Taking Incomplete Information Seriously: The Misunderstanding of John Harsanyi

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A Tripartite Distinction of Informational Assumptions in Economic Analysis

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   - Everything is "common knowledge" among economic agents
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2. Complete but Imperfect Information

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   - No common knowledge assumptions at all
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A Pessimistic Assessment

von Neumann and Morgenstern "Theory of Games and Economic Behavior" 1944

...we cannot avoid the assumption that all subjects under consideration are completely informed about the physical characteristics of the situation in which they operate

Aumann (1987) wrote "The common knowledge assumption underlies all of game theory and much of economic theory. Whatever be the model under discussion ... the model itself must be assumed common knowledge; otherwise the model is insufficiently specified, and the analysis incoherent."
incomplete Information is not a problem
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we can incorporate any incomplete information without loss of
generality!
there is a set of states $\Theta$ that we care about
John Harsanyi Part 1: Type Spaces

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- two players, Ann and Bob (generalize straightforwardly to many players)
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- write $\pi_A(t_B, \theta|t_A)$ for the probability that type $t_A$ of Ann assigns to both Bob being type $t_B$ and the state being $\theta$; so we have
  \[ \pi_A : T_A \rightarrow \Delta(T_B \times \Theta) \]
  and analogously
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  - in game theory, it can encompass payoffs but also the rules of the game....
  - in economic model, it can encompass preferences, technology, etc...
Ann is characterized by...

So Ann is characterized by this infinite sequence of such higher order beliefs, or universal types

"universal type space" satisfies

We can assume that this structure is common knowledge

Incomplete information is not a problem after all!
John Harsanyi Part 2: Universal Type Spaces

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  - how incomplete information can be re-visited recognizing that implicit common knowledge assumptions are a real issue
  - make those *implicit* common knowledge assumptions *explicit* and relax them
  - taking higher-order beliefs seriously

*Game theory....is deficient to the extent that it assumes other features to be common knowledge, such as one agent’s probability assessment about another’s preferences or information.*

*I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality.*"
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key but subtle observation: relaxing common knowledge is equivalent to allowing richer type spaces
Application 1: Strategic Complementarities

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- Many economic problems have "strategic complementarities" and thus, when modelled as a perfect information game, multiple equilibria
- E.g., currency crises, bank runs, financial crises, demand externalities…..
- Strategic complementarities are important but what are the implications of multiple equilibria for empirical work, policy analysis or comparative statics more generally?
- CLAIM: It is important for lots of applied economic analysis to think about the implications of relaxing common knowledge assumptions in coordination games
Today’s Talk:

- Simplest Example of a Coordination Game...
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- Ann chooses row, Bob chooses column

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If \( 0 < \theta_A < 1 \) and \( 0 < \theta_B < 1 \), then this game has multiple Nash equilibria.
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- Bob knows $\theta_B$ but forms conjecture about $\theta_A$ by Bayes updating...
- Minor variant of Carlsson and van Damme (1993)
Suppose Ann and Bob follow strategies of the form: invest if $\theta_1 \geq \theta^*$.
Risk Dominance

- Suppose Ann and Bob follow strategies of the form: invest if $\theta_I \geq \theta^*$
- Suppose $\theta_A = \theta^*$
Risk Dominance

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- Ann attaches probability $\frac{1}{2}$ to $\theta_B \geq \theta^*$

For this to be an equilibrium, we must have $\theta = \frac{1}{2}$.
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- The "risk dominant" action is always played in this equilibrium
  - for small noise, the risk dominant Nash equilibrium of the perfect information game is almost always played
Unique Rationalizable Play

- In fact, the unique "rationalizable" in this game is to invest if and only if $\theta_i \geq \frac{1}{2}$
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PROOF:
In fact, the unique "rationalizable" in this game is to invest if and only if $\theta_I \geq \frac{1}{2}$

**PROOF:**

- Let $\bar{\theta}$ be the largest value of $\theta_I$ at which it is rationalizable for either player to not invest.
In fact, the unique "rationalizable" in this game is to invest if and only if $\theta_I \geq \frac{1}{2}$.

**PROOF:**

- Let $\theta$ be the largest value of $\theta_I$ at which it is rationalizable for either player to not invest.
- Suppose $\theta > \frac{1}{2}$.
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- When $\theta_A = \bar{\theta}$, she assigns probability $\frac{1}{2}$ to $\theta_B > \bar{\theta}$.
- Her expected payoff to investing is at least $\bar{\theta} - \frac{1}{2} > 0$. 
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  - In particular, suppose that not invest is rationalizable for Ann when $\theta_A = \bar{\theta}$ and invest is uniquely rationalizable for Bob whenever $\theta_B > \bar{\theta}$
  - When $\theta_A = \bar{\theta}$, she assigns probability $\frac{1}{2}$ to $\theta_B > \bar{\theta}$
  - Her expected payoff to investing is at least $\bar{\theta} - \frac{1}{2} > 0$
  - ....a contradiction
Suppose that state $\omega$ is distributed according to smooth density $g(\cdot)$. 

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Global Games

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- If $\sigma \approx 0$, then Ann always attaches probability $\approx \frac{1}{2}$ to $\theta_B \leq \theta_A$
- As $\sigma \to 0$, unique rationalizable outcome has each player invest if and only if $\theta_I \geq \frac{1}{2}$
Global Games Extensions

- INTERPRETATION: Relaxing strong and unjustified assumption of common knowledge of payoffs generates intuitive prediction
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Further extends to general supermodular games.
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Global Games Critique

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Let’s go back to basics and examine our coordination game without making common knowledge assumptions.... or at least making fewer common knowledge assumptions....
An Important Restriction on Type Space: Private Values

- suppose that Ann’s preferences are summarized by a parameter $\theta_A \in \Theta_A$ (known to Ann), and similarly for Bob ("private values")
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- Ann’s universal type space is $T^*_A \approx \Theta_A \times \Delta (T^*_B)$

- Is a subset of our first universal type space
Relaxing Common Knowledge Assumptions in Coordination Game

- Suppose Ann is almost sure that $\theta_B \approx \frac{3}{4}$
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what can we say about strategic behavior?
Rubinstein 89, Weinstein and Yildiz 07
Suppose that the state may be "good" with \((\theta_A, \theta_B) = (\frac{3}{4}, \frac{3}{4})\):

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but Bob may have a dominant strategy to not invest, so the state is "bad", with \((\theta_A, \theta_B) = (\frac{3}{4}, -1)\):

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Electronic Mail Game on Steroids

- If the state is good, Bob sends a message to Ann, reporting that the state is good.
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  - If a player does not receive a confirmation of his/her message, he/she thinks that the other player did not receive his/her message with probability $1 - \varepsilon$. 
Infection Argument

- If Ann receives many confirmations, she is "close" (formally, in the product topology on the universal type space) to common knowledge that game is \((\theta_A, \theta_B) = \left(\frac{3}{4}, \frac{3}{4}\right)\)
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- "On steroids" relative to Rubinstein 89 become we didn’t impose the common prior assumption
- Weinstein Yildiz 07 show that this logic is completely general: (roughly) any action that is rationalizable in a perfect information game is uniquely rationalizable for a nearby type in the product topology
von Neumann was right about one thing: cannot do much without making common knowledge assumptions....
Bad News?

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- but let us be sophisticated about what common knowledge assumptions we make
Bad News?

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- but let us be sophisticated about what common knowledge assumptions we make
- we don’t have assume perfect information or nothing
In the (private value) universal type space, a player’s "rank belief" is the probability that she assigns to her return to investment being higher than another player’s.
Good News?

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Informationally Robust Analysis

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- why is this useful (and feasible)?
Informationally Robust Analysis

1. Robust Predictions
Informationally Robust Analysis

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2. Robust Identification
Informationally Robust Analysis

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2. Robust Identification
3. Information Design
Example: Third Degree Price Discrimination

- Bergemann, Brooks and Morris (2015)
Example: Third Degree Price Discrimination

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- Demand curve for a good represents single unit demand of a continuum of consumers

Two special cases:
- no information = uniform price monopoly
- producer charges uniform monopoly price, giving consumer surplus $u$ and producer surplus $\pi$

Full information = perfect price discrimination
- consumer gets zero surplus and producer extracts efficient surplus $w > \pi + u$

Robust Prediction: What can we say about all (consumer surplus, producer surplus) pairs that can arise?
Example: Third Degree Price Discrimination

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    - consumer gets zero surplus and producer extracts efficient surplus $w^* > \pi^* + u^*$
- Robust Prediction: What can we say about all (consumer surplus, producer surplus) pairs that can arise?
A Pictorial Characterization

![Graph showing Consumer surplus and Producer surplus]
The Uniform Price Monopoly

- producer charges (uniform) monopoly price
- consumers get positive consumer surplus, socially inefficient allocation
First Degree Price Discrimination: Perfect Discrimination

- producer extracts full surplus
- consumers get zero surplus, but socially efficient allocation
Welfare Bounds: Voluntary Participation

Consumer surplus is at least zero

Producer surplus

\( \pi^* \)

\( w^* \)

Consumer surplus

\( u^* \)

0
Welfare Bounds: Nonnegative Value of Information

Producer gets at least uniform price profit

\[ \pi^* \]

\[ w^* \]

0

Consumer surplus
Welfare Bounds: Social Surplus

Total surplus is bounded by efficient outcome

\[ \begin{align*}
\text{Producer surplus} & \quad \text{Consumer surplus} \\
\pi^* & \quad w^* \\
0 & \quad 0
\end{align*} \]
What is the feasible surplus set?

The diagram illustrates the relationship between consumer surplus and producer surplus in a market. The feasible surplus set is represented by the shaded area, which is bounded by the lines indicating consumer surplus and producer surplus. The set must be convex, as indicated by the shape of the shaded area.
Main Result: No More Robust Predictions!

\[ \text{Main result} \]

\[ \begin{align*}
\text{Producer surplus} & \quad w^* \\
\text{Consumer surplus} & \quad \pi^* \\
\end{align*} \]
Example

- $\frac{1}{3}$ of consumers have valuation 1, $\frac{1}{3}$ have valuation 2 and $\frac{1}{3}$ have valuation 3

- optimal prices:

$$p^* = 2$$
$$p^* = 3$$
$$p^* = 1$$
A segmentation of the three value uniform aggregate market:

<table>
<thead>
<tr>
<th></th>
<th>( v = 1 )</th>
<th>( v = 2 )</th>
<th>( v = 3 )</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>market 1</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>market 2</td>
<td>0</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>market 3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>total</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
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</table>
"Extremal Segmentation"

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</thead>
<tbody>
<tr>
<td>( {1, 2, 3} )</td>
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<td>( \frac{1}{3} )</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>( {2, 3} )</td>
<td>0</td>
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<td>( \frac{1}{6} )</td>
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<tr>
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</table>

- price 2 is optimal in all markets
- in fact, seller is always indifferent between all prices in the support of the market
- this is always possible to do (this is the meat of our general argument)
Geometry of Extremal Markets

- extremal segment $x^S$: seller is indifferent between all prices in the support of $S$
an optimal policy: always charge lowest price in the support of every segment:

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<th>(v = 3)</th>
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Social Surplus Minimizing Segmentation

- all incentive constraints in the support are binding
- another optimal policy: always charge highest price in each segment:

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Robust Predictions

- In this example, surprisingly weak
Robust Predictions

- In this example, surprisingly weak
- In other settings, there are... e.g., first price auction
Robust Identification

- What can be inferred from prices about valuations?
Robust Identification

- What can be inferred from prices about valuations?
- Very little.....
Consider the problem of an "information designer" who could pick (and commit to) an information structure to give to the monopolist.
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Consider the problem of an "information designer" who could pick (and commit to) an information structure to give to the monopolist. If the designer had the joint interest of consumers in mind he would pick the bottom right hand corner. Compare Kamenica and Gentzkow (2011).
Wilson (1987): (more complete quote)

Game theory has a great advantage in explicitly analyzing the consequences of trading rules that presumably are really common knowledge; it is deficient to the extent that it assumes other features to be common knowledge, such as one agent’s probability assessment about another’s preferences or information.

I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality.”
The Misunderstanding of John Harsanyi and Mechanism Design

- Mechanism design

We would really like to assume that there is complete information about the game/mechanism. It is particularly desirable to relax common knowledge assumptions about the environment, because optimal mechanisms are otherwise too finely tuned.

Contrast this with economic theory/game theory; really important to relax common knowledge of the mechanism (John Sutton and IO). Common knowledge of the environment is maybe (at least a bit) less of a problem.

One response to misunderstanding: do not address "incomplete information", focus on simple mechanisms, computational constraints, worst case analysis, etc...

Another response: take relaxing common knowledge assumptions seriously and allowing real incomplete information in mechanism design. This may rationalize simple/detail-free mechanisms (and suggest new questions).
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Type Space Restrictions = Implicit Common Knowledge
Assumptions

- Common to assume:
Type Space Restrictions $=$ Implicit Common Knowledge Assumptions

- Common to assume:
  1. naive type space (identify types with payoff parameters)
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  3. and either *independence* or beliefs determine payoff parameters (*BDP*: Neeman 2004) implied by generic beliefs on naive type space

Sometimes implicitly or explicitly trying to implement on all types spaces in some class (e.g., all naive common prior independent type spaces)

Implementing on the universal type space is the same (modulo technicalities) as implementing on all types spaces
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  - There is not full surplus extraction on the universal type space
  - Take a position on which types in the universal type space are relevant
Funny Result 2: Prior Extraction

- Consider a public goods problem with private values and budget balanced transfers. Two key public good results:

- Not possible to implement efficient choice in dominant strategies.
- Possible to implement efficient choice in (Bayes) Nash equilibrium with independent types, AGV (see also Arrow).

But what if the prior is not known? Two responses:

- Back to dominant strategies and negative results.
- Prior extraction: ask players to report their common prior and shoot them if they report something different.

Alternative nuanced response: relax union of common prior naive type spaces assumption to universal type space. Nuanced conclusion:

- Implementation of the efficient outcome in Bayes Nash equilibrium on universal type space may or may not be equivalent to dominant strategies implementation.
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Relaxing Private Values Assumption

- Maintained common knowledge assumption in discussion so far: private values

\[ v_A = \theta_A + \gamma \theta_B \]

Analogously, Bob's value is

\[ v_B = \theta_B + \gamma \theta_A \]
Relaxing Private Values Assumption

- Maintained common knowledge assumption in discussion so far: private values
- Let’s relax this assumption

Suppose that values are interdependent:

- Ann’s value of an object is $v_A = \theta_A + \gamma \theta_B$ for some $0 < \gamma < 1$
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Three Interpretations

1. $\theta_A$ is Ann’s consumption value but it is possible that Ann will have to re-sell to Bob, extracting proportion $\gamma$ of Bob’s value
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2. Ann and Bob each have a signal that confounds a common value and private value component (cannot be distinguished).
Implicit Common Knowledge Assumptions and Interdependent Values

In example, we have single good interdependent values example, we had

\[ v_A = \theta_A + \gamma \theta_B \]  
\[ v_B = \theta_B + \gamma \theta_A \]
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\[ v_A = \theta_A + \gamma \theta_B \quad \text{and} \quad v_B = \theta_B + \gamma \theta_A \]

- By linear algebra, we have

\[ \theta_A = \frac{1}{1 - \gamma^2} (v_A - \gamma v_B) \quad \text{and} \quad \theta_B = \frac{1}{1 - \gamma^2} (v_B - \gamma v_A) \]
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- Whether this makes sense depends on the interpretation
Should actually distinguish "higher order preference types", e.g.,

1. first order valuation: Ann’s unconditional value of an object,

Thus we are looking higher order preferences over acts
Should actually distinguish "higher order preference types", e.g.,

1. first order valuation: Ann’s unconditional value of an object,
2. second order belief and valuation:
   - Ann’s belief about Bob’s first order valuation
   - Ann’s valuation conditional on Bob’s first order valuations
3. third order belief and valuation:
   - Ann’s belief about Bob’s second order type
   - Ann’s valuation conditional on Bob’s second order type
4. and so on

Thus we are looking higher order preferences over acts
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1. first order valuation: Ann’s unconditional value of an object,
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Canonical Preference Higher-Order Preference Types

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1. first order valuation: Ann’s unconditional value of an object,
2. second order belief and valuation:
   - Ann’s belief about Bob’s first order valuation
   - Ann’s valuation conditional on Bob’s first order valuations

Thus we are looking higher order preferences over acts
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4. and so on

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Universal Higher-Order Preference Type Space

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- Higher order preference types correspond exactly to what would be learnt about players
- Selling on the higher-order preference type space is complicated
Conclusion

- Incomplete information has not been fully incorporated into economic analysis.
- Results are driven by implicit common knowledge whose role is sometimes not well understood.
- But relaxing all common knowledge assumptions may be possible but unhelpful.
- Focus on which are reasonable common knowledge assumptions and make them explicit.