Response to Saez and Zucman (2020)’s “Comments on Smith Zidar Zwick (2020)”

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October 7, 2021
## Contents

1 Executive Summary ........................................... 1

2 Our Revision Process ........................................ 5

I Reply to “Comments” ........................................ 7

3 Estimating Fixed Income Wealth .............................. 9
   3.1 Excerpts from “Comments” ............................... 10

4 Estimating Public Equity Wealth .............................. 27
   4.1 Relevant Excerpts from “Comments” ..................... 28

5 Comparing Capitalization Estimates to Other Data ....... 39
   5.1 Relevant Excerpts from “Comments” ..................... 40

6 Other Comments .............................................. 49
   6.1 Relevant Excerpts from “Comments” ..................... 50

7 Theoretical Analysis of Capitalization Bias ............... 61
   7.1 The Arguments of Saez and Zucman (2020, Appendix A) .............................. 61
      7.1.1 Proving a mathematical result .......................... 61
      7.1.2 Applying the mathematical result to justify SZ’s argument ............... 62
      7.1.3 Which interest rate to use? A Comparison of SZ and SZZ estimates ........... 63
      7.1.4 Four Issues with SZ’s theoretical argument ....................... 64
      7.1.5 Appendix: Proofs ........................................ 64
   7.2 Relaxing assumptions ...................................... 66
      7.2.1 Allowing for $\text{cov} (\beta_i, r_i) \neq 0$ ................................ 66
      7.2.2 Second-order approximation to the formula ................... 67
   7.3 Other useful expressions .................................. 68
      7.3.1 Formulae for quantifying the biases ...................... 68
      7.3.2 SZ’s Implicit assumptions on the distribution of interest income $(r_i \beta_i W_i)$ and wealth $(W_i)$ ........................................ 68

II Reply to “Revising the Revisionists” ....................... 71

8 Estimating Fixed Income Wealth .............................. 73
   8.1 Relevant Excerpts from “Revising the Revisionists” ........ 74
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Estimating Public Equity Wealth</td>
<td>83</td>
</tr>
<tr>
<td>10 Other Differences and Reconciliation</td>
<td>87</td>
</tr>
<tr>
<td>11 Comparing Capitalization Estimates to Other Data</td>
<td>91</td>
</tr>
<tr>
<td>9.1 Relevant Excerpts from “Revising the Revisionists”</td>
<td>84</td>
</tr>
<tr>
<td>10.1 Relevant Excerpts from “Revising the Revisionists”</td>
<td>88</td>
</tr>
<tr>
<td>11.1 Relevant Excerpts from “Revising the Revisionists”</td>
<td>92</td>
</tr>
</tbody>
</table>
Chapter 1

Executive Summary

We identified 112 distinct comments in the SZ (2020) report, not including the theoretical appendix, and an additional 33 comments in “Revising the Revisionists.” We reply to each comment and provide a detailed discussion of the theoretical analysis of capitalization bias in the body of this document.

Overall, SZ make four main points. We summarize our responses below.

1. **SZZ under-estimate the interest-bearing assets of the wealthy because they assume the interest rate earned by the rich is much higher than in the existing evidence.**
   
   • The revised draft provides substantial new evidence on interest rates of the rich from (a) the universe of taxable interest sources linked to owners using de-identified data from income tax records spanning 2001–2016, (b) estimates of risk exposure by group, (c) election filings and surveys of ultrahigh net worth individuals, and (d) interviews with wealth managers and leading practitioners.
   
   • The **new information return data** allow us to disaggregate taxable interest income into subcomponents. These new disaggregated data reveal that rich individuals earn a much larger share of their interest income in the tax data in higher-yielding forms (such as boutique investment partnerships of distressed debt or mezzanine funds). We also estimate rates more accurately since we can use source and recipient information when determining rates. For example, we assign interest rates on fixed income payments from partnerships by using firm-level tax returns of fixed income partnerships, which provide total interest payments and balance sheet information on fixed income assets. This assignment process also reveals differences within sources of fixed income by group, which we document and incorporate into our interest rate measure. Overall, the combination of disaggregated flow data and more accurate assignment of rates for each flow type allows us to contribute meaningful new evidence on the degree of heterogeneity in returns.
   
   • We develop **new methods** that use the covariance structure of interest rates, assets, and returns. These new methods allow us to estimate return heterogeneity, cross-validate the information return approach, and conduct inference about the key issue—the degree of heterogeneity. We find that the top wealth group has much stronger exposure to credit risk, which results in around a 3 times higher rate of return on fixed income at the top relative to average returns in recent years. We can also conduct inference and can reject the SZ baseline approach that these rates are equal and the SZ (2020) updated ratio of 1.4.
   
   • Past work has likely underestimated rates of return at the top because the interest rate is measured with a denominator that includes too many assets (specifically, fixed income and
money market mutual funds). These assets pay non-qualified dividends, not interest, so should not be estimated by capitalizing interest flows. Removing these non-taxable-interest-generating assets from the denominator increases the rate of return in 2016 in the SCF for the top 0.1% wealth group from 2.3% (s.e. = 0.4%) to 3.9% (s.e. = 1.0%). Because we are measuring return heterogeneity with population data, our estimates are substantially more precise than those derived from either the SCF due to sampling error or the estate tax due to volatility from mortality rates and small sample sizes. In the SCF and estate tax data, it is also not possible to isolate the boutique funds that we find are key for generating the bulk of interest income for those at the very top. Consequently, disaggregating and separately capitalizing these flows is not possible in these other data sets.

• We confirmed these concerns with interviews with practitioners and data experts.

2. The SZZ methodology, which almost fully ignores capital gains, is not appropriate to capture billionaire wealth and delivers estimates equal to only 57% of Forbes.

• The revised version augment the data with Forbes, so includes Forbes 400 by construction. Due to their relative size—Forbes individuals collectively account for 2.8% of total household wealth in 2016—and overlap with our estimates—owners of private businesses or dividend-paying public companies account for 77% of collective Forbes wealth in 2016—we find that incorporating the Forbes data has only a modest effect on our overall top share estimates.

• We estimate the weight placed on capital gains by minimizing the distance between top equity wealth shares in the data and in the equity wealth model. We also provide complementary evidence using a regression approach, which shows that dividends are much more informative than capital gains for predicting equity wealth. We use both approaches to estimate heterogeneity by group and find no evidence that the ultra wealthy have much lower dividend rates (i.e., the best fitting weight on dividends in both approaches exceeds .9 for groups going all the way into the tail).

• The original SZ method also cannot fit equity wealth of Forbes individuals. It only matches aggregate Forbes wealth by assigning $1.9T of fixed income wealth to the top 0.001%, which accounts for 55% of their portfolios. Our approach finds 48% in C-corporations, 30% in pass-throughs, and 14% in fixed income, in line with external data on ultrarich portfolios.

3. The SZZ methodology fails to capture the level of top-end wealth recorded in the official Survey of Consumer Finances and its 2001–2016 rise.

• The SZZ series matches top 0.01% and 0.1% harmonized SCF shares in level, trend, and are much closer in portfolio composition than the SZ series (Figures 1 and 15).

• We show that adjusting for private business aggregate values closes the remaining gap for the top 1% and brings portfolio composition into alignment. We provide evidence that self-reported business valuations in the SCF are likely overstated relative to market prices.

• The trend in the SCF is moderated once the 2019 data are considered, suggesting the 2016 survey presented a local maximum. Confidence intervals from the SCF show the gap between our estimates and the SCF in the 2001–2016 trend is smaller than asserted, especially once the data sets are harmonized in terms of unit and wealth component definitions.

4. SZZ do not properly discuss previous work that asked the same question, used the same data, applied similar key assumptions, and obtained similar results.
• We expanded discussion to ensure proper credit is given to previous work, to clarify how our methods and assumptions differ from previous work, and to show the extent to which the results differ from previous work.

• We provide new data and methods to complement and enhance previous work measuring and modeling heterogeneous returns in fixed income, public equity, and private business.
  
  (a) We use new tax data and novel risk-exposure methods to find that the interest rate on fixed income at the top is approximately 3.5 times higher than the average (with a confidence interval of 2.8-4.3). This degree of return heterogeneity is much higher than in prior work in the US. It also affects conclusions about wealth shares and composition, especially in the top tail, which is hard to study with precision given the small survey samples of 6,200 observations per year in the SCF and limitations of estate tax data.
  
  (b) Our bottom-up estimates of pass-through business wealth use firm-level tax data to value the full population of firms. Relative to the SCF, we find lower estimates of pass-through business wealth, especially for owners in the P99-P99.9 of the wealth distribution. Twenty percent of total pass-through business wealth accrues to those with losses. Our tax data approach is also valuable relative to Forbes, which uses lower quality inputs to estimate the wealth of private business owners. Of the 400 Forbes individuals, 241 are primarily private business owners. Their collective wealth represents around 40% of total Forbes wealth in recent years.
  
  (c) We combine our new tax data on fixed income and private business returns—which are the two biggest sources of difference between capitalized estimates and the SCF—with refined estimates of housing wealth, pension wealth, and C-corporation equity wealth to deliver comprehensive wealth share estimates based on tax data.

• We also highlight the new data that we previously integrated to model heterogeneity in housing, private business, pensions, and to add Social Security wealth.

We hope the efforts undertaken in this revision will help to pave the way for a new consensus view of the recent evolution of wealth shares that builds on important insights from SZ’s prior work.
Chapter 2

Our Revision Process

We sought to address the four main comments summarized above by probing the robustness of findings across multiple data sets and contexts, by collecting new data when needed, and by talking to economic actors to validate our approach. In our revision, we took the following steps:

1. **New population-level data on fixed income portfolio and interest rate heterogeneity.** We marshal new data on the universe of taxable interest sources linked to owners using de-identified data from the population of income tax returns spanning 2001–2016. These data cover all information returns that report taxable interest (Forms 1099-INT, 1065-K1 for partnerships, 1120S-K1 for S-corporations, 1041-K1 for trusts) and allow breakdowns of 1099-INT payments via financial institutions versus private loans versus savings bonds.

   The full sample comprises 3,166,087,481 source-owner-year observations (respectively, 2.8B, 120M, 110M, 27M, 21M, and 7.4M from banks, savings bonds, partnerships, S-corporations, estates, and loans). In 2016, the sample comprises 140,682,577 source-owner observations on 2,378,896 distinct sources and 64,716,434 distinct owners.

   To measure rates of return on fixed income assets, for each income component, we estimate a rate of return using tax data when possible and supplement these estimates with other data when necessary. For boutique sources of income, we construct new data that link the population of interest-paying partnerships (Form 1065) to their owners (via Form 1065-K1). For private loans, we link the SOI corporate sample (Form 1120 and 1120S) to the payees for their interest payments (via Form 1099-INT).

2. **New methods for estimating return heterogeneity.** We develop an alternative approach to measuring interest rate heterogeneity by estimating a risk-factor model of the covariance structure of interest rates, assets, and returns. These new methods allow us to estimate the degree of heterogeneity in the years when the information returns are not available, cross-validate the information return approach, and conduct inference about the degree of heterogeneity.

3. **Interviews with fixed income practitioners.** We conducted interviews with eight wealth management experts from multiple financial institutions, including specialists in family office portfolio management for the ultrahigh net worth individuals at one of the largest fixed income asset managers in the world.

4. **Portfolio insights from politician election filings and family office surveys.** We obtained from voluntary public disclosures the detailed tax returns with attachments for high wealth politicians.
We draw additional insights from the UBS-Camden survey of family offices representing the investment arms for several hundred ultrahigh net worth families.

5. **Historical Forbes data at the individual level going back to 1982 to augment capitalized series.** We gathered historical Forbes data and use hand-collected data on the source of Forbes business wealth in 2016 to compute Forbes-augmented top wealth and portfolio shares that account for overlap between capitalized estimates and Forbes.

6. **Comprehensive review of SCF literature, additional investigation using SCF data, and interviews of SCF experts.** To identify discrepancies between estimates and how they can be reconciled, we read or re-read 25 articles by SCF experts on total wealth and top wealth shares, in addition to all SCF bulletin articles since 1989. Based on this review, we conducted new analysis exploring the definition and sources of interest-income-generating assets and the valuation of private business in the SCF. We then conducted follow-up interviews with Jesse Bricker, Alice Henriques Volz, and Kevin Moore to discuss key areas of disagreement and ambiguities in the SCF data, as well as to clarify outstanding questions for key items in the Financial Accounts.

7. **Comprehensive review of SZ comments.** We carefully reviewed each comment in the SZ (2020) report, as well as subsequent iterations and additions in the SZ (2020) Revisionists article. We identified 47 comments pertaining to the estimation of fixed income wealth, 17 comments pertaining to the estimation of public equity wealth, 22 comments pertaining to the comparison between capitalized estimates and those in other sources (especially the SCF), and 26 additional comments. We reviewed and extended the theoretical arguments offered by SZ on the appropriate assumptions for capitalization. In Revisionists, we identified 33 additional comments, including those that overlap from the SZ (2020) report.
Part I

Reply to “Comments”
Chapter 3

Estimating Fixed Income Wealth
3.1 Excerpts from “Comments”

1. “SZZ under-estimate the interest-bearing assets of the wealthy because they assume the interest rate earned by the rich is much higher than in the existing evidence.” (p.1)

   • We provide new data and new methods to estimate the degree of return heterogeneity on fixed income. New disaggregated fixed income data from 3.2 billion information tax returns reveal that rich individuals earn a much larger share of their interest income in higher-yielding forms (such as boutique investment partnerships of distressed debt or mezzanine funds). Consistent with this evidence, new estimates using the covariance structure of interest income, assets, and returns indicate that the top wealth group has much stronger exposure to credit risk, which results in an approximately three times higher rate of return on fixed income at the top relative to average returns in recent years.

   • We describe why our approach outperforms prior alternatives for measuring interest rate heterogeneity, including the SCF and estate tax returns.

      – Past work has likely underestimated rates of return at the top because the interest rate is measured with a denominator that includes too many assets (specifically, fixed income and money market mutual funds). These assets pay non-qualified dividends, not interest, so should not be estimated by capitalizing interest flows. Removing these non-taxable-interest-generating assets from the denominator increases the rate of return in 2016 in the SCF for the top 0.1% wealth group from 2.3% (s.e.=0.4%) to 3.9% (s.e.=1.0%).

      – Because we are measuring return heterogeneity with population data, our estimates are substantially more precise than those derived from either the SCF due to sampling error or the estate tax due to volatility from mortality rates and small sample sizes.

      – In the SCF and estate tax data, it is not possible to isolate the boutique funds that we find are key for generating the bulk of interest income for those at the very top. Consequently, disaggregating and separately capitalizing these flows is not possible in these other data sets. In contrast, our data permits us to characterize and incorporate heterogeneity across fixed income sources and further into the top tail.

   • Our novel risk-exposure approach permits us to generate standard errors for characterizing uncertainty in rates of return and capitalized wealth estimates. We formally test and reject the equal returns rate and the Revisionists ratio of returns of 1.4 (Figure 5A).

2. “A similar assumption was implemented and investigated in supplementary series constructed in Saez and Zucman (2016, Appendix Tables B40, B41, B41b, and B41c, discussed pp. 549–551) and in Bricker et al. (2018).” (p.1)

   We discuss the relation between our approach and this prior work in detail. Ultimately, prior approaches do not account for the degree of heterogeneity we find in the data.

   We mention SZ’s appendix analysis in the introduction:

   To their credit, SZ do consider robustness analysis that assigns top groups modestly higher interest rates, which bring capitalization estimates down, although they use the equal-return approach for their headline results and subsequently in PSZ.

   We discuss in detail in Section 3, Section 9, and Appendix I how our approach compares to past work. For example:
The headline results in SZ and PSZ assume no heterogeneity in fixed income returns. However, as we noted in the introduction, SZ 2016 do include some robustness series that assume modestly higher rates at the top. For example, they present a two-tier model that assigns some a capitalization factor that is based on the US 10-year treasury rate:

\[ \beta_{t}^{fix,UST} = \begin{cases} \beta_{t}^{fix,UST,bot} & \text{if original wealth rank } \geq 99 \\ \beta_{t}^{fix,UST,top} = \frac{1}{r_{t}^{fix,UST}} \sum_{i \in \text{top}} \hat{a}_{i t}^{fix} & \text{otherwise} \end{cases} \tag{3.1} \]

where \( y_{it}^{fix} \) is taxable interest income, \( a_{t}^{total,fix} \) is total household fixed income assets from the Financial Accounts, \( \hat{a}_{it}^{fix} \) is the fixed income wealth estimate, and original wealth is \( \hat{a}_{it}^{fix} + \sum_{k} \hat{a}_{kt}^{fix} \) where \( k \) are the other types of wealth. Note that the baseline equal-return fixed income wealth estimate \( \hat{a}_{it}^{fix} \) is used to determine the wealth rank. While the UST10 approach improves model fit relative to the equal-returns approach (Figure 5D), it underperforms our estimates by overstating estimated wealth, especially for the top 0.1% and top 0.01%.

SZ 2016 also present a robustness series that uses a top rate from estate tax data. This series follows the same approach but replaces \( r_{t}^{fix,UST} \) with \( r_{t}^{fix,estate} \) for the top group, although this rate isn't weighted and has several other limitations. The estate tax rate estimate has a denominator that includes too many assets—specifically, fixed income and money market mutual funds—which are more prevalent at the top, which biases the rate down. There is also considerable uncertainty due to small samples in the estate tax data. Moreover, in the SCF data and estate tax data, it is not possible to isolate the boutique funds that we show generate the bulk of interest income for those at the very top in recent years. Consequently, disaggregating and separately capitalizing these flows is not possible in these other data sets.

We also detail differences in fixed income formulae in Appendix M to make it easier for readers to compare approaches.

As for Bricker et al. (2018) (BHH), we discuss this important work more prominently in the abstract, introduction, data, comparison section, and Appendix:

- In the introduction, we note that “Building on the capitalization approach in SZ and Piketty,

\[ \hat{a}_{it}^{fix} = \beta_{t}^{fix} \times y_{it}^{fix}, \text{ where } \beta_{t}^{fix} = \frac{1}{\bar{r}_{t}^{fix}} = \frac{a_{t}^{total,fix}}{\sum y_{it}^{fix}} \text{ is the capitalization factor for all, } \bar{r}_{t}^{fix} = \frac{\sum y_{it}^{fix}}{\sum a_{it}^{fix}} \text{ is the equal-return interest rate, } y_{it}^{fix} \text{ is taxable interest income, and } a_{t}^{total,fix} \text{ is total household fixed income assets from the Financial Accounts.} \]

\[ \text{For the estate tax returns, we also apply inverse mortality rates, which is needed to estimate rates of return for the living. SZ advocate applying this approach “one should weight matched estate-income observation by the inverse of the mortality rate conditional on age, gender, and wealth. We leave this difficult task to future research.” (p. 549)} \]

\[ \text{Indeed, SZ 2016 cite this limitation as well: “We retain our baseline top 0.1% wealth share estimate because only a few hundred non-married individuals die with estates above $20 million each year. As a result, there is likely significant noise in the annual series, making it difficult to make a precise and systematic inference of the true interest premium at the top.” (p. 550)} \]
Saez and Zucman (2018) (PSZ) and on insights in Bricker, Henriques and Hansen (2018) (BHH), we combine new data that links people to the sources of capital income and the firms they own with new methods that estimate the degree of heterogeneity within asset classes when mapping income flows to wealth."

• In the introduction, we also note that “BHH (2018) show that adjusting for top-1% heterogeneity in interest rates narrows the gap between the SCF and the capitalization approach for most of the gap for the top 1% (e.g., BHH Figure 6) and about one third of the gap (e.g., Appendix Figure 14) for the top 0.1%.”

Key to capitalization is having the right measure of interest rates. BHH use estimates of interest rate heterogeneity from (a) estate tax returns in Saez and Zucman (thus, this is not an independent test of SZ’s appendix analysis); from (b) SCF interest and wealth data alone; and from (c) a matched data set that combines 6,200 SCF responses per year with administrative tax data from 2002-2016.

We show that all three sources suffer from weaknesses that our approach overcomes. As mentioned above, each suffers from using an imperfect denominator, a pooled measure in the numerator, and small samples, leaving considerable uncertainty about which interest rates should be used for capitalization and for whom. For instance, removing non-taxable-interest-generating assets from the denominator increases the rate of return in 2016 in the SCF for the top 0.1% wealth group from 2.3% (s.e.=0.4%) to 3.9% (s.e.=1.0%). We confirmed this specific point in conversation with Bricker, Henriques, and Moore.

Overall, given the sensitivity of wealth estimates to assumptions about the degree of return heterogeneity, we believe that providing new estimates of this key input (based on novel data and methods) advances our understanding of wealth inequality in America.

3. “Saez and Zucman (2016, Appendix Tables B40, B41, and B41b) construct series where the top 1% by total income earn the 10-year Treasury yield. Bricker et al. (2018, Figure 4) construct series where the top 1% by interest income earn the 10-year Treasury yield.” (p.1, footnote 2)

• Correct. We find the degree of heterogeneity when ranked either by wealth, non-interest wealth, or total income exceeds that implied by the 10-year Treasury yield approach for the richest groups within the top 1%. As we emphasize, a crucial difference is our ability to go further into the tail, where fixed income portfolios look quite different in terms of credit risk relative to those in the P99-99.9 group.

4. “SZZ do not provide new data or theory to support the case for switching to these alternative assumptions; their implicit justification for applying very high interest rates at the top is conceptually wrong; and their methodological changes lead to large inconsistencies with the other sources about top-end US wealth.” (p.2)

• We collected data from several new sources to improve the evidence used to support our approach. See comments #1, #3, and #4 in Chapter 2 for a description of these data.

• We developed a novel approach to estimate return heterogeneity. See comment #2 in Chapter 2 for a description.

• Both our new data and new methods provide substantial support for our approach to modeling return heterogeneity.
• Our prior justification was based on past work, Bricker, Henriques, Krimmel, and Sabelhaus (2016) (BHKS), and the capitalization model used by the SCF to develop its sampling frame. We no longer rely on this past work to justify our approach.

• As the new evidence shows, the Aaa corporate rate was a reasonable choice for those at the top of the interest income distribution. Nevertheless, we no longer use interest income ranks to estimate return heterogeneity or assign interest rates.

We address specific claims that our prior approach was “conceptually wrong” in Chapter 7. These claims require strong assumptions and may not apply in several empirically relevant cases.

• In terms of fixed income wealth, our estimates are consistent with other sources of top-end wealth. In contrast, the SZ and PSZ estimates show dramatic increases in fixed income concentration in recent years, such that the top 0.1%, 0.01%, and 0.001% fixed income portfolio shares reach 42%, 49%, and 55%, respectively. We are not aware of any evidence suggesting the fixed income shares of the ultrarich come close to these levels.

• We discuss inconsistencies between our data and the SCF in terms of private business valuations below.

5. “the high interest rate for the wealthy that SZZ assume is inconsistent with the existing evidence in the datasets where both income and wealth are visible—matched estates-income tax data and the Survey of Consumer Finances (SCF). These data show that the top 0.1% wealthiest Americans have an average interest rate much lower than implied by the SZZ methodology.” (p.2)

This comment is addressed above. To summarize:

• The measures using these data are not reliable sources for determining the right interest rate. Many of the assets in the denominator generate non-qualified dividends, not interest, so including them biases the interest rate downward.

• The SCF does not collect enough information to separate non-interest-generating assets definitively. However, careful separation of taxable-interest-generating assets suggests the top 0.1% interest rate are much higher than the average rate (Figure 6).

• At the same time, standard errors in the SCF make it hard to be sure, especially for the top 0.1% and above (Figure 6). For this reason, we collect new data and develop a methodology for estimating the interest rate.

• The same issues of imperfect denominator and sampling uncertainty affect the estate tax data (Appendix Figure A.7).

6. “in 2016, Americans who died with more than $20 million in wealth had an interest rate of 1.4%: for any $1 in interest in their income tax return, they had $70 in bonds and other interest-bearing assets in their estate.” (p.2)

• As noted above, this interest rate is biased downward due to the inclusion of non-interest-generating assets in the denominator. In addition, it is subject to considerable uncertainty due to mortality rates (see Appendix Figure A.7). See also the discussion from SZ: “only a few hundred non-married individuals die with estates above $20 million each year. As a result, there is likely significant noise in the annual series, making it difficult to make a precise and systematic inference of the true interest premium at the top. (p.550).”
7. “in the SZZ methodology Americans with more than $20 million in wealth are assigned an interest rate of 3.3% in 2016: for any $1 in interest in their income tax return, SZZ assign them $30 in interest-bearing assets. As a result, SZZ under-estimate the interest-bearing assets owned by the wealthy.” (p.2)

- This statement is false. In the prior version, many top households received a lower rate because many households with significant wealth did not have sufficient interest income to receive the high rate.
- In the current version, we use new data to estimate top interest rates. Figure 4C shows their interest rates and 5B shows the implied capitalization factors for these groups. In 2016, top 0.1% households ranked by wealth have an interest rate of 2.2% and the top 0.01% have an interest rate of 3.4%, which imply capitalization factors of 45 and 29, respectively.
- These interest rates are consistent with other sources on the expected fixed income returns for the wealthy. Using these capitalization factors in the SCF delivers predicted taxable fixed income wealth levels that fit actual levels better than alternative approaches (Figure 5D).

8. “SZZ respond to this objection by pointing out that although the interest rate they assume is much higher than the interest rate \( r \) of people at the top of the wealth distribution, it is similar to the interest rate \( \bar{r} \) of people at the top of the interest income distribution. So, which interest rate should conceptually be used to capitalize income when rates of returns are heterogeneous and correlated with wealth, \( r \) or \( \bar{r} \)? We formally demonstrate below that the answer is \( r \), and that using \( \bar{r} \), as SZZ do, generates large biases. The SZZ methodology under-estimates top wealth shares by construction.” (p.2)

- We no longer use the interest income distribution to assign interest rates, so this critique is no longer relevant.
- At the same time, the formal analysis in the SZ note depends on several strong assumptions that are not empirically supported and may not apply in several empirically relevant cases. See Chapter 7.
- We show that the interest rate for the wealthy that we estimate is consistent with evidence on interest rates and the composition of fixed income and total portfolios at the top.

9. “The SZZ methodology assumes that the wealthy earn a much higher interest rate than the rest of the population. This assumption is inconsistent with the evidence where one can observe both income and wealth, which shows that the wealthy in recent years have earned a slightly higher, but not much higher, interest rate than the rest of the population.” (p.4)

- This statement is based on an incorrect categorization of fixed income assets, which includes many assets that do not generate taxable interest in the denominator, biasing the estimated interest rates downward.
- We use the population of interest information returns for six types of interest-paying entities to show this fact arises because of meaningful differences in taxable interest portfolios across the income and wealth distribution. We provide further evidence from ultrahigh-net-worth portfolios and from publicly available tax returns for ultrarich politicians in Section 3.3.
- We provide further evidence from new methods to estimate risk exposure by wealth group.
10. “We formally demonstrate below that...[c]apitalizing income using the interest rate of top interest-income earners delivers downwards biased estimates of top wealth shares by construction.” (p.4)

• As noted above and in Chapter 7, this result depends on several strong assumptions that make it difficult to draw certain conclusions. In addition, this concern is no longer relevant because do not use interest income ranks to estimate or assign interest rates in capitalization.

11. “Saez and Zucman (2016) match estates tax returns filed over the period 1997–2012 with income tax returns the year before death (see Saez and Zucman p. 547–551, in particular Figure V.B). They find interest rates of 3.0% on average over 2001–2008 for estates above $10 million (close to the aggregate rate of 2.7% on average), but higher-than-average interest rates at the top over 2009–2011 (2.2% for estates above $20 million vs. 1.4% on aggregate, a difference of a factor of 1.6).” (p.4)

• These interest rates are derived from dividing taxable interest by too broad a base of assets, which includes many fixed income assets that pay owners in non-qualified dividends and not interest. The bias from this issue is of uncertain magnitude, but could easily cause the interest rate to be understated substantially.
• We discuss above and in Appendix L additional reasons to be cautious when drawing inferences from the estate tax data.
• Appendix Figure A.7 presents our attempt to correct the denominator in estate-tax-derived interest rates, while also bootstrapping confidence intervals that account for sampling uncertainty due to mortality. The interest rates for those with high net worth in our capitalized wealth estimates are well within the range of interest rates in these confidence intervals, which are quite wide.

12. “Bricker, Henriques & Hansen (2018, Table 1, col. 2) find similar patterns in the SCF: interest rates for the top one percent richest households of 3.0% on average for the 2001, 2004, and 2007 SCF (close to the average SCF interest rate of 2.8% over these 3 years), but higher-than-average interest rates at the top in 2010, 2013 and 2016 SCF (2.0% for the top one percent wealthiest households vs. 1.5% on average in the SCF, a difference of a factor of 1.3).” (p.4)

• As with SZ, BHH also use too broad a base to measure interest rates in the SCF. Conversations with this coauthor team and replication of their interest rate series confirm this issue. Removing these non-taxable-interest-generating assets from the denominator increases the rate of return in 2016 in the SCF for the top 0.1% wealth group from 2.3% (s.e. = 0.4%) to 3.9% (s.e. = 1.0%).
• Furthermore, BHH also do not focus on the top 0.1% or top 0.01% in their analysis. We find considerable heterogeneity within the top 1%, which is important to account for in capitalization.
• In the SCF, it is also possible to measure these interest rates with standard errors (Figure 6), which show that our interest rates are well within the range supported by the SCF data (2.0% to 5.8% in 2016 for the top 0.1%).

13. “SZZ successfully replicate these findings, update the series to 2016, and do not analyze any new data source on the interest rate of the wealthy in the US.” (p.4)
• We have endeavored to expand the frontier of knowledge on this important issue by bringing data from numerous sources as described above. Our data is the most comprehensive yet analyzed on this question and speaks directly to the proper capitalization of taxable interest. We focus on heterogeneity within this category for exactly the individual tax returns used to generate capitalized estimates, removing many degrees of freedom from attempts to draw inferences from the SCF, estate tax data, or other sources.

14. “Saez and Zucman (2016) constructed supplementary series showing how the level, trend, and composition of top wealth shares is affected when higher interest rates—the 10-year Treasury yield or the rate seen in matched estates-income tax data—are assumed at the top.” (p.4)

• Agreed. As noted above, we find it is necessary to account for more heterogeneity than that implied by using the 10-year Treasury yield or SZ’s estate tax series. We acknowledge this prior work in the draft.
• The data we bring to bear to estimate the appropriate interest rate improves on the data used in past work along several dimensions.

15. “Appendix Tables B40, B41, B41b in Saez and Zucman (2016) assign the 10-year Treasury yield to the top 1% by income; Appendix Table B41c assigns the interest premium seen in matched estates-income tax data to the top 0.1% by wealth; the results are discussed in Saez and Zucman (2016, pp. 549-551).” (p.4, footnote 6)

• We agree. We acknowledge these robustness tests were done in our introduction.

16. “Bricker et al. (2018) analyzed how the Saez and Zucman (2016) benchmark estimates are affected when assuming that the top 1% by wealth (or the top 1% by total income, or the top 1% by interest income) earn the 10-year Treasury yield.” (p.4)

• We agree. In the revised draft, we credit this important contribution and describe how we advance the frontier relative to the important work of BHH. We now discuss the important work of BHH more prominently in the abstract, introduction, data section, comparison section, and Appendix (e.g., Appendix L.1).

(a) We use new tax data and novel risk-exposure methods to find that the interest rate on fixed income at the top is approximately 3.5 times higher than the average (with a confidence interval of 2.8-4.3). This degree of return heterogeneity is much higher than in BHH and other prior work in the US. It also affects conclusions about wealth shares and composition, especially in the top tail, which BHH are not able to study with precision given the small survey samples of 6,200 observations per year.

(b) Our bottom-up estimates of pass-through business wealth use firm-level tax data to value the full population of firms. Relative to the SCF, which underlies BHH’s analysis, we find lower estimates of pass-through business wealth, especially for owners in the P99-P99.9 of the wealth distribution. We discuss these differences in Section 4.3, Section 9.2, and Appendix L.1. Our tax data approach is also valuable relative to Forbes, which uses lower quality inputs to estimate the wealth of private business owners. Of the 400 Forbes individuals, 241 are primarily private business owners. Their collective wealth represents around 40% of total Forbes wealth in recent years.
(c) We combine our new tax data on fixed income and private business returns—which are 
the two biggest sources of difference between capitalized estimates and the SCF—with 
refined estimates of housing wealth, pension wealth, and C-corporation equity wealth 
to deliver comprehensive wealth share estimates based on tax data.

17. “Saez and Zucman (2016, Appendix Tables B40, B41, and B41b) construct series where the 
top 1% by total income earn the 10-year Treasury yield, while SZZ rank by interest income. 
Bricker et al. (2018, Figure 4) rank by interest income as SZZ.” (p.4–5, footnote 7) 
The revised version no longer ranks by interest income when estimating or assigning interest rates.

- BHH do rank by interest income but do not consider heterogeneity within the top 1%, which 
  we show is material because much of the heterogeneity appears within the top 1%.
- As noted, BHH do not account for the too-broad definition of taxable-interest bearing assets.

18. “As shown by the figure below, over the 2001-2016 period, the Moody’s Aaa rate (plain red) 
was always much higher than the interest rate for the top 0.1% richest people observed 
both in matched estates-income tax data and in the SCF (dashed red), by a factor of 2.0 
on average. Similarly, the 10-year Treasury yield (plain blue) was always higher that the 
interest rate for the next 0.9% richest people observed both in matched estates-income tax 
data and in the SCF (dashed blue), by a factor of 1.5 on average. The top 0.1% wealthiest 
Americans in the SZZ methodology (purple line) have an interest rate very close to the 
Moody’s rate, much higher than the real-world top 0.1% wealthiest Americans.” (p.5) 

- The interest rates in the SCF and estate tax data are downward-biased for reasons mentioned 
  above.
- Additionally, SZ’s graph cited here displays an “SZZ rate” that is not from our data and does 
  not match what we actually found in the prior version. It is incorrectly labeled.
- Figure 4C shows our preferred interest rates over time for different groups and compares 
  them to capital market rates.

19. “SZZ top 0.1% by wealth” (p.5, Figure 1 legend) 

- We are not sure how this was computed but it is not from our paper or data, and should not 
  be labeled “SZZ.”

20. “They never formally demonstrate that capitalizing interest using \( \bar{r} \) [the interest rate of 
people at the top of the interest income distribution] leads to unbiased estimates, or even 
informally discuss why this might be the case, or what the biases with such a method might 
be.” (p.5–6) 

- In the prior draft, we presented evidence showing our approach improved fit, formally mea-
  sured as the MSE for particular groups, relative to alternatives.
- In the current draft, we overhauled this analysis. We also do not use \( \bar{r} \) in our capitalization 
  approach. Among other robustness checks, to demonstrate the improved fit of our approach, 
  we present analyses comparing actual wealth to predicted wealth under our specification, 
  the equal returns specification in SZ, and the updated specification in SZ2020 (Figure 5D).
21. “SZZ also defend their approach in their appendix J by arguing that it is “more practically useful” to apply heterogeneous returns to bins of interest income, since interest income is observable in the data and wealth is not. But this argument conflates two issues—what is the conceptually correct \( \bar{r} \) to apply, and how, practically, to implement the capitalization method—and is also incorrect. Conceptually, as we show below, the interest rate to apply is the interest rate of the wealthy. Practically, one can apply this rate to the wealthy by proceeding by iteration. Using \( \bar{r} \) is neither correct conceptually, nor more practically useful than alternatives.” (p.6, footnote 8)

- We no longer use \( \bar{r} \) in our capitalization model.
- The iterative approach proposed here also entails some assumptions. Because it uses a biased method to assign wealth ranks, this approach can generate empirical bias in predicted fixed income wealth.
- The revised version takes a couple approaches to improve on both methods. When initially assigning rates to individuals, we use the best available ranks for each approach. When determining rates using the minimum distance approach, we rank people by wealth rank excluding fixed income wealth or “non-fixed income wealth,” because it is not possible to rank by fixed income wealth before estimating it. Note that in the SCF, more than 90% of wealth at the top is in non-fixed-income assets (Figure 15). For the information return approach for interest income from partnerships, we rank by adjusted gross income (AGI) just for assignment of interest rates from the population because non-fixed-income wealth estimates are not available for these data. Figure 4B shows that interest rate estimates for non-fixed-income wealth and AGI ranks are similar. It also compares these results with the main ranking, which is by wealth. Rates of return that rank by AGI display moderately greater heterogeneity in absolute terms though the differences are similar in relative terms.

22. “using \( \bar{r} \) to capitalize interest income when interest rates are heterogeneous generates a first-order downward bias in top wealth shares.” (p.6)

- We no longer use \( \bar{r} \) so this statement no longer applies. As described in Chapter 7, this statement depends on strong assumptions that may not apply in empirically relevant cases.

23. “the fact that top-interest income earners have a high interest rate \( \bar{r} \) is unsurprising. It’s selection: people are in the top 0.1% of the interest income distribution precisely because they have high interest rate. It’s also a consequence of measurement errors. In the 2001 SCF, for example, the top 0.1% interest income earners have a 20% interest rate (SZZ Figure A.15 Panel B)—an implausibly high rate due to data inconsistencies, such as misclassification of non-interest income as interest or under-reporting of interest-bearing assets. The high interest rate observed at the top of the interest income distribution is a mechanical consequence of idiosyncratic returns, whether these returns are real or reflect measurement errors.” (p.6)

- The comment attributes high interest income to idiosyncratic classical measurement error. If true, we would not expect to see high interest income strongly associated with other wealth components, nor would we expect to see high interest income reflect measurable differences in underlying fixed income portfolios. However, we see both these patterns in our linked tax data. High interest income therefore typically reflects systematic underlying portfolio
heterogeneity that is correlated with wealth ranks. Failing to model this heterogeneity leads to wealth estimation bias when capitalizing interest income.

- Furthermore, the assertion that 20% is an implausible interest rate neglects a few factors. First, measurement error in numerator and denominator in the SCF that may be systematic. For example, if high interest incomes come from private loans or from boutique investments that are more likely to be held at the top and are hard to capture given the granularity of SCF questions. Second, the top 0.1% in the SCF is a relatively small group and most of these people do not have much fixed income wealth relative to the other components of their portfolios. Thus, there are large standard errors around the 20% number that should be remembered (see e.g., Figure 6). Third, it is possible that some high interest income individuals have accumulated long maturity bonds during the early 1980s, which were still paying very high interest rates to recipients in the late 1990s and early 2000s. Moreover, interest income also includes all accumulated interest paid when recipients redeem savings bonds, which would cause measured interest income to be affected by lumpy realizations.

24. “Zero first-order bias requires using \( r_c = r \), i.e., requires using the average interest rate of the wealthy, \( r \), to capitalize interest.” (p.7)

- This statement holds under particular assumptions. See Chapter 7.

25. “If as done by SZZ, one capitalizes interest using the average interest rate of top \( p \) interest earners, \( \bar{r} \), there will be a first order bias in the estimated top wealth shares. For example, with \( \beta = 0.25 \) and \( r/\bar{r} \) of 0.33, then \( sh_{\bar{r}} = 0.83 \cdot sh_{r} \). Instead of a true top 0.1% wealth share of 18%, the SZZ methodology of capitalizing interest using \( \bar{r} \) delivers a biased estimated top 0.1% wealth share of 15%.” (p.7)

- This statement holds under particular assumptions. See Chapter 7.

26. “In 2016, \( r/\bar{r} = 0.3 \) in matched estates-income data and 0.5 in the SCF for the top 0.1% (SZZ Figure A.15)” (p.7, footnote 9)

- See our replies above on why these data may be unreliable.

27. “The benchmark Saez and Zucman (2016) series use \( r_c = r_m \). SZZ use \( r_c = \bar{r} \) the Moody’s Aaa rate, a rate which is 4.6 times larger than the average interest rate \( r_m \) in 2016. In both matched estates-income tax data and the SCF, the average interest rate of the wealthy, \( r \), is close to \( r_m \) before the Great Recession, and averages around 1.4 times \( r_m \) over 2008–2016. This calls for using \( r_c = r = 1.4 \times r_m \) after the Great Recession, i.e., for increasing the interest rate of the wealthy by a factor of around 1.4 compared to the benchmark Saez and Zucman (2016) methodology. SZZ increase the interest rate of the wealthy by a factor of 4.6 in 2016.” (p.7)

- See our comments above on why this ratio is likely biased downward in the SCF and estates data.

- Figure 5A presents the point estimates and standard errors of a key ratio of the top 0.1% rate relative to the equal-returns rate. As you note, this ratio is the crux of the debate because it summarizes the degree of heterogeneity. We find that the ratio’s value is around 3.5 in recent years. This estimate indicates that the top 0.1% group enjoyed three and a half times the
rate of return on fixed-income assets as the average return in the economy. Moreover, we can reject the null hypothesis that the top group earned the equal-returns rate. The confidence interval for this key ratio of top-to-average returns ranges from 2.8 to 4.3 in 2016.

- Therefore, prior approaches that assign the top group the average rate of return will overstate top returns by 180% to 360% in 2016, thereby substantially overstating top fixed income assets. Our estimates also reject the SZ2020 approach, which assumes the top group’s return exceeds the equal-returns rate by a factor of 1.4.

28. “For the top 0.1% in the SCF, \( r/r_m \) is slightly higher (1.57). Top-end interest rates are noisy due to small sample sizes, and interest rates in the SCF are upward biased, probably more so at the very top than on average (biasing \( r/r_m \) upwards)” (p.7, footnote 10, repeated on p.16, footnote 25)

- We can measure this noise using SCF sampling weights and standard inference procedures. As noted above, our interest rates are well within the range supported by the SCF data (2% to 6% in 2016 for the top 0.1%).
- Note also the issue of interest-generating asset classification remains a source of bias in using the SCF to discipline the choice of capitalization factor. We show that the approach described in this comment understates top 0.1% interest rates by 70 percent. Figure 6B shows top rate to aggregate ratios after fixing the issue with non-interest-generating assets in the denominator.
- We use population data to obtain much more precise estimates of interest rate heterogeneity.


- While moving in the right direction, we believe this ratio likely still biases several aspects of wealth distribution estimates. First, the ratio is likely too low for top wealth holders. Figure 5A shows it falls outside the 95% confidence interval from our estimates. Second, setting a fixed ratio over the 2008–2016 period will induce a mechanical trend in the estimated wealth series, if the true ratio is increasing over this time period, as we find.

30. “Capitalizing interest using \( r \) instead of \( \bar{r} \) increases the SZZ top 0.1% wealth share by 2.9 points in 2016, from 14.3% to 17.2%.” (p.8)

- Figure 5C compares our preferred estimates to those from PSZ with 2018 aggregate definitions, with updated definitions, and to alternative capitalization approaches. The gap between our preferred estimate and the PSZ estimate is 4.1 percentage points.

31. “In 2016, the average interest rate of the wealthy in matched estates-income tax data \( r \) is 1.3%, the average rate used by SZZ to capitalize the interest of the top 0.1% is the Moody’s rate \( r_c = 3.7\% \), so that with \( \beta = 0.25 \) there is a bias of 1 - \( \beta (1 - r/r_c) = 0.83 \). Starting from the SZZ top 0.1% wealth share of 14.3%, getting rid of the bias by setting \( r_c = r \) increases the estimated top 0.1% wealth share to 14.3%/0.83 = 17.2%. Note that the bias generated when using the interest rate of the top interest earners is larger than the bias of opposite sign generated when using the homogeneous macro interest rate \( r_m \). Over 2008–2016, \( r/r_m \) has averaged around 1.4, so that with \( \beta = 0.25 \), homogeneous capitalization leads to overestimating top wealth shares by 1 - \( \beta (1 - r/r_m) = 1.10 \).” (p.8, footnote 12)
• Again, this claim holds under particular assumptions discussed in Chapter 7. Nevertheless, the critique is no longer relevant because we do not use interest income ranks to assign interest rates.

32. “This single adjustment to the SZZ methodology closes 67% of the gap between SZZ and the benchmark estimates of Saez and Zucman (2016) (top 0.1% wealth share of 18.6% in 2016), while being consistent with the interest-rate differential observed between the wealthy and less wealthy.” (p.8)

• We have a new analysis that shows how our adjustments individually contribute to deviations from PSZ (Appendix Tables B.9 and B.10).

33. “Saez and Zucman (2016, p. 550) called for monitoring the evolution of the interest rate differential observed in the post-Great Recession years (as few post-Great Recession years were still available at the time of their study, which ended in 2012, making it hard to assess whether the differential observed in matched estates-income tax data post-2008 reflected statistical noise or a real phenomenon) and to adjust the capitalization method accordingly if need be.” (p.15)

• We are sympathetic to this point.

• Capitalization factors start to deviate from those implied by other capital market rates as early as 2001, when deposit rates fall below 1%.

• SZ highlight the “truly enormous increase in the concentration of interest income,” during which the “[t]op 0.01% interest income earners had 2.6% of all taxable interest in 1980; in 2012, they had 10 times more, that is, 27.3%. . . This is primarily due to the increase in the concentration of bonds...” (p.550–551)

Our information returns analysis suggests this owes to a few forces evolving over this time. First, the rise of money market mutual funds and fixed income mutual funds, which distribute income to owners in the form of dividends, not interest. Second, the decline of deposit rates to close to zero, which caused many individuals to stop receiving interest information returns from banks. This change in perceived participation in savings accounts does not appear in the SCF, which asks directly about participation in various financial instruments.

The interest rate for many households fell close to zero, while those at the top continued to derive interest primarily from non-bank deposit sources, including boutique investment funds with interest rates that were typically much higher than those earned by deposit holders and private loans with possibly even higher interest rates. We are aware of no datasets that show rapidly increasing shares of fixed income wealth at the top, nor rapid reallocation of wealth toward fixed income during the low interest rate period. The data instead point toward a much larger role for heterogeneous returns biasing fixed income wealth estimates in recent years, relative to an increase in the concentration of bonds.

34. “The matched estates-income data covering the years 2012–2016 have generally confirmed the presence of an interest rate premium at the top. Over 2008–2016, the interest rate of estates above $10 million has been equal to around 1.4–1.5 times the macro interest rate on average, see Figure 2 above. SCF data show similar patterns, with \( r/r_m \) of 1.36 on average for the top 1% over 2007–2016. These new data points call for a more moderate adjustment than the one proposed in the Saez and Zucman (2016) Appendix Table B41c series that used \( r/r_m = 1.6 \)” (p.15–16)
As noted above, estate tax data also likely understate the interest rate differential because of asset classification problems, and interest rate estimates are very sensitive to mortality rates and small sample sizes.

35. “See Saez and Zucman (2016, p. 550): “We retain our baseline top 0.1% wealth share estimate because only a few hundred non-married individuals die with estates above $20 million each year. As a result, there is likely significant noise in the annual series, making it difficult to make a precise and systematic inference of the true interest premium at the top. Looking forward, should new evidence show that taxable returns rise or fall with wealth, then it would become necessary to specifically account for this fact—and similarly when applying the capitalization technique to other countries.” (p.15–16, footnote 23)

We hope our new evidence enables future work to account for the observed return heterogeneity in capitalized wealth estimates and other data products that build on this work (e.g., distributional national income and tax progressivity estimates).

36. “For the 2012–2016 period, we use the interest rates reported by SZZ In their Figure A.15 Panel A. Young wealthy decedents contribute to these estimates with a high weight, since estate data are weighted by the inverse mortality rate by age and gender to be representative of the entire population—a valuable addition in SZZ compared to Saez and Zucman (2016) who did not weight by the inverse mortality rate; see Saez and Zucman (2016, pp. 549) and SZZ footnote 43. The weighted results happen to be similar to the unweighted results, showing that young wealthy decedents have low returns like elderly decedents.” (p.16, footnote 24)

Given the uncertainty in estate tax estimates from mortality rates, we are cautious to draw strong conclusions about relative heterogeneity in returns by age from this data.

37. “SZZ put too much faith in the top-end interest rates seen in the SCF, which are overstated by construction. Interest rates in the SCF are upward biased because what is recorded as interest-bearing assets in the SCF only includes a fraction of interest-generating assets. All the interest-generating assets of pass-through businesses (the bank deposits, notes receivable, bonds, etc., owned by S-corporations and partnerships) generate interest for their individual owners, because interest flows to their 1040, and respondents are asked about interest as reported in their 1040. But these assets are typically counted as business assets in the SCF—not as bank accounts, notes receivable, etc., owned by households.” (p.18)

Our information returns approach addresses this issue directly. Ultimately, the measurement issue in the SCF is a problem of underestimation due to the inclusion of assets that don’t generate interest income, such as fixed income mutual funds. The reason is that fixed income mutual funds are much larger in aggregate than fixed income assets held by non-financial pass-throughs.

Overall, interest income accounts for 3% of all income distributed to individuals from the pass-through sector. We are not aware of evidence suggesting the bank deposits of actively managed private businesses are large enough to counteract the downward bias in prior work from including non-interest-generating fixed income funds in the SCF interest rate denominator.
Based on conversations with SCF experts, the passively-held financial partnerships that generate the bulk of interest for top owners are more likely to be included in other items in the SCF, in particular, the other managed assets category, which includes both equity and debt investments delegated to investment professionals. Aggregate SCF assets in this category in 2016 are in line with our estimates for boutique assets held at the top and substantially smaller than aggregates implied by applying the equal-returns rate or the SZ2020 1.4X approach.

38. “In 2011, Cooper et al. (2016) report that about $22 billion was paid as taxable interest by partnerships to individuals (around 7% of all partnership income paid to individuals is interest (Figure 8), and about 35% of partnership income is paid to individuals (Figure 3), so out of a total of $895 billion in partnership income in 2011 (p. 122), 0.07 x 0.35 x $895 = $22 billion was paid as interest to households). In addition, according to IRS statistics, an extra $11 billion was distributed by S-corporations as interest, for a total of $33 billion in pass-through interest income, i.e., 28% of the total taxable interest income in 1040s ($120 billion in 2011). A negligible amount of pass-through interest income was tax-exempt (partnerships and S-corporations owned 80 billion in tax-exempt securities, generating about $3 billion in interest in 2011).” (p.18, footnote 31)

These figures are approximately in line with the aggregates we report in the paper based on information returns. As noted above, most of this interest income comes from financial entities that are passively held, so are not the same as the active private businesses that predominate the top in the SCF.

The cite to Cooper et al. (2016) is slightly incorrect, as the 35% number appears to be a typo (should be 31.5%).

39. “Note also that S-corporations plus partnerships owned about $3 trillion in interest-bearing assets (with some uncertainty as bonds cannot be separated from equities), and S-corporations had an average interest rate of about 1% on their interest-bearing assets, close to the macro aggregate rate.” (p.18, footnote 31, continued)

These numbers are too imprecise to be especially informative. We should note that comparing the average interest rate for holdings of S-corporations seems strange when trying to evaluate the portfolios of rich people. Aside from financial firms (a small share of S-corporation activity), S-corporations are mainly operating companies that presumably hold fixed income assets mainly for immediate liquidity. Their desires and risk tolerance for this asset class are quite different and likely result in lower interest rates than high net worth individuals.

In addition, it is not possible to separate interest-generating and non-interest-generating holdings from the balance sheet data for these firms, so the S-corporation interest rates are likely biased downward as well.

The same considerations apply when estimating interest rates for large public companies, like Apple, using SEC filings.

40. “Close to 30% of taxable interest income earned by households flows from pass-through businesses in recent years, meaning that that SCF interest rates are overstated by a factor of 1.4 (and potentially more at the top where pass-through income is prevalent). For instance,
instead of 1.67% in 2016, the top 1% interest is closer to 1.2%, consistent with the rate seen in matched estates-income tax data.” (p.18–19)

- This is a relatively small source of bias compared to the bias from including non-interest-generating fixed income assets in the denominator of the SCF interest rate.

41. “Note that to some extent the same bias exists in matched estates-income tax data (although less severe), meaning that the interest rate computed in these data should probably be seen as upper bounds.” (p.19)

- This is a relatively small source of bias compared to the bias from including non-interest-generating fixed income assets in the denominator of the SCF interest rate.

42. “The most valuable empirical finding in SZZ is Figure A.15 Panel A (updating Figure VB in Saez and Zucman, 2016), showing the interest rate of the wealthy, \( r \), in the only administrative data source where it can be seen—matched estates-income tax data. However, the graph is illegible, as SZZ increase the y-axis to 20%, despite the fact that the top value in the graph is 7.6% (see graph below). Presumably this is to make the axis consistent with the right panel (Figure A.15 Panel B) that shows the interest rate of top-interest earners \( \bar{r} \). Inflating the y-axis, however, obscures the large gap between \( r \) and the top-end rates assumed by SZZ. It also illustrates the key problem in the SZZ approach, namely that \( \bar{r} / r \) is very large—which means that using \( \bar{r} \) to capitalize interest generates severely downwards biased top wealth shares” (p.19)

- We apologize for the illegible graph, which was displayed to show how ranking by income versus ranking by wealth matters for estimating interest rates by group. Appendix Figure A.7 contains a legible version of estate tax interest rates for different groups. In this figure, we correct for the issue of estate tax interest rates estimated with the wrong assets in the denominator. We also plot standard errors that account for uncertainty in sampling and mortality rates.

43. “Following Kopczuk (2015) and Bricker et al. (2016), SZZ puzzle over the fact that the benchmark estimates of Saez and Zucman (2016), anchored to the Financial Accounts totals, have more interest-bearing assets at the top than the SCF. This misses a key difference in asset definition across these 2 sources. In the Financial Accounts, the interest-bearing assets of domestic hedge funds and private equity funds are recorded as interest-bearing assets of households, whereas they are typically business assets in the SCF (e.g., for the funds’ general partners).” (p.20)

- Given that most of the assets of hedge funds and private equity are held by pensions, endowments, other non-profits, and foreigners, this comment suggests that, if anything, the USFA based estimates may overstate these assets.

- See also our response to the next comment and to comment #37 for a discussion of where interest-bearing assets in the partnership sector likely appear in the SCF.

- Differences in asset definition across data sets cannot account for the dramatically rising portfolio concentration in fixed income at the top in the equal returns series and SZ’s subsequent iterations.
• Ultimately, we believe it is possible to use reasonable, empirically sound interest rates applied to capitalize respective interest income flows to generate aggregates that align well with the SCF.

44. “This turns out to matter quantitatively, because hedge fund and private equity fund managers play a prominent role at the top of the wealth distribution, and these funds hold large amounts of interest-bearing assets. In 2016, according to IRS tabulations of partnership tax returns, financial partnerships owned $729 billion in cash, $190 billion in trade notes and accounts receivable, $150 billion in US government obligations, $71 billion in mortgage and real estate loans, $39 billion in tax-exempt bonds, and $2 trillion in other current assets (listed equities, corporate and foreign bonds, etc.). Assuming one-third of other current assets were interest-bearing, financial partnerships had $1.8 trillion in interest-bearing assets, of which half with a close to zero interest rate. A considerable fraction of these assets belong to the top 0.1%.” (p.20)

• While we agree there are many such managers at the top, they are far from the majority. See, for example, SYZZ (2019). Alternatively, Kaplan and Rauh’s (2009) bottom-up estimate finds that just 5% of top 0.1% earners are in this industry.

• This comment also appears to misunderstand the nature of private equity partnerships. Fund managers do not “own” most of the assets of these funds. In fact, they only own a right to upside profits (distributed as capital gains) and a right to management fees. Most of the assets are owned by limited partners, who primarily comprise tax-exempt institutions. This can be seen in Cooper et al, as relatively little income going to individuals from partnerships takes the form of interest or dividends. To the extent fund managers derive their wealth from owning these firms, it is mostly not due to the fixed income assets these firms hold.

45. “Non-profits and foreigners typically invest in offshore hedge funds, which are excluded from these statistics and from the Financial Accounts household balance sheet aggregates. Wealthy US individuals typically invest in onshore funds, which are captured by these statistics.” (p.20, footnote 33)

• This assertion appears unsupported by the data. Cooper et al show a significant share of income among recognizable entity types accrues to tax-exempts and foreigners. Excluding ordinary income, which mostly accrues to individuals, the tax-exempt and foreign share of income is even larger.

46. “An approach based on the Financial Accounts must generate more interest-bearing assets at the top than in the SCF.” (p.20)

• This statement is too strong. It depends on how one uses the SCF.

• The logic does not account for the rapidly rising concentration of fixed income within top portfolios in the equal returns series, whereas failure to account for portfolio heterogeneity can account for both.

47. “It is apparent that what gets counted as interest-bearing assets in the SCF vs. the Financial Accounts is different when one compares the SCF vs. Financial Accounts totals. “Time deposits and short-term investments”, by far the largest form of interest-bearing assets and one of the easiest form of wealth to capture in a survey, are consistently twice larger in the
Financial Accounts than the conceptually similar category in the SCF (Batty et al. 2019, Table 1). The SCF coverage has deteriorated over time, from 61% on average over 1989–2001 to 47% over 2004–2016.” (p.20, footnote 34)

- This fact may well reflect the relative importance of non-individual-owned deposits in the Financial Accounts, as the hedge fund sector and non profit sectors grew relatively faster than the overall economy.

- In Section 9.2 and Appendix L.1, we discuss this fact as a source of potential reconciliation across our approach and the SCF.
Chapter 4

Estimating Public Equity Wealth
4.1 Relevant Excerpts from “Comments”

1. “[The] SZZ methodology under-estimates billionaire equity wealth, because SZZ infer equity wealth based on dividend income despite the fact that the wealthiest Americans often own equities that do not pay dividends.” (p.1)

• We respectfully disagree with the accuracy of this fact.
  – We classified each individual in the Forbes 400 list in 2016 to address this concern. Of the top 400 individuals in Forbes in 2016, 68 out of 159 public equity owners did not receive dividends, 91 out of 159 did receive dividends, and the other 241 individuals were private business owners.\(^1\)
  – We found that owners of private businesses or dividend-paying public companies account for 77% of collective Forbes wealth in 2016. Therefore, our capitalized wealth estimates, which use firm-level administrative data to value pass-through businesses (rather than less accurate self-reported information in Forbes for these firms) as well as a mix of dividends and capital gains for public equity owners, are likely substantially more informative about the wealth of the ultrarich than this statement suggests.

• That said, we do agree that Forbes concerns (regarding non-dividend-generating C-corporation equity) are important and have overhauled this part of the paper. We now augment our data to include the estimates from Forbes since 1982. Our baseline approach follows Bricker, Hansen, and Volz (2019)—we add the Forbes 400 members and adjust the sampling weights to account for overlap between capitalized estimates and the additional observations from Forbes. We also compare this baseline approach to an approach that replaces the top 400 estimates in the raw capitalized data with the Forbes estimates (see Figure 8D and Figure 11).
  – Due to their relative size—Forbes individuals collectively account for 2.8% of total household wealth in 2016—and overlap with our capitalized estimates—owners of private businesses or dividend-paying public companies account for 77% of collective Forbes wealth in 2016—we find that incorporating the Forbes data has only a modest effect on our overall top share estimates.
  – In 2016, we estimate that the top 0.001% tax units—those with more than $590M—collectively have $3.1T of wealth, which includes the impact of blending Forbes into our data. Without blending, this group would have $2.66T. Replacing the top 400 capitalized tax units with Forbes estimates gives $3.53T. We show the effects of these alternatives on top wealth shares in Figure 8D. Appendix L.3 provides additional discussion.

• The statement incorrectly asserts that we use dividend income alone. We place 90% weight on dividends and 10% weight on capital gains. Because aggregate capital gains are 230% of aggregate dividends on average (and 260% since 2001), the dollar weight on capital gains is approximately 20%. In contrast, SZ’s approach of equally weighting dividends and realized capital gains effectively places 70% weight on capital gains in dollar terms.

• To argue our approach is biased, SZ offer a few anecdotes but no systematic evidence. In contrast, we present a number of tests of bias using different weights on dividends and capital gains.

\(^1\)We find similar results within the top 50. Of the top 50 ranked individuals in Forbes, 13 out of 29 public equity owners did not receive dividends, 16 out of 29 did receive dividends, and the other 21 individuals were private business owners. To be clear, “receiving dividends” means that the company the individual owns (e.g., Microsoft) did pay dividends in 2016 according to publicly available data.
4.1. RELEVANT EXCERPTS FROM “COMMENTS”

gains and our approach is based on the wealth-weighted bias-minimizing solution.

• Capitalization with equal weight on dividends and capital gains also does not solve the problem of understating the wealth of the “wealthiest Americans [who] often own equities that do not pay dividends.” See our reply to #4 in this chapter for a detailed example.

• Our approach also considerably improves estimates of non-C-corporation equity wealth because we estimate pass-through equity wealth using firm-level information. According to the SCF in 2016, even among decamillionaires, the share of private business wealth in pass-through form is 87%, while the share in C-corporation form is 13%. Thus, in recent years, valuing private C-corporations appears relatively unimportant for valuing private business wealth at the top.

2. “SZ Z capitalize equity wealth using almost entirely dividends (90% weight on dividends, 10% weight on capital gains) instead of using a combination of dividends and realized capital gains as in Saez and Zucman (2016) benchmark series. Saez and Zucman (2016) had extensively analyzed this issue and reported supplementary series using only dividends (Appendix Tables B36, B37 and B37b discussed pp. 534–535).” (p.1)

• See our response to the previous comment on why it’s not correct to interpret our model as “using almost entirely dividends.”

• Our weight is based on a wealth-weighted bias-minimizing estimate. SZ’s mixed approach underperforms our approach in bias-minimization. The reason is that they remove the bias from capital gains for determining wealth groups, but retain the bias for estimating wealth for a particular person’s flows.

• SZ do not provide evidence supporting their approach versus alternatives. Instead, they argue that using dividends only doesn’t matter that much for top wealth shares: “[W]ether one disregards capital gains, fully capitalizes them, or adopts the mixed method does not affect the results much. The reason is that groups who receive lots of dividends also receive lots of capital gains, so that allocating the total amount of household equity wealth on the basis of how dividends alone or the sum of dividends and gains are distributed across groups makes little difference. . . Our baseline estimates are always close to those obtained by capitalizing dividends only. (p.535)”

• The extent of bias in SZ’s supplementary series is understated because wealth shares reflect concentration of other components of wealth, such as fixed income, which is quite important at the top in all of the auxiliary series they report. In addition, because the reported auxiliary series rerank individuals, they present a muddied picture of bias in the wealth model itself. We show how changing the model changes the estimates for a particular group in Tables B.9 and B.10 as well as Figure 8C.

3. “Saez and Zucman (2016) chose a ‘mixed method’ of using dividends only for ranking and dividends plus capital gains to estimate wealth shares based on a careful analysis of private foundation data (Appendix Figure C5 discussed in p. 542) where this ‘mixed method’ proved to be the best. It was also the method that came closest to match the Forbes 400. Hence the SZZ method of using (pretty much) only dividends is neither new nor an improvement.” (p.1–2)

• While the data from foundations is interesting, it has several limitations that undermine its reliability for our purposes.
SZ estimate two series. The first uses total foundation wealth divided by total capital income reported by foundations (interest on savings and cash + dividends and interest + rent + positive kg). Note that dividends and interest are not separately reported. The second changes the capitalization factor by excluding positive kg from the formula. The test is to compare top shares estimated using the equivalent of the mixed method for this data and comparing it to actual top foundation wealth shares, after reranking who comprises the top group. This test understates the bias from placing too much weight on capital gains when estimating equity wealth for a few reasons.

• First, foundation balance sheets are not broken out into sufficient detail to permit a test of equity wealth estimation. The only test that can be done is to see whether changing this one component changes estimated top wealth shares.

• Second, foundation income flows do not permit a test of the appropriate weight on capital gains versus dividends when predicting equity wealth or foundation wealth because foundations do not report these income flows separately. Thus, SZ do not conduct such a test.

• As SZ note, there are other reasons to worry that extrapolating from foundations data can inform the estimation of wealth for high net worth individuals: “foundations have minimum spending rules that might lead them to have different realization patterns than wealthy families, and they are tax exempt” (p.544).

• We note a third reason, which is that foundations have longer time horizons than high net worth individuals and foundation wealth typically comes after high net worth individuals have diversified concentrated positions in businesses and gifted the proceeds to the foundations. So the portfolios of foundations are likely to be quite different than the typical high net worth individual. Evidence from family office surveys supports this conjecture.

If realized gains are less common for high net worth individuals than for foundations, the foundations data will tend to overstate the usefulness of using realized gains to estimate wealth for individuals.

• Finally, as discussed in Chapter 8 below, updating SZ’s original capitalization exercise for foundations shows that the actual wealth series and capitalized series diverge after 2010. Thus, it is not clear that the data from foundations provides strong support for the mixed method approach in recent years.

• As to matching the Forbes 400, SZ don’t present evidence their approach comes close to matching Forbes 400 wealth in terms of equity wealth. Instead, they find their approach matches the overall level of billionaire wealth. This fact owes more to the biased estimation of fixed income wealth. In the equal returns series, the top 0.001% have 55% of their portfolios in fixed income.

• Ultimately, the capitalization approach will struggle to match the wealth of those atypical billionaires at the top, such as Buffett, whose taxable income including realized capital gains is small relative to their wealth. This is why we attempt to quantify the bias using the overlapping samples approach of Bricker, Hansen, and Volz (2020), which directly augments the capitalized series with those people at the very top whom we are unlikely to match using income data alone.

4. “5 of the top 10 richest Americans—Jeff Bezos (Amazon), Mark Zuckerberg (Facebook), Warren Buffett (Berkshire Hathaway), Sergey Brin (Alphabet), and Larry Page (Alphabet),
collectively worth more than $250 billion in 2016—were the main shareholders of corporations that did not pay dividends in 2016. The SZZ methodology assigns them a negligible amount of wealth relative to their true wealth.” (p.3)

- Both our capitalization methodology and the SZ methodology will assign these unique individuals a “negligible” amount of equity wealth.
  - For example, consider Jeff Bezos. Saez and Zucman (2020a) note that our prior approach underestimates wealth for those like Bezos who realize a small portion of capital gains. “According to SEC Form 4 public records, in 2016 Jeff Bezos sold around 2 million Amazon stocks at a price of around $700, resulting in up to 1.4 billion in capital gains. In the SZZ methodology, the implied equity wealth is $1.4B \times 4 = $5.6 billion. That same year, Bezos’s stake in Amazon was valued at around $60 billion.” (p.8–9). However, this issue is equally relevant for the approach in SZ and PSZ. The capitalization factor for $\alpha = .5$ in 2016 is 26, so Bezos's estimated wealth in SZ capitalization approach is 13 times $1.4B = $18.2 billion. Rather than illustrating that our approach is dramatically inferior to SZ’s, the example shows that no approach to capitalization will get Bezos close to right. The case of Warren Buffett is even more extreme, with the SZ approach assigning stock wealth equal to just over 2% of his listed stake in Berkshire Hathaway. Moreover, to the extent past attempts to capitalize tax data have delivered top wealth that matches Forbes (as in SZ and PSZ), these estimates have been driven by massive amounts of fixed income wealth rather than equity wealth.

- We should note that the anecdote may overstate the bias in our approach. The other 5 are likely much better estimated using our approach. These include Bill Gates (number 1 on the Forbes list in 2016), Larry Ellison of Oracle, Michael Bloomberg (pass-through rich), and the Koch brothers (likely mix of pass-through and C-corporations, mostly private). The next three are heirs to the Wal-Mart, which is a dividend paying C-corporation. These anecdotes can help explain why a more rigorous sample reconstruction that augments the capitalized series accounting for overlap only increases top shares modestly.
  - Of the top 50 ranked individuals in Forbes, 13 out of 29 public equity owners did not receive dividends, 16 out of 29 did receive dividends, and the other 21 individuals were private business owners.
  - As mentioned above, owners of private businesses or dividend-paying public companies account for 77% of collective Forbes wealth in 2016.

- We should not overemphasize the importance of these individuals.
  - These 5 account for just 0.3% of total wealth in the US and 11% of Forbes wealth in 2016.
  - The Forbes 400 have considerable wealth ($2.4T in 2016), but the total wealth of the P99-99.9 and P99.9-99.99 tax unit groups exceeds this amount by factors of 6.2 and 3.0, respectively. Of course, Forbes members are much wealthier on average: these groups respectively contain 1.5 million and 150 thousand tax units, whereas Forbes represents only 400. Our top 0.001% group contains $3.1T of wealth, which includes the impact of blending Forbes into our data.

5. “SZZ under-estimate billionaire wealth by about 40%. In 2016, US billionaires owned 3.0 trillion in wealth according to Forbes, 3.1 trillion in the benchmark Saez and Zucman (2016)
series, and 1.7 trillion according to SZZ. The benchmark Saez and Zucman (2016) methodology is consistent with the evidence on billionaire wealth; the SZZ methodology is not.” (p.3)

- Following the advice of the editor, we no longer analyze ultra millionaire wealth taxes with surtaxes for billionaires, so we focus on reporting wealth and counts for top groups. The highest of which is the top 0.001%, which represents those with more than $590M in 2016. As mentioned above in reply to #1 in this chapter, our estimate for the collective wealth of this group is $3.13T. Without blending, this group would have $2.66T. Replacing the top 400 capitalized tax units with Forbes estimates gives $3.53T. We show the effects of these alternatives on top wealth shares in Figure 8D.

- With a Pareto parameter of 1.4, the amount of wealth between the Forbes 2016 cutoff of $1.7B and $590M is \( \frac{(1700/590)^{1.4} - 1}{1.4 - 1} \) = 52% of the wealth above $1.7B in Forbes, which is $2.4T. Thus, the collective wealth of those with wealth above $590M is $2.4T \times (1.52) = $3.6T.

- This difference between our estimate of $3.1T and $3.6T is sensitive to the Pareto parameter, which is somewhat uncertain as it varies depending on the cutoff of the 400\(^{th}\) person and depends on distributional assumptions. It also depends on the accuracy of the Forbes $2.4T estimate in 2016; if Forbes is overstated by 15% (i.e., the true number is $2.06T), then there is no discrepancy between our baseline estimate and the Pareto-extrapolated estimate.

- There are several reasons why the Forbes wealth estimates are uncertain.
  
  (a) When Raub, Johnson and Newcomb (2010) link the Forbes 400 data to the estate tax data, they only find about half of that wealth in the administrative data. It’s hard to determine how much of this gap is due to tax avoidance and evasion, which are also likely quite substantial.

  (b) Given the publicity associated with placing onto the Forbes list, it is possible that individuals exaggerate their wealth (Kopczuk, 2015). There are several well-known cases of substantially exaggerated private business values in the Forbes list.\(^2\)

  (c) Many of the Forbes 400, those in the Bloomberg billionaires list, or top 400 units in the SCF have substantial shares of wealth in private firms, which are difficult to value.\(^3\)

  (d) One contribution of our approach is that our private firm values are based on firm-level administrative data and capital market valuation multiples, which are likely more accurate than estimates based on harder to verify self-reported estimates.

- In terms of billionaire wealth, our estimates imply the following:
  
  In terms of the collective wealth of billionaires, our estimates before blending in the Forbes data amount to $1.93T in 2016 held by 750 tax units. Of these 750 tax units, 330 have capitalized wealth estimates above the 2016 Forbes threshold of $1.7B and their collective wealth is $1.4T. The other 420 tax units that have between one billion and the Forbes threshold of $1.7B collectively hold $530B in wealth.\(^4\)


\(^3\)Indeed the Bloomberg list has an accuracy rating system that reflects these difficulties: [https://www.bloomberg.com/billionaires/methodology/]

\(^4\)With a Pareto parameter of 1.4, the amount of wealth between the Forbes 2016 cutoff of $1.7B and $1B is
4.1. RELEVANT EXCERPTS FROM “COMMENTS”

If we replace the top 400 tax units with estimates from Forbes, we’d find that these 750 tax units collectively held $2.83T in wealth.

Our preferred approach that includes the Forbes-augmented series using the Bricker Hansen Volz (2019) method gives 786 billionaires with $2.44T in collective wealth. Since this blending method effectively averages wealth estimates in Forbes with capitalized estimates in bins with overlap, it does not add the full $2.4T in Forbes in 2016.

One important robustness exercise that we describe above is adding an estimate of $220B of non-dividend generating C-corporation wealth to our preferred estimates, which results in an estimate of $2.67T in collective wealth of 786 billionaires.

We show how all of these approaches affect top share estimates in Figure 8D.

- Given the importance of pass-through business wealth—which represents around one trillion dollars in Forbes wealth in 2016—and the uncertainty in the Forbes estimates, our preferred approach is to use the blending approach of BHV rather than replace the top 400 capitalized estimate with the Forbes data. Many of these tax units may be in the data already and it is not clear that the accuracy of top wealth shares would improve. Ultimately, we think it is best to describe what we do and provide a number of alternative approaches to help readers understand the potential magnitudes of different adjustments to account for Forbes wealth.

- It is useful to have alternative estimates from Forbes because of the necessary limitations of their methodology, which is subject to self-reporting bias as illustrated by recent cases including Wilbur Ross, Kylie Jenner, and others.

- In addition, the evidence that we present about self-reported values of pass-through business wealth in the SCF—which substantially exceed estimates based on public company multiples and private firm sales—supports the possibility that Forbes wealth estimates may be overstated, especially for the 241 private business owners who comprise the majority of individuals in the Forbes 400.

- More aggressive inclusion of Forbes wealth (that properly accounts for overlaps with capitalized and blended estimates) would deliver top share estimates in the ballpark of our imputation intervals shown in Figure 16. Figure 8D and Figure 11 help illustrate this point.

- The original SZ (2016) method also cannot fit equity wealth of Forbes individuals. It only matches aggregate Forbes wealth by assigning $1.9T of fixed income wealth to the top 0.001, which accounts for 55% of their portfolios. Our approach finds 48% in C-corporations, 30% in pass-throughs, and 14% in fixed income, in line with external data on ultrarich portfolios.

6. “The SZZ methodology, which almost fully ignores capital gains, is not appropriate to capture billionaire wealth and delivers estimates equal to only 57% of Forbes.” (p.3)

- We do not ignore capital gains.

- The problem with capturing the wealth of people like Buffett is not about the weight on realized capital gains. Even if we put 100% weight on realized gains, the wealth estimate for him would be a small fraction of his true wealth.

- The problem is that most of their income is unrealized capital gains. The only way to get these rare cases into the estimates is via augmentation of the underlying data using other sources, which is what we do.

\[
\frac{(1700/1000)^{1.4-1} - 1}{1} = 23\% \text{ of the wealth above } $1.7B \text{ in Forbes, which is } $560B \text{ (when using the $2.4T estimate for Forbes wealth in 2016 for those with more than $1.7B).}
\]
• We discuss this point in Appendix L.3:
  – “Saez and Zucman (2020a) note that our approach underestimates wealth for those like Bezos who realize a small portion of capital gains. According to SEC Form 4 public records, in 2016 Jeff Bezos sold around 2 million Amazon stocks at a price of around $700, resulting in up to 1.4 billion in capital gains. In the SZZ methodology, the implied equity wealth is $4 \times 1.4$ billion = $5.6$ billion. That same year, Bezos’s stake in Amazon was valued at around $60$ billion.’’(p.8–9). However, this issue is equally relevant for the approach in SZ and PSZ. The capitalization factor for $\alpha = .5$ in 2016 is 26, so Bezos’s estimated wealth in SZ capitalization approach is 13 times $1.4B = 18.2$ billion. Rather than illustrating that our approach is dramatically inferior to SZ’s, the example shows that no approach to capitalization will get Bezos close to right. The case of Warren Buffett is even more extreme, with the SZ approach assigning stock wealth equal to just over 2% of his listed stake in Berkshire Hathaway. Moreover, to the extent past attempts to capitalize tax data have delivered top wealth that matches Forbes (as in SZ and PSZ), these estimates have been driven by massive amounts of fixed income wealth rather than equity wealth.”

7. “According to SZZ (2020, p. 32) billionaires owned $1.7$ trillion in wealth in 2016. According to Forbes, the top 400 wealthiest Americans (who had wealth above $1.7$ billion) owned $2.4$ trillion in 2016. Billionaires with more than 1 billion and less than $1.7$ billion add close to an extra $600$ billion, for a total billionaire wealth of around $3$ trillion. SZZ only capture 57% of that amount.” (p.8)

• We addressed this comment in reply to #5 in this chapter.

8. “There are two ways to arrive at the $600$ billion number for non-Forbes 400 billionaire wealth. First, one can make the classical assumption that the tail of the wealth distribution is Pareto distributed. As the average wealth of the Forbes 400 in 2016 ($6.0$b) was 3.5 times the threshold to belong to the Forbes 400 ($1.7$b), the corresponding Pareto parameter is $a = 3.5/(3.5 - 1) = 1.4$. Standard calculations imply that the wealth between $1$bn and $1.7$bn is $[(1.7/1)^{(a-1)} - 1] = 23.6\%$ of the wealth above $1.7$bn, i.e., $567$ billion. Second, one can look at SCF data: the public-use 2016 SCF file, which by construction excludes the Forbes 400, has $583$ billion in billionaire wealth (with by construction wealth below 1.7 billion).” (p.8, footnote 13)

• As mentioned in reply to #5, our estimates for this group line up closely.
• Regarding the SCF, our estimates in Figure 13C show that top 0.01% C-corporation wealth exceeds that of the harmonized SCF, which includes Forbes. In addition, our top share estimates in Figure 1 also often exceed or rival the SCF plus Forbes series for the top 0.1% and top 0.01% groups in recent years.

9. “SZZ under-estimate billionaire wealth because their methodology is not appropriate to estimate equity wealth (in non-pass through businesses), the key form of wealth at the top.” (p.8)

• Non-pass-through equity wealth is less central at the top than this comment implies.
• As mentioned in reply #1, we classified each individual in the Forbes 400 list in 2016. Of the top 400 individuals in Forbes in 2016, 68 out of 159 public equity owners did not receive
dividends, 91 out of 159 did receive dividends, and the other 241 individuals were private business owners.

- We find that owners of private businesses or dividend-paying public companies account for 77% of collective Forbes wealth in 2016. Therefore, our capitalized wealth estimates, which use firm-level administrative data to value pass-through businesses (rather than less accurate self-reported information in Forbes for these firms) as well as a mix of dividends and capital gains for public equity owners, are likely substantially more informative about the wealth of the ultra rich than this statement suggests.

- The core issue is that Bezos, Zuckerberg, Buffett, etc. are not representative of top wealth on either person-weighted or dollar-weighted terms. See our reply to #4 in this chapter for additional details.

10. “SZZ put a very low weight (10%) on capital gains (vs. 90% on dividends) to estimate equity wealth. A similar assumption was implemented and investigated in supplementary series constructed in Saez and Zucman (2016, Appendix Tables B36, B37 and B37b discussed pp. 534–535) that put a 0% weight on capital gains (vs. 100% on dividends)” (p.8)

- SZ’s analysis understates the bias from putting too much weight on capital gains, for reasons discussed above.

- SZ never present direct tests of the equity wealth estimation model they use. Instead, they focus on one-off perturbations to the equity wealth model and then look at how top shares change.

- For example, their supplementary series do not correct the issue with capitalizing fixed income assets. Given fixed income wealth is so important for top wealth in the SZ series, using this estimate will dampen the influence of biased equity estimates on overall top wealth shares.

- Also reranking will tend to mitigate the bias to top shares from individual wealth model changes as shown by Tables B.9 and B.10.

11. “this [similar] assumption was found to underestimate top end wealth, because many of the wealthiest Americans are major shareholders of companies that do not pay dividends (Amazon, Google, Facebook, Berkshire Hathaway, etc.).” (p.8)

- Any capitalization approach will fail in these cases. The difference between 0%, 10%, 50%, and 100% weight on capital gains is second order to the fact that Buffett doesn’t sell any shares.

- As mentioned in reply #1 in this chapter, we went through the Forbes 400 more systematically to determine whether public company owners on the Forbes list owned companies that paid dividends in 2016. Of the top 50 ranked by wealth, we found that 54% of the collective wealth represented public company owners whose companies paid dividends in 2016. Of the top 400, this share is 56%. Thus, this statement overstates the importance of non-dividend-generating C-corporation equity wealth in Forbes. We estimate the importance and account for overlap with the blending approach in our analysis and robustness checks described above.

12. “In 2016 the capitalization factor for dividends plus capital gains is 40 in the SZZ methodology, which means that people are assigned 36 times their dividends plus 4 times their
capital gains in equity wealth. Someone with 0 dividend gets 4 times his realized capital gains in equity wealth. According to SEC Form 4 public records, in 2016 Jeff Bezos sold around 2 million Amazon stocks at a price of around $700, resulting in up to 1.4 billion in capital gains. In the SZZ methodology, the implied equity wealth is $4 \times 1.4 \text{ billion} = 5.6 \text{ billion}. That same year, Bezos’s stake in Amazon was valued at around $60 \text{ billion}.” (p.8–9)

- The SZ approach similarly underperforms.
- We discuss this point in Appendix L.3:
  - “Saez and Zucman (2020a) note that our approach underestimates wealth for those like Bezos who realize a small portion of capital gains. According to SEC Form 4 public records, in 2016 Jeff Bezos sold around 2 million Amazon stocks at a price of around $700, resulting in up to 1.4 billion in capital gains. In the SZZ methodology, the implied equity wealth is $4 \times 1.4 \text{ billion} = 5.6 \text{ billion}. That same year, Bezos’s stake in Amazon was valued at around $60 \text{ billion}.” (p.8–9). However, this issue is equally relevant for the approach in SZ and PSZ. The capitalization factor for $\alpha = .5$ in 2016 is 26, so Bezos’s estimated wealth in the SZ capitalization approach is 13 times $1.4B = 18.2 \text{ billion}$. Rather than illustrating that our approach is dramatically inferior to SZ’s, the example shows that no approach to capitalization will get Bezos close to right. The case of Warren Buffett is even more extreme, with the SZ approach assigning stock wealth equal to just over 2% of his listed stake in Berkshire Hathaway. Moreover, to the extent past attempts to capitalize tax data have delivered top wealth that matches Forbes (as in SZ and PSZ), these estimates have been driven by massive amounts of fixed income wealth rather than equity wealth.”

- This example illustrates that no approach to capitalization will get Bezos close to right, not that our approach is dramatically inferior to SZ’s approach.
- The ultra-wealthy C-corporation owners of firms that don’t pay dividends is one of the reasons why we augment the data with Forbes data. Overall, as mentioned above, our estimates of the share of total wealth accounted for by the top 0.01% in the form of C-corporation wealth exceeds the share in the harmonized SCF plus Forbes (Figure 13).

13. “To take another example, in 2016 Warren Buffett disclosed he had adjusted gross income of $11,563,931 in 2015. Assuming this all came from capital gains (as Berkshire Hathaway does not pay dividends), the implied wealth in the SZZ model is $50 \text{ million}. That year, Buffett’s stake in Berkshire Hathaway was worth about $60 \text{ billion}.” (p.9)

- The implied wealth in the SZ model, applying a capitalization factor of 24 for $\alpha = .5$ in 2015, is $139 \text{ million}$.

14. “Forbes + SCF is the estimate of billionaire wealth obtained by appending the Forbes 400 to the SCF, which by construction excludes the Forbes 400.” (p.10, Figure 3 Caption)

- It should be noted that this refers to the public-use SCF, as the private SCF has some overlap with Forbes (Bricker, Hansen, Volz, 2019).

15. “As shown by [Figure 3], the benchmark Saez and Zucman (2016) capitalization method captures close to 100% of the amount of billionaire wealth implied by Forbes. By contrast, the SZZ methodology, which under-estimates top-end equity wealth, under-estimates billionaire wealth by almost 2.” (p.9)
4.1. RELEVANT EXCERPTS FROM “COMMENTS”

• Comment addressed above in reply to #1.

16. “Saez and Zucman (2016) opted for a method with a higher weight on capital gains precisely because it did a better job at matching Forbes. The gap between SZZ and Forbes is the sign of an issue in the SZZ methodology.” (p.10)

• Thank you for pushing us to think harder about the aggregate wealth of the top 400.
• We were unable to find a discussion in SZ of how putting higher weight on capital gains helps to match Forbes.
• As noted above, the key reason the SZ aggregates match Forbes is fixed income wealth at the top, which Figure 14A shows is 55% in the PSZ estimates for the top 0.001% and substantially smaller in all other datasets. SZ and all capitalization approaches fail to estimate the wealth of the exceptional cases such as Bezos and Buffett.

17. “Correcting the SZZ estimates so that they match the aggregate billionaire wealth implied by Forbes, without making any other correction, increases the SZZ top 0.1% wealth share by 1.7 points in 2016, closing 40% of the gap with the benchmark Saez and Zucman (2016) series.” (p.10)

• See Figures 8C and 8D for the exact exercise of changing C-corporation models and treatment of Forbes on top shares.
Chapter 5

Comparing Capitalization Estimates to Other Data
5.1 Relevant Excerpts from “Comments”

1. “the SZZ estimates are inconsistent with the 2016 level and the 2001-2016 rise of top-end US wealth seen in the official Federal Reserve estimates (Survey of Consumer Finances, Distributional Financial Account) and Forbes.” (p.1)

   • Our preferred estimates match the level and trend of the harmonized SCF + Forbes series for the top 0.01% and top 0.1% (Figure 1).
   • Our estimates are lower for the top 1% in 2016 (Figure 1). Most of this difference comes from higher pass-through business values, especially among P99-99.9 owners of businesses with less than $50M in revenue, in the SCF relative to our bottom-up approach and the Financial Accounts. Owners of these firms report business values that substantially exceed sales, profit, and book multiples in Compustat. We provide more detail in the text and Appendix L.1:
      – “For example, Appendix Table B.4 shows that the average market value to sales ratio in the SCF is 2.6 and 2.5 for those in the P99-99.9 and top 0.1% of net worth, which is much higher than the market to sales ratio of 1.8 in Compustat. Similar valuation premia appear for ratios relative to profits (22.6 and 18.2 vs. 16.3) and cost basis (8 and 9.5 vs. either 3 or 6.5 depending on whether the measure of cost basis in Compustat is book equity or net capital). These facts also contrast with evidence we present on liquidity discounts for private targets in large firm acquisitions (Appendix J), evidence on private market sales data for mid-market firms (Bhandari and McGrattan, 2021), and the literature estimating private firm sales discounts (Officer, 2007), all of which point toward considerable private firm discounts.”
   • Adjusting these business values (e.g., by using the Financial Accounts aggregates in Appendix Figures A.24 and A.25) can close the gap between our top 1% estimates and those in the SCF. This force also explains why the DFA measures of top 1% shares are closer to our estimates.
   • Moreover, in terms of composition, Figure 15 shows that the composition of wealth in our estimates line up quite well with those of the SCF.
   • In terms of changes over time, Figures 1 and 13 illustrate that our top share series are more consistent with the trends in the SCF, which do not show a sharp increase in top shares due to fixed income at the top (Figure 13). As mentioned above, the key adjustment to close gaps in our series is pass-through business.
   • Figure A.22 also shows that the 1989-2016 difference illustrates much closer correspondence with our estimates than with those of PSZ.
   • In addition, Figure 16 shows how many perturbations relative to our baseline model fall within the standard errors of the top 0.1% harmonized SCF confidence intervals, which is not true for the headline estimates of PSZ.

2. “According to the SCF (which by construction excludes the Forbes 400) appended to the Forbes 400 list, a wealth tax at a rate of 2% above $50 million and 3% above $1 billion would have generated $202 billion in revenue in 2016. According to SZZ the same wealth tax would have raised only $117 billion.” (p.1)

   • The revised manuscript focuses on top wealth and composition, and no longer analyzes the wealth tax.
5.1. RELEVANT EXCERPTS FROM “COMMENTS”

- A key input into these calculations is the collective wealth of the ultra rich, which we discuss in detail in reply #5 in Chapter 5.
- As mentioned above, our approach now incorporates Forbes 400.
- One reason for different revenues is that aggregate wealth in the SCF exceeds that in the Financial Accounts by $10 trillion in 2016 (Batty et al., 2019), so comparing estimates in the two series can be misleading given the differences in aggregates across the two sources.

3. “According to the official Survey of Consumer Finances data, the top 1% wealth share rose 6.2 points between 2001 and 2016. According to the official Distributional Financial Accounts, the top 1% wealth share rose 5.2 points over the same period. According to the benchmark Saez and Zucman (2016) estimates, the top 1% wealth share (for adult individuals) rose 5.4 points. By contrast, according to SZZ the top 1% wealth share rose only 1.4 points over the same period. SZZ provide no evidence that the Federal Reserve overestimates the rise of wealth inequality since 2001.” (p.3)

- This comment is addressed in #1 in this section.

4. “Anyone can download the public-use 2016 SCF micro-file—which by construction excludes the Forbes 400—append the 2016 Forbes 400 list, and simulate the mechanical revenue from a wealth tax at rate of 2% above 50 million and 3% above 1 billion in 2016, assuming no tax avoidance. The result is $202 billion. Using the benchmark Saez and Zucman (2016) methodology, the same wealth tax would have generated a similar amount, $190 billion. According to SZZ however, the same wealth tax would have raised only 117 billion (SZZ, p. 32). SZZ do not provide a logically consistent explanation for this gap.” (p.3)

- This comment is addressed in #2 in this section.

5. “The SZZ methodology fails to capture the level of top-end wealth recorded in the official Survey of Consumer Finances and its 2001–2016 rise. As a result SZZ under-estimate wealth tax revenues by almost 50%.” (p.3)

- This comment is addressed in #1 and #2 in this section.

6. “SZZ do not provide evidence that the Forbes 400 aggregate is wrong.” (p.9)

- When Raub, Johnson and Newcomb (2010) link the Forbes 400 data to the estate tax data, they only find about half of that wealth in the administrative data. It’s hard to determine how much of this gap is due to tax avoidance and evasion, which are also likely quite substantial.
- Nonetheless, the revised version augments the data with the Forbes 400 data as described in replies #1, #4, and #5 in Chapter 4. We also discuss the basis for uncertainty in the Forbes estimates in the main draft and in Appendix L.3.

7. “They note that private businesses are hard to value—but diversified portfolios of stocks and bonds, for which there is no public information, are even harder to capture and likely to be missed by Forbes.” (p.9)

- We respectfully disagree. The evidence that we present about self-reported values of pass-through business wealth in the SCF—which substantially exceed estimates based on public
company multiples and private firm sales—supports the possibility that Forbes wealth estimates may be overstated, especially for the 241 private business owners who comprise the majority of individuals in the Forbes 400.

8. “They refer to the Bloomberg billionaire index in their footnote 60—but do not mention that the Bloomberg index finds as much and sometimes even more wealth at the top than Forbes. Even the small point on Pareto coefficients does not reflect the current state of knowledge.” (p.8–9)

- The revised paper augments the data with Forbes, so it matches the Bloomberg series well.
- The revised version of the paper focuses on Pareto shares for the very top of the distribution, and no longer reports a top 10% share since Pareto parameter estimates are local to the top and extending estimate to the top decile requires more extrapolation. At the top of the distribution, where the local Pareto parameters are more appropriate, the implied top 0.1% share and top 1% shares are much closer to the SZZ estimates than SZ estimates.

9. “SZZ also claim that “in terms of the number of billionaires and their collective wealth, statistics from the Forbes 400 fall in between our estimates and those from the equal-return approach of Saez and Zucman (2016),” suggesting that the benchmark Saez and Zucman (2016) methodology delivers too high results for billionaire wealth. However, as Figure 3 above shows, the benchmark Saez and Zucman (2016) methodology delivers an estimate of billionaire wealth very similar to the one implied by Forbes, $3.14T in 2016. A close estimate ($3.27 trillion) can be obtained in the publicly available Distributional National Accounts micro-files of Piketty, Saez and Zucman (2018), which are blurred at the top for confidentiality reasons.” (p.10, footnote 14)

- The close estimate from PSZ does not validate the SZ approach or contribute new evidence since the two papers use very similar methodologies.
- The estimate of billionaire wealth in the revised version, which augments the data with Forbes, is discussed in detail in reply #5 in Chapter 4.

10. “Anyone can download the public-use 2016 SCF micro-file—which by construction excludes the Forbes 400—append the 2016 Forbes 400 list, and simulate the mechanical revenue from a wealth tax at rate of 2% above 50 million and 3% above 1 billion in 2016, assuming no tax avoidance. The result is $202 billion. The code is below... [code]” (p.10–11)

- This comment is addressed in #2 in this section.

11. “The same result can be obtained without running any code using the wealth tax calculator published by Vox, which also uses the SCF appended to the Forbes 400.” (p.11)

- Getting the same result from the same methods does not provide additional corroborating evidence.
- We address this issue in the prior answer that discusses the SCF plus Forbes estimates.

12. “According to the SZZ methodology (SZZ, p.32), the same wealth tax, with the same assumption about avoidance, would have raised only 117 billion in 2016. This is only 58% of the SCF + Forbes estimate for the same year. this discrepancy is due to the fact that SZZ
5.1. **RELEVANT EXCERPTS FROM “COMMENTS”**

have both less billionaire wealth than in Forbes + SCF (see point 2 above), and less wealth for tax units with wealth between $50 million and $1 billion than in the SCF, in both cases by a factor of 1.7. **SZZ under-estimate the wealth of tax units with more than $50 million by a factor of 1.7 compared to existing sources. By contrast, the benchmark Saez and Zucman (2016) methodology is consistent with both the level of billionaire wealth found in Forbes + SCF, and the level of wealth for tax units with net wealth between $50 million and $1 billion seen in the SCF.” (p.11)

- We address this comment in reply #1 and #5 in Chapter 4.
- The gap between SZ and SZZ is primarily due to large top fixed income estimates in the SZ series—the baseline SZ numbers imply that over half of the portfolio of the top 0.001% is in fixed income—which are not consistent with any other data on the wealthy such as portfolio shares in other data sets or data on family offices.

13. **“SZZ (p. 42) suggest that the Survey of Consumer Finances over-estimates top-end business wealth. Business assets would have to be over-estimated by a factor of 3 to reconcile the SCF and SZZ estimates of top-end wealth. Instead of owning business assets worth $6 trillion, families with net wealth above $50 million (slightly above the top 0.1% threshold, $43.2 million in 2016) would have to own business assets worth only $2 trillion in 2016. SZZ do not provide evidence to support the notion that the business assets of top 0.1% families are over-estimated by a factor of 3 in the SCF.”** (p.12)

- We agree that business assets for the top 0.1% in the SCF are not over-estimated by a factor of 3. However, our estimates do not imply this degree of overvaluation.
- As mentioned above, SCF firm owners report business values that substantially exceed sales, profit, and book multiples in Compustat. The overvaluation relative to public firms also contrast with evidence we present on liquidity discounts for private targets in large firm acquisitions (Appendix J), evidence on private market sales data for mid-market firms (Bhandari and McGrattan, 2021), and the literature estimating private firm sales discounts (Officer, 2007), all of which point toward considerable private firm discounts on the order of 10–30%.
- Appendix Figure A.19 shows that we estimate top 0.1% pass-through business wealth of $3.3T and C-corporation equity wealth of $4.2T in 2016. If 20% of C-corporation equity wealth is private business, then our overall estimate of private business for this group would be approximately $4.1T. Some assets that are categorized as private business in the SCF, such as rental housing, appear in other categories in our data, which would further increase our private business estimate.

Given the evidence we present suggests private business discounts on the order of 10–30% and possibly higher, we believe our estimates are consistent with the SCF after taking a relatively modest amount of overvaluation into account. Indeed, Appendix Figures A.24 and A.25 show that scaling SCF private business to match Financial Accounts totals (which are below our estimates) fully closes the top 1% gap between our series and the SCF and leads our top 0.1% and top 0.01% shares to modestly exceed the harmonized SCF series.

- We also note that our pass-through estimates exceed the PSZ estimates by $1T for the top 0.1%. PSZ’s estimates would only match the SCF private business levels if half of the SCF private business assets (approximately $3T) represent fixed income claims, which appears implausible.
CHAPTER 5. COMPARING CAPITALIZATION ESTIMATES TO OTHER DATA

14. “SZZ seem to hesitate between claiming that the SCF over-estimates top-end wealth, and claiming that their results are consistent with the SCF. For instance on p. 42 SZZ write that “Our preferred series closely fits the most comparable equal-split SCF series that makes all adjustments, trending similarly and matching the levels of the top 1% and rising somewhat above the top 0.1% in the 2000s.” This assertion is impossible to understand, given that the top 1% and top 0.1% “most comparable equal-split SCF wealth series” rise as much as the benchmark Saez and Zucman (2016) equal-split series from 2001 to 2016, while the top 1% and top 0.1% SZZ “preferred series” barely increase from 2001 to 2016.” (p.12, footnote 17)

- Thank you for pushing us to be clearer on our assessment of the SCF estimates and trends.
- In contrast to the initial draft, we now only present a harmonized SCF that includes the Forbes 400 and measures top shares in the same units as the capitalized estimates.
- As noted above, we present evidence that top-wealth shares in the harmonized SCF match ours in terms of levels, trends, and composition for the top 0.01% and top 0.1% quite well. See Figures 1, 13, and 15.
- As noted above, the top-1%-share discrepancy seems to be eliminated by an adjustment of aggregate pass-through business values.
- We hope that Figures 1 and 12 make it possible to understand our view that we match the top SCF shares well (as well as their analogues that scale down pass-through business wealth in the SCF in Appendix Figures A.24 and A.25).
- In terms of increases from 2001-2016, the top 0.01% increases 3.0 p.p in PSZ (which is entirely due to a 3 p.p. increase in fixed income), 1.2 p.p. in our estimates, and 2.2 pp in the harmonized SCF. Of the 2.2 p.p. increase in top 0.01% shares, 1.7 p.p. were due to an increase in pass-through business in the SCF. Thus, a smaller increase in pass-through business, due partly to lower estimates of the levels, helps reconcile the post-2000 differences between SZZ and the harmonized SCF.
- The patterns in the top 0.1% and top 1% groups are similar. That is, the 4.1 and 5.4 p.p. increases in PSZ series were more than driven by 4.5 and 5.9 p.p. increases due to fixed income, respectively. In contrast, the top 0.1% and top 1% shares in the SCF of 5.8 and 3.7 were largely driven by increases in pass-through business representing 3.1 and 4.1 p.p. of total household wealth. Scaled down pass-through business growth brings the SCF changes since 2001 closer to our changes of 2.1 and 4.2 p.p. for the top 0.1% and top 1% shares.

15. “this potential reconciliation is logically inconsistent. If the SCF over-estimates the wealth of the top 0.1% by a factor of 1.7 because it over-estimates the business assets of top-end families by a factor of 3, then the portfolio shares observed in the SCF are biased. The shares of all-non-business assets in the portfolios of top 0.1% families must be multiplied by 1.7, while the share of business assets must be divided by 3/1.7 = 1.7. The SZZ methodology, however, is based on fitting the observed portfolio shares at the top-end in the SCF (and other auxiliary moments in the SCF, such as 2001–2016 changes in portfolio shares, the interest rate of the rich, etc.); see, e.g., SZZ Table 3.” (p.12–13)

- The point that overstated pass-through business estimates affecting portfolio shares is well taken.
• Appendix Figure A.25 presents harmonized SCF portfolio shares after scaling the pass-through business aggregates to match the Financial Accounts.

• To be clear, the methodology in SZZ does not use top portfolio shares as a target for estimation. For example, the C-corporation estimation uses the share of total C-corporation wealth that is held by a group like the top 1% of wealth holders.

• We provide Figure 15 and other portfolio shares to help readers assess implications for other moments of different modeling and capitalization assumptions but do not target those directly. Moreover, one can use the data reported in Figure 15 and other portfolio share series to make further adjustments if they are interested in evaluating the consequences of scaling down one of the components.

16. “The argument that business assets are too high in the SCF results in a logical contradiction: if the SCF over-estimates business assets, it does not make sense to fit the portfolio shares observed at the top-end of the SCF.” (p.13)

• This comment was addressed in response to the previous two comments.

17. “Even if the SCF over-estimates business assets and hence has biased portfolio shares, part of the SZZ Table 3 validation exercise could still have merit if the SCF has the correct level of interest-bearing assets at the top. However, what is counted as interest-bearing assets in the SCF vs. in the Financial Accounts is different, so that one should not aim at matching the SCF level of interest-bearing assets as SZZ do. In the Financial Accounts, the interest-bearing assets owned through domestic hedge funds and private equity funds are recorded as interest-bearing assets, whereas they are typically business assets in the SCF (e.g., for the funds’ general partners). This turns out to matter quantitatively, because hedge fund and private equity fund managers play a prominent role at the top of the wealth distribution, and these funds hold large amounts of interest-bearing assets.” (p.13, footnote 18)

• See the replies to #37 and #44 in Chapter 3, #11 in Chapter 8, #5 in Chapter 9, which discuss why this claim about interest-bearing assets in private business for fund general partners is likely incorrect.

• We also provide top wealth and portfolio shares when scaling SCF Fixed Income to match the Financial Accounts in Appendix A.26 and A.27.

• It shows that the PSZ series is a substantial outlier in terms of wealth shares, top fixed income levels, and the evolution of both.

18. “SZZ take as gospel auxiliary moments in the SCF that are either mis-measured (such as top-end interest rates…) or not comparable across sources (such as portfolio compositions…) while discarding the level of wealth found in the SCF—which is what the SCF aims at capturing accurately.” (p.13)

• We respectfully disagree with the characterization that the SCF is mainly useful for accurately measuring the level of wealth. For example, the detailed information in the survey on family balance sheets suggest it is quite focused on accurately measuring portfolio shares. We believe the SCF portfolio shares, especially in the harmonized series, contain valuable information that can help validate different models.
CHAPTER 5. COMPARING CAPITALIZATION ESTIMATES TO OTHER DATA

- Rather than discard SCF aggregates, one goal of our paper is to shed more light on key discrepancies between the aggregates in the SCF and the Financial Accounts. One can also usefully make adjustments to the SCF, such as our adjustment to pass-through firm valuations in Figure A.24 and A.25, and use the resulting SCF series to try to understand the evolution and composition of top wealth under alternative models for private business valuation.

- In Section 9, we write about the many uses of the SCF: “Overall, the SCF is a crucial input into the wealth inequality debate. It allows researchers using income tax data to say more than they otherwise could, provides a benchmark for inequality research, contains detailed portfolio information that is unavailable in other data sets, and enables analysis by characteristics (such as race) that cannot be studied elsewhere. At the same time, the SCF is of course too small of a sample for some things, for example, estimating precise top shares within the top 1%, characterizing private businesses held at the top, unpacking the portfolios and returns of the ultra rich, and the geography of wealth.”

19. “the SCF may over-estimate top-end wealth. Using the benchmark Saez and Zucman (2016) methodology, a 2% wealth tax above $50 million and 3% above $1 billion would have generated slightly less revenue than implied by the SCF + Forbes, 190 billion vs. 202 billion in 2016. The reason is that total net wealth in the SCF exceeds total net wealth in the Financial Accounts, by about 10% (see Batty et al., 2019, Table 1). Saez and Zucman (2019) take a conservative approach to estimate wealth tax revenues and re-scale the SCF so that the SCF + Forbes aggregate matches the Financial Accounts total. This reduces the tax base by 10%–15% compared to the raw SCF + Forbes tax base. The notion that the SCF over-estimates top-end wealth by a factor of close to 2, however, has no empirical basis.” (p.13)

- Figure 1 shows that our top 0.01% and top 0.1% wealth share estimates exceed those of the harmonized SCF plus Forbes series.

- We address estimates of the collective wealth of the ultra rich in reply #1 and #5 in Chapter 4.

20. “Since the SZZ top 0.1% wealth share is close to the benchmark Saez and Zucman (2016) series before 2001 and only diverges from the early 2000s on, the inconsistency of SZZ with the available evidence for the 2001–2016 period is problematic…SZZ do not discuss or provide evidence that the Federal Reserve over-estimates the rise of inequality since 2001. In their Sections (10.3.1 and 10.3.3) comparing their results to the Federal Reserve data, and in their Appendix J, SZZ note that there are a number of conceptual differences between the Federal Reserve series and theirs (e.g., the unit of observation and the treatment of defined benefit pensions). These conceptual differences explain some of the SZZ vs. SCF difference in the level of the top 1% and top 0.1% wealth shares, but they do not explain the large differences in the 2001–2016 rise of these top shares.” (p.17)

- This comment is addressed in #1 in this section.

21. “Most of the difference between the rise of inequality according to SZZ vs. the Federal Reserve comes from the trend for the top 0.1%, not the next 0.9%. According to the official SCF results, the top 0.1% wealth share has increased 4.0 points between 2001 and 2016 (5.0 points when adding the Forbes 400), vs. 0.9 point in SZZ. The Distributional Financial Accounts do not report statistics for the top 0.1%.” (p.17, footnote 30)
This comment is addressed in #1 in this section.

22. “A recurring theme in SZZ (p. 9, p. 20, p. 38, pp. 44–45) is that the Financial Accounts substantially under-estimate the total value of business assets in the United States. SZZ, however, end up with essentially the same value for total business assets. The SZZ preferred estimate for the wealth of sole proprietorships + partnership + S-corporations + private C-corporations is 85% of national income in 2016 (SZZ Figure 6), of which 14% of national income is for private C-corporations (assumed to be 20% of all C-corporations). The conceptually equivalent number in the Financial Accounts is 79% of national income (partnerships plus sole proprietorships: 50%; S-corporations: 20%, private C-corporations: 9% of national income). Should we think of the SZZ estimates as being too low, or are the Financial Accounts totals accurate after all?” (p.19)

- Figure 2A plots our preferred estimates as a share of national income and compares them to the SZ 2020 Financial Accounts pass-through estimate which is lower in recent years.
- Figure 7A provides additional detail and shows how our estimates compare to those in the Financial Accounts and SCF.
Chapter 6

Other Comments
6.1 Relevant Excerpts from “Comments”

1. “SZZ do not properly discuss previous work that asked the same question, used the same data, applied similar key assumptions, and obtained similar results.” (p.1)

- Thanks for pushing us to improve the discussion of previous work. The introduction includes the following:
  - Prior work shows that allowing for interest rate heterogeneity materially reduces capitalized wealth shares in recent years. Kopczuk (2015) suggests that return heterogeneity is especially important when average returns are close to zero. Fagereng, Guiso, Malacrino and Pistaferri (2016) also challenge the equal returns assumption using administrative records from Norway to construct individual rates of return and show how this assumption biases the trend upward. Bricker, Henriques, Krimmel and Sabelhaus (2016) (BHKS) show that assigning the top 1% to have a higher interest rate—while also augmenting the SCF with the wealth of the Forbes 400 and reconciling the unit of measurement in the SCF from households to tax units—can close most of the gap between the SCF and capitalization series for the top 1% but leaves some gap unexplained for the top 0.1%. Building on this work with income tax data matched to the SCF, BHH show that adjusting for top-1% heterogeneity in interest rates narrows most of the gap between the SCF and the capitalization approach for the top 1% (e.g., BHH Figure 6) and about one third of the gap for the top 0.1% (e.g., Appendix Figure 14). To their credit, SZ do consider robustness analysis that assigns top groups modestly higher interest rates, which bring capitalization estimates down, although they use the equal-return approach for their headline results and subsequently in PSZ.
  - We discuss the relationship between our work and contemporaneous and subsequent work, including SZ, PSZ, BHKS, BHH, and Saez and Zucman (2020b), in Appendix L. The revisions in Saez and Zucman (2020b) result in a similar top 0.1% share compared to the SZ series (Appendix Figure A.1), partly because they account for a smaller degree of return heterogeneity than we find for fixed income (Figure 5A).

- We also discuss prior work in the sections for each asset class such as fixed income.
- We clarify our contribution relative to prior work.
  - By combining new data that links people to the sources of capital income and the firms they own with new methods that estimate the degree of heterogeneity, we substantially improve the estimates of rates of return and aggregate values of fixed income and private business wealth, which are the two biggest sources of discrepancies in wealth estimates in recent decades.
  - We provide direct evidence on the central assumption for capitalization—the extent of heterogeneity in returns. We find substantially more return heterogeneity than prior work. We document how detailed portfolio differences lead to return heterogeneity in fixed income and how accounting for human capital and business losses generate return heterogeneity for pass-through firms. Our new evidence draws from the universe of tax returns that generate taxable interest, dividends, capital gains, and pass-through business income.
  - Our new methods deliver standard errors that can be used to evaluate different heterogeneous-return assumptions and generate confidence intervals around capitalized wealth estimates.
Overall, we provide new wealth estimates, new evidence on rates of return, and a systematic analysis of the issues most consequential for capitalized wealth estimates.

2. “Under the benchmark Saez and Zucman (2016) methodology, the share of wealth owned by the top 0.1% richest adults rose from 6.5% in 1978 to 18.6% in 2016 (+12.1 points). Under the modified methodology used by SZZ, the share of wealth owned by the top 0.1% richest adults rose from 6.6% in 1978 to 14.3% in 2016 (+7.7 points). The different capitalization of interest explains the vast majority of the gap.” (p.2)

Table B.9 and B.10 present a systematic version of this exercise. While our estimated fixed income return heterogeneity is important, there are also important contributions from pass-through business, C-corporation equity, as well as housing and pensions for those outside the very top.

In addition, we plot top wealth share series that result from perturbing the preferred specification to include alternatives for each asset class in Figures 5C, 8C, 8D, 9C, and A.15.

3. “SZZ implement 3 other changes relative to the benchmark Saez and Zucman (2016) methodology: (i) housing... (ii) pensions... (iii) pass-through businesses... These changes, while useful to provide a more granular picture of wealth, do not materially affect the level of top wealth shares or their trend, since pensions and housing wealth are small at the top, and business assets turn out to be similar in size and distribution in SZZ as in the benchmark Saez and Zucman (2016) series” (p.2, footnote 3)

Table B.9 and B.10 show different approaches, such as changing wealth ranks due to allowing for business losses, can matter for top share estimates.

In addition, the perturbation figures mentioned in the previous reply show that these other categories do affect top shares as well.

4. “SZZ also include an estimate of unfunded defined benefit pensions in their measure of wealth (in contrast to Saez and Zucman 2016 who only include funded pension), which reduces wealth concentration but does not affect the level of top-end wealth nor wealth tax revenue estimates.” (p.2, footnote 3)

We include unfunded DB pensions for consistency with and similar reasons as Bricker, Henriques, Krimmel and Sabelhaus (2016), Bricker, Henriques and Hansen (2018), and the DFA.

We clearly note how aggregate wealth differs and present series that show the effects of using PSZ alternatives:

- “Aggregate wealth is 77 percentage points of national income higher than in PSZ, of which 40 p.p., 15 p.p., 10 p.p., and 12 p.p. are from unfunded defined benefit pensions, our bottom-up pass-through estimates, adjustments to non-mortgage debt, and residual updates.”

- “We plot an additional measure of pension and pass-through business wealth to compare our measures to those in other work. We show a pension series that excludes the unfunded portion of defined benefit pension wealth. We also show the Financial Accounts pass-through measure as defined in Saez and Zucman (2020b). Appendix Figure A.4 compares aggregates derived from the Financial Accounts in PSZ to those in the updated series with updated definitions in Saez and Zucman (2020b).”
• Figure A.15C plots the consequences of different pension approaches for the top 0.1% share.

5. “In their Section 10.3.2 and appendix J, SZZ instead provide a biased overview of the literature on the reliability of the Forbes ranking. They note that some billionaires have overstated their wealth to Forbes—but do not mention that Forbes misses some billionaires, since people above the Forbes 400 threshold but who do not appear in Forbes have been sampled by the SCF (Batty et al., 2019, Appendix F).” (p.9)

• Thanks very much for this point. We added it to our discussion of uncertainty in Forbes estimates in Appendix L.3.

6. “SZZ claim that the Forbes-400-based Pareto parameter of 1.4 in 2016 implies top 0.1%, top 1%, and top 10% wealth shares that line up more closely with their preferred estimates of top shares than with the Saez and Zucman (2016) benchmark estimates. However, the literature emphasizes that the Pareto approximation is only valid locally and that the Pareto coefficient is not constant from billionaires down to the top 10% threshold; see, e.g., Blanchet, Fournier, Piketty (2017) and references therein.” (p.10)

• This comment is addressed is addressed in #8 in Chapter 5.

7. “fixing two issues in the SZZ series closes 90% of the gap between the SZZ top 0.1% wealth share and the benchmark Saez and Zucman (2016) estimate:

• Using the conceptually correct interest rate to capitalize interest income increases the SZZ top 0.1% wealth share from 14.3% to 17.2% in 2016.

• Matching the amount of billionaire wealth implied by Forbes further increases the SZZ top 0.1% wealth share to 18.1%.

…They also bring the SZZ top 0.1% wealth share within less than 5% of the Saez and Zucman (2016) benchmark estimate (18.1 / 18.6 = 97%). In both level and trend, the SZZ top 0.1% wealth share becomes almost identical to the Saez and Zucman (2016) series.” (p.10)

• The evidence in the revised version—both using disaggregated fixed income data from billions of source-owner links and from new methods that match the covariance structure of the data—show that this assertion is not supported. We find that using the “conceptually correct” and empirically best fitting interest rates result in wealth shares that are well below the SZ series.

• Augmenting the data with Forbes does not increase top shares to 18%. We find it affects top shares by much less than that due in part to overlap with our pass-through business estimates. Owners of private businesses or dividend-paying public companies account for 77% of collective Forbes wealth in 2016. Therefore, adding the full amount of Forbes wealth will lead to considerable overcorrection since much of it is already reflected in capitalized estimates.

• Tables B.9 and B.10 walk through this exercise more systematically as mentioned above.

• We carefully describe how we arrive at these estimates in Appendix I and discuss the results in the main text as well as Appendix L.3.
8. “Coming after the interest rate correction, the billionaire correction adds less than 1.7 points (namely, about 1 point), because the interest rate correction already increases billionaire wealth.” (p.10, footnote 15)
   - Tables B.9 and B.10 walk through this exercise more systemically as mentioned above.

9. “SZZ do not adequately discuss the previous literature that asked the same question (how are the benchmark Saez and Zucman, 2016, estimates of US wealth inequality affected by returns heterogeneity?), used the same data, made similar key assumptions, and obtained similar results. SZZ are not the first to investigate the consequences of return heterogeneity for estimates of US wealth inequality based on income tax data—a fact that their current draft fails to make clear.” (p.13)
   - This comment is addressed in #1 in this section.

10. “SZZ do not appropriately discuss the closely related work by Bricker et al. (2018). Compared to Bricker et al. (2018), SZZ add the assumption that the top 0.1% by interest earn an even higher interest rate, the Moody’s Aaa corporate bond yield.” (p.13)
   - Thank you for pushing us to be clearer on innovations relative to Bricker et al (2018, 2019) (BHH and BHV, respectively).
     First, we discuss this important work more prominently in the abstract, introduction, data, comparison section, and Appendix.
     - In the introduction, we note that “Building on the capitalization approach in SZ and Piketty, Saez and Zucman (2018) (PSZ) and on insights in Bricker, Henriques and Hansen (2018) (BHH), we combine new data that links people to the sources of capital income and the firms they own with new methods that estimate the degree of heterogeneity within asset classes when mapping income flows to wealth.”
     - In the introduction, we also note that “BHH (2018) show that adjusting for top-1% heterogeneity in interest rates narrows the gap between the SCF and the capitalization approach for most of the gap for the top 1% (e.g., BHH Figure 6) and about one third of the gap (e.g., Appendix Figure 14) for the top 0.1%.”

   - **Fixed Income Estimates.** In terms of our first innovation, we build on their insight about the importance of heterogenous returns on fixed income by using new data and methods to estimate the degree of return heterogeneity on fixed income.
     Key to capitalization is having the right measure of interest rates. BHH use estimates of interest rate heterogeneity from (a) estate tax returns in Saez and Zucman, from (b) SCF interest and wealth data alone, and from (c) a matched data set that combines 6,200 SCF responses per year with administrative tax data from 2002-2016.
     We show that all three sources suffer from weaknesses that our approach overcomes. As mentioned above, each suffers from using the wrong denominator, a pooled measure in the numerator, and small samples, leaving considerable uncertainty about which interest rates should be used for capitalization and for whom. For instance, removing non-taxable-interest-generating assets from the denominator increases the rate of return in 2016 in the SCF for the top 0.1% wealth group from 2.3% (s.e.=0.4%) to 3.9% (s.e.=1.0%).
     Overall, given the sensitivity of wealth estimates to assumptions about the degree of return heterogeneity, we believe that providing new estimates of return heterogeneity (based on novel data and methods) advances our understanding of wealth inequality in America.
• **Pass-through Estimates.** A second innovation are the bottom-up estimates of pass-through business wealth using administrative firm-level data. As described above, pass-through business values in the SCF appear overvalued relative to Compustat multiples and market transactions for private firms. BHH reconcile capitalized top shares with the SCF under the assumption that SCF private business values are unbiased. Given the evidence we present on private business values, it’s unclear whether the fixed income refinements in BHH are sufficient to reconcile capitalized estimates with the SCF.

Whereas BHH find a relatively small role for reranking in affecting capitalized wealth estimates with return heterogeneity, we find a larger role for reranking because we identify a significant amount of pass-through wealth among those with low or negative taxable incomes.

• **Incorporating Other Components into Capitalized Estimates.** BHH integrate defined benefit pensions and BHV blend the Forbes 400 into the SCF. We adapt this approach to the tax data. We then show how augmenting capitalized estimates with Forbes data affects top wealth estimates and compares to other approaches.

11. “[t]he SZZ top 0.1% wealth share is close to the Bricker et al. (2018) estimate. The reader of SZZ cannot know this, since although Bricker et al. (2018) are cited by SZZ, their methodology and quantitative results are never discussed.” (p.14)

   • This comment is addressed in #10 in this section.

12. “SZZ briefly allude to aspects of Bricker et al. (2018) in their footnote 40, failing to note that Bricker et al. (2018) do not only focus on the top 1% but also investigate the implication of heterogeneous returns for the top 0.1% wealth share (see Bricker et al., 2018, figure 14), the main focus of SZZ.” (p.14, footnote 19)

   • This comment is addressed in #10 in this section.

13. “After quoting Bricker et al. (2018) together with a paper about Norway, SZZ (p. 6) write that “Our contribution is to build on these insights by implementing proposed adjustments in the tax data and combining them with other first-order refinements to all other major asset categories.” However, (i) Bricker et al. (2018) also implemented their insights in the US tax data, and (ii) the “other first-order refinements” of SZZ, whether individually or taken altogether, have a second-order effect on the level, trend, and composition of top wealth shares. 80% of the difference between the SZZ top 0.1% wealth share and the benchmark Saez and Zucman (2016) estimate is due to the different capitalization of interest income, an issue thoroughly investigated in previous work.” (p.14)

   • This comment is addressed in #1 in this section.

14. “As shown by SZZ Figure 14.B, the SZZ refinements of housing wealth increases top 0.1% wealth by 0.1 trillion in 2016 (0.15% of total wealth), reduces pension wealth by 0.2 trillion (0.3% of total wealth), increases business wealth by 0.4 trillion (0.6% of total wealth, see point 5.8 below), and reduces public equity wealth by 1.0 trillion (1.4% of total wealth), for a total net effect of -0.9% of total wealth. These adjustments are individually and collectively second-order compared to the benchmark Saez and Zucman (2016) top 0.1% wealth share of 18.6% in 2016.” (p.14, footnote 20)
6.1. RELEVANT EXCERPTS FROM “COMMENTS”

- This comment is addressed in #2 in this section.

15. “Using the 10-year Treasury yield to capitalize interest for the entire top 1%, as in Bricker et al. (2018), makes the SZZ series virtually identical to Bricker et al.’s (2018, Figure 14) top 0.1% wealth share.” (p.15)

- We respectfully disagree with this assertion based on new evidence on returns in the tail.
- Figure 5B compares our information return and minimum distance estimates to the top-Treasury-rate approach, showing they differ in terms of capitalization factors.

16. “One small methodological difference between SZZ and Bricker et al. (2018, Figure 14) is that for the purpose of estimating the top 0.1% wealth share, Bricker et al. (2018) capitalize interest income ranking people by wealth, while SZZ capitalize interest income ranking people by interest income. However, Bricker et al. (2018, Figure 4) show that for a given assumption about the interest rate, ranking by interest income vs. ranking by wealth makes virtually no difference.” (p.15, footnote 21)

- As discussed in Chapter 3, Bricker et al.’s measure of interest rates includes non-taxable-interest generating assets in the denominator, which leads to downward bias in measured rates. In addition, in the revised version, we do not rank by interest income but instead use disaggregated data from information returns and a risk-exposure approach.

17. “SZZ do note mention that Saez and Zucman (2016) constructed and discussed detailed appendix series showing how the level, trend, and composition of top wealth shares are affected when higher interest rates at the top are assumed. Appendix Tables B40, B41, B41b in Saez and Zucman (2016) assign the 10-year Treasury yield to the top 1% by income. Appendix Table B41c assigns the interest premium seen in matched estates-income tax data to the top 0.1% by wealth. These series are discussed pp. 549-551 of the published Saez and Zucman (2016) paper. The SZZ top 0.1% wealth share tracks the top 0.1% wealth share in Saez and Zucman’s (2016) Appendix Table B41c, see Figure below. SZZ misrepresent our work in their Section 10.1.1 (p. 35) by claiming these series were constructed in subsequent work of ours—they were in fact constructed and discussed in our original paper.” (p.15)

- We have edited this sentence to ensure that the timing of these appendix graphs is unambiguous.
- The initial manuscript in section 10.1.1 said “Saez and Zucman (2019a) have subsequently acknowledged this approach introduces bias and make an adjustment to the equal-returns specification by adopting a higher interest rate for their top wealth group.”
- We have edited references to clarify that the revision refers to the SZ headline estimates, rather than in a supplemental appendix series. We mention the existence of the appendix series in the introduction. We stress that incorporating higher rates for the wealthy in the headline series is key, as their baseline underlies the influential and often cited wealth and income inequality statistics.
- The initial version was referring to the following quotes that discuss the SZ 2016 paper relative to the Brookings paper (Fall, 2019) and Revisionists (October 2020), respectively (with our emphasis added in bold):
– SZ, 2016, pg 550: “We retain our baseline top 0.1% wealth share estimate because only a few hundred non-married individuals die with estates above $20 million each year. As a result, there is likely significant noise in the annual series, making it difficult to make a precise and systematic inference of the true interest premium at the top. Looking forward, should new evidence show that taxable returns rise or fall with wealth, then it would become necessary to specifically account for this fact—and similarly when applying the capitalization technique to other countries.”

– SZ, 2016, page 551: “If wealthy individuals were able to report abnormally high or low taxable returns in a systematic way, then assuming a constant capitalization factor within asset class would produce biased top wealth shares. In practice, however, taxable rates of returns appear to be roughly flat across wealth groups, the key condition for our method to produce unbiased results. The richest individuals might have recently benefited from an interest rate premium, perhaps leading to some overestimation of top wealth shares since 2008”

– Progressive Wealth Taxation (Fall 2019), page 452: “Overall, while somewhat noisy, the SCF data confirm the estate income tax data which shows that the interest rate for the wealthy tracks pretty closely the aggregate interest rate but is slightly higher. When interest rates are very low, as in recent years, this small difference translates into a significant difference in capitalization factors. Therefore, we revise the capitalization method to incorporate these empirical findings as we did in the earlier sensitivity analysis presented in Saez and Zucman (2016, 547–51 and appendix tables B41, B41b, and B41c). As in the Saez and Zucman (2016) appendix B41c series, we apply higher interest rates to the top 0.1 percent to match the interest rate differential observed in matched estate income tax returns for estates above $20 million. Concretely, this correction reduces the fixed-income claims owned by the top 0.1 percent by a factor of about two in recent years, consistent with the more recent SCF evidence depicted in figure 3.”

– Revising Revisionists (October 2020): page 2-3: “Our revised wealth series incorporate a higher interest rate for the wealthy than for the average household since 2008, consistent with the evidence from matched estates-income tax data from 1997 to 2012 analyzed in Saez and Zucman (2016) and extended to 2016 in Smith, Zidar and Zwick (2019). As a result, in our revised series interest-bearing assets play a smaller role in the portfolios of the rich than previously reported”

18. “SZZ also claim they “make a methodological contribution by clarifying how capitalization works in practice and by emphasizing both heterogeneity and the concomitant uncertainty that arises.” These issues were discussed in Saez and Zucman (2016), both conceptually and empirically, e.g., in Section III.B.1 titled “How the Capitalization Technique Works,” Section IV titled “Pros and Cons of the Capitalization Method”—including Section IV.A titled “Idiosyncratic Returns,” Section IV.B titled “Returns Correlated with Wealth,” and Section IV.E, e.g., discussion starting with “to assess the quantitative implication of the interest rate differential seen in matched estates-income tax data…” SZZ contribute no new data source or theory to these questions.” (p.15)

• Thanks for pushing us to clarify our contribution. We describe it in #1 of this section.

19. “We shared all our programs and data infrastructure with SZZ. We welcome replications
and extensions of our work and put all our code online for that purpose. It is essential for scientific progress, however, to clearly acknowledge prior work.” (p.15)

• Thank you for sharing programs and data with us and making them publicly available online, they have helped advance our understanding of inequality substantially.
• We hope the edits discussed above address your concern about clearly acknowledging prior work.

20. “SZZ misrepresent their key finding in their abstract. They write: “We find that the top 0.1% share of wealth increased from 7% to 14% from 1978 to 2016. While this rise is half as large as prior estimates…” In fact, the true ratio is not half but 64%.” (p.16–17)

• We updated the abstract and introduction to be more precise on this point and to report the results using our revised series:
  – Abstract: “From 1989 to 2016, the top 1%, 0.1%, and 0.01% wealth shares increased by 7.6, 5.1, and 3.0 percentage points, respectively, to 31.5%, 15.0%, and 7.0%. While these changes are less dramatic than some prior estimates, wealth is very concentrated: the top 1% holds nearly as much wealth as either the bottom 90% or the “P90-99” class.”
  – Introduction: “We find less wealth concentration relative to the equal-returns, individual-level approach in PSZ, especially at the very top. Figure 1A shows that the top 0.1% wealth share in 2016 is 15% under our approach, and around 20% in PSZ. Top 1% and 0.01% shares fall by 24 percent and 36 percent, respectively, leaving the recent wealth estimates above the estate tax series and closer to the SCF. The growth in top wealth shares is also less dramatic, especially in the tail. For example, our approach reduces the growth in top 0.01% shares since 1989 by 45%.”

21. “SZZ provide an inconsistent treatment of unfunded defined benefit pensions. SZZ include $1.9 trillion in unfunded defined benefit pension wealth in their series (SZZ, footnote 6 p. 8), 30% of the total amount of unfunded defined benefit pensions recorded in the Financial Accounts ($6.5 trillion in 2016). However, when they compare their series to the SCF, SZZ add all unfunded defined benefit pensions to the SCF ($6.5 trillion in 2016). Unfunded pensions have increased since 2001 (from $2.1 trillion in 2001, 22% of national income, to 6.5 trillion in 2016, 40% of national income), and almost none of this wealth goes to the top. As a result, the comparison between the SZZ top wealth shares and the SCF top wealth shares (adjusted to incorporate unfunded pensions) reported in Figure A.16 are biased.” (p.17)

• Thank you for pushing us to be clearer and more consistent in the treatment of unfunded defined benefit pensions. In our revised draft, the $1.9T figure is no longer relevant. We describe the contribution of unfunded DB pensions in #4 in this section.

22. “SZZ do not provide any justification for including 1.9 trillion in unfunded defined benefit pensions (30% of the Financial Accounts aggregate) in their series while including 100% of the Financial Accounts aggregate in the SCF. In both cases the same source, Sabelhaus and Henriques Volz (2019), is cited. Sabelhaus and Henriques Volz (2019) do not provide estimates for unfunded DB pensions separately from funded DB pensions. The Saez and Zucman (2016) series do not include unfunded defined benefit pensions (for the reasons discussed in Saez and Zucman 2016, p. 525–526).” (p.17–18)
23. “After fixing the identified biases in the SZZ methodology and adopting a consistent treatment of unfunded defined benefit pensions, the SZZ top 0.1% wealth share series is identical to the benchmark Saez and Zucman (2016) estimate. For instance, starting from the SZZ top 0.1% wealth share of 14.3% in 2016, removing the bias in the interest capitalization adds 2.9 points and matching the evidence on billionaire wealth adds close to an extra point, bringing the SZZ top 0.1% wealth share to 18.1% (see point 2 above). Removing unfunded defined benefit pensions increases the top 0.1% wealth share to 18.6%, identical to the Saez and Zucman (2016) estimate. Alternatively, starting from the Saez and Zuman (2016) level of 18.6% and incorporating 1.9 trillion in unfunded defined benefit pensions as in SZZ, the top 0.1% wealth share falls to 18.1%, identical to the SZZ estimate corrected for interest and billionaires. Whether unfunded pensions are included in wealth is immaterial for the level of top-end wealth and wealth tax revenues. Therefore, after fixing the identified biases in the SZZ methodology, the SZZ series generate the same amount of wealth tax revenue as the benchmark Saez and Zucman (2016) series, and are reconciled with the Federal Reserve data on the level of top-end wealth.” (p.18)

24. “Another recurring theme in SZZ is that business assets at the top play a more important role in their series than in the benchmark Saez and Zucman (2016) series (e.g., p. 3: “We find a larger role for private business wealth and a smaller role for fixed income wealth”). In fact, this finding is the mechanical consequence of using an inconsistent definition of business wealth. In their series, SZZ assume that 20% of C-corporation equity wealth corresponds to private equity, and add that amount to business wealth. When they compare their results to the benchmark Saez and Zucman (2016) series, they do not make that adjustment. When 20% of C-corporation equity wealth is added to business wealth in the benchmark Saez and Zucman (2016) series, there is a comparable amount of business wealth in the top 0.1%: 3.9 trillion in 2016 in SZZ vs. 3.5 trillion in equal-returns series (see SZZ Figure 14 Panel B.).” (p.20)

25. “SZZ under-estimate pension wealth at the top. SZZ have $1.9 trillion in pension wealth for the top 1% in 2016. This is lower than in the existing evidence. According to the SCF (which only includes defined contribution pensions) supplemented by the Sabelhaus and Henriques-Volz (2019) estimates of defined benefit pensions, the top 1% wealthiest tax units had $2.9 trillion in pension wealth in 2016, and the top 1% adults had $2.6 trillion (SZZ Figure 14 Panel C.).” (p.20–21)
6.1. RELEVANT EXCERPTS FROM “COMMENTS”

- Figure A.18D compares the evolution of pension wealth held by top groups. Note that following a comment by another referee, we updated the pension model, so the specific statistics from the previous manuscript have changed.

26. “SZZ rely on taxable pension distributions to allocate the defined-contribution pension wealth of retirees (see SZZ p. 39), despite the fact that some of the more concentrated forms of defined contribution pensions (e.g., Roth IRAs) do not generate taxable income. To match the level of pension wealth at the top seen in the SCF, it is necessary to allocate part of pension wealth proportionally to non-taxable pension income, as done in Saez and Zucman (2016). One may debate what weight to put on non-taxable pension distributions (vs. taxable distributions), but a 0 weight, as used by SZZ, fails to match the SCF evidence on the distribution of DC pension—the only evidence on the distribution of DC pension wealth in the US.” (p.21)

- SZ include nontaxable pension rollovers in their measure of pension income, which tends to overstate the concentration of pension wealth because rollovers are stock rather than flow measures and disproportionately accrue to the top.

- In contrast, we only use taxable pension distributions to estimate pension wealth. We compare our estimates of pension wealth with the harmonized SCF in Figure 13 and other figures, which show a closer relationship between our estimate in Panel D than that of PSZ.

- SZ 2020 update this treatment and reduce but do not eliminate the weight on rollovers. Chapter 10, comment #4, provides more detail on this update.
Chapter 7

Theoretical Analysis of Capitalization Bias

Although the fixed income capitalization approach no longer uses a capitalization approach that ranks by interest income, which is the primary issue in question in the theoretical analysis of capitalization bias in SZ (2020), the following discussion highlights the crucial features and assumptions underlying the arguments offered in the theoretical analysis of capitalization bias in SZ (2020). It shows that SZ’s theoretical arguments depend on strong assumptions and may not apply in several empirically relevant cases.

This appendix chapter shows that addressing the following issues can reverse SZ’s argument that favors using \( r \) (for those at the top of the wealth distribution) rather than using \( \bar{r} \), which is the rate of return of people at the top of the interest income distribution. These four issues are: (1) Assuming independence between \( W_i \) and \( (\beta_i, r_i) \), (2) assuming zero correlation between \( \beta_i \) and \( r_i \), (3) using a first-order approximation with very low interest rates, and (4) SZ’s numerical example is not robust.

After presenting the theoretical arguments in SZ in Section 7.1, these four issues are described in more detail in Section 7.1.4. We then show the consequences of relaxing some of these assumptions in Section 7.2. Finally, we present some additional expressions and implied restrictions (on the distribution of interest income and wealth) for the benefit of researchers who may be interested in testing some of these restrictions empirically in future work.

7.1 The Arguments of Saez and Zucman (2020, Appendix A)

We focus on the top \( p \) percentile people of the wealth distribution, and we are interested in how much wealth they have. Let \( W_i \) be person \( i \)’s total wealth. We define \( \beta_i \) as the share of interest bearing assets in person \( i \)’s portfolio, so that \( \beta_i W_i \) is the amount of interest-bearing assets and \( (1 - \beta_i)W_i \) is the other assets. \( r_i \) is the rate of return on bonds of person \( i \) so that the interest income is \( r_i \beta_i W_i \). In particular, let \( r \) denote the average rate of return in the top \( p \) percentile of the wealth distribution, and let \( \bar{r} \) be the average rate of return in the top \( p \) percentile of the interest income distribution. This part of the debate is about which average interest rate is more appropriate for inferring the wealth distribution.

7.1.1 Proving a mathematical result

SZ (2020) provide a formal model to justify the use of \( r \). Their argument relies on a lemma from a formal model. We first lay out assumptions and then state their mathematical result. First, they assume a Pareto wealth distribution at the top.

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\(^1\)We thank Atsushi Yamagishi for outstanding research assistance on this chapter.
Assumption 1. **Pareto wealth distribution at the top**: \( Pr(W_i ≥ W) = 1 − F(W) = p_0 · (W_{p_0}/W)^α \), where \( α \) is the shape parameter and \( p_0^{1/α}W_{p_0} \) is the scale parameter.

In words, the top \( p_0 \) percentile of the wealth distribution (i.e., those above \( W_{p_0} \)) is Pareto with the shape parameter \( α \). SZ (2020) consider a mean-preserving transformation of the wealth distribution. Formally, letting \( W'_i = e_iW_i \), they impose the following on \( e_i \):

**Assumption 2. Mean-preserving transformation of the wealth distribution**: Let \( W'_i = e_iW_i \) be a transformation of wealth distribution, where (i) \( E(W'_i) = E(W_i) \), (ii) \( e_i \) is bounded (i.e., \( e_i < \bar{e} \) where \( \bar{e} \) is finite), and (iii) \( e_i \) is independent of \( W_i \).

(i) comes for free as long as the focus is on the mean-preserving transformation. (ii) is also a technical assumption given that there is no infinite wealth in reality. However, (iii) is crucial and can be restrictive. Essentially, this implies that the transformation must be orthogonal to the amount of wealth, so that the transformation is equivalent to adding a random noise to wealth distribution.\(^2\)

The final assumption is on the distribution of parameters \( (W_i, β_i, r_i) \):

**Assumption 3. Restrictions on the distribution of parameters**: At the top of the wealth distribution, (i) the distribution of \( β_i \) and \( r_i \) conditional on \( W_i \) converges at the top of the wealth distribution (ii) \( β_i \) and \( r_i \) are iid.

Although it is somewhat unclear what Assumption 3(i) exactly means since the term “convergence” is used ambiguously, most likely, this assumption means that the joint distribution of \( (β_i, r_i) \) is independent of \( W_i \).\(^3\) However, it seems contradictory to evidence that the interest rate might be higher for the super wealthy people.\(^4\) Importantly, SZ’s mathematical result cannot be applied to the arguments in section 7.1.2 without imposing the independence between \( W_i \) and \( (β_i, r_i) \) at the top. Assumption 3(ii) is not needed for the lemma in this section, but it is invoked in applying the lemma to SZ’s argument (see equation (7.2) and the accompanying proof in this note).

Based on these assumptions, SZ prove the following lemma. To make the exposition simple, we break SZ’s lemma into the following two lemmas. Proofs are presented in the Appendix to this note.

**Lemma 1. Part 1 of SZ’s (2020, Appendix A) lemma**: Suppose Assumptions 1, 2, and 3 hold. Then, the right tail of the distribution of \( W'_i \) is also Pareto with the shape parameter \( α \) and the scale parameter \( (p_0E(e_0^α))^{1/α}W_{p_0} \).

**Lemma 2. Part 2 of SZ’s (2020, Appendix A) lemma**: Suppose Assumptions 1, 2, and 3 hold. Then, the wealth share of the top \( p \) percentile is

\[
sh'_p = sh_p · (E(e_0^α))^{1/α},
\]

where \( sh_p \) denotes the top \( p \) percentile’s wealth share under \( W_i \) and \( sh'_p \) is that under \( W'_i \). In words, the top wealth share for \( W_0 \) is corrected with the power mean of \( e_i \) with coefficient \( α \).

### 7.1.2 Applying the mathematical result to justify SZ’s argument

SZ then apply the above Lemma to SZ and SZZ estimators.

**Case 1: Capitalization by \( r \) (SZ)**: Suppose we capitalize using \( r \), the average interest rate among the top \( p \) percentile wealth holders. Person \( i \)’s interest income is \( r_iβ_iW_i \). If we use \( r \) to recover the

\(^2\)To see this, note that \( ln(W'_i) = ln(W_i) + ln(e_i) \). The independence between \( W_i \) and \( e_i \) implies that \( ln(e_i) \) is a random perturbation.

\(^3\)In footnote 43, SZ state that the distribution of \( W_i \) is independent of \( r_i \) and \( β_i \) in the upper tail.

\(^4\)This argument also appears in Saez and Zucman (2016, QJE), as shown in #17 in section 6.1 above.
amount of interest-bearing assets, it is \( r_i \beta_i W_i / r \). Person \( i \) also holds \((1 - \beta_i)W_i\) of other assets, which are assumed to be perfectly observable. Letting \( e_i = \beta_i r_i / r + 1 - \beta_i \) and applying lemma 2, we have

\[
sh_p^{SZ} = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i / r)^a])^{1/a} \geq sh_p, \text{ where } r = E(r_i) \tag{7.1}
\]

The inequality implies an upward bias of the top share.\(^5\)

**Case 2: Capitalization by \( \bar{r} \) (SZZ):** Suppose we capitalize using \( \bar{r} \), the average interest rate among the top \( p \) percentile interest income earners. Letting \( e_i = \beta_i r_i / \bar{r} + 1 - \beta_i \) and applying lemma 2, we have

\[
sh_p^{SZZ} = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i / \bar{r})^a])^{1/a}, \text{ where } \bar{r} = \frac{Er_i^a}{Er_i^{a-1}} \geq r \tag{7.2}
\]

The proof that \( \bar{r} = \frac{Er_i^a}{Er_i^{a-1}} \) is given in the Appendix to this note.\(^6\) Since \( \bar{r} > r \) would understate the wealth share more than the SZ approach does, we cannot, in general, determine whether SZZ overestimate or underestimate the true wealth share.

### 7.1.3 Which interest rate to use? A Comparison of SZ and SZZ estimates.

Based on (7.1) and (7.2), SZ compare the performance of the two estimators in the following special cases:

**Case 1:** Pareto shape parameter \( a = 1 \). In this case, the power mean in (7.1) and (7.2) becomes the simple mean. Thus, SZ gives the exact estimate while SZZ underestimates the wealth share. However, \( a = 1.4 - 1.5 \) is the relevant values in the US. Larger \( a \) might favor the SZZ approach since SZZ’s high \( \bar{r} \) might counteract the overestimation arising from the use of the power the mean in (7.1) and (7.2).\(^7\)

**Case 2:** Small bond share (\( \beta_i \approx 0 \)). In this case, the bias in (7.1) is second order while it is first order in (7.2). Thus, SZ gives the exact estimate while SZZ underestimates the wealth share.

Let us first look at SZ. With \( \beta_i \approx 0 \), we have \((1 + \beta_i (r_i / r - 1))^a \approx 1 + a \beta_i (r_i / r - 1)\) by the first-order approximation.\(^8\) Applying this formula to (7.1), we get \((E[(1 - \beta_i + \beta_i r_i / r)^a])^{1/a} \approx (1 + a \beta E(r_i / r - 1))^{1/a} = 1\), implying that SZ has no bias.\(^9\)

Applying the same argument to SZZ, we have \((E[(1 - \beta_i + \beta_i r_i / \bar{r})^a])^{1/a} \approx (1 + a \beta E(\bar{r}/r - 1))^{1/a} \approx 1 - \beta (1 - \bar{r}/r)\) (the last expression uses first-order approximation again). The intuition given by SZ is as follows: when the share of bonds is small, ranking with or without estimated bonds is pretty much the same, and therefore to a first approximation the SZZ method just creates a straight downwards bias.

**Case 3:** An illustrative numerical example: Let \( r_i \) take \( \bar{r} \) with probability \( \lambda \) and 0 otherwise. The average rate \( r = \lambda \bar{r} \) while the rate on high interest earners is \( \bar{r} \). Also assume \( \beta_i = \beta \ \forall i \). Substituting

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\(^5\)This overestimation result comes from the property of the power mean, where the weights are the equal sampling weight \( 1/n \). See, for example, the following Wikipedia page [https://en.wikipedia.org/wiki/Generalized_mean](https://en.wikipedia.org/wiki/Generalized_mean). Also see Schaumberg (1988) “Another Proof of the Inequality between Power Means” The College Mathematics Journal, 19(1), 56-58.

\(^6\)While \( \bar{r} \geq r \) is almost obvious from the definition, it can be formally proven in this context. Note that \( E(r_i^n) \geq E(r_i^{n-1})E(r_i) \) because the \( \text{cov}(r_i^{n-1}, r_i) \geq 0 \) for \( a > 1 \). Dividing the inequality by \( E(r_i^{n-1}) \) and noting \( \bar{r} = \frac{Er_i^n}{Er_i^{n-1}} \) and \( r = E(r_i) \) proves \( \bar{r} \geq r \).

\(^7\)See footnote 44 of SZ (2020) on this point.

\(^8\)Higher order terms might be important, especially given \( r \) is small.

\(^9\)Note that the expectation is taken separately for \( \beta_i \) and \( r_i \), that is, independence of \( r_i \) and \( \beta_i \) is invoked.
these into (7.1) and (7.2),

$$sh_{SZ}^p = sh_p \cdot \left( (1 - \lambda) (1 - \beta)^a + \lambda \cdot \left[ 1 - \beta + \frac{\beta}{\lambda} \right]^a \right)^{\frac{1}{a}}$$

$$sh_{PZZ}^p = sh_p \cdot \left( (1 - \lambda) (1 - \beta)^a + \lambda \right)^{\frac{1}{a}}$$

SZ use $\lambda = 0.5$ and $a = 1.5$. When $\beta = 0.2$, they find that SZ approach introduces only 1% of upward bias, while SZZ approach introduces 10% downward bias. When $\beta = 0.4$, SZ approach yields 4% upward bias while SZZ approach leads to 19% downward bias. SZ mention that this result is because the first order approximation is still valid even if $\beta$ deviates from zero.

### 7.1.4 Four Issues with SZ’s theoretical argument

Having gone through the argument of SZ, we think addressing the following issues can reverse SZ’s argument that favors $r$ rather than $\bar{r}$.

1. Independence between $W_i$ and $(\beta_i, r_i)$. Without the independence, Lemma 1 and 2 (defined below) can no longer be used since $e_i = \beta_i r_i / r + 1 - \beta_i$ is not independent of $W_i$.

2. No correlation between $\beta_i$ and $r_i$. Relaxation of this assumption changes the sign of biases. Most notably, Case 2 of section 7.1.3 relies on this assumption.

3. The quality of first-order approximations, which is relevant since $r$ is potentially low.

4. The extent of overestimation in SZ’s approach (equation 7.1). In some cases, SZZ’s high interest rate might counteract SZ’s overestimation bias and perform better (although this does not happen in SZ’s numerical example).

### 7.1.5 Appendix: Proofs

**Proof of Lemma 1**

Let $F^\prime()$ be the distribution function of $W_i^\prime$ (NOTE: $F^\prime$ does not mean a density function). Let $G()$ denote the distribution function of $e_i$, which is assumed to be independent of $W_i$ (Assumption 3(i)).\(^{10}\)

For a given value of $W$, we have

$$1 - F^\prime(W) = Pr(e_i W_i \geq W) = \int_{e_i=0}^{\hat{e}} Pr(W_i \geq W/e_i) dG = \int_{e_i=0}^{\hat{e}} p_0 \cdot (W_{p0}/W)^a dG,$$

where the second equality uses the independence of $W_i$ and $e_i$, and the third equality uses Assumption 1.\(^{11}\)

Putting parameters outside of the integral, we have

$$1 - F^\prime(W) = p_0 (W_{p0}/W)^a \int_{e_i=0}^{\hat{e}} e_i^a dG = (1 - F(W)) \cdot E(e_i^a).$$

\(^{10}\)SZ use the notation $E()$, but we avoid this notation since it conflicts with the expectation operator.

\(^{11}\)Here, we assume $W_i/\hat{e} \geq W_{p0}$ so that the assumption of Pareto distribution at the top (i.e., Assumption 1) can be used here.
From this, we can explicitly write down \( F'(W) \), showing that \( W' \) follows Pareto distribution with the shape parameter \( a \) and the scale parameter \((p_0E(e_i^a))^{1/a} W_{p_0}^{a}\).

**Proof of Lemma 2**

Let \( W_p \) (resp. \( W'_p \)) be such that \( 1 - F(W_p) = p \) (resp. \( 1 - F'(W'_p) = p \)). Then,

\[
1 - F(W_p) = 1 - F'(W'_p) = p_0 E(e_i^a)(W_{p_0}/W_p)^a = p_0 (E(e_i^a)^{1/a} W_{p_0}/W_p)^a = 1 - F(W'_p/(E(e_i^a))^{1/a})
\]

so that \( W_p = W'_p/(E(e_i^a))^{1/a} \). Hence, the amount of the wealth top \( p \) percentile possesses under \( W'_p \) is \((E(e_i^a))^{1/a}\) times that under \( W_i \). Since the total amount of the wealth in the economy is unaffected by the mean-preserving transformation, the wealth share is written as \( sh'_p \geq sh_p \cdot (E(e_i^a))^{1/a} \).

**Proof of \( \bar{r} = \frac{E r_i^a}{E r_i^{a-1}} \) in (7.2)**

Let \( y_p \) be the \( p \) percentile threshold in the distribution of interest income \( y_i = ri \beta_i W_i \). Let \( H \) be the joint distribution function of \((r_i, \beta_i)\).

Then,

\[
\bar{r} = \frac{\int r_i \beta_i W_i \geq y_p dHdF}{\int r_i \beta_i W_i \geq y_p \beta_i W_i dFdH} = \frac{\int r_i \beta_i (\int W_i \geq y_p/(r_i \beta_i) W_i dF)dH}{\int r_i \beta_i (\int W_i \geq y_p/(r_i \beta_i) W_i dF)dH}
\]

Noting that \( dF = p_0 a W_i^a dW_i^{1+a} \), we have

\[
\int r_i \beta_i (\int W_i \geq y_p/(r_i \beta_i) W_i dF)dH = \int r_i \beta_i (\int W_i \geq y_p/(r_i \beta_i) p_0 a W_i^a dW_i^{1+a} W_i dF)dH
\]

\[
= \int r_i \beta_i (\int W_i \geq y_p/(r_i \beta_i) W_i^{-a} dW_i^{1+a}) dH = \int r_i \beta_i (\int W_i \geq y_p/(r_i \beta_i) W_i^{-a} dW_i^{1+a}) \frac{\infty}{\int r_i \beta_i} dH
\]

where we have assumed \( a > 1 \) to obtain the last equality.

Now we use the independence of \( r_i \) and \( \beta_i \) (Assumption 3(ii)). Let \( R \) and \( B \) be the corresponding distribution functions so that \( H(\beta, r) = B(\beta)R(r) \). Then,

\[
\int r_i \beta_i a \beta_i dH = \int r_i \beta_i (\int \beta_i a \beta_i dB)dR = \int r_i a \beta_i dR = \frac{E(r_i^a)}{E(r_i^{a-1})}.
\]

---

12 We impose the independence of \((r_i, \beta_i)\) later in this proof, but we keep this generality for now.
CHAPTER 7. THEORETICAL ANALYSIS OF CAPITALIZATION BIAS

7.2 Relaxing assumptions

7.2.1 Allowing for $\text{cov}(\beta_i, r_i) \neq 0$

We consider a potential correlation between $\beta_i$ and $r_i$. Indeed, we might naturally expect $\text{cov}(\beta_i, r_i) > 0$ because those with higher interest rate $r_i$ would want to hold more interest-bearing assets in their portfolio. First, note that Lemmas 1 and 2 do not utilize $\text{cov}(\beta_i, r_i) = 0$ (please see the proofs in section 1.5). Thus, formulae (7.1) and (7.2) continue to apply. However, the formulae in cases 1 and 2 (section 7.1.3) are modified in the following way. In sum, the SZ formula always overestimates the wealth share while the SZZ formula provide smaller estimate than the SZ does. Whether the SZZ lead to underestimation is ambiguous.

**Modified Case 1: Pareto shape parameter ($a = 1$).**

**Modifying SZ formula:**

The coefficient of the formula (7.1) is $1 - E(\beta_i) + E(\beta_i r_i)/r = 1 + E(\beta_i)(E(r_i)/r - 1) + \text{cov}(\beta_i, r_i)/r = 1 + \text{cov}(\beta_i, r_i)/r > 1$.

Thus, unlike the case of the no correlation, the SZ formula still exhibits an upward bias. The upward bias might be intuitive: given a certain amount of the interest income, $\beta$ between where $\sigma \beta$ observed interest income might not translate into a huge aggregate wealth, both due to high having a lot of interest-bearing assets have a high interest rate. It implies that that even a sizable +1.5), Thus, formulae (7.1) and (7.2) continue to apply.

$\text{cov}(\beta_i, r_i)$ denotes the standard deviation. Thus, the upward bias is significant when (i) the correlation between $\beta_i$ and $r_i$ is large, (ii) the average interest rate $r$ is small, (iii) the variation of $\beta_i$ is large, and (iv) the variation of $r_i$ is large.

**Modifying SZZ formula:**

The coefficient of the formula (7.2) becomes $1 - E(\beta_i) + E(\beta_i r_i)/\bar{r} = 1 + E(\beta_i)(\bar{r}/\bar{r} - 1) + \text{cov}(\beta_i, r_i)/\bar{r}$.

Since the second term is negative while the third term is positive, we cannot determine whether SZZ underestimates or overestimates the wealth. However, since $\text{cov}(\beta_i, r_i)/\bar{r} < \text{cov}(\beta_i, r_i)/r$, the SZZ formula gives a smaller estimate than the SZ formula. Given that SZ always overestimates the wealth under $\text{cov}(\beta_i, r_i) > 0$ and the magnitude of the overestimation might be large, the SZZ approach might significantly mitigate the upward bias. The intuition is as follows. Since the SZZ estimate uses the interest rate from the top interest income earners, it might better measure the interest rate under $\text{cov}(\beta_i, r_i) > 0$. To see this point, suppose $W_i$ is (almost) the same for everyone at the top and $\rho(\beta_i, r_i) = 1$ (perfect correlation). In this case, focusing on the interest income distribution to form the capitalization rate accurately measures the relevant interest rate because the interest income distribution is perfectly informative of the ranking of $r_i$. On the other hand, forming the capitalization rate based on the ranking of $W_i$, which is uninformative of $r_i$, does not take into account the fact that high interest income earners tend to have higher interest rate, resulting in the upward bias.

**Modified Case 2: Small bond share ($\beta_i \approx 0$).**

**Modifying SZ formula:**

First consider the SZ formula (7.1). With $\beta_i \approx 0$, we have $((1 + \beta_i(r_i/r - 1)))^a \approx 1 + a\beta_i(r_i/r - 1)$ by the first-order approximation.

---

13However, the explicit formula for $\bar{r}$ in (7.2) no longer applies because it invokes the independence between $\beta_i$ and $r_i$.

14Since we do not observe the individual $r_i$, however, we still have to rely on an average interest rate like $\bar{r}$.
7.2. RELAXING ASSUMPTIONS

Thus, unlike the case of the no correlation, the SZ formula still exhibits an upward bias. Except for $a$ in the second term and that entire expression is to the power of $1/a$, the expression is essentially the same as Case 1.

**Modifying SZZ formula:**
Following the same calculation as above, we have

\[
\left( E\left(1 - \beta_i + \beta_i r_i/r\right)^a \right)^{1/2} \approx \left( E\left[1 + a\beta_i(r_i/r - 1)\right]\right)^{1/2} = (1 + aE(\beta_i r_i/r - aE(\beta_i)))^{1/2} = (1 + aE(\beta_i)/E(r_i/r - 1) + (a/r)\text{cov}(\beta_i, r_i))^{1/2} = (1 + (a/r)\text{cov}(\beta_i, r_i))^{1/2} > 1.
\]

This expression is almost the same as Case 1, except that the second and third terms have $a$ and the entire expression is to the power of $1/a$. Therefore, the properties of the SZZ formula in this case is analogous to that in Case 1.

**The case of negative correlation** ($\text{cov}(\beta, r) < 0$).

Our argument so far assumes the positive correlation. When $\text{cov}(\beta, r) < 0$, the SZ formula in cases 1 and 2 underestimates the true wealth. The SZZ formula in cases 1 and 2 also underestimates the true wealth. However, we cannot determine whether SZ or SZZ formula has the larger negative bias. To illustrate this, consider the formula in case 1: $1 + E(\beta_i)/E(r_i/r - 1) + \text{cov}(\beta_i, r_i)/\hat{r}$. On one hand, the presence of the second term pushes down the SZZ estimate compared with the SZ. On the other hand, the third term is smaller in absolute value since $\hat{r} > r$. This result is in contrast to the case of positive correlation because in the positive correlation case, we can show that SZZ always yields smaller wealth estimate than SZ.

### 7.2.2 Second-order approximation to the formula

Case 2 of section 7.1.3 involves a first-order approximation. Here, we derive the formula using the second-order approximation. By extending the results of the previous subsection, we allow for the correlation between $\beta_i$ and $r_i$. Suppose $\beta_i(r_i/r - 1)$ is close to zero (assuming $\beta_i = 0$ achieves this purpose in the case 2 of section 7.1.3). Then, we have $(1 + \beta_i(r_i/r - 1))^a \approx 1 + a\beta_i(r_i/r - 1) + \frac{a(a-1)}{2}r_i^2(r_i/r - 1)^2$ by the second-order approximation around 1.

**Modifying SZ formula:**
Using the second-order approximation, we have

\[
\left( E\left[(1 - \beta_i + \beta_i r_i/r)^a \right]^3 \right)^{1/3} \approx \left( E\left[1 + a\beta_i(r_i/r - 1) + \frac{a(a-1)}{2}r_i^2(r_i/r - 1)^2 \right]\right)^{1/3} = (1 + aE(\beta_i r_i/r) = aE(\beta_i))/E(r_i/r - 1) + (a/r)\text{cov}(\beta_i, r_i))^{1/3} = (1 + (a/r)\text{cov}(\beta_i, r_i))^{1/3}.
\]

The first and second terms are the first-order impacts. As for the second-order terms, the third term
is positive. The sign of the fourth and fifth terms depend on the correlation between $\beta_i$ and $r_i$. If $\text{cov}(\beta_i, r_i) > 0$, the fourth term is positive while the fifth term is negative. Since there are conflicting terms, we cannot generally determine the sign.

In the special case where $\beta_i$ and $r_i$ are independent, we have $\text{cov}(\beta_i, r_i) = \text{cov}(\beta^2_i, r_i) = \text{cov}(\beta^2_i, r_i^2) = 0$. Thus, the formula becomes $1 + \frac{a(a-1)}{2}E(\beta^2_i)E(r^2_i/r^2 + 1 - 2r_i/r) > 1$, so that SZ has a second-order positive bias.

Modifying SZZ formula:

Using the second-order approximation, we have

$$
(1 - \beta_i + \beta_i r_i/\bar{r})^2 \approx (1 + a\beta_i(r_i/\bar{r} - 1) + \frac{a(a-1)}{2}\beta^2_i(r_i/\bar{r} - 1)^2)^2
$$

$$
= (1 + aE(\beta_i r_i/\bar{r}) - aE(\beta_i) + \frac{a(a-1)}{2}E(\beta^2_i r_i^2/\bar{r}^2) - a(a-1)E(\beta^2_i r_i/\bar{r}) + \frac{a(a-1)}{2}E(\beta^2_i))^2
$$

$$
= [1 + aE(\beta_i)(r_i/\bar{r} - 1) + (a/\bar{r})\text{cov}(\beta_i, r_i) + \frac{a(a-1)}{2}E(\beta^2_i)E(r^2_i/\bar{r}^2 + 1 - 2r_i/\bar{r})
+ \frac{a(a-1)}{2F^2}\text{cov}(\beta^2_i, r^2_i) - \frac{a(a-1)}{\bar{r}}\text{cov}(\beta^2_i, r_i)]
$$

Except for the second term, we cannot determine the sign of each term. Thus, the sign of the bias of the SZZ formula is ambiguous. Again, if we invoke the independence of $\beta_i$ and $r_i$, we have $[1 + aE(\beta_i)(r_i/\bar{r} - 1) + \frac{a(a-1)}{2}E(\beta^2_i)E(r^2_i/\bar{r}^2 + 1 - 2r_i/\bar{r})]^2$. The second term is a negative first-order bias while the third term is a second-order bias that is ambiguous in sign.

### 7.3 Other useful expressions

#### 7.3.1 Formulae for quantifying the biases

We derive the expressions for biases. We focus on the general SZ and SZZ formulae (7.1) and (7.2).

**Bias in SZ estimate:**

$$
\text{bias}_{SZ} = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i/\bar{r})^2])^{\frac{1}{2}} - 1.
$$

(7.3)

The bias expression becomes neat if $a = 1$: $\text{bias}_{SZ} = sh_p \cdot \text{cov}(\beta_i, r_i)/r$.

**Bias in SZZ estimate:**

$$
\text{bias}_{SZZ} = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i/\bar{r})^2])^{\frac{1}{2}} - 1.
$$

(7.4)

The bias expression becomes neat if $a = 1$ : $\text{bias}_{SZZ} = E(\beta_i)(r_i/\bar{r} - 1) + \text{cov}(\beta_i, r_i)/\bar{r}$.

#### 7.3.2 SZ’s Implicit assumptions on the distribution of interest income ($r_i\beta_iW_i$) and wealth ($W_i$)

**Pareto distribution of the interest income** Assumption 1 imposes a distributional assumption on $W_i$. Combined with other SZ’s assumptions, we can derive the explicit distribution of $r_i\beta_iW_i$ (interest in-
come) using Lemma 1 from SZ.\(^{16}\) Let \(e_i = r_i \beta_i / E(r_i \beta_i)\). This \(e_i\) satisfies all the requirements in Assumption 2.\(^{17}\) Thus, by applying Lemma 1, we know

\[
\frac{r_i \beta_i W_i}{E(r_i \beta_i)} \sim \text{Pareto with shape parameter } a \text{ and the scale parameter } (p_0 E(e_i^a)^{1/a}) W_{p_0}
\]

Note that \(E(r_i \beta_i) = E(r_i \beta_i W_i / W_i)\) so that it can be estimated as long as the interest income and the wealth are observed. Since \(r_i \beta_i\) can be backed out for each \(i\), \(e_i\) is observable for each individual. Thus, all the parameters of Pareto distribution can be estimated. If the (right tail of the) interest income distribution does not follow such Pareto distribution, either Assumption 1 or Assumption 3(i) must be false. Since we can separately test Assumption 1 by looking at the distribution of \(W_i\), the test essentially allows us to see the plausibility of Assumption 3(i).

**Testing Assumption 3(i).** Without relying on Assumption 1, we consider the test of Assumption 3(i). Dividing \(\beta_i r_i W_i\) by \(W_i\), we get \(\beta_i r_i\). Assumption 3(i) implies that this should be independent of \(W_i\).

**Testing Assumption 3(ii) when either \(\beta_i\) or \(r_i\) is observed in data.** Without relying on Assumption 1, we consider the test of Assumption 3(ii). Suppose w.l.o.g. that \(\beta_i\) is observed. Then, we can calculate back \(r_i\) by \(\beta_i r_i W_i / \beta_i W_i\). Due to Assumption 3(ii), the constructed \(r_i\) should be independent of the observed \(\beta_i\).

\(^{16}\)The joint density of \(r_i \beta_i W_i = x\) and \(W_i = y\) is the Pareto density of \(W_i\) (from Assumption 1) times the density of \(r_i \beta_i W_i\) conditional on \(W_i = y\) (i.e., \(dH(x/y)\)). The integration of the joint density should give the distribution function, but I haven’t figured out a neat expression for this.

\(^{17}\)\(E(e_i) = 1\) so that part (i) holds (Note that \(E(e_i W_i) = E(e_i) E(W_i)\) by Assumption 3(i)). Regarding part (ii), note first that \(\beta_i \leq 1\). Since infinite interest rate \(r_i\) is hard to imagine in reality, part (ii) holds. Finally, Part (iii) also holds under Assumption 3(i).
Part II

Reply to “Revising the Revisionists”
Chapter 8

Estimating Fixed Income Wealth
8.1 Relevant Excerpts from “Revising the Revisionists”

1. “In Smith, Zidar and Zwick (2020), the interest rate assigned to the wealthy is higher than in the datasets where both income and wealth can be observed, leading to downward biased top wealth shares” (p.1)
   - This comment is addressed in #1 in Chapter 3.

2. “The difference in SZZ relative to earlier work is to use the Moody’s Aaa rate at the very top. The Moody’s Aaa rate is an index of high-quality corporate bonds with maturity of at least 20 years. It has averaged 6.0% over 2000–2009 and 4.2% over 2010–2016. As one moves up the wealth distribution, all tax units end up being in the top 0.1% of the interest income distribution (e.g., virtually all the top 0.01% by wealth ends up in the top 0.1% by interest, a group 10 times more numerous) and thus all the wealthiest tax units in the SZZ methodology end up having the Moody’s Aaa rate.” (p.6)
   - This comment is addressed in #7 in Chapter 3.

3. “Do the wealthiest investors earn the Moody’s Aaa yield on average on their interest-bearing assets? A systematic investigation of all the available evidence on this issue delivers a clear answer: ‘no.’” (p.7)
   - We present new evidence based on new data and new methods that suggests our estimates of the rate of returns for the wealthiest are indeed appropriate.
   - There are several limitations of SZ’s investigation of the available evidence:
     (a) This assessment is inconsistent with tax-data based evidence above from 3.2 billion information returns, the risk-exposure estimates, and qualitative evidence in Section 3 of the revised manuscript.
     (b) SZ’s foundations analysis does not provide strong evidence against our approach for three reasons. First, updating their original capitalization exercise for foundations shows that the actual wealth series and capitalized series diverge in the low interest rate years after 2010 (Revisiting Evidence from Foundations A). Thus, it is not clear that the data from foundations provides strong support for the equal-returns approach in recent years. Second, it is generally not possible to separate income flows for foundations into those generating taxable interest versus non-taxable interest, non-qualified dividends, or traditional dividends. It is also not possible to partition fixed income assets into these respective categories. As a result, one cannot use data from foundations to measure the interest rate that would be appropriate for capitalizing taxable interest in individual tax data. Data presented in SZ (2020) from the Gates Foundation similarly does not permit an interest rate measured with the appropriate denominator, namely, which excludes fixed income assets that do not generate taxable interest income. Third, looking at foundation balance sheets over time, the fixed income share of assets has fallen over time from 40% in the 1990s to below 20% in 2016 (Revisiting Evidence from Foundations B). This portfolio share evidence lines up closely with our estimates for the top 1%, 0.1%, 0.01%, and 0.001%, supporting our overall approach. In contrast, the fixed income portfolio shares in PSZ and subsequent revisions show increasing portfolio concentration in the past decade.
Revisiting Evidence from Foundations

A. Capitalized and actual wealth shares diverge after 2010

B. Fixed income wealth shares fall over time

(c) There are issues with the way interest rates in the SCF and estate tax data are measured, because they include assets that do not generate taxable interest. See, for example, #1 and #11 in Chapter 3, as well as the following reply.

(d) The evidence on corporate balance sheets, while interesting, is much less informative than direct tax data on wealth individuals from information returns and from our minimum distance approach. Large firms have substantially different needs like meeting payroll and inventory and cash management.

4. “Over the 2001-2016 period, the interest rate observed among the largest estates was always much below the Moody’s Aaa rate. It was also almost always below the 10-year Treasury yield.” (p.7)

- This comment is addressed in #1 and #11 in Chapter 3.
- In addition, the limitations of the estate tax series raised by SZ still apply. Specifically, SZ defended not using the estate tax series at the top on the following grounds: “We retain our baseline top 0.1% wealth share estimate because only a few hundred non-married individuals die with estates above $20 million each year. As a result, there is likely significant noise in the annual series, making it difficult to make a precise and systematic inference of the true interest premium at the top.” (p.550, emphasis added)

5. “Our re-analysis of the SCF has uncovered a key issue when estimating interest rates in this survey. It is not possible in the SCF to identify the fixed-income claims held by pass-through businesses (the bank deposits, notes receivable, bonds, loans, repurchase agreements, etc., owned by S-corporations and partnerships). These assets generate taxable interest for their owners in the SCF, because interest flows to their individual income tax return, and respondents are asked about interest as reported on their individual income tax return. But these assets are typically counted as business assets in the SCF—not as bank deposits, bonds, loans, etc., owned by households. This means that interest rates estimated in the SCF are upward biased. Moreover, because partnerships and S-corporation assets are highly concentrated, SCF interest rates are more upward biased at the top.” (p.7)
• This comment is addressed in #37 in Chapter 3.

6. “If all the corresponding assets are classified as business wealth in the SCF, average SCF interest rates are over-stated by a factor of 1.4, more so at the top where pass-through income is prevalent, with a bias rising over time.” (p.7)

• This comment is addressed in #40 in Chapter 3.

• The SCF does not present evidence that the composition of business assets has tilted toward fixed income assets, nor that the share of fixed income assets at the top has shifted toward fixed income held through mostly actively-managed private business. Thus, the claim seems to depend on strong and unsupported assumptions.

7. “The interest rate of the top 1% wealthiest households is consistently 1.4 times higher than the overall interest rate after the Great Recession. Taking into account the fact that interest earned via pass-throughs is likely concentrated among the wealthy would reduce the small interest rate premium of wealthy households in the SCF. The interest rate of the rich is always well below the Moody’s Aaa rate.” (p.7)

• This comment is addressed in #27 in Chapter 3.

8. “There is no data source where wealthy investors earn an average interest rate remotely close to the Moody’s Aaa rate, no matter how wealthy the investor.” (p.11)

• The revised draft provides substantial new evidence on interest rates of the rich from (a) the universe of taxable interest sources linked to owners using de-identified data from income tax records spanning 2001–2016, (b) estimates of risk exposure by group, (c) election filings and surveys of ultrahigh net worth individuals, and (d) interviews with wealth managers and leading practitioners.

• The new information return data allow us to disaggregate taxable interest income into subcomponents. These data cover all information returns that report taxable interest (Forms 1099-INT, 1065-K1, 1120S-K1, 1041-K1) and allow breakdowns of 1099-INT payments via financial institutions versus private loans versus savings bonds. These new disaggregated data reveal that rich individuals earn a much larger share of their interest income in the tax data in higher yielding forms (such as boutique investment partnerships of distressed debt or mezzanine funds).\(^1\) We also estimate rates more accurately since we can use source and recipient information when determining rates. For example, we assign interest rates on fixed income payments from partnerships by using firm-level information returns of fixed income partnerships, which provide total interest payments and balance sheet information on fixed income assets. This assignment process also reveals differences within sources of fixed income by group, which we document and incorporate into our interest rate measure. Overall, the combination of disaggregated flow data and more accurate assignment of rates for each flow type allows us to contribute meaningful new evidence on the degree of heterogeneity by returns.

• We develop new methods that use the covariance structure of interest rates, assets, and returns. These new methods allow us to estimate the degree of heterogeneity, cross-validate the information return approach, and conduct inference about the key issue—the degree of

\(^1\)Appendix Table B.1 provides interest rates by common fund names of these boutique firms.
heterogeneity. We find that the top wealth group has much stronger exposure to credit risk, which results in a 3 times higher rate of return on fixed income at the top relative to average returns in recent years. Our confidence intervals exclude the SZ baseline approach that these rates are equal and the SZ (2020) updated ratio of returns of 1.4.

- The prior basis for the claim about understated interest rates is not valid because the interest rates cited as evidence were measured with a denominator that includes too many assets—specifically, fixed income and money market mutual funds—which are more prevalent at the top. Fixing this issue increases top 0.1% interest rates in 2016 by 70% in the SCF. We show in Figure 6 how correcting for these issues in the SCF data affects SCF interest rates for top 0.01, 0.1, and top 1%, respectively. There is also considerable uncertainty due to small samples in both the SCF and estate tax data, which implies a 95% bound on the SCF interest rate in 2016 for the top 0.1% is roughly 2% to 6%. Moreover, in the SCF data and estate tax data, it is not possible to isolate the boutique funds that we show generate the bulk of interest income for those at the very top in recent years. Consequently, disaggregating and separately capitalizing these flows is not possible in these other data sets.

- We confirmed these concerns with interviews with practitioners and data experts. We conducted interviews with eight wealth management experts from multiple financial institutions, including specialists in family office portfolio management for the ultrahigh net worth individuals at one of the largest fixed income asset managers in the world. We summarize this qualitative evidence in Section 3.3 of the revised manuscript.

9. “Whether one looks at matched-estates income tax (covering individuals with wealth in the tens and hundreds of millions of dollars), S-corporations (with wealth in the hundreds of millions of dollars), listed corporations (with wealth in the billions of dollars), the wealthiest foundation (with wealth in the tens of billions of dollars), or the wealthiest corporation (with wealth in the hundreds of billions of dollars), the interest rate is in a range of 1.0%–1.7% in 2016.” (p.11-12)

- This comment is addressed by reply to #3 in this section.

10. “Why do the wealthiest investors earn less than the Moody’s Aaa rate on average? The Moody’s Aaa rate is only representative of a small share of all interest-bearing assets. First, it only includes corporate bonds; it excludes government bonds that pay lower yields for a given maturity. Second, it only includes corporate bonds with a maturity of 20 years or more. Bonds with a shorter maturities pay an interest rate significantly lower. This explains why the Moody’s Aaa rate is higher than other corporate bond benchmarks that include shorter maturities. For example, the ICE Bank of America AAA US Corporate Index yield was 2.57% in 2016. Third, wealthy investors own (directly and indirectly through partnerships) interest-bearing assets other than bonds, such as saving accounts, certificates of deposits, and repurchase agreements, which pay lower interest rates. Wealthy investors value the liquidity and low price risk of short-term assets, explaining why they invest only a fraction of their fixed-income portfolio into long-term corporate bonds.” (p.12)

- The information return approach systematically addresses this issue using linked tax data. For example, Figure 3C shows it is not consistent with data on fixed income portfolios, which show that the bulk of interest income at the top comes from higher-yielding boutique sources.
• The evidence from PIMCO and family offices as well as public disclosures in Section 3.3 of the revised manuscript also contrast with the assertions in this comment. In particular, the fixed income portfolios of the ultrarich feature exposure to substantial credit risk. This fact also emerges from our CMD estimates for top fixed income portfolio exposures.

• Treasury rates, savings accounts, and AAA corporate bonds have essentially no credit risk, thus generate lower yields than other fixed income assets despite their longer maturities. Appendix Table B.2 compares the interest rate distributions for boutique funds and private loans to that for different groups of corporate bonds. The partnership and private loan interest rate distributions are quite similar to each other and overlap with corporate bond distributions for bonds with mid-tier and lower credit ratings. These bonds almost always have shorter durations than AAA bonds, on the order of 5 to 10 years. Nevertheless, they have considerably higher yields because of the credit risk exposure of these bonds.

11. “Note that the relatively low average interest rate of the wealthiest investors is consistent with the high rates of return recorded by certain fixed-income investments funds. For instance, a hedge fund can borrow money and invest in low-yield bonds. With a high enough leverage, the return from such an investment strategy can be high, despite the fact that the fund only holds low-yield securities. For the capitalization method, what matters is the interest rate on assets and the interest rate on liabilities separately, because interest received and interest paid by hedge funds both pass through to investors, and hedge funds’ assets and liabilities both show up separately in the household balance sheet of the Financial Accounts.” (p.12)

• In Section 3.3, we present evidence on the types of underlying assets held by fixed income financial partnerships and mutual funds, including their credit characteristics, expected returns, and strategy names. Overall, it does not appear that the fixed income strategies of the ultrarich focus on levered investments in low-yield securities.

Instead, ultrarich fixed income investors take substantial credit risk, appearing to invest in subordinate securities in private equity and real estate transactions, mezzanine and distressed debt, mortgage servicing rights, foreign bonds, and so on. The yields we estimate for these funds overlap with those for investment-grade and near-investment grade corporate bonds held by fixed income mutual funds.

• Separately, while it appears empirically not to be the first order concern for high net worth fixed income investors, it is likely true that some funds generate high returns through leverage. However, it requires strong assumptions to use capitalization and assign all the assets this way, such as by assuming the debt holders of these funds are the same as the equity holders. If that assumption does not hold, which is plausible given the different risk characteristics of senior and junior claims on these funds, then it will be more appropriate to assign the fund’s net worth (i.e., assets minus debt) in proportion to interest payments received by the fund’s equity holders and separately assign the debt claims in proportion to interest payments received by the fund’s debt holders.

12. “SZZ defend their approach in their appendix J by arguing that it is “more practically useful” to apply heterogeneous returns to bins of interest income, since interest income is observable in the data and wealth is not. But this argument conflates two issues—what is the conceptually correct return to apply, and how, practically, to implement the capitalization method. Conceptually, as we show below, the interest rate to apply is the interest rate of
8.1. RELEVANT EXCERPTS FROM “REVISING THE REVISIONISTS”

the wealthy. Practically, one can apply this rate to the wealthy by proceeding by iteration, as explained in Bricker et al. (2018, pp. 17-18) and done in the updated Saez and Zucman (2016) series presented in this paper (see section 5.1 below)” (p.13)

• This issue is addressed in #21 in Chapter 3.

13. “We now show conceptually and empirically that using the interest rate of top interest earners to capitalize interest income when interest rates are heterogeneous is conceptually incorrect and generates a first-order downward bias in top wealth shares. [...] To preview the conclusion: using as SZZ the rate on return of high interest earners generates a downward bias in estimated top wealth shares that is first order in the share of bonds in the wealth of the wealthy. By contrast, using the rate of return of the wealthy generates an upward bias that is only second order in the share of bonds in the wealth of the wealthy. Therefore, zero first-order bias requires using r, the interest rate of the wealthy, as in the updated Saez and Zucman (2016) series presented in this paper.”

• This issue is addressed in Chapters 3 and 7.

14. “We now confirm this theory by applying it to US foundations. Foundation data are more adapted than the SCF, because, as we have seen, it is impossible to measure the value of all assets generating taxable interest in the SCF.” (p.17)

• This comment is addressed in #3 in this section and in Chapter 4.

15. “The fourth series in yellow triangles depicts the interest rate of the top 1% interest-earning foundations (this is \( \bar{r} \) of the theory). This interest rate is much higher due to selection as high interest income selects both on savings accounts size and interest rate. It is comparable in magnitude to the Moody’s Aaa interest rate (depicted in dashed line as the fifth series). This graph is similar to the SCF results presented by SZZ showing that top interest-earners have a much higher interest rate than the wealthy. It has the advantage of being cleaner, as the interest reported by foundations corresponds exactly to the asset class of savings deposits, allowing us to provide a compelling test of the various capitalization methodologies.” (p.18)

• We no longer use ranks in the interest income distribution to assign rates of return, so the yellow series is no longer relevant.

• Our information returns analysis shows that the bulk of interest income flows at the top are due to higher-yielding boutique assets.

• In addition, there are several limitations of the foundations analysis that we discuss in #3 in this section. In particular, it is generally not possible to separate income flows for foundations into those generating taxable interest versus non-taxable interest, non-qualified dividends, or traditional dividends. It is also not possible to partition fixed income assets into these respective categories. As a result, one cannot use data from foundations to measure the interest rate that would be appropriate for capitalizing taxable interest in individual tax data.

16. “We can compare the amount of fixed-income claims owned by the top 1% according to SZZ and according to the Federal Reserve Distributional Financial Accounts in 2016 (Figure 13). In both cases, fixed-income claims include checkable deposits and currency, time deposits and short-term investments, money market fund shares, debt securities, loans, and
the fraction of mutual fund assets invested in bonds and loans, minus fixed-income claims held in individual retirement accounts (which are part of pension wealth). In both cases, the same aggregate ($15.0 trillion in mid-2016, i.e., the official Financial Accounts total for fixed-income claims) is distributed across the population. The SZZ series (which are among equal-split adults) can be directly compared to DFA estimates (which are based on households), because the share of wealth owned by the top 1% wealthiest adults is very close to the share of wealth owned by the top 1% households. As shown by Figure 13, SZZ only capture 68% of the fixed-income claims owned by the top 1% recorded in the Distributional Financial Accounts. The $2.2 trillion gap represents 2.9% of household wealth in 2016. This gap explains the bulk of the difference between the SZZ top 1% wealth share and the DFA top 1% wealth share.” (p.19)

- Thanks for this comment. The previous version presented results for equal split individuals compared to households and with fixed income categories imperfectly aligned across data sets.
- Overall, our top 1% estimates match those from the DFA more closely than the PSZ estimates in terms of levels (Appendix Figures A.19 and A.20), trends (Appendix Figure A.21), and portfolio shares (Figure 15 and Appendix Figure A.18).
- The revision shows that our fixed income aggregates for the top 1% are in fact somewhat higher than the equivalent concept for the DFA (when defining fixed income to include fixed income mutual funds to better align with the DFA concept) and comparing the top 1% of tax units to the DFA's top 1%. See, for example, Figure 13 and Appendix Figures A.19 and A.20. If we were able to adjust the number of households in the DFA to match the number of tax units in our top 1%, these numbers would align even more closely.
- However, given our fixed income portfolio shares for the top 1% are somewhat higher than the DFA (Figure 15 and Appendix Figure A.18), there would remain a modest discrepancy, which ultimately owes to lower fixed income concentration in the SCF than in our estimates. Appendix Figures A.26 and A.27 show scaling SCF deposits up to match Financial Accounts totals can move SCF portfolio shares toward our estimates. Our interpretation of this difference is that taxable interest from banks mixes income from deposits with income from wealth management accounts, which increases concentration in taxable interest flows at the top relative to what we might find if we could further disaggregate these flows.
- Thus, using the DFA as a benchmark suggests that our estimates are not allocating too little fixed income to the top 1%.

17. “We can use the theoretical formulas established in Section 2.2.2 to quantify the bias caused by using \( \bar{r} \). In 2016, the average interest rate of the wealthy (estates above $20 million) in matched estates-income tax data \( r \) is 1.4%, the rate used by SZZ to capitalize interest at the top is the Moody’s rate \( \bar{r} = 3.67\% \), so that with a share of fixed-income claims in wealth \( \beta = 0.25 \) there is a first-order bias of \( 1 – \beta (1 - r/\bar{r}) = 0.85 \). Starting from the SZZ top 0.1% wealth share of 14.3%, getting rid of the bias by capitalizing interest using the conceptually correct rate, \( r = 1.4\% \), increases the estimated top 0.1% wealth share to \( 14.3%/0.85 = 16.9\% \), i.e., it adds 2.6 points of total wealth.” (p.20)

- This issue is addressed in Chapter 7 above.

18. “SZZ do not report the interest rate of the wealthy generated by their model and their code is not publicly available. We replicate their model by using our own updated code (which
among other things has more business wealth concentration than in our original estimates; see Section 5.1), estimating equity wealth as in SZZ (i.e., with a weight of 10% on capital gains and 90% on dividends, see below) and capitalizing interest as in SZZ. According to these computations, the top 0.1% ranked by wealth using the SZZ methodology has an interest rate of about 2.5% in 2016.” (p.21)

- This exercise does not incorporate several important components, such as our private business estimates, which allocate 20% of the overall value to wealthy individuals who report tax losses. It also uses the prior version’s fixed income model and other components, which we revised substantially in the updated manuscript.

- In terms of code, we did provide SZ with relevant code snippets (e.g., Nov 20, 2020 or August 12, 2020 emails), and answered an extensive series of questions via email.

- Preparing a full replication file is something we will do following the standard publication process—we have been focused on revising the draft and replying to the 6 detailed referee reports plus editorial comments.

19. “After re-ranking, the interest rate of the top 0.1% by wealth is still much higher than the interest rate actually earned by the top 0.1% wealthiest Americans according to the existing evidence analyzed above. According to the matched estates-income tax data analyzed by Saez and Zucman (2016) and extended to 2016 by SZZ, Americans who died with more than $20 million in wealth (a threshold close to the top 0.1% threshold) had an interest rate of 1.4% in 2016, 1.8 times lower than the interest rate of the top 0.1% by wealth in the SZZ methodology. Thus, SZZ under-estimate the fixed-income claims owned by the top 0.1% by a factor of about 1.8 in 2016.” (p.21)

- This comment is addressed in reply #4 in this section.

20. “We can compute top wealth share using the conceptually correct interest rate (i.e., the interest rate of the wealthy, $r$, as proved in Section 2.2.2), and then keeping everything else the same, implement instead the SZZ methodology of ranking by interest and applying the conceptually incorrect $(\bar{r})$. Specifically, we run our updated income capitalization code which, among other things, applies heterogeneous interest rates to tax units ranked by estimated wealth iteratively and at the top uses the observed $r$ of the top 0.1% by wealth (as seen in matched estates-income tax data); see Section 5.1. We find that implementing the SZZ method (in lieu of this conceptually correct method) to estimate fixed-income claims reduces the top 0.1% wealth share by close to 2.0 points in 2016, consistent with the theoretical predictions. This explains half of the 4 points gap between SZZ and the updated estimates of Saez and Zucman (2016)” (p.21)

- We no longer use $\bar{r}$ so this statement no longer applies. As described in Chapter 7, this “conceptually correct” statement depends on strong assumptions that may not apply in empirically relevant cases.

- Figure 5C compares our preferred estimates to those from PSZ with 2018 aggregate definitions, with updated definitions, and to alternative capitalization approaches. The gap between our preferred estimate and the PSZ estimate is 4.1 percentage points.

- Table B.9 and B.10 provide a systematic analysis of the effects of changing different model approaches and assumptions.
21. “We revise our estimation of interest-bearing assets by factoring in the interest rate premium observed in matched estates-income tax data. Consistent with the theory developed in Section 2.2.2, we capitalize interest at the top using the observed interest rate of the wealthy. Specifically, we capitalize the interest of the top 1% wealthiest tax units using an interest rate equal to 1.15 times the average interest rate between 2003 and 2007, and equal to 1.4 times the average interest rate starting in 2008. We apply these heterogeneous interest rates to tax units ranked by wealth, proceeding by iteration. That is, we first construct wealth using homogeneous returns, and then reconstruct wealth using heterogeneous returns, ranking tax units by wealth estimated in the first step.” (p.46)

- This comment is addressed in reply #4 in this section in terms of issues with estate tax series. As noted above, estate tax data also likely understate the interest rate differential because of asset classification problems, and interest rate estimates are very sensitive to mortality rates and small sample sizes.
- The new evidence that we present using information returns and using classical minimum distance indicates that this approach is inadequate and can be rejected by the data.
Chapter 9

Estimating Public Equity Wealth
CHAPTER 9. ESTIMATING PUBLIC EQUITY WEALTH

9.1 Relevant Excerpts from “Revising the Revisionists”

1. “equities are capitalized using almost only dividends, which dramatically underestimates the wealth of billionaires relative to the Forbes 400 list” (p.2)
   • This comment is addressed in #1 in Chapter 4.

2. “Estimating equity wealth based on dividend income as done by SZZ does not allow one to capture top-end wealth accurately, because the wealthiest Americans often own equities that do not pay dividends. For instance, 5 of the top 10 richest Americans—Jeff Bezos (Amazon), Mark Zuckerberg (Facebook), Warren Buffet (Berkshire Hathaway), Sergey Brin (Alphabet), and Larry Page (Alphabet), collectively worth more than $250 billion in 2016—were the main shareholders of corporations that did not pay dividends in 2016. The SZZ methodology assigns them a negligible amount of wealth relative to their true wealth.” (p.22)
   • This comment is addressed in #4 in Chapter 4.

3. “SZZ under-estimate billionaire wealth by about 40% relative to the existing evidence. According to SZZ (2020, p. 32) billionaires owned $1.7 trillion in wealth in 2016. According to Forbes, the top 400 wealthiest Americans (who had wealth above $1.7 billion) owned $2.4 trillion in 2016. Billionaires with more than 1 billion and less than $1.7 billion add close to an extra $600 billion, for a total billionaire wealth of around $3 trillion. As shown by Figure 15, the benchmark Saez and Zucman (2016) capitalization method captures close to 100% of the amount of billionaire wealth implied by Forbes. By contrast, the SZZ methodology, which under-estimates top-end equity wealth, under-estimates billionaire wealth by almost 50%.” (p.22)
   • This comment is addressed in #5 in Chapter 4.

4. “Forbes is certainly not perfect. However, to learn about the wealth of the richest American, the Forbes approach of looking at ownership in large businesses to figure out the true wealth of Jeff Bezos, Mark Zuckerberg, Warren Buffet, Sergey Brin, Larry Page, Elon Musk, Michael Dell, etc., is obviously superior to trying to infer wealth from dividends that haven’t yet materialized. Saez and Zucman (2016) opted for a method with a higher weight on capital gains precisely because it did a good job at matching Forbes.” (p.22-23)
   • This comment is addressed in #1 in Chapter 4.

5. “SZZ justify their methodology by pointing out that in the Survey of Consumer Finances, putting a weight of 10% on capital gains minimizes mean-square error. There are two issues with this argument. First, mean-square error is not the relevant statistic to assess the reliability of various capitalization methods. If only billionaires own a lot of equities that pay no dividends, putting a 90% weight on dividends can minimize mean-square-error for a given person’s wealth in the SCF while severely under-estimating the equity wealth of billionaires, who own about 40% of the equity wealth of the top 0.1%. Second, as noted above for interest, there is an inconsistency between income flows and assets in the SCF. Equities held via pass-through businesses (e.g., hedge funds) are typically not recorded as equities but as business assets. However, the dividends and capital gains these equities generate
are recorded as such. Dividends flowing from pass-throughs are relatively small (about 14% of all dividends reported on 1040s), but capital gains flowing from pass-throughs are large, about a third of all capital gains reported on 1040s. Since a large fraction of capital gains arise from assets that are not classified as corporate equities (but as business wealth), capital gains in the SCF may not be as predictive of equity ownership as they actually are.” (p.23)

• The revised version updates the approach to target top wealth shares as described above in Chapter 4 and in section 5 of the manuscript.

• Note that realized capital gains from x5712 correspond to the tax form question (i.e., lines 13-14 from form 1040), so the SCF lines up with information on capital gains. Note also that we include the subset of private business that is organized as C-corporations (less than 20% of private business wealth in the SCF in recent years) when estimating the optimal weight to apply on capital gains. The critique about other forms of capital gains is one of the reasons why putting equal weight on capital gains may not fit the data as well—these gains can reflect other asset holdings besides public stock and that’s what our approach is picking up.

• The point about hedge funds also conflates two categories of wealth that appear in different places in the SCF. The private business line primarily includes actively managed businesses that are closely held, whereas the bulk of capital gains distributed via partnerships comes from passively held investments in which the owner is one of many limited partners. Per conversations with SCF experts, this latter category likely appears in the “other managed assets” variable in the SCF, not in the private business variable. When the respondent is a general partner, it may appear in either variable or even be split between both variables. Unfortunately, the SCF does not provide sufficient information to unpack these cases. Because general partners also receive ordinary income, they will be allocated substantial pass-through business wealth in our model, so we are not missing their wealth in our estimates.

See also #37 in Chapter 3, which discusses the difference between private business and other managed assets in the context of fixed income assets.

• Consistent with the idea that most pass-through capital gains represent passive investments, Appendix Figure A.10 shows that, at the very top, the share of information return dividends coming from pass-throughs is similar to the share of capital gains coming from pass-throughs. That is, even though the overall share of dividends from pass-throughs is lower than for capital gains (as you correctly highlight), these shares converge further up the distribution.

6. “We now adjust equity wealth at the very top to match the amount of billionaire wealth implied by Forbes each year. Between 1982 and 2005, we adjust the equity wealth of the top 400 so that total top 400 wealth matches Forbes (reducing equity wealth proportionally in the rest of the distribution). We make no correction before 1982 (in 1982 the share of wealth owned by the top 400 is small according to Forbes, less than 1% of aggregate wealth vs. more than 3% in recent years). Starting in 2006 we implement the same correction but for a group slightly larger than the top 400, namely billionaires (estimated using the Forbes 400 and Pareto-interpolation techniques). This adjustment is motivated by the fact that the capitalization method, which infers equity wealth based on dividends and realized capital gains, does not accurately capture the wealth of billionaires who receive no dividend and
barely realize any capital gains. This problem has become more severe in recent years with the rise of giant tech companies that do not distribute dividends yet. Six of the ten wealthiest Americans in July 2020—Jeff Bezos, Mark Zuckerberg, Warren Buffett, Larry Page, Sergei Brin, and Elon Musk—collectively worth around $500 billion in July 2020 (0.5% of total US wealth wealth), are major shareholders of corporations that do not pay dividends.” (p.50)

- This approach likely overstates top public equity wealth because a considerable share of Forbes wealth is in the form of pass-through firms. In addition, of the top 50 individuals on the Forbes list who own public companies, more than half of them (weighted-by-wealth) owned companies that pay dividends.
Chapter 10

Other Differences and Reconciliation
10.1 Relevant Excerpts from “Revising the Revisionists”

1. “The methodology used by SZZ to estimate business wealth (other than equity in listed equities) at the top has two downward biases. First, there is a conceptual issue in the estimation of equity in large pass-through businesses. SZZ estimate equity in pass-through businesses (S-corporations, partnerships) by capitalizing business profits, assets, and sales, using capitalization factors observed for listed firms in the same sector. In their benchmark methodology, they divide reported pass-through profits by 4, following Smith et al. (2019) who found that three-quarters of pass-through business profits reflect labor income. However, the main estimates of Smith et al. (2019) are based on an equal-weighted sample of pass-through businesses, in which a doctor’s practice enters with the same weight as a businesses with $1 billion in assets. As we show in Section 4, there is little empirical reason to believe that in large pass-through businesses, which for all intents and purposes are similar to large listed corporations where profits are pure capital income, three-quarters of business profits are disguised wages.” (p.23-24)

   • Thanks for this comment. We now provide robustness to allow for the adjustment for labor income to be 0% rather than 75%. Appendix Figure A.15 shows this increases the top 0.1% wealth share by approximately 0.25 percentage points, which provides an approximate upper bound for alternative models that condition the human-capital adjustment on firm size. Thus, the potential bias from assuming the same parameter for all firms appears quantitatively small.

2. “Second, there is a specific issue with the estimation of equity in real estate pass-through businesses. For the purpose of capitalizing profits, profits are defined by SZZ as one quarter of business income, plus interest paid, plus depreciation. In the real estate sector, the bulk of interest and depreciation is not reported on forms 1120S (for S-corporations) or 1065 (for partnerships), but expensed on form 8825. However, SZZ do not include interest or depreciation expensed on form 8825 to compute real-estate pass-through profits (while such interest and depreciation is included for listed firms). Figure 16 shows that for real-estate S-corporations, the SZZ measure or profits (and thus the SZZ income-based valuation) is too low by a factor of 5. A similar problem arises for the sales-based valuation. For the purpose of capitalizing sales, sales are defined as business receipts as reported on forms 1120S and 1065. However, in the real estate sector a large fraction of sales is gross rents, which are not on form 1120S or 1065 but on form 8825. This issue matters quantitatively because real estate S-corporations and especially real-estate partnerships are large and their ownership highly concentrated.” (p.24)

   • Thanks very much for this point. Our previous appendix table, on which this comment is based, excluded one of the real estate NAICS codes due to an issue with NAICS harmonization over time. In the revision, we were able to generate estimates using updated data that corrected this issue. The revised table (Appendix Table B.6) shows that “Lessors of real estate” (NAICS 5311) and “Activities related to real estate” (NAICS 5313) collectively account for $730B of pass-through firm value according to our estimates and rank as the #2 and #6 four-digit industries in terms of aggregate size. Thus, this important sector appears to be receiving substantial weight in our overall pass-through estimates.

   • We note that, because these partnerships tend to generate very little ordinary business income for their owners, the SZ and PSZ methodologies allocate very little wealth to their
owners. Our approach therefore substantially improves on past work for this sector. Nevertheless, we believe a deeper dive into real estate and finance partnerships could be a very interesting avenue for future research.

3. (Reconciliation) “There is a gap of 4.3 points between the top 0.1% wealth share estimated by SZZ (14.3% among equal split adults in 2016) and the original Saez and Zucman (2016) estimate (18.6% among equal-split adults in 2016). After the revision we implement in this paper (see Section 5.1), the gap is reduced to 4 point. This remaining gap can be reduced to zero as follows. First, theory and evidence suggest that the fixed-income claims of the top 0.1% estimated by SZZ are too low by about 2 points of total wealth. Second, the SZZ estimates of billionaire wealth are lower than those implied by Forbes by 1.7 point. Finally, two identified downward biases in business wealth lead to an under-estimation of the top 0.1% wealth share, by perhaps 0.5—1 point. Using the conceptually correct rate to capitalize interest income, upgrading the values of real-estate and other pass-through businesses, and matching Forbes billionaire wealth (should any gap remain after the first two corrections) would close virtually all the gap between SZZ and the original and updated Saez and Zucman (2016) top 0.1% wealth share.” (p.24-25)

- Figure A.15 shows how different pass through assumptions affect our top wealth shares.
- Table B.9 and B.10 provide a more systematic perturbation analysis using the updated estimates.

4. “We improve the treatment of pension wealth. Our previous methodology aimed at matching the amount of wealth found in the SCF for the top 10%, but did not specifically target the top 1%. We now match the amount of pension wealth owned by the top 1%. To do so we reduced the weight put on non-taxable pension distributions (vs. taxable pensions). Specifically, we now allocate 60% of pension wealth proportionally to taxable pension distributions, 30% proportionally to wages and 10% proportionally to non-taxable pension distributions (e.g., Roth IRA distributions). This allows us to match the amount of pension wealth seen in the SCF supplemented by the Sabelhaus and Henriquez-Volz (2019) estimates of defined benefit pensions. This revision reduces the top 1% wealth share by 1–1.5 points in recent years. The share of pension wealth in the total wealth of the top 1% in now the same as in the Distributional Financial Accounts.” (p.51)

- SZ’s reduction in the weight on rollovers in the 2020 work move in the direction of our pension model. However, there are still differences in terms of matching the life-cycle of pensions.
Chapter 11

Comparing Capitalization Estimates to Other Data
11.1 Relevant Excerpts from “Revising the Revisionists”

1. “According to the public-use Survey of Consumer Finances data, the top 0.1% wealth share has increased by 4.3 points between 2001 and 2016. The SCF by design excludes the Forbes 400.Appending the Forbes 400 to the public-use SCF data, the top 0.1% wealth share has increased by 5.0 points. This increase is in line with the rise found using the Saez and Zucman (2016) methodology, but not consistent with the quasi-stagnation in SZZ.” (p.5)

   • This comment is addressed in #14 in Chapter 5.

2. “the SZZ estimates are not consistent with the rise in the top 1% wealth share observed since the turn of the 21st century in the official Federal Reserve data on wealth inequality. According to the official SCF results, the top 1% wealth share rose 6.2 points between 2001 and 2016, and by 6.7 points when adding the Forbes 400. According to the Federal Reserve Distributional Financial Accounts, the top 1% wealth share rose 5.2 points over the same period. By contrast, according to SZZ the top 1% wealth share rose only 1.4 points.” (p.5)

   • This comment is addressed in #14 in Chapter 5.