu_t^H k_t^H = \alpha y_t - \frac{\sigma \mu_t^H}{q_t - \xi} \left( \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right),
\]

where the last equality holds whether the collateral constraint for the high type binds or not.\(^{46}\)

Because labor market is frictionless, the share of labor income is exactly \(\theta\). Due to frictions in capital allocations, the share of capital measured at market price of capital \(q_t\) is smaller than \(\alpha\) when the collateral constraint strictly binds for the high type (i.e., \(\mu_t^H > 0\)).

Plug the expression for profit back into the household budget constraint, we have

\[
[U_{c,t} c_t + U_{h,t} h_t - U_{c,t} (1 - \alpha - \theta) y_t] + U_{c,t} \left( \frac{B_t}{P_t} + a_t \right) = U_{c,t} \left( 1 + \frac{\sigma \mu_t^H}{q_t - \xi} \right) \left( \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right).
\]

Taking conditional expectation in date \(t - 1\) on both sides of the equation, and use the Euler

\(^{46}\)To see this, if the collateral constraint binds for the high type, we have

\[
k_t^H = \frac{1}{q_t - \xi} \left( \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right).
\]

If the collateral constraint does not bind for the high type, we have

\[
\mu_t^H = 0.
\]
equations for bond and capital (8) and (7), we arrive at the flow implementability condition in equation (15):

$$\beta E_{t-1} [U_{c,t}c_t + U_{h,t}h_t - U_{c,t}(1 - \alpha - \theta)y_t] + \beta E_{t-1}U_{c,t} (a_t + b_t) = U_{c,t-1}(a_{t-1} + b_{t-1}).$$

**Collateral constraint.** Combining the government budget constraint (9) and the expression for tax rate (12), we can express the outstanding value of debt at the beginning of period $t$ by

$$r^b_t b_{t-1} = \theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t.$$

Substituting for $r^b_t b_{t-1}$ in the collateral constraint, we arrive at the form of collateral constraint in equation (16):

$$x_t a_{t-1} (q_t - \xi) \leq q_t a_{t-1} + \left( \theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t \right).$$

**Proof of the “if”**

To prove the “if” part, we need to show that if allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t=0}^\infty$ satisfy the set of constrains in Lemma 1, they also satisfy the set of competitive equilibrium conditions. However, competitive equilibrium conditions also involve prices and policy instruments. We first show here how to construct prices and policy instruments from the set of allocations in the Ramsey problem.

The wage rate $w_t$, price of capital $q_t$, multiplier on high type’s collateral constraint $\mu_t^H$, tax rate $\tau_t$ are implied by equations (11), (13), (14) and (12) respectively.

The only remaining price is the real return on debt $r_t^b$, can be backed out through the government budget constraint (9), i.e.,

$$r_t^b = \frac{\theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t}{b_{t-1}}.$$

It is straightforward to check that allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t=0}^\infty$, prices $w_t$, $q_t$, and policy instruments $r_t^b$ satisfy the competitive equilibrium conditions. $\square$
C The deterministic model and the long-run level of debt

In this subsection we discuss the deterministic model and the determination of long-run debt level. Figure A.1 plot the policy functions of debt issuance, tax rate and real interest rate as a function of outstanding debt.

The first blue-dashed line indicates a stable deterministic steady state. The amount of debt in this steady state is smaller than the amount that relaxes the collateral constraint of the high-type bankers indicated by the second blue-dashed line. The determination of the long-run debt level contrasts sharply with the random walk behavior of debt in Barro (1979) and Aiyagari et al. (2002). This result reflects the tradeoff between the two key distortions confronted by the Ramsey planner. By reducing the level of debt, the planner tightens the bankers’ collateral constraint, which exacerbates the inefficiency in capital reallocation and investment. On the other hand, the tightening of the collateral constraint increases the bankers’ willingness to hold government debt, reduces the interest rate on debt and alleviates tax distortions. The steady-state level of debt is determined by balancing of these two effects.\footnote{At the debt level where the collateral constraint just binds, the distortion from tightening the collateral constraint is second order, while the tax-saving effect is first order. Therefore it is optimal for the government to reduce debt to the steady state level where the collateral constraint strictly binds.}

Note the debt level indicated by the third blue-dashed line, above which the policy function of debt-issuance converges to the 45 degree line, and the optimal policy stops to wind down debt. To reduce debt, the government needs to impose higher taxes in the short run. When the initial debt is too high, this short-run cost of higher taxes together with the long-run cost of tighter constraints outweighs the long-run benefit of lower interest rate and lower taxes.
D Proof of Proposition 1 (Ramsey policy without financial friction)

It is convenient to prove Proposition 1 using the sequence-problem formulation. The Ramsey problem without financial frictions is

$$\max_{a_t, h_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( c_t - \frac{\chi h_t^{1+\epsilon}}{1+\epsilon} \right)$$

subject to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \theta y_t - h_t^{1+\epsilon} - g_t \right) = r_0 b_{-1}, \quad (34)$$

$$\beta \mathbb{E}_t q(a_t, h_{t+1}, x^*) = 1, \quad (35)$$

$$c_t + a_t + g_t = (1 - \delta)a_{t-1} + \Gamma(x^*)a^\alpha_{t-1} h_t^\theta, \quad (36)$$

where equations (34) to (36) are the inter-temporal government budget constraint, the Euler equation of capital, and the social resource constraint, respectively. $x^*$ is the first best value of capital allocation $x$. In the absence of any financial friction, physical capital is allocated optimally across two types of bankers in each time period. As a result, aggregate TFP is always at the maximum $\Gamma(x^*)$. In the text below we save on notation by denoting $\Gamma^* = \Gamma(x^*)$.

By substituting for consumption $c_t$ in the objection function using the resource constraint and substituting for capital price $q_t$ using the capital demand condition, the Ramsey policy could be written as

$$\max_{a_t, h_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \Gamma^* F(a_{t-1}, h_t) - a_t + (1 - \delta)a_{t-1} - g_t - \frac{\chi h_t^{1+\epsilon}}{1+\epsilon} \right]$$

subject to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \theta \Gamma^* F(a_{t-1}, h_t) - h_t^{1+\epsilon} - g_t \right] = r_0 b_{-1}, \quad [\omega]$$

$$\beta \mathbb{E}_t \left[ 1 - \delta + \Gamma(x^*) F_\alpha(a_t, h_{t+1}) \right] = 1. \quad [\gamma_t]$$

In the initial period $t = 0$, the government wants to engineer infinite price level and make $r_0 b_{-1} = 0$ in order to completely monetize the stock of government debt. Following the literature, we assume that the initial price level and therefore the initial return on government debt $r_0$ is given.
The first order condition for capital $a_t$ ($t \geq 0$) is

$$[\partial a_t] (1 + \theta \omega) \beta \mathbb{E}_t \Gamma^* F_a (a_t, h_{t+1}) - 1 + \beta (1 - \delta)$$

$$+ \beta \mathbb{E}_t \gamma_{t+1} \Gamma^* F_{aa} (a_t, h_{t+1}) = 0.$$

The first order condition for $h_t$ ($t > 0$) is

$$[\partial h_t] (1 + \theta \omega) \Gamma^* F_h (a_{t-1}, h_t) - [1 + (1 + \epsilon) \omega] h_t^\epsilon + \gamma_{t-1} \Gamma^* F_{ah} (a_{t-1}, h_t) = 0.$$

Government consumption shock $g_t$ does not enter either the first order conditions of the Ramsey planner or the Euler equation of the household, which suggests that $h_t$, $a_t$ are independent of $g_t$. More formally, assume $h_t = h^*$, $a_t = a^*$, and $\gamma_t = \gamma^*$, where $h^*$, $s^*$, and $\gamma^*$ are constant and independent of time $t$ and state $g^t$. Then the first order conditions become

$$(1 + \theta \omega) \beta \Gamma^* F_a (a^*, h^*) - 1 + \beta (1 - \delta) + \beta \gamma^* \Gamma^* F_{aa} (a^*, h^*) = 0,$$

$$(1 + \theta \omega) \beta \Gamma^* F_h (a^*, h^*) - [1 + (1 + \epsilon) \omega] h^\epsilon + \gamma^* \Gamma^* F_{ah} (a^*, h^*) = 0.$$

The Euler equation of capital becomes

$$\beta [1 - \delta + \Gamma^* F_a (a^*, h^*)] = 1.$$

Therefore, $a^*$, $h^*$, and $\gamma^*$ are pinned down by the above three equations as functions of parameters and multiplier $\omega$, and $\omega$ is determined by making the inter-temporal government budget constraint holds. It immediately follows that tax rate $\tau_t$ is also constant across states

$$\tau_t = 1 - \frac{h^\epsilon}{F_h (a^*, h^*)}.$$

Government debt $b_t$ equals the discounted sum of expected future primary surplus.

$$b_t = \mathbb{E}_t \sum_{s \geq t+1} \beta^{s-t} \left[ \theta F (a^*, h^*) - h^{s+\epsilon} - g_s \right].$$
If $g_t$ process is i.i.d., then $b_t$ is constant across states; if $g_t$ follows a first-order Markov process, then $b_t$ is a function of $g_t$ and also follows a first-order Markov process.

The real return on government debt $r_t^b$ is

$$r_t^b = \frac{\sum_{s \geq t} \beta^{s-t} \left[ \theta F(a^*, h^*) - h^{s+1+\epsilon} - g_s \right]}{\sum_{s \geq t} \beta^{s-t+1} \left[ \theta F(a^*, h^*) - h^{s+1+\epsilon} - g_s \right]}.$$ 

The same as $b_t$, when $g_t$ is a first-order Markov process, $r_t^b$ only depends on current state $g_t$, not on the entire history.

E Accuracy of solution

The quantitative results presented in section 5 are based on a log-linear approximation to the first-order conditions of the Ramsey problem. For the simplified model under the assumption of quasi-linear utility, we have computed numerical solutions using global methods. The availability of global solution allows us to evaluate the accuracy of the log-linear solution at least for the case with quasi-linear utility. Table A.1 shows the maximum percentage deviation of the linear solution from the global solution. It shows that the quantitative results obtained using the global solution and the log-linear approximation are very similar. The most noticeable difference concerns the capital and tax rate, with a percentage difference around 0.72%.

Table A.1: Maximum percentage deviation of the linear solution from the global solution

<table>
<thead>
<tr>
<th>Real interest rate</th>
<th>Tax rate</th>
<th>Debt issuance</th>
<th>Labor</th>
<th>Capital</th>
<th>$x$</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0008</td>
<td>0.0073</td>
<td>0.0008</td>
<td>0.0018</td>
<td>0.0072</td>
<td>0.0002</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Next, we gauge the accuracy of the first-order approximation to the Ramsey problem with general utility functions by comparing it to the results based on a second-order approximation. Figure A.2 compares the responses of policies and allocations to a one standard-deviation government spending shock in the first- and second-order approximations. At least for the size of government expenditure shocks experienced by the U.S. economy, the first- and second-order approximations are remarkably similar.
Figure A.2: Comparing the responses of policies and allocations to a one standard-deviation government spending shock in the first- and second-order approximations.

F The Initial-period Ramsey Problem

The initial-period value function involves only two state variables: outstanding capital stock \( a \) and government spending shock \( g \). In the initial period the government is not bound by any precommitment, therefore \( \lambda \) drops out as a state variable. In addition, initial debt level is not a state variable because government can adjusts inflation rate freely to monetize initial debt. The value function \( U(a,g) \) satisfies the following Bellman equation.

\[
U(a,g) = \max_{b'(g),a'(g),h(g),x(g),c(g),\lambda'(g)} \mathbb{E} \left[ \frac{c(g)^{1-\rho}}{1-\rho} - \lambda \frac{h(g)^{1+\epsilon}}{1+\epsilon} + \beta V(a'(g),b'(g),\lambda'(g),g \mid g) \right],
\]

where the maximization is subject to

\[
c(g) + g + a'(g) = (1 - \delta)a + \Gamma(x(g))a^\alpha h(g)^\theta,
\]

\[
x(g)a[q(g) - \xi] \leq q(g)a + \left[ \theta y(g) + \frac{U_h(g)}{U_c(g)} h(g) + b'(g) - g \right],
\]

\[
\mu^H \geq 0 \quad \text{and the household complementary slackness condition, and}
\]

\[
\lambda'(g) = U_c(g).
\]
Compared with the continuation problem after the initial period, in the initial period the government is not bound by the flow implementability constraint and the Euler equation in the previous period.

The initial-period problem in this model contrasts sharply with that in a frictionless model. In a frictionless model, it is well known that a government finds it optimal to confiscate the entire stock of government debt by generating an infinite price level. By doing this the government reduces the future tax distortions. In our model, monetizing debt has the constraint-tightening effect, and monetizing the entire stock of debt is generally not optimal.

_An example under assumption of quasi-linear utility._ We numerically solve the initial period Ramsey problem under the assumption that the household utility function is quasi-linear. Figure A.3 plots the time path of the deterministic Ramsey policy starting from the initial period, where initial capital stock equals its steady-state level. Parameters take the same values as in Section 4.

![Figure A.3: The time path of the Ramsey optimal policy starting from the initial period in a deterministic model. We assume that the household utility function is quasi-linear.](image)

Due to lack of commitment in the initial period, the Ramsey government uses inflation to adjust the real value of debt such that real debt in the initial period drops to about half of the long-run steady-state level. It then gradually increases and converges to the steady state. Along this path of growing debt, labor tax rate and real interest rate also increase. Importantly, in our model a positive amount (about half of the steady-state level) of debt can be sustained in the initial period, in contrast with a frictionless model. This result relates to the literature on sovereign debt default that domestic banking sector’s exposure to sovereign debt provides a commitment device for the government (Gennaioli et al., 2014; Sosa-Padilla, 2012).
G Derivations of the fiscal financing decompositions

G.1 One-period nominal government debt

In this subsection, we derive the equation of the fiscal financing decomposition (equation 29) under the assumption that the government debt is a one-period nominal debt.

Suppose the government spending shock occurs in date $t$ when the economy was at the steady state. We start from the government budget constraint

$$T_t + b_t = r^{by} b_{t-1} + g_t,$$

where $T_t$ is the labor tax revenue, that is, $T_t = \tau_t w_t h_t$. By linearizing this equation we get

$$\tilde{b}_{t-1} = \frac{1}{\tilde{r}^b} \tilde{b}_t - \frac{1}{\tilde{r}^b} \tilde{r}^{by}_t - \frac{\tilde{g}_t}{\tilde{r}^b} \tilde{g}_t + \frac{\tilde{T}_t}{\tilde{r}^b} \tilde{T}_t,$$

where we use $\bar{X}$, $\hat{X}$, and $\tilde{X}$ to denote the steady state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable $X$, respectively. By iterating this equation forward, we get

$$\tilde{b}_{t-1} = -\sum_{s=t}^{\infty} \frac{1}{(\tilde{r}^b)_{s-t+1}} \tilde{r}^{by}_s - \sum_{s=t}^{\infty} \frac{1}{(\tilde{r}^b)_{s-t+1}} \tilde{g}_s \tilde{b}_s + \sum_{s=t}^{\infty} \frac{1}{(\tilde{r}^b)_{s-t+1}} \tilde{T}_s \tilde{b}_s.$$

Using the fact that $\tilde{b}_{t-1} = 0$, we arrive at

$$\sum_{s=t}^{\infty} \frac{1}{(\tilde{r}^b)_{s-t+1}} \tilde{g}_s \tilde{b}_s = \sum_{s=t}^{\infty} \frac{1}{(\tilde{r}^b)_{s-t+1}} \tilde{T}_s \tilde{b}_s - \sum_{s=t}^{\infty} \frac{1}{(\tilde{r}^b)_{s-t+1}} \tilde{r}^{by}_s. \tag{42}$$

In period $t$ when the shock occurs, the real return of debt $r^b_t$ follows the Fisher equation

$$r^b_t = \frac{\tilde{R}^B_{t-1}}{\tilde{\pi}_t}.$$

By linearizing the Fisher equation and using the fact that the nominal interest rate is pre-determined ($\tilde{R}^B_{t-1} = 0$), we get

$$r^{by}_t = -\frac{\tilde{r}^b}{\tilde{\pi}_t} \hat{\pi}_t.$$
Combining this equation with equation (42), we arrive at the fiscal financing decomposition condition (29), that is,

$$\sum_{s=t}^{\infty} \frac{1}{(\bar r b)_{s-t+1}} \bar g_s = \sum_{s=t}^{\infty} \frac{1}{(\bar r b)_{s-t+1}} \bar T_s b = \frac{1}{\bar \pi} \hat \pi_t - \sum_{s=t+1}^{\infty} \frac{1}{(\bar r b)_{s-t+1}} \hat r_s^b.$$

G.2 Long-term nominal government debt

In this subsection, we derive the equation of the fiscal financing decomposition (equation 32) under the assumption that the government debt is a long-term nominal debt. In this case, equation (42) still holds, but the period-\(t\) return on nominal debt becomes

$$r_t^b = \frac{1 + (1 - \eta) \bar Q_t^B}{Q_{t-1}^B \bar \pi_t}.$$

(43)

Linearizing this equation and using the fact that $\hat Q_{t-1}^B = 0$, we have

$$r_t^b = \frac{1 - \eta}{\bar \pi} \hat Q_t^B - \frac{\hat r_t^b}{\bar \pi t}.$$

Intuitively, the real return on debt depends on the nominal bond price and the inflation rate in the current period. The nominal bond price $Q_t^B$ is in turn a function of the future real interest rates and inflation rates, which can be shown by iterating $\hat Q_t^B$ forward using a linearized version of equation (43):

$$\hat Q_t^B = \sum_{s=t+1}^{\infty} \left( \frac{1 - \eta}{\bar r_b \bar \pi} \right)^{s-t-1} \left( \frac{\bar \pi_s}{\bar \pi} - \frac{\hat r_s^b}{\hat r_b} \right).$$

Therefore, we can express the ex-post return $r_t^b$ as a function of current and future inflation rates and future interest rates:

$$r_t^b = \sum_{s=t+1}^{\infty} \frac{(1 - \eta)^{s-t}}{(\bar r_b)^{s-t-1}} \left( \frac{\bar \pi_s}{\bar \pi} - \frac{\hat r_s^b}{\hat r_b} \right) - \frac{\hat r_b}{\bar \pi_t}.$$

where we use the fact that $\bar \pi = 1$. By combining this equation with equation (42), we arrive at the fiscal financing decomposition condition (32).