A Bargain Might Not Exist:
How the Distribution of Power Causes War

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September 16, 2015

Abstract

This paper challenges the conventional wisdom about bargaining and war. If more than two players bargain, or if war can end in stalemate, then a bargain that all actors prefer to war may not exist—even if there are no information problems, commitment problems, principal agent problems, alliance dynamics, or indivisible goods. Existing models generate a bargaining range because they artificially constrain the technology of power. Consequently, current theories constitute only a subset of the possible rationalist explanations for war. This paper suggests how the nature of power, states, and military technology affect the possibility of peace.

1 Introduction

We study war as an aberration. War puzzles us because we see its waste, and this waste makes us wonder how rational human beings fail to avoid its devastation. In a celebrated article, James Fearon (1995) sets precisely this puzzle at the heart of the study of war: if actors can anticipate a war's outcome, he argues, then the costs of war should induce them to settle their differences peaceably rather than incur the costs of war; there should always exist some *ex ante* bargain that makes all parties better off. To explain war, therefore, requires explaining the impediments to a peaceful solution. Now, the bargaining model of war has become the leading approach to violent conflict. Using the model as a foundation, political scientists have developed a fully coherent theory relating the origins of war to its termination.[1]

This paper challenges the scope of the bargaining model of war. Its thesis is simple: the costs of war do not guarantee that a mutually preferred bargain exists. Consequently, existing rationalist explanations for war are incomplete. I examine two common assumptions which have been largely ignored as innocuous: first, that the world contains only two players; second, that war cannot end in a draw. Changing either assumption will change the basic result of Fearon's analysis.

The bargaining model's main conclusions derive from two-player games, since these are tractable and easily show why peace should prevail. Most theorists then presume that their conclusions will generalize beyond the two-player context, but this presumption is dangerous and remains unproven. Power is simple among two players, and this simplicity makes a satisfactory distribution of goods straightforward: if one state wins with probability p and the other with probability $1 - p$, then each state's share of the pie will be roughly p and $1 - p$. But among three or more players power is complex, and such complexity can preclude a bargain which all prefer to war.

Consider the classic child's game of Rock-Paper-Scissors. In this game, rock is weak to paper, paper is weak to scissors, and scissors is weak to rock. Each element is thus strong against one type and weak against another. Asking which element is strongest does not make sense, but neither does saying they are all equally strong; instead, the only sensible way to describe the game is that each element is strong against some and weak against others. Because of this complexity, in a world of many actors, trying to distribute goods proportional to A's strength against B may make the distribution of goods between A and C disproportional. Indeed, it may be impossible to satisfy all power relationships simultaneously. Since peace is only possible when the distribution of goods approximates the distribution of power, if this approximation does not hold, then at least one state can gain by attempting to seize another state's goods by force—leading to war.

This problem can take many forms. In its simplest shape, because among several players power is more complex than p and $1 - p$, the sum of three or more states' relative power can exceed one. Thus, even though war is costly, the sum of all players' demands can exceed the sum of all the world's goods. In such a scenario, peace might be impossible. In more complex forms, the rock-paper-scissors problem can make the distribution of international wealth and influence difficult to predict. Between two actors, wealth will tend to track relative power, but between three or more actors, this may no longer be the case. For instance, one state, while much stronger than its neighbor, might be more vulnerable to a third party, and consequently it might enjoy less wealth than its weaker neighbor. Knowing that paper beats rock is not enough: we must also account for their relationships to scissors.

The second problematic assumption of the bargaining model makes the transfer of wealth more straightforward than in reality it is. When we assume that war cannot end in a draw, we assume that war must end either peacefully or in the capture of all the world's goods by one state. This assumption implies that altering the distribution of wealth cannot alter states' payoffs to war—they remain merely $p-c$ and $1-p-c$.[2] The same is not true if war can end in a draw, with neither state seizing the other's wealth. In this instance, redistributing a state's share of the pie can increase its payoff to war: its payoff to a complete victory remains the same, but its payoff to a draw increases (since in the case of a draw it retains what it currently has). A state which enjoys a disproportionately large share of the pie might therefore prefer to fight an enemy rather than cede goods until that enemy is satisfied. In this instance, the dissatisfied enemy will attack, seeking to revise the *status quo* by force.

So why does it matter? If the costliness of war does not guarantee that a mutually preferred bargain exists, then we have barely begun to explore the possible rationalist explanations for conflict. War might result purely from the initial distribution of power and wealth, even in a world without uncertainty or power transitions. The number of states might increase the likelihood of war, and the available military technologies might constrain peace in more ways than the traditional offense/defense balance suggests. As well, the very kinds of actors and regimes that populate the international system might push rational states toward conflict, and the distinctions between great, middle, and minor powers might shape how such states behave and the types of wars they fight. And all of these could be rationalist explanations for war—if we recognize that a bargain might not exist.

According to extant models, given war's inefficiency, there are at most six rational explanations for violent conflict: information problems, commitment problems, issue indivisibilities, the future cost of arming, principal agent problems, and coalition dynamics.[3] Other explanations are secondary: when inefficiency inheres in conflict, what drives empires to expand or states to become revisionist is less interesting and less important since the states should still be able to avoid war. Thus, incomplete information and, most important of all, commitment problems have become the key rationalist explanations for conflict.

The rise of the bargaining model effectively terminated our longstanding emphasis on the balance of power. At least since Waltz' *Theory* (1979), a crucial question in the study of international politics had always been how the distribution of power affected the likelihood of war. Scholars probed the implications of bipolarity and multipolarity, seeking the origins of great power conflict in states' quest for hegemony.[4] Many theorists simply assumed that an imbalance of power would produce war. The crucial move of the bargaining model was to show that, even if a power imbalance exists, then states should still be able to reach a peaceful bargain that made them both better off. As well, Powell (1996) showed that the likelihood of war and the distribution of power were essentially unrelated. Instead, the key issue was the speed and magnitude with which the distribution of power was changing (Powell, 2006). After all, according to the theory, no matter what shape the distribution of power might take, a bargaining range must still exist. Therefore, an explanation for conflict must lie, not in the distribution of power, but in the way states perceive that power or in the way that power changes.

But is this satisfying? Consider two classic subjects of international politics, conquest and appeasement. If the strong do what they will, why do the weak ever resist? If a strong state can conquer with odds of 90%, why not cede 90% to its demands? The question relates to a longstanding puzzle in the literature on war: if commitment problems cause wars when power shifts, then it must be the declining power which provokes conflict; most wars of conquest, however, are undertaken by rising states.[5] But why did conquered states, especially ones with little hope of success, not simply submit? If a bargain always exists, why is appeasement a dangerous policy?

Intuitively, we know the answer. If an aggressor threatens war unless another nation makes concessions, why would that aggressor refrain from demanding additional concessions in the future? Like a blackmailer who has once extorted wealth from a victim, in an anarchic world what guarantees he will not extort the victim again? In the standard bargaining model of war, this fear is unfounded, since conceding a division of the pie proportional to an aggressor's strength will satisfy it, leading to peace. In this approach the division of wealth changes, but states retain the same power over it. Consequently, appeasement only becomes a problem when transferring goods and territory strengthens an aggressor, making it able to demand more. But this conception seems too narrow: the problem of appeasement is not that concessions will strengthen an opponent or somehow whet its appetite but that, once yielded, the concessions will not satisfy.

Current bargaining models of war fail to recognize this problem because they miss an important aspect of bargaining. When resources are transferred from one state to another, or even from one person to another, the new owner gains additional power over them. In its crudest form, this can be imagined as gold moving from one treasure hoard to another, but it applies equally to land and other resources. The crucial point is this: the balance of power over different hoards is not the same. Rome will have more power over a wagonload of gold among its hills than along the Rhine; altering the distribution of wealth necessarily alters the distribution of power over that wealth. The standard model does not accommodate this possibility; once we allow for it, it turns out that states will often prefer to fight to defend what they have rather than gradually cede it to an aggressive rival. That is, it turns out that a mutually preferred settlement between states will often not exist.

This paper challenges the conventional wisdom of the bargaining model of war. While seemingly a comprehensive catalogue of rationalist explanations for war, in fact the model has unwittingly excluded many of the most important causes from consideration. To explain a war, before relying on informational or commitment problems, we must first ask whether a bargain is even possible. In the next section, I show that a bargaining range is not guaranteed when three or more players exist, even if the game is in all other ways identical to classic two-player models. In particular, I prove that a bargain might not exist whenever at least one state is better off fighting its enemies one-by-one rather than all at the same time.

The third section explores more broadly how a rock-paper-scissors dynamic can emerge under conditions of stalemate, both causing war and making the distribution of wealth unpredictable. The fourth section goes on to prove that, if our models permit war to end in stalemate, then even in two-player games a bargaining range will likely not exist. The fifth section uses the model to derive comparative statics predicting how the character of military technology will affect the possibility of peace. A final section concludes.

2 When Several Players Bargain, a Bargain Might not Exist

In a world containing at least three states, there might not exist a bargain which all prefer to war. This claim arouses scepticism; after all, it challenges perhaps the most entrenched belief about war in modern political science, a belief well-established in two decades of theoretical and empirical research. Considerable scepticism is warranted. Nevertheless, the claim should not be entirely shocking, since very few models have actually asked whether the dynamics of the classic two-player game extend to games with several players. We have only presumed that our findings generalize. As I show, this presumption is too hasty.

Multilateral bargaining has always posed difficulties to mathematical theories of international relations. Indeed, studying war within such a multilateral context is a startlingly unexplored field of inquiry.[6] The most natural approach is to use coalitional (cooperative) game theory, but this method typically requires the assumption that bargains are binding—an assumption fundamentally incompatible with the anarchy of international politics.[7] Some recent efforts attempt to combine cooperative and noncooperative game theory to study multiplayer games, and Krainin (2014) succeeds in predicting balancing/bandwagoning behavior from three-player games, but this remains a relatively young area of study.[8] Older efforts tend to constrain games narrowly in order to focus on a particular problem, for instance, cooperation within alliances.[9] But *all* of these treatments examine multilateral bargaining within the context of alliances. This choice, I think, is too quick: before we consider multiplayer games with alliances, we need to understand the basic dilemmas of multiplayer games themselves.

In this paper's model I will disallow alliances. I do this for two reasons: first, in order to study the problems created by having several players bargain, even if they cannot ally; second, because we already know alliances can cause war. My purpose in this paper is to show that an extension of the bargaining model to several players eliminates its main result. I therefore strive to maintain the assumptions of the two-player model whenever possible, including the non-existence of alliances. This strictness will show that the propositions follow from the simple fact of having several players, not from any coalition dynamics.[10] The effects of alliances are understudied, particularly from the vantage of formal theory. Nevertheless, before studying alliances, we must first understand the world without them. Thus, I will show that, even if alliances are impossible, a peaceful bargain may still be infeasible among several players.

The problem arises through the complex nature of power. Between only two states, power is simple: the odds that A wins against B is one minus the probability that B wins against A. But among several states, things are not so straightforward. States come in kinds, and some kinds are stronger against some than against others. For instance, even if two countries spend equally on their militaries, but only one has a navy, then they will not enjoy the same odds of victory against an island nation. Power is in some sense intransitive: knowing the strength of one actor against another says relatively little about the strength of each against a third. Indeed, as in rock-paper-scissors, A might be strong against B, and B against C, but knowing so does not even imply that A is stronger than C. As a result, it can prove difficult, perhaps impossible, to distribute goods to reflect such a complicated distribution of power. This paper's third section explores this problem in greater depth; for now, I focus only on its simplest manifestation—when the sum of all players' payoffs to war exceeds the total goods to divide.

Two countries have no choice how and in what order they fight their enemies: they can fight a duel or not at all. But three or more actors do have such a choice. In such a setting, one player might be stronger if it fights rivals in one way, and another player might be stronger if it fights them in another way. Consequently, the sum of their individual power might actually exceed 1. The problem is not that a bargain all actors prefer to *a* war does not exist; it usually does. Rather, the problem is that a bargain all actors prefer to *all* wars does not exist. Between only two players, there is only one way to make war (A versus B). Between three players, there are seven ways each player could make war on the others (A versus B then C, A versus C then B, or A versus B and C; similarly B and similarly C). Between many players, there are enormously many different wars they might fight. For peace to prevail, A might demand a share of the pie equal to its utility from fighting against B and then against C, but B might demand a share of the pie equal to its utility from fighting against C and then A (or against them both simultaneously), and there is no reason to believe these demands sum to less than one.[11]

For instance, before the First World War, Germany expected to fight two very different wars on its western and eastern fronts. If it could fight a war against France and then fight against Russia, Germany expected it would almost certainly emerge victorious. This was, of course, the famous Schlieffen Plan. By contrast, for France and Russia, if they fought Germany at the same time, then even if later they fought each other, their odds of victory were still considerably improved. For the sake of argument, let us put Germany's perceived odds of success under the Schlieffen Plan at $\frac{1}{2}$, even if in reality we now know they were much lower; also, let us put the odds of Russian or French success in an all out war at $\frac{1}{3}$. In symbols,

$$p(Ge, Fr)p(Ge, Rus) = \frac{1}{2}$$
$$p(Ge, Fr \cup Rus) = \frac{1}{3}$$
$$p(Fr \cup Rus, Ge)p(Fr, Rus) = p(Rus \cup Fr, Ge)p(Rus, Fr) = \frac{1}{3}$$

If war costs c, then Germany's peacetime demand would be $\frac{1}{2} - c$, and France and Russia's demands would each be $\frac{1}{3} - c$. But the sum of these demands is $1\frac{1}{6} - 3c$, which means that, unless $c > \frac{1}{18}$, a bargain that all states prefer to war does not exist.

Importantly, the Franco-Russian alliance does *not* drive this result: a bargaining range might not exist even if France and Russia fought each other while fighting Germany.[12] Instead, the dynamic which precludes a bargain is the existence of different paths for fighting to take. Since actors' demands will depend on the maximum they could get from any one path, and since different paths might maximize different players' odds of victory, then the sum of these maximal demands can exceed one. With only two players, there is only one path, and so the sum of their maximal odds is always one; with many players, this sum might exceed one—and in fact, it usually will.

The example also offers a new way to think about offensive advantages. Traditionally, scholars have conceived offensive advantages as returns to surprise attack. But in the example (though not in the actual war), the odds of a German victory against France do not depend on who strikes first: France is just as likely to defeat Germany in a one-on-one struggle when it starts a war as when it does not. The offensive advantage accrues, not from the ability to alter the balance of power through a surprise attack, but from the ability to determine, in part, the subsequent course of conflict. Germany cannot change its odds in a one-on-one war with France, but it can increase its odds of fighting such a war—that is, of separating the war with the French from the war with the Russians. A key role of technology in causing (or preventing) war might thus be the degree to which it enables (or constrains) the ability of aggressive states to choose how they fight their enemies.

In the remainder of this section, I offer a general model of war involving any number of players; I then deduce the necessary and sufficient conditions under which a bargain will not exist. In developing this model, I change only the number of players; in all other ways, it remains identical to extant two-player games. Indeed, every two-player model in which war is a costly lottery is part of the game I present. Thus, the impossibility of peace will result, not from any new ways of specifying power or bargaining, but purely from the introduction of additional players. I also show how previous multiplayer models of war have never produced this result because of the equations they have used to model power.

2.1 General Model

Consider a game. Call the set of players in this game N, and assume that N contains at least two elements. These players are bargaining over a good of unit size. The game has potentially infinite rounds. Each player $i \in N$ begins each round with an endowment $w_i^t \geq 0$, where t denotes the round, subject to the constraint that the sum of all states' endowments is equal to the total pie, $\sum_{i \in N} w_i^t = 1$. Conceptually, w_i captures a player's current share of the world's goods (hence why their sum equals one), which it can keep or bargain away as it pleases.

Next, Nature offers each player an $x_i^t \geq 0$, with t again denoting the round and the sum of the offers again equal to the total pie to divide, $\sum_{i \in N} x_i^t = 1$. Call the vector of these offers X^t and assume that X^t is known in advance for any possible path the game might take. (Thus, players operate under complete and perfect information.) For convenience, I refer to X^1 as X with generic element x_i; I also refer to players' initial endowments simply as w_i. Conceptually, X captures a bargain which might emerge from some process of negotiation. I abstract from this process in order to highlight how peace might be impossible no matter how we formalize bargaining. Since the precise specification of the bargaining process itself can inhibit peace,[13] I leave it opaque so that, in any game, if a bargain is possible under some specification, it is possible in the model. My goal will be to show not that war is possible in some equilibrium, but that in any subgame perfect equilibrium with any negotiation process, there is no bargain that could induce states to remain at peace. In some sense, then, this modeling choice has maximized the possibility of peace, biasing the model against the claim I wish to make.

After Nature offers X^t, players decide simultaneously between three options: each player can accept X^t, allow X^t, or reject X^t. These options represent a state's willingness to accept a distribution of goods forever, its willingness to allow a redistribution of goods to take place while reserving the right to reject it later, and its rejecting a distribution and challenging it by force. Theoretically, the distinction between accepting and allowing an offer makes it possible for a redistribution of goods not to terminate the game; thus, no settlement can bind a player when that player might one day prefer to overturn it, allowing the model to approximate more closely the anarchy of international relations.

If all states accept X^t, the game terminates and each player receives x_i^t as its payoff, less any costs from war; acceptance thus happens when all states are, in some sense, satisfied with the present bargain. If not all players accept X^t but no player rejects X^t, then the game continues to the next round, and players' endowments become $w_i^{t+1} = x_i^t$. That is, the game continues whenever some players refuse to accept forever the current bargain X^t but no player challenges that agreement by force. Conceptually, this stage captures a bargain taking effect—for instance, a tribute paid by a vassal to a suzerain or a transfer of territory between two empires. In the propositions, the goal will be to show that, under many circumstances, no bargain exists which can satisfy all players.

If a player i rejects X^t, she must do so by attacking some players $J \subseteq N \setminus \{i\}$, with $J \neq \emptyset$. If at least one player rejects X^t, then a war results and proceeds as follows. After players declare whom they wish to attack, with some probability $\gamma > 0$ the war resolves immediately, and with probability $1 - \gamma$ actors not involved in the war are given the chance to enter it, and actors involved in the war are given the chance to attack additional players. No matter how many players reject X^t, a single war results between all players who rejected X^t, were attacked by a player, or chose to enter. For instance, if A attacks B, and B attacks C, then a war results between A, B, and C.After the war, losing states exit the game, receiving a payoff of 0 less their costs from war. If only one state is left in the game after a war, then the game terminates and that state receives a payoff of 1 less his costs of war. Each war costs all participants $(n-1)c$, where n captures the number of players involved and c is a constant greater than zero. For instance, in a game of three players, if all three fight, and only one emerges victorious, then the winner receives $1 - 2c$, and the losers each receive $-2c$. Note that the precise specification of how costs accrue is immaterial to the propositions; what matters is that, even though war destroys part of the pie, still a mutually preferred bargain might not exist.

A war resolves according to a function p describing the distribution of power, where

$p(J, S)$ describes the probability that a set of countries J survives a war among countries S, with $J \subseteq S$ and $J \neq \emptyset$. Since at least one country must emerge from a war, note that, for any $S \subseteq N$, with S containing at least two elements, $\sum_{J \subseteq S} p(J, S) = 1$. For instance, $p(A, AB)$ is the probability that A emerges victorious from a war between A and B, and $p(AB, AB)$ is the probability neither can defeat the other; moreover, it must be true that $p(A, AB) + p(B, AB) + p(AB, AB) = 1$. Note that, because p is fixed, no power transition exists, and so no commitment problems will drive the propositions' results.

Finally, it is technically necessary to specify what happens when multiple players survive a war. For the first proposition, this specification will not matter, as I will assume that only one player ever survives. For the second proposition, I will use the possibility to capture the idea of stalemate; therefore, assume that players who have once survived a common war cannot attack each other in the future. For instance, if A and B both emerge from a war, in future rounds A can never attack B, and B can never attack A. If no surviving player can attack any of the others, then the game terminates and players receive their present endowments as payoffs, less their costs from war.

So that the model with many players is identical to existing work on two-player models, for the first proposition assume that $p(J, S) > 0$ only if J contains exactly one element; I refer to this condition by saying that *stalemate is impossible*, since only one state can emerge from a war. This restriction makes even tacit alliances impossible; it also implies that the first proposition is not driven by coalitional dynamics or the problem of an empty core. As a consequence, because peace will be impossible for many levels of c, war results because no bargain can satisfy all distributions of power simultaneously. Note that relaxing this assumption would make war even more likely, as the second proposition shows.

Since N, p, and c define the key attributes of the game, for simplicity, we can refer to a game simply as (N, p, c). To define our subject of interest, call a game *peaceful* if there exists an X such that in some subgame perfect equilibrium (SPNE), players always accept X. Thus, if a game is *never peaceful* (or not peaceful), then there must exist no subgame perfect equilibrium in which players would all accept some X. In other words, if a game is *never* *peaceful*, then there is no equilibrium in which war does not occur with some probability.

The first proposition establishes a sufficient condition under which a multiplayer game is never peaceful. This condition I refer to as *divide-and-conquer* (DAC). Conceptually, *divide-and-conquer* means that at least one state is more likely to defeat its enemies if it fights them piecemeal rather than all at the same time. This condition requires in no way that its enemies cooperate, only that a country is better off facing them singly than simultaneously. As I discuss later, this condition is impossible to satisfy using traditional ways to mathematize power between states; as a consequence, the result in the first proposition has never before been produced.

Formally, *divide-and-conquer* requires that, for at least one player $i \in N$, for all subsets of players $J, K \subset N$, with J and K each containing at least two elements, i prefers to fight those subsets separately, i.e.

$$J \cup K = N, \;\; J \cap K = \{i\}$$
$$p(i, J)p(i, K) > p(i, N)$$

The first proposition follows immediately.

Proposition 1. *If stalemate is impossible, then for any set of players N and distribution of power p there exists a game (N, p, c) which is never peaceful* <u>*if and only if*</u> *N and p satisfy divide-and-conquer.*

Intuitively, if *divide-and-conquer* holds, then the sum of state power will exceed one. Since $\sum_{i \in N} p(i, N) = 1$, if for one player $p(i, J)p(i, K) > p(i, N)$, then that player's expected gains from war added to the expected gains of every other state, which are at least $p(i, N)$, will exceed one. Thus, it will always be possible to find a level of cost to ensure that the sum of all states' expected utilities from war still exceed one, in which case, a bargain cannot exist. (For complete proof, see appendix.)

As a condition on the distribution of power, *divide-and-conquer* seems to describe the international system most of the time. It captures how we intuitively think the world works:

a state facing many enemies cannot bring its best materiel to bear against all at the same time; it cannot take advantage of economies of scale; it cannot give any theater its full attention; &c. The Schlieffen Plan had its origins in precisely this dynamic—that Germany could defeat France and Russia individually but not possibly together. More generally, the principle of concentration, the most basic maxim of warfare, would seem to imply that divide-and-conquer must always hold.[14] As a condition on international relations, it is, at the very least, intuitively plausible.

But this condition cannot be satisfied by the standard contest function. When determining the odds a state wins a war, its likelihood of victory is generally a variation of

$$p(i, J) = \frac{m_i^\alpha}{\sum_{j \in J} m_j^\alpha},$$

where m captures a state's total materiel and α augments this materiel for returns to scale.[15] We favor this equation for its proportionality and its convenience; it seems as good as any other. Nonetheless, despite a certain intuitive sense, the equation unwittingly ensures that a peaceful bargain always exists. It can easily be seen that the definition implies that, because $m_j^\alpha m_k^\alpha \geq 0$, then the inequality

$$\frac{m_i^\alpha}{m_i^\alpha + m_j^\alpha + m_k^\alpha} \geq \frac{m_i^\alpha}{m_i^\alpha + m_j^\alpha} \frac{m_i^\alpha}{m_i^\alpha + m_k^\alpha}$$

is guaranteed to hold, so that $p(i, ijk) \geq p(i, ij)p(i, ik)$ always. In other words, the standard contest function implies that states are more likely to win wars when they face all their opponents at the same time than when they face them one-by-one. This implication contradicts the received wisdom of military strategists and statesmen alike. But admitting this tendency into our models entails the possibility that no mutually preferred bargain exists in many-player worlds.

In addition, we might desire to assume that, for any state, its odds of ultimate victory are always greater when facing its enemies piecemeal: any state i would prefer to fight j and k separately rather than simultaneously, so that $p(i, ijk) < p(i, ij)p(i, ik)$ for all

$i, j, k \in N, i \neq j \neq k$. Unfortunately, such an assumption is inconsistent with the assumption that stalemate is impossible, i.e. that $p(J, S) > 0$ only when J contains exactly one element. To prove this, it will suffice to consider the three-person case. If we suppose that both assumptions are true, then it must also be true that

$$1 = p(i, ijk) + p(j, ijk) + p(k, ijk)$$
$$1 < p(i, ij)p(i, ik) + p(j, ij)p(j, jk) + p(k, ik)p(k, jk)$$

But the formula is maximized for any probabilities at $p(i, ik) = p(i, jk) = 1$, which even if it were possible would cause a contradiction. Thus, the assumptions must be inconsistent. Therefore, if we think that assuming *divide-and-conquer* would better model the world, we must begin to allow for the possibility of stalemated conflict.

3 The Rock-Paper-Scissors Problem

In games of many players, the possibility of stalemate greatly exacerbates the complexity of power—what we might call the Rock-Paper-Scissors problem in international relations. This problem increases the chances of war and makes the distribution of goods difficult to predict from country/country comparisons. Because a clear ranking of countries by power is impossible (as in Rock-Paper-Scissors, where no absolute strength can be discerned), the distribution of wealth, prestige, and influence will depend not only on the present character of power but also on the past distributions of goods. In fact, under the same distribution of power, it is quite possible to observe radically different patterns of consumption.

To see this, consider the following variation on Rock-Paper-Scissors. Suppose that actors can be either Regional Powers (R), Middle Powers (M), or Guerrillas (G). Suppose further that regional powers, while they are strong against middle powers, are unable to defeat guerrilla forces in others' territories. Likewise, let us say that middle powers, while weak against their larger neighbors, are relatively strong against domestic guerrillas (perhaps because of their more intimate knowledge of the terrain and culture). Finally, suppose that guerrillas, while weak against the state they're fighting, are quite able to resist any foreign enemy. Without knowing the initial distribution of wealth and influence, it is entirely consistent to observe a world in which the regional power consumes everything or in which the guerrillas consume everything; it is equally consistent to observe a world in which peace is certain or in which war is unavoidable. Indeed, even though the middle power is much stronger than its guerrilla enemies, its vulnerability to outside rivals might make it enjoy far less wealth and influence than those same guerrillas. Likewise, though the regional power seems obviously the strongest actor in the system, it might consume much less than we would otherwise expect.

This claim can be easily demonstrated in a few examples. Suppose that there are three actors, one of each type (regional, middle, and guerrilla), and that their odds of victory are as follows:

$$p(R, RM) = \frac{3}{4} \qquad p(M, RM) = \frac{1}{4}$$
$$p(R, RG) = 0 \qquad p(G, RG) = 0$$
$$p(M, GM) = \frac{1}{2} \qquad p(G, GM) = \frac{1}{4}$$
$$p(RG, RGM) = 1$$

Consider a series of initial distributions of wealth, each of which will produce an outcome radically different from all the others. First, suppose that initially guerrilla forces control $\frac{1}{3}$ of the goods the actors are disputing while the middle power controls $\frac{2}{3}$. (A convenient illustration would be to say that guerrillas control one-third of country M's territory.) In this situation, the actors agree on a distribution assigning $\frac{1}{2}$ of the goods to the regional power, $\frac{1}{3}$ to the guerrillas, and $\frac{1}{6}$ to the middle power. In other words, the guerrillas, though manifestly weaker than the middle power against which they're fighting, nonetheless enjoy a much larger share of the pie. If we alter the middle power's initial share, this result changes. For instance, if the middle power initially controls all of its territory, then a peaceful equilibrium does not exist: it will fight the guerrillas until a stalemate or a decisive victory is reached, and then it will peacefully yield an appropriate share of the pie to its regional rival. In this situation, the guerrilla forces come out worst of the three players. By contrast, if the guerrillas begin the game by controlling almost all of the pie, they will come out best of the three, though war may prove unavoidable. And so forth. These results are summarized below. (The order is $\{Regional, Middle, Guerrilla\}$.)[16]

Init. Dist.	Exp Payoffs	Peace?
$\{0, \frac{2}{3}, \frac{1}{3}\}$	$\{\frac{1}{2}, \frac{1}{6}, \frac{1}{3}\}$	peace
$\{0, 1, 0\}$	$\{\frac{9}{16}, \frac{3}{16}, \frac{1}{4}\}$	war
$\{0, 0, 1\}$	$\{\frac{3}{8}, \frac{1}{8}, \frac{1}{2}\}$	war
$\{1, 0, 0\}$	$\{\frac{3}{4}, \frac{3}{16}, \frac{1}{4}\}$	war

In this example, war results because the distribution of goods must satisfy multiple power relationships simultaneously. Among two players, the middle power would necessarily consume more than the guerrillas, but the introduction of a third player changes the game. Guerrillas become less willing to cede wealth because the middle power will in turn have to yield a portion to the regional hegemon (whom the guerrillas are quite able to defy). Similarly, the middle power becomes less willing to fight to retake guerrilla territory if it fears the regional power will just extort it further. The same applies to the regional actor, who will not cede wealth to its weaker rival for fear that rival will use some of it to appease the guerrillas. Depending on the initial setup of the game, war can thus become inevitable.

While I have used here the language of guerrilla war and regional power, the same logic applies equally to other realms. The Rock-Paper-Scissors problem will arise wherever the technology of power is to some degree non-fungible, that is, wherever weapons and strategies are less effective against some enemies than against others. This non-fungibility will tend to make power relationships less straightforward, consequently making it more difficult to find an outcome satisfactory to all.

I have outlined only one instance of the Rock-Paper-Scissors problem. I suspect there are many more, and that examining the complexities of power between several countries will greatly enhance our understanding of interstate and civil conflict. Nevertheless, I turn now to a more immediate cause of war. The previous section argued that stalemate is mathematically necessary in order to reflect conventional military wisdom. This section showed how, by including stalemate, numerous problems arise in predicting the character of international politics. The following section will show that, even in the traditional two-player setting, the possibility of stalemate can make war inevitable. Including its possibility must therefore fundamentally alter how we understand bargaining between states, since it implies that a bargain might not exist. Its incorporation into our models can help us understand some of the longest-standing puzzles confronting the bargaining model of war.

4 When Stalemate is Possible, a Bargain Might not Exist

Clausewitz made a mistake. *On War* argues that war is a duel in which "each tries through physical force to compel the other to do his will...to make him incapable of further resistance" (1976, 75). This approach underpins virtually all bargaining models of war: unless players can reach a negotiated settlement, eventually every war must end in the decisive defeat of one party.[17] While this modeling choice has borne great fruit, it misses an important source of conflict, not least because it poorly captures the bargaining dynamic between asymmetric rivals; to assume that war must end decisively is to generalize to all forms of conflict what is really only true of great power war on the European Continent. When a small country defies a great power, it does not entertain the hope of decisively defeating its enemy; the Boers had no illusions of taking London and decapitating the queen. Rather, a weak country hopes to deny an enemy the ability to continue hostilities (or, in the interim, convince an enemy that war would prove too costly). Thus, Clausewitz' mistake is this: for one side, the purpose of fighting might not be to render an enemy incapable of further resistance; rather, it might be to render him incapable of further aggression.

A country may lose its ability to threaten its opponent without also losing the ability to defend itself. For instance, an empire might be able to destroy a small state with probability .9, but this ability does not imply the small state can destroy the empire with probability .1. It makes little sense to suggest that the odds of a Roman victory in Gaul or an American victory in Vietnam were 1 minus the odds of Washington or Rome getting sacked, or that the probability Alexander could conquer India was one minus the probability that India could conquer Macedon. These examples may seem obvious or even trivial, but granting them must upend how we model war. If war can end in stalemate, then states do not wholly risk themselves when they fight wars, and this insurance can prevent a bargain.

In a two-player game, stalemate means simply that neither player wins. There is some probability that A wins, and some probability B wins, and some probability that both players walk away with their wallets intact (minus the costs of playing). It turns out that this chance of a draw, of keeping one's stake, increases the willingness of actors to engage in conflict.

In the standard approach, if players do not reach a negotiated settlement, war can end in only two ways: player A decisively defeats B, or B decisively defeats A. If B cedes goods to A, then it does not alter A's payoff to war: the payoffs to each outcome remain constant (namely, either $1 - c$ or $-c$) and the probability of each outcome remains constant, as well (namely, p and $1 - p$). If we allow for stalemate, for a state to keep its goods without necessarily defeating its opponent, then this constancy no longer holds. As B cedes goods to A, A's payoff to war increases: A's payoff for a decisive victory remains constant $(1 - c)$, but its payoff to a stalemated outcome grows larger, since it now owns a larger share of the pie. Consequently, even though power remains constant, by ceding goods to A, B increases A's payoff to war and thus the share of goods A is able to demand. Rather than continually cede goods to A until A is finally satisfied, B might prefer simply to fight initially in defense of what it has.

To see this, consider an example. Say that two countries, call them Athens and Melos, each possess \$10. Suppose that Athens is strong, so that the odds Athens can defeat Melos and take all its wealth are $\frac{1}{2}$; on the other hand, Melos possesses no offensive capability, so that it could never seize Athens' wealth by force. If the countries ever do fight a war, its outcome is decisive. Suppose a war costs each player \$1. Initially, Athens' utility from fighting a war is \$14, i.e. its present share (\$10) which Melos could never seize plus its expected gains from war, which are its odds of taking Melos' share less the costs of conflict ($\frac{1}{2}\$10 - \$1$). Thus, Melos must yield to Athens at least \$4, leaving Melos with \$6. But then, Athens' utility from fighting a war is \$16 ($\$14 + \frac{1}{2}\$6 - \$1$), and so Melos must again yield to Athens, and so on, until Melos is left only with \$2, when Athens will be content. Thus, in a final settlement Melos will consume \$2 and Athens will consume \$18. But, were Melos simply to fight initially, Melos could consume \$4 in expectation, which is strictly more than its payoff from a series of negotiations. Consequently, Melos will reject Athens' demands and fight.

The example illustrates the basic problem of appeasement. Since international politics are anarchic, no negotiation is final—an aggressor can always demand additional concessions after some have been made. Its ability to do so is not tied to a commitment problem as the literature conceives it: there is no future power shift or offensive advantage to undermine the current bargain, nor does transferring wealth and territory increase power. Rather, concession itself enables an enemy to demand more. Thus, appeasement is dangerous because a rival might not be satisfied with what it currently demands. Strangely, this problem is tied inextricably to that of stalemate, to the ability of a country to keep what it has, not by decisively defeating a rival, but by exhausting it. In the example, I use mismatched rivals to illustrate the point, but stalemate can eliminate a bargain between virtually any opponents.

To see this, reconsider the general model of section two but with only two players. This focus on the two-player game will show how changing the single assumption in the classic model undermines its results; it will also show how the change can generate novel predictions about the world. Therefore, assume that N contains only two elements, and call these players A and B. For convenience, I will refer to A and B's power as $p_A = p(A, AB)$ and $p_B = p(B, AB)$; I will also refer to the probability of stalemate as $p_S = p(AB, AB)$. We can state the second proposition thus:

Proposition 2. *A two-player game is peaceful for all costs of war c <u>if and only if</u> either*

1. *stalemate is impossible,* $p_S = 0$

2. *stalemate is guaranteed,* $p_S = 1$

3. *the initial distribution of wealth is exactly proportional to relative power,* $w_A = \frac{p_A}{p_A + p_B}$

This proposition shows that the previous example generalizes to almost all situations in which states might find themselves. Specifically, it claims that, if stalemate is possible but not certain, then, except only in the unlikely event that players' initial endowments precisely align with their odds of victory, for some costs of war c the probability of war is nonzero in all subgame perfect equilibria for all possible bargains states might reach. In other words, if two players avoid war, it is almost certainly because war is sufficiently expensive.

In the case of interest where $0 < p_S < 1$, for a state i to accept a settlement, it must be true that its share of the pie x_i is greater than what it could attain from allowing the settlement to take effect and then fighting in the next round, i.e.

$$x_i \geq p_i + p_S x_i - c$$

Moreover, for a state to prefer to allow or accept a settlement rather than reject it and fight immediately, it must also be true that its share of the bargain is more than what it could gain through war, i.e.

$$x_i \geq p_i + p_S w_i - c$$

Thus, for peace,

$$x_i \geq p_i + p_S max[x_i, w_i] - c$$

which means that, unless $x_i = w_i$, then x_A and x_B can sum to more than one for sufficiently small c. (For complete proof, see appendix.) In short, unless each actor enjoys an amount of wealth exactly proportional to its power, when c is small enough one state will always prefer to fight to defend its present share of the pie.

5 Peace, Revision, and the Nature of Power

The introduction of stalemate allows us to engage formally the literature on status quo and revisionist states. If the distribution of wealth does not exactly mirror the distribution of power, then one state must be in some sense 'dissatisfied.' In particular, a state is dissatisfied if its share of the world's wealth is less than its share of the world's power. When confronted by a dissatisfied rival, a satisfied state has three options: it can ignore its rival; it can cede enough wealth to satiate its rival; or it can deny its rival and fight to defend what it has. Ignoring a rival is possible only when the costs of war are very high, so that an enemy cannot benefit from a war of aggression. Appeasing an enemy is what the standard bargaining model of war predicts under perfect information and constant power. But defying an enemy is what most often happens. This paper explains why.

Ceding wealth to a dissatisfied state increases its payoff to war. Its payoff to a decisive victory remains constant (p_A); its payoff to a stalemate increases ($p_S * x_A$). Thus, a bargain which will satisfy a rival today might not satisfy him tomorrow. In the standard model, the sum of players' demands must be less than one. But if stalemate is possible, then this sum can exceed one if the costs of war are sufficiently small, in which case a bargain does not exist.

In this way, the possibility of peace is tied to the costliness of war. If costs are very high, a satisfied state ignores its rival's dissatisfaction. If costs are high but not too high, a satisfied state appeases its rival by ceding some of its wealth. But if costs are not high, then a satisfied state would prefer to fight on the chance that it can successfully defend the wealth it has. And so this question naturally follows: at what level of cost does peace become impossible to sustain? That is, at one point does a dissatisfied state become revisionist, challenging the status quo by force?

The remainder of this section seeks to answer this question, at least in part. It argues that the level of cost necessary to prevent war increases when i) a satisfied state grows wealthier; ii) a dissatisfied state grows stronger; or iii) states' offensive power declines (or states' defensive power rises). We may denote this minimum cost necessary to sustain peace as $\bar{c}$. This term allows us to study a system's stability: the lower is $\bar{c}$, the more stable is the game, since peace can be sustained in more circumstances. We can see that

$$\bar{c} = \frac{1}{2} p_S |x_A - w_A|$$

where x_A is the bargain closest to w_A both players could accept. Clearly, the more a peaceful bargain must diverge from the present distribution of wealth, the higher the costs of war necessary to induce a satisfied state to accept it. If c is not high enough, the satisfied state will instead refuse to appease its rival, and the dissatisfied rival will seek to revise the *status quo*. Formally, if we assume that A is the dissatisfied state, so that the bargain it accepts is more valuable to it than the present distribution of goods—i.e. $x_A > w_A$, or equivalently, $p_A > (1 - p_S)w_A$—then we can state the first claim like so:

Corollary 2.1. *The instability of the system $\bar{c}$ is increasing in the wealth w_B of the satisfied state.*

Thus, the wider the divergence between a country's power and its present share of the world's goods, the higher the costs of war necessary to prevent conflict. This result aligns with current expectations from the bargaining model, e.g.(Powell, 1999); where it differs is that no information or commitment problems are necessary to impel states to conflict. The divergence between power and the *status quo* matters, not because it exacerbates other problems, but because it creates one itself. Curiously, this prediction also aligns with power-preponderance models, which predict that war becomes more likely as a revisionist state rises.[18] So, while previous work has seen these two theories in tension, in fact they share a common logic.

The second and third claims are not quite so straightforward. A bit of algebraic manipulation allows us to rewrite the previous equation as

$$\bar{c} = p_S \frac{[p_A - (1 - p_S)w_A]}{2 - p_S}$$

The second claim shows that, even if a dissatisfied state A increases its power through purely defensive means, it still increases $\bar{c}$ and so jeopardizes the peace. By contrast, even if a satisfied state B increases its power through purely offensive means, it still decreases $\bar{c}$ and so stabilizes the peace.

Corollary 2.2. *If the offensive power p_A of the dissatisfied state remains constant, then the instability of the system $\bar{c}$ is increasing in its defensive power p_S. On the other hand, if the odds of stalemate p_S remain constant, then the system's instability $\bar{c}$ is decreasing in the offensive power p_B of the satisfied state.*

Intuitively, as a dissatisfied actor grows stronger, the costs of war necessary to sustain the *status quo* must also increase. This is true even if the dissatisfied actor only strengthens its defensive power, since even this action increases the share of the pie it can demand. Mathematically, if we hold p_A and w_B constant, then we can take the partial derivative of $\bar{c}$ with respect to p_S (states' purely defensive power), which is

$$\frac{(2-p_S)p_S w_A + 2(1-p_S)[p_A - (1-p_S)w_A]}{(2-p_S)^2}$$

So long as A remains dissatisfied (so long as $p_A > (1-p_S)w_A$), then this partial derivative is necessarily positive. The opposite also holds. If a satisfied state B increases its power, even through purely offensive means, then it decreases the costs necessary to preserve peace. Formally, if we hold p_S and w_B constant, so that only B's offensive power p_B changes, we can take the partial derivative of $\bar{c}$ with respect to p_B, which is obviously negative (assuming A remains dissatisfied).Thus, when a satisfied country builds new offensive weapons, so long as it remains satisfied, it has only strengthened the peace.[19]

The third claim is perhaps the most surprising of all: in most circumstances, offensive weapons are more stabilizing than defensive ones. States become revisionist because a satisfied power would rather fight to retain its excess share of the world's goods than appease a rival; war results because a dissatisfied power may be unable to overturn the *status quo*. Thus, the more states are able to defeat each other decisively, the more willing they are to transfer goods between themselves in order to avoid conflict.

We can formalize this claim by comparing similar balances of power, but vary to what extent the system favors the offense versus the defense:

Corollary 2.3. *If states' relative power* $\frac{p_A}{p_A+p_B}$ *remains constant, then the instability of the system* $\bar{c}$ *is increasing in the odds of stalemate over* $p_S \in [0, 2-\sqrt{2}]$ *and decreasing over* $p_S \in [2-\sqrt{2}, 1]$.

Essentially, if the balance of power, $\frac{p_A}{p_A+p_B}$, in two systems is the same, but in one the defense is stronger (p_S is higher), then so long as the defense is not too strong, the more offensive system is more stable. Again, we can see this by taking the partial derivative of $\bar{c}$ with respect to p_S, this time holding $\frac{p_A}{p_A+p_B}$ and w_B constant. If we first rewrite $\bar{c}$ as

$$\bar{c} = p_S(1-p_S)(2-p_S)^{-1}\big(\frac{p_A}{p_A+p_B} - w_A\big)$$

then we can see that the partial derivative in question is

$$(2-p_S)^{-2}\big(\frac{p_A}{p_A+p_B} - w_A\big)(2-4p_S+p_S^2)$$

which is positive for all $p_S \leq 2-\sqrt{2}$, and so $\bar{c}$ is usually decreasing in the strength of offensive weapons.[20]

6 Conclusion

We cannot assume, without consideration, that a bargain exists that all states prefer to conflict. Indeed, peace might be impossible even with perfect information, no commitment problems, no alliance dynamics, no principal agent problems, and perfectly divisible goods. If we extend the traditional bargaining model to three or more players, then if at least one player prefers to divide its enemies rather than fight them all at once, a bargain will often not exist. Previous studies of multilateral games have not produced this result because they have implicitly assumed that states would prefer to fight all their enemies simultaneously. In this world of many actors, war results because the complexity of power makes it impossible to agree on a distribution of goods satisfactory to all. We must therefore apply two-player models with caution, for even bilateral wars occur in a multilateral context, and conclusions reached in two-player settings might not generalize to games of many players.

Even in the traditional two-player game, if we introduce the possibility of stalemate—i.e. the possibility that neither player wins—then once again a bargain might not exist. In this scenario, an actor who enjoys disproportionately more wealth has an incentive *not* to transfer that wealth in order to appease a dissatisfied rival, in which case the rival will seek to revise the *status quo* by force. The ability of a system to resist such pressures for revisionist war is a function of military technology: the more indecisive a war is likely to be, the less one state is willing to satisfy the demands of another. Modes of warfare which may be less destructive or more materially costly, but which do not erode with defeat, lead more often to peace than stronger weapons and doctrines which diminish after a failed offensive or extended campaign. If after several years a state can still threaten victory, then peace will be easier to sustain than if it must win swiftly or not at all. And so follows a strange conclusion: if the inability of states to score a decisive victory causes war, then the rising tide of peace in the modern world may be a product of its flattening: the more able are countries to project power into all corners of the earth, the less likely is violent conflict.

The emphasis in these predictions is on indecisive war. For actual negotiations, this emphasis carries a strong imperative: for peace to succeed, a state must retain the power to retake what it surrenders. The more a country loses power over what it yields, the more likely is conflict. Successful bargaining may thus depend, not on finding out each other's relative strength, but on discovering creative ways to transfer assets.

A final note on military power. We treat technology too blithely. How we theorize power strongly affects the conclusions our models generate, whether we realize it or not. The distribution of power cannot be easily summarized by a single factor m_i capturing its total materiel; rather, a state's power is always relative to its opponent. One form of violence will prove more effective against some enemies than against others: a naval power's might will figure more prominently against a maritime opponent than a landlocked one; similarly, democracies may find it more difficult than autocracies to fight guerrilla wars (or *vice versa*). Power cannot be aggregated into a single term, either in formal models or in statistical analyses. Instead, our models must begin to account for the complexity of power—because this complexity can cause a war.

Commitment problems, information problems, and all the rest remain reasonable explanations of conflict. But they are not exhaustive. A host of rationalist explanations for war exist, both within the traditional two-player context and beyond it, and they are almost entirely unexplored. This expanse should give political scientists hope, but it should also give us pause. The world contains far more rational sources of conflict than we ever imagined. We have begun to think of peace as usual, as the expected outcome when difficult problems do not intervene, but this view may be fundamentally naive. If there is peace, we should be surprised—and we should look carefully for its source. For the distribution of power matters. It matters more than principals and agents, it matters more than information, and it matters more than sudden changes in strength. All these determine whether states can accept a bargain, but power determines whether a bargain exists. When studying international security, the distribution of power must once again become our first quantity of interest.

Notes

[1]For an older but invaluable overview of the literature, see Reiter (2003). For the most comprehensive statements of the theory's key findings, see especially Fearon (1995), Slantchev (2003), Powell (2006), Leventoğlu and Slantchev (2007), and Fey and Ramsay (2011).

[2]Assuming that altering a state's share does not alter its power to avoid a commitment problem; see Fearon (1996).

[3]The first three are famously outlined in Fearon (1995). The fourth is sometimes mentioned but rarely examined in-depth; for two exceptions, see Jackson and Morelli (2009) and Coe (2012). The last I address in the second section of this paper.

[4]For the classic works, see Gilpin (1981) and Kennedy (1987). This line of inquiry persisted even into the late 1990s. In its strongest forms, theorists suggested that, because all states seek security, all states will seek hegemony, and in this quest for hegemony they will necessarily come into conflict; see Mearsheimer (2001). Subtler versions would divide states between revisionist and *status quo* powers—see e.g. Schweller (1994) or Kydd (1997)—but would still assume that aggressive intentions necessitate conflict.

[5]A.F.K. Organski wrote in 1968 (p294-5): "Nations with preponderant power have indeed dominated their neighbors, but they have not been the ones to start the major wars that have marked recent history. That role has fallen almost without exception to the weaker side." This puzzle remains unanswered.

[6]Jackson and Morelli (2011) note this surprising inattention; in the *Handbook of the Political Economy of War*, multilateral bargaining has only one citation—to an economist.

[7]For the classic cooperative approach, see Niou and Ordeshook (1986). For a more recent cooperative approach to alliance formation, see Sandler (1999).

[8]This finding amends that of Skaperdas (1998), which suggests weak actors usually balance.

[9]See especially Sandler (1993) and Sandler and Hartley (2001). Garfinkel (2004) is unusually general, though the actors in the model remain fairly constrained.

[10]To my knowledge, this result has never before been proven. It is akin to a working paper by Max Gallop, which shows how shared interests can drive states to war (assuming any settlement cannot be renegotiated), but is far more general in its treatment of interests and power. In particular, I show that the distribution of power itself, not its combination with the shared interests of states, can compel a conflict; I also allow for postwar settlements to be overturned.

[11]As discussed later, the standard contest function implicitly assumes these demands must be less than one and so has not produced this paper's result. Similarly, though Krainin (2014) does not assume the contest function in its general model, it does assume that the sum of players' unaligned payoffs must be less than the goods to divide (p418).

[12]Using the above numbers, a bargaining range might not exist so long as German odds of victory in a free-for-all remained less than one-half.

[13]See Powell (2004) and Smith and Stam (2004); also Fey and Ramsay (2011)

[14]It certainly describes power on the battlefield, where it is better to face half an enemy's army and then face the other half rather than face all at the same time. Sun Tzu (1963, 80, 98) urges a general to: "divide him [one's enemy]...if I concentrate while he divides, I can use my entire strength to attack a fraction of his."

[15]For instance, a variation on this formula is used by Powell (1999), Wagner (2007), and more recently by Gallop (2015). Stam (1996, 21) characterizes previous empirical work as, effectively, employing this same equation. It also figures in some two-player models, though less commonly; see for instance the example in Meirowitz and Sartori (2008, 344).

[16]These portions can vary as the costs of war increase. I have set them at zero to dramatize the dynamic. When the costs of war are zero, for a given distribution the peaceful equilibrium, if it exists, is unique.

[17]For a sampling of papers employing this assumption, see Fearon (1995), Fearon (1996), Powell (2002), Filson and Werner (2002), Slantchev (2003), Powell (2004), Powell (2006), Leventoğlu and Slantchev (2007), Fey and Ramsay (2007), and Slantchev and Tarar (2011). While Powell (1999, 203) recognizes that assuming away stalemate may be problematic, until now no work has investigated how its possibility changes the bargaining model's main results. The only partial exception is Stam (1996), which explores the empirical implications if war can end in a draw, but even in his model draws only result when states choose to stop fighting, not because they can no longer achieve victory.

[18]"War is most likely when the power of the dissatisfied challenger and its allies begins to approximate the power of those who support the *status quo*" (Organski, 1968, 370).

[19]This conclusion contradicts much of the received wisdom in political science but not in history or military science: "If we are to look for 'threats to peace' in the world, therefore, we should not look in the first place at armaments...rather for those elements in international society...who have the greatest incentive to disturb the existing order...the greater that ability [to disturb order], and the less the capacity of defenders of the *status quo* to deter them, the more precarious peace is likely to be...Stability comes from relationship between forces; not from their overall numbers...All depends on who increases their armaments and for what purpose" (Howard, 1983, 155, 160)

[20]Incidentally, the exception to this rule is telling: when the probability of stalemate is very high, reducing it makes war more likely. For instance, if two countries were once beyond each other's reach, but one abruptly finds itself vulnerable to the other, then war may be inevitable. This finding might help explain how technological advances in the early periods of globalization made wars of imperial conquest inevitable.

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Appendix

(For review; not necessary if published.)

6.1 Proof of Proposition 1

I first prove that if DAC holds then a c can always be found to make (N, p, c) *never peaceful.* Suppose that DAC is satisfied. Choose a player $i \in N$ and sets J, K such that $J \cup K = N$, $J \cap K = \{i\}$, and $p(i, J)p(i, K) > p(i, N)$. Suppose that a peaceful equilibrium exists. In this equilibrium, if i deviated to fight in the first round, attacking all other states in J, then i could gain an expected utility at least as much as $\gamma\big[p(i, J)[p(i, K) - (n_K - 1)c] - (n_J - 1)c\big] + (1-\gamma)(p(i, N) - (n_N - 1)c$, where n_J and n_K denote the number of players in sets J and K, respectively. Similarly, every other player $j \in N \setminus \{i\}$ could gain at least $p(j, N) - (n_N - 1)c$ by deviating and attacking all players simultaneously. Therefore, each must receive a share of the pie x_i or x_j in excess of these payoffs. Thus,

$$\begin{aligned} 1 = \sum_{\nu \in N} x_\nu > & \gamma\big[p(i, J)[p(i, K) - (n_K - 1)c] - (n_J - 1)c\big] + (1-\gamma)(p(i, N) - (n_N - 1)c \\ & + \sum_{j \in N \setminus \{i\}} [p(j, N) - (n_N - 1)c] \end{aligned}$$

By DAC, $p(i, J)p(i, K) > p(i, N)$. Call the difference between the left and right side k. Thus,

$$1 = \sum_{\nu \in N} x_i > 1 + \gamma k - n_N(n_N - 1)c$$

But clearly we can choose any c satisfying $0 < c \leq \frac{\gamma k}{n_N(n_N-1)}$ to make the inequality false, causing a contradiction. Therefore, the equilibrium cannot be peaceful for all c.

To complete the proof, I show that if there exists a c such that (N, p, c) is never peaceful, then the game must satisfy DAC. Suppose that a game does not satisfy DAC. Consider a strategy set that is a variant of the classic grim trigger. Let Nature offer each player an $x_i = p(i, N)$ and an $x_i^t = p(i, R)$ in any subsequent round, where R is the set of remaining players; each player i accepts an offer if and only if $x_i \geq p(i, N) - (n_N - 1)c$ in the first round, but in subsequent rounds every player rejects all offers and instead attacks all remaining players. Moreover, if given the chance to attack after a war is declared, every party involved in the war will also attack every other player. Thus, war is guaranteed amongst all remaining players in the second round or in the first round if the war does not resolve immediately. Moreover, since it is guaranteed no matter what any one player does, no player has an incentive to deviate in the second round. For any player i, the most he could gain by deviating in the first round is thus what he could attain by fighting some subset J of N in the first round, then fighting the remaining subset of N (call it K) in the second. But because DAC is not satisfied, $p(i, N) \geq p(i, J)p(i, K)$, and so i could be no better off than if it fought all its rivals in the first round. As its payoff to such a fight is strictly less than the x_i Nature offers, then no player i desires to deviate, implying that the strategy set is an equilibrium. But then, the game (N, p, c) is peaceful. Thus, if (N, p, c) is not peaceful, then the game must satisfy DAC. This completes the proof.

6.2 Proof of Proposition 2

If $p_S = 0$, the game is the same as extant models. If $p_S = 1$, any stream of offers can be accepted in equilibrium. Therefore, suppose $0 < p_S < 1$. Suppose players accept an initial settlement X in equilibrium. Either $x_A < w_A$, $x_A > w_A$, or $x_A = w_A$. Case 1: suppose the first. Then it must be true that

$$\frac{p_B - c}{1 - p_S} \leq x_B = 1 - x_A \leq 1 - p_A - p_S w_A + c$$

$$p_B - c \leq (1 - p_S)[1 - p_A - p_S w_A + c]$$

$$c \geq \frac{p_S(p_A + w_A - p_S w_A)}{2 - p_S}$$

But since the right term is positive, it is always possible to find a c less than it, and so a c must exist to make this equilibrium impossible.

The second case is similar. For the third, suppose that $x_A = w_A$, but that $w_A \neq \frac{p_A}{p_A + p_B}$. Without loss of generality, suppose $w_A < \frac{p_A}{p_A + p_B}$, and call the difference k. For players to accept peace, it must be true that $x_A = w_A \geq \frac{p_A - c}{p_A + p_B}$. This implies that

$$c \geq k(p_A + p_B),$$

which is not guaranteed to hold. But then, there always exists a c to make the equilibrium impossible.

Finally, if $w_A = \frac{p_A}{p_A + p_B}$, then the stream of offers assigning $x_i^t = w_i^t$ in every round can be accepted in equilibrium. This completes the proof.