Input-Output Structure and Trade Elasticity Preliminary*

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Abstract

This paper studies how disaggregated input-output interactions shape trade and welfare responses to changes in trade costs. I consider a model with a large number of products linked through a general "snakes and spiders" network. The central feature of the model which also makes it highly intractable is endogenous formation of comparative advantage. To overcome this complexity challenge, I obtain a perturbation solution in terms of intuitive summary statistics. I find that, in contrast to the composite intermediate good structure often employed in the literature, imperfect supplier diversification transforms fundamental comparative advantage in two ways. First, as exogenous productivity differences accumulate along supply chains, endogenous variation in relative costs is increasing in the level of trade frictions. Second, comparative advantage of upstream and downstream industries becomes positively correlated. The first effect generates larger welfare gains from trade and also raises the average product-level import share. The second effect, however, is trade-reducing: the tendency of comparative-advantage industries to source disproportionately from each other increases the aggregate home bias. Such comparative advantage spillovers are relatively strong for moderate trade costs, dominating the average import share effect, but decay fast closer to autarky. As a result, the elasticity of trade is generally non-monotone in the level of trade costs.

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1 Introduction

How does the input-output (I-O) structure of the economy affect the response of trade flows and welfare to changes in trade barriers? Once we acknowledge the great complexity of the world's production architecture in which goods are linked through an intricate "snakes and spiders" network, approaching this question is quite challenging even within stylized competitive environments. Thus, in spite of clear microfoundations, models of supply chains struggle with providing a transparent link between outcome variables, such as welfare and trade flows, and model fundamentals which include trade costs and the pattern of I-O interactions. Gaining intuitive insight typically involves either shutting down particular channels (what happens if trade is in final goods only) or semi-informal arguments (when countries exchange both final and intermediate goods, the impact of trade frictions is magnified by multiple border crossing). The lack of understanding of which I-O characteristics are essential in the trade context may be uncomfortable given the growing attention to international production fragmentation.[1] Related to this, little is known about potential biases arising from various simplifications employed in the literature, such as a composite intermediate good assumption which is a common way to avoid specifying the detailed structure of linkages while introducing trade in inputs.

As identified in this paper, a major source of complications in studying supply chain trade lies not in I-O linkages per se, but in their interplay with comparative advantage (CA). My benchmark theoretical model has two symmetric countries, a large number of products, $i = 1..N$, and features a log-linear relationship between relative marginal costs on the one hand and relative exogenous cost shifters and relative prices[2] on the other:

$$\log \frac{MC_i^*}{MC_i} = \log \frac{Z_i^*}{Z_i} + \beta \sum_{i'=1}^{N} \omega_{ii'} \log \frac{P_{i'}^*}{P_{i'}}, \tag{1.1}$$

where β is the input share and $\omega_{ii'}$ is the share of intermediate spending by product i on product i'. As common in multi-industry models, the distribution of relative marginal costs is the central object that determines trade flows and the gains from trade.

[1] The ongoing surge in vertical trade studies is driven by a variety of factors, starting from a basic empirical regularity that trade flows are dominated by intermediate goods. See Johnson (2014) for a review of recent developments.

[2] Prices are decoupled from marginal costs because of trade. All goods enter production function as well as preferences as Armington CES bundles aggregating the corresponding domestic and imported varieties. This structure nests the case of perfect substitutes when the Armington elasticity is infinite. Although this latter case activates the extensive margin and excludes two-way trade, its difference in terms of aggregate outcomes is quantitative rather than qualitative.

In two cases $-$ when intermediate inputs are absent ($\beta = 0$) and with a composite, product-invariant intermediate good $-$ heterogeneity in relative marginal costs is shaped directly by fundamental CA (variation in relative exogenous costs) independently of trade barriers. In general, however, the I-O structure interacts with trade costs to transform fundamental CA. Depending on the level of trade integration, the differences in fundamental productivities accumulate along production chains, which makes marginal cost heterogeneity endogenous to trade frictions. My goal is to investigate how this endogenous CA formation depends on the pattern of I-O linkages and how it maps into trade flows and welfare. Given that under general form of linkages functional form assumptions do not bring much or any tractability, overcoming the model's black box nature is not straightforward. However, the following simple observation allows moving forward.

Although I-O interactions transform CA, they do not create it out of nothing. In particular, in the absence of exogenous technology heterogeneity, there is no relative variation in endogenous marginal costs under any specification of cost shares $\omega_{ii'}$ in (1.1). The model then collapses into (effectively) one-product economy with only intra-industry trade. In fact, this neutrality result applies more broadly to multiple asymmetric countries and non-constant I-O coefficients. Proposition 3.1 considers a multi-country multi-product economy in which product-level trade flows satisfy a gravity equation and states that, without fundamental CA forces, the standard constant-elasticity gravity holds at the aggregate level under very general I-O structure. Despite the fact that multiple border crossing makes import prices move more than one-for-one with trade costs, the *relative* prices are only affected by I-O when it interacts with CA.[3] For its constructive value, Proposition 3.1 suggests the gravity framework as a natural point of departure for studying the effects of CA (as well as other forms of product-level heterogeneity). In parallel to macroeconomics, the gravity system can be viewed as a "steady state" to which the economy converges in the absence of shocks. To fully exploit this analogy, I adopt the perturbation methodology to modeling cross-section variation in international trade.[4]

I obtain a second-order approximation for the weighted variance of relative marginal costs as a function of trade barriers and a few summary statistics for the I-O matrix and

[3] In Section 3 I discuss the connection of this result to the existing literature. Most importantly, Yi (2003, 2010) introduced the idea that international production sharing can magnify the responses of trade flows to changes in trade costs. For the role of CA, a special case of Proposition 3.1 is present in French (2015) who does not focus on I-O linkages and employs a composite intermediate good structure.

[4] Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) use a similar methodological idea that network interactions can be studied by perturbing a cross-section of homogenous economic units. While the main distinction of my work is in exploring features of the model that are specific to international trade, in Section 3 I also discuss technical differences with this paper.

then show that this object is sufficient to capture second-order departures of trade flows and welfare from the gravity benchmark. My three main results are the following.

First, heterogeneity in relative marginal costs is magnified by I-O interactions and increases with the level of trade frictions. Under free trade, when international production sharing is not constrained, producers in each country face the same input prices and the only source of heterogeneity in relative marginal costs is variation is fundamental productivities. When trade is restricted, firms increasingly rely on domestic intermediate inputs; the differences in fundamental productivities accumulate along production chains, resulting in stronger marginal cost heterogeneity. In other words, due to trade barriers, firms in different countries face different input prices, which contributes to marginal cost variation. This mechanism depends crucially on the extent of supplier diversification in the economy which is captured by the weighted average product-level Herfindahl of cost shares. In case of perfect supplier diversification (for example, under complete I-O network), there is no amplification of exogenous cost heterogeneity because variation in per unit total material costs vanishes for each particular product. At the same time, when products concentrate their intermediate spending on narrows subsets of specialized inputs, trade barriers cause the variance of relative marginal costs to be greater than the variance of relative exogenous costs. This result highlights the importance of the "spider" dimension of production networks.

Second, the combined welfare effects of CA and production sharing are approximated with an additional term in the formula by Arkolakis, Costinot, and Rodriguez-Clare (2012), henceforth ACR. This term is precisely the variance of relative marginal costs multiplied by a measure of trade integration (a function of trade costs and the Armington elasticity only). Therefore, holding other model primitives fixed, a particular I-O structure generates a higher welfare if and only if it corresponds to stronger marginal cost heterogeneity. Thus, perfect supplier diversification gives the lowest gains from trade. This suggests that imposing a composite intermediate good assumption and ignoring the disaggregated nature of I-O linkages will tend to underestimate the gains from trade.[5]

Third, the effects of CA and production sharing on trade flows are more nuanced. Marginal cost heterogeneity affects trade flows by two channels. One is the average product-level import share and the other is CA spillovers expressed as a covariance between upstream and downstream industries' performance. The second channel arises from an accounting decomposition and intuitively means that products sourcing from CA in-

[5] In Section 5 I show that this is indeed the case in the context of my model, both when trade costs are observed and when they need to be recovered.

dustries tend to produce more compared to autarky. Importantly, this second channel (CA spillovers) decreases trade flows for a given level of trade barriers or, in other words, amplifies the home bias.[6] In a second-order approximation, the average import share is proportional to the variance of relative marginal costs, while the strength of CA spillovers is proportional to the derivative of this variance with respect to trade costs. Analyzing the behavior of these two channels, I find that, compared to the complete network, the role of imperfect supplier diversification in a general I-O structure with disaggregated interactions is to amplify the effect of small trade costs, but to generate relatively more trade when trade costs are high. Although variation in relative marginal costs increases in the level of trade frictions, this happens slowly near free trade, so that the trade-increasing effect of stronger heterogeneity is dominated by the trade-decreasing effect of CA spillovers. When trade costs are high, however, these spillovers decay fast, so that their negative contribution to trade flows becomes weaker relative to the positive contribution of stronger marginal cost heterogeneity.[7]

To summarize, the present paper establishes a theoretical result that imperfect supplier diversification magnifies the gains from trade, while also amplifying the home bias for moderate trade costs. The latter effect occurs due to CA spillovers. What can we say about the extent of supplier diversification and the presence of CA spillovers in the data? Appendix A.1 presents some evidence which is based on the B.E.A. detailed I-O table and trade flows for 2007. Figure 11 gives the histogram of cost shares Herfindahl indices for 388 industries, $HHI_i = \sum_j \omega_{ij}^2$ where ω_{ij} is the share of product j in intermediate spending of industry i. The average value[8] of 0.12 indicates that supplier diversification at this level of aggregation is quite far from perfect. For different measures of industries' CA, I calculate the upstream CA for industry i as a linear combination of its suppliers' CA with the weights equal to the corresponding input shares. I find that, first, consistent

[6] To get a clearer understanding of this effect, it is helpful to compare domestic expenditure by consumers (D^F for final) and firms (D^I for intermediate). Intuitively, D^F is dominated by consumers' purchases of goods in which a country has a comparative advantage, while D^I mostly counts intermediate spending on comparative-advantage goods *by* comparative-advantage industries. This "two-sided selection" increases D^I when comparative advantage in downstream and upstream industries is positively correlated. Interestingly, home bias for intermediate expenditure can be present even under free trade, due to self-sourcing (I-O matrix diagonal) or positive correlation between upstream and downstream exogenous productivities.

[7] Although perturbation approximations in this paper are derived for a finite Armington elasticity, numerically I find qualitatively similar results when domestic and imported goods are perfect substitutes. In particular, for a two-stage production structure similar to Yi (2010), the home bias is magnified (compared to roundabout production Eaton and Kortum, 2002) only locally for moderate trade costs.

[8] This paper shows that the (weighted) average Herfindahl index of supplier diversification is the key summary statistic for the I-O matrix as it captures how fundamental CA is magnified by I-O linkages.

with nontrivial concentration of intermediate spending, there is substantial variation in upstream CA. Second, there is a positive and statistically significant relationship between upstream and downstream CA. This evidence is consistent with the model's implication that CA spillovers endogenously arise under costly trade.

Additional results. [preliminary] Within the model context, I study under what conditions the gains from trade can be estimated based on disaggregated trade data alone (that is, without having the detailed production data) and the role of correct I-O structure specification. I find that ignoring disaggregated nature of linkages by assuming a composite intermediate good introduces a downward bias in the welfare gains estimates.

Literature

Multistage production. My paper relates closely to the multistage production literature originated by Yi (2003, 2010). Subsequent developments include Johnson and Moxnes (2013), Connolly and Yi (2014), Arkolakis and Ramanarayanan (2009), and Kim (2013). These models rely on specific, highly stylized patterns of the production network by assuming two-stage production and/or composite intermediate good. While Baldwin and Venables (2013) draw attention to a more general "snakes and spiders" structure, little progress is made in terms of general understanding of supply chain trade because of limited tractability. The perturbation approach of this paper allows overcoming this challenge and obtaining analytical characterizations.

Multisector gravity. Another strand of related research is the multisector gravity literature, e.g. Caliendo and Parro (2015), Costinot and Rodriguez-Clare (2014), Ossa (2015). The focus of this literature is mainly quantitative; the role of international production sharing is only demonstrated by shutting down trade in inputs and comparing the resulting trade and welfare effects. My work is complementary to this line of research. On the one hand, I contribute to the theoretical understanding of the mechanisms operating in these papers, unifying them with the multistage production literature. On the other hand, I focus on highly disaggregated I-O interactions, while in the multisector gravity literature the number of sectors is relatively small. As my benchmark model assumes uncorrelated cost shocks to particular industries, it does not incorporate large industrial clusters and therefore does not, in general, nest the setup of, for example, Caliendo and Parro (2015).[9]

Endogenous formation of CA. The central conceptual idea of my paper, so far underappreciated in the literature, is that variation in relative costs is endogenously determined

[9] See Section 3 for a further discussion. In particular, Section 3.4 outlines a multisector extension of my benchmark model.

together with international production sharing. A related argument made in the literature, see Koopman, Wang, and Wei (2014), is that refining the concept of revealed CA needs to control for I-O links. I rather emphasize that the *strength* of CA forces is co-determined with vertical specialization, which, to the best of my knowledge, is a new angle. At the same time, endogenous CA formation with feedback effects is examined in several other contexts, including external economies of scale in Grossman and Rossi-Hansberg (2010); intra-industry heterogeneity interacted with Heckscher-Ohlin forces in Bernard, Redding, and Schott (2007); directed innovation in Somale (2014); horizontal multinational production in a multisector setting in Alviarez (2015).

The rest of the paper is organized as follows. Section 2 presents the benchmark two-country model. While several basic results can be obtained analytically, further progress requires using some type of approximation. Section 3 establishes a neutrality result which suggests perturbation with respect to CA as a way forward. Section 4 then provides a perturbation solution to the model. Section 5 (preliminary) contains extensions and additional results. Concluding remarks are given in Section 6.

2 Model

The world economy consists of two symmetric countries, Home and Foreign. Different products (or industries) are indexed $i = 1..N$. For most results I will focus on the limit case as $N \to \infty$. Formally, I consider a sequence of economies with increasing number of products, so all variables are indexed by N which I suppress for brevity as it causes no confusion. I start with describing the closed economy setup which is similar to Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Saleh (2012).

Throughout the paper, I denotes the identity matrix and $\mathbf{1}$ denotes the column vector of ones. For some vector x, D_x denotes the diagonal matrix generated by x. Both $D_x a = D_a x$ and $x \circ a$ are used for element-wise multiplication of vectors x and a.

2.1 Closed Economy

Environment

Consumers maximize Cobb-Douglas preferences parameterized with the vector of expenditure shares α:

$$U=\prod_{i=1}^{N}\left(\frac{c_i}{\alpha_i}\right)^{\alpha_i}, \tag{2.1}$$

with $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$.

Production function is also Cobb-Douglas: the amount q_i of good i is produced using labor l_i and intermediate inputs m_{ij}:

$$q_i=\exp\left(-z_i\right)\left(\frac{l_i}{1-\beta}\right)^{1-\beta}\prod_{j=1..N}\left(\frac{m_{ij}}{\beta\omega_{ij}}\right)^{\beta\omega_{ij}}, \tag{2.2}$$

where z_i is a cost shock, $0 \leq \beta < 1$ is the total share of intermediate inputs, and $0 \leq \omega_{ij} \leq 1$ are the shares of particular intermediates. Constant returns to scale are imposed, so $\sum_j \omega_{ij} = 1$ for each i. The matrix Ω of ω_{ij} is referred to as the I-O matrix.[10]

The triple (α, β, Ω) defines the I-O structure of the economy.[11] Apart from a disaggregation condition, formulated in Assumption 1 below, this I-O structure can be arbitrary. Some goods may be intermediate inputs only ($\alpha_i = 0$), while other ones can only enter final consumption (good j is a pure final good if $\omega_{ij} = 0$ for all i). More generally, the model accommodates any pattern of "snakes and spiders" linkages, nesting, in particular, two-stage production in Yi (2010)[12] and the complete symmetric I-O network with $\omega_{ij} = \frac{1}{N}$ for all i, j.

Consumers inelastically supply L units of labor and competition is perfect in all markets. After normalizing the nominal wage $W = 1$, cost minimization for (2.2) gives a linear relationship between log marginal costs mc_i and log prices p_j:

$$mc_i = z_i + \beta\sum\nolimits_j \omega_{ij}p_j, \quad \text{or} \quad mc = z + \beta\Omega p. \tag{2.3}$$

In competitive equilibrium, prices are equal to marginal costs, so

$$mc = p = \left(I-\beta\Omega\right)^{-1} z. \tag{2.4}$$

With additional normalization $L = 1$, so that $GDP = WL = 1$, the vector of consumption expenditure is $\alpha \cdot GDP = \alpha$. Let b denote the vector of total expenditure or

[10] An alternative convention would be to call $\beta\Omega$ the I-O matrix. Its element (i, j) would then be the share of product j in product i's total costs rather than material costs. As long as this section restricts attention to a common input share β, it is straightforward to switch between these two interpretations.

[11] Recall that we consider a sequence of economies with increasing number of products, so the full notation has α and Ω indexed by N. That is, consumption shares and I-O matrices form a sequence $\{\alpha^{(N)}, \Omega^{(N)}\}_{N=1}^{\infty}$. The input share β is kept fixed.

[12] See Section 2.3 below.

sales (the two are equal due to constant returns to scale and perfect competition). The sales of industry i consist of final expenditure α_i and intermediate expenditure by other industries:

$$b_i = \alpha_i + \sum_j \beta\omega_{ji}b_j, \quad \text{or} \quad b = \beta\Omega' b + \alpha, \tag{2.5}$$

which implies that

$$b = (I - \beta\Omega')^{-1}\alpha. \tag{2.6}$$

Since total labor income should equal a fraction $(1-\beta)$ of aggregate sales $Y = \sum_i b_i$, it holds that $WL = (1-\beta)Y$, and therefore $\sum_i b_i = \frac{1}{1-\beta}$. This (autarky) sales vector b is often called "the influence vector",[13] as it summarizes the importance of particular industries (or, more generally in network analysis, "nodes"). In autarky, b_i equals expenditure on product i relative to GDP, or $(1-\beta)b_i$ equals the share of industry i in aggregate sales.

As the aggregate (log) price index for (2.1) is equal to $\overline{p} = \sum_i \alpha_i p_i$, it is expressed in terms of cost shocks z simply as[14]

$$\overline{p} = \alpha' p = b' z. \tag{2.7}$$

Expression (2.7) shows that in the closed economy, for welfare determination, the influence vector b summarizes the entire I-O structure. This will be no longer true with trade, as economies with exactly the same autarky sales shares can behave very differently depending on other characteristics of Ω.

Disaggregation (LLN assumption)

To obtain sharp analytical results, this paper relies on the law of large numbers which can be applied to highly disaggregated economy. Holding the size of the economy fixed (aggregate sales $Y = \sum_i b_i = \frac{1}{1-\beta}$), consider increasing the level of disaggregation by letting the number of products N grow to infinity. To formalize this disaggregation, it is natural to require that the size of the largest industry in the limit shrinks to zero (yet it may decline slowly):

[13] Usually after normalizing its components to sum up to one. I refer to b as the influence vector without such a normalization.

[14] Combining (2.4) and (2.6), $\alpha' p = \alpha' (I - \beta\Omega)^{-1} z = b' z$. This relationship is derived in Acemoglu et al. (2012) for $\alpha = \frac{1}{N}\mathbf{1}$ and in Baqaee (2015) for a general α.

$$\lim_{N\to\infty} \max_i \{b_i\} = 0. \tag{2.8}$$

By looking at (2.7), this condition says that welfare is less and less affected by shocks to individual products as these products are being defined more and more narrowly: $\frac{\partial \overline{p}}{\partial z_i} = b_i \to 0$ for all i. Stated in the form (2.8), the disaggregation condition has a simple interpretation, but the actual law of large numbers results will use an equivalent formulation in terms of the Euclidian norm:[15]

$$\lim_{N\to\infty} \|b\|_2 = \lim_{N\to\infty} \left(\sum\nolimits_i b_i^2\right)^{1/2} = 0. \tag{2.9}$$

Condition (2.8) or (2.9) is a joint restriction on the I-O structure determined by (α, β, Ω). For technical reasons, an additional restriction on Ω is imposed [in the current draft; is likely to be redundant]: if some vector of (alternative) consumption shares $\widetilde{\alpha}$ is diversified in the sense that $\|\widetilde{\alpha}\|_2 \to 0$, the corresponding influence vector is also diversified:

$$\lim_{N\to\infty} \|\widetilde{\alpha}\|_2 = 0 \Rightarrow \lim_{N\to\infty} ||\widetilde{b}||_2 = 0, \text{ where } \widetilde{b} = \left(I - \beta\Omega'\right)^{-1} \widetilde{\alpha}. \tag{2.10}$$

To state a formal assumption, reintroduce for a moment the full notation in which all vectors and matrices that change size are indexed with superscript (N).

Assumption 1. *The sequence* $\{\alpha^{(N)}, \Omega^{(N)}\}_{N=1}^{\infty}$ *satisfies:*
(i) (2.9) holds for the corresponding influence vectors $\{b^{(N)}\}_{N=1}^{\infty}$.
(ii) For any alternative consumption shares $\{\widetilde{\alpha}^{(N)}\}_{N=1}^{\infty}$, *the implication (2.10) holds.*

Summary statistics for the I-O structure

In the volatility literature, such or Acemoglu et al. (2012), the influence vector's norm $\|b\|_2$ is the key object that determines how fast the aggregate volatility declines with the level of disaggregation.[16] The present paper is concerned with comparative advantage, which is cross-sectional rather than time-series variation. Under Cobb-Douglas preferences and technology, cross-sectional productivity dispersion is inconsequential (from (2.7), welfare does not depend on the distribution of z_i beyond its first moment), but in the open economy it affects trade flows and the gains from trade. To characterize the open-economy

[15] In particular, (2.8) implies (2.9) since $\sum_i b_i^2 \leq \max_i\{b_i\} \sum_i b_i = \frac{1}{1-\beta} \max_i\{b_i\} \to 0$. The converse is also obvious: $\sum_i b_i^2 \geq \left(\max_i\{b_i\}\right)^2$, so a failure of (2.8) produces a contradiction with (2.9).

[16] Gabaix (2011), not in the I-O context, provides a similar result that aggregate volatility depends on the sales Herfindahl.

effects of productivity heterogeneity, I now introduce the following list of summary statistics for the I-O structure. Each index takes some product-level statistic and calculates the weighted average across all industries with the weights $(1-\beta)\,b_i$. First, the average Herfindahl index of supplier diversification:

$$HHI = (1-\beta)\sum_i b_i \sum_j \omega_{ij}^2. \tag{2.11}$$

This index is the average of product-level input shares Herfindahls $HHI_i = \sum_j \omega_{ij}^2$. It shows, on average, how dispersed intermediate spending of individual producers is. A zero limit value of HHI, which is the case, for example, for complete and symmetric I-O network with all $\omega_{ij} = \frac{1}{N}$, means perfect diversification of suppliers. The prices of input bundles $p_i^M = \sum_j \omega_{ij} p_j$ become deterministic for all industries, so the randomness in the marginal cost mc_i only comes from idiosyncratic cost shock z_i. Note that HHI in (2.11) contrasts to the sales Herfindahl defined as $\sum_i \left[(1-\beta)\,b_i\right]^2$. While the latter is a macro-level statistic describing concentration of total sales, the former is a micro-level characteristic in the sense that it captures diversification of intermediate spending by individual industries.

Second, two diagonal, or "self-sourcing" indices:

$$SSI = (1-\beta)\sum_i b_i\omega_{ii} \text{ and } SSI_2 = (1-\beta)\sum_i b_i\omega_{ii}^2. \tag{2.12}$$

Finally, the "short loop index" which captures the extent of reciprocal sourcing (how often industries supply their immediate suppliers, including themselves):

$$SLI = (1-\beta)\sum_i b_i \sum_j \omega_{ij}\omega_{ji}. \tag{2.13}$$

In the closed Cobb-Douglas economy, these indices have no welfare interpretation, but Appendix A.3 demonstrates their role in case of CES preferences and technology. In the open economy, these summary statistics determine how variation in relative exogenous productivities shapes variation in endogenous relative marginal costs.

2.2 Open Economy: Setup and General Relationships

Consider two countries, Home and Foreign, which share the same fundamentals except for the cost shock vectors z and z^* which have a symmetric distribution. The Foreign variables have asterisk notation. Unless otherwise noted, all further results in this section are derived under the assumption that the pairs of cost shocks (z_i, z_i^*) are iid across products (yet z_i may be correlated with z_i^* for the same i) with a pdf $g(z_i, z_i^*) = g(z_i^*, z_i)$ restricted such that $\eta_i = z_i^* - z_i$ has four finite moments. Labor endowments are $L = L^* = 1$. It will be shown that as $N \to \infty$ the law of large numbers implied by Assumption 1 allows normalizing the two (equilibrium) wages to one: $W = W^* = 1$.[17]

The model incorporates two motives for trade. One is Ricardian cross-industry heterogeneity as the two countries may differ in relative efficiency in producing different goods. Second, the model incorporates intra-industry trade by allowing for product differentiation in the Armington fashion: each product i enters consumption or production as a CES bundle x_i which aggregates the corresponding domestic and imported varieties x_{id} and x_{im} with the elasticity of substitution $\rho > 1$:

$$x_i = \begin{cases} \left(x_{id}^{1-1/\rho} + x_{im}^{1-1/\rho}\right)^{\frac{\rho}{1-\rho}}, & \rho < \infty \\ x_{id} + x_{im}, & \rho = \infty \end{cases} \tag{2.14}$$

While this paper is primarily concerned with the effects of CA across industries, intra-industry trade is introduced for both tractability and realism, as the presence of two-way trade even for narrowly defined products is well-documented. As known in the literature, the Armington formulation of intra-industry trade can be replaced with the one proposed by Eaton and Kortum (2002), henceforth EK. Moreover, it can be easily replaced with a Krugman (1980) monopolistic competition specification, but only without free entry (avoiding corner solutions).[18] Although the Armington formulation involves the simplest notation, the choice of interpretation may depend on the level of aggregation.

Costs, prices, and domestic shares

International trade involves symmetric iceberg costs $\exp(\tau)$, so free trade corresponds to log trade cost $\tau = 0$. Domestic trade is assumed frictionless. The log price index for good i in Home is

$$\begin{aligned} p_i &= \min\{mc_i, mc_i^* + \tau\} \quad \text{for } \rho = \infty \\ p_i &= mc_i + \tfrac{1}{\rho-1}\log(\lambda_i) \quad \text{for } \rho < \infty, \end{aligned} \tag{2.15}$$

where λ_i is the domestic share for good i. Denoting the difference in log marginal costs

[17] Specifically, equal wages imply that trade imbalances go to zero in the probability limit. In the current draft, such a law of large numbers result is proved for a perturbation approximation and near free trade.

[18] In multi-industry monopolistic competition models, e.g. Romalis (2004), fixed costs generally lead to zero entry in most competitive-disadvantage industries, with the cutoff being endogenous to trade frictions.

$\xi_i = mc_i^* - mc_i$,

$$\lambda_i = \frac{1}{1 + e^{(1-\rho)(\tau+\xi_i)}}. \text{ Similarly, } \lambda_i^* = \frac{1}{1 + e^{(1-\rho)(\tau-\xi_i)}}. \tag{2.16}$$

The following table summarizes the main notation:

Variable	Definition
mc_i	log of marginal costs for (2.2)
p_i	log of price index (2.15)
ξ_i	$mc_i^* - mc_i$
η_i	$z_i^* - z_i$
λ_i	domestic share in (2.16)
y_i	sales of industry i

One key model relationship is for the marginal costs. (2.3) and (2.15) imply

$$mc_i = z_i + \beta \sum\nolimits_j \omega_{ij} p_j, \text{ or } \xi_i = \eta_i + \beta \sum\nolimits_j \omega_{ij} \left[p_j^* - p_j\right]. \tag{2.17}$$

In the matrix notation, also using that $p_j^* - p_j = \xi_j + \frac{1}{\rho-1} \log\left(\frac{\lambda_j^*}{\lambda_j}\right)$,

$$\xi = \eta + \beta\Omega \left[\xi + \tfrac{1}{\rho-1} \log\left(\frac{\lambda^*}{\lambda}\right)\right], \tag{2.18}$$

where $\log\left(\frac{\lambda^*}{\lambda}\right)$ is element-wise. Expression (2.18) captures how η, the difference in exogenous cost shocks, both directly and indirectly determines ξ, the difference in endogenous marginal costs. Thus, a change in η_j affects ξ_i directly for $i = j$ and also indirectly by changing relative input prices. Differentiating (2.17) with respect to η_j,[19]

$$\frac{d\xi_i}{d\eta_j} = 1_{\{i=j\}} + \beta \sum\nolimits_{i'} \omega_{ii'} \left(\lambda_{i'} + \lambda_{i'}^* - 1\right) \frac{d\xi_{i'}}{d\eta_j}, \tag{2.19}$$

where $0 \leq \lambda_{i'} + \lambda_{i'}^* - 1 \leq 1$ is an inverse measure of trade intensity in industry i'. Under free trade this measure is zero, so indirect effects coming with $\frac{d\xi_{i'}}{d\eta_j}$ do not contribute to $\frac{d\xi_i}{d\eta_j}$. This is also seen immediately from $p_j^* = p_j$ at $\tau = 0$: as production sharing is not constrained under free trade, input prices are the same in the two countries and therefore marginal cost differences only arise from exogenous cost shocks. With costly trade, the sum of domestic shares exceeds one, $\lambda_{i'} + \lambda_{i'}^* - 1 > 0$, and fundamental cost differences η_j propagate downstream (in other words, ξ_i is determined by upstream η_j). In general,

[19] Use that $d\lambda_j = (\rho - 1)\lambda_j (1 - \lambda_j) d\xi_j$ and $d\lambda_j^* = -(\rho - 1)\lambda_j^* \left(1 - \lambda_j^*\right) d\xi_j$.

keeping track of such CA spillovers which depend on trade costs is a challenging task; Section 4 uses a perturbation approximation to obtain a parsimonious characterization of relative marginal costs ξ and relate it to trade flows and welfare.

Sales and trade flows

Recall that autarky sales are given by the influence vector b in (2.6). The open economy sales vectors y and y^* generally differ from b because of specialization. To determine y and y^*, consider the global I-O structure that links different industries in the two countries. For a clearer exposition, reintroduce GDP, C and C^*, which is total consumer expenditure. The global I-O balance takes the form

$$\underbrace{\begin{pmatrix} y \\ y^* \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} D_\lambda \alpha \cdot C + D_{\mathbf{1}-\lambda^*} \alpha \cdot C^* \\ D_{\lambda^*} \alpha \cdot C^* + D_{\mathbf{1}-\lambda} \alpha \cdot C \end{pmatrix}}_{\mathbf{f}} + \underbrace{\begin{bmatrix} D_\lambda \beta \Omega' & D_{\mathbf{1}-\lambda^*} \beta \Omega' \\ D_{\mathbf{1}-\lambda} \beta \Omega' & D_{\lambda^*} \beta \Omega' \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} y \\ y^* \end{pmatrix}}_{\mathbf{y}}, \tag{2.20}$$

in which $\mathbf{y}$ is the vector of world sales, $\mathbf{f}$ is the vector of world final consumption, and $\mathbf{A}$ is the global I-O matrix. In particular, final consumption of Home-produced goods comes from Home demand $D_\lambda \alpha \cdot C$ and Foreign demand $D_{\mathbf{1}-\lambda^*} \alpha \cdot C^*$. To produce y, Home industries need to spend the vector $\beta \Omega' y$ on intermediate inputs, of which $D_\lambda \beta \Omega' y$ goes into demand for Home-produced goods and $D_{\mathbf{1}-\lambda} \beta \Omega' y$ into demand for Foreign-produced goods.

Applying $C = WL = C^* = W^* L^* = 1$, which is symmetry plus normalization, Appendix A.4 derives that

$$y + y^* = 2b, \tag{2.21}$$

$$y = b + \left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}} \Omega'\right)^{-1} D_b \left(\lambda - \lambda^*\right), \text{ and} \tag{2.22}$$

$$y^* = b + \left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}} \Omega'\right)^{-1} D_b \left(\lambda^* - \lambda\right). \tag{2.23}$$

Specific to Cobb-Douglas, the sum of world sales is proportional to the influence vector b regardless of the level of trade costs. Expressions (2.22)-(2.23) show that departures of open-economy sales in each country from their autarky levels b are linked to $(\lambda - \lambda^*)$, which is a measure of cost advantage. (From (2.16), higher domestic shares in Home relative to Foreign correspond to higher marginal cost differences in favor of Home). The diagonal matrix $D_{\lambda+\lambda^*-\mathbf{1}}$ captures the (inverse) degree of trade integration: it is zero under free trade ($\tau = 0$ implies $\lambda_i + \lambda_i^* = 1$ for each i) and converges to I as $\tau \to \infty$. Interestingly, it enters the Leontief-inverse-type matrix $\left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1}$ as if controlling the input share. Section 4 shows that this phenomenon is related to the fact that international production sharing affects variation in relative marginal costs.

Aggregate domestic and import expenditure by Home consumers (final consumption) and producers (intermediate consumption) is

$$D^F = \lambda'\alpha; \quad M^F = 1 - D^F; \quad D^I = \lambda'\beta\Omega' y; \quad M^I = \tfrac{\beta}{1-\beta} - D^I, \tag{2.24}$$

where total final expenditure is $D^F + M^F = WL = 1$ and total intermediate expenditure is $D^I + M^I = \frac{\beta}{1-\beta}$.

Total domestic and import expenditure in Home is

$$D = D^F + D^I = \lambda'\alpha + \lambda'\beta\Omega' y \text{ and } M = M^F + M^I = \tfrac{1}{1-\beta} - D. \tag{2.25}$$

As an accounting decomposition,[20] domestic expenditure can be represented as

$$D = \sum\nolimits_i b_i \lambda_i + \beta \sum\nolimits_i \left(y_i - b_i\right) \lambda_i^{Upstream}, \tag{2.26}$$

where $\lambda_i^{Upstream} = \sum_j \omega_{ij}\lambda_j$ is the average domestic share among suppliers of industry i and the term $\sum_i \left(y_i - b_i\right)\lambda_i^{Upstream}$ captures the covariance between performance of individual industries and cost advantage of their suppliers. In Section 4, this covariance term is shown to be positive for $0 < \tau < \infty$, meaning that products which source from cost-advantage industries (with high domestic shares) tend to sell more relative to autarky. As seen from (2.26), such CA spillovers increase the home bias above the level based on the average domestic share. This reflects the conceptual difference between final and intermediate expenditure. Intuitively, D^F is dominated by consumers' purchases of goods in which Home has comparative advantage, while D^I counts intermediate spending on comparative-advantage goods *by* comparative-advantage industries. This two-sided selection is minimal under free trade because input prices are the same in Home and Foreign. However, as shown in Section 2.3, it does not vanish completely in the presence of self-sourcing.

Welfare

[20] Write $D^I = \lambda'\beta\Omega' y = \sum_i y_i \left(\beta \sum_j \omega_{ij}\lambda_j\right) = \beta\sum_i \left(y_i - b_i + b_i\right)\lambda_i^{Upstream}$. It further equals to $\beta\sum_i \left(y_i - b_i\right)\lambda_i^{Upstream} + \beta b'\Omega\lambda = \beta\sum_i \left(y_i - b_i\right)\lambda_i^{Upstream} + \left(b-\alpha\right)'\lambda$. Combining with $D^F = \lambda'\alpha$, $D = \lambda' b + \beta\sum_i \left(y_i - b_i\right)\lambda_i^{Upstream}$.

Welfare (the real wage) is the inverse of the aggregate price index for consumers which is, in logs,

$$\overline{p} = \alpha' p = b' \left[z + \tfrac{1}{\rho-1} \log(\lambda) \right]. \tag{2.27}$$

This expression[21] implies that welfare in a trade equilibrium relative to the autarky level, or the gains from trade, is given by a multi-industry ACR formula

$$GfT = \exp\left[\sum\nolimits_i b_i \tfrac{1}{1-\rho} \log(\lambda_i) \right] = \prod\nolimits_i \lambda_i^{\frac{b_i}{1-\rho}}. \tag{2.28}$$

Expression (2.28) holds more generally at the country level in the Cobb-Douglas perfect competition environments.[22] This formula is equivalent to expression (28) in Costinot and Rodriguez-Clare (2014), to expression (15) in Caliendo and Parro (2015), and to a related expression in Ossa (2015).[23]

While (2.28), developed in the multisector gravity literature, provides a measuring tool for evaluating the gains from trade, two issues require further exploration. First, in terms of theory, (2.28) hides the actual mechanisms through which I-O interactions affect the gains from trade. This is because domestic shares are determined by heterogeneity in production costs which is endogenous to trade costs and the I-O structure. Using a perturbation technique, Section 4 demonstrates that the gains from trade are parsimoniously related to heterogeneity in exogenous cost shocks and a few summary statistics for the I-O matrix. Second, applying (2.28) at disaggregated level is problematic with currently available data because production and use data is limited or missing. (Trade data is available at more detailed levels, but trade data alone does not allow calculating domestic expenditure shares.) In Section 5 I discuss how to explore variation in disaggregated trade flows without having disaggregated production data in order to give an approximate estimate for the gains from trade.

An interesting feature of this setup is that when trade is either absent or completely frictionless, welfare does not depend on the I-O matrix Ω as long as Assumption 1 (the law of large numbers) is imposed. This is directly seen in (2.27) for the case of autarky when

[21] Obtained from (2.3) and (2.15): $p = mc + \log\left(\lambda^{\frac{1}{\rho-1}}\right) = z + \beta\Omega p + \log\left(\lambda^{\frac{1}{\rho-1}}\right)$, which means that $p = (I-\beta\Omega)^{-1}\left[z + \log\left(\lambda^{\frac{1}{\rho-1}}\right)\right]$. Multiplying both sides by α' and using the definition of b in (2.6) yields (2.27).

[22] In particular, with heterogeneity in the elasticities of substitution, trade costs and labor shares across products and for multiple asymmetric countries that may have different I-O matrix coefficients.

[23] A clear connection to the influence vector is less common in the literature; one paper that also emphasizes this link is Albrecht and Tombe (2015).

$\lambda_i = 1$ for all i. By Lemma 1A, $\overline{p} = b'z \underset{p}{\rightarrow} \frac{1}{1-\beta} E(z_i)$. Under free trade ($\tau = 0$), Home and Foreign industries pay the same prices for intermediate inputs, so (2.17) implies $\xi = \eta$ and λ_i's, which now only depend on iid η_i's, are iid as well. Therefore, by the same lemma, $\sum_i b_i \log(\lambda_i)$ also converges to an expectation that is proportional to $\frac{1}{1-\beta}$ but does not depend on Ω.

Appendix A.5 proves the following result that relates local changes in welfare to aggregate trade flows.

Proposition 2.1. *In the probability limit, a change in the log price index is equal to total imports (relative to GDP=1)*

$$\frac{d}{d\tau}\overline{p} = M. \tag{2.29}$$

In fact, Appendix A.5 proves a more general result that, keeping the wages fixed, cost minimization and utility maximization alone imply $\frac{d}{d\tau}\left(\frac{1}{2}\overline{p} + \frac{1}{2}\overline{p}^*\right) = \frac{M+M^*}{World\ GDP}$. The role of the law of large numbers is only to ensure that in equilibrium $W = W^* = 1$ for any trade costs, as well as $\overline{p} = \overline{p}^*$ and $M = M^*$.

Proposition 2.1 is a symmetric-countries version of the main result in Fan, Lai, and Qi (2014), yet it allows a more general I-O structure.[24] Equation (2.29) is used below in assessing how the I-O links can amplify the effects of trade costs, but it can also be of some independent interest. In particular, (2.29) has all trade flows counted equally, independently of the value added content.

We can define the aggregate domestic share $\overline{\lambda} = \frac{D}{Y}$ and the "trade elasticity"

$$\epsilon(\tau) = -\frac{d}{d\tau}\log\left(\frac{1-\overline{\lambda}}{\overline{\lambda}}\right). \tag{2.30}$$

Appendix A.6 shows that (2.29) implies that $\frac{d\overline{p}}{d\log(\overline{\lambda})} = \frac{1}{(1-\beta)\epsilon(\tau)}$, so the ACR interpretation of the trade elasticity holds locally.[25]

The fact that welfare in autarky and under free trade does not depend on Ω, combined with Proposition 2.1, implies that it is not possible that some I-O matrix $\Omega^{(1)}$ generates uniformly (over τ) more or less trade than another I-O matrix $\Omega^{(2)}$. This result contrasts with the notion that multistage production as in Yi (2010) magnifies the effect of trade costs. The impossibility of such uniform magnification is formally stated as

[24] Fan, Lai, and Qi (2014) only consider multistage production without "spider" linkages. Results of this type also appear in Burstein and Cravino (2012), Atkeson and Burstein (2010), and Allen, Arkolakis, and Takahashi (2014).

[25] Brooks and Pujolas (2014) work on generalizing the ACR formula for the cases when log-linear gravity does not hold because of non-homotheticities.

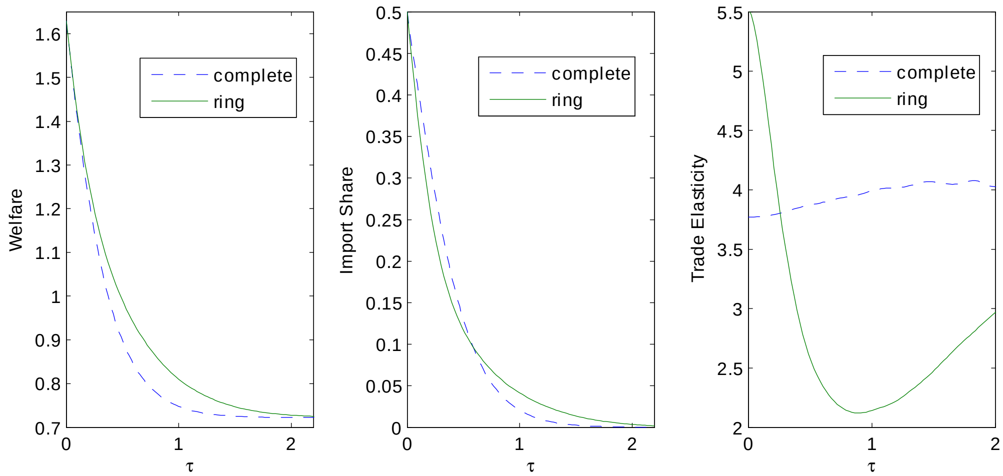

Figure 1: Complete symmetric network vs. Ring

$$\overline{p}^{Autarky} - \overline{p}^{Free\ Trade} = \int_0^\infty M\left(\Omega^{(1)}\right) d\tau = \int_0^\infty M\left(\Omega^{(2)}\right) d\tau \text{ for any } \Omega^{(1)}, \Omega^{(2)}. \qquad (2.31)$$

This is illustrated in Figure 1 which compares the complete and symmetric I-O network ($\omega_{ij} = \frac{1}{N}$) with the ring ($\omega_{N1} = 1$, $\omega_{ij} = 1$ for $i = 1..N-1$, $j = i+1$ and $\omega_{ij} = 0$ otherwise). In both cases, $\alpha = \frac{1}{N}\mathbf{1}$, productivities e^{-z_i} and $e^{-z_i^*}$ and independently Frechet distributed with $\theta = 4$, and the Armington elasticity is $\rho = 15$. The ring generates uniformly higher welfare, which is achieved with lower import shares for small trade costs but with higher import shares when trade costs are large. The area between the two import share lines integrates to zero.[26] While the next subsection provides an exact characterization of trade flows near $\tau = 0$, understanding these differentiated welfare and trade responses to changes in trade costs will require a perturbation approximation which would be able to relate the two components of (2.26) to the pattern of I-O linkages. Note also that the elasticity of trade (2.30) is non-monotone for the ring (for the complete network, $\epsilon(\tau)$ would be equal $\theta = 4$ if $\rho = \infty$, which is the EK setup).

[26] As $\int_0^\infty M\left(\Omega^{(1)}\right) d\tau = \int_0^\infty M\left(\Omega^{(2)}\right) d\tau$, which is written in terms of import-to-GDP ratios, the same holds for trade shares which are $\frac{M}{Y} = (1-\beta) M$.

2.3 Local Analysis Near Free Trade

Even with Cobb-Douglas preferences and technology and maximum symmetry, the model of general I-O interactions remains extremely complex, which motivates the use of perturbation approach in Section 4. However, the model's solution simplifies dramatically when trade is frictionless, so that an exact, rather than approximate, characterization is available. In this subsection I highlight two key results of the local analysis near $\tau = 0$, while all technical discussion is delegated to Appendix A.7.

Free-trade home bias

Given the symmetry, it is not surprising that under free trade consumers spend one half of their income (GDP, which is normalized to one) on domestically produced goods:

$$D^F = \lambda' \alpha \underset{p}{\rightarrow} \tfrac{1}{2}. \tag{2.32}$$

For producers, however, the situation is different. Of the total intermediate expenditure $\frac{\beta}{1-\beta}$, firms spend on domestic products

$$D^I = \lambda' \beta \Omega' y \underset{p}{\rightarrow} \tfrac{1}{2} \tfrac{\beta}{1-\beta} \left[1 + SSI \cdot var\left(\varepsilon_i\right)\right], \tag{2.33}$$

where ε_i, defined in (A.30), is a monotone transformation of η_i. Domestic intermediate expenditure D^I is strictly greater than import of intermediates $M^I = \frac{\beta}{1-\beta} - D^I$ as long as $var\left(\varepsilon_i\right) > 0$ (there is comparative advantage) and $SSI > 0$ (industries' spending on their own output[27] is nonnegligible). This free-trade home bias arises endogenously from the I-O interactions, as opposed to the one exogenously introduced into preferences in some studies. (See the discussion after (2.26) for intuition.)

More generally, $D^I > M^I$ despite $\tau = 0$ occurs under positive correlation between upstream and downstream cost shock differences η_i, of which self-sourcing is a primitive example.[28] As I show below, the effects of such correlation can be quite strong with limited supplier diversification (large HHI), so that small trade costs are recovered from high home shares. The issue, however, is that correlation in exogenous cost shocks is hard to discipline within my framework.[29] Nevertheless, one implication of this paper is that firm-level studies of supply chain trade should pay special attention to the upstream-

[27] In the open economy, such "self-sourcing" means that industries use CES bundles (2.14) aggregating output of these same industries from the two countries. The effect is present for any $\rho > 1$ and it is the strongest when $\rho = \infty$ as $var\left(\varepsilon_i\right)$ attains its maximum value 1.

[28] In principle, negative correlation may induce $D^I < M^I$.

[29] Section 5 provides an extension in which upstream and downstream cost shocks are correlated via reduced-form knowledge spillovers.

downstream correlation in technology.

Trade elasticity

In Appendix A.7 I consider separately the case $\rho < \infty$ and the case of perfect substitutes $\rho = \infty$. Although this paper focuses on the former, local analysis near free trade provides an opportunity to consider high values of ρ as well as very strong cross-product productivity heterogeneity.

One result that allows making a clear connection to existing literature, is for perfect substitutes when productivity e^{-z_i} is Frechet distributed with the dispersion parameter θ. In the absence of self-sourcing ($SSI = 0$), the relative import spending near free trade satisfies

$$\log\left(\frac{M}{D}\right) \underset{p}{\rightarrow} -\theta\left[1+\beta^2\left(HHI + 2SLI\right)\right]\tau + o\left(\tau\right). \tag{2.34}$$

The elasticity of trade (the negative of the slope is the above expression) in this case is $\epsilon = \theta\left[1+\beta^2\left(HHI+2SLI\right)\right]$. Indeed, as previously argued in the literature, the elasticity of trade $\epsilon \geq \theta$ can be amplified by the presence of I-O linkages. Near free trade, this amplification is only determined by supplier diversification and reciprocal sourcing (short loops).

This case of perfect substitutes $\rho = \infty$ is mainly of theoretical interest, since two-way trade is present in the data even at highly disaggregated level. Yet considering (2.34) for different I-O matrices Ω helps to develop a unified understanding of the existing models. With Cobb-Douglas technology and preferences, the EK model with continuum of goods can be discretized as having $\alpha = \frac{1}{N}\mathbf{1}_N$ and $\omega_{ij} = \frac{1}{N}$. This complete symmetric I-O network has $b = \frac{1}{1-\beta}\frac{1}{N}\mathbf{1}_N$, so $HHI = \frac{1}{N} \rightarrow 0$ and $SLI = \frac{1}{N^2} \rightarrow 0$. The I-O structure in Yi (2010) has two stages of production. Each stage 1 good $i = 1..\frac{N}{2}$ is used by one and only one stage 2 good $j = \frac{N}{2} + i$. Consumption shares are $\alpha = [\mathbf{0}_{N/2}\ \frac{1}{N/2}\mathbf{1}_{N/2}]'$, as only stage 2 goods are consumed. Also, a composite bundle of stage 2 goods is used by each stage 1 product. The I-O matrix is therefore

$$\Omega^{Yi} = \begin{pmatrix} \mathbf{0}_{\frac{N}{2},\frac{N}{2}} & \frac{1}{N/2}\mathbf{1}_{\frac{N}{2},\frac{N}{2}} \\ I_{\frac{N}{2},\frac{N}{2}} & \mathbf{0}_{\frac{N}{2},\frac{N}{2}} \end{pmatrix},$$

for which $b = \left(I - \beta\Omega'\right)^{-1}\alpha = \frac{2}{N}\frac{1}{1-\beta^2}[\beta\mathbf{1}_{N/2}\ \ \mathbf{1}_{N/2}]'$. While stage 2 goods have perfect supplier diversification, stage 1 goods use only one product each. The average supplier diversification is $HHI = (1-\beta)\sum_i b_i \sum_j \omega_{ij}^2 = \frac{1}{1+\beta}\left(1 + \frac{\beta}{N/2}\right) \rightarrow \frac{1}{1+\beta} > 0$, which increases the trade elasticity in (2.34) relative to EK. Expression (2.34) also shows that several tractable modifications of Kei-Mu Yi's two-stage model, such as Arkolakis and Ramanarayanan (2009) and Kim (2013), change the original setup dramatically by imposing perfect diversification.[30] Yet the role of (2.34) in theoretical understanding of supply chain trade is quite limited. First, the economics behind this result is still not clear. Moreover, it would be wrong to conclude that such amplification holds globally, that is for any trade costs.

3 Neutrality Result

This section establishes a neutrality result that, in the absence of CA, the detailed structure of I-O linkages does not matter for trade shares and the gains from trade. I use this finding to argue that the commonly cited "amplification logic" (the impact of trade costs is magnified by multiple border crossing) is incomplete. On the constructive side, the neutrality result suggests the no-CA case as a basis for perturbation, which I explore in Section 4.

3.1 I-O Trade and Gravity

By looking at equation (2.18), one can guess that $\eta = 0$ implies $\xi = 0$ regardless of Ω. Without variation in relative fundamental productivities, there is no variation in relative marginal costs. In other words, although I-O interactions transform CA, they do not create it out of nothing. This guess is verified by observing that $\xi = 0$ implies $\lambda = \lambda^*$ in (2.16) and therefore $p = p^*$.[31] The model then effectively collapses to a one-product Armington economy in which trade flows satisfy a log-linear gravity equation and the welfare gains are given by the ACR formula.

To appreciate this gravity implication, it is worth formulating this proposition for multiple asymmetric countries. I ask what is the maximum level of generality − in particular in terms of the I-O structure − that is consistent with aggregate-level gravity. The detailed description of the environment is given in Appendix A.8. The setup of Section 2 is maintained with the following generalizations:

(i) The world economy consists of countries $k = 1..K$ and products (industries) $i = 1..N$. Bilateral iceberg trade costs are $t_{kk'}$ (uniform across products) and country-industry

[30] In Arkolakis and Ramanarayanan (2009), each stage-two product uses a composite bundle of stage-one goods.

[31] Such a proof by construction also requires showing uniqueness of ξ, which I do in a more general setting of Proposition 3.1.

cost shifters (inverse TFP) are Z_{ik}.

(ii) Consumer preferences are CES with potentially country-specific utility weights.

(iii) Industry-specific material bundles are CES.

(iv) Exogenous trade imbalances are allowed.

Crucially, the common Armington elasticity ρ is maintained and as well as the common intermediate share β. This setup, in general, features both intra-industry and inter-industry trade. Under the chosen interpretation, the former is driven by Armington product differentiation as in Anderson and van Wincoop (2003), but it can equivalently be related to technology heterogeneity within individual industries as in EK.[32] Inter-industry trade arises from asymmetries between countries and generally precludes one from specifying the gravity equation at the aggregate level. The following proposition shows that the root of this gravity failure lies in the presence of CA forces, meaning variation in relative exogenous productivities. In the setting of Appendix A.8, the gravity structure is not affected by I-O linkages as long as relative cost shifters

$$\frac{Z_{ik}/Z_{i'k}}{Z_{ik'}/Z_{i'k'}} \tag{3.1}$$

do not vary across country-industry combinations $(k, k'; i, i')$, which is equivalent to decomposition of Z_{ik} into industry and country fixed effects. Formally, Appendix A.8 proves

Proposition 3.1. *If the cost shifters in (A.42) satisfy $Z_{ik} = \widetilde{Z}_i \widehat{Z}_k$, then there exist a set of importer fixed effects $\{FM_k\}_{k=1}^K$ and a set of exporter fixed effects $\{FX_{k'}\}_{k'=1}^K$ such that equilibrium trade flows $X_{kk'}$ satisfy the log-linear gravity system with trade elasticity $\rho - 1$:*

$$\log X_{kk'} = FM_k + FX_{k'} + (1-\rho)\log t_{kk'}. \tag{3.2}$$

If in addition trade is balanced, the gains from trade for country k are given by the ACR formula

$$\overline{\lambda}_k^{\frac{1}{(1-\rho)(1-\beta)}}, \tag{3.3}$$

where $\overline{\lambda}_k$ is the share of country k's expenditure spent on domestic goods.

A special case of this result is known in the literature due to French (2015) who considers a multi-industry EK model in which all industries use the same composite

[32] It is well known that EK yields the same set of equilibrium equations. In this case, the Frechet dispersion parameter θ plays the role of $\rho - 1$ and heterogeneity in Z_{ik} relates to heterogeneity in location parameters T_{ik}. Caliendo and Parro (2015) and most of the other multisector gravity literature employ the EK setup for modeling intra-sectoral trade, while Ossa (2015) also uses the Armington formulation.

intermediate good (same as in final consumption). Imposing such uniformity is a common way to avoid specifying the detailed I-O structure when introducing intermediate goods. Under this assumption, material costs cannot by construction affect variation in relative marginal costs, which rules out any interaction between CA forces and I-O linkages.[33]

Proposition 3.1 states that in the absence of CA the elasticity of trade is completely unaffected by I-O interactions. For any input share β and I-O shares $\omega_{ii'}$,[34] the value $\rho - 1$ determines the proportional change in trade flows resulting from a given change in trade costs conditional of the fixed effects. The latter qualification, standard in the gravity literature, is equivalently replaced by considering a Head and Ries (2001) type ratio:

$$\rho - 1 = -\frac{\partial \log X_{kk'}}{\partial \log t_{kk'}} = -\frac{d \log \frac{X_{kk'} X_{k'k}}{X_{kk} X_{k'k'}}}{d \log \frac{t_{kk'} t_{k'k}}{t_{kk} t_{k'k'}}}, \tag{3.4}$$

in which partial differentiation "∂" means keeping constant FM_k and $FX_{k'}$ in (3.2), while total differentiation "d" does not restrict the sources of variation in trade flows.[35]

The intuition behind this neutrality result is simple. It is true that multiple border crossing makes the prices of traded goods change more than one-for-one with trade costs, and more so for a higher input share β. But the *relative* prices (import vs. domestic) need not be affected by I-O links because all products in all locations use some imported intermediates. Since trade flows depend on relative rather than absolute prices, the possibility of no amplification is not surprising. More specifically, the proof shows that no relative variation in exogenous productivities implies no relative variation in marginal costs. Bilateral trade shares are then uniform across products, hence there are no composition

[33] Although not focusing on I-O interactions, the result in French (2015) is not to be downplayed, however; even without intermediate goods, the gravity structure relies critically on a flat CA pattern. That is, CA forces can only operate within – not between – industries once intra-industry trade is modeled as in EK. In other words, French (2015) emphasizes that gravity is correctly specified at a certain level of aggregation. This is, of course, implicitly understood in multisector models, yet they face a further heterogeneity challenge because they only include a relatively small number of sectors.

[34] To emphasize the generality of production architecture consistent with Proposition 3.1, note that an arbitrary pattern of "snakes and spiders" linkages is allowed, nesting, for example, multistage production as in Yi (2010). At the same time, variation in labor shares across industries is assumed away, as it translates into variation in relative costs across countries unless all wages are equal. (In case of symmetric countries, the gravity holds with industry-specific labor shares.)

[35] Note that one needs to be careful when defining the concept of (partial equilibrium) trade elasticity outside the context of log-linear gravity. There are examples in the literature when this concept is well-defined but the second equality in (3.4) does not hold, e.g. translog gravity in Novy (2013). In general, however, using the "∂" symbol can be problematic, in particular under I-O interactions. In case of two symmetric countries the elasticity of trade which is *defined* by the last expression in (3.4) happens to (locally) preserve the ACR interpretation.

effects and the product-level trade elasticity is unchanged under aggregation.

In terms of welfare gains, the entire effect of the I-O structure is summarized by the share of intermediates β. When firms and not only consumers have access to the world market, the gains from trade are magnified due to compounding effect, which is already present in EK. As long as the share of material inputs in the data is quite large, accounting for tradable intermediates is of first-order importance for estimating the gains from trade.[36]

Going back to the benchmark model with two symmetric countries, we have $k \in \{H, F\}$, $\widehat{Z}_H = \widehat{Z}_F$, and the absence of fundamental CA means $\log \frac{Z_{iF}}{Z_{iH}} = \eta_i = 0$ for all products. It implies that domestic shares in Home and Foreign are equal and uniform across products: $\lambda_i = \lambda_i^* = \lambda_0 = \frac{1}{1+e^{(1-\rho)\tau}}$. Imports are then $M = \frac{1}{1-\beta}(1 - \lambda_0)$, so both partial equilibrium and general equilibrium trade elasticities do not depend on the I-O matrix.

3.2 Connection to the Literature

From the theory perspective, Proposition 3.1 can be viewed as a counterexample for the "conventional wisdom" on the role of international production sharing in magnifying the effects of trade costs. Since Yi (2003), the theoretical understanding of supply chain trade has been centered on "multiple border crossing", a concept which may at first look self-explaining. As tariffs and transportation cost are applied multiple times, production fragmentation should make trade flows more responsive to changes in trade barriers. Despite the awareness that input trade itself it not sufficient to generate such amplification,[37] the literature continues to refer to this semi-informal logic, e.g. Johnson and Moxnes (2013), Connolly and Yi (2014). Furthermore, Koopman, Wang, and Wei (2014) and Miroudot, Rouzet, and Spinelli (2013), among others, use international I-O tables to calculate the cumulative burden of tariffs. This is supposed to be informative of the role of supply chains in magnifying the effects of trade costs, yet the general equilibrium interpretation of such numbers is not clear.

If multiple border crossing itself does not necessarily amplify the impact of trade costs, then what is the nature this amplification? Yi (2010) notes that, unlike the roundabout

[36] Melitz and Redding (2014) and Samuelson (2001) point out that the gains from trade in inputs may potentially be infinite. In Table 4.1 in Costinot and Rodriges-Clair (2014) the estimated average gains from trade range from 4.4-15.3% in models without intermediates (including multiple sectors) to 26.9-40% in models with intermediates.

[37] For example, Yi (2010) mentions that the model in Eaton and Kortum (2002) has no magnification effect despite the presence of I-O linkages. See footnote 14 in his paper.

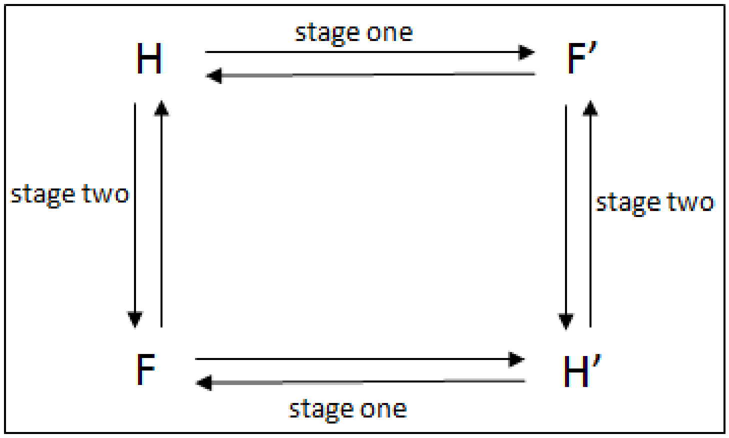

Figure 2: Alternative scheme of world trade in which correlation in marginal costs across countries is removed.

scheme in EK, the *structure of production* in his model is not invariant to changes in trade barriers. What exactly stands behind this endogenous production structure, however, is not perfectly clear.[38] In Section 4 I use a perturbation approximation to demonstrate that at the heart of the multistage model lies endogenous relative cost heterogeneity which is present because of imperfect supplier diversification (each stage-two product uses only one stage-one input). I find that, compared to the EK benchmark, the home bias is only magnified for moderate trade costs.

The perturbation results of Section 4 are obtained for a finite Armington elasticity, while the setup in Yi (2010) corresponds to the case of perfect substitutes $\rho = \infty$. This difference, however, is quantitative rather than qualitative. To demonstrate that endogenous relative cost heterogeneity (and CA spillovers that arise together with it) is key in the original Yi's environment, we can consider the following thought experiment. [Full description to be provided in the Appendix.] Suppose that each of the original two countries, H and F, has its "double", H' and F', and trade takes place as shown in Figure 2. In particular, country H trades in stage-two (finished) goods with F, while it exchanges stage-one goods with F'.

All the intuition from Yi (2010) on multiple border crossing as well as co-location that changes in response to trade liberalization applies to this artificial setup. Yet this alternative model behaves very differently: the trade elasticity for stage-one products is

[38] For a formal analytical result, Yi (2010) assumes that components are always produced in the region where finished products are consumed, an assumption that is not maintained in the calibration part of the paper. The original model does not admit a closed-form solution, which limits the intuitive understanding of its general equilibrium forces.

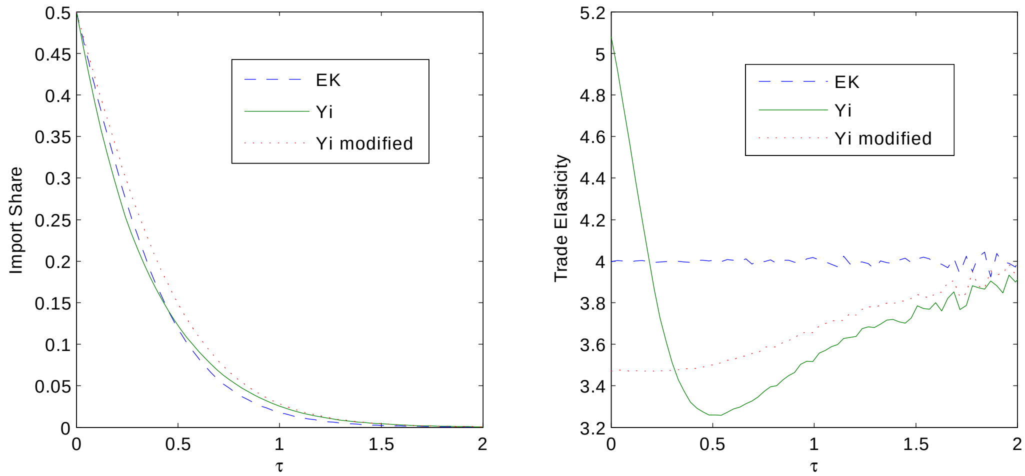

Figure 3: The effects of changing the trade scheme as in Figure 2.

constant as in EK, while for total trade flows it is monotonically *increasing* in the level of trade costs (that is, trade flows are less sensitive to trade barriers near the point of frictionless trade). The trade scheme in Figure 2 changes the original setting by removing marginal cost correlation through which production sharing affects heterogeneity in relative marginal costs. The precise result (which is also specific to Frechet distribution) is that the variance of log marginal costs differences does not depend on the level of trade frictions for both stage-one and stage-two goods. Another manifestation of this is the absence of CA spillovers as the covariance term in (2.26) becomes zero: unlike in Yi (2010), stage-two goods that rely on domestically supplied components are not produced more compared to autarky. As a result, the home bias is not magnified in the alternative setup. Figure 3 gives an illustration.

3.3 Perturbation

The interesting content of Proposition 3.1 lies in the fact that a very general form of I-O linkages is allowed. Beyond this, however, all sorts of cross-industry heterogeneity are excluded: no CA, common ρ, β and $t_{kk'}$ across products. It is easy to show that any of that breaks the gravity structure and the ACR formula. And yet the gravity model is too valuable because of its tractability (and significant empirical success) to dismiss it so quickly. Instead, Proposition 3.1 can be viewed as to suggest that the effects of product-level heterogeneity may be characterized as deviations from the gravity benchmark. The main methodological contribution of this paper is to formalize and explore this idea with a perturbation approach.

While common in macroeconomics,[39] especially in modeling dynamics, perturbation techniques which would go beyond the standard low-dimensional comparative statics exercises are not widespread in international trade. The insight is that such techniques can be very helpful in dealing with cross-sectional heterogeneity which is otherwise difficult to approach because of rich interactions between different industries. The general idea that network interactions can be studied by perturbing a cross-section of homogenous economic units is not new. Methodologically related to my study is the work by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015).[40] The novelty of the present paper is to apply perturbation techniques to cross-industry heterogeneity in international trade and to explore the model features specific to supply chain trade.

Until Section 5 I focus on CA and then incorporate other forms of heterogeneity.

Once the decomposition of cost shifters Z_{ik} into industry and country fixed effects $\widetilde{Z}_i$ and $\widehat{Z}_k$ does not hold, there is fundamental CA which interacts with I-O linkages and trade costs to shape endogenous heterogeneity in relative marginal costs which determines trade flows and welfare. Reflecting the great complexity of the real world production architecture, approaching this interplay between CA and I-O is challenging even within a stylized competitive framework. We can represent Z_{ik} in the form

$$\log Z_{ik} = \log \widetilde{Z}_i + \log \widehat{Z}_k + \delta z_{ik}, \tag{3.5}$$

where z_{ik} is residual variation and δ is the perturbation parameter. The strategy is then to approximate the outcome variables of interest, such as trade flows, welfare or optimal taxes, with Taylor series in δ around $\delta = 0$, which in many circumstances is a simple case. For aggregate outcomes, particularly sharp results can be obtained when the number of industries is large. In this case, assuming that residual cost shocks z_{ik} come from a

[39] Perturbation techniques originated in natural science and were introduced to macroeconomics in Judd and Guu (1997). See Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) for a recent exploration.

[40] The authors provide first- and second-order Taylor series approximation to a generic aggregate variable determined by network interaction between individual nodes which are subject to idiosyncratic shocks. Technically, my analysis in Section 3 is very similar, except that it involves differentiating with a single perturbation parameter rather than calculating all mixed derivatives with respect to idiosyncratic shocks. Using a single perturbation parameter in my case is more convenient, especially for higher orders of approximation and under multiple sources of heterogeneity. More substantially, it accommodates easily more general settings which may not fit into Equation (1) in their paper because of nonadditive shocks (which is in case of CES technology) or additional nonlinearity specific to international trade. Finally, while most results in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) are stated in terms of Leontief inverse coefficients, I provide more parsimonious and intuitive summary statistics for the I-O matrix, such as the average level of supplier diversification.

probability distribution, the law of large numbers allows characterizing the effects of CA parsimoniously with a few moments of z_{ik} and a few summary statistics for the I-O matrix.

3.4 Multisector Extension

The focus of this paper is theoretical; the main goal is to understand the mechanisms of supply chain trade and provide a clear connection between outcome variables and model fundamentals. Given that functional form assumptions do not buy much (or any) tractability under general I-O interactions, for several questions the perturbation approach appears to be the only way forward.

For practical purposes, the perturbation approach outlined above can be helpful in incorporating product-level heterogeneity in face of data and computation limitations. There is currently a considerable mismatch between import-export data available at HS6+ level of disaggregation and more aggregate production and intermediate use data. Because of this, the existing multisector models, e.g. Caliendo and Parro (2015), cannot be applied to disaggregated trade flows,[41] so we need more flexible methods that are robust to data limitations.

As demonstrated in Section 5, aggregate trade shares together with variation in product-level trade flows can be used to identify trade costs and the (unobserved) variance of residual cost shocks in (3.5). The aggregate-level ACR formula can then be adjusted to account for product-level heterogeneity. In other words, unlike the existing multisector models, this method does not condition on technology and trade flows at disaggregated level, but instead relies on the law of large numbers. (It still conditions on the entire I-O matrix, but that reduces to using just a few summary statistics.) An obvious disadvantage of this approach is that potentially valuable information is lost by not using particular values of sector-level trade flows and production which we do have at about two-digit level. A more elaborate approach would therefore incorporate some information on large individual sectors. The next proposition demonstrates that sector-level rather than aggregate-level gravity can serve as a basis for perturbation. [Appendix **??** to provide details]

Proposition 3.2. *If the cost shifters satisfy $Z_{ik} = \widetilde{Z}_i \widehat{Z}_{s(i)k}$, then equilibrium trade flows satisfy the log-linear gravity system in each sector s.*

[41] For example, evaluating the gains from trade requires domestic expenditure shares which cannot be computed based on imports and exports alone without using production data (which is limited or unavailable for narrowly defined products).

4 Perturbation Solution

We can represent the vector of exogenous cost differences η as $\eta = \delta\eta$ and write (2.18) as

$$\xi = \delta\eta + \beta\Omega\left[\xi + \log\left(\frac{\lambda^*}{\lambda}\right)^{\frac{1}{\rho-1}}\right]. \tag{4.1}$$

Scalar δ is the perturbation parameter. The actual model corresponds to $\delta = 1$, while the case $\delta = 0$ is particularly simple, as discussed in the previous section. The strategy is to obtain approximations for various outcome variables of interest as Taylor expansions with respect to δ around $\delta = 0$. From (2.18), disabling CA by setting $\delta = 0$ implies $\xi = 0$ and the log price index (2.27) becomes

$$\overline{p} = \overline{p}^{Autarky} - \tfrac{1}{1-\beta}\tfrac{1}{\rho-1}\log\left(1/\lambda_0\right), \tag{4.2}$$

where $\lambda_0 = \frac{1}{1+e^{(1-\rho)\tau}}$ is the no-CA home share (common for all products). Total import becomes simply $M_0 = (1-\lambda_0)/(1-\beta)$, and the trade elasticity equals $\epsilon = \rho - 1$.

4.1 Endogenous Relative Cost Heterogeneity

First, I show that the difference in log marginal costs (4.1) satisfies

$$\xi = \left(I - \beta\left(2\lambda_0 - 1\right)\Omega\right)^{-1}\eta + o\left(\delta^2\right), \tag{4.3}$$

where the next term of the expansion is cubic.[42] Note that $\delta = 1$ is used in the first term, as it corresponds to the actual η.[43] The derivation is provided in Appendix A.9.

Note that the exact expressions for ξ in autarky and under free trade are

$$\xi^A = \left(I - \beta\Omega\right)^{-1}\eta \text{ and } \xi^{FT} = \eta. \tag{4.4}$$

Related to (2.22)-(2.23), (4.3) nests (4.4) as if trade costs affect the input share. The case $\tau = 0$, $\lambda_0 = \frac{1}{2}$ corresponds to ξ^{FT}, while $\tau = \infty$, $\lambda_0 = 1$ gives ξ^A. (See production sharing interpretation below.)

[42] Higher order: $\frac{d^3\xi}{d\delta^3} = 2\left(\rho-1\right)^2\left(1-2\lambda_0\right)\lambda_0\left(1-\lambda_0\right)\beta\left(I-\beta\left(2\lambda_0-1\right)\Omega\right)^{-1}\Omega\left(\frac{d\xi}{d\delta}\right)^3$ and $\frac{d^4\xi}{d\delta^4} = 0$.

[43] For a useful alternative interpretation, we can represent the true η as $\delta\widetilde{\eta}$ imposing a normalization $var\left(\widetilde{\eta}_i\right) = 1$. The actual model then has $var\left(\eta_i\right) = \sigma_\eta^2 = \delta^2$, so the interpretation of the perturbation parameter becomes the standard deviation of exogenous cost differences η_i. The chosen formulation (replacing η with $\delta\eta$, so that the actual model has $\delta = 1$) is convenient in more general cases as it allows dealing with multiple sources of heterogeneity with a single perturbation parameter. It still preserves the interpretation of residual terms of the form $o(\delta^k)$ as $o(\sigma_\eta^k)$.

Expression (4.3) is immediately informative as it highlights that heterogeneity in production costs is endogenous to trade barriers (which enter λ_0). Let $\widetilde{var}(\xi)$ denote the weighted average variance of ξ_i with the weights given by the autarky sales shares $(1-\beta)\, b_i$. Namely, $\widetilde{var}(\xi) = (1-\beta)\sum_i b_i \xi_i^2 - \left[(1-\beta)\sum_i b_i \xi_i\right]^2$. Appendix A.9 uses (4.3) to derive the second-order approximation to $\widetilde{var}(\xi)$ which, in the probability limit over realizations of η_i, is

$$\widetilde{var}(\xi)^{2nd} = (1-\beta)\, tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right]\sigma_\eta^2, \tag{4.5}$$

where $0 \leq \widetilde{\beta} = \beta\,(2\lambda_0 - 1) \leq \beta$ and σ_η^2 is the variance of η_i.

Under frictionless trade, $\lambda_0 = \frac{1}{2}$ and $\widetilde{\beta} = 0$, $\widetilde{var}(\xi)$ is minimal and equals to the variance of exogenous cost shock differences η_i. Intuitively, when international production sharing is not limited by trade barriers, producers in all countries face the same input costs and the only source of heterogeneity in relative marginal costs is the variation is fundamental productivities. When trade is restricted, firms increasingly rely on domestic intermediate inputs; the differences in fundamental productivities accumulate along production chains,[44] resulting in stronger heterogeneity in marginal costs. Formally,[45]

$$\frac{\partial}{\partial\tau} tr\left[\left(I-\widetilde{\beta}(\tau)\,\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}(\tau)\,\Omega\right)^{-1}\right] \geq 0. \tag{4.6}$$

Expression (4.5) can be used to understand which properties of the I-O structure determine cross-country heterogeneity in production costs (which ultimately determines trade flows and welfare). Appendix A.11 derives several properties of the trace in (4.5). First, its bounds are:

$$1 \leq (1-\beta)\, tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right] \leq \frac{1}{\left(1-\widetilde{\beta}\right)^2}. \tag{4.7}$$

The upper bound is achieved for the pure diagonal I-O matrix $\Omega = I$, and the lower bound is achieved in the limit as $N \to \infty$ when $\omega_{ij} = \frac{1}{N}$ for all i, j (which means perfect diversification of suppliers as $N \to \infty$). Second, one general approximation for this trace is:

[44] To see this accumulation, expand (4.3) as $\xi = \eta + \beta\,(2\lambda_0 - 1)\,\Omega\eta + \beta^2\,(2\lambda_0-1)^2\,\Omega^2\eta + ... + o\left(\delta^2\right)$. Differences in (log) marginal costs ξ_i come from exogenous cost differences η_i themselves, their immediate upstream combinations $\sum_j \omega_{ij}\eta_j$, and so on. (The second and subsequent terms represent the expansion of input price differences in (2.18).) These upstream combinations are multiplied by the powers of $(2\lambda_0 - 1) \in [0, 1]$ which is an inverse measure of trade openness.

[45] See Appendix A.10.

$$(1-\beta)\, tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right] \simeq 1+2\widetilde{\beta}SSI+\widetilde{\beta}^2\left(HHI+2SLI\right), \qquad (4.8)$$

which works well for small $\widetilde{\beta} = \beta(2\lambda_0 - 1)$ but becomes inaccurate if both λ_0 and β are close to 1 because it ingores higher order (in $\widetilde{\beta}$) terms. Finally, if the I-O network is generated randomly such that productions loops are negligible for large N, there is another approximation that works for any $\widetilde{\beta} < 1$:

$$(1-\beta)\, tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right] \simeq \left(1-\widetilde{\beta}^2 HHI\right)^{-1}. \qquad (4.9)$$

Appendix A.11 does not prove this relationship but demonstretes that it holds in sumulations and explains the nature of this result.

These results mean that heterogeneity in marginal costs depends, first of all, on the degree of supplier diversification captured by the average input shares Herfindahl HHI. For a given total input share β and cost shock dispersion σ_η^2, relying on more suppliers diversifies away the randomness in total input costs, see the intuition after (2.11). Second, the variance in relative marginal costs is affected by production loops, starting from self-sourcing captured with SSI. Think about an I-O structure in which every product has one supplier. In the ring case (no loops shorter than N) the marginal costs in autarky are $mc_1 = z_1 + \beta mc_2 = z_1 + \beta z_2 + ...$ As z_1 appears again in the summation only multiplied by $\beta^N \to 0$, the loop structure can be ignored. Each industry's log marginal costs are given by a linear combination of all cost shocks in the economy. The industries effectively spread their intermediate spending across the entire economy, which implies a great deal of diversification. In the presence of (short) loops, there are fewer uncorrelated shocks in the expansion of mc_i, so the variance of marginal costs is larger. This marginal cost dispersion is inconsequential in the closed economy under Cobb-Douglas preferences and technology, but in the open economy it affects the gains from trade.

4.2 Welfare

The following proposition, proved in Appendix A.12, gives the second-order approximation for the gains from trade.

Proposition 4.1. *The second-order approximation to the change in log price index* $g =$

$\overline{p}^{Autarky} - \overline{p}$ *equals in the probability limit to*

$$g^{2nd} = \frac{1}{1-\beta}\left[\underbrace{\log\left(\lambda_0^{\frac{1}{1-\rho}}\right)}_{Intra\text{-}industry} + \underbrace{\frac{1}{2}(\rho-1)\lambda_0(1-\lambda_0)\widetilde{var}(\xi)^{2nd}}_{Inter\text{-}industry}\right]. \tag{4.10}$$

With this proposition we can relate the (log) gains from trade to heterogeneity in relative marginal costs and – armed with (4.5) and its approximations (4.8) and (4.9) – to the primitives of the model. Welfare gains from trade consist of two components. The pure intra-industry part, which in our formulation is due to the Armington love for variety, is the gravity-ACR benchmark that would be the case without cross-industry heterogeneity. The inter-industry component captures the contribution of CA forces. It is represented as the variance of relative marginal costs times $\lambda_0(1-\lambda_0)$ which is a measure of trade openness. Both parts are magnified by $\frac{1}{1-\beta}$, reflecting the compounding effect of supply chain trade on the welfare gains.

Looking deeper into the nature of CA forces, variation in relative marginal costs arises from fundamental technology heterogeneity, captured with σ_η^2 in (4.5), that is transformed by the I-O structure and trade barriers. Depending on the degree of supplier diversification (and also the presence of short production loops) on the one hand and the level of trade costs on the other, exogenous heterogeneity is amplified by the possibility that fundamental differences accumulate along production chains:

$$\widetilde{var}(\xi)^{2nd} \geq \sigma_\eta^2. \tag{4.11}$$

The equality in (4.11) is achieved when trade is frictionless and under perfect diversification of suppliers ($HHI = 0$).

Figure 4 provides an illustration. In particular, it shows that the welfare ranking of different I-O structures corresponds to the ranking of relative cost heterogeneity. Compared to the complete I-O network, the ring structure has imperfect supplier diversification ($HHI = 1$) and generates stronger heterogeneity which results in a higher welfare for all $\tau > 0$. The diagonal I-O structure ($\Omega = I$), in addition to imperfect diversification, has also extreme self-sourcing ($HHI = SSI = 1$), resulting in very strong amplification of exogenous heterogeneity and therefore even higher welfare.

Another idea that we can learn from (4.10) and (4.5) is that observed (or recovered) heterogeneity cannot, in general, be considered structural, in particular independent of

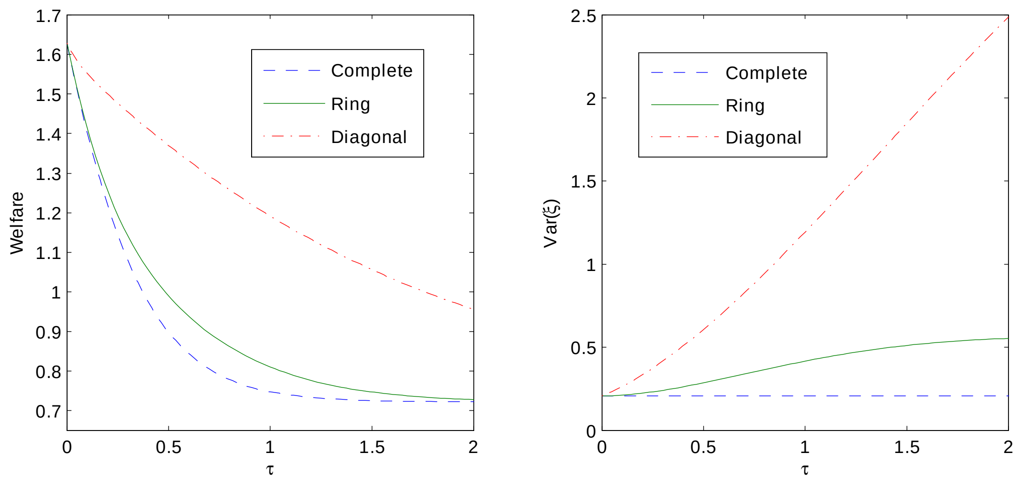

Figure 4: Welfare $\exp(-\overline{p})$ and variance of relative marginal costs $\widetilde{var}\,(\xi)$ for different I-O networks. The flat dashed line is $\widetilde{var}\,(\xi) = \sigma_\eta^2$ for complete network.

trade costs. A (misspecified) model with perfect supplier diversification, such as having the complete I-O network, can match a model with granular (industry-level) linkages by adjusting exogenous variation σ_η^2, but it will fail to predict the effects of changes that affect variation in (endogenous) marginal costs. In Section 5 I study the role of correctly specifying the I-O structure under different available information and, more generally, econometric issues related to this model.

Trade flows

By now we understand the welfare part of Figure 1. Imperfect supplier diversification in case of the ring I-O structure generates greater variation in relative marginal costs for any $\tau > 0$. The trade part is more nuanced. Note that (2.26) can also be written as

$$M = \sum\nolimits_i b_i\,(1-\lambda_i) - \beta \sum\nolimits_i (y_i - b_i)\,\lambda_i^{Upstream}, \tag{4.12}$$

where, again, $\lambda_i^{Upstream} = \sum_j \omega_{ij}\lambda_j$. Total imports are given by a linear combination of industry-level import shares minus a term capturing downstream spillovers of CA.

The following proposition, proved in Appendix A.13, gives a second-order approximation for the two components of total imports (4.12).

Proposition 4.2. *Denoting "≈" the second-order approximation in the probability limit,*

$$\sum\nolimits_i b_i\,(1-\lambda_i) \approx \tfrac{1}{1-\beta}\left[1 - \lambda_0 + \tfrac{1}{2}(\rho-1)^2\,(2\lambda_0 - 1)\,\lambda_0\,(1-\lambda_0)\,\widetilde{var}\,(\xi)^{2nd}\right]$$

$$\beta \sum\nolimits_i (y_i - b_i)\, \lambda_i^{Upstream} \approx \tfrac{1}{1-\beta}\tfrac{1}{2}(\rho - 1)\,\lambda_0 (1 - \lambda_0)\, \frac{d\widetilde{var}(\xi)^{2nd}}{d\tau}.$$

To finally explain the trade part of Figure 1, we want to understand how supplier diversification differentially impacts trade flows for high and low trade costs.[46] Denote $M^{(1)}$ the average import share component $\sum_i b_i (1 - \lambda_i)$ and $M^{(2)}$ the covariance component $\beta \sum_i (y_i - b_i)\, \lambda_i^{Upstream}$. The complete symmetric network $\left(\Omega = \frac{1}{N}\mathbf{11'}\right)$ is straightforward. The trace involved in $\widetilde{var}(\xi)^{2nd}$ does not depend on trade costs and $\widetilde{var}(\xi)^{2nd} = \sigma_z^2$, so $M^{(1)} = \frac{1}{1-\beta}\left[1 - \lambda_0 + \frac{1}{2}(\rho - 1)^2 (2\lambda_0 - 1)\,\lambda_0 (1 - \lambda_0)\,\sigma_z^2\right]$ and $M^{(2)} = 0$. Because of imperfect diversification, the ring has $\widetilde{var}(\xi)^{2nd} > \sigma_z^2$ (for $\tau > 0$) and therefore a larger contribution $M^{(1)}$. At the same time, $M^{(2)}$ contributes negatively. Which effect dominates when?

Denote $\Delta M^{(1)}$ the difference between $M^{(1)}$ for a given I-O structure and the complete symmetric network, and similarly $\Delta M^{(2)}$. The last part of Appendix A.13 shows that

$$\lim_{\tau \to \infty} \frac{\Delta M^{(2)}}{\Delta M^{(1)}} = 0 \;\text{ and }\; \lim_{\tau \to 0} \frac{\Delta M^{(1)}}{\Delta M^{(2)}} = 0. \tag{4.13}$$

It means that the covariance between downstream output and upstream cost advantage plays a relatively small role when trade costs are high, but it dominates the direct heterogeneity effect $M^{(1)}$ for low trade costs.

Therefore, compared to the benchmark of complete and symmetric network,[47] the role of imperfect supplier diversification in a general I-O network with disaggregated interactions is to amplify the effect of small trade costs, but to generate relatively more trade when trade costs are high. Although variation in relative marginal costs increases in the level of trade frictions, this happens slowly near free trade, so that the trade-increasing effect of stronger heterogeneity is dominated by the trade-decreasing effect of downstream CA spillovers. When trade costs are high, however, these spillovers decay fast, so that their negative contribution to trade flows becomes weaker relative to the positive contribution of stronger marginal cost heterogeneity. This is illustrated in Figure 5. Note that self-sourcing (diagonal elements of the I-O matrix) plays a special role in determining trade flows. As discussed earlier, even free trade equilibrium exhibits home bias in the sense that import shares are below one half in a two-country economy. The two right panels of Figure 5 provide an illustration.

[46] Here we keep σ_z^2 fixed and consider different I-O structures.

[47] Such stylized roundabout production structure is often assumed in the literature to avoid specifying the detailed I-O network when introducing intermediate goods.

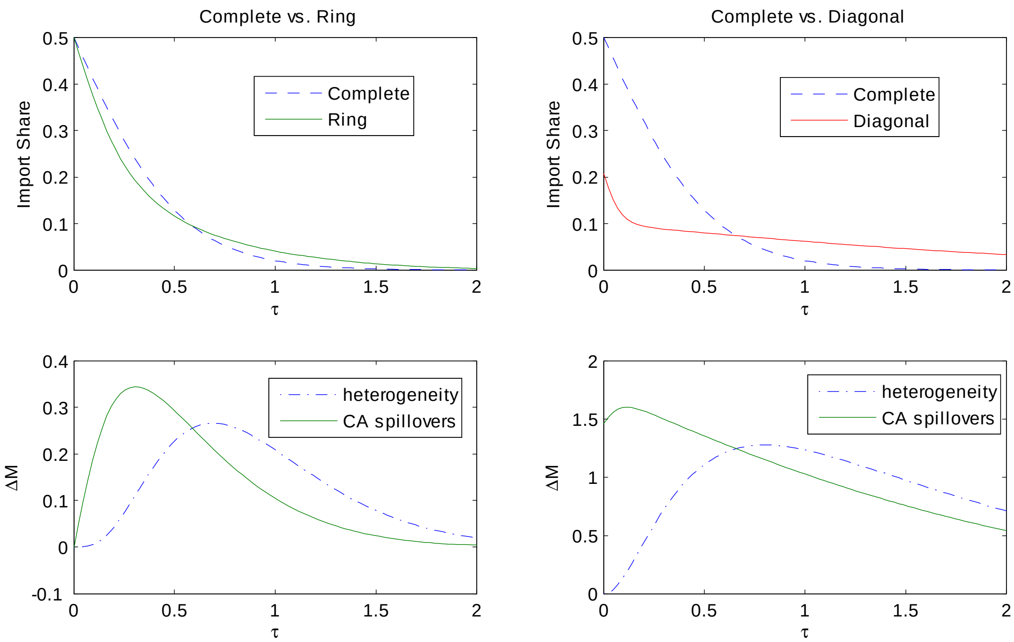

Figure 5: Comparing trade flows for different I-O networks."Heterogeneity" corresponds to differences in average domestic shares $\sum_i b_i^{Comp} \lambda_i^{Comp} - \sum_i b_i^{Ring\ (Diag)} \lambda_i^{Ring\ (Diag)}$. "CA spillovers" correspond to covariance terms $\beta \sum_i (y_i - b_i) \lambda_i^{Upstream}$.

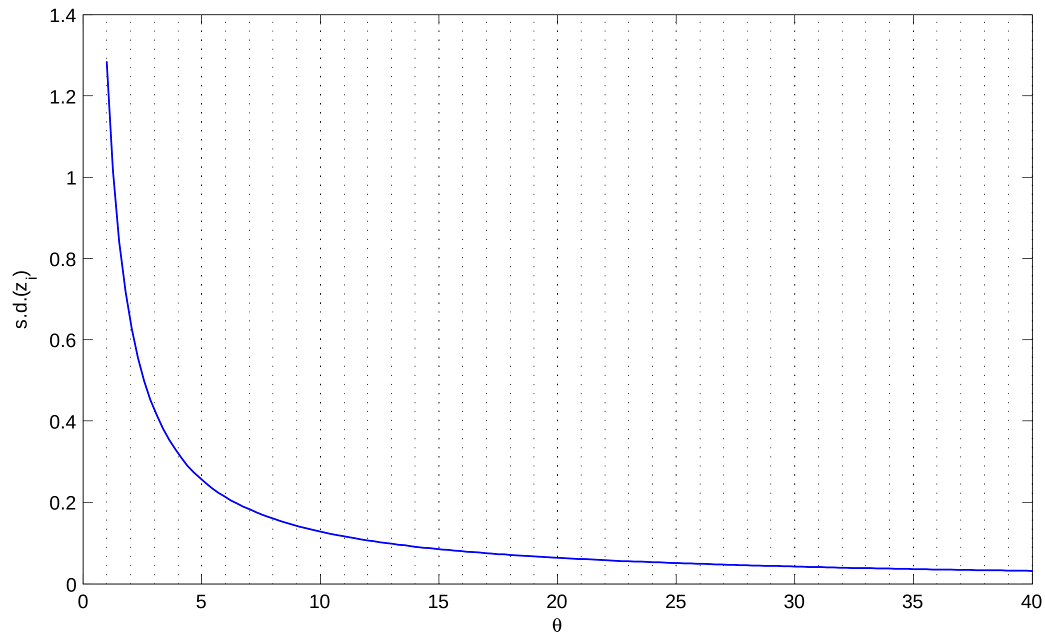

Figure 6: Standard deviation of z_i for different values of θ.

4.3 Approximation quality

The perturbation approximations obtained above are quantitatively good when intra-industry trade is sufficiently strong relative to inter-industry trade. This is the case when productivity heterogeneity driven by cost shock variation σ_z^2 is moderate and the Armington elasticity ρ is small. What is the relevant range of σ_z? From Table 6 in Costinot, Donaldson, and Komunjer (2012), the standard deviation of residual cost shifters (after controlling for country and sector fixed effects) is $\sigma_z = 0.1259$. Given the high level of aggregation in that paper, this number can be viewed as a lower bound for more disaggregated data. [A more relevant estimate can be obtained from Hanson and Muendler (2014)] For a reference, Figure 6 provides the values of σ_z for Frechet-distributed productivity with the dispersion parameter θ.

In Broda and Weinstein (2006), the median elasticity of substitution among varieties ranges from 2.2 to 3.7, while Feenstra, Luck, Obstfeld, and Russ (2014) find even lower values. At the same time, the distribution of industry-level elasticities has a well-represented right tail, driving the average value to as high as 17. To address this concern, in Section 5 I add product-specific shock to ρ, which, combined with product-specific shock to trade costs, can also account the presence of negligible trade flows (the problem of zeros).

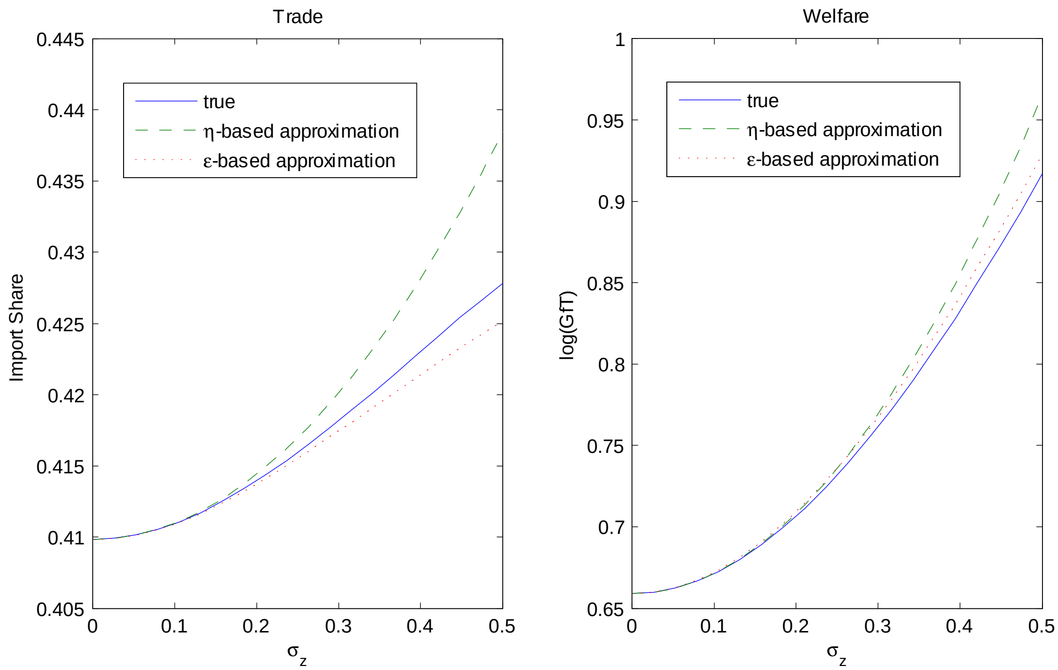

Figure 7: $\rho = 3$

Figure 7 and Figure 8 illustrate approximation quality for $\rho = 3$ and $\rho = 5$ for the ring I-O structure. Other parameters are set as $\beta = 0.6$ and $\tau = \log(1.2)$. "η-based approximation" corresponds to the results of Proposition 4.1 and Proposition 4.2. For higher values of σ_z as well as ρ, approximation quality is improved if the exogenous heterogeneity is expressed in terms ε_i, defined in (A.30), rather than η_i.

5 Measuring the Gains from Trade [Preliminary]

This section is concerned with measuring the gains from trade within the model environment of Section 2. First, I discuss data requirements and identification issues that arise is relation to Proposition 4.1. Second, I study the role of correctly specifying the I-O structure. Finally, I provide an extension of the basic model in order to account for additional sources of cross-industry heterogeneity, which I show is consequential for evaluating the welfare effects of trade.

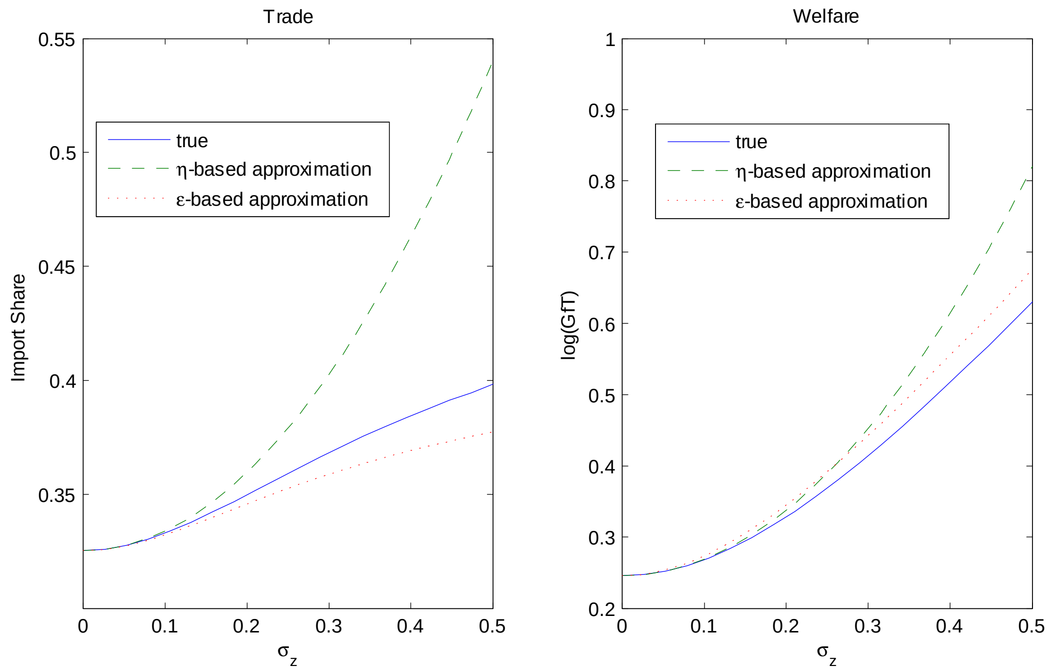

Figure 8: $\rho = 5$

5.1 Data Requirements and Identification

In the context of the model in Section 2, suppose that we know the share of intermediate inputs β, the elasticity of substitution ρ and the list of summary statistics for the I-O matrix involved in (4.8)-(4.9).[48] In addition to these, the gains from trade formula (4.10) requires two other fundamental parameters: the trade cost τ and the variance of exogenous relative cost shocks σ_η^2. Suppose that τ and σ_η^2 are unknown, which is the empirically relevant case. In this situation, what data is needed to implement (4.10)? I show that fundamental parameters can be recovered with trade data alone, that is, without disaggregated production data.

Simple case: complete symmetric I-O network

Consider first a simplified version of the model with $\Omega = \frac{1}{N}\mathbf{11}'$, so that the influence vector is $b = \frac{1}{1-\beta}\alpha$ (autarky sales are proportional to consumption shares) and $\xi = \eta$ (input prices do not contribute to differences in marginal costs because of perfect diversification of suppliers). The second-order approximation for the gains from trade is

[48] In principle, these statistics can be estimated based on a limited sample of products. The weights $(1-\beta)\, b_i$ in (2.11)-(2.13) are impossible to calculate without knowing the full I-O matrix, but they can be proxied with the sales shares.

$$\sum_i b_i \log\left(\lambda_i^{\frac{1}{1-\rho}}\right) = \frac{1}{1-\beta}\left[\log\left(\lambda_0^{\frac{1}{1-\rho}}\right) + \frac{1}{2}(\rho-1)\lambda_0(1-\lambda_0)\widetilde{var}(\eta)\right],$$

where $\widetilde{var}(\eta) = \sum_i \alpha_i var(\eta_i)$, which reduces to σ_η^2 in case of iid η_i. For this simple I-O structure, however, the iid assumption can be relaxed a little at no extra costs, by allowing $var(\eta_i)$ to differ across products i and even depend on α_i.

The second-order perturbation approximation for aggregate import is

$$M = \sum_i b_i(1-\lambda_i) = \frac{1}{1-\beta}\left[1-\lambda_0 + \frac{1}{2}(\rho-1)^2(2\lambda_0-1)\lambda_0(1-\lambda_0)\widetilde{var}(\eta)\right]. \tag{5.1}$$

How can we use the information on product-level trade flows $m_i = b_i(1-\lambda_i)$ and $m_i^* = b_i(1-\lambda_i^*)$, $i = 1..N$, to recover both λ_0 (and hence τ as $\lambda_0 = \frac{1}{1+e^{(1-\rho)\tau}}$) and $\widetilde{var}(\eta)$? The variance of net trade flows $\frac{1}{N}\sum_i(m_i - m_i^*)^2$ does not work because it is affected by variation in α_i's. At the same time, controlling for unobserved expenditure shares by taking the ratios $\frac{m_i^*}{m_i}$ can only recover σ_η^2 under iid η_i but not $\widetilde{var}(\eta)$ in the general case. The task is to find the right balance between not trying to control for expenditure shares at all and eliminating them completely. I find that it is achieved by constructing moments analogous to measures of probability distribution divergence, such as Kullback-Leibler, Rényi, or Hellinger. Thus, I derive the second-order approximation for the following Kullback-Leibler-type statistic, see Appendix.. for details:

$$KL = \sum_i\left(m_i \log\frac{m_i}{m_i^*} + m_i^* \log\frac{m_i^*}{m_i}\right) = \frac{4}{1-\beta}(\rho-1)^2\lambda_0^2(1-\lambda_0)\widetilde{var}(\eta). \tag{5.2}$$

The following proposition states that aggregate imports M together with the divergence measure KL can uniquely recover $\widetilde{var}(\eta)$ and τ.

Proposition 5.1. *For any given* $M \in \left(0, \frac{1}{2}\frac{1}{1-\beta}\right)$ *and* $KL \in \left(0, \overline{KL}\right)$, *there is a unique combination of* $\tau > 0$ *and* $\widetilde{var}(\eta) > 0$ *that solves (5.1)-(5.2).*

Proof: see Appendix..

While the condition for total imports is formal (for any $\tau \in (0,\infty)$ it always holds that M is positive and does not exceed one half on aggregate expenditure), the condition $KL < \overline{KL}$ is restrictive. The levels of KL above a certain threshold (that is, strong enough heterogeneity in product-level trade flows) can recover $\lambda_0 > 1$ which is not consistent with

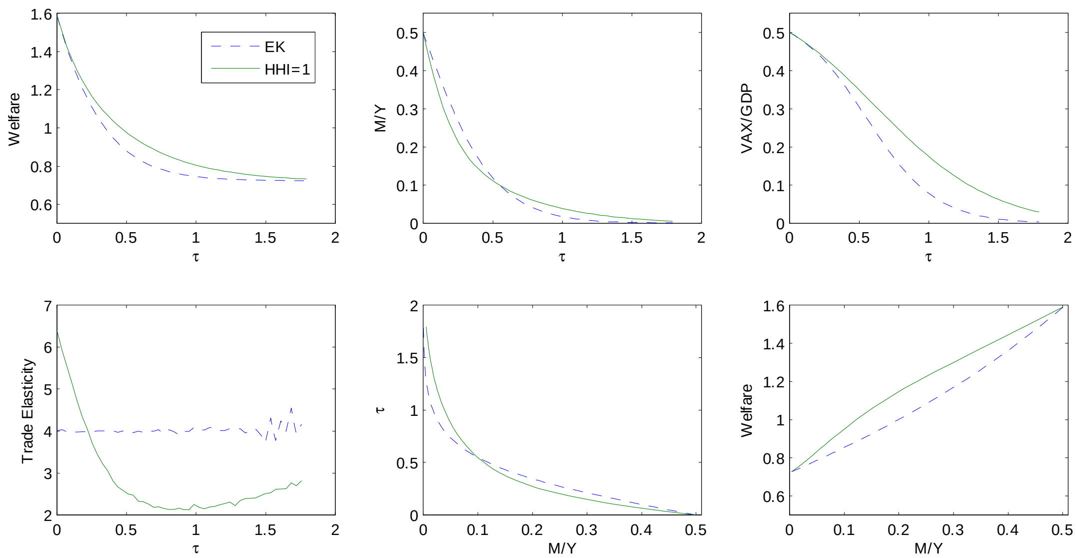

Figure 9: Comparing to the EK benchmark

any (real) value of τ. This reflects a natural limitation of the perturbation method that its quantitative validity is restricted to relatively moderate amounts of variation it is trying to approximate.

General I-O structure

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5.2 The Role of Correct I-O Specification

Analytical results for perturbation

...

Numerical results for perfect substitutes ($\rho = \infty$)

Although the perturbation results above are derived for finite ρ, the qualitative insights extend to the case of perfect substitutes. Figure 9 illustrates the role of disaggregated I-O interactions by comparing an I-O structure in which every industry has one randomly chosen supplier ($HHI = 1$) to the EK benchmark ($\omega_{ij} = \frac{1}{N}$ for all i, j, $HHI \to 0$). In both cases, $\alpha = \frac{1}{N}\mathbf{1}$, the goods from Home and Foreign are perfect substitutes, the share of intermediates is $\beta = 0.8$, and the Frechet parameter is $\theta = 4$. I simulate the model with one million goods. The random graph I-O structure is implemented by using sparse matrices. For the EK, I use that $b = \frac{1}{1-\beta}\frac{1}{N}\mathbf{1}$, import is $b'(\mathbf{1}-\lambda)$, and λ_i is the indicator that exogenously given $-\eta_i$ does not exceed τ.

The top left panel of Figure 9 shows that welfare declines faster in the EK benchmark as τ increases. For any given $0 < \tau < \infty$, the I-O structure with positive HHI has a larger variance of marginal cost differences, which translates into larger gains from trade. As the countries move toward autarky, trade flows are first smaller with $HHI = 1$, but eventually become larger (central top panel). The absence of monotone amplification necessarily follows from the fact that the area between the two lines integrates to zero, which is a consequence of Proposition 2.1. For the value added trade flows, however, the EK has uniformly less trade (top left panel). The elasticity of trade in case $HHI = 1$ is first larger than θ, consistent with (2.34), but becomes smaller for large trade costs. To my knowledge, this non-monotone behavior of the trade elasticity has not been recognized in the literature, although it is present, for example, in Yi (2010). The last two panels illustrate that, conditional on the known θ, the two models read the observed trade shares differently. Suppose the true model is the one with $HHI = 1$. Ignoring the micro-level nature of I-O linkages and incorrectly assuming the EK structure can recover either larger or lower trade costs, but always underestimates the gains from trade.

What if the true θ is unknown? Assume again that the true I-O structure is the one with $HHI = 1$. Suppose that we observe trade flows and the variance of log price differences. These two moments recover the combination $\left(\widehat{\theta}, \widehat{\tau}\right)$. Figure 10 illustrates the importance of correctly specifying the I-O structure is this case. For the reference, the case of known θ, in which the information on price dispersion is ignored, is included for the EK. In both cases, incorrectly assuming the complete symmetric I-O network of EK leads to underestimation of the gains from trade.

6 Conclusion

This paper introduces a perturbation approach to comparative advantage and other forms of cross-industry heterogeneity which allows studying complex input-output networks with a tractable extension of the gravity model. The new method is instrumental in theoretical analysis of global value chains and refining the level of aggregation in face of data limitations. I develop a model that accommodates a nearly arbitrary pattern of "snakes and spiders" linkages between a large number of products. Despite this rich production architecture, trade flows and welfare are related to model fundamentals in a transparent and intuitive way. In particular, I derive summary statistics for the input-output structure needed to characterize the effects of international production sharing. The key feature of the model is endogenous variation in relative costs that is determined simultaneously

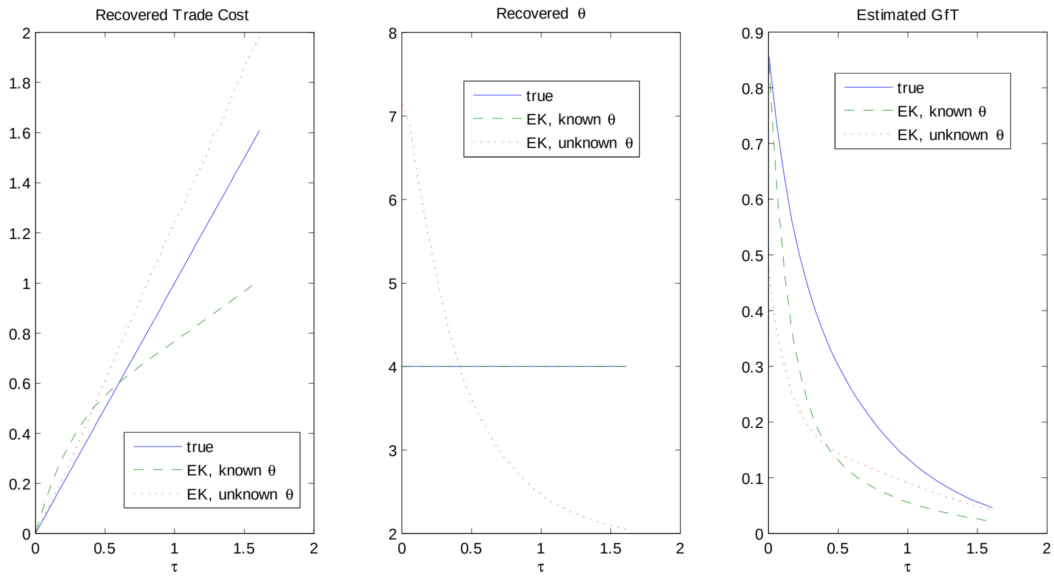

Figure 10: Misspecification

with vertical specialization. This mechanism primarily depends on average supplier diversification and is also influenced by short production loops. For this reason, accounting for disaggregated network structure is essential and cannot be replaced with a composite intermediate good assumption often imposed in the literature. Within the model framework, I discuss how the gains from trade can be evaluated when production and use data for narrowly defined industries is missing but trade data is available.

One central finding of the paper is that I-O interactions amplify the effects of trade costs through CA spillovers. It implies that supply chain models that feature CA forces should pay special attention to the "spider" dimension of the I-O network (the number of suppliers and input shares diversification). In contrast, assuming a composite intermediate good structure does not allow the impact of trade costs on trade flows to be magnified and also leads to underestimating the gains from trade.

For subsequent research, the perturbation approach developed in this paper is more general and can be applied to other questions. In particular, in my current work I use it to study optimal trade policy. In general, characterizing optimal trade policy in the spirit of Costinot, Donaldson, Vogel, and Werning (2015) is hard in case of complicated I-O interactions since the entire pattern of CA is affected by both import and export taxes. However, in the absence of CA and other forms of heterogeneity, the I-O structure is irrelevant beyond the (common) input share and all trade taxes implementing the

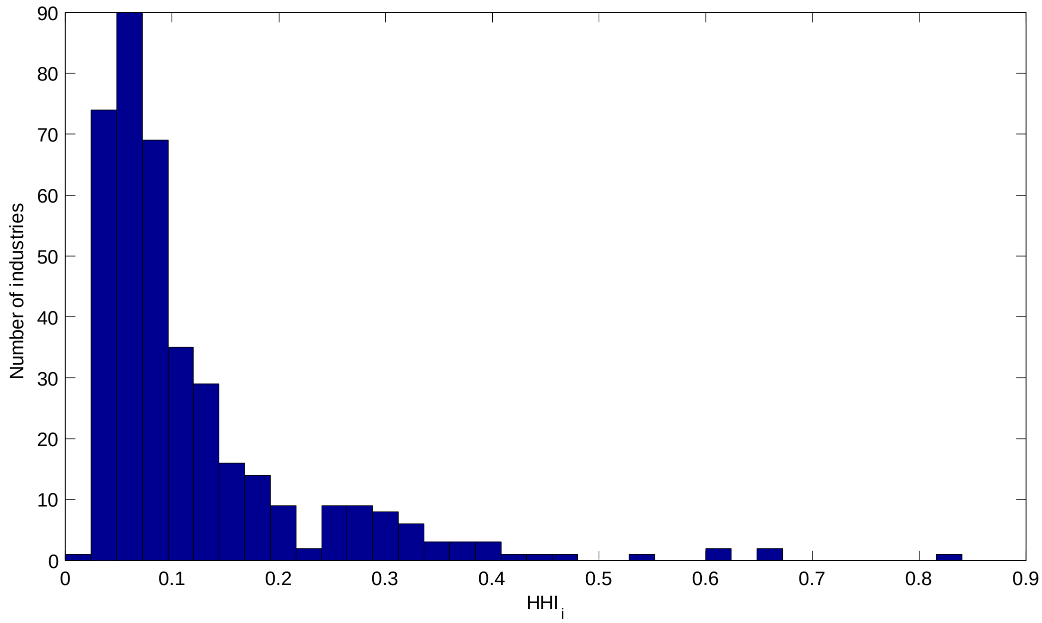

Figure 11: Histogram of industry-level cost shares Herfindahl indices.

optimal allocation are uniform. Based on this observation, the actual optimal taxes can be characterized analytically in term of deviations from the uniform benchmark.

A Appendix

A.1 Evidence

Figure 12. Based on the B.E.A. detailed I-O table and trade flows for 2007, I calculate a measure of industry i's CA as $\frac{\text{Export}_i - \text{Import}_i}{\text{Export}_i + \text{Import}_i}$ and industry i's upstream CA as a linear combination of suppliers' CA with the weights equal to the corresponding input shares. Taking the ratio of net exports to total trade flows is supposed to net out variation attributable to heterogeneity in trade costs. The result is that there is nontrivial variation in upstream CA (which is a case against perfect diversification) and it has predictive power for downstream CA. Total 333 observations after deleting industries with $\text{Export}_i = \text{Import}_i = 0$. Correlation coefficient 0.305 and $R^2 = 0.106$. Similar results if CA is measured as the difference between actual and auratky sales shares which corresponds to specialization.

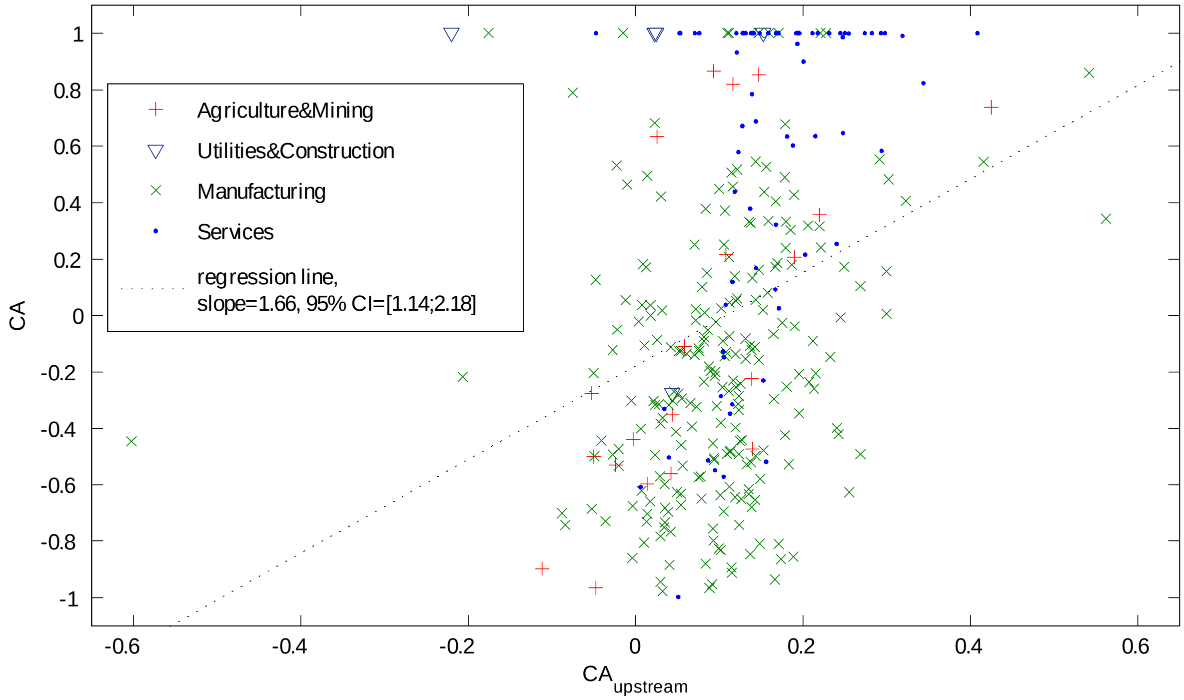

Figure 12: CA spillovers

A.2 LLN Lemma

Let $\{A^{(N)}\}_{N=1}^{\infty}$ be a sequence of quadratic deterministic $N \times N$ matrices with entries $a_{ij}^{(N)}$. Let $u_i^{(N)}$, $1 \leq i \leq N \leq \infty$, be iid random variables that form a sequence of random vectors $\{u^{(N)} = (u_1^{(N)}..u_N^{(N)})'\}_{N=1}^{\infty}$. To simplify notation, suppress from now on the "N" superscript in $A^{(N)}$ and $u^{(N)}$. Assume that each component of u has four finite moments – μ_1, μ_2, μ_3, and μ_4 – and $\mu_1 = 0$.

Lemma 1. *If* $\lim_{N\to\infty} tr\,(A'A) = 0$, *then* $u'Au \underset{p}{\to} \mu_2 tr\,(A)$.

Proof:

As u_i are iid, $E\,[u'Au] = \sum_{ij} a_{ij} E\,[u_i u_j] = \sum_i a_{ii} E\,[u_i^2] = \mu_2 tr\,(A)$. For symmetric $\widetilde{A}$, a textbook result[49] for the variance of quadratic form is that

$$Var\left[u'\widetilde{A}u\right] = 2\mu_2^2 tr\left(\widetilde{A}^2\right) + \left(\mu_4 - 3\mu_2^2\right)\sum\nolimits_i \widetilde{a}_{ii}^2. \tag{A.1}$$

In general nonsymmetric case, $\widetilde{A}$ needs to be replaced with $\frac{1}{2}\,(A + A')$, which gives

[49] See Seber and Lee (2002).

$$Var\left[u'Au\right] = \mu_2^2 tr\left(A^2 + A'A\right) + \left(\mu_4 - 3\mu_2^2\right)\sum\nolimits_i a_{ii}^2. \tag{A.2}$$

We want to show that $tr\left(A'A\right) \to 0$ implies $Var\left[u'Au\right] \to 0$. Obviously, $0 \leq \sum_i a_{ii}^2 \leq \sum_{ij} a_{ij}^2 = tr\left(A'A\right) \to 0$, so we only need to show that $tr\left(A^2\right) \to 0$. This is indeed the case since $|tr\left(A^2\right)| \leq tr\left(A'A\right)$ holds because of the Cauchy-Schwarz inequality.[50] Now as $E\left[u'Au\right] = \mu_2 tr\left(A\right)$ and $Var\left[u'Au\right] \to 0$, $u'Au \underset{p}{\to} \mu_2 tr\left(A\right)$. ■

Lemma 1 is the main LLN tool of this paper. Several proofs also use a simpler result for linear forms which is well known. I provide it below for completeness.

Let $\{a^{(N)}\}_{N=1}^{\infty}$ be a sequence of deterministic $N \times 1$ vectors whose components sum up to a constant $\overline{a}$: $\sum\limits_{i=1}^{N} a_i^{(N)} = \overline{a}$ for all N. Random variables $u_i^{(N)}$, $1 \leq i \leq N \leq \infty$, are defined as before. Assume that each component of u has two finite moments, μ_1 and μ_2.

Lemma 1A. *If* $\lim\limits_{N\to\infty} \|a\|_2 = 0$, *then* $a'u \underset{p}{\to} \overline{a}\mu_1$.

Proof:

First, $E\left[a'u\right] = \sum_i a_i E\left[u_i\right] = \overline{a}\mu_1$. Second, as u_i are iid, $Var\left[a'u\right] = \sum_i a_i^2 Var\left[u_i\right] = \mu_2 \cdot \|a\|_2^2 \to 0$. The two imply that $a'u \underset{p}{\to} \overline{a}\mu_1$. ■

A.3 Closed Economy with CES

Setup: CES modification

Consumers have CES preferences with the elasticity of substitution σ:

$$U = \left(\sum_{i=1}^{N} \alpha_i^{\frac{1}{\sigma}} c_i^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{A.3}$$

Industries produce with technology that is Cobb-Douglas over labor and industry-specific composite intermediate input, which in turn is a CES aggregate with the same elasticity:

$$q_{ik} = e^{-z_i}\left(\frac{l_i}{1-\beta}\right)^{1-\beta}\left(\frac{1}{\beta}\left(\sum_{j=1}^{N} \omega_{ij}^{\frac{1}{\sigma}} m_{ij}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right)^{\beta}. \tag{A.4}$$

The log price index for (A.3) is now

[50] In the space of $N \times N$ real matrices, $< A_1, A_2 >= tr\left(A_1 A_2'\right)$ is an inner product with induced norm $\sqrt{tr\left(AA'\right)}$. Therefore, $\left(tr\left(A_1 A_2'\right)\right)^2 \leq tr\left(A_1 A_1'\right) \cdot tr\left(A_2 A_2'\right)$. Taking $A_1 = A$ and $A_2 = A'$, we have $\left(tr\left(AA\right)\right)^2 \leq tr\left(AA'\right) \cdot tr\left(A'A\right) = \left(tr\left(A'A\right)\right)^2$, or $|tr\left(A^2\right)| \leq tr\left(A'A\right)$.

$$\bar{p} = \frac{1}{1-\sigma} \log \left(\sum_{i=1}^{N} \alpha_i e^{(1-\sigma)p_i} \right), \tag{A.5}$$

where log prices (marginal costs) are given by the following system, $i = 1..N$:

$$p_i = z_i + \frac{\beta}{1-\sigma} \log \left(\sum_{j=1}^{N} \omega_{ij} e^{(1-\sigma)p_j} \right). \tag{A.6}$$

Perturbation

Let us introduce into (A.6) a perturbation parameter δ by replacing "z_i" with "δz_i". The value $\delta = 1$ corresponds to the actual model, while the case $\delta = 0$ is particularly simple because then $p_i = 0$ for all i and hence $\bar{p} = 0$ as well. We will now combine a second-order perturbation approximation to (A.5) with the law of large numbers in order to investigate the interaction between cross-industry productivity variation and the I-O structure. For this purpose we assume that z_i are iid with four finite moments, in particular $E(z_i) = 0$ and $E(z_i^2) = \sigma_z^2$.

We proceed by totally differentiating (A.5) and (A.6) with respect to δ. Let $\dot{x}$ denote $\frac{dx}{d\delta}$ and $\ddot{x}$ denote $\frac{d^2x}{d\delta^2}$ for generic variable x. For (A.5),

$$\dot{\bar{p}} = \frac{\sum_i \alpha_i e^{(1-\sigma)p_i} \dot{p}_i}{\sum_i \alpha_i e^{(1-\sigma)p_i}}. \tag{A.7}$$

Differentiating again with respect to δ gives, evaluated at $p_i = 0$,

$$\ddot{\bar{p}} = \sum\nolimits_i \alpha_i \ddot{p}_i + (1-\sigma) \left[\sum\nolimits_i \alpha_i \dot{p}_i^2 - \left(\sum\nolimits_i \alpha_i \dot{p}_i \right)^2 \right]. \tag{A.8}$$

Next differentiate (A.6) $-$ in which z_i is represented as δz_i $-$ to obtain $\dot{p}_i$and $\ddot{p}_i$:[51]

$$\dot{p}_i = z_i + \beta \frac{\sum_j \omega_{ij} e^{(1-\sigma)p_j} \dot{p}_j}{\sum_j \omega_{ij} e^{(1-\sigma)p_j}}. \tag{A.9}$$

Evaluated at $p_i = 0$, it reduces to $\dot{p}_i = z_i + \beta \sum_j \omega_{ij} \dot{p}_j$, from which we have

$$\dot{p} = (I - \beta\Omega)^{-1} z, \tag{A.10}$$

which means that the first-order approximation to log prices under CES is given by exact

[51] Without I-O interactions ($\beta = 0$) log prices are $p_i = \delta z_i$, so (A.8) has $\dot{p}_i = z_i$, $\ddot{p}_i = 0$ and therefore $\bar{p} \underset{p}{\to} \frac{1}{2}(1-\sigma)\sigma_z^2$. (See the general case below for a formal law of large numbers argument.)

log prices under Cobb-Douglas (2.4). Evaluating (A.7) at $\delta = 0$ (or $p_i = 0$ for all i) gives $\dot{\overline{p}} = \alpha' \dot{p} = \alpha' (I - \beta\Omega)^{-1} z = b'z$, therefore the first-order approximation to (A.5) is the exact Cobb-Douglas log price index (2.7) plus a residual term $o(\delta)$:[52]

$$\overline{p} = b'z + o(\delta). \tag{A.11}$$

Differentiating (A.9) with respect to δ again gives (evaluated at $p_j = 0$)

$$\ddot{p}_i = \beta \sum_j \omega_{ij} \ddot{p}_j + \beta (1 - \sigma) \left[\sum_j \omega_{ij} \dot{p}_j^2 - \left(\sum_j \omega_{ij} \dot{p}_j \right)^2 \right]. \tag{A.12}$$

In vector-matrix notation (A.12) is $\ddot{p} = \beta\Omega\ \ddot{p} + \beta(1-\sigma)\left[\Omega \dot{p}^2 - \left(\Omega \dot{p}\right)^2\right]$, where $\dot{p}^2$ and $\left(\Omega\dot{p}\right)^2$ are element-wise. Therefore,

$$\ddot{p} = \beta (1 - \sigma) (I - \beta\Omega)^{-1} \left[\Omega \dot{p}^2 - \left(\Omega \dot{p} \right)^2 \right]. \tag{A.13}$$

We can now construct the second-order approximation to the log price index (A.5). To get a more intuitive expression, introduce $\overline{p}^{CD}$, $var_\alpha\left(p^{CD}\right)$ and $var_\Omega\left(p^{CD}\right)$ to denote the log price index and two measures of cross-industry price variation in the Cobb-Douglas case. Specifically,

$$\overline{p}^{CD} = b'z, \qquad var_\alpha\left(p^{CD}\right) = \sum_i \alpha_i \dot{p}_i^2 - \left(\sum_i \alpha_i \dot{p}_i \right)^2, \text{ and}$$

$$var_\Omega\left(p^{CD}\right) = (1 - \beta) \sum_i b_i \left[\sum_j \omega_{ij} \dot{p}_j^2 - \left(\sum_j \omega_{ij} \dot{p}_j \right)^2 \right].$$

In words, $var_\alpha\left(p^{CD}\right)$ is the weighted variance of consumer (log) prices under Cobb-Douglas and $var_\Omega\left(p^{CD}\right)$ is the weighted average (with the weights $(1-\beta) b_i$ that are shares of particular industries in total expenditure) of $\left[\sum_j \omega_{ij} \dot{p}_j^2 - \left(\sum_j \omega_{ij} \dot{p}_j\right)^2\right]$ which is wighted variance of input prices for industry i under Cobb-Douglas. With this notation, we have from (A.13) that[53]

$$\alpha' \ddot{p} = \beta (1 - \sigma) b' \left[\Omega \dot{p}^2 - \left(\Omega \dot{p} \right)^2 \right] = (1 - \sigma) \frac{\beta}{1 - \beta} var_\Omega\left(p^{CD}\right). \tag{A.14}$$

[52] An intermediate step is to write $\overline{p} = b'z \cdot \delta + o(\delta)$ and set $\delta = 1$, which corresponds to the actual model.

[53] Using that $\alpha' (I - \beta\Omega)^{-1} = b'$.

The second-order approximation for the CES log price index (A.5) is therefore

$$\overline{p}^{CES} = \overline{p}^{CD} + \frac{1-\sigma}{2}\left[var_{\alpha}\left(p^{CD}\right) + \frac{\beta}{1-\beta}var_{\Omega}\left(p^{CD}\right)\right] + o\left(\delta^2\right). \tag{A.15}$$

This formula reflects that when different products are substitutes ($\sigma > 1$), there are consumption gains from variation in consumer prices as well as productivity gains from variation in input prices. The final part of this appendix looks at the asymptotic behavior of (A.15) as the number of industries N goes to infinity.

Law of large numbers

The next step is to apply the law of large numbers which in our context in based on Lemma 1 and Lemma 1A from Appendix A.2. First consider the term $\sum_i \alpha_i \dot{p}_i$in (A.8). Using (A.10), it is written as

$$\sum\nolimits_i \alpha_i \dot{p}_i = \alpha' \left(I - \beta\Omega\right)^{-1} z = b'z \underset{p}{\rightarrow} 0, \tag{A.16}$$

which follows from (2.9) by Lemma 1A.

For (A.14), use that $b = \beta\Omega' b + \alpha$, so it reduces to

$$\alpha'\ddot{p} = (1-\sigma)\left[(b-\alpha)'\dot{p}^2 - \beta b'\left(\Omega\dot{p}\right)^2\right]. \tag{A.17}$$

Now in (A.8) we have that $(1-\sigma)\,\alpha'\dot{p}^2$ cancels out, so, also with (A.16), it becomes[54]

$$\ddot{\overline{p}} = (1-\sigma)\left[b'\dot{p}^2 - \beta b'\left(\Omega\dot{p}\right)^2\right], \tag{A.18}$$

The first term $b'\dot{p}^2$ is

$$\dot{p}' D_b \dot{p} = z' \left(I - \beta\Omega'\right)^{-1} D_b \left(I - \beta\Omega\right)^{-1} z, \tag{A.19}$$

and the term $b'\left(\Omega\dot{p}\right)^2$ is

$$\dot{p}'\Omega' D_b \Omega\dot{p} = z' \left(I - \beta\Omega'\right)^{-1} \Omega' D_b \Omega \left(I - \beta\Omega\right)^{-1} z. \tag{A.20}$$

Denote $A_1 = \left(I - \beta\Omega'\right)^{-1} D_b \left(I - \beta\Omega\right)^{-1}$ and $A_2 = \left(I - \beta\Omega'\right)^{-1} \Omega' D_b \Omega \left(I - \beta\Omega\right)^{-1}$. To

[54] The first term in this expression does not represent the combined effect of consumption and production gains from price variation – the sum $var_{\alpha}\left(p^{CD}\right) + \frac{\beta}{1-\beta}var_{\Omega}\left(p^{CD}\right)$ in (A.15) – as it double counts variation in input prices. The second term gives the necessary correction. [To be clarified with variance decomposition. Simple illustration for the pure diagonal I-O structure with $\Omega = I$.]

apply Lemma 1 to (A.19) and (A.20) we need to show that (2.9) implies that $tr(A_1'A_1) \to 0$ and $tr(A_2'A_2) \to 0$ as $N \to \infty$. Observe that for any non-negative matrix its trace is less of equal to the sum of its elements. Also using that $\Omega\mathbf{1} = \mathbf{1}$, so that $(I - \beta\Omega)^{-1}\mathbf{1} = \frac{1}{1-\beta}\mathbf{1}$, we have

$$tr\left(A_1'A_1\right) \leq \mathbf{1}'\left(I-\beta\Omega'\right)^{-1} D_b\left(I-\beta\Omega\right)^{-1}\left(I-\beta\Omega'\right)^{-1} D_b\left(I-\beta\Omega\right)^{-1}\mathbf{1}$$

$$= \tfrac{1}{(1-\beta)^2} b'\left(I-\beta\Omega\right)^{-1}\left(I-\beta\Omega'\right)^{-1} b \leq const \cdot \widetilde{\alpha}'\left(I-\beta\Omega\right)^{-1}\left(I-\beta\Omega'\right)^{-1}\widetilde{\alpha}$$

$$= const \cdot \widetilde{b}'\widetilde{b} = const \cdot \left\|\widetilde{b}\right\|_2^2 \to 0.$$

The second line replaces b with $\widetilde{\alpha} \propto b$ ("alternative consumption shares" in part (ii) of Assumption 1 for which $\|\widetilde{\alpha}\|_2 \to 0$ because $\|b\|_2 \to 0$). The argument for $tr(A_2'A_2) \to 0$ is similar.

Now by Lemma 1 $\dot{p}' D_b \dot{p} \underset{p}{\to} tr(A_1)\sigma_z^2$ and $\dot{p}'\Omega' D_b \Omega \dot{p} \underset{p}{\to} tr(A_2)\sigma_z^2$ and therefore $\ddot{\overline{p}} \underset{p}{\to} (1-\sigma)\left[tr(A_1) - \beta tr(A_2)\right]\sigma_z^2$. The second-order approximation to (A.5) in the probability limit is

$$\overline{p}^{(2nd)} = \frac{1}{2}(1-\sigma)\left[tr(A_1) - \beta tr(A_2)\right]\sigma_z^2. \tag{A.21}$$

Expanding the matrices A_1 and A_2, these traces can be expressed with statistics HHI, SSI, SLI and the terms involving $\beta^{l+m} tr(\Omega^{l\prime} D_b \Omega^m)$. [to be completed]

A.4 Sales

From (2.20) with $C = 1$ and $C^* = 1$, the two sales vectors are determined by the system

$$\begin{aligned} y &= D_\lambda\left(\alpha + \beta\Omega' y\right) + D_{\mathbf{1}-\lambda^*}\left(\alpha + \beta\Omega' y^*\right) \\ y^* &= D_{\lambda^*}\left(\alpha + \beta\Omega' y^*\right) + D_{\mathbf{1}-\lambda}\left(\alpha + \beta\Omega' y\right) \end{aligned}. \tag{A.22}$$

Taking the sum gives $y + y^* = 2\alpha + \beta\Omega'(y + y^*)$, or $y + y^* = (I - \beta\Omega')^{-1} 2\alpha = 2b$, according to (2.6) which defines the autarky sales vector b. Substituting $y^* = 2b - y$ into the first equation in (A.22) yields

$$y = \left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1}\left[D_\alpha\left(\lambda + \lambda^* - \mathbf{1}\right) + 2D_b\left(\mathbf{1} - \lambda^*\right)\right], \tag{A.23}$$

which further implies that $y - y^* = \left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1} 2D_b\left(\lambda - \lambda^*\right)$. Summing it with $y + y^* = 2b$ results in

$$y = b + \left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1} D_b\left(\lambda - \lambda^*\right). \quad \text{(A.24)}$$

Value-added trade flows, such as defined in Johnson and Noguera (2012), are obtained from (2.20) by setting $C = 0$ or $C^* = 0$, solving for the corresponding sales y and y^* and then summing the involved value added which (by industry) is $(1-\beta)\,y$ and $(1-\beta)\,y^*$.

.

A.5 Proof of Proposition 2.1

Although the shortest way to prove this proposition is by applying the envelope theorem to the planner's problem, I provide a direct proof which has useful intermediate results and, more generally, demonstrates the mechanics of the model. From (2.27),

$$\overline{p} + \overline{p}^* = b'\left[z + z^* + \log\left(\lambda^{\frac{1}{\rho-1}}\right) + \log\left(\lambda^{*\frac{1}{\rho-1}}\right)\right]. \quad \text{(A.25)}$$

For some variable x_i and $\lambda_i = 1/\left(1 + e^{(1-\rho)x_i}\right)$, we have $d\lambda_i = (\rho - 1)\, e^{(1-\rho)x_i}/\left(1 + e^{(1-\rho)x_i}\right)^2 dx_i$, so

$$d\lambda_i = (\rho - 1)\,\lambda_i\,(1 - \lambda_i)\,dx_i, \quad \text{(A.26)}$$

which is used repeatedly in proofs. Then, differentiating (A.25),

$$d\left(\overline{p} + \overline{p}^*\right) = \sum\nolimits_i b_i\left[(1-\lambda_i)\left(d\tau + d\xi_i\right) + \left(1-\lambda_i^*\right)\left(d\tau - d\xi_i\right)\right]$$

$$= \sum\nolimits_i b_i\left[\left(2 - \lambda_i - \lambda_i^*\right)d\tau + \left(\lambda_i^* - \lambda_i\right)d\xi_i\right] = \tfrac{2d\tau}{1-\beta} - b'\left(\lambda + \lambda^*\right)d\tau - \left(\lambda - \lambda^*\right)' D_b d\xi. \quad \text{(A.27)}$$

From (2.18),

$$d\xi = \beta\Omega\left[\left(\lambda + \lambda^* - \mathbf{1}\right)\circ d\xi + \left(\lambda - \lambda^*\right)d\tau\right], \text{ or}$$

$$d\xi = \left(I - \beta\Omega D_{\lambda+\lambda^*-\mathbf{1}}\right)^{-1}\beta\Omega\left(\lambda - \lambda^*\right)d\tau. \quad \text{(A.28)}$$

Together (A.27) and (A.28) imply

$$\frac{d\left(\overline{p}+\overline{p}^*\right)}{d\tau}=\tfrac{2}{1-\beta}-b'\left(\lambda+\lambda^*\right)-\left(\lambda-\lambda^*\right)'D_b\left(I-\beta\Omega D_{\lambda+\lambda^*-\mathbf{1}}\right)^{-1}\beta\Omega\left(\lambda-\lambda^*\right). \quad \text{(A.29)}$$

Denote $B=\left(I-\beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1}$. For trade flows, we have

$$M+M^*=\tfrac{2}{1-\beta}-\alpha'\left(\lambda+\lambda^*\right)-\lambda'\beta\Omega'y-\lambda^{*\prime}\beta\Omega'y^*$$

$$=\tfrac{2}{1-\beta}-\alpha'\left(\lambda+\lambda^*\right)-\lambda'\beta\Omega'B\left[D_\alpha\left(\lambda+\lambda^*-\mathbf{1}\right)+2D_b\left(\mathbf{1}-\lambda^*\right)\right]-$$

$$-\lambda^{*\prime}\beta\Omega'^{-1}\left[D_\alpha\left(\lambda+\lambda^*-\mathbf{1}\right)+2D_b\left(\mathbf{1}-\lambda\right)\right]=\tfrac{2}{1-\beta}-b'\left(\lambda+\lambda^*\right)-\left(\lambda-\lambda^*\right)'\beta\Omega'BD_b\left(\lambda-\lambda^*\right)$$

$$=\frac{d\left(\overline{p}+\overline{p}^*\right)}{d\tau}.$$

A.6 The Local ACR Result

We have

$$\frac{d}{d\tau}\overline{p}=M=\frac{1}{1-\beta}-D=\frac{1-\overline{\lambda}}{1-\beta}.$$

Also,

$$\epsilon\left(\tau\right)=-\frac{d}{d\tau}\log\left[\left(1-\overline{\lambda}\right)/\overline{\lambda}\right]=\frac{1}{\overline{\lambda}\left(1-\overline{\lambda}\right)}\frac{d\overline{\lambda}}{d\tau}.$$

Therefore,

$$\frac{d\overline{p}}{d\log\left(\overline{\lambda}\right)}=\frac{\frac{1-\overline{\lambda}}{1-\beta}d\tau}{d\overline{\lambda}/\overline{\lambda}}=\frac{1}{\left(1-\beta\right)}\left(\frac{1}{\overline{\lambda}\left(1-\overline{\lambda}\right)}d\overline{\lambda}\right)^{-1}=\frac{1}{\left(1-\beta\right)\epsilon\left(\tau\right)}.$$

A.7 Local Analysis Near Free Trade

Under free trade, i.e. $\tau=0$, we have $\xi=\eta$: Home and Foreign industries face the same prices for intermediate inputs, so the difference in marginal costs only arises from the difference in cost shocks z_i and z_i^*. Another major simplification is that $\lambda_i+\lambda_i^*=1$ for all i. It is convenient to introduce the following transformation of η_i:

$$\varepsilon_i = (\lambda_i - \lambda_i^*)\,|_{\tau=0} = \frac{1-e^{(1-\rho)\eta_i}}{1+e^{(1-\rho)\eta_i}} = \tanh\left(\tfrac{\rho-1}{2}\eta_i\right). \tag{A.30}$$

Several properties of ε_i are useful:

(i) $\varepsilon_i \in [-1,1]$. $\varepsilon_i \in \{-1,1\}$ if $\rho = \infty$.

(ii) $\mathbf{E}[\varepsilon_i] = \mathbf{E}[\varepsilon_i^3] = 0$ (due to the symmetry).

(iii) $\mathbf{E}[\varepsilon_i \varepsilon_j] = 0$ $\forall i \neq j$.

Denote $\mu_2 = \mathbf{E}[\varepsilon_i^2]$ and $\mu_4 = \mathbf{E}[\varepsilon_i^4]$.

With this definition, domestic shares at $\tau = 0$ are

$$\lambda = \tfrac{1}{2}\left(\mathbf{1}+\varepsilon\right) \quad \text{and } \lambda^* = \tfrac{1}{2}\left(\mathbf{1}-\varepsilon\right), \tag{A.31}$$

and sales are

$$y = D_b\left(\mathbf{1}+\varepsilon\right) \text{ and } y^* = D_b\left(\mathbf{1}-\varepsilon\right). \tag{A.32}$$

First of all, these simplifications allow showing that labor market clearing holds asymptotically for $W = W^* = 1$ in the sense that $p\lim L_D = L$ and $p\lim L_D^* = L^*$ where L_D and L_D^* denote aggregate labor demand:

$$L_D = \frac{1}{W}\left(1-\beta\right)\mathbf{1}'y \text{ and } L_D^* = \frac{1}{W^*}\left(1-\beta\right)\mathbf{1}'y^*. \tag{A.33}$$

In fact, $L_D + L_D^* = L + L^* = 2$ holds as identity (simply from that fact that $y + y^* = 2b$), so it is sufficient to demonstrate that $p\lim\left(L_D - L_D^*\right) = 0$.

From Appendix A.4, $y - y^* = \left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1} 2D_b\left(\lambda - \lambda^*\right)$. Therefore,

$$L_D - L_D^* = 2\left(1-\beta\right)\mathbf{1}'\left(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega'\right)^{-1} D_b\left(\lambda - \lambda^*\right). \tag{A.34}$$

Using (A.31), it reduces to

$$L_D - L_D^* = 2\left(1-\beta\right) b'\varepsilon \underset{p}{\rightarrow} 0, \tag{A.35}$$

which is by Lemma 1A.

Free-Trade Home Bias

Expenditure on domestically produced goods:

$$D^F = \lambda'\alpha = \tfrac{1}{2} + \textstyle\sum_i \alpha_i \varepsilon_i \underset{p}{\rightarrow} \tfrac{1}{2},$$

which is again by Lemma 1A. (An intermediate step is to show that $\|b\|_2 \to 0$ implies $\|\alpha\|_2 \to 0$. This follows from $b = (I - \beta\Omega')^{-1}\alpha \geq \alpha$.)

For producers

$$D^I = \lambda'\beta\Omega' y = \tfrac{1}{2}(\mathbf{1}+\varepsilon)'\beta\Omega' D_b(\mathbf{1}+\varepsilon) \underset{p}{\to} \tfrac{1}{2}\tfrac{\beta}{1-\beta}(1 + SSI \cdot \mu_2), \tag{A.36}$$

which is obtained by expanding

$D^I = \frac{1}{2}(\mathbf{1}+\varepsilon)'\beta\Omega' D_b(\mathbf{1}+\varepsilon) = \frac{1}{2}[\mathbf{1}'\beta\Omega' D_b\mathbf{1} + \mathbf{1}'\beta\Omega' D_b\varepsilon + \varepsilon'\beta\Omega' D_b\mathbf{1} + \varepsilon'\beta\Omega' D_b\varepsilon]$

The first term is $\mathbf{1}'\beta\Omega' D_b\mathbf{1} = \frac{\beta}{1-\beta}$. The second and the third have probability limit zero by Lemma 1A. For the last term, we have $tr(\Omega' D_b D_b \Omega) \leq \mathbf{1}'\Omega' D_b D_b \Omega\mathbf{1} = \|b\|_2^2 \to 0$, then by Lemma 1 $\varepsilon'\beta\Omega' D_b\varepsilon \underset{p}{\to} tr(\beta\Omega' D_b)\mu_2 = \frac{\beta}{1-\beta} SSI \cdot \mu_2$.

Trade Elasticity

I consider separately the case $\rho < \infty$ and the case of perfect substitutes $\rho = \infty$.

Proposition A.1. *For $\rho < \infty$, at $\tau = 0$*

$$\frac{dD}{d\tau} \underset{p}{\to} \tfrac{\rho-1}{4}\tfrac{1}{1-\beta}\left[1 - \mu_2 + \beta^2\left(\mu_2(1-\mu_2)[HHI + 2SLI] - \left(\mu_4 - \mu_2^2\right)3SSI_2\right)\right]. \tag{A.37}$$

The proof is provided below at the end.

In the absence of self-sourcing, $SSI = 0$, the case of perfect substitutes can be characterized by taking the limit in (A.37). Denote $f()$ the pdf of η_i. Under a weak regularity condition on the distribution of cost shocks we have

$$\lim_{\rho\to\infty}(\rho-1)(1-\mu_2) = \lim_{\rho\to\infty}(\rho-1)\mu_2(1-\mu_2) = 4f(0), \text{ and}$$

$$\frac{dD}{d\tau} \underset{p}{\to} \tfrac{1}{1-\beta} f(0)\left[1 + \beta^2(HHI + 2SLI)\right]. \tag{A.38}$$

Once we know the levels and the derivatives of trade flows, we can calculate the trade elasticity. Note that $\log\left(\frac{1+\kappa_1 x + o(x)}{a - \kappa_1 x + o(x)}\right) = -\log(a) + \frac{a+1}{a}\kappa_1 x + o(x)$. Thus, for example, for $SSI = 0$ (no free-trade home bias) and $\rho = \infty$,

$$\log\left(\frac{M}{D}\right) \underset{p}{\to} -4f(0)\left[1 + \beta^2(HHI + 2SLI)\right]\tau + o(\tau). \tag{A.39}$$

If productivities $\exp(-z_i)$ and $\exp(-z_i^*)$ are independently Frechet distributed, $4f(0)$ equals to the dispersion parameter θ. The elasticity of trade in this case is

$$\epsilon = \theta\left[1+\beta^2\left(HHI+2SLI\right)\right].$$

Proof of Proposition A.1

Let $\dot{x}$ denote $\frac{dx}{d\tau}$ and $\ddot{x}$ denote $\frac{d^2x}{d\tau^2}$. World domestic spending is $D+D^*=\frac{2}{1-\beta}-M+M^*$. As shown in Appendix A.5,

$$\dot{\xi}=\left(I-\beta\Omega D_{\lambda+\lambda^*-\mathbf{1}}\right)^{-1}\beta\Omega\left(\lambda-\lambda^*\right)\text{ and }D+D^*=b'\left(\lambda+\lambda^*\right)+\dot{\xi}'D_b\left(\lambda-\lambda^*\right).$$

From the same Appendix, applying $\lambda=\frac{1}{2}\left(\mathbf{1}+\varepsilon\right)$ and $\lambda^*=\frac{1}{2}\left(\mathbf{1}-\varepsilon\right)$, we have $\dot{\lambda}=\frac{\rho-1}{4}D_{\mathbf{1}-\varepsilon^2}\left(\mathbf{1}+\beta\Omega\varepsilon\right)$ and $\dot{\lambda}^*=\frac{\rho-1}{4}D_{\mathbf{1}-\varepsilon^2}\left(\mathbf{1}-\beta\Omega\varepsilon\right)$. Therefore,

$$\dot{\lambda}+\dot{\lambda}^*=\frac{\rho-1}{2}\left(\mathbf{1}-\varepsilon^2\right)\text{ and }\dot{\lambda}-\dot{\lambda}^*=\frac{\rho-1}{2}D_{\mathbf{1}-\varepsilon^2}\beta\Omega\varepsilon.$$

$$\ddot{\xi}=\beta^2\left(\rho-1\right)\Omega D_{\mathbf{1}-\varepsilon^2}\Omega\varepsilon.$$

$$\dot{D}+\dot{D}^*=b'\left(\dot{\lambda}+\dot{\lambda}^*\right)+\ddot{\xi}'D_b\left(\lambda-\lambda^*\right)+\dot{\xi}'D_b\left(\dot{\lambda}-\dot{\lambda}^*\right)$$

$$=\frac{\rho-1}{2}b'\left(\mathbf{1}-\varepsilon^2\right)+\varepsilon'D_b\beta^2\left(\rho-1\right)\Omega D_{\mathbf{1}-\varepsilon^2}\Omega\varepsilon+\varepsilon'\beta\Omega'D_b\frac{\rho-1}{2}D_{\mathbf{1}-\varepsilon^2}\beta\Omega\varepsilon. \tag{A.40}$$

The first term in (A.40) is $\frac{\rho-1}{2}\sum_i b_i\left(1-\varepsilon_i^2\right)\underset{p}{\rightarrow}\frac{\rho-1}{2}\frac{1}{1-\beta}\left(1-\mu_2\right)$. Fot the second term, expand

$$\left(1-\beta\right)\varepsilon'D_b\Omega D_{\mathbf{1}-\varepsilon^2}\Omega\varepsilon=\left(1-\beta\right)\sum_i\left(1-\varepsilon_i^2\right)\left(\sum_j\omega_{ij}\varepsilon_j\right)\left(\sum_k\omega_{ki}\varepsilon_k b_k\right),$$

getting $SLI\cdot\mu_2\left(1-\mu_2\right)-SSI_2\left[\mu_4-\mu_2^2\right]$. For the third term in (A.40),

$$\left(1-\beta\right)\varepsilon'\Omega'D_bD_{\mathbf{1}-\varepsilon^2}\Omega\varepsilon=\left(1-\beta\right)\sum_i b_i\left(1-\varepsilon_i^2\right)\left[\sum_j\omega_{ij}\varepsilon_j\right]^2,$$

which gives $HHI\cdot\mu_2\left(1-\mu_2\right)-SSI_2\left[\mu_4-\mu_2^2\right]$.

Now we can compute derivatives of domestic expenditure evaluated at $\tau=0$.

$$\dot{D}+\dot{D}^*\underset{p}{\rightarrow}\frac{\rho-1}{2}\frac{1}{1-\beta}\left(1-\mu_2\right)+$$

$$+\frac{\beta^2(\rho-1)}{1-\beta}\left(SLI\cdot\mu_2\left(1-\mu_2\right)-SSI_2\left[\mu_4-\mu_2^2\right]\right)+$$

$$+\frac{\beta^2(\rho-1)}{2(1-\beta)}\left(HHI\cdot\mu_2\left(1-\mu_2\right)-SSI_2\left[\mu_4-\mu_2^2\right]\right),\text{ so}$$

$$\dot{D}+\dot{D}^*\underset{p}{\to}\frac{\rho-1}{2}\frac{1}{1-\beta}\left[1-\mu_2+\beta^2\left(\mu_2\left(1-\mu_2\right)\left[HHI+2SLI\right]-\left(\mu_4-\mu_2^2\right)3SSI_2\right)\right],$$

which completes the proof (LLN details to be provided). ■

A.8 Neutrality Result

Consider the world economy consisting of countries $k = 1..K$ and products (industries) $i = 1..N$. Consumers maximize CES preferences with the elasticity of substitution σ:

$$U_k=\left(\sum_{i=1}^{N}\alpha_{ik}^{\frac{1}{\sigma}}c_{ik}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},\tag{A.41}$$

where $\alpha_{ik}\geq 0$ are utility weights (potentially country-specific), $\sum_i \alpha_{ik}=1$ for each k.[55]

Technology is Cobb-Douglas over labor and industry-specific composite intermediate input:

$$q_{ik}=\frac{1}{Z_{ik}}\left(\frac{L_{ik}}{1-\beta}\right)^{1-\beta}\left(\frac{M_{ik}}{\beta}\right)^{\beta},\tag{A.42}$$

with cost shifters Z_{ik}, input shares $0\leq\beta<1$ (uniform across products and countries), and materials being CES bundles with industry-specific elasticities of substitution γ_i:

$$M_{ik}=\left(\sum_{i'=1}^{N}\omega_{ii'}^{\frac{1}{\gamma_i}}m_{ii'k}^{1-\frac{1}{\gamma_i}}\right)^{\frac{\gamma_i}{\gamma_i-1}},\tag{A.43}$$

in which the weights $\omega_{ii'}\geq 0$, $\sum_{i'}\omega_{ii'}=1$ for each i, are common across countries.

For each product i, both consumption and intermediate inputs are Armington CES bundles aggregating country-specific varieties with the elasticity of substitution $1<\rho<\infty$:

$$x_{ik}=\left(\sum_{k'=1}^{K}x_{ikk'}^{1-\frac{1}{\rho}}\right)^{\frac{\rho}{\rho-1}},\tag{A.44}$$

[55] This normalization allows considering a countable set of products ($N=\infty$), as the level of real consumption U_k remains bounded and positive. Similar applies to technology in (A.43).

where x_{ik} stands for either consumption of product i or its use as an intermediate input in country k and $x_{ikk'}$ denotes the involved amount of variety coming from country k'.

Trade is subject to iceberg costs $t_{kk'} \geq 1$ (for importer country k and exporter country k') which are uniform across all goods.

Consumers in country k inelastically supply L_k units of labor. Competitive equilibrium is defined in the usual way: a set of prices and allocations such that consumers maximize utility (subject to the standard budget constraint potentially allowing exogenous trade imbalances), producers maximize profit, and all markets clear.

Proof of Proposition 3.1

Our goal is to show that any competitive equilibrium, once it exists, has trade flows satisfying log-linear gravity. For some given wages $w_k > 0$, consider cost minimization by producers. Denote μ_{ik} the equilibrium marginal cost of producing good i in country k. That is the unit cost associated with (A.42)-(A.44) when producers optimally combine labor and materials from different industries and source countries. The key to the proof is showing that no relative variation in exogenous cost shifters implies no relative variation in (endogenously determined) marginal costs:

$$Z_{ik} = \widetilde{Z}_i \widehat{Z}_k \;\Rightarrow\; \mu_{ik} = \widetilde{\mu}_i \widehat{\mu}_k. \tag{A.45}$$

We will first establish that (conditional on wages) there can be only one positive solution to μ_{ik} and then will use $Z_{ik} = \widetilde{Z}_i \widehat{Z}_k$ to construct μ_{ik} in the form $\widetilde{\mu}_i \widehat{\mu}_k$.

Step 1. Uniqueness of μ_{ik}.

From the Armington aggregator (A.44), the price index for intermediate input i in country k is equal to

$$P_{ik} = \left(\sum\nolimits_{k'} \left(t_{kk'} \mu_{ik'} \right)^{1-\rho} \right)^{1/(1-\rho)}. \tag{A.46}$$

From (A.42) and (A.43), the marginal cost of producing good i in country k is

$$\mu_{ik} = Z_{ik} w_k^{1-\beta} \left(P_{ik}^M \right)^{\beta}, \tag{A.47}$$

where the price index for materials (i.e. composite intermediate input) is

$$P_{ik}^M = \left(\sum\nolimits_{i'} \omega_{ii'} P_{i'k}^{1-\gamma_i} \right)^{1/(1-\gamma_i)}. \tag{A.48}$$

Combining (A.46)-(A.48),

$$\mu_{ik} = Z_{ik} w_k^{1-\beta} \left[\sum\nolimits_{i'} \omega_{ii'} \left(\sum\nolimits_{k'} \left(t_{kk'} \mu_{i'k'} \right)^{1-\rho} \right)^{\frac{1-\gamma_i}{1-\rho}} \right]^{\beta/(1-\gamma_i)} . \tag{A.49}$$

For $\boldsymbol{\mu}$ the matrix of μ_{ik} define $f : \mathbb{R}^{NK}_{++} \to \mathbb{R}^{NK}$ such that $f_{ik}(\boldsymbol{\mu}) = g_{ik}(\boldsymbol{\mu}) - h_{ik}(\boldsymbol{\mu})$ where $g_{ik}(\boldsymbol{\mu}) = Z_{ik} w_k^{1-\beta} \left[\sum_{i'} \omega_{ii'} \left(\sum_{k'} \left(t_{kk'} \mu_{i'k'} \right)^{1-\rho} \right)^{\frac{1-\gamma_i}{1-\rho}} \right]^{\beta/(1-\gamma_i)}$ and $h_{ik}(\boldsymbol{\mu}) = \mu_{ik}$. Note that $h_{ik}(\boldsymbol{\mu})$ is homogenous of degree one and $g_{ik}(\boldsymbol{\mu})$ is homogenous of degree $\beta < 1$. We have that $f(\boldsymbol{\mu})$ satisfies gross substitution: $\frac{\partial f_{ik}(\boldsymbol{\mu})}{\partial \mu_{i'k'}} > 0$ for $i \neq i'$, $k \neq k'$. Then Theorem 2 from Allen, Arkolakis, and Li (2015)[56] implies that there is at most one solution to $f(\boldsymbol{\mu}) = 0$. Therefore, (A.49) has at most one positive solution.

Step 2. Constructing $\widetilde{\mu}_i$ *and* $\widehat{\mu}_k$.

Let's construct $\widetilde{\mu}_i$ and $\widehat{\mu}_k$ such that $\mu_{ik} = \widetilde{\mu}_i \widehat{\mu}_k$ indeed holds. The price index (A.46) is

$$P_{ik} = \left(\sum\nolimits_{k'} \left(t_{kk'} \mu_{ik'} \right)^{1-\rho} \right)^{1/(1-\rho)} = \left(\sum\nolimits_{k'} \left(t_{kk'} \widetilde{\mu}_i \widehat{\mu}_{k'} \right)^{1-\rho} \right)^{1/(1-\rho)}$$

$$= \widetilde{\mu}_i \underbrace{\left(\sum\nolimits_{k'} \left(t_{kk'} \widehat{\mu}_{k'} \right)^{1-\rho} \right)^{1/(1-\rho)}}_{\overline{\mu}_k \text{ (access to suppliers)}} . \tag{A.50}$$

Then the price index for materials (A.48) is

$$P^M_{ik} = \left(\sum\nolimits_{i'} \omega_{ii'} P_{i'k}^{1-\gamma_i} \right)^{1/(1-\gamma_i)} = \left(\sum\nolimits_{i'} \omega_{ii'} \left(\widetilde{\mu}_{i'} \overline{\mu}_k \right)^{1-\gamma_i} \right)^{1/(1-\gamma_i)}$$

$$= \overline{\mu}_k \underbrace{\left(\sum\nolimits_{i'} \omega_{ii'} \widetilde{\mu}_{i'}^{1-\gamma_i} \right)^{1/(1-\gamma_i)}}_{\breve{\mu}_i \text{ (material requirement)}} . \tag{A.51}$$

Now (A.49) can be written as

$$\mu_{ik} = Z_{ik} w_k^{1-\beta} \left(P^M_{ik} \right)^{\beta} = \widetilde{Z}_i \widehat{Z}_k w_k^{1-\beta} \left(\overline{\mu}_k \breve{\mu}_i \right)^{\beta} = \underbrace{\widetilde{Z}_i \breve{\mu}_i^{\beta}}_{\widetilde{\mu}_i} \cdot \underbrace{\widehat{Z}_k w_k^{1-\beta} \overline{\mu}_k^{\beta}}_{\widehat{\mu}_k} , \tag{A.52}$$

which means that industry and country "fixed effects" $\widetilde{\mu}_i$ and $\widehat{\mu}_k$ are given by

$$\widetilde{\mu}_i = \widetilde{Z}_i \breve{\mu}_i^{\beta} = \widetilde{Z}_i \left(\sum\nolimits_{i'} \omega_{ii'} \widetilde{\mu}_{i'}^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)} \text{ and} \tag{A.53}$$

[56] July 2015 draft.

$$\widehat{\mu}_k = \widehat{Z}_k w_k^{1-\beta} \overline{\mu}_k^{\beta} = \widehat{Z}_k w_k^{1-\beta} \left(\sum_{k'} \left(t_{kk'} \widehat{\mu}_{k'} \right)^{1-\rho} \right)^{\beta/(1-\rho)}. \tag{A.54}$$

Existence of solutions to (A.53)-(A.54) follows from Lemma 1 in Allen, Arkolakis, and Li (2015). Note that it is sufficient to prove existence for (A.53) because (A.54) can be viewed as its special case with $\gamma_i = \rho$. The next argument follows closely Allen, Arkolakis, and Li (2015) in their proof of Theorem 1 in Alvarez and Lucas (2007).

Define $f : \mathbb{R}_{++}^N \to \mathbb{R}^N$ such that $f_i(\widetilde{\mu}) = \widetilde{Z}_i \left(\sum_{i'} \omega_{ii'} \widetilde{\mu}_{i'}^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)} - \widetilde{\mu}_i$ and $F : \mathbb{R}_{++}^{N+1} \to \mathbb{R}^N$ such that $F_i\left(\widetilde{\mu}', \widetilde{\mu}_i\right) = \widetilde{Z}_i \left(\sum_{i'} \omega_{ii'} \left(\widetilde{\mu}_{i'}'\right)^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)} - \widetilde{\mu}_i$.[57] For this function F, we have the three conditions of Lemma 1 in Allen, Arkolakis, and Li (2015) satisfied:

(i) For all $\widetilde{\mu}' \in \mathbb{R}_{++}^N$, there exists $\widetilde{\mu}_i$ such that $F_i\left(\widetilde{\mu}', \widetilde{\mu}_i\right) = 0$. (Just set $\widetilde{\mu}_i = \widetilde{Z}_i \left(\sum_{i'} \omega_{ii'} \left(\widetilde{\mu}_{i'}'\right)^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)}$.)

(ii) $\frac{\partial F_i\left(\widetilde{\mu}', \widetilde{\mu}_i\right)}{\partial \widetilde{\mu}_i} \frac{\partial F_i\left(\widetilde{\mu}', \widetilde{\mu}_i\right)}{\partial \widetilde{\mu}_j'} < 0$ for all j. (Obvious since $\frac{\partial F_i\left(\widetilde{\mu}', \widetilde{\mu}_i\right)}{\partial \widetilde{\mu}_i} = -1$ and $\frac{\partial F_i\left(\widetilde{\mu}', \widetilde{\mu}_i\right)}{\partial \widetilde{\mu}_j'} > 0$.)

(iii) There exists $\widetilde{\mu}'$ such that for $\widetilde{\mu}_i$ defined in $F_i\left(s\widetilde{\mu}', \widetilde{\mu}_i\right) = 0$, $\widetilde{\mu}_i = o(s)$. (From $F_i\left(s\widetilde{\mu}', \widetilde{\mu}_i\right) = 0$, $\widetilde{\mu}_i \propto s^{\beta}$.)

Once (i)-(iii) hold, a solution to $f(\widetilde{\mu}) = 0$ exists, so (A.53)-(A.54) have positive solutions. Thus we have shown that, given wages and cost shifters $Z_{ik} = \widetilde{Z}_i \widehat{Z}_k$, there exist equilibrium marginal costs in the form $\mu_{ik} = \widetilde{\mu}_i \widehat{\mu}_k$. Combining this result with previously established uniqueness[58] of μ_{ik}, we conclude that the implication (A.45) indeed holds.

Step 3. Trade shares and gravity.

Denote X_{ik} the total expenditure of country k on product i and $X_{ikk'}$ its expenditure on product i coming from country k'. Using (A.50), we obtain a strong result that trade shares $\pi_{ikk'} = X_{ikk'}/X_{ik}$, which are common for producers and consumers, are not product-specific:

$$\pi_{ikk'} = \left(\frac{t_{kk'} \mu_{ik'}}{P_{ik}} \right)^{1-\rho} = \left(\frac{t_{kk'} \widetilde{\mu}_i \widehat{\mu}_{k'}}{\widetilde{\mu}_i \overline{\mu}_k} \right)^{1-\rho} = \left(\frac{t_{kk'} \widehat{\mu}_{k'}}{\overline{\mu}_k} \right)^{1-\rho}. \tag{A.55}$$

The proof is completed by aggregating $X_{ikk'}$ over i:

$$X_{kk'} = \sum_i X_{ikk'} = \sum_i \pi_{ikk'} X_{ik} = \left(\sum_i X_{ik} \right) \left(\frac{t_{kk'} \widehat{\mu}_{k'}}{\overline{\mu}_k} \right)^{1-\rho}. \tag{A.56}$$

Therefore, the importer fixed effect is $FM_k = \log\left(X_k \overline{\mu}_k^{\rho-1}\right)$, where $X_k = \sum_i X_{ik}$ is the

[57] A note on notation: $\widetilde{\mu}$ and $\widetilde{\mu}'$ denote two vectors. No matrix transpose appears in this proof.

[58] Step 1 is needed because uniqueness for (A.53)-(A.54) alone does not rule out the logical possibility of having marginal costs not in the form $\mu_{ik} = \widetilde{\mu}_i \widehat{\mu}_k$.

total expenditure of country k, and the exporter fixed effect is $FX_{k'} = \log\left(\widehat{\mu}_{k'}^{1-\rho}\right)$.

Step 4. Gains from trade.

Denote λ_k the home share of country k, which is π_{ikk} in (A.55). Combining $\widehat{\mu}_k = \widehat{Z}_k w_k^{1-\beta} \overline{\mu}_k^{\beta}$ and $\lambda_k = \left(\frac{t_{kk}\widehat{\mu}_k}{\overline{\mu}_k}\right)^{1-\rho}$, we can express the "access to suppliers" index $\overline{\mu}_k$ as

$$\overline{\mu}_k = w_k \left(t_{kk}\widehat{Z}_k \lambda_k^{\frac{1}{\rho-1}}\right)^{\frac{1}{1-\beta}}. \tag{A.57}$$

The price index for aggregate final consumption in country k is

$$\begin{aligned} P_k &= \left(\sum\nolimits_i \alpha_{ik} P_{ik}^{1-\sigma}\right)^{1/(1-\sigma)} = \left(\sum\nolimits_i \alpha_{ik} \left(\widetilde{\mu}_i \overline{\mu}_k\right)^{1-\sigma}\right)^{1/(1-\sigma)} \\ &= \overline{\mu}_k \underbrace{\left(\sum\nolimits_i \alpha_{ik} \widetilde{\mu}_i^{1-\sigma}\right)^{1/(1-\sigma)}}_{\Gamma_k} = w_k \lambda_k^{\frac{1}{(\rho-1)(1-\beta)}} \left(t_{kk}\widehat{Z}_k\right)^{\frac{1}{1-\beta}} \Gamma_k. \end{aligned} \tag{A.58}$$

From this expression, the real wage w_k/P_k is proportional to $\lambda_k^{\frac{1}{(1-\rho)(1-\beta)}}$ which summarizes the effect of trade (Γ_k is exogenous). Once trade is balanced, welfare in this setup is given by the real wage. Hence the relative loss in welfare from moving to autarky with $\lambda_k = 1$ (which is changing $t_{kk'}$, $k' \neq k$, from the current levels to infinity) is given by $\lambda_k^{\frac{1}{(\rho-1)(1-\beta)}}$.
■

A.9 Derivation of (4.3) and (4.5)

Totally differentiating $\xi = \delta\eta + \beta\Omega\left[\xi + \log\left(\frac{\lambda^*}{\lambda}\right)^{\frac{1}{\rho-1}}\right]$,

$$d\xi = \eta d\delta + \beta\Omega\left[d\xi + \tfrac{1}{\rho-1}\left(\frac{d\lambda^*}{\lambda^*} - \frac{d\lambda}{\lambda}\right)\right],$$

which after applying (A.26) becomes

$$\frac{d\xi}{d\delta} = \eta + \beta\Omega\left[\frac{d\xi}{d\delta} + (\mathbf{1} - \lambda^*) \circ \frac{-d\xi}{d\delta} - (\mathbf{1} - \lambda) \circ \frac{d\xi}{d\delta}\right]. \tag{A.59}$$

Evaluating (A.59) at $\delta = 0$ with $\lambda_i = \lambda_i^* = \lambda_0$ gives

$$\frac{d\xi}{d\delta} = \eta + \beta\left(2\lambda_0 - 1\right)\Omega\frac{d\xi}{d\delta}, \text{ or } \frac{d\xi}{d\delta} = \left(I - \beta\left(2\lambda_0 - 1\right)\Omega\right)^{-1}\eta.$$

Differentiating (A.59) again with respect to δ produces

$$\frac{d^2\xi}{d\delta^2} = \beta\Omega\left[\frac{d^2\xi}{d\delta^2}\circ(\lambda+\lambda^*-\mathbf{1})+(\rho-1)\left(\frac{d\xi}{d\delta}\right)^2\circ(\lambda\circ(1-\lambda)-\lambda^*\circ(1-\lambda^*))\right],$$

from which $\frac{d^2\xi_i}{d\delta^2}=0$ at $\delta=0$.

Derivation of (4.5) ■

$$\widetilde{var}(\xi) = (1-\beta)\textstyle\sum_i b_i\xi_i^2 - \left[(1-\beta)\sum_i b_i\xi_i\right]^2 = (1-\beta)\,\xi' D_b\xi - \left[(1-\beta)\,b'\xi\right]^2.$$

$$\frac{d\widetilde{var}(\xi)}{d\delta} = (1-\beta)\,2\left(\frac{d\xi}{d\delta}\right)' D_b\xi - 2(1-\beta)^2 b'\xi\cdot b'\frac{d\xi}{d\delta}, \qquad \text{(A.60)}$$

which evaluated at $\delta=0$ (so $\xi=0$) is 0. Differentiating (A.60) again wrt δ and evaluating at $\delta=0$: (using that $\frac{d^2\xi}{d\delta^2}=0$)

$$\frac{d^2\widetilde{var}(\xi)}{d\delta^2} = (1-\beta)\,2\left(\frac{d\xi}{d\delta}\right)' D_b\frac{d\xi}{d\delta} - 2(1-\beta)^2\left(b'\frac{d\xi}{d\delta}\right)^2. \qquad \text{(A.61)}$$

Denote $B=(I-\beta(2\lambda_0-1)\Omega)^{-1}$. The second term in (A.60) involves $b'B\eta=\eta'B'b$. Note first that $B'b\le(I-\beta\Omega')^{-1}b$ because $\beta(2\lambda_0-1)\le\beta$. Consider $\widetilde{\alpha}=(1-\beta)\,b$ for which $\|\widetilde{\alpha}\|_2\to 0$ by part (i) of Assumption 1. Part (ii) of Assumption 1 implies that $\left\|(I-\beta\Omega')^{-1}\widetilde{\alpha}\right\|_2\to 0$, so $\|B'b\|_2\to 0$. Therefore, Lemma 1A applies to $b'B\eta$ and $b'B\eta\underset{p}{\to}0$.

The first term in (A.60) is $(1-\beta)\,2\eta'B'D_bB\eta$. First, $tr\left[B'D_bB\right]\le tr\left[(I-\beta\Omega')^{-1}D_b(I-\beta\Omega)^{-1}\right]$ because $\beta(2\lambda_0-1)\le\beta$. As shown in Appendix A.3, for $A_1=(I-\beta\Omega')^{-1}D_b(I-\beta\Omega)^{-1}$, $tr(A_1'A_1)\to 0$ under Assumption 1, so Lemma 1 applies to $\eta'B'D_bB\eta$ and $\eta'B'D_bB\eta\underset{p}{\to}tr(B'D_bB)\,\sigma_\eta^2$. The probability limit of the second-order approximation to $\widetilde{var}(\xi)$ is therefore

$$\widetilde{var}(\xi)^{2nd} = (1-\beta)\,tr\left[B'D_bB\right]\sigma_\eta^2.$$

A.10 Derivation of (4.6)

Denote $B=\left(I-\widetilde{\beta}(\tau)\,\Omega\right)^{-1}$. $\frac{d}{d\tau}tr\left[B'D_bB\right]=tr\left[\frac{dB'}{d\tau}D_bB+B'D_b\frac{dB}{d\tau}\right]=2tr\left[B'D_b\frac{dB}{d\tau}\right]$. Using that $\frac{dA^{-1}}{d\tau}=-A^{-1}\frac{dA}{d\tau}A^{-1}$, $tr\left[B'D_b\frac{dB}{d\tau}\right]=\frac{d\widetilde{\beta}(\tau)}{d\tau}tr\left[B'D_bB\Omega B\right]\ge 0$, as $\widetilde{\beta}(\tau)=\beta(2\lambda_0-1)$ is increasing in τ and $B'D_bB\Omega B$ is non-negative.

,.,\

A.11 Trace Properties

Bounds. First, from the fact that, for any non-negative matrix A, $tr\,(A) \leq \mathbf{1}'A\mathbf{1}$ and from expanding $\left(I-\widetilde{\beta}\Omega\right)^{-1}$ the bounds on the trace in (4.5) are obtained:

$$1 \leq (1-\beta)\,tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right] \leq \frac{1}{\left(1-\widetilde{\beta}\right)^2}.$$

The upper bound is achieved for the pure diagonal I-O matrix $\Omega = I$, and the lower bound is achieved[59] in the limit as $N \to \infty$ when $\omega_{ij} = \frac{1}{N}$ for all i, j.

General approximation for this trace is obtaited by expanding $\left(I-\widetilde{\beta}\Omega\right)^{-1}$ as the sum $\sum_k \widetilde{\beta}^k \Omega^k$, so that

$$(1-\beta)\,tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right] = (1-\beta)\sum\nolimits_{l,m} \beta^{l+m} tr(\Omega^{l\prime} D_b \Omega^m).$$

Ignoring the terms containing $\widetilde{\beta}^3$ and higher powers, we get $1+2\widetilde{\beta}SSI+\widetilde{\beta}^2\,(HHI+2SLI)$.

Random networks. In simulations I find that when the I-O structure is generated as a random graph in which each industry has some number (random or deterministic) of randomly chosen suppliers, it appears that $tr(\Omega^{l\prime} D_b \Omega^m) \approx 0$ for $l \neq m$. This reflects the fact that production loops tend to be very weak in such randomly generated networks as the number of nodes grows large. More surprisingly, it appears that $tr(\Omega^{m\prime} D_b \Omega^m) \approx HHI^m$, provided that product-level input shares Herfindahls $HHI_i = \sum_j \omega_{ij}^2$ and b_i are uncorrelated. This is satisfied when suppliers for different industries are assigned independently. (Across industries $i = 1..N$, the ax ante probability of becoming a supplier for a given industry i' can vary, but it should be unrelated to HHI_i). Therefore, another trace approximation which works well for randomly generated networks in which loops tend to be negligible is

$$(1-\beta)\,tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right] \simeq 1+\widetilde{\beta}^2 HHI+.. = (1-\widetilde{\beta}^2 HHI)^{-1}. \quad \text{(A.62)}$$

[59] For $\Omega = \frac{1}{N}\mathbf{1}\mathbf{1}'$, $\left(I-\widetilde{\beta}\Omega\right)^{-1} = I + \frac{\widetilde{\beta}\frac{1}{N}}{1-\widetilde{\beta}}\mathbf{1}\mathbf{1}'$. The trace in (4.7) is then $\frac{1}{1-\beta}$ plus terms proportional to $\frac{1}{N}$.

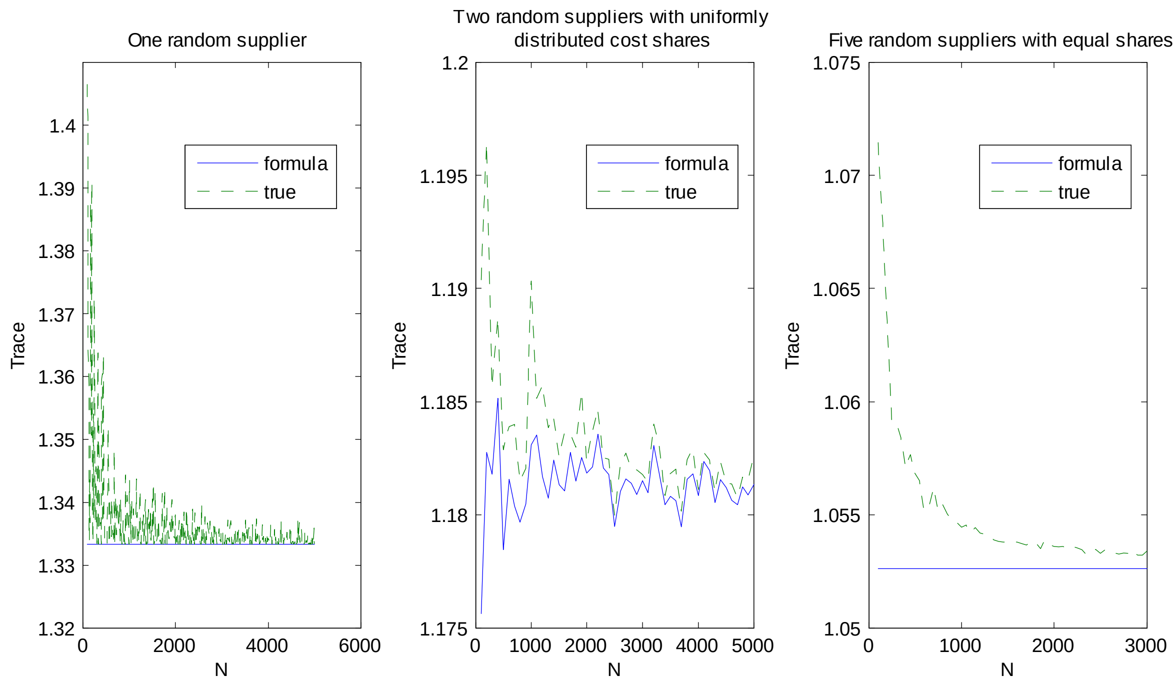

Figure 13: Approximation quality of formula (A.62).

Figure 13 illustrates that this approximation becomes near perfect for large N. For a fixed $\beta = 0.5$, it considers several I-O specifications which differ in the degree of supplier diversification but suppliers are chosen randomly and independently for each industry. The true value is $(1-\beta)\, tr\left[\left(I-\widetilde{\beta}\Omega'\right)^{-1} D_b \left(I-\widetilde{\beta}\Omega\right)^{-1}\right]$, while the formula is $(1-\widetilde{\beta}^2 HHI)$ $^{-1}$.

A.12 Proof of Proposition (4.1)

We have the gains from trade $g = \overline{p}^{Autarky} - \overline{p} = -b' \log\left(\lambda^{\frac{1}{\rho-1}}\right)$. At $\delta = 0$, $g = \frac{1}{1-\beta} \log\left(\lambda_0^{\frac{1}{1-\rho}}\right)$.

Let $\dot{x}$ denote $\frac{dx}{d\delta}$ and $\ddot{x}$ denote $\frac{d^2x}{d\delta^2}$ for generic variable x. Recall (2.18), so $\dot{\lambda}_i = (\rho-1)\lambda_i (1-\lambda_i)\dot{\xi}_i$ and

$$\dot{g} = -\sum_i b_i (1-\lambda_i)\dot{\xi}_i. \tag{A.63}$$

Evaluated at $\delta = 0$, it is $(1-\lambda_0)\, b'\dot{\xi} = (1-\lambda_0)\, b' \left(I-\widetilde{\beta}\Omega\right)^{-1}\eta$. As argued after (A.61),

$b'\left(I-\widetilde{\beta}\Omega\right)^{-1}\eta \underset{p}{\rightarrow} 0.$

Differentiating (A.63) again,

$$\ddot{g}=\sum_{i} b_i\left(\rho-1\right)\lambda_i\left(1-\lambda_i\right)\left(\dot{\xi}_i\right)^2-\sum_{i} b_i\left(1-\lambda_i\right)\ddot{\xi}_i. \tag{A.64}$$

Evaluating at $\delta=0$ (with $\ddot{\xi}_i=0$),

$$\ddot{g}=\left(\rho-1\right)\lambda_0\left(1-\lambda_0\right)\left(\dot{\xi}\right)' D_b\dot{\xi}.$$

As shown after (A.61), $\left(\dot{\xi}\right)' D_b\dot{\xi} \underset{p}{\rightarrow} tr\left(B'D_bB\right)\sigma_\eta^2$ with $B=\left(I-\widetilde{\beta}\Omega\right)^{-1}$. In other words, the probability limit of the second-order approximation to g is

$$g^{2nd}=\frac{1}{1-\beta}\left[\log\left(\lambda_0^{\frac{1}{1-\rho}}\right)+\frac{1}{2}\left(\rho-1\right)\lambda_0\left(1-\lambda_0\right)\widetilde{var}\left(\xi\right)^{2nd}\right].$$

A.13 Proof of Proposition (4.2)

Using that $\frac{d\lambda_0}{d\tau}=\left(\rho-1\right)\lambda_0\left(1-\lambda_0\right)$,

$$M^{2nd}=-\frac{d}{d\tau}g^{2nd}=$$

$$\frac{-1}{1-\beta}\left[-\left(1-\lambda_0\right)+\frac{1}{2}\left(\rho-1\right)^2\left(1-2\lambda_0\right)\lambda_0\left(1-\lambda_0\right)\widetilde{var}\left(\xi\right)^{2nd}+\frac{1}{2}\left(\rho-1\right)\lambda_0\left(1-\lambda_0\right)\frac{d\widetilde{var}\left(\xi\right)^{2nd}}{d\tau}\right].$$

Now consider $b'\left(\mathbf{1}-\lambda\right)$. Its first-order approximation is $-\left(\rho-1\right)\sum_i b_i\lambda_i\left(1-\lambda_i\right)\dot{\xi}_i$ and its second-order approximation (evaluated at $\delta=0$) is

$$-\left(\rho-1\right)^2\sum_{i} b_i\left(1-2\lambda_i\right)\lambda_i\left(1-\lambda_i\right)\left(\dot{\xi}_i\right)^2-\left(\rho-1\right)\sum_{i} b_i\lambda_i\left(1-\lambda_i\right)\ddot{\xi}_i=$$

$$\left(\rho-1\right)^2\left(2\lambda_0-1\right)\lambda_0\left(1-\lambda_0\right)\sum_{i} b_i\left(\dot{\xi}_i\right)^2.$$

Comparing to Appendix (4.1), the second-order approximation for $b'\left(\mathbf{1}-\lambda\right)$ is $\frac{1-\lambda_0}{1-\beta}$ +

$\frac{1}{1-\beta}\frac{1}{2}\left(\rho-1\right)^{2}\left(2\lambda_{0}-1\right)\lambda_{0}\left(1-\lambda_{0}\right)\widetilde{var}\left(\xi\right)^{2nd}$.

Comparing $M^{(1)}$ and $M^{(2)}$

Let $\widetilde{var}\left(\xi\right)^{2nd}$ be for a given I-O network that has non-zero HHI, while its value for the complete symmetric network is σ_z^2. We have $M^{(1)}=\frac{1}{1-\beta}\left[1-\lambda_{0}+\frac{1}{2}\left(\rho-1\right)^{2}\left(2\lambda_{0}-1\right)\lambda_{0}\left(1-\lambda_{0}\right)\widetilde{var}\left(\xi\right)^{2nd}\right.$ so

$$\Delta M^{(1)}=\frac{1}{1-\beta}\frac{1}{2}\left(\rho-1\right)^{2}\left(2\lambda_{0}-1\right)\lambda_{0}\left(1-\lambda_{0}\right)\left[\widetilde{var}\left(\xi\right)^{2nd}-\sigma_z^2\right].$$

Similarly, $\frac{d\widetilde{var}(\xi)^{2nd}}{d\tau}$ is for a given network, while for the complete network benchmark this value is zero:

$$\Delta M^{(2)}=\frac{1}{1-\beta}\frac{1}{2}\left(\rho-1\right)\lambda_{0}\left(1-\lambda_{0}\right)\frac{d\widetilde{var}\left(\xi\right)^{2nd}}{d\tau}.$$

Note that $\frac{d\widetilde{var}(\xi)^{2nd}}{d\tau}=\frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}\frac{d\lambda_0}{d\tau}=\left(\rho-1\right)\lambda_{0}\left(1-\lambda_{0}\right)\frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}$, so

$$\Delta M^{(2)}=\frac{1}{1-\beta}\frac{1}{2}\left(\rho-1\right)^{2}\lambda_{0}^{2}\left(1-\lambda_{0}\right)^{2}\frac{d\widetilde{var}\left(\xi\right)^{2nd}}{d\lambda_0}.$$

Now we can see that the covariance term $M^{(2)}$ is very weak for high trade costs, as it is proportional to $\left(1-\lambda_0\right)^2$, and is negligible relative to $M^{(1)}$:

$$\lim_{\tau\to\infty}\frac{\Delta M^{(2)}}{\Delta M^{(1)}}=\lim_{\tau\to\infty}\frac{-\lambda_{0}\left(1-\lambda_{0}\right)\frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}}{\left(2\lambda_{0}-1\right)\left[\widetilde{var}\left(\xi\right)^{2nd}-\sigma_z^2\right]}=0, \qquad \text{(A.65)}$$

because $\frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}$ remains bounded and $\lim_{\tau\to\infty}\left[\widetilde{var}\left(\xi\right)^{2nd}-\sigma_z^2\right]>0$.

Near free trade ($\lambda_0=\frac{1}{2}$), in contrast, the covariance contribution of $M^{(2)}$ dominates, because $\widetilde{var}\left(\xi\right)^{2nd}$ in $M^{(1)}$ departs slowly from its free-trade level σ_z^2. To see this, we can apply approximation (4.8), which is precise for small trade costs: $1+2\widetilde{\beta}SSI+\widetilde{\beta}^2\left(HHI+2SLI\right)$

$$\widetilde{var}\left(\xi\right)^{2nd}=\left[1+2\widetilde{\beta}SSI+\widetilde{\beta}^2\left(HHI+2SLI\right)\right]\sigma_z^2+o(\widetilde{\beta}^3),$$

where $\widetilde{\beta}=\beta\left(2\lambda_0-1\right)$.

We have in approximation $\Delta M^{(1)}=\frac{1}{1-\beta}\frac{1}{2}\left(\rho-1\right)^{2}\left(2\lambda_{0}-1\right)\lambda_{0}\left(1-\lambda_{0}\right)\left[2\widetilde{\beta}SSI+\widetilde{\beta}^2\left(HHI+2SLI\right)\right]\sigma_z^2$ or

$$\Delta M^{(1)} = \tfrac{1}{1-\beta}\frac{1}{2}\left(\rho-1\right)^2\beta\left(2\lambda_0-1\right)^2\lambda_0\left(1-\lambda_0\right)\left[2SSI+\widetilde{\beta}\left(HHI+2SLI\right)\right].$$

Also $\frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}$ is approximated as $\left[2SSI+2\widetilde{\beta}\left(HHI+2SLI\right)\right]\frac{d\widetilde{\beta}}{d\lambda_0}\sigma_z^2 = 4\beta\left[SSI+\widetilde{\beta}\left(HHI+2SLI\right)\right]\sigma_z^2$ and then

$$\Delta M^{(2)} = \tfrac{1}{1-\beta}2\beta\left(\rho-1\right)^2\lambda_0^2\left(1-\lambda_0\right)^2\left[SSI+\widetilde{\beta}\left(HHI+2SLI\right)\right]\sigma_z^2.$$

Consider two cases. If $SSI > 0$, then $\Delta M^{(2)}$ clearly dominates $\Delta M^{(1)}$ at $\tau \to 0$ because $\Delta M^{(2)}$ remains positive while $\Delta M^{(1)} \to 0$ as $\lambda_0 \to \frac{1}{2}$. If $SSI = 0$, then

$$\lim_{\tau\to 0}\frac{\Delta M^{(1)}}{\Delta M^{(2)}} = \lim_{\tau\to 0}\frac{\frac{1}{2}\left(2\lambda_0-1\right)}{2\beta\lambda_0\left(1-\lambda_0\right)} = 0.$$

A.14 Proof of Proposition

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■

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